Let p be a point of a surface S with tangent plane T_pS. Suppose that Π is a plane passing p and orthogonal to T_pS with $\gamma = \Pi \cap S$ (called a normal section). Prove that the geodesic curvature $k_g(p)$ of γ at p is zero.

Proof. Since $\Pi \perp T_pS, \overrightarrow{n} \perp T_pS$, and the curve $\gamma \subseteq \Pi$. Then $\gamma', \overrightarrow{n}$ at point p is an orthognal basis of the plane Π. Since γ is a planar curve, γ'' is the linear combination of γ' and $\overrightarrow{n}$. We choose a unit-speed parametrization and $\gamma'' \perp \gamma'$. Then γ'' is parallel with $\overrightarrow{n} \Rightarrow k_g = \gamma'' \circ (N \times \gamma') = 0$ $\square$

Suppose the surface Σ contains a straight line ℓ, show that for any point $p \in \ell$, the Gaussian curvature $K(p) \leq 0$

Proof. The normal curvature k_n for the straight line is zero. While k_n is between k_1 and k_2, we have $K(p) = k_1 k_2 \leq 0$. $\square$