Morphological Image Processing

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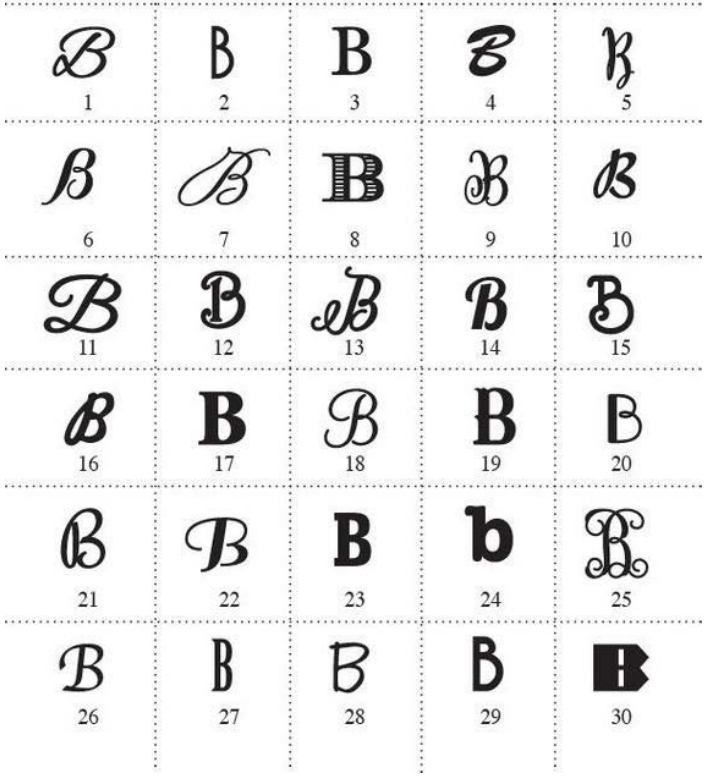
Introduction

What Is Morphology?

- Morph: shape
- Morphology: study of shapes
- In the context of image processing
  - Input: binary images
  - Output: processed binary images
    - Denoising
    - Thinning
    - Etc.

Example

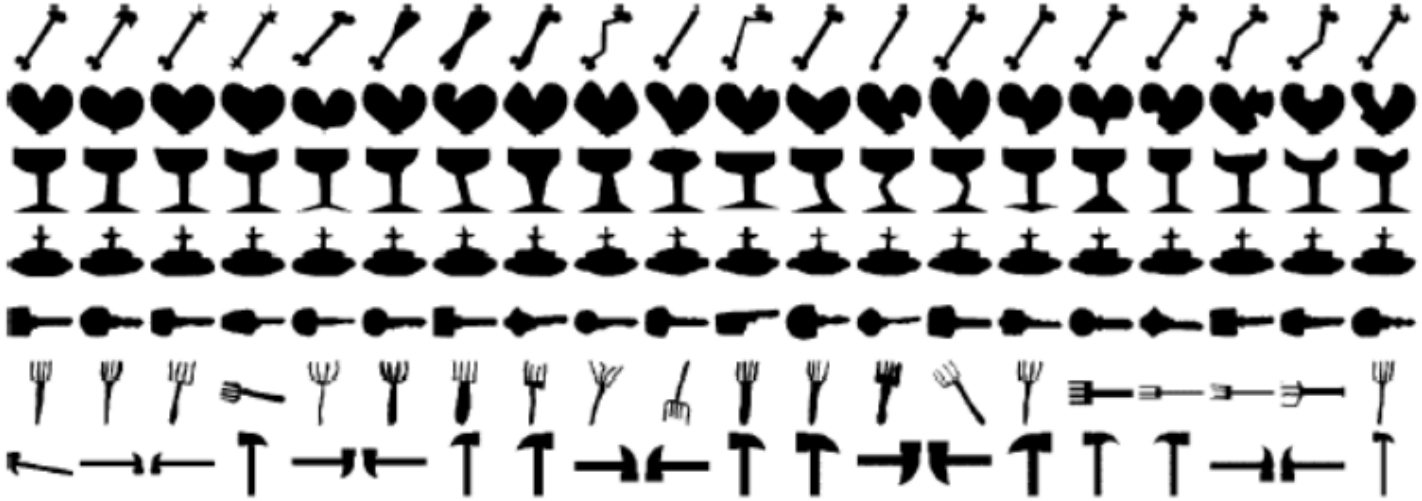
- Essential shape of an image
  - It has nothing to do with the stroke width



Morphological Processing

- Some objects contain shapes formed by line segments, arcs and curves
- Applications
  - Optical character recognition (OCR)
  - Fingerprint recognition
  - Shape retrieval
  - Etc.

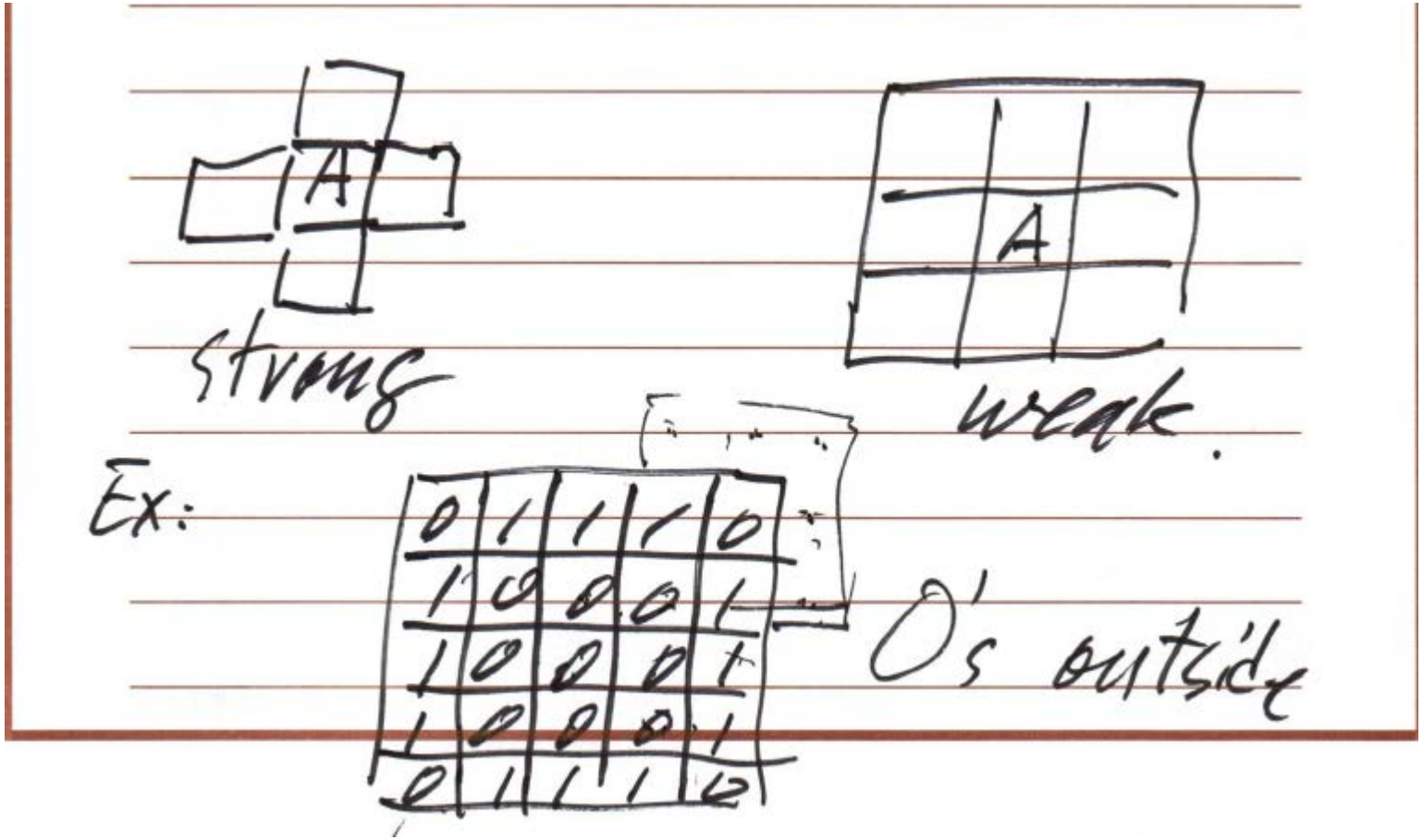
MPEG-7 Shape Dataset



Binary Image Connectivity

- 1: object pixel (black)
- 0: background pixel (white)
- 4-connectivity:
  - A pixel is 4-connected if its value is the same as one (or more) of its four nearest neighbors
- 8-connectivity:
  - A pixel is 8-connected if its value is the same as one (or more) of its eight nearest neighbors

Example

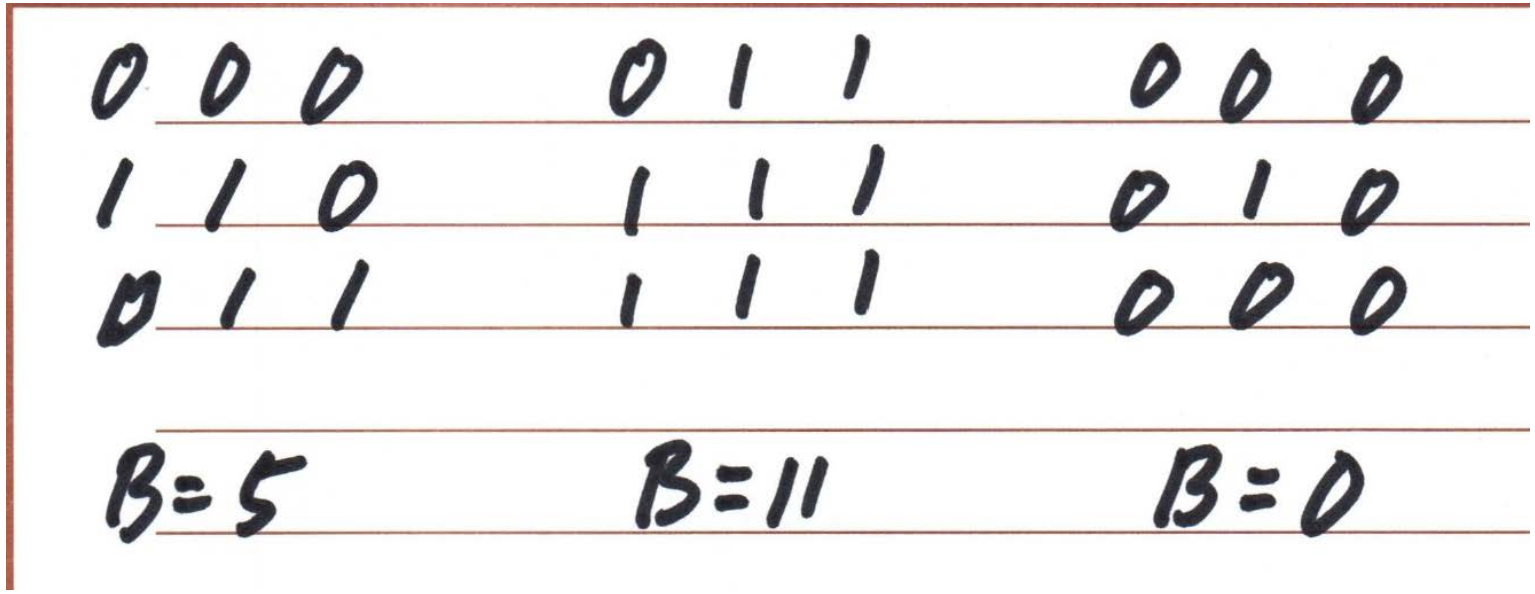


Object Counting

- How many objects in the last example?
  - 4-connectivity rule
    - No. of objects: 4
    - No. of background regions: 2
  - 8-connectivity rule
    - No. of objects: 1
    - No. of background regions: 1
  - Hybrid connectivity rule
    - 8-connectivity for objects and 4-connectivity for background
    - No. of objects: 1
    - No. of background regions: 2

Another Connectivity Measure: Bond

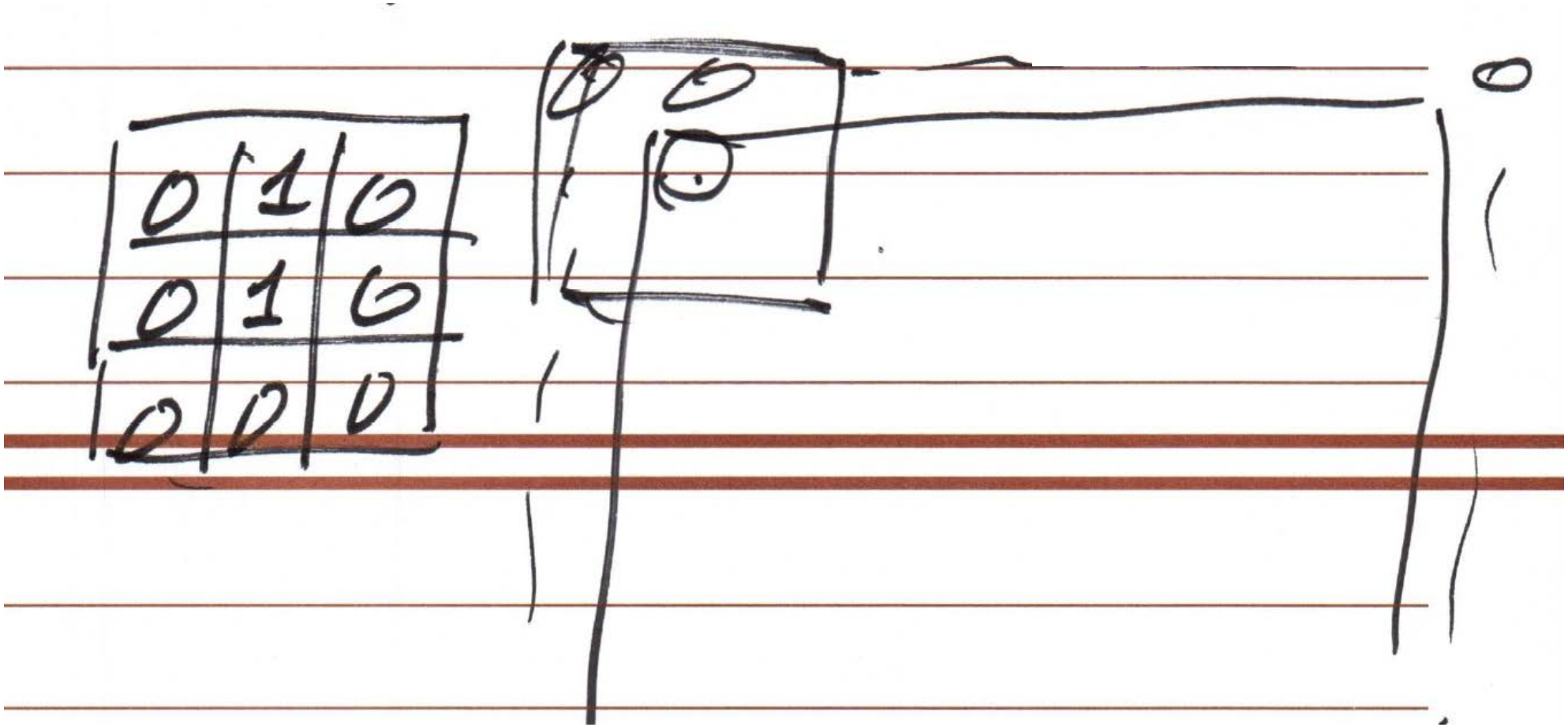
- Side connectivity: 2 pts.
- Corner connectivity: 1 pt.
- Bond= 2 x (no. of the same side neighbors)
  + 1 x (no. of the same corner neighbors)
- Example:



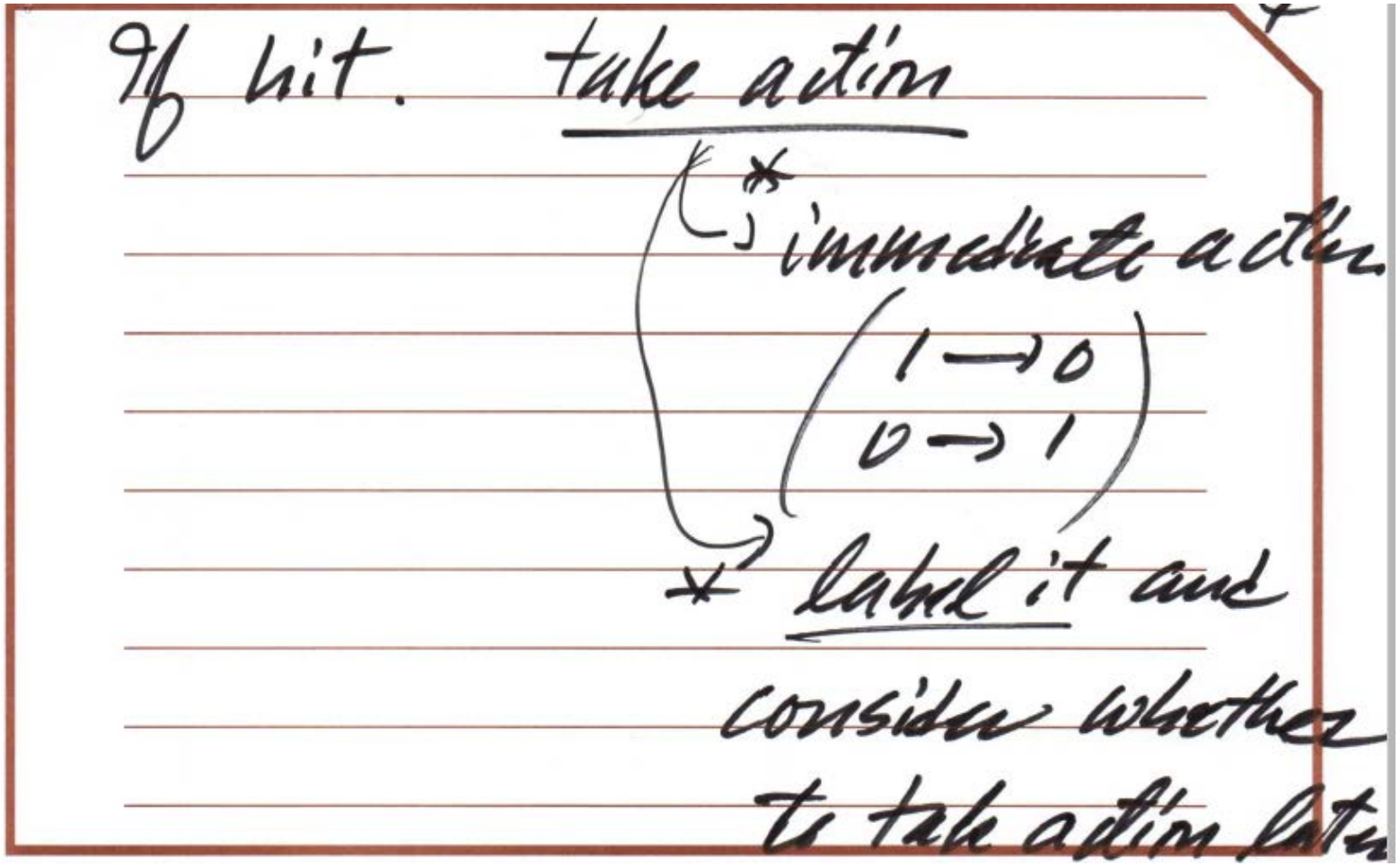
Basic Morphological Filters

Hit or Miss Morphological Filters

- Use an odd-size mask (typically 3x3) to scan a binary image
- Pre-define a set of hit masks
- If the underlying patch pattern matches one of the hit masks, it is called a “hit”. Otherwise, it is called a “miss”
- Action:
  - Hit -> take action on the central pixel (usually, change 0 to 1, change 1 to 0)
  - Miss -> no action on the central pixel (copy the central pixel value to the same location of the output image)


0 1 0
0 1 0
0 0 0
0
1

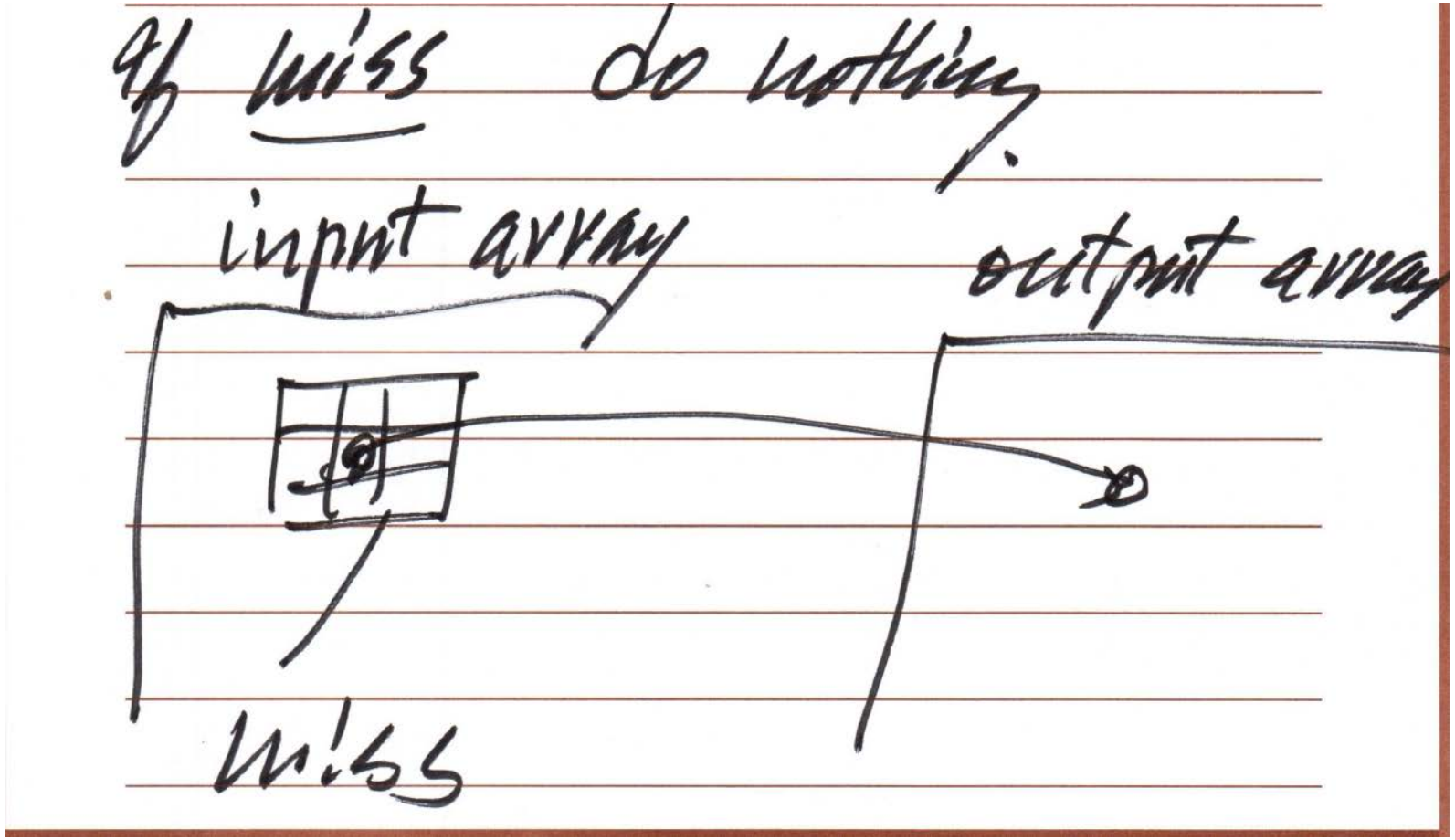
Hit



Simple Filter

Advanced Filter

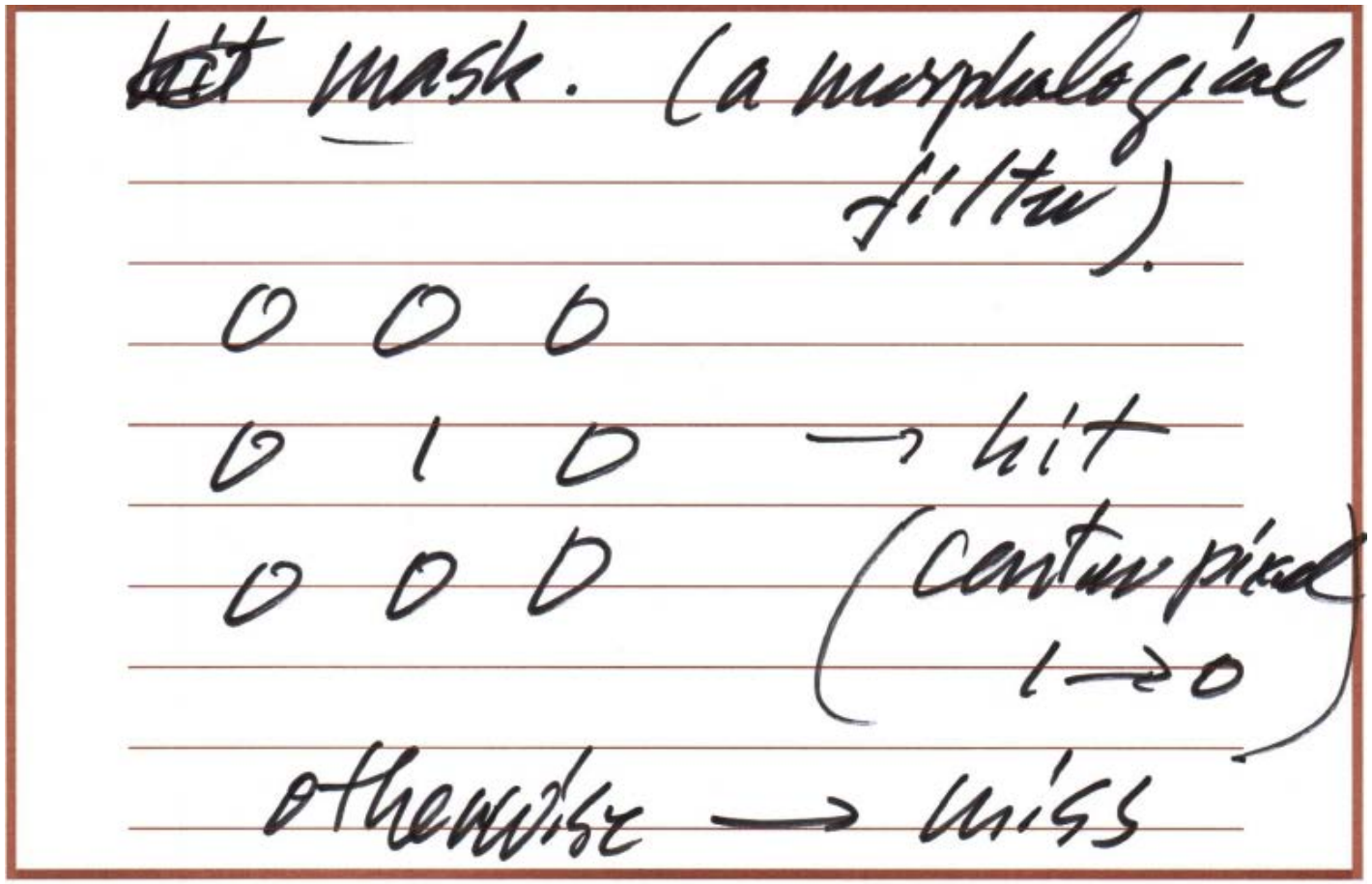
Miss



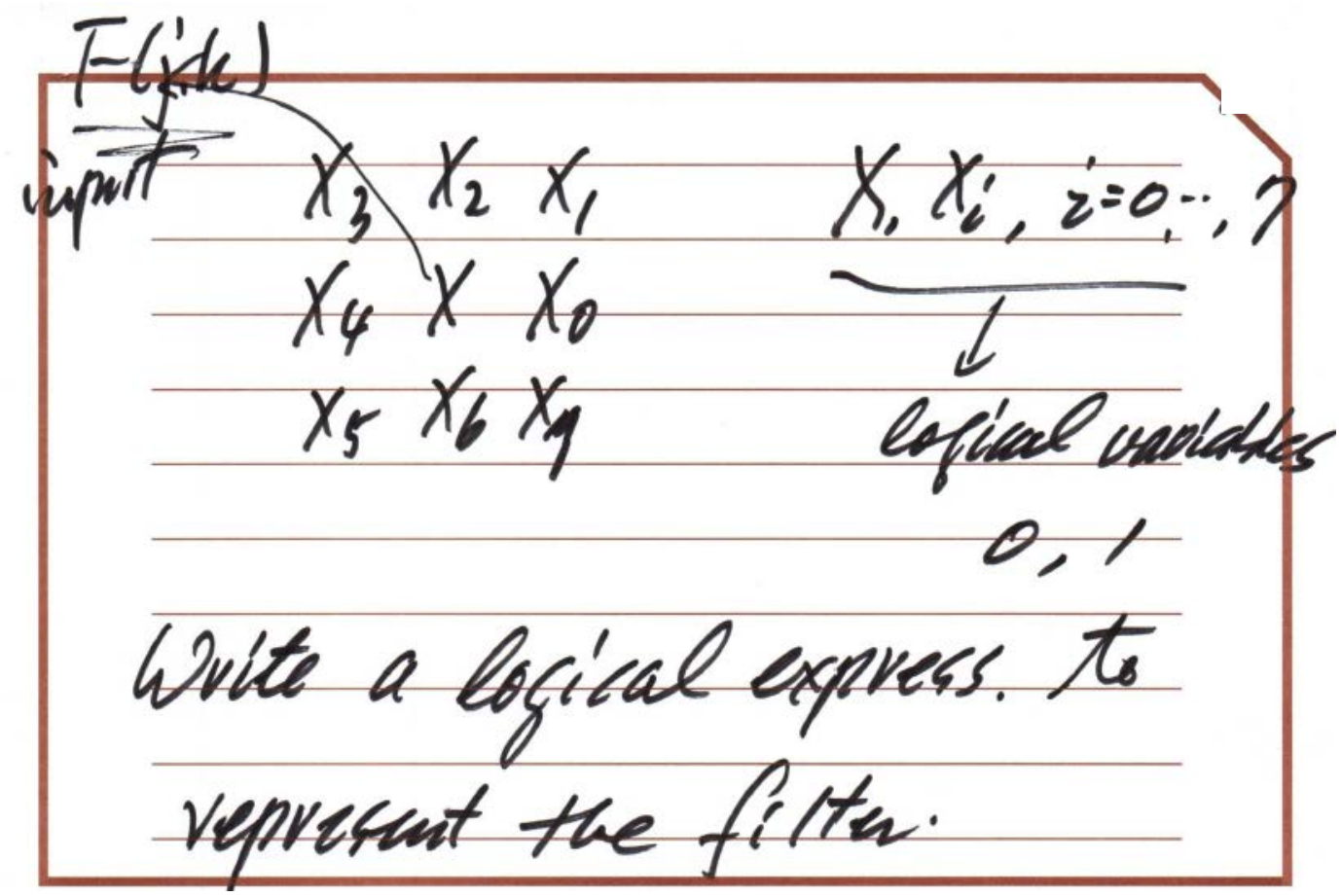
Example: Isolated Dots Removal

- Isolated black dots can be viewed as noise in black/white images

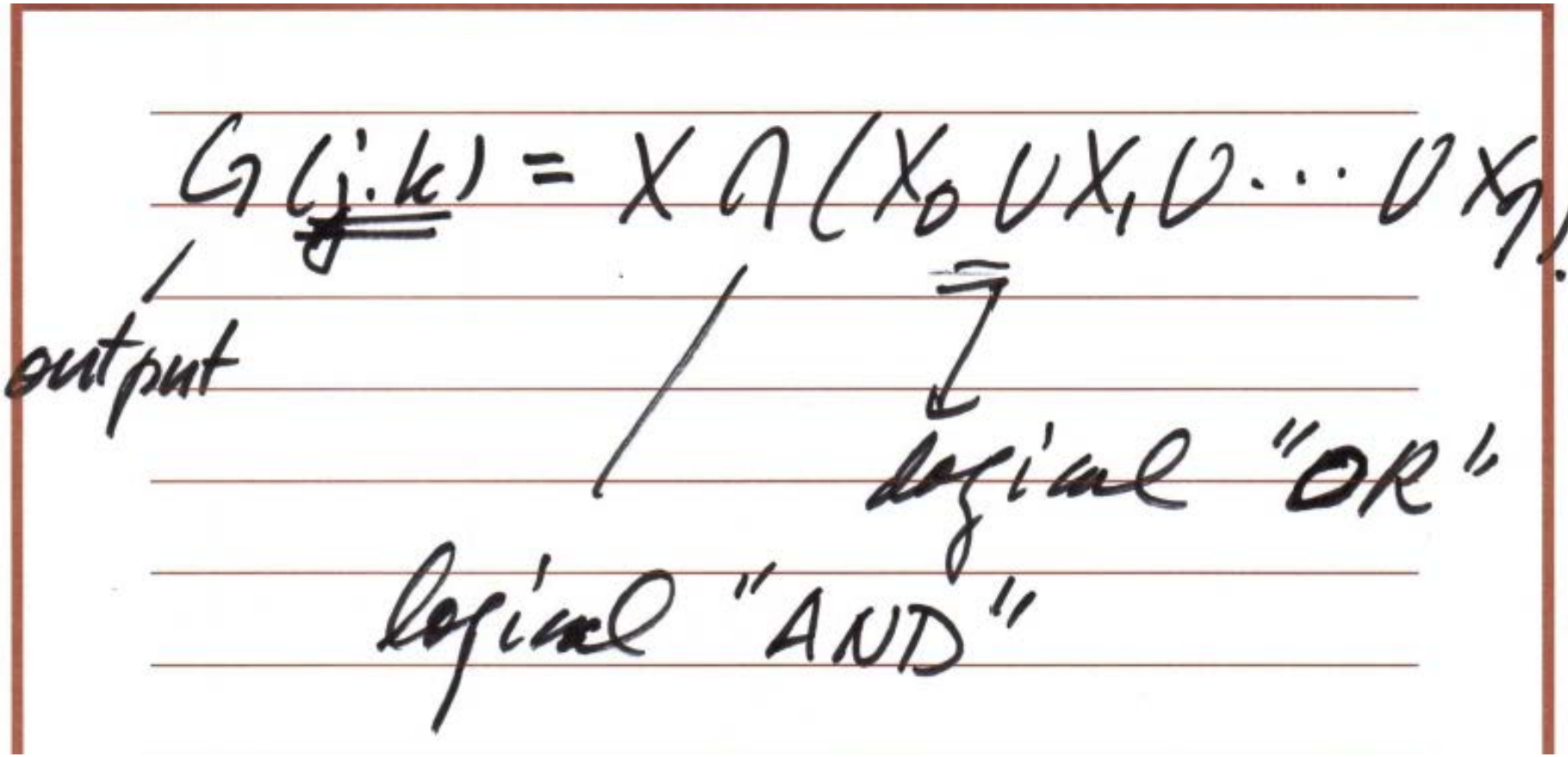
Mask Design


bit mask. (a morphological filter).
0 0 0
0 1 0 → hit
0 0 0
(center pixel 1→0)
otherwise → miss

Mathematical Representation of Morphological Filters



Logical Expression of Noise Removal Filter

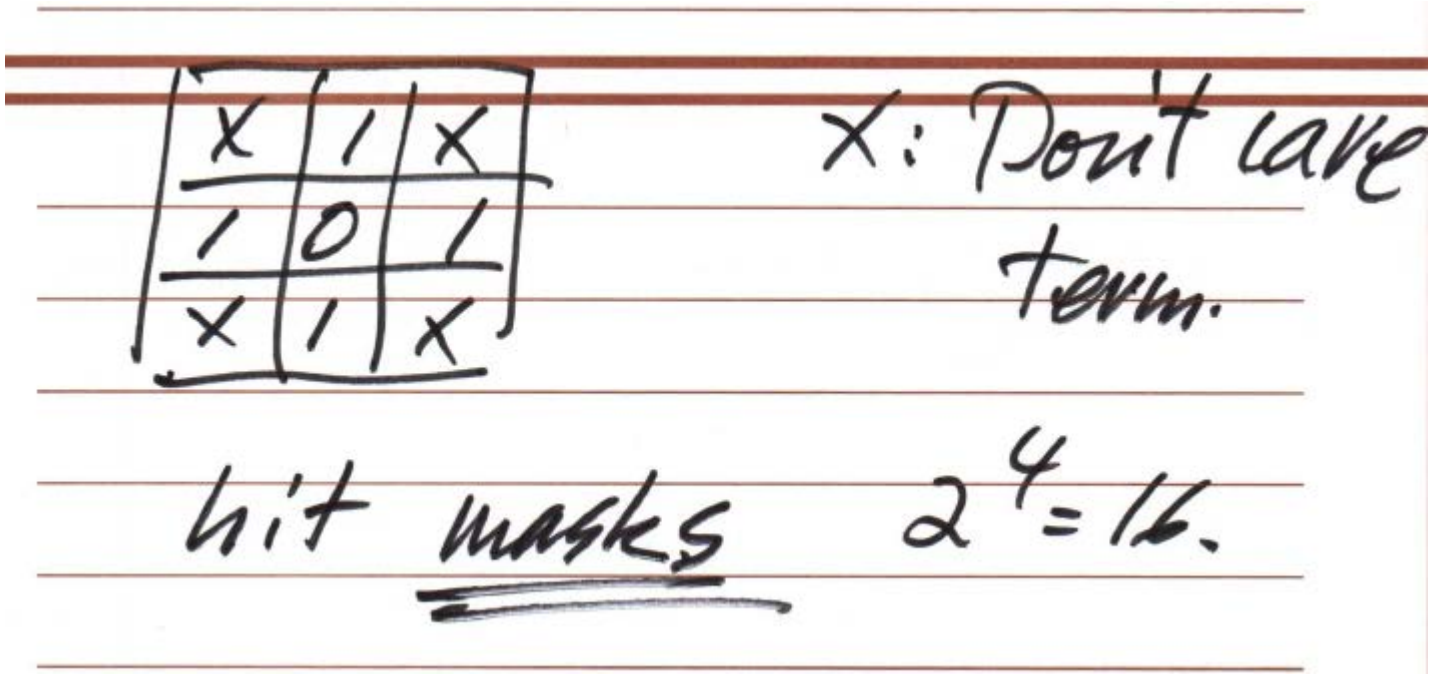


Simple Morphological Filters

- Additive Filters
  - Action: Converting “0” (white, background) in the input image to “1” (black, foreground) in the output image
- Subtractive Filters
  - Action: Converting “1” (black, foreground) in the input image to “0” (white, background) in the output image

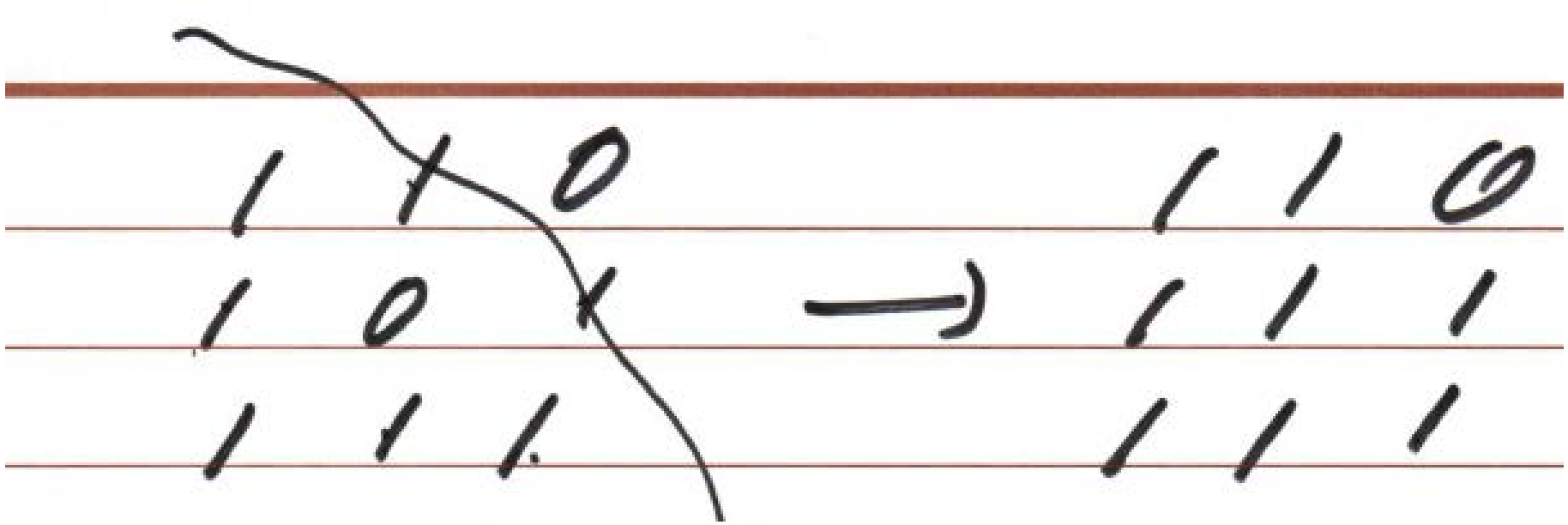
Example of Additive Filters (1)

- Interior Fill



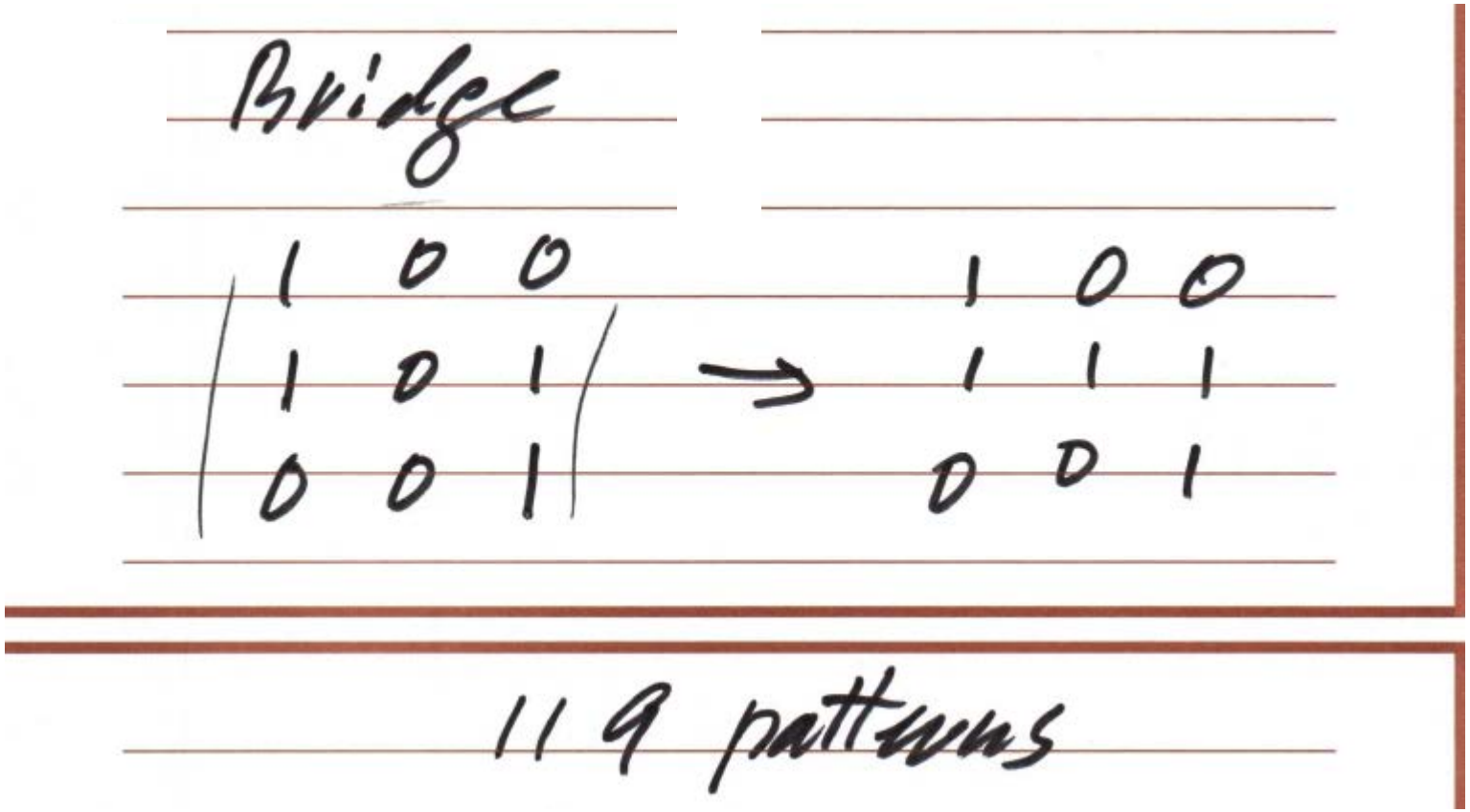
Example of Additive Filters (2)

- Diagonal Fill



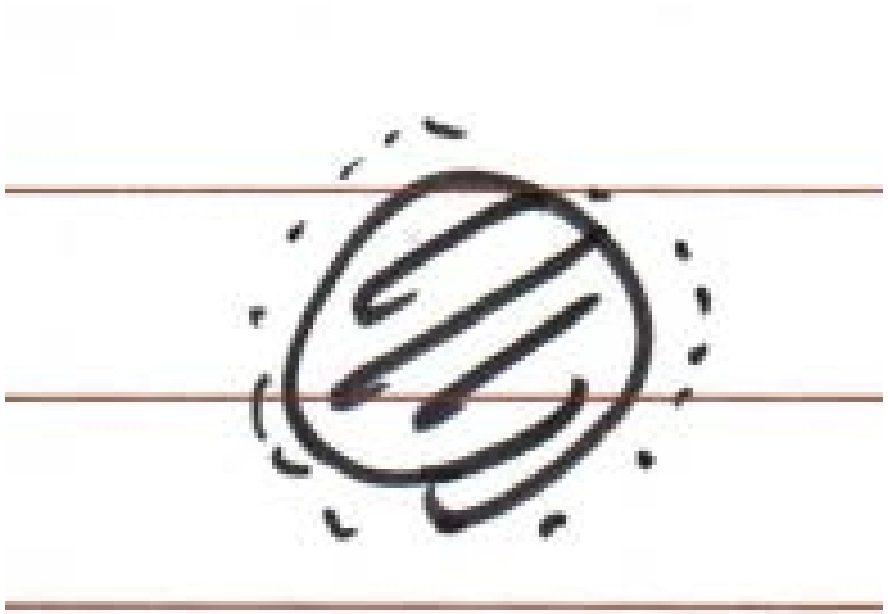
Example of Additive Filters (3)

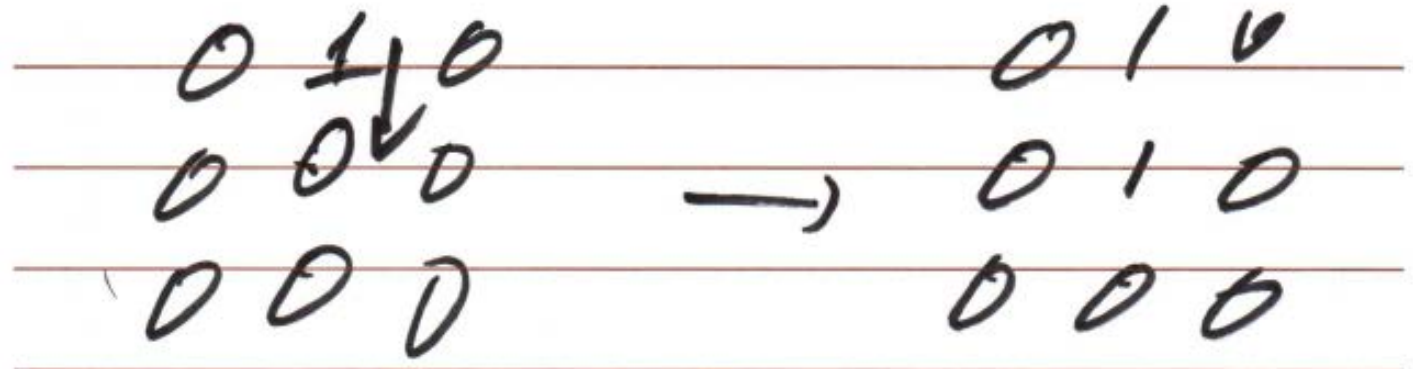
- Bridge



Example of Additive Filters (4)

- Eight-Neighbor Dilation
  - Goal: grow the size of an object

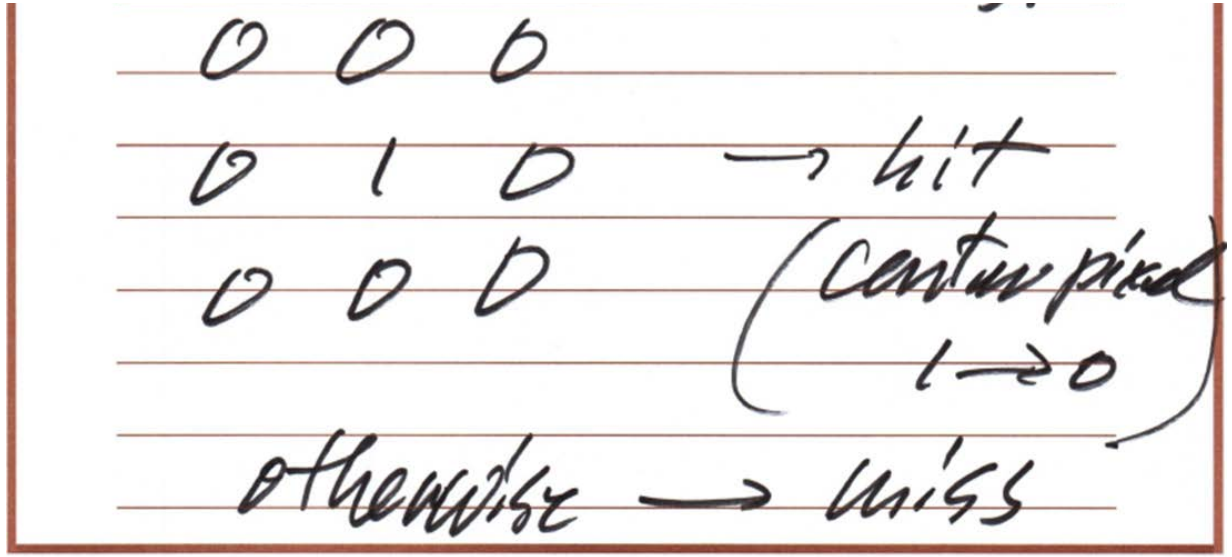




any of 8 neighbors is "me"

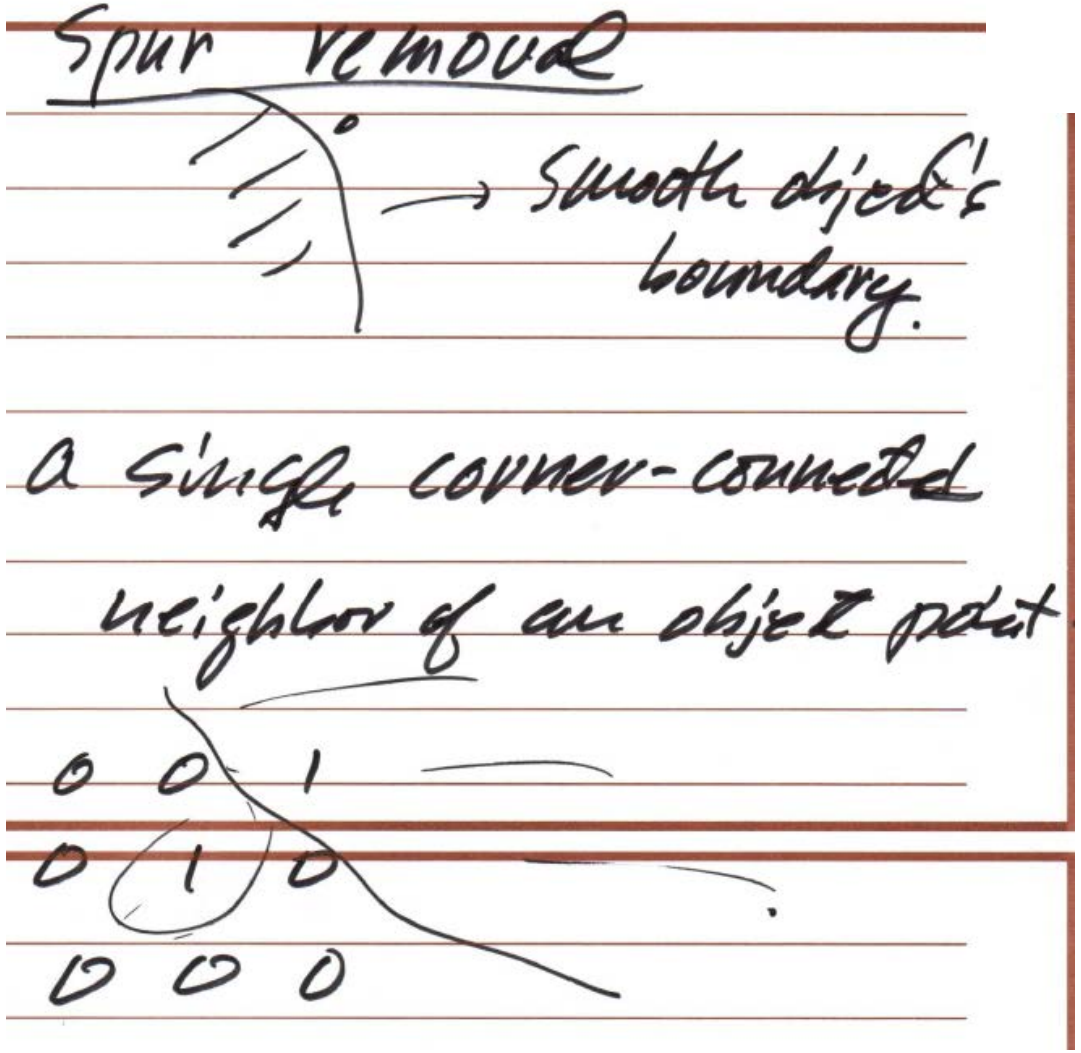
Example of Subtractive Filters (1)

- Isolated pixel removal



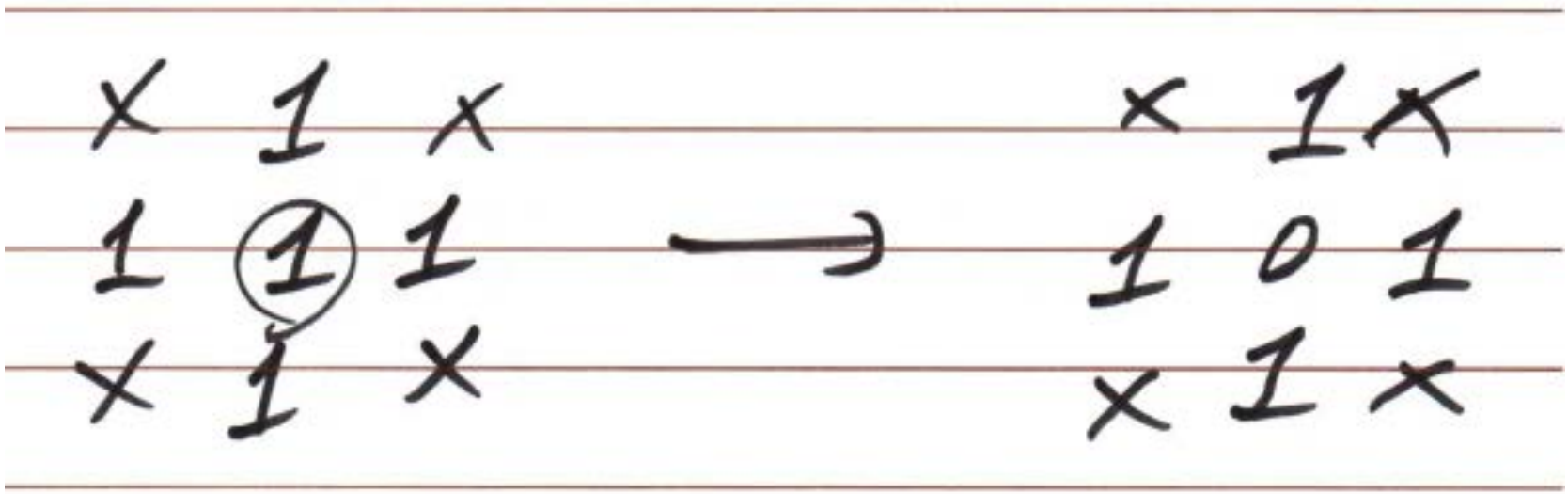
Example of Subtractive Filters (2)

- Spur removal

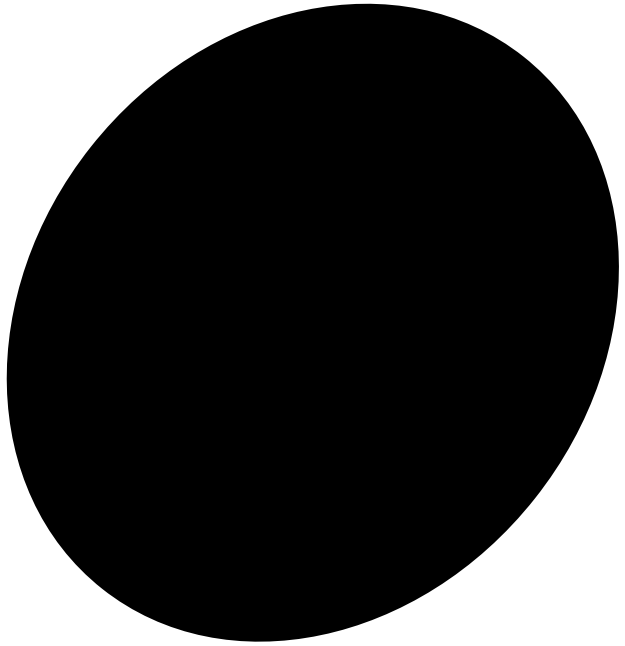


Example of Subtractive Filters (3)

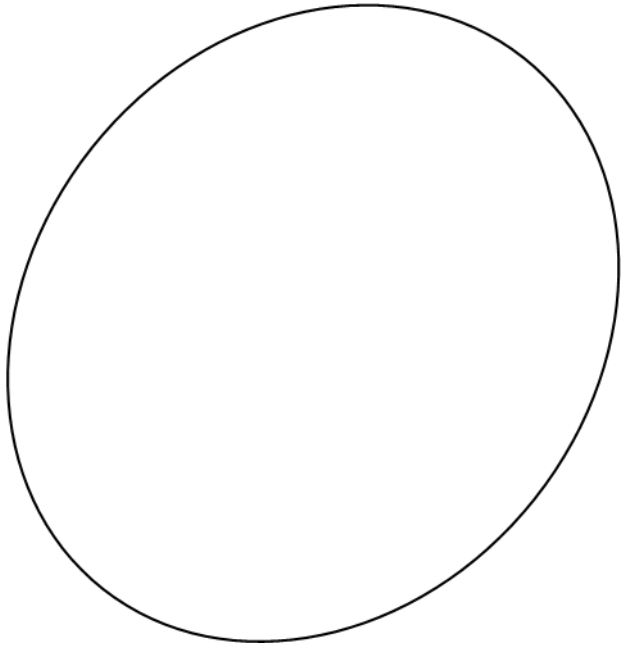
- Interior Pixel Removal



Overall Effect of Interior Pixel Removal



Input Image



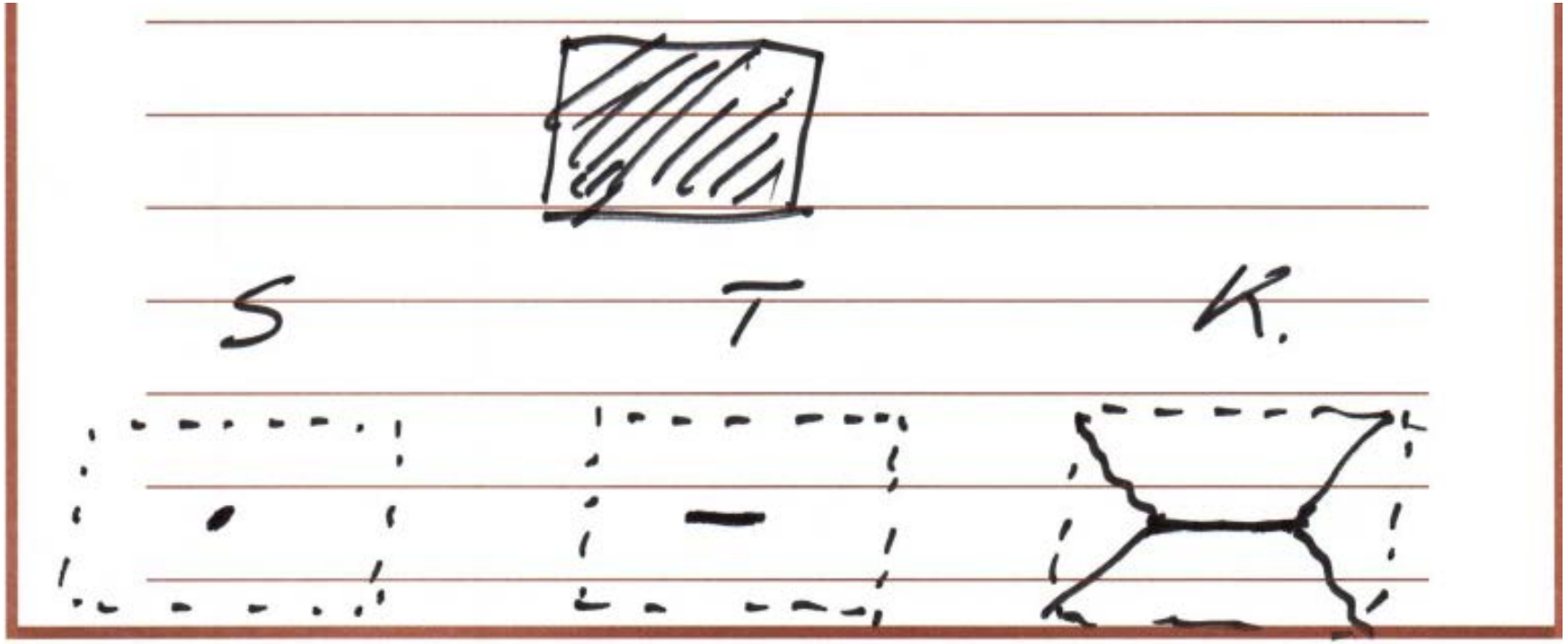
Output Image

Advanced Morphological Filters

Advanced Morphological Filters

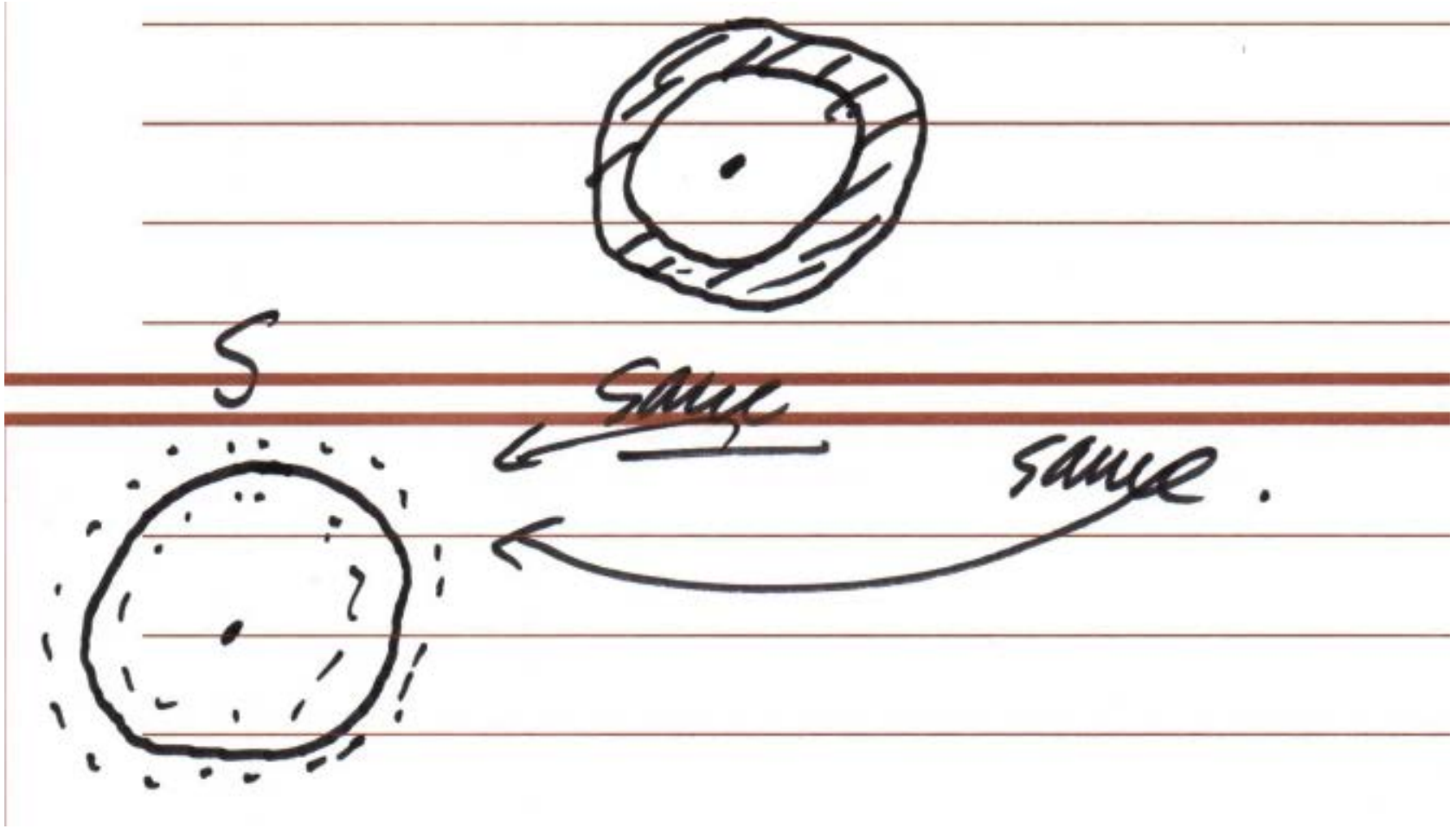
- Three subtractive filters
  - Shrinking
  - Thinning
  - Skeletonizing
- One additive filter
  - Thickening

Examples (1)

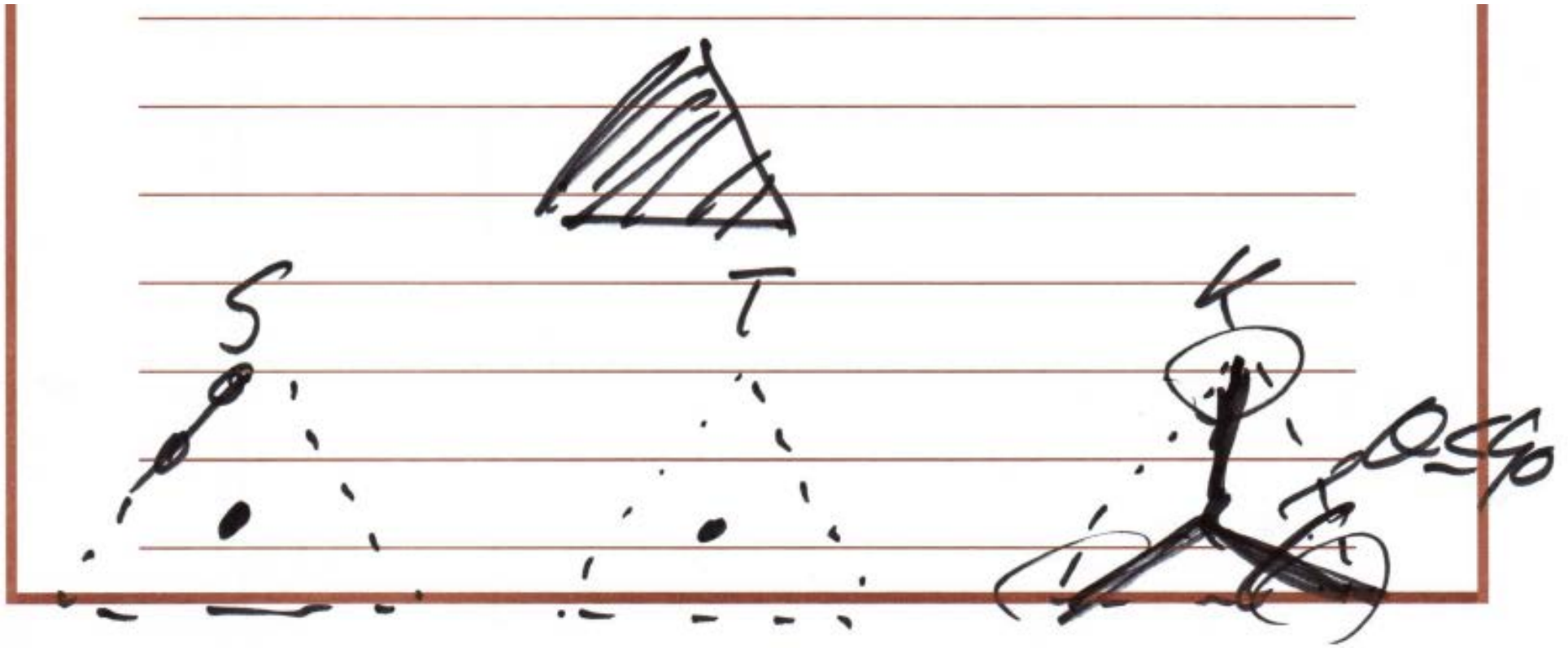


Examples (2)

Examples (3)

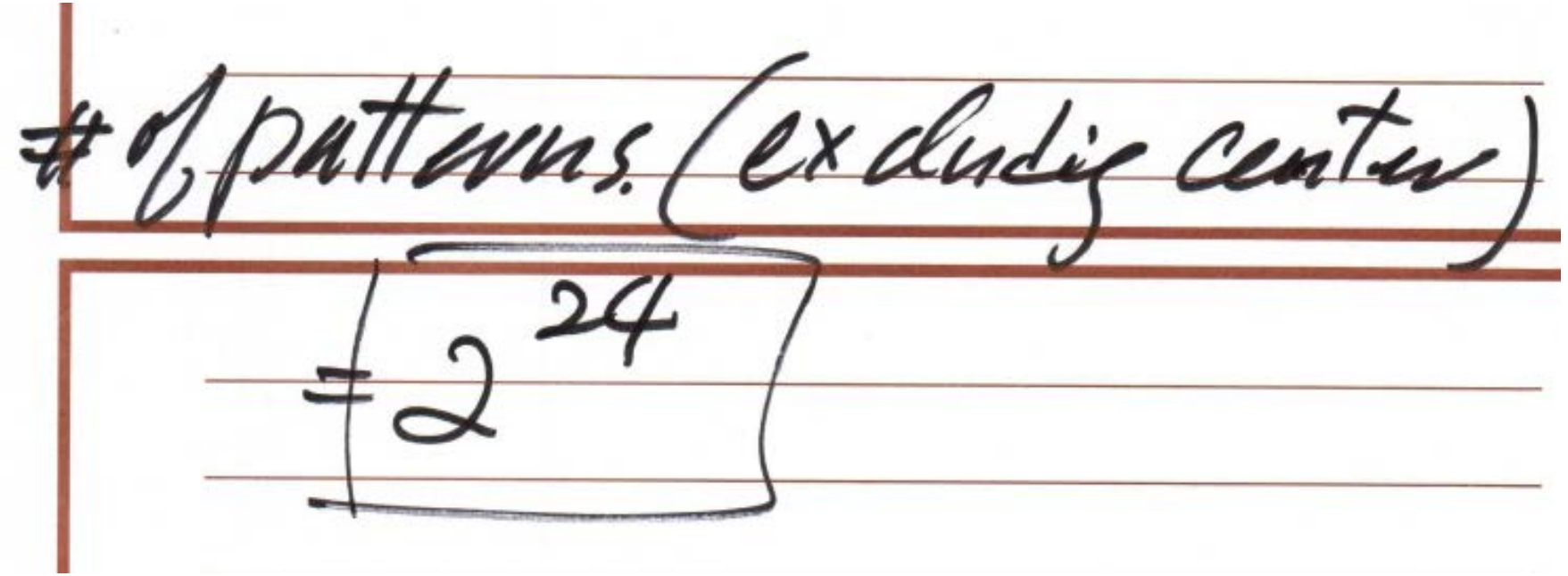


Examples (4)



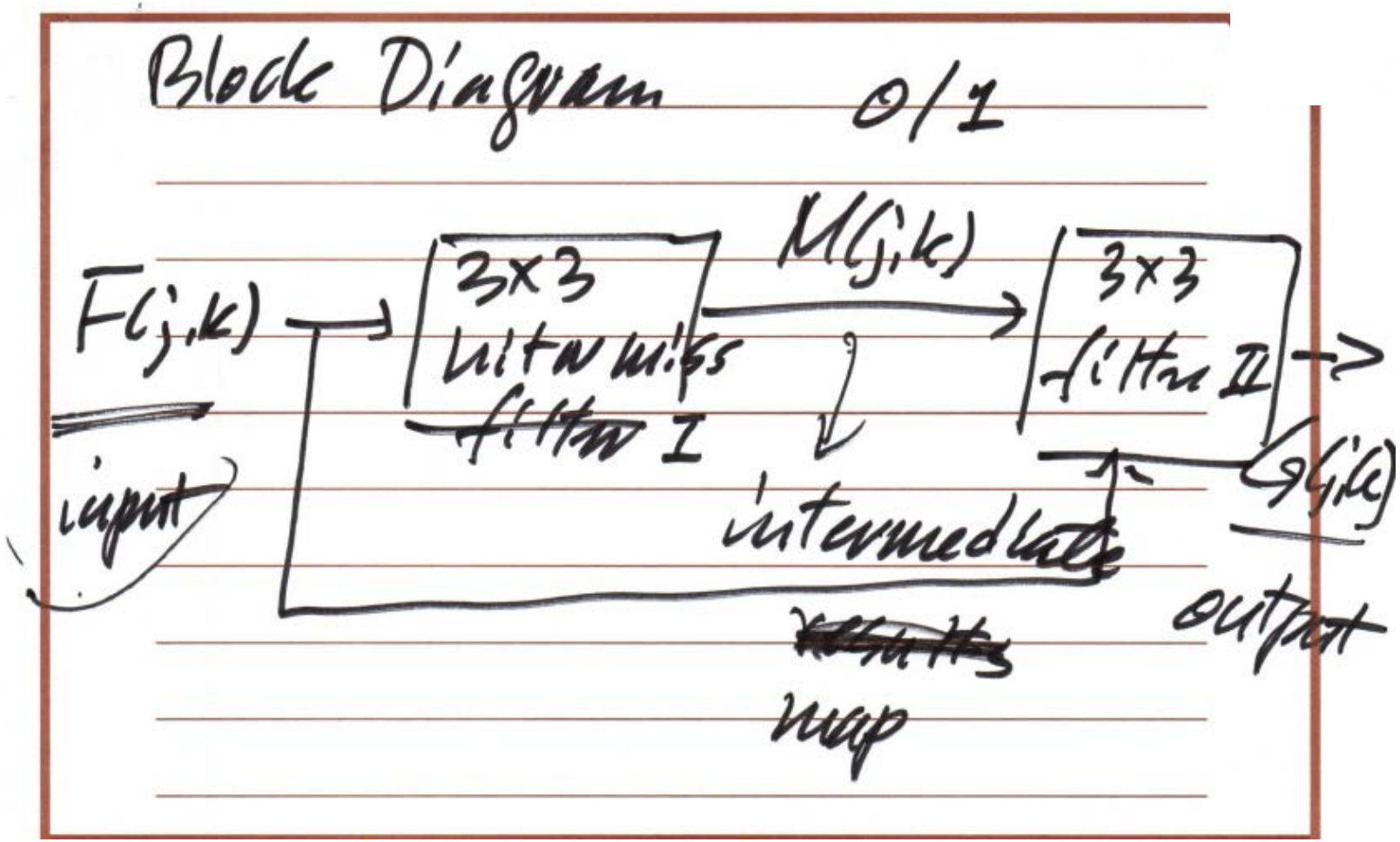
One-Stage Filter Design

- If we adopt the single-stage hit-or-miss filter solution, the filter size has to be of 5x5=25

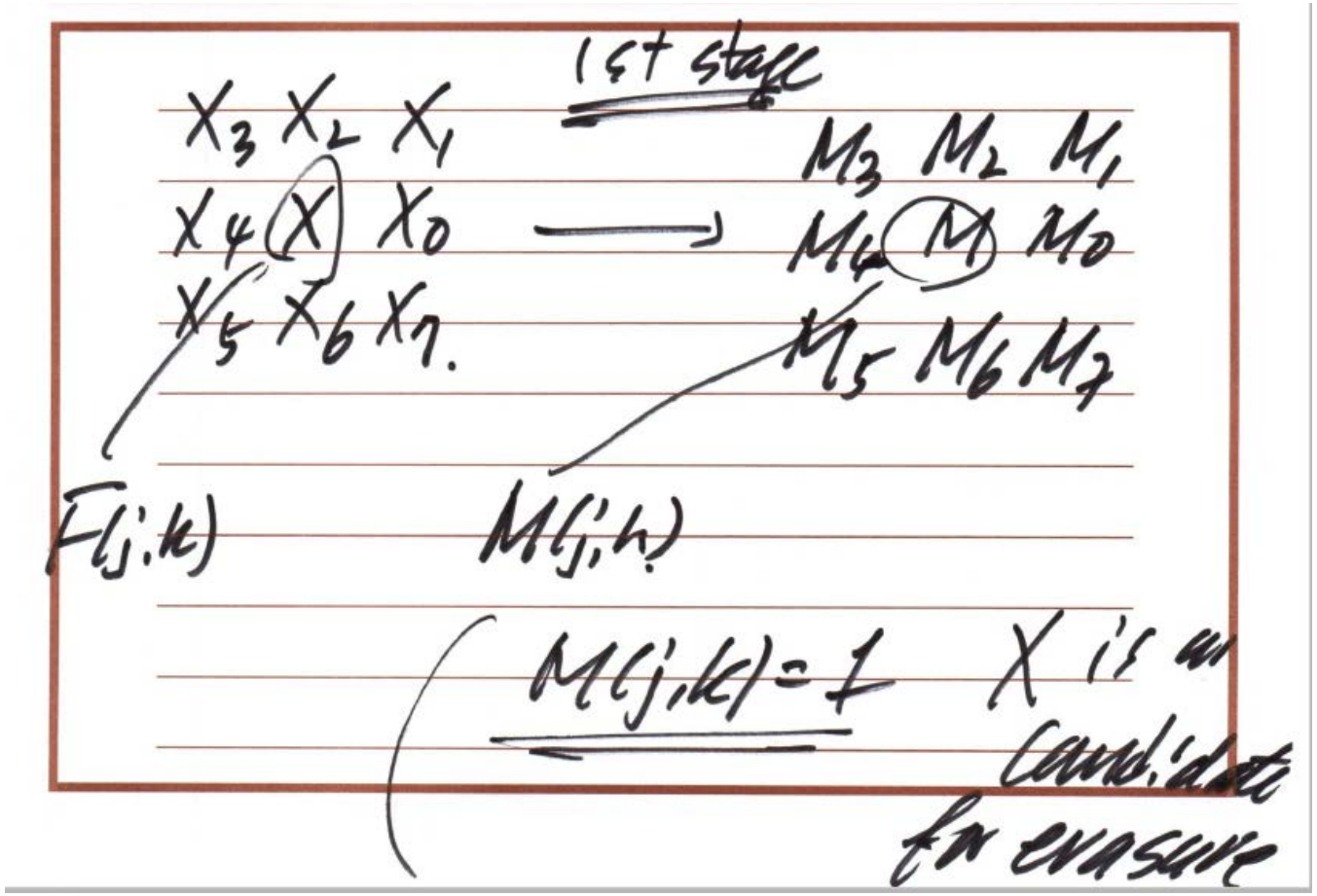


Two-Stage Filter Design

- To simplify the design process, we decompose the one-stage 5x5 filter to two stages in cascade, where each stage consists of a 3x3 filter

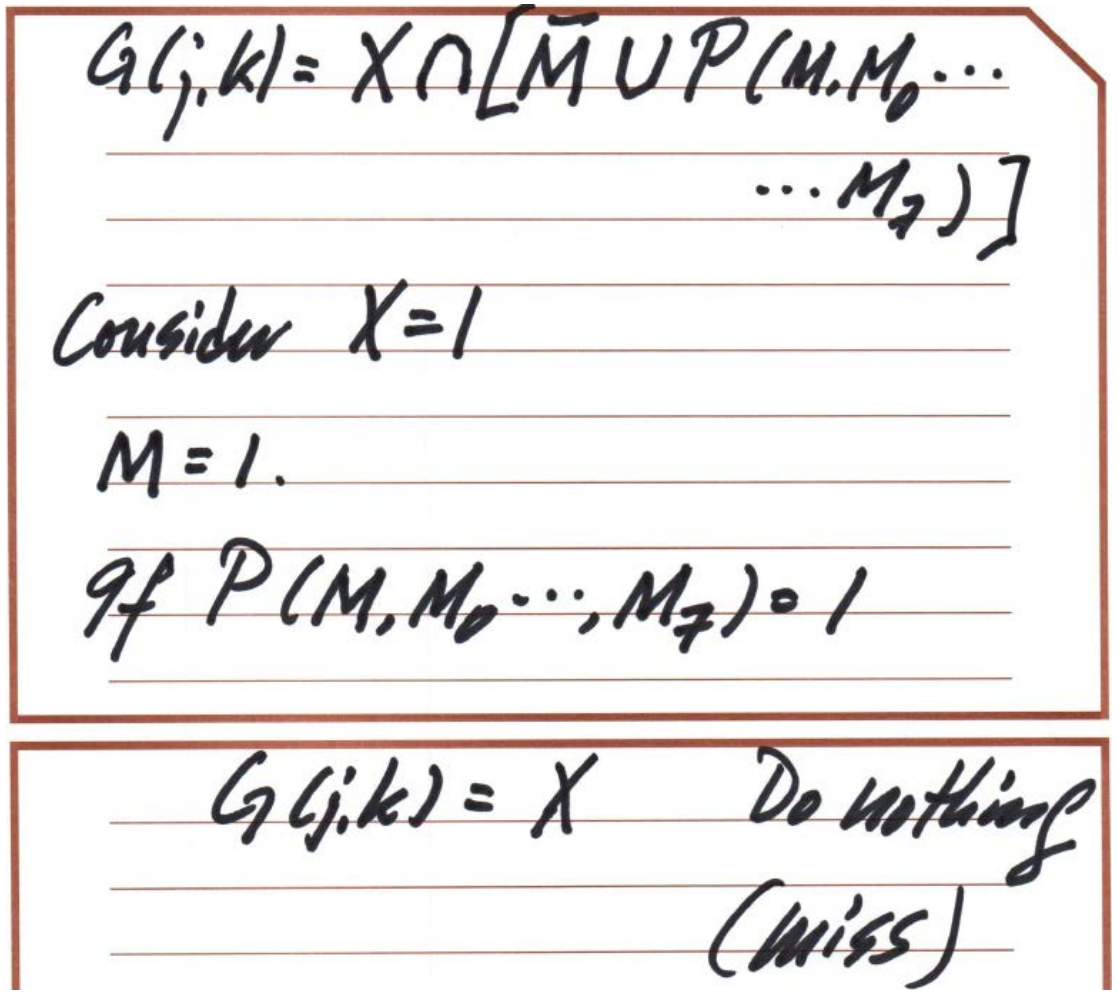


Purpose of 1st Stage Design



M(j,k)=0. X is not a candidate for erasure.

Purpose of 2nd Stage Design (1)



Purpose of 2nd Stage Design (2)

If $P(M, M_0 \cdots, M_3) = 0$

$G(j,k) = X \cap 0 = 0$

Evasure. (bit)

$P(M, M_0 \cdots M_3) = 1$

Evasure inhibiting logical variable.

First Stage (or M) Hit Masks (1)

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TABLE 14.3-1. Shrink, Thin and Skeletonize Conditional Mark Patterns [M = 1 if hit]

Table	Bond	Pattern							
		0 0 1	1 0 0	0 0 0	0 0 0				
S	1	0 1 0	0 1 0	0 1 0	0 1 0				
		0 0 0	0 0 0	1 0 0	0 0 1				
		0 0 0	0 1 0	0 0 0	0 0 0				
S	2	0 1 1	0 1 0	1 1 0	0 1 0				
		0 0 0	0 0 0	0 0 0	0 1 0				
		0 0 1	0 1 1	1 1 0	1 0 0	0 0 0	0 0 0	0 0 0	0 0 0
S	3	0 1 1	0 1 0	0 1 0	1 1 0	1 1 0	0 1 0	0 1 0	0 1 1
		0 0 0	0 0 0	0 0 0	0 0 0	1 0 0	1 1 0	0 1 1	0 0 1
		0 1 0	0 1 0	0 0 0	0 0 0				
TK	4	0 1 1	1 1 0	1 1 0	0 1 1				
		0 0 0	0 0 0	0 1 0	0 1 0				
		0 0 1	1 1 1	1 0 0	0 0 0				
STK	4	0 1 1	0 1 0	1 1 0	0 1 0				
		0 0 1	0 0 0	1 0 0	1 1 1				

(Continued)

First Stage (or M) Hit Masks (2)

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		1 1 0	0 1 0	0 1 1	0 0 1
ST	5	0 1 1	0 1 1	1 1 0	0 1 1
		0 0 0	0 0 1	0 0 0	0 1 0

		0 1 1	1 1 0	0 0 0	0 0 0
ST	5	0 1 1	1 1 0	1 1 0	0 1 1
		0 0 0	0 0 0	1 1 0	0 1 1

		1 1 0	0 1 1
ST	6	0 1 1	1 1 0
		0 0 1	1 0 0

		1 1 1	0 1 1	1 1 1	1 1 0	1 0 0	0 0 0	0 0 0	0 0 1
STK	6	0 1 1	0 1 1	1 1 0	1 1 0	1 1 0	1 1 0	0 1 1	0 1 1
		0 0 0	0 0 1	0 0 0	1 0 0	1 1 0	1 1 1	1 1 1	0 1 1

(Continued)

First Stage (or M) Hit Masks (3)

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TABLE 14.3-1. (Continued)

Table	Bond	Pattern							
		1 1 1	1 1 1	1 0 0	0 0 1				
STK	7	0 1 1	1 1 0	1 1 0	0 1 1				
		0 0 1	1 0 0	1 1 1	1 1 1				
		0 1 1	1 1 1	1 1 0	0 0 0				
STK	8	0 1 1	1 1 1	1 1 0	1 1 1				
		0 1 1	0 0 0	1 1 0	1 1 1				
		1 1 1	0 1 1	1 1 1	1 1 1	1 1 1	1 1 0	1 0 0	0 0 1
STK	9	0 1 1	0 1 1	1 1 1	1 1 1	1 1 0	1 1 0	1 1 1	1 1 1
		0 1 1	1 1 1	1 0 0	0 0 1	1 1 0	1 1 1	1 1 1	1 1 1
		1 1 1	1 1 1	1 1 1	1 0 1				
STK	10	0 1 1	1 1 1	1 1 0	1 1 1				
		1 1 1	1 0 1	1 1 1	1 1 1				
		1 1 1	1 1 1	1 1 0	0 1 1				
K	11	1 1 1	1 1 1	1 1 1	1 1 1				
		0 1 1	1 1 0	1 1 1	1 1 1				

Second Stage (or P) Hit Masks for Shrinking and Thinning (1)

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TABLE 14.3-2. Shrink and Thin Unconditional Mark Patterns
[$P(M, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7) = 1$ if hit] [a]

Pattern							
Spur		Single 4-connection					
0 0 *M*	*M*0 0	0 0 0	0 0 0				
0 *M*0	0 *M*0	0 *M*0	0 *MM*				
0 0 0	0 0 0	0 *M*0	0 0 0				
L Cluster							
0 0 *M*	0 *MM*	*MM*0	*M*0 0	0 0 0	0 0 0	0 0 0	0 0 0
0 *MM*	0 *M*0	0 *M*0	*MM*0	*MM*0	0 *M*0	0 *M*0	0 *MM*
0 0 0	0 0 0	0 0 0	0 0 0	*M*0 0	*MM*0	0 *MM*	0 0 *M*
4-Connected offset							
0 *MM*	*MM*0	0 *M*0	0 0 *M*				
*MM*0	0 *MM*	0 *MM*	0 *MM*				
0 0 0	0 0 0	0 0 *M*	0 *M*0				
Spur corner cluster							
0 *A M*	*MB* 0	0 0 *M*	*M*0 0				
0 *MB*	*A M*0	*A M*0	0 *MB*				
*M*0 0	0 0 *M*	*MB* 0	0 *A M*				

(Continued)

Second Stage (or P) Hit Masks for Shrinking and Thinning (2)

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Corner cluster

$M\,M\,D$
$M\,M\,D$
$D\,D\,D$

Tee branch

$D\,M\,0$	$0\,M\,D$	$0\,0\,D$	$D\,0\,0$	$D\,M\,D$	$0\,M\,0$	$0\,M\,0$	$D\,M\,D$
$M\,M\,M$	$M\,M\,M$	$M\,M\,M$	$M\,M\,M$	$M\,M\,0$	$M\,M\,0$	$0\,M\,M$	$0\,M\,M$
$D\,0\,0$	$0\,0\,D$	$0\,M\,D$	$D\,M\,0$	$0\,M\,0$	$D\,M\,D$	$D\,M\,D$	$0\,M\,0$

Vee branch

$M\,D\,M$	$M\,D\,C$	$C\,B\,A$	$A\,D\,M$
$D\,M\,D$	$D\,M\,B$	$D\,M\,D$	$B\,M\,D$
$A\,B\,C$	$M\,D\,A$	$M\,D\,M$	$C\,D\,M$

Diagonal branch

$D\,M\,0$	$0\,M\,D$	$D\,0\,M$	$M\,0\,D$
$0\,M\,M$	$M\,M\,0$	$M\,M\,0$	$0\,M\,M$
$M\,0\,D$	$D\,0\,M$	$0\,M\,D$	$D\,M\,0$

[a] $A \cup B \cup C = 1 \quad D = 0 \cup 1 \quad A \cup B = 1.$

§0, 1§

P-Hit Masks for Skeletonizing (1)

TABLE 14.3-3. Skeletonize Unconditional Mark Patterns
$[P(M, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7) = 1$ if hit$]^a$

Pattern												
Spur												
0	0	0		0	0	0		0	0	M		M	0	0
0	M	0		0	M	0		0	M	0		0	M	0
0	0	M		M	0	0		0	0	0		0	0	0
Single 4-connection												
0	0	0		0	0	0		0	0	0		0	M	0
0	M	0		0	M	M		M	M	0		0	M	0
0	M	0		0	0	0		0	0	0		0	0	0
L corner												
0	M	0		0	M	0		0	0	0		0	0	0
0	M	M		M	M	0		0	M	M		M	M	0
0	0	0		0	0	0		0	M	0		0	M	0
Corner cluster												
M	M	D		D	D	D						
M	M	D		D	M	M						
D	D	D		D	M	M						

(Continued)

P-Hit Masks for Skeletonizing (2)

Tee branch

D	M	D	D	M	D	D	D	D	D	M	D
M	M	M	M	M	D	M	M	M	D	M	M
D	D	D	D	M	D	D	M	D	D	M	D

Vee branch

M	D	M	M	D	C	C	B	A	A	D	M
D	M	D	D	M	B	D	M	D	B	M	D
A	B	C	M	D	A	M	D	M	C	D	M

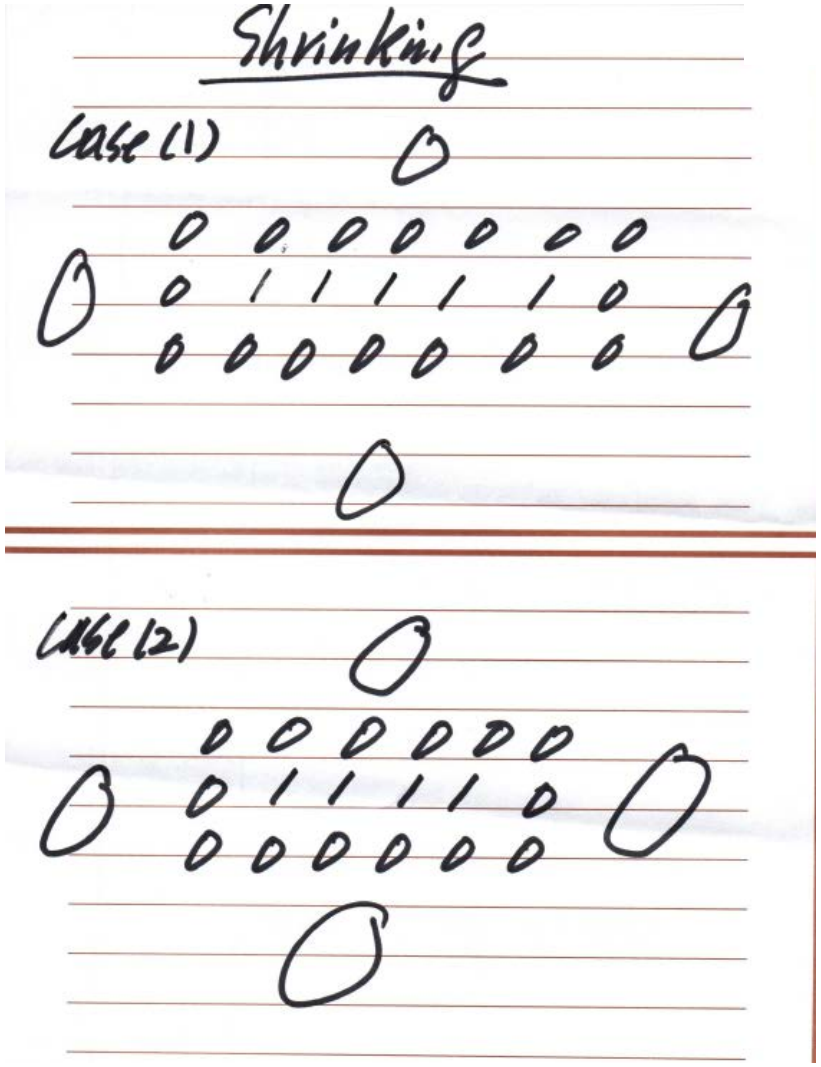
Diagonal branch

D	M	0	0	M	D	D	0	M	M	0	D
0	M	M	M	M	0	M	M	0	0	M	M
M	0	D	D	0	M	0	M	D	D	M	0

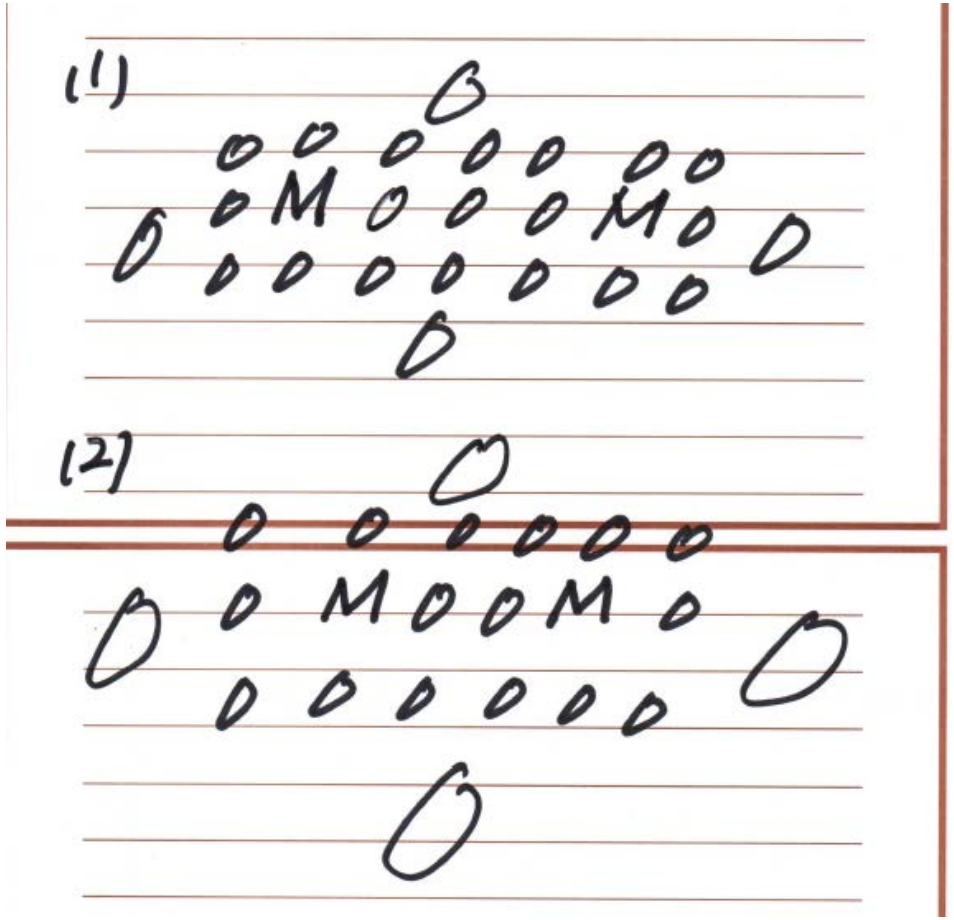
[a] $A \cup B \cup C = 1 \quad D = 0 \cup 1$.

Why Two-Stage Design?

- Consider the following two cases:

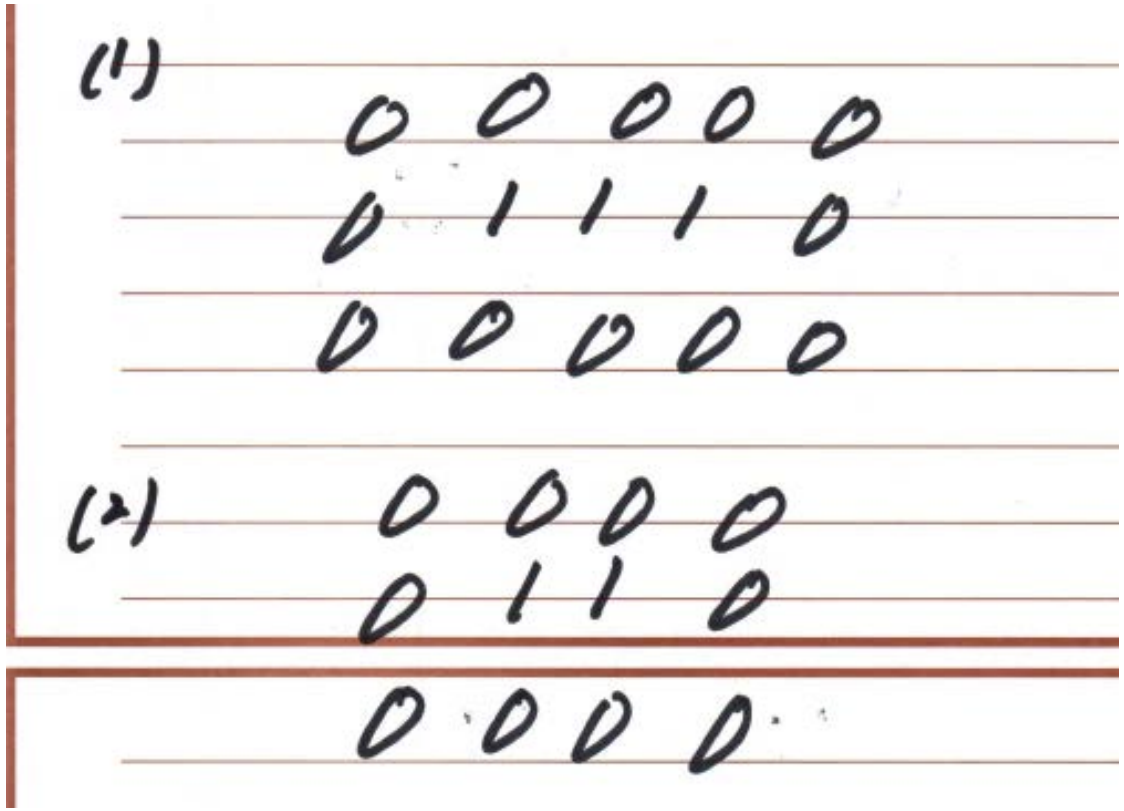


Iteration #1, M Filters

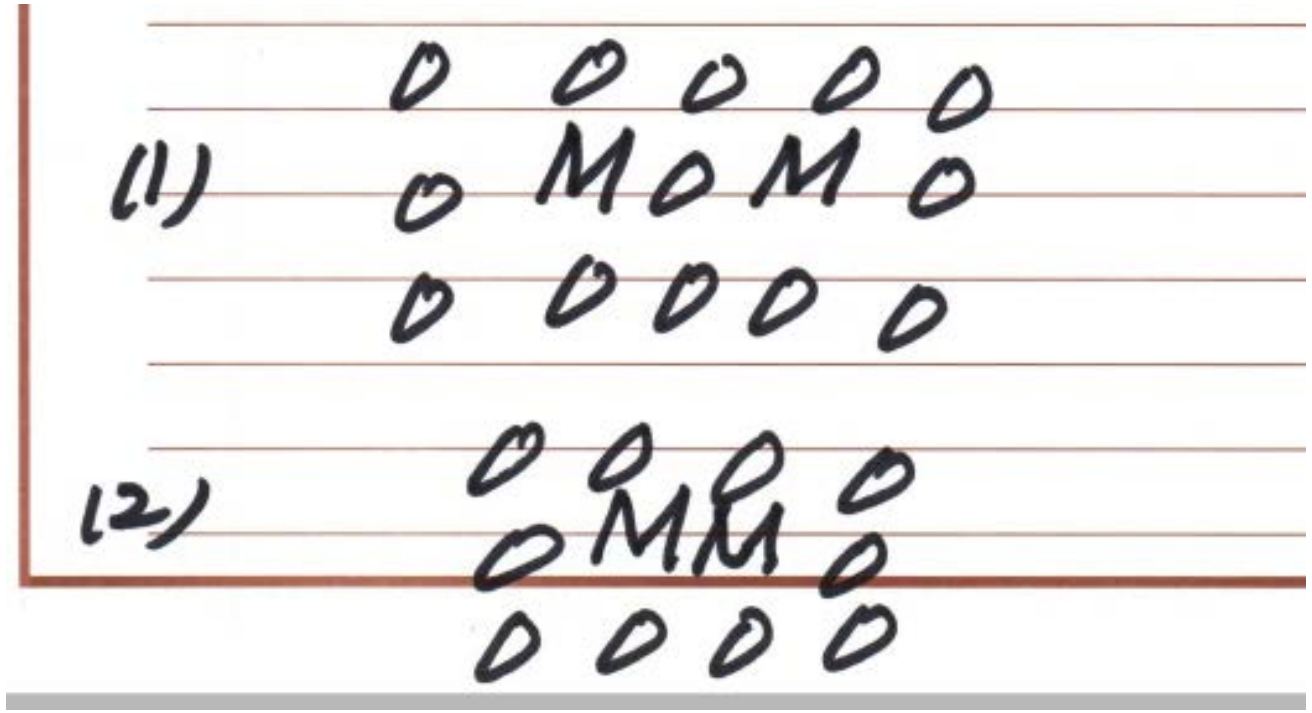


Iteration #1, P Filters

- No hit P filters - erasure is allowed
- Results:

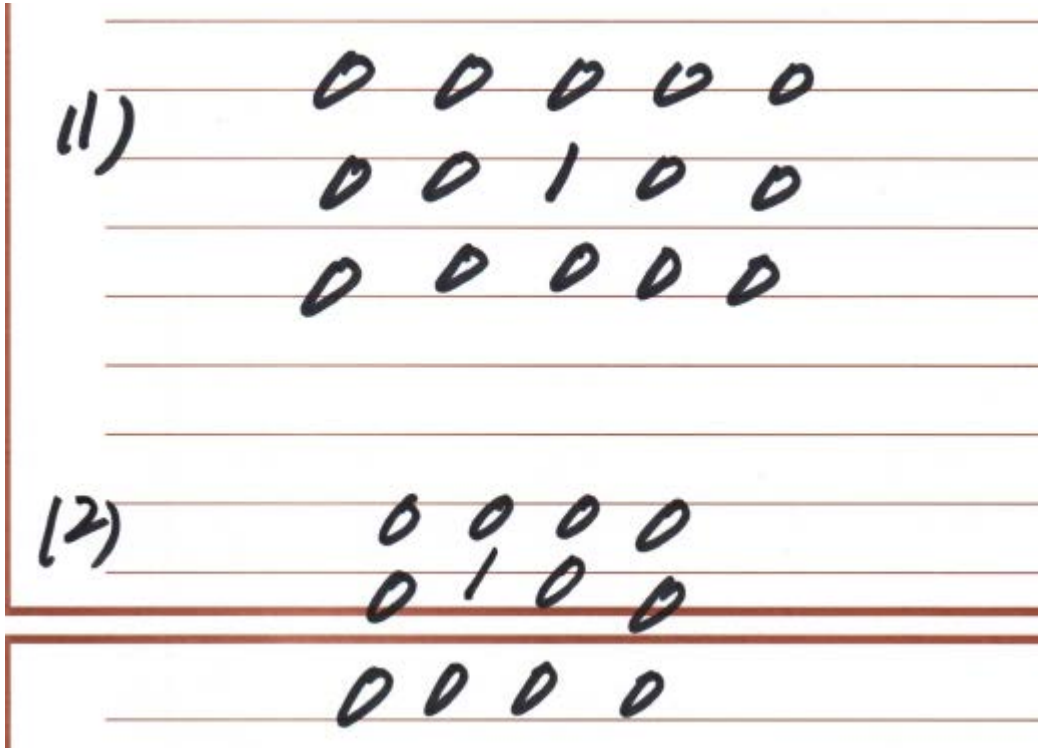


Iteration #2, M Filters



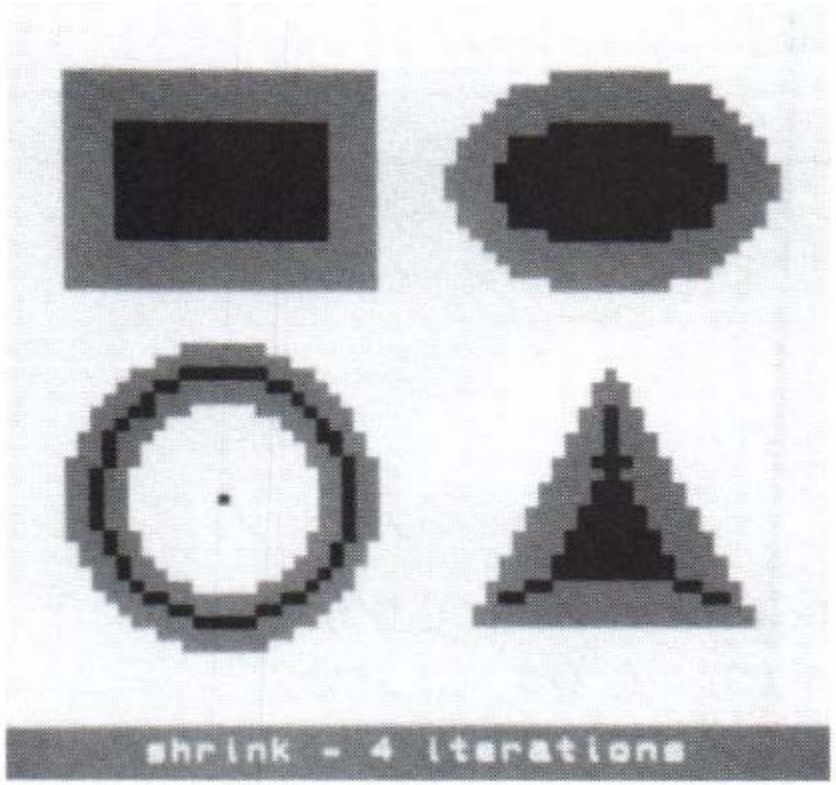
Iteration #2, P Filters

- No hit P filters in case (1) - erasure is allowed
- One hit P filter in case (2) – erasure in left M position is inhibited
- Results:

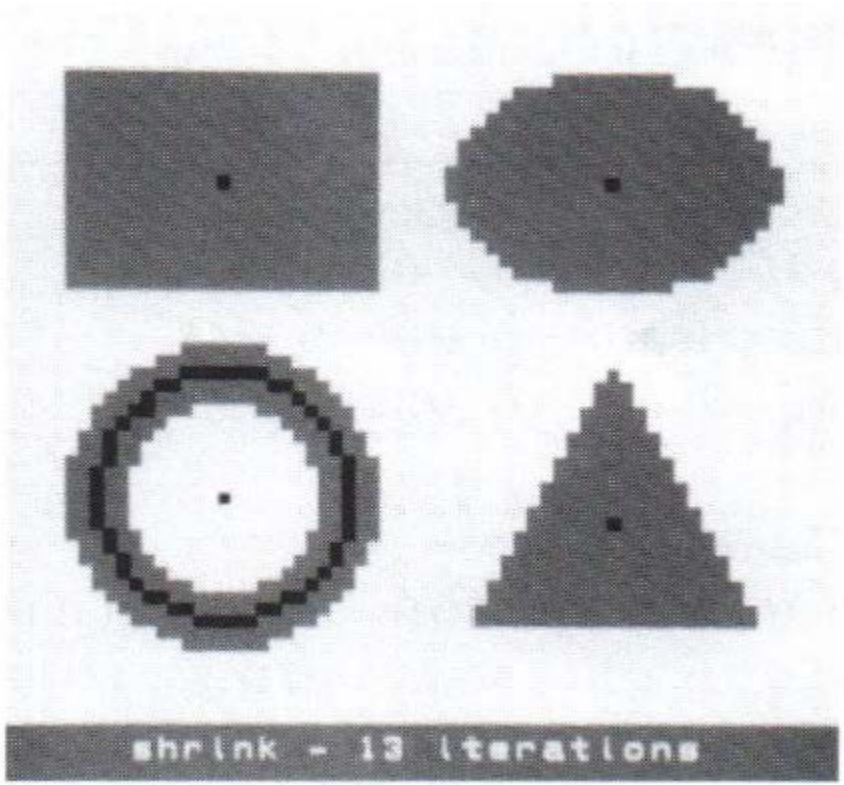


- No more change in future iterations

Iterative Application of Shrinking Filters Until Convergence

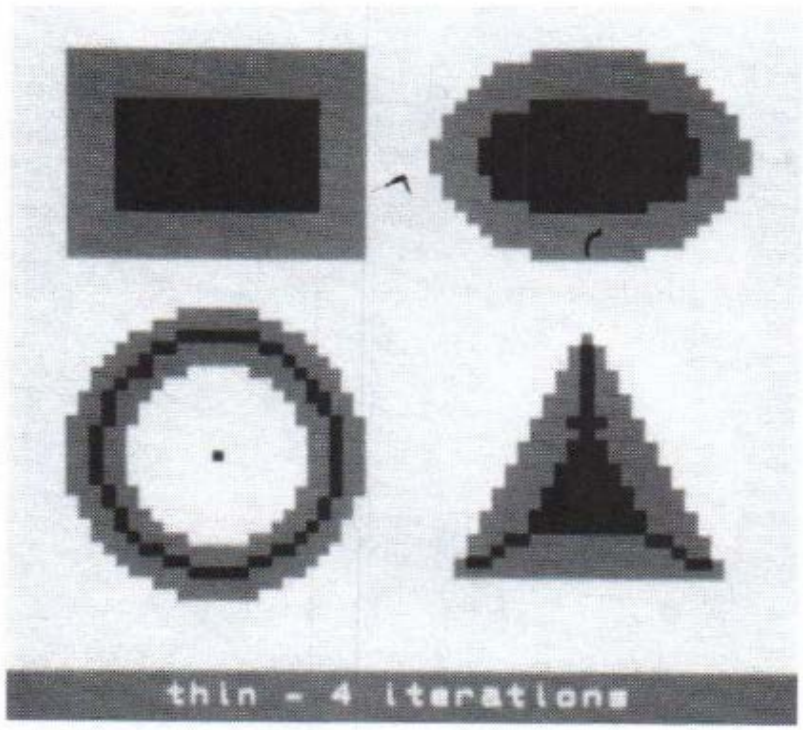


(*a*) Four iterations

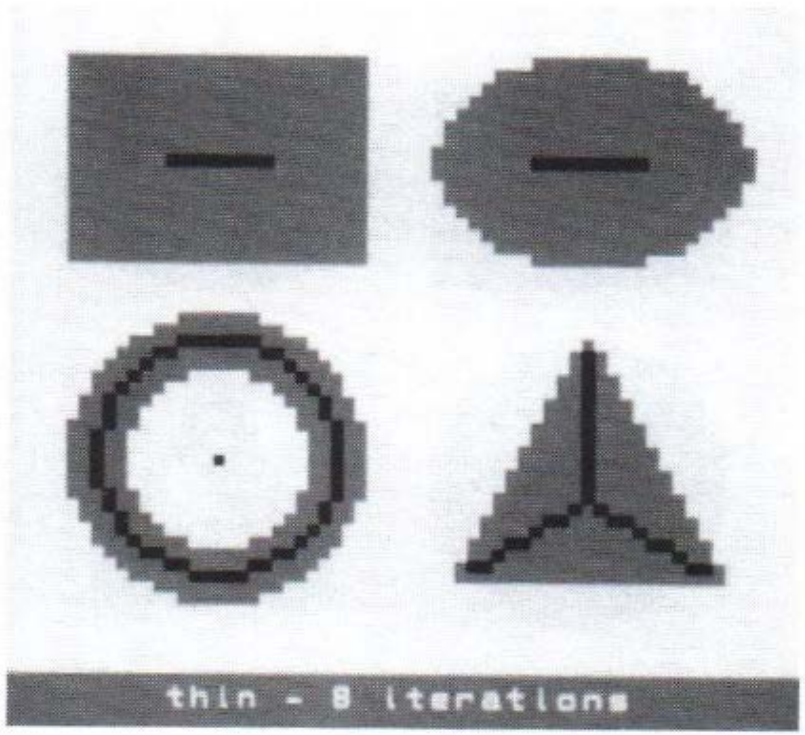


(*b*) Thirteen iterations

Iterative Application of Thinning Filters Until Convergence

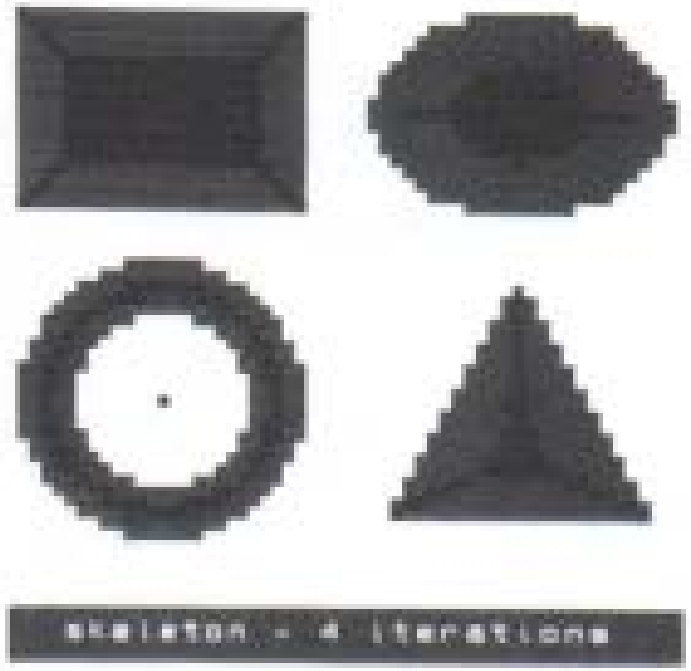


(*a*) Four iterations

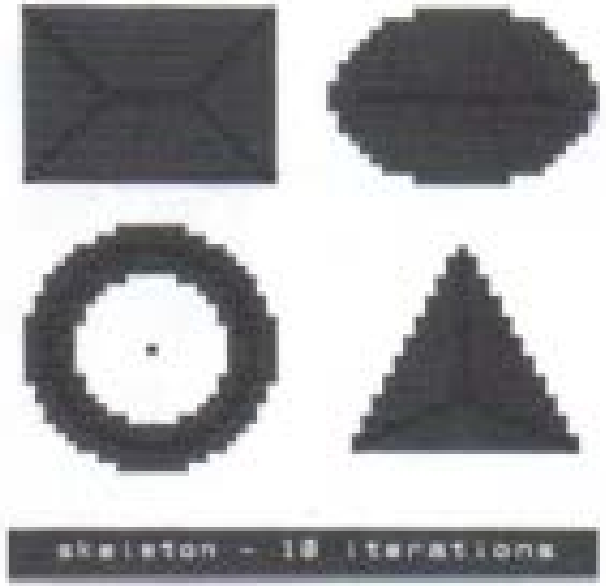


(*b*) Eight iterations

Iterative Application of Skeletonizing Filters Until Convergence

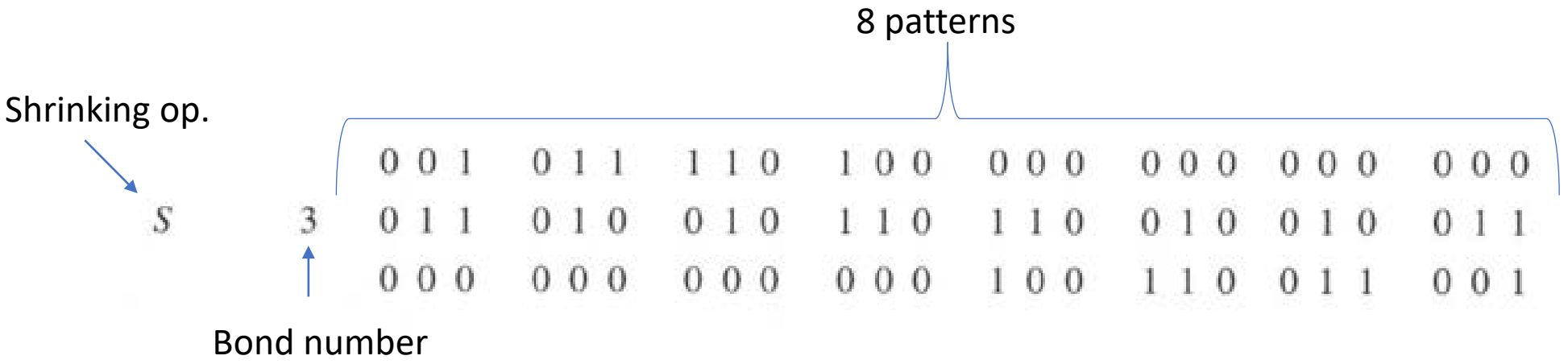


(*a*) Four iterations



(*b*) Ten iterations

Implementation of Morphological Filters



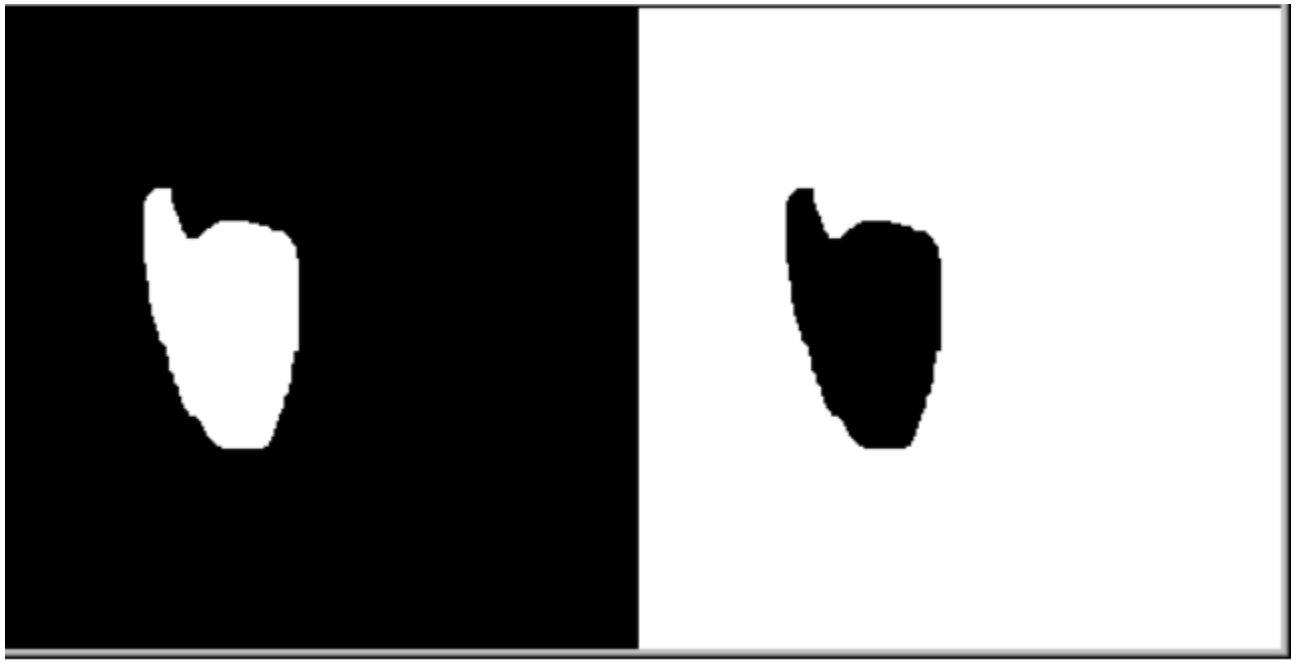
- Check filter type and bond number filter
- Center pixel always takes value “1” (if it is “0”, skip)
- Encode the eight neighbors with a binary sequence (bit-string)
- Begin with East, counter-clockwise
  - 11000000, 01100000, 00110000, 00011000
  - 00001100, 00000110, 00000011, 10000001

Image-Set-Based Morphology

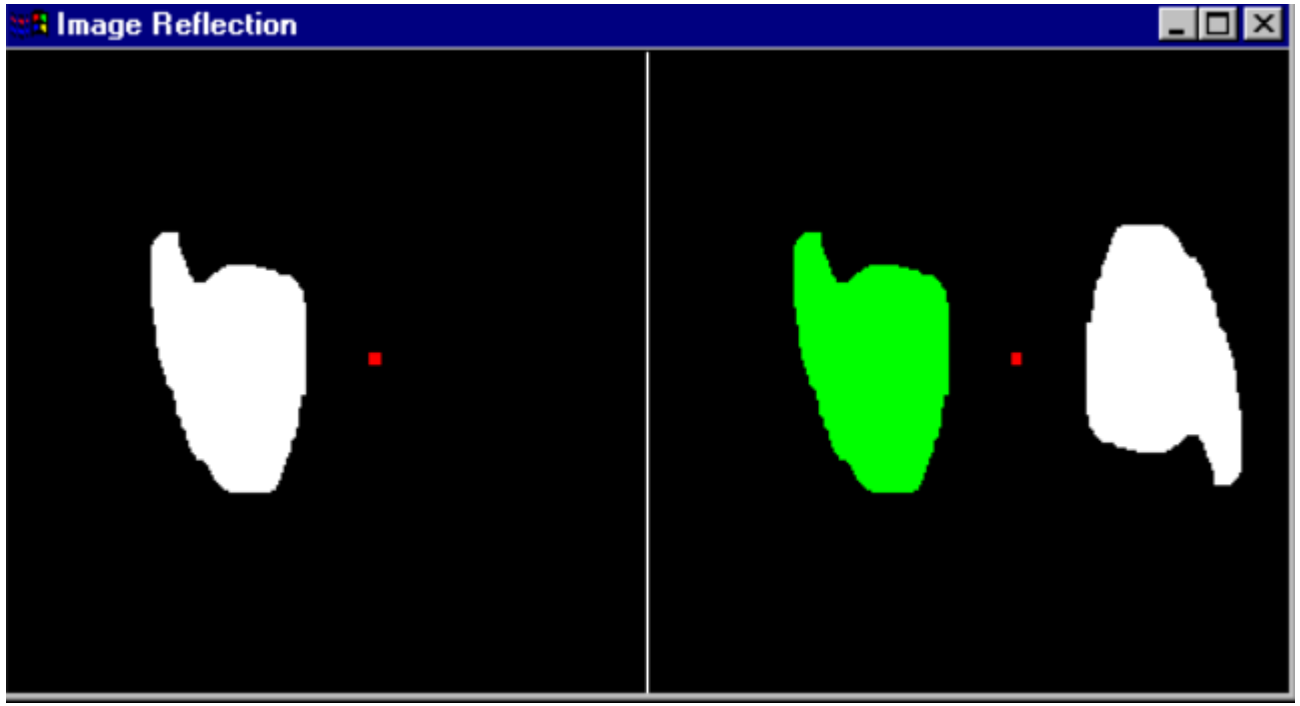
Example: Universal Set, Object Set and Complement Set

Object Set A

Complement of Object Set A



Example: Image Reflection



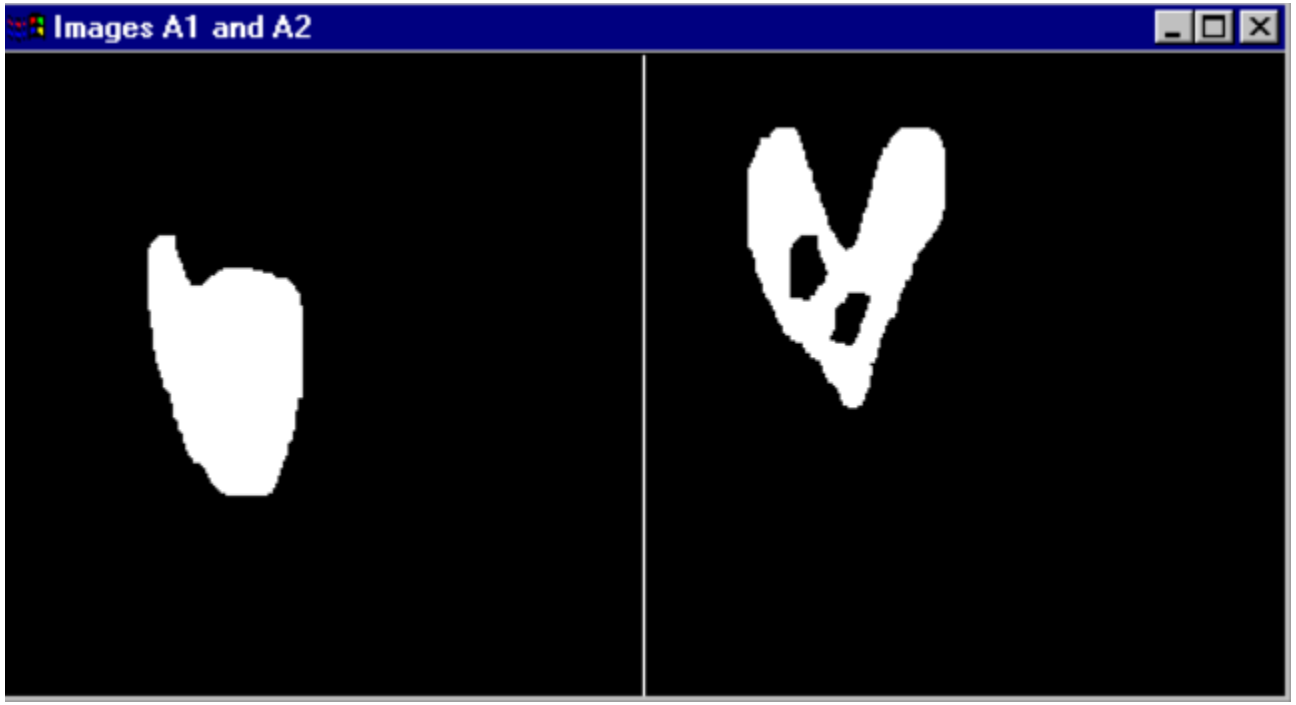
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Image Reflection, Union and Intersection

Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

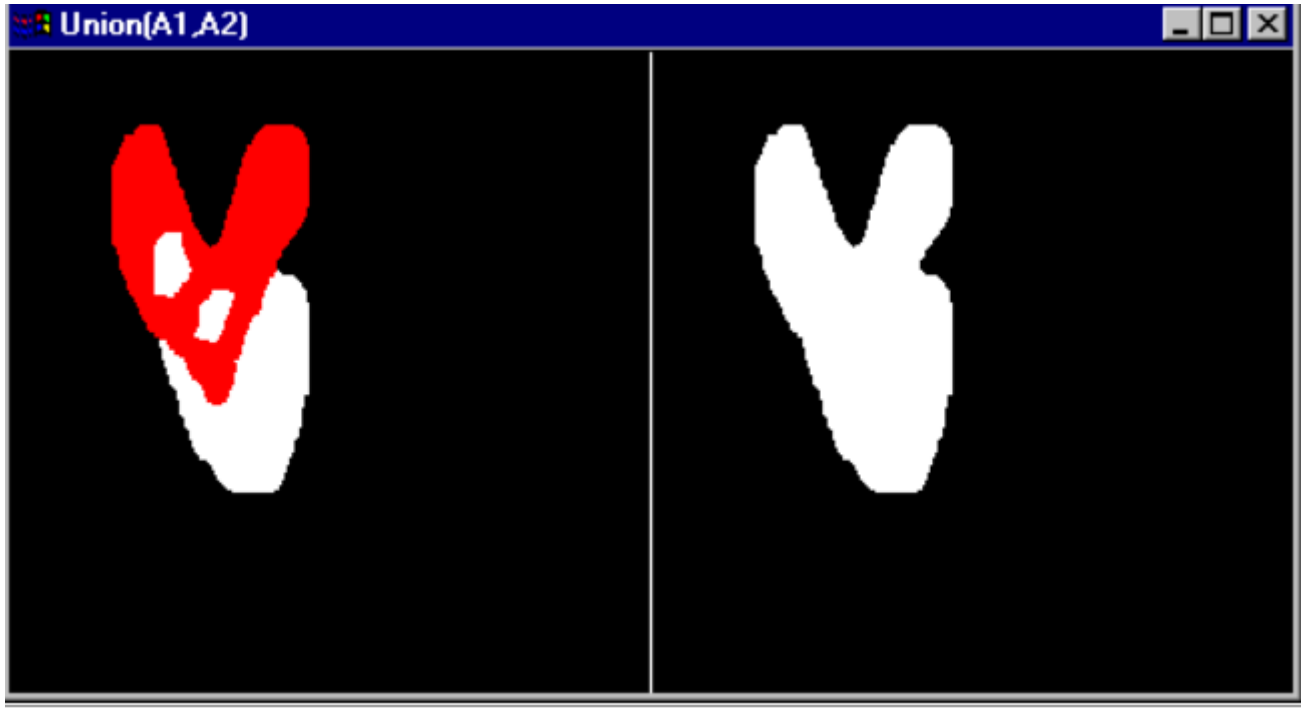
Example: Two Image Sets

A1 A2



Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

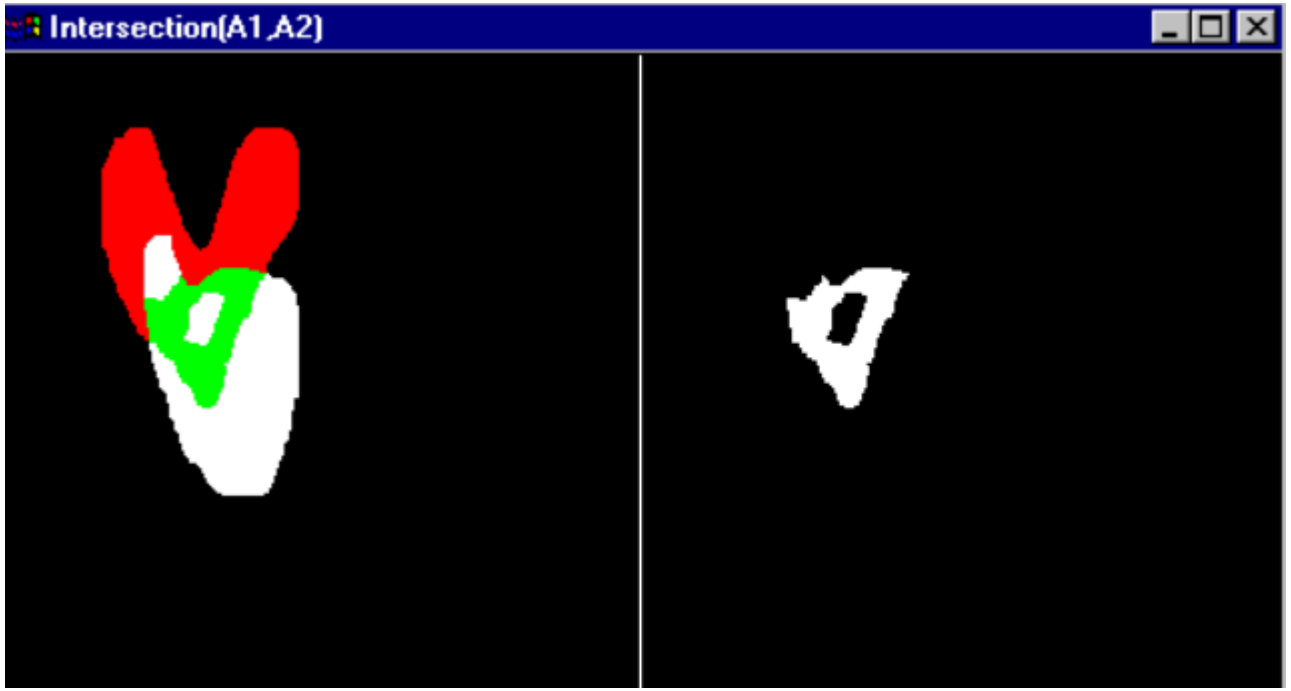
Example: Union of Two Image Sets



Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

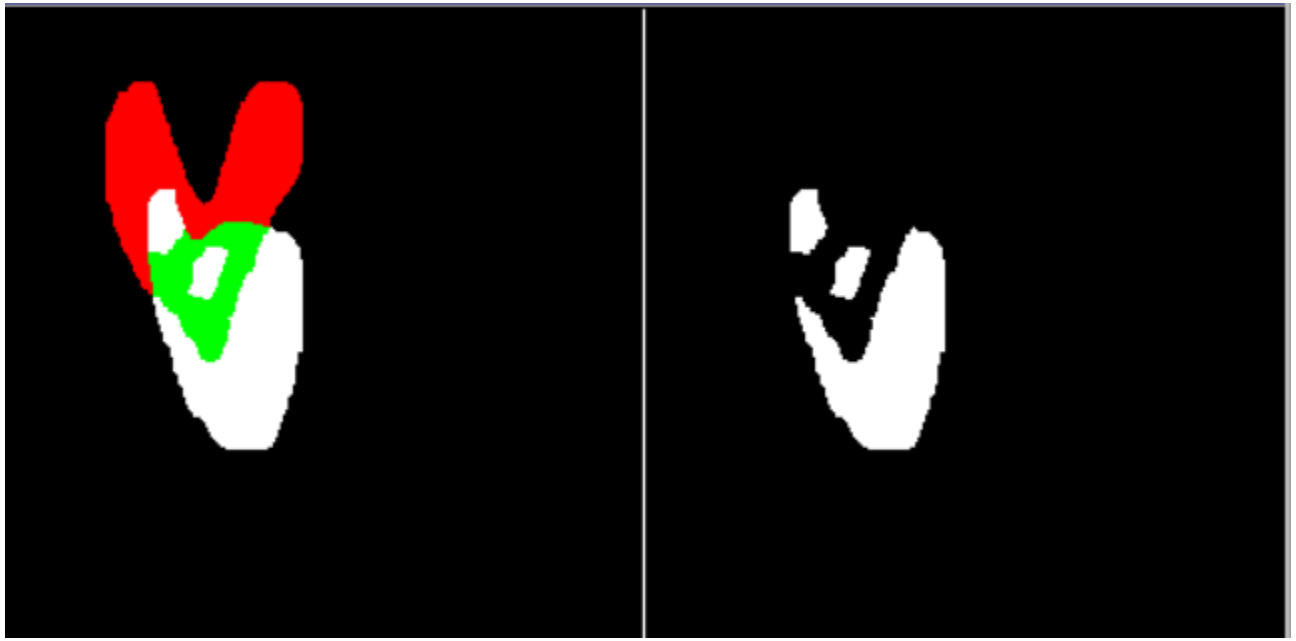
Example: Intersection and Differences of Two Image Sets

Green: Intersection of A1 and A2
Red: A2-A1
White: A1-A2



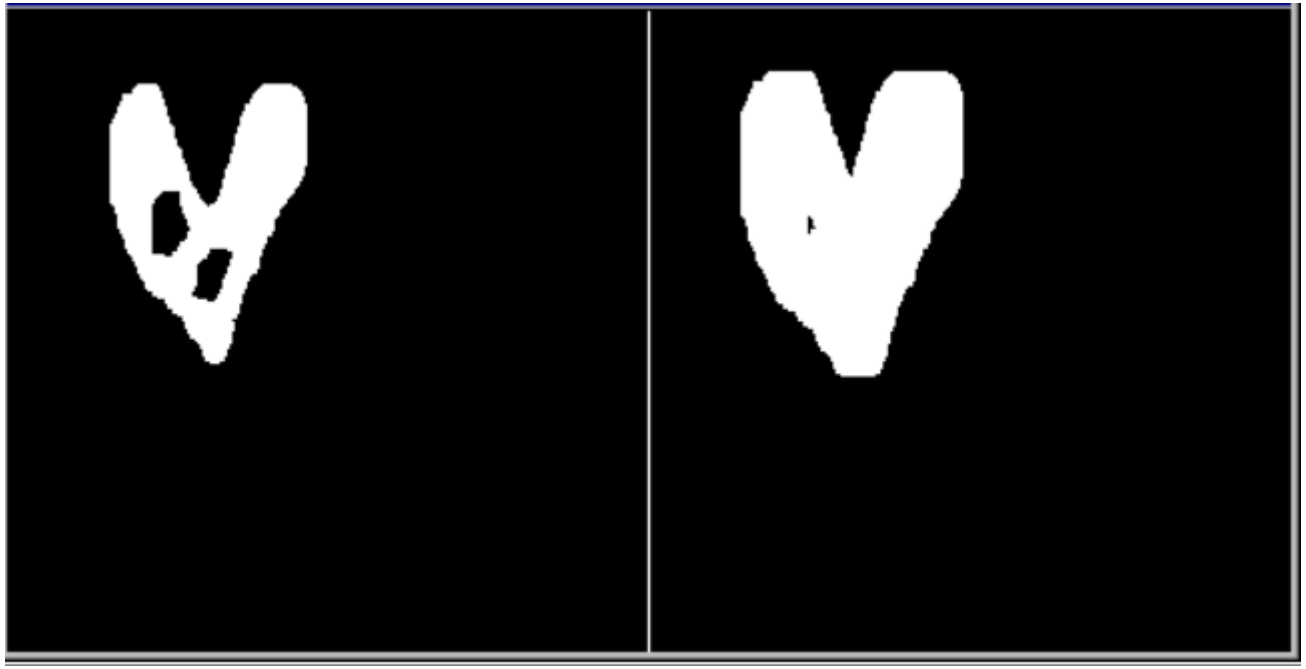
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: XOR



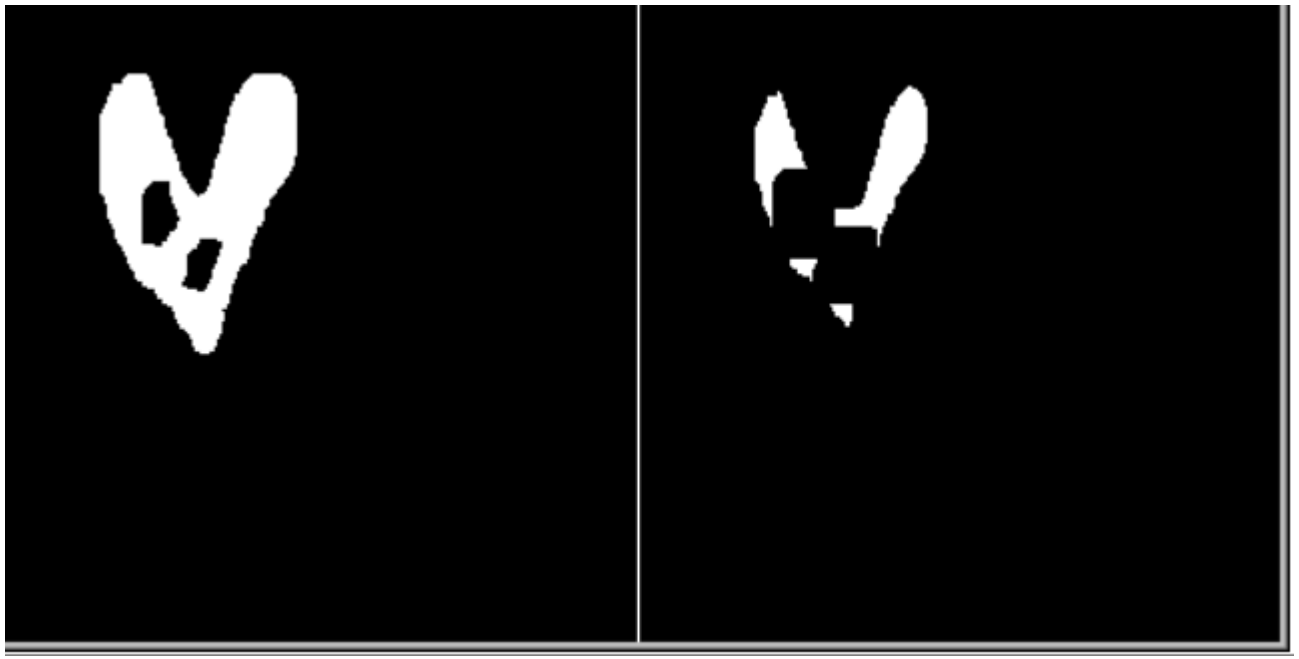
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Object Dilation



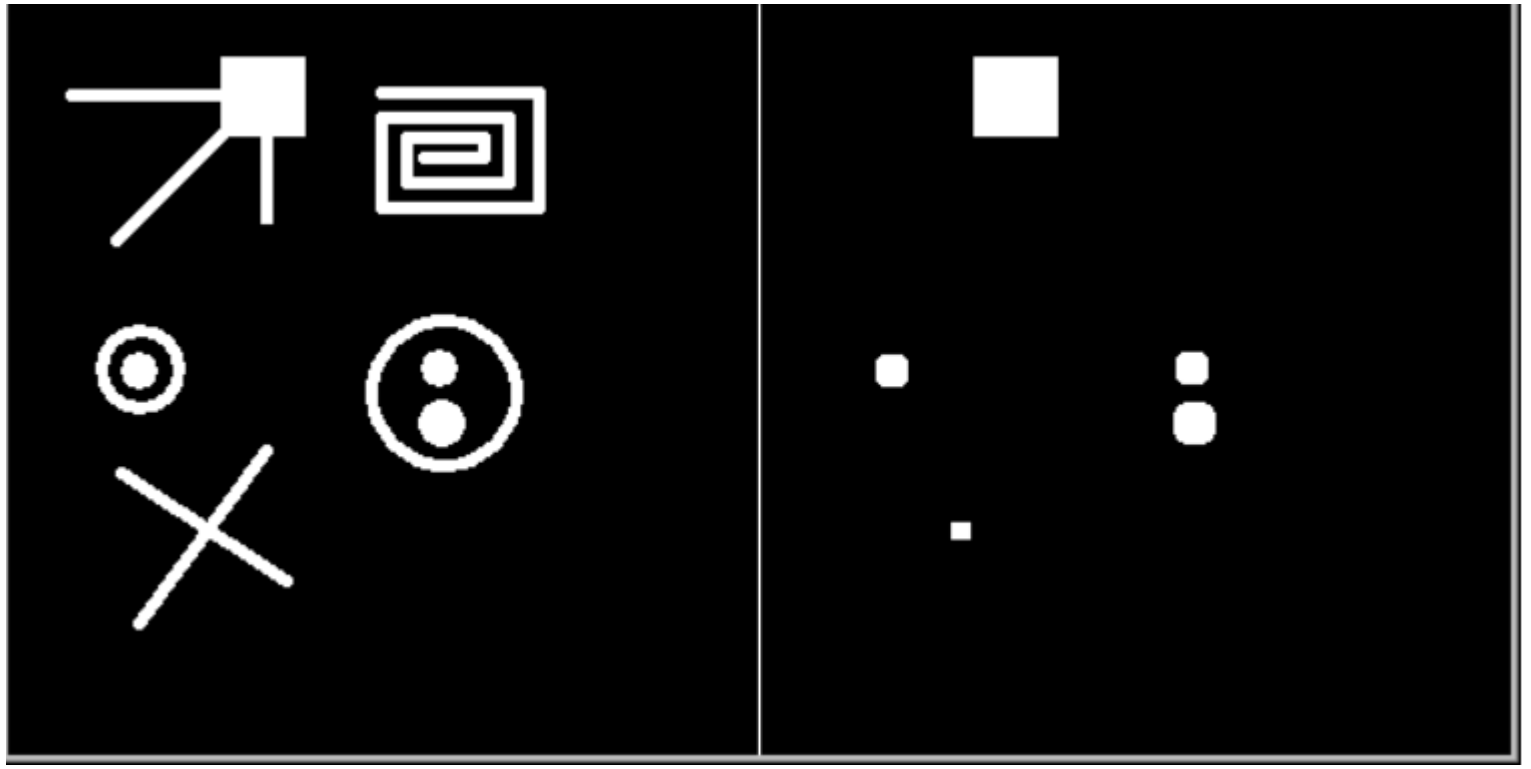
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Object Erosion



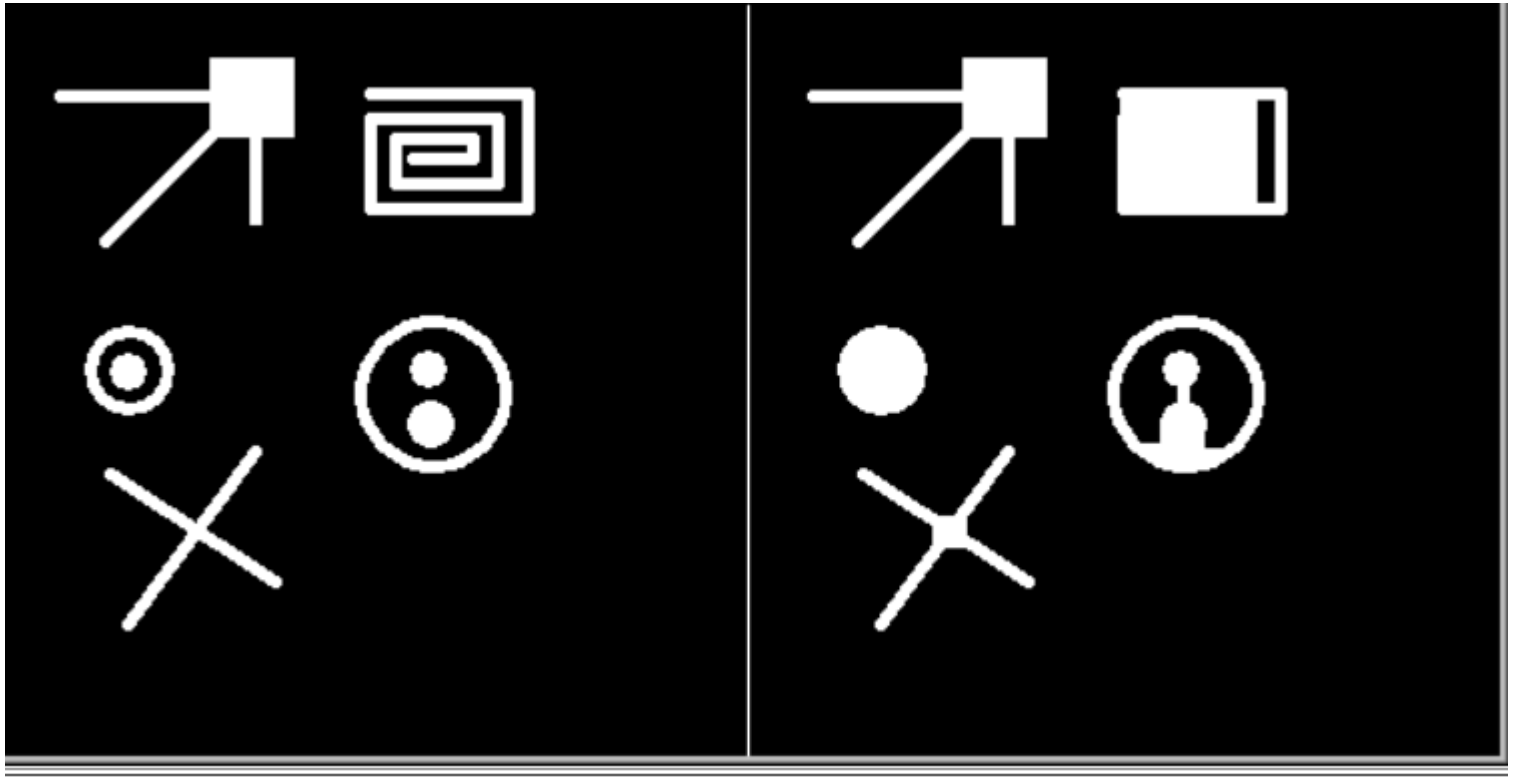
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Opening



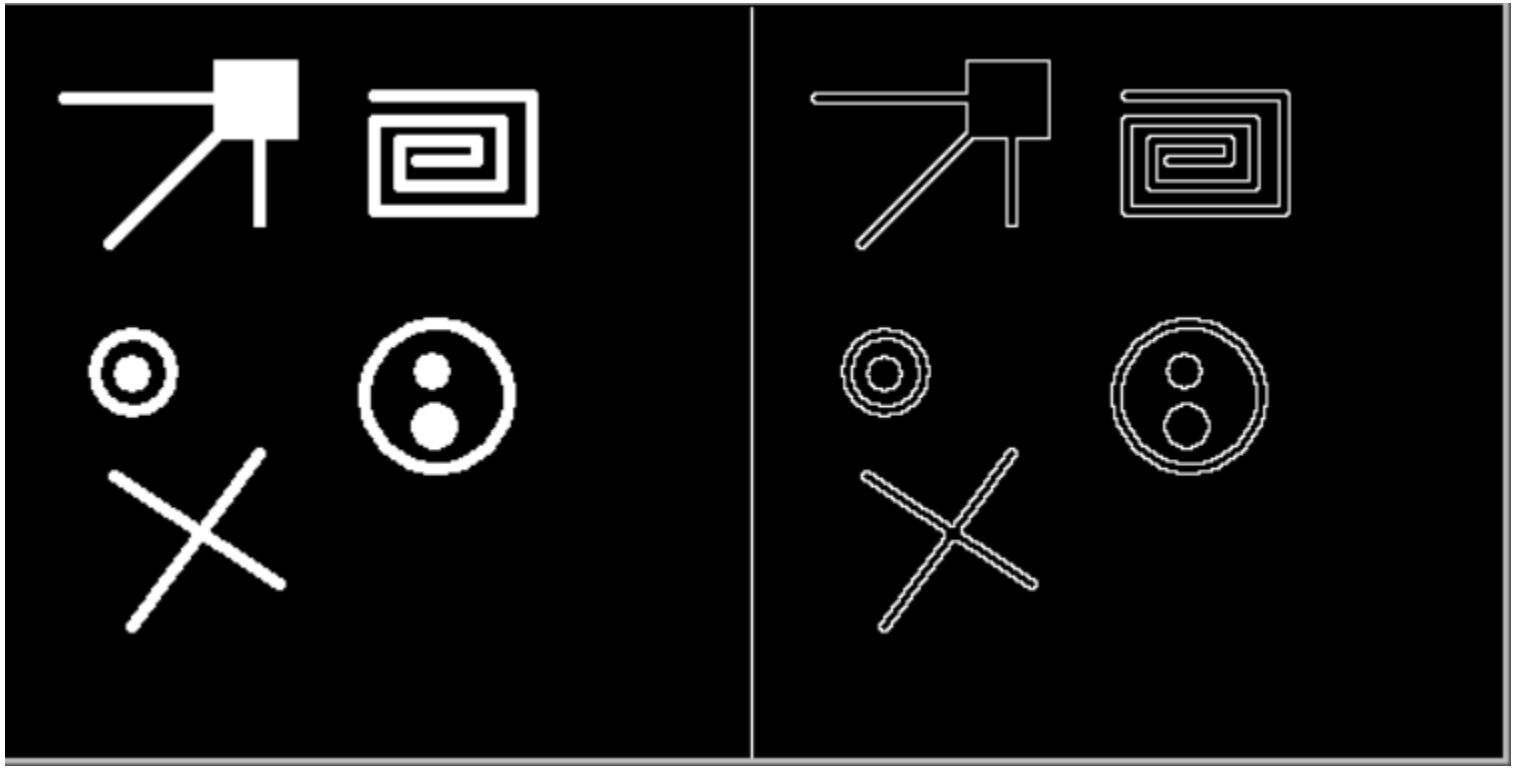
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Closing



Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Boundary Extraction

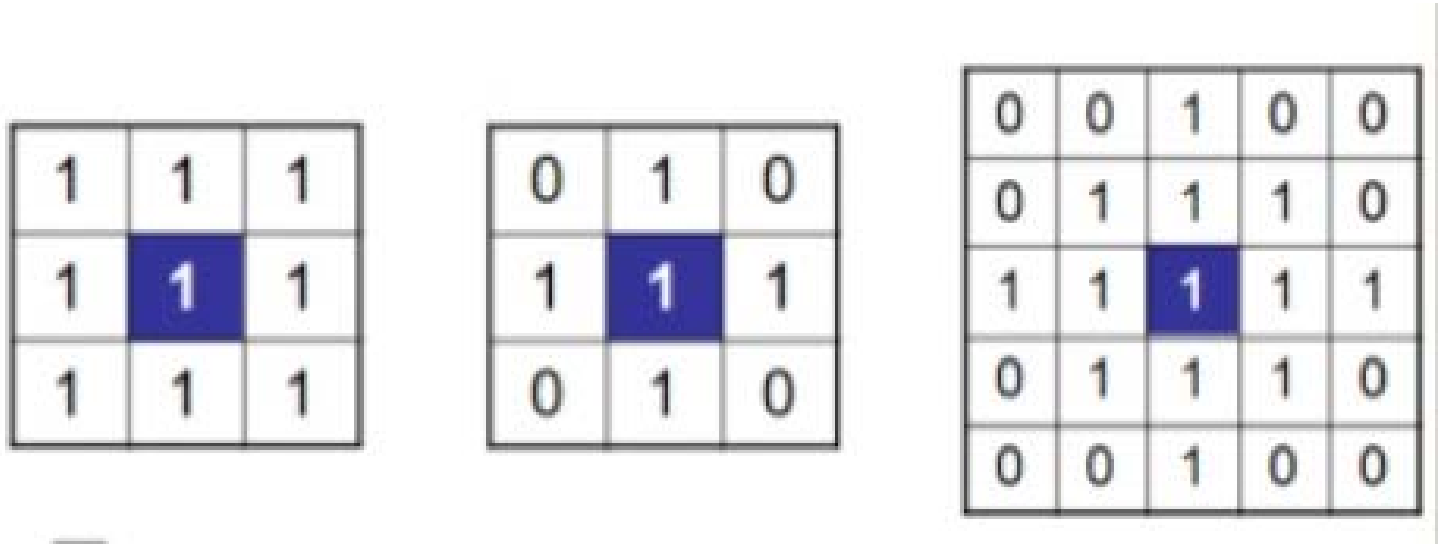


Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Morphological Filter Design with Structuring Elements

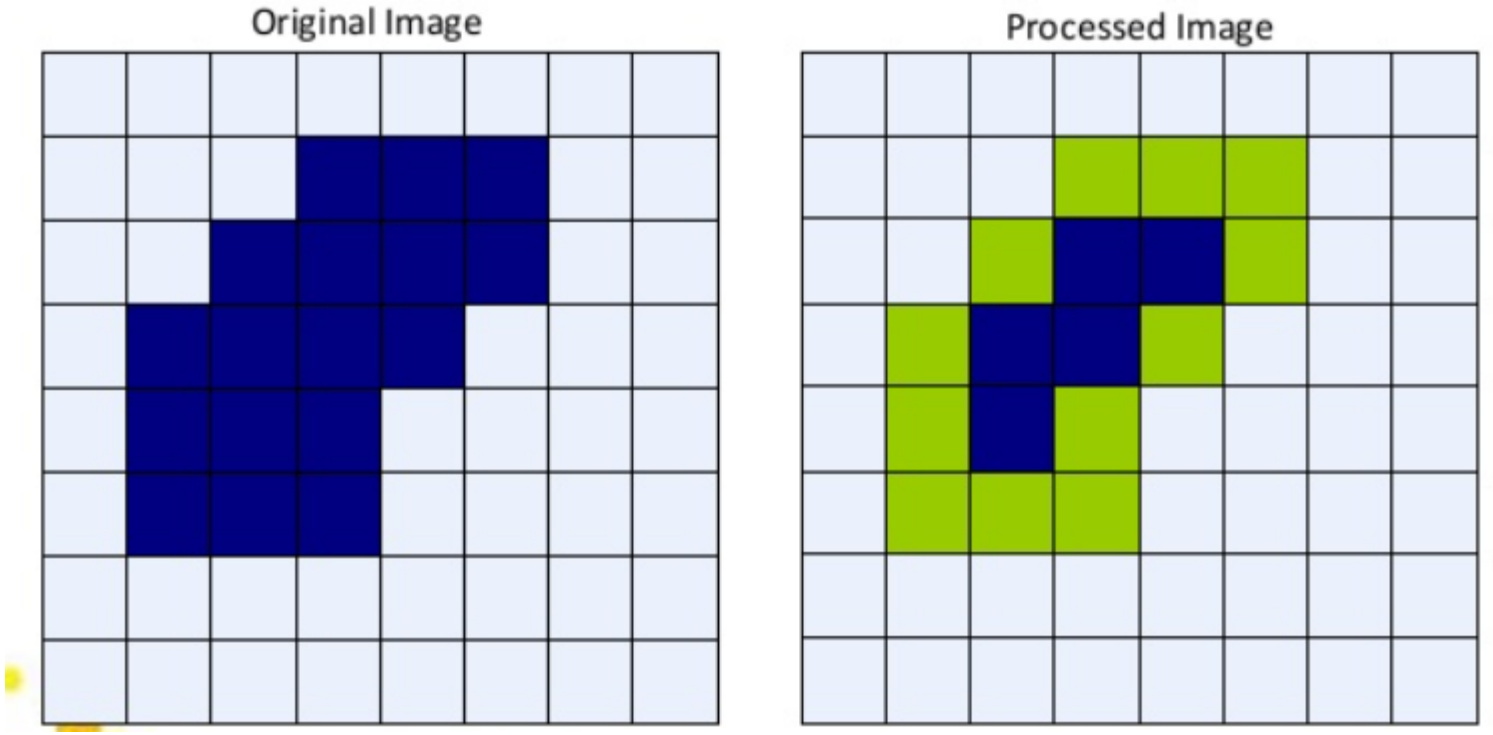
Object: Set A, Structuring Element: Set B

Example of Structuring Elements

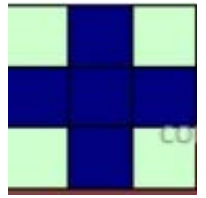


Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Erosion with Structuring Element



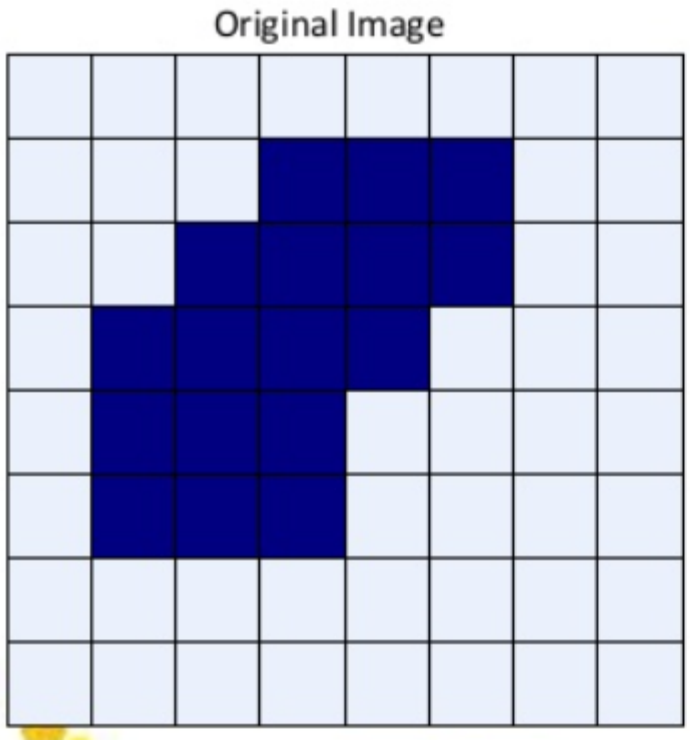
Structuring Element

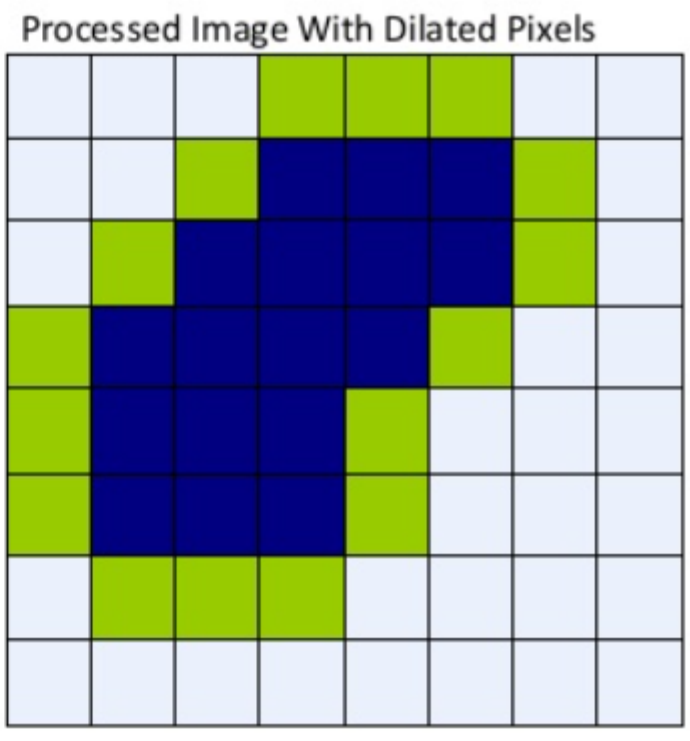


Use the center of SE to scan the object image
If hit, include the center pixel in the output image

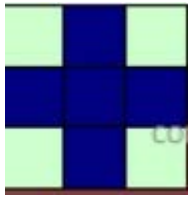
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Dilation with Structuring Element





Structuring Element



Use the center of SE to scan the object image
Include the union of the two in the output image

Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Erosion and Dilation with Structuring Element

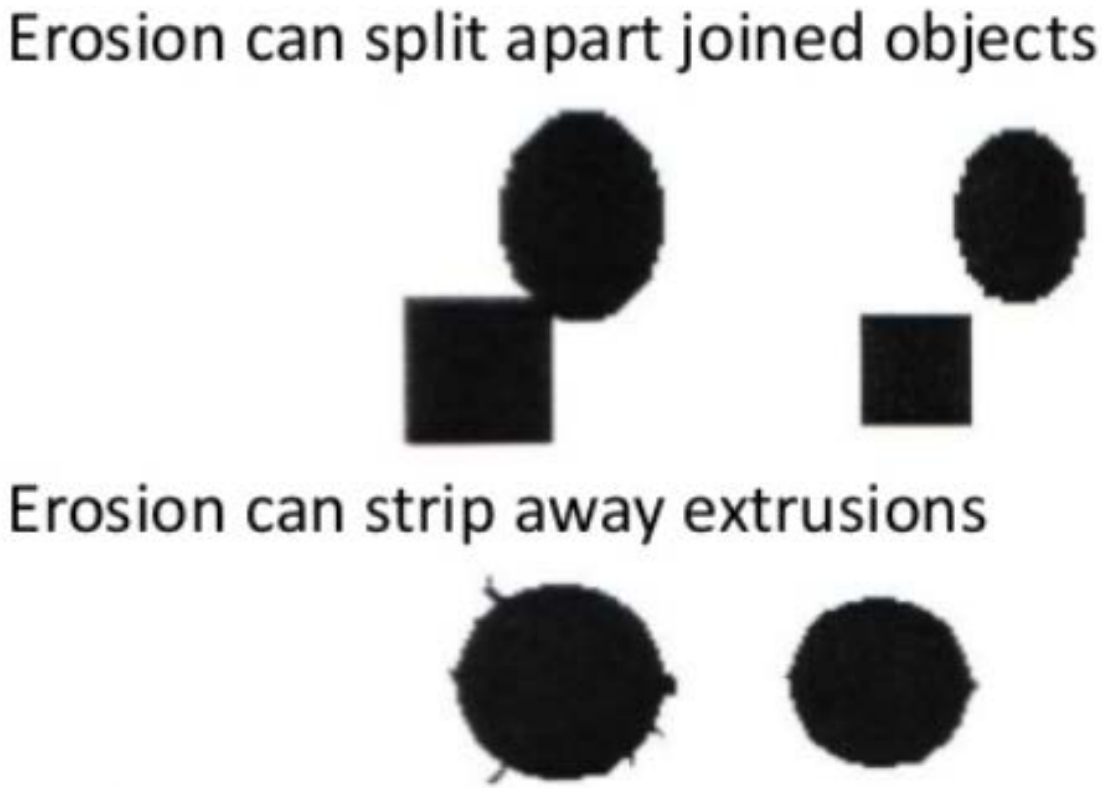
Original image

Erosion by 3*3 square structuring element

Erosion by 5*5 square structuring element

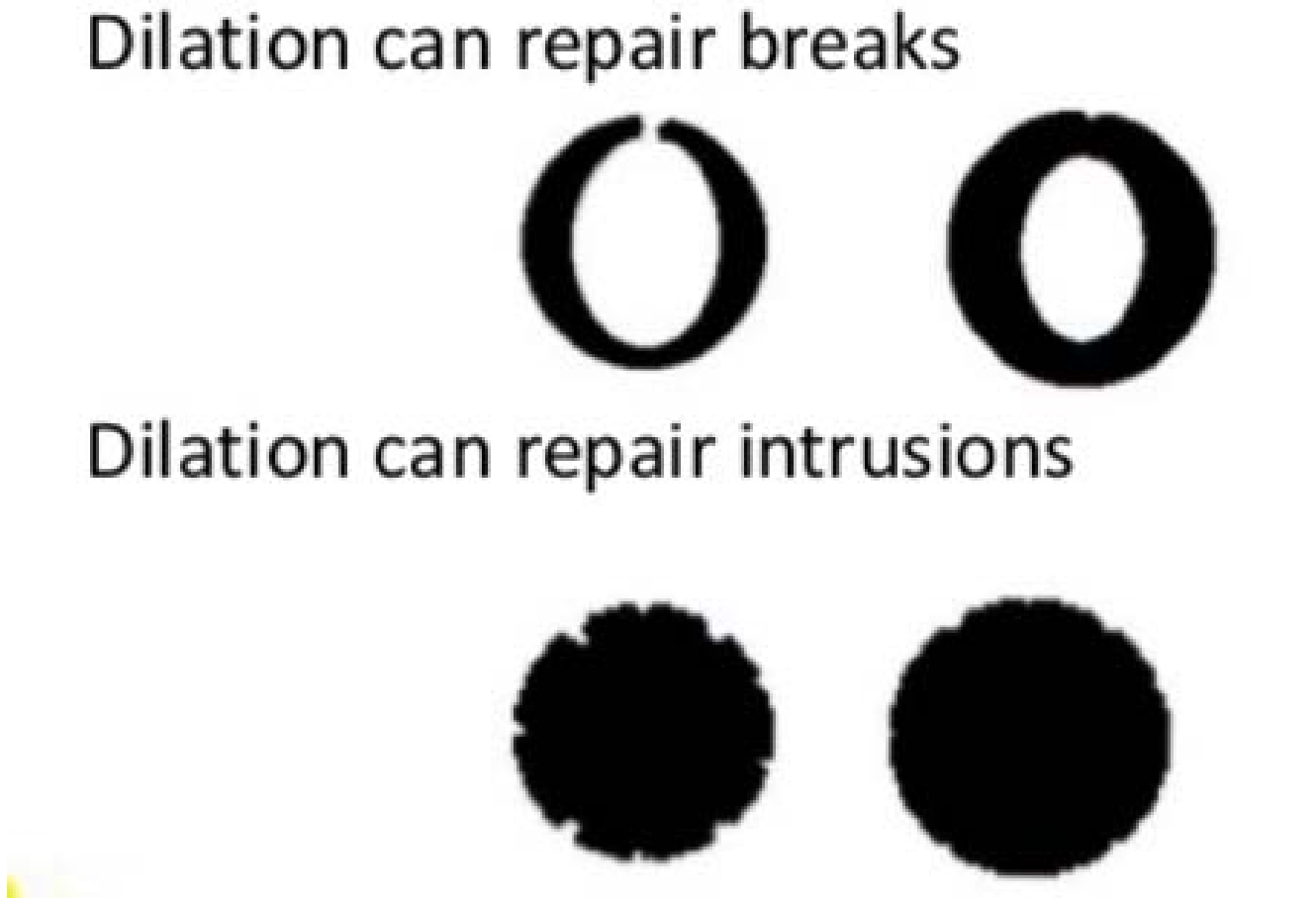
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Erosion Effect



Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Dilation Effect



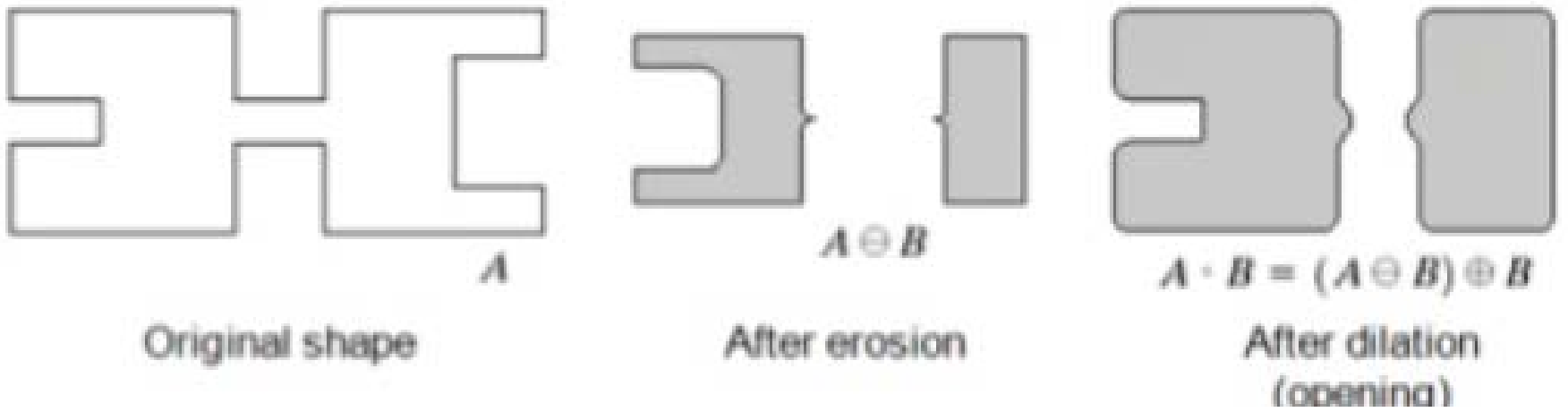
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Opening

– The opening of image f by structuring element s, denoted $f \circ s$ is simply an erosion followed by a dilation

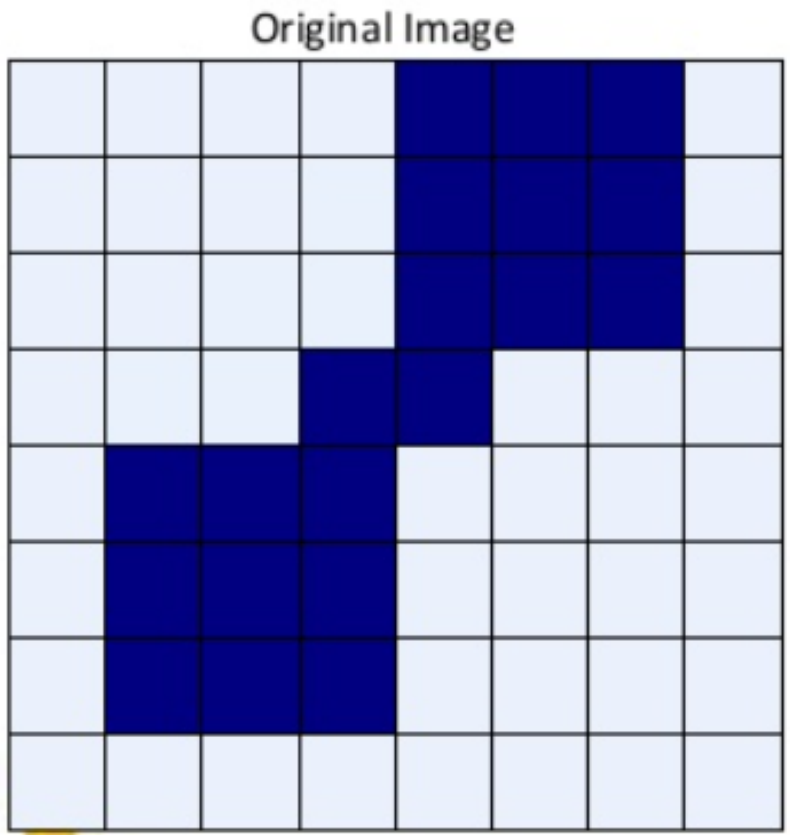
$$f \circ s = (f \ominus s) \oplus s$$

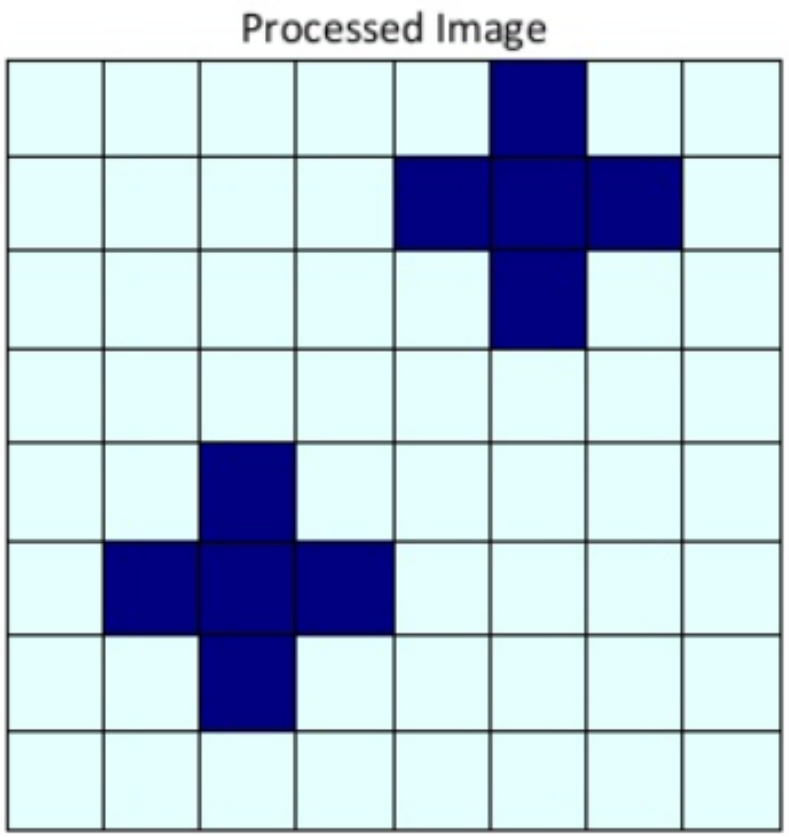
A disk-shaped SE is used



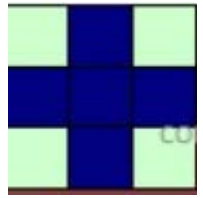
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Examples of Opening (1)



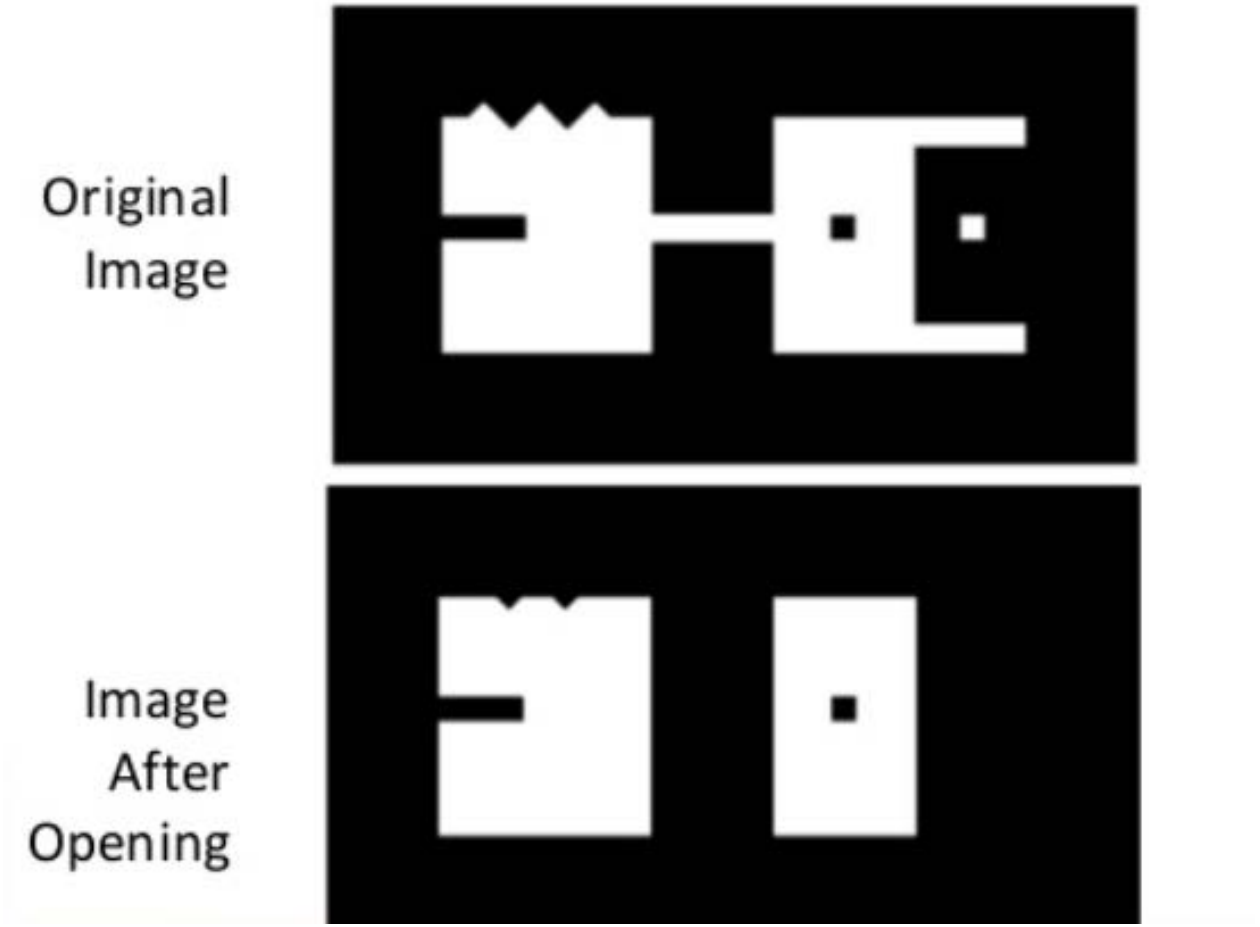


Structuring Element



Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Examples of Opening (2)



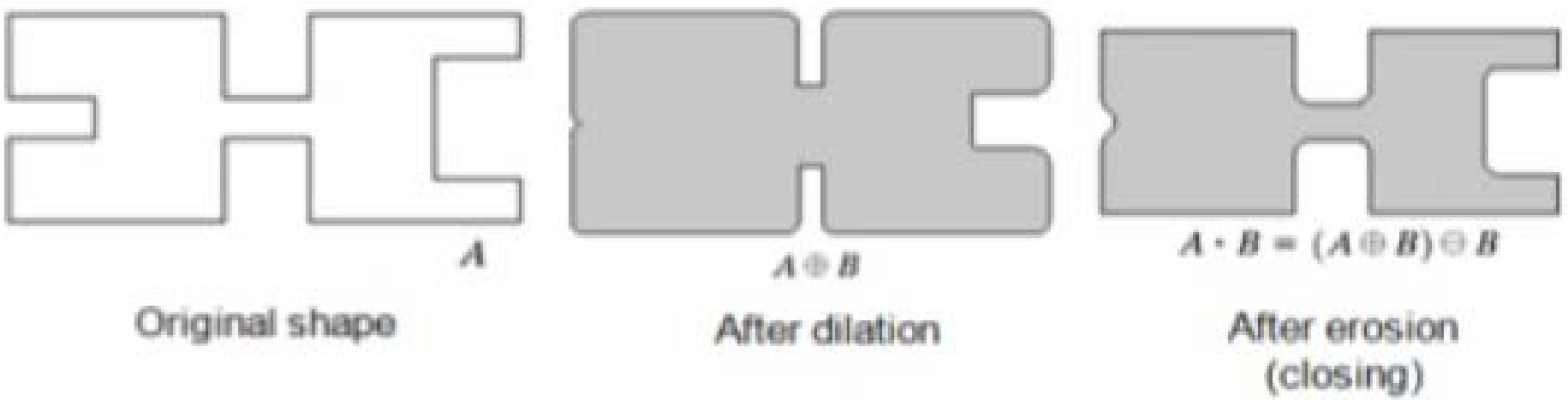
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Closing

– The closing of image f by structuring element s, denoted $f \bullet s$ is simply a dilation followed by an erosion.

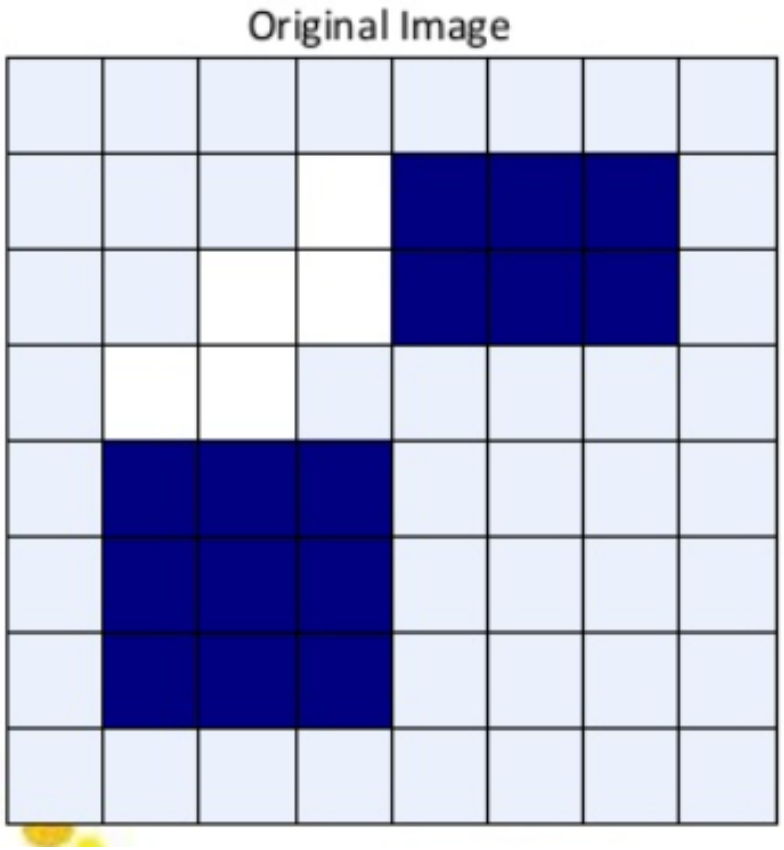
$$f \bullet s = (f \oplus s) \ominus s$$

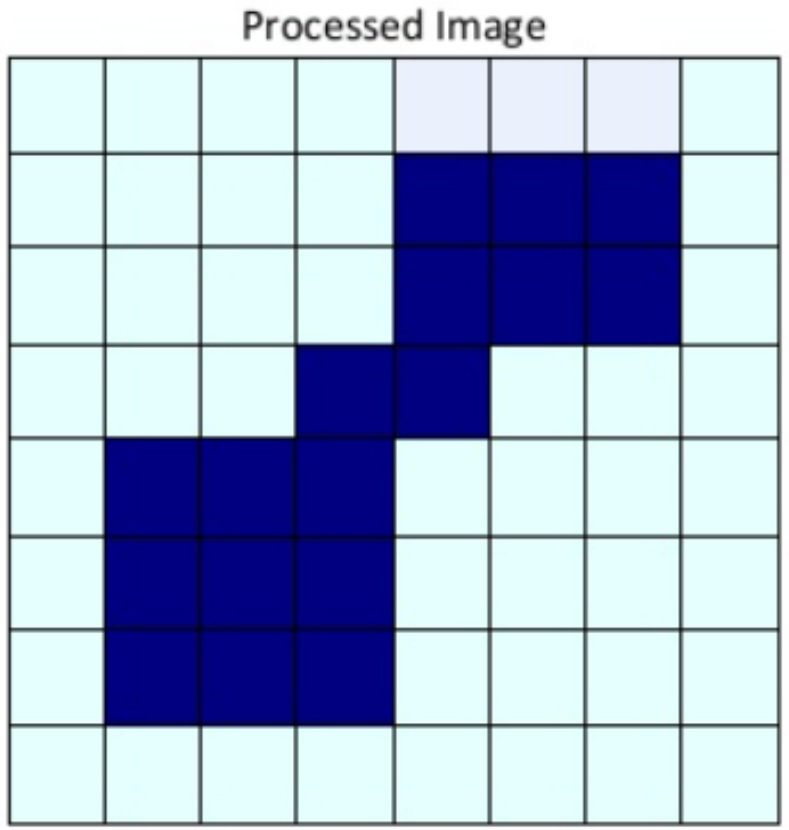
A disk-shaped SE is used



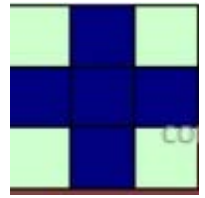
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Examples of Closing (1)



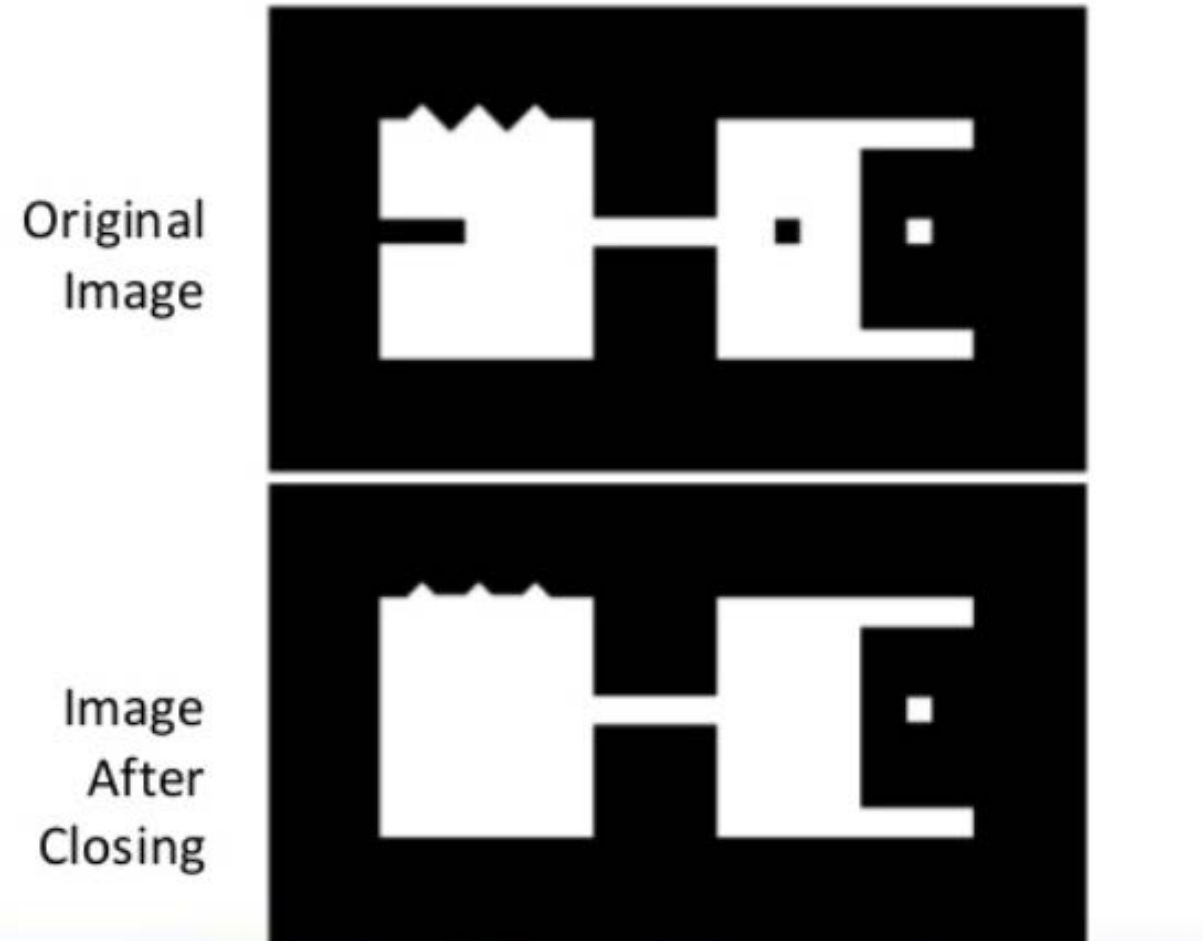


Structuring Element



Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Examples of Closing (2)



Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Qualitative Description of Opening and Closing

- Opening
  - Smooth the contour of an object, break narrow isthmuses and eliminate thin protrusions
- Closing
  - Fuse narrow breaks and long thin gulfs, eliminate small holes and fill gaps in the contours