Classic Edge Detection: 1^{st} and 2^{nd} Order Derivative Edge Detectors

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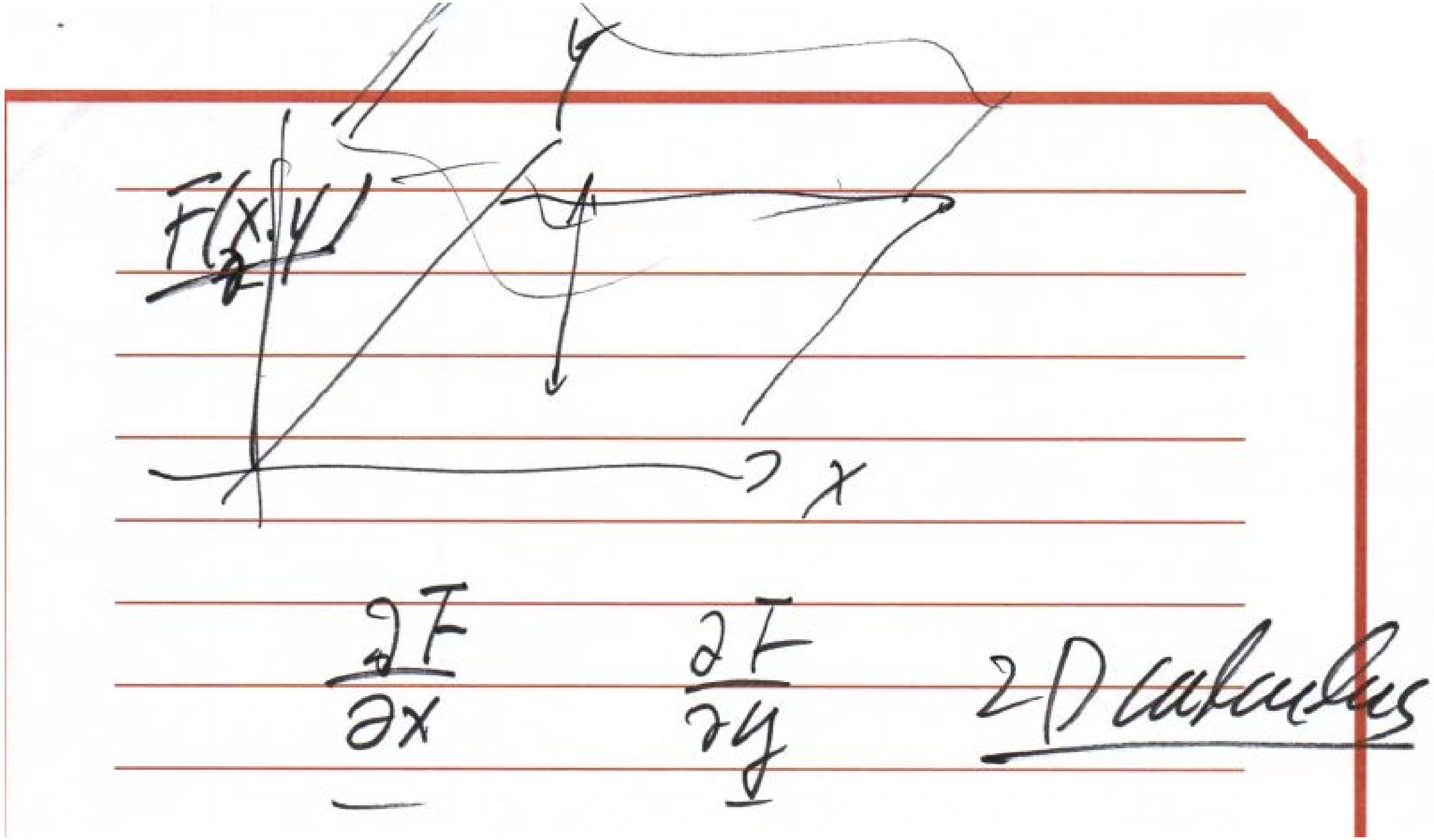
Two Main Branches of Image Processing

- Image/Video Compression
  - Still image compression 1980
    - JPEG, JPEG 2000
  - Video compression 1990-2020
    - MPEG-1, MPEG-2, MPEG-4, H.264/AVC, H.265/HEVC, H.266/VVC
- Image Understanding
  - Image analysis (low-level vision tasks)
    - Edge detection, segmentation, etc.
  - Computer vision (high-level vision tasks)
    - Object recognition, activity recognition, etc.
  - Slow progress from 1980-2010
  - Rapid progress in the last decade (leveraging a large amount of labeled data)

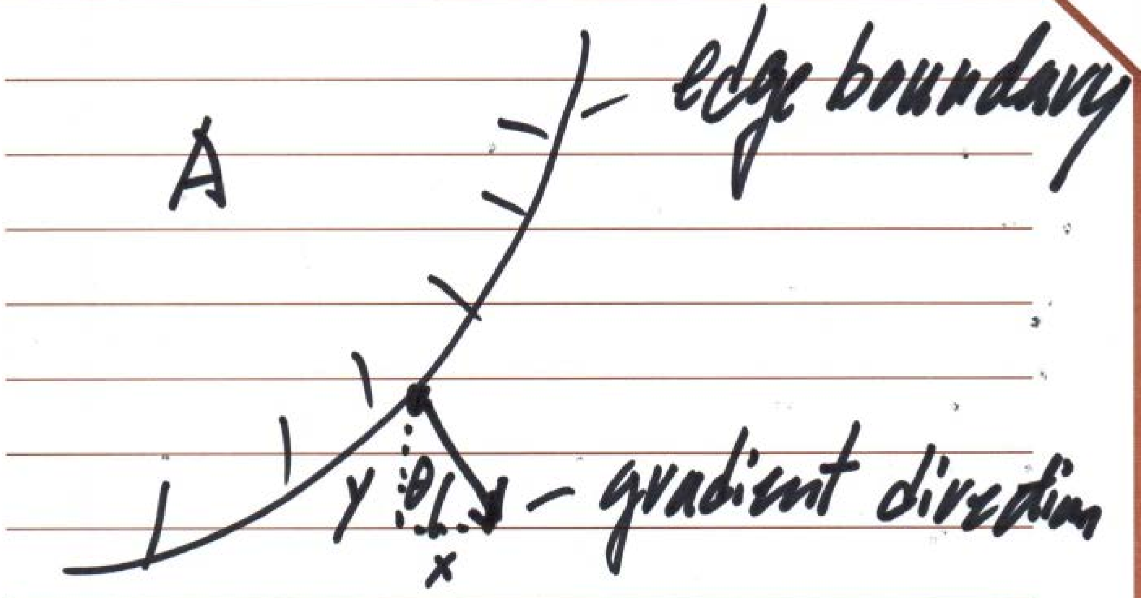
Classic Edge Detection Methods

- 1^{st} Order Derivative Method
- 2^{nd} Order Derivative Method
- Canny Edge Detection (1986)

1st Order Derivative Edge Detector (1)



1st Order Derivative Edge Detector (2)



$$\frac{dF}{dr} = \frac{\partial F}{\partial x}\frac{dx}{dr} + \frac{\partial F}{\partial y}\frac{dy}{dr}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

1st Order Derivative Edge Detector (3)

$$\frac{dF}{dv} = \frac{\partial F}{\partial x}\cos\theta + \frac{\partial F}{\partial y}\sin\theta$$

$$= \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}\right)^T \begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$$

magnitude is 1

$\frac{dF}{dv}$ reaches max. if

$$\begin{pmatrix}\frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y}\end{pmatrix} \parallel \begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$$

1st Order Derivative Edge Detector (4)

$$\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}\right) \quad \text{Orientation}$$

1st Order Derivative Edge Detector (5)

$$\max \left| \frac{dI}{dr} \right| = \left\| \begin{pmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{pmatrix} \right\|$$

$$= \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

Gradient magnitude.

1st Order Derivative Edge Detector (6)

$$\frac{\partial F}{\partial x} \approx F(j+1,k) - F(j,k)$$

j

$F(j,k-1)$

y

$F(j,k)$ $F(j+1,k)$

k

$$\frac{\partial F}{\partial y} \approx -F(j,k) + F(j,k-1)$$

1st Order Derivative Edge Detector (7)

A_0	A_1	A_2
A_7	$F(j,k)$	A_3
A_6	A_5	A_4

Table 15.2-9

Compass gradient template:

1^{st} Order Derivative Edge Detector (8)

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Operator	Row gradient	Column gradient
Pixel difference	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Separated pixel difference	$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
Roberts	$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Prewitt	$\frac{1}{3}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$	$\frac{1}{3}\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
Sobel	$\frac{1}{4}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$	$\frac{1}{4}\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$
Frei–Chen	$\frac{1}{2+\sqrt{2}}\begin{bmatrix} 1 & 0 & -1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & 0 & -1 \end{bmatrix}$	$\frac{1}{2+\sqrt{2}}\begin{bmatrix} -1 & -\sqrt{2} & -1 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$

FIGURE 15.2-6. Impulse response arrays for 3 × 3 orthogonal differential gradient edge operators.

DSP:
- Impulse response
- Convolution

DIP:
- Image filter
- Correlation (or elementwise multiplication)

1st Order Derivative Edge Detector (9)

Most famous 1st-order derivative based edge detector.

Sobel Edge Detector.

$$\frac{\partial F}{\partial x} \approx \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial F}{\partial y} \approx \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

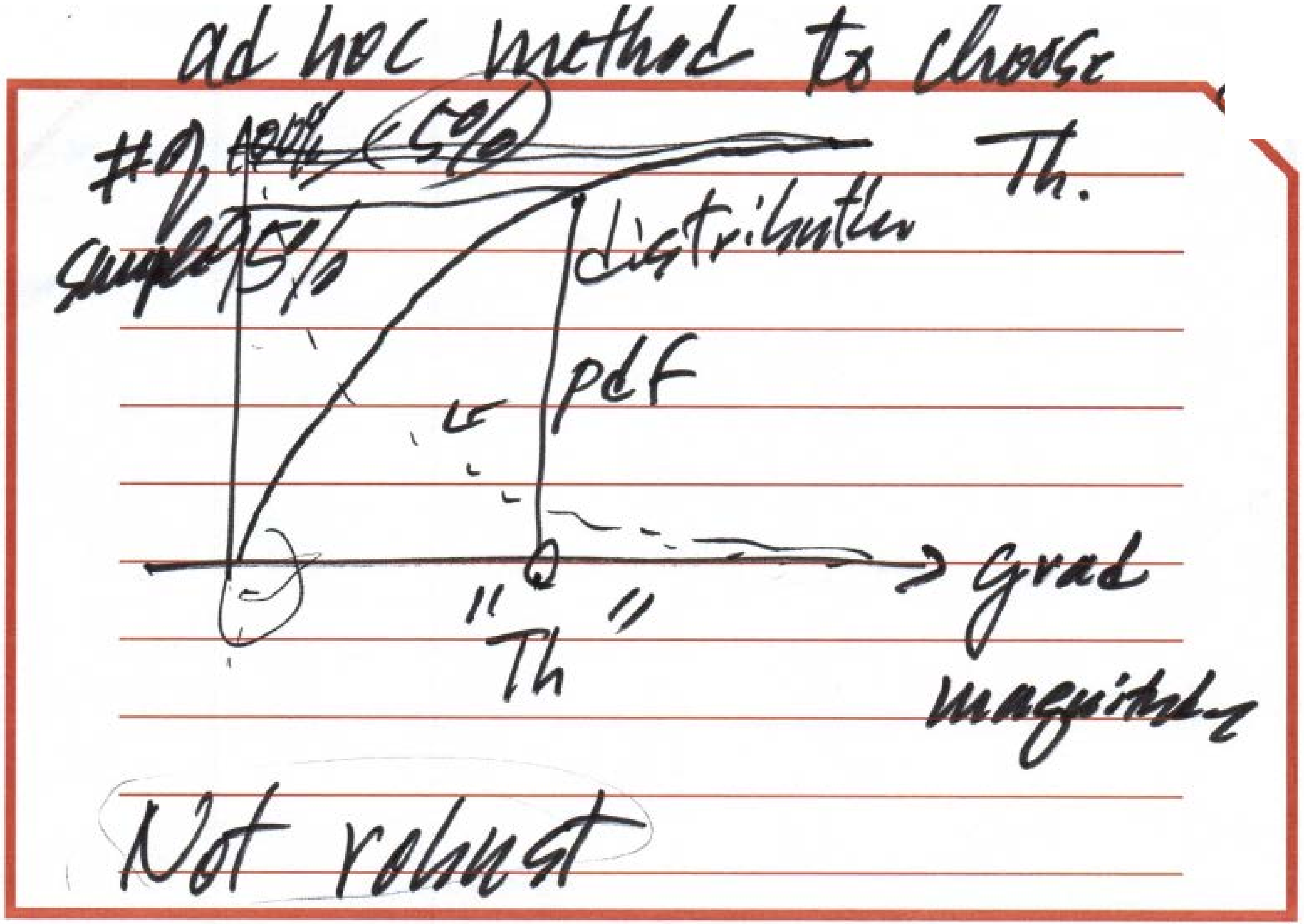
1st Order Derivative Edge Detector (10)

Edge Detection

if $\left\| \dfrac{dF}{dr} \right\|_{(j,k)} \geq Th.$

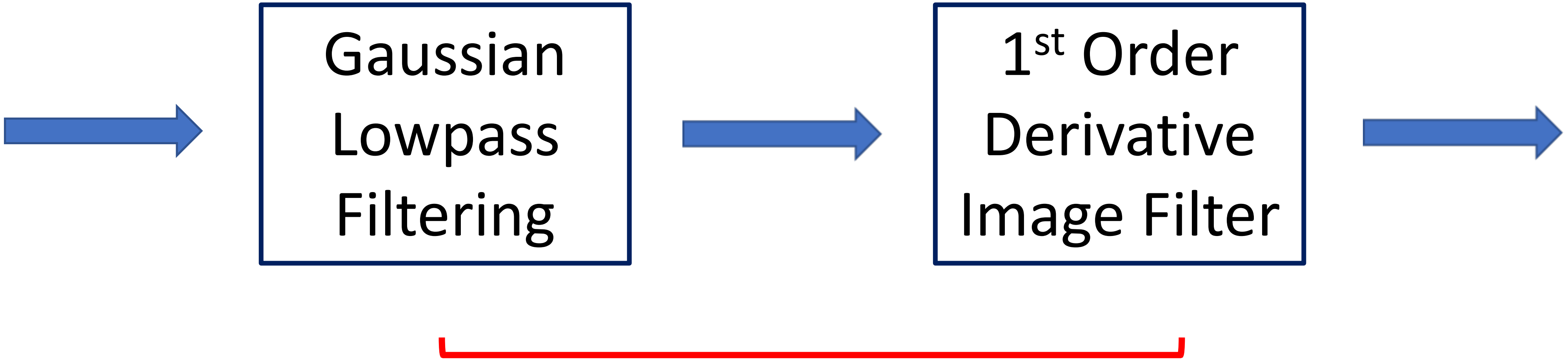
(j,k) is an edge point

1st Order Derivative Edge Detector (11)



1st Order Derivative Edge Detector (12)

- Differencing filters often amplify noise
- To suppress noise, we have



Compound Filter: DoG (Derivative of Gaussian)

1^{st} Order Derivative Edge Detector (13)

- Example of Compound Filters

$$\mathbf{H}_R = \frac{1}{34}\begin{bmatrix} 1 & 1 & 1 & 0 & -1 & -1 & -1 \\ 1 & 2 & 2 & 0 & -2 & -2 & -1 \\ 1 & 2 & 3 & 0 & -3 & -2 & -1 \\ 1 & 2 & 3 & 0 & -3 & -2 & -1 \\ 1 & 2 & 3 & 0 & -3 & -2 & -1 \\ 1 & 2 & 2 & 0 & -2 & -2 & -1 \\ 1 & 1 & 1 & 0 & -1 & -1 & -1 \end{bmatrix}$$

1^{st} Order Derivative Edge Detector (14)

Directional Edge Detector

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Gradient direction	Prewitt compass gradient	Kirsch	Robinson 3-level	Robinson 5-level
East $\mathbf{H}_1$	$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & -1 \\ 1 & 1 & -1 \end{bmatrix}$	$\begin{bmatrix} 5 & -3 & -3 \\ 5 & 0 & -3 \\ 5 & -3 & -3 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$
Northeast $\mathbf{H}_2$	$\begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -3 & -3 & -3 \\ 5 & 0 & -3 \\ 5 & 5 & -3 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$
North $\mathbf{H}_3$	$\begin{bmatrix} -1 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -3 & -3 & -3 \\ -3 & 0 & -3 \\ 5 & 5 & 5 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$
Northwest $\mathbf{H}_4$	$\begin{bmatrix} -1 & -1 & 1 \\ -1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -3 & -3 & -3 \\ -3 & 0 & 5 \\ -3 & 5 & 5 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$
West $\mathbf{H}_5$	$\begin{bmatrix} -1 & 1 & 1 \\ -1 & -2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -3 & -3 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & 5 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$
Southwest $\mathbf{H}_6$	$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$	$\begin{bmatrix} -3 & 5 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & -3 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$
South $\mathbf{H}_7$	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ -1 & -1 & -1 \end{bmatrix}$	$\begin{bmatrix} 5 & 5 & 5 \\ -3 & 0 & -3 \\ -3 & -3 & -3 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$
Southeast $\mathbf{H}_8$	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & -1 & -1 \end{bmatrix}$	$\begin{bmatrix} 5 & 5 & -3 \\ 5 & 0 & -3 \\ -3 & -3 & -3 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}$	$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -2 \end{bmatrix}$
Scale factor	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{1}{4}$

FIGURE 15.2-9. Template gradient 3×3 impulse response arrays.

1^{st} Order Derivative Edge Detector (15)

Directional Edge Detector

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$$\frac{1}{1000}\begin{bmatrix} 100 & 100 & 0 & -100 & -100 \\ 100 & 100 & 0 & -100 & -100 \\ 100 & 100 & 0 & -100 & -100 \\ 100 & 100 & 0 & -100 & -100 \\ 100 & 100 & 0 & -100 & -100 \end{bmatrix}$$

0 degrees

$$\frac{1}{1102}\begin{bmatrix} 100 & -32 & -100 & -100 & -100 \\ 100 & 78 & -92 & -100 & -100 \\ 100 & 100 & 0 & -100 & -100 \\ 100 & 100 & 92 & -78 & -100 \\ 100 & 100 & 100 & 32 & -100 \end{bmatrix}$$

30 degrees

$$\frac{1}{1102}\begin{bmatrix} -100 & -100 & -100 & -100 & -100 \\ 32 & -78 & -100 & -100 & -100 \\ 100 & 92 & 0 & -92 & -100 \\ 100 & 100 & 100 & 78 & -32 \\ 100 & 100 & 100 & 100 & 100 \end{bmatrix}$$

60 degrees

$$\frac{1}{1000}\begin{bmatrix} -100 & -100 & -100 & -100 & -100 \\ -100 & -100 & -100 & -100 & -100 \\ 0 & 0 & 0 & 0 & 0 \\ 100 & 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 & 100 \end{bmatrix}$$

90 degrees

$$\frac{1}{1102}\begin{bmatrix} -100 & -100 & -100 & -100 & -100 \\ -100 & -100 & -100 & -78 & 32 \\ -100 & -92 & 0 & 92 & 100 \\ -32 & 78 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 & 100 \end{bmatrix}$$

120 degrees

$$\frac{1}{1102}\begin{bmatrix} -100 & -100 & -100 & -32 & 100 \\ -100 & -100 & -92 & 78 & 100 \\ -100 & -100 & 0 & 100 & 100 \\ -100 & -78 & 92 & 100 & 100 \\ -100 & 32 & 100 & 100 & 100 \end{bmatrix}$$

150 degrees

$$\frac{1}{1000}\begin{bmatrix} -100 & -100 & 0 & 100 & 100 \\ -100 & -100 & 0 & 100 & 100 \\ -100 & -100 & 0 & 100 & 100 \\ -100 & -100 & 0 & 100 & 100 \\ -100 & -100 & 0 & 100 & 100 \end{bmatrix}$$

180 degrees

$$\frac{1}{1102}\begin{bmatrix} -100 & 32 & 100 & 100 & 100 \\ -100 & -78 & 92 & 100 & 100 \\ -100 & -100 & 0 & 100 & 100 \\ -100 & -100 & -92 & 78 & 100 \\ -100 & -100 & -100 & -32 & 100 \end{bmatrix}$$

210 degrees

$$\frac{1}{1102}\begin{bmatrix} 100 & 100 & 100 & 100 & 100 \\ -32 & 78 & 100 & 100 & 100 \\ -100 & -92 & 0 & 92 & 100 \\ -100 & -100 & -100 & -78 & 32 \\ -100 & -100 & -100 & -100 & -100 \end{bmatrix}$$

240 degrees

$$\frac{1}{1000}\begin{bmatrix} 100 & 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 & 100 \\ 0 & 0 & 0 & 0 & 0 \\ -100 & -100 & -100 & -100 & -100 \\ -100 & -100 & -100 & -100 & -100 \end{bmatrix}$$

270 degrees

$$\frac{1}{1102}\begin{bmatrix} 100 & 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 78 & -32 \\ 100 & 92 & 0 & -92 & -100 \\ 32 & -78 & -100 & -100 & -100 \\ -100 & -100 & -100 & -100 & -100 \end{bmatrix}$$

300 degrees

$$\frac{1}{1102}\begin{bmatrix} 100 & 100 & 100 & 32 & -100 \\ 100 & 100 & 92 & -78 & -100 \\ 100 & 100 & 0 & -100 & -100 \\ 100 & 78 & -92 & -100 & -100 \\ 100 & -32 & -100 & -100 & -100 \end{bmatrix}$$

330 degrees

FIGURE 15.2-11. Nevatia–Babu template gradient impulse response arrays.

2nd Order Derivative Edge Detector (1)

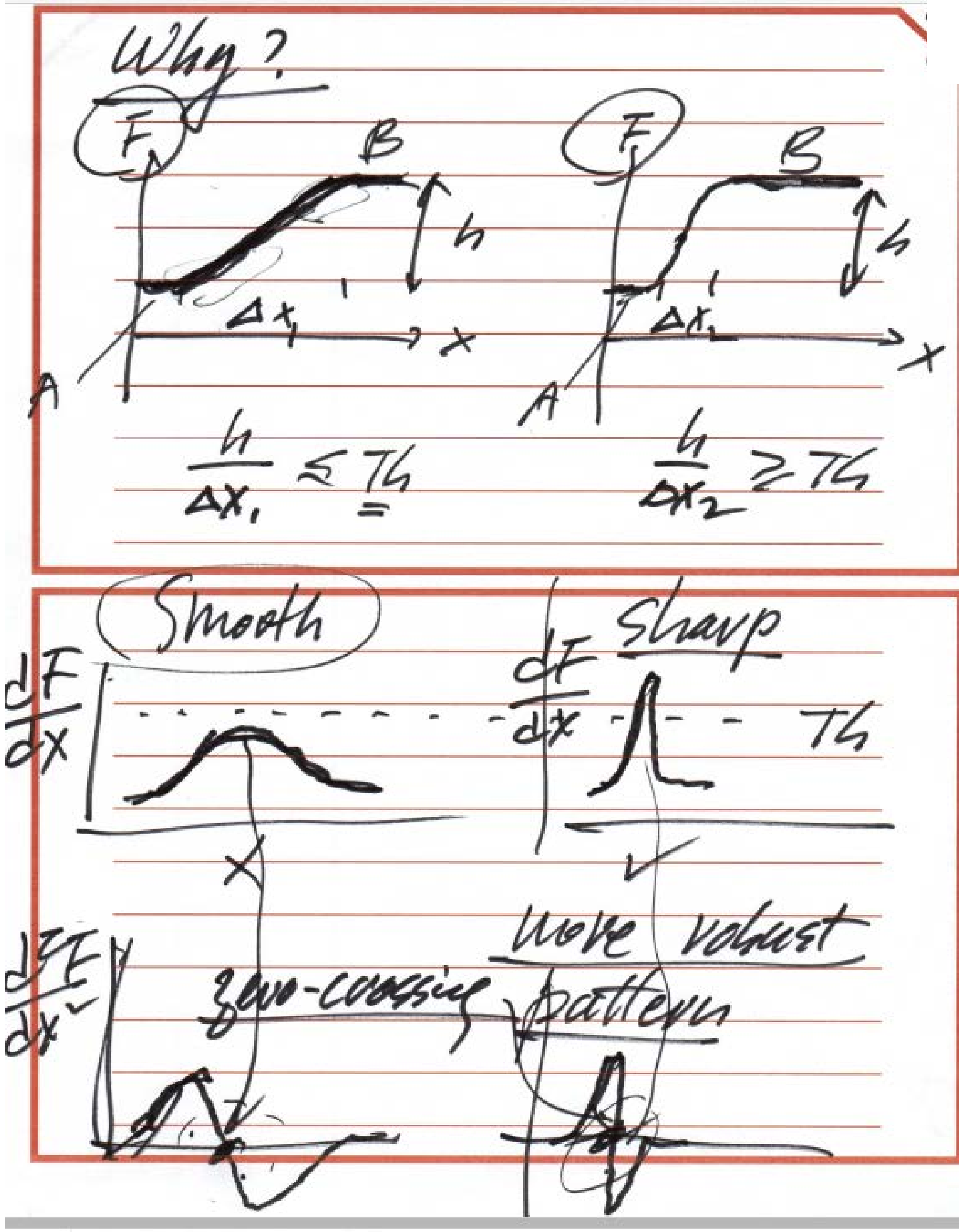
David Marr

2nd-order derivatives.

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \leftarrow \text{Laplacian operator}$$

2nd Order Derivative Edge Detector (2)



2^{nd} Order Derivative Edge Detector (3)

discrete Laplacian

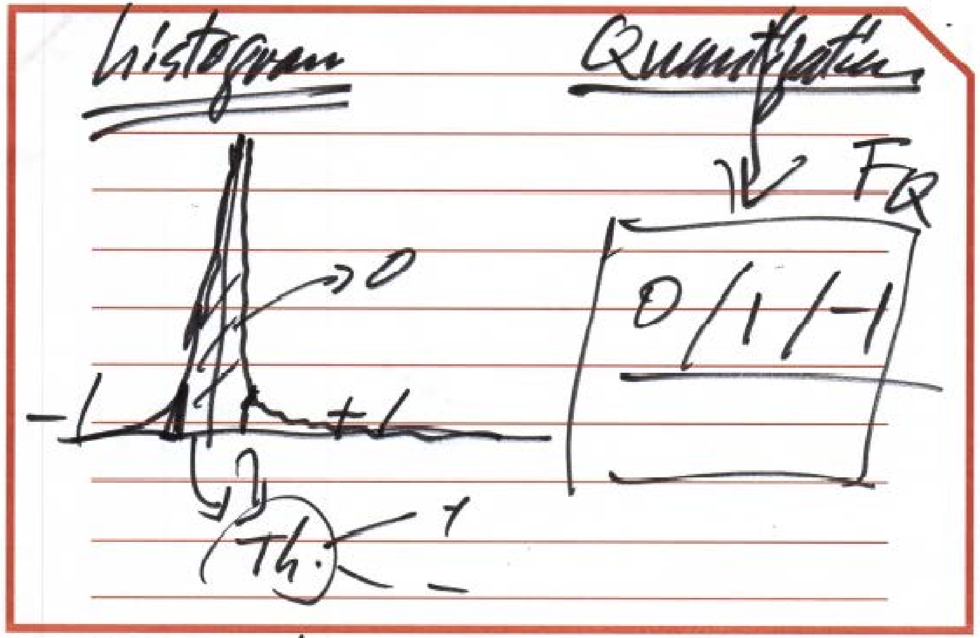
$$\frac{1}{4}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \Delta_4$$

$$-\frac{\partial^2 F}{\partial x^2} \approx \begin{bmatrix} -1 & 2 & -1 \end{bmatrix}$$

$$+$$

$$-\frac{\partial^2 F}{\partial y^2} \approx \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

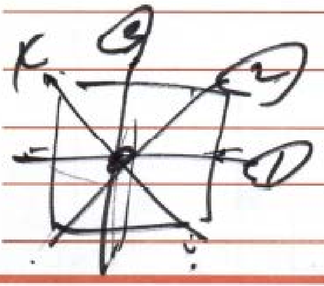
2nd Order Derivative Edge Detector (4)



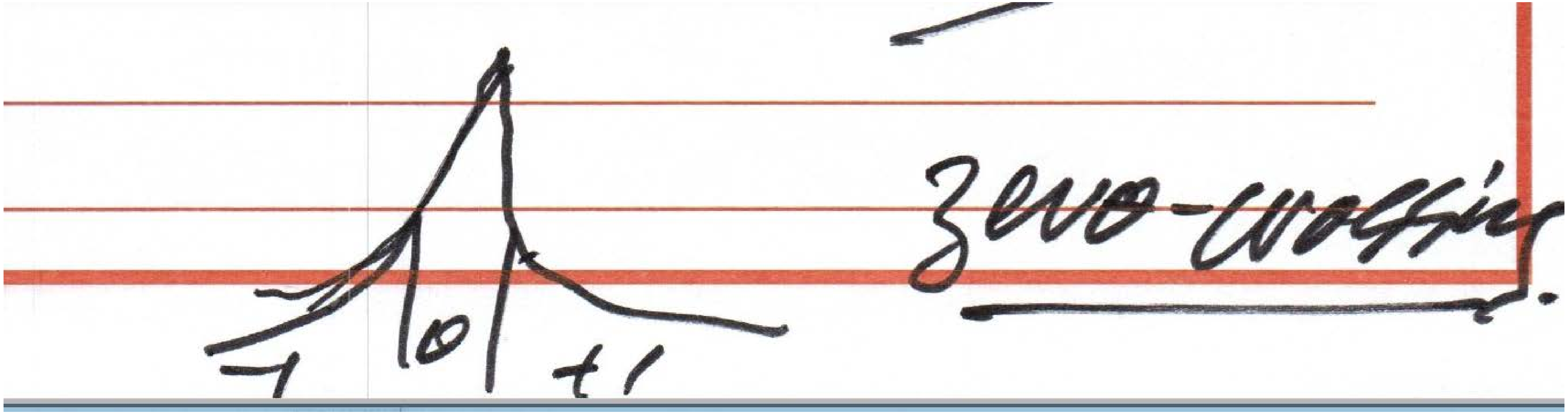
$|\Delta F| < Th \rightarrow \Delta F_Q = 0$

$\Delta F \geq Th \rightarrow \Delta F_Q = 1$

$\Delta F < -Th \rightarrow \Delta F_Q = -1$

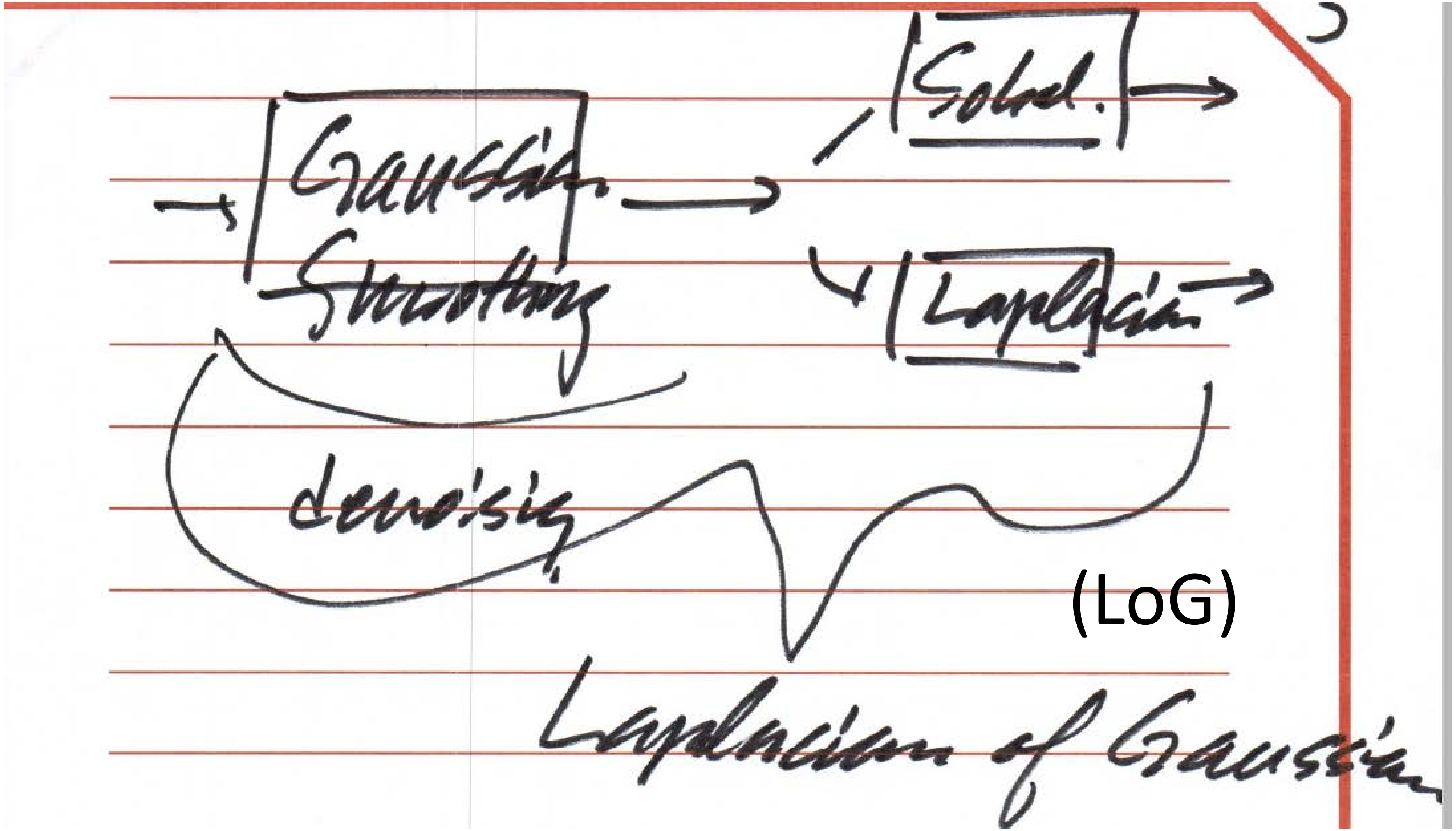


2nd Order Derivative Edge Detector (6)



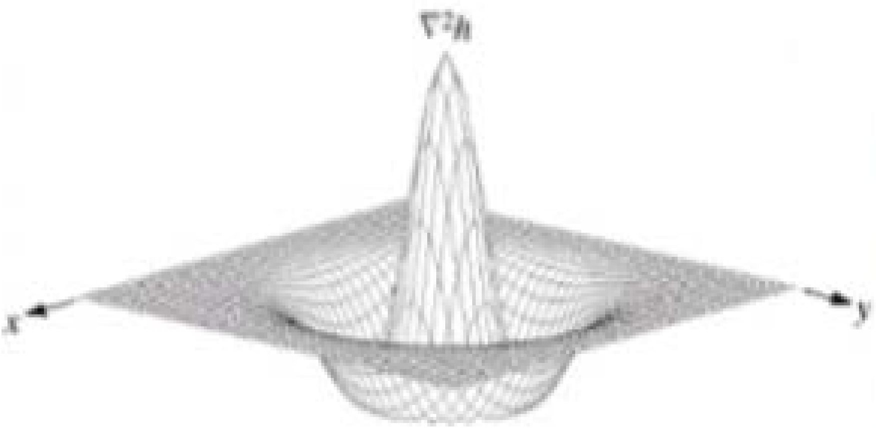
2nd Order Derivative Edge Detector (7)

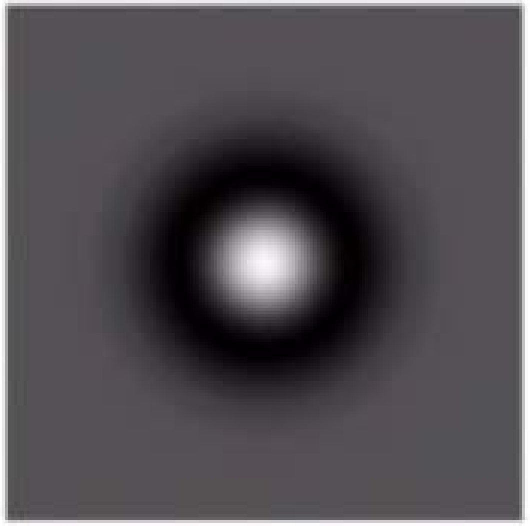
- Denoising followed by edge detection



2nd Order Derivative Edge Detector (8)

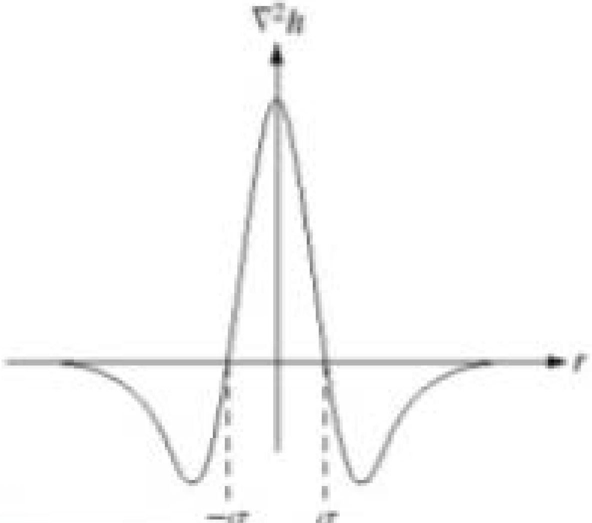
- Laplacian of Gaussian (LoG) filters
  - also known as (a.k.a.) the Mexican hat filter





Original Laplacian

0	−1	0
−1	4	−1
0	−1	0



0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0