Announcements

- Project 0: Python Tutorial
  - JOJ is accessible
  - Due next Monday
  - Don’t wait for last moment!

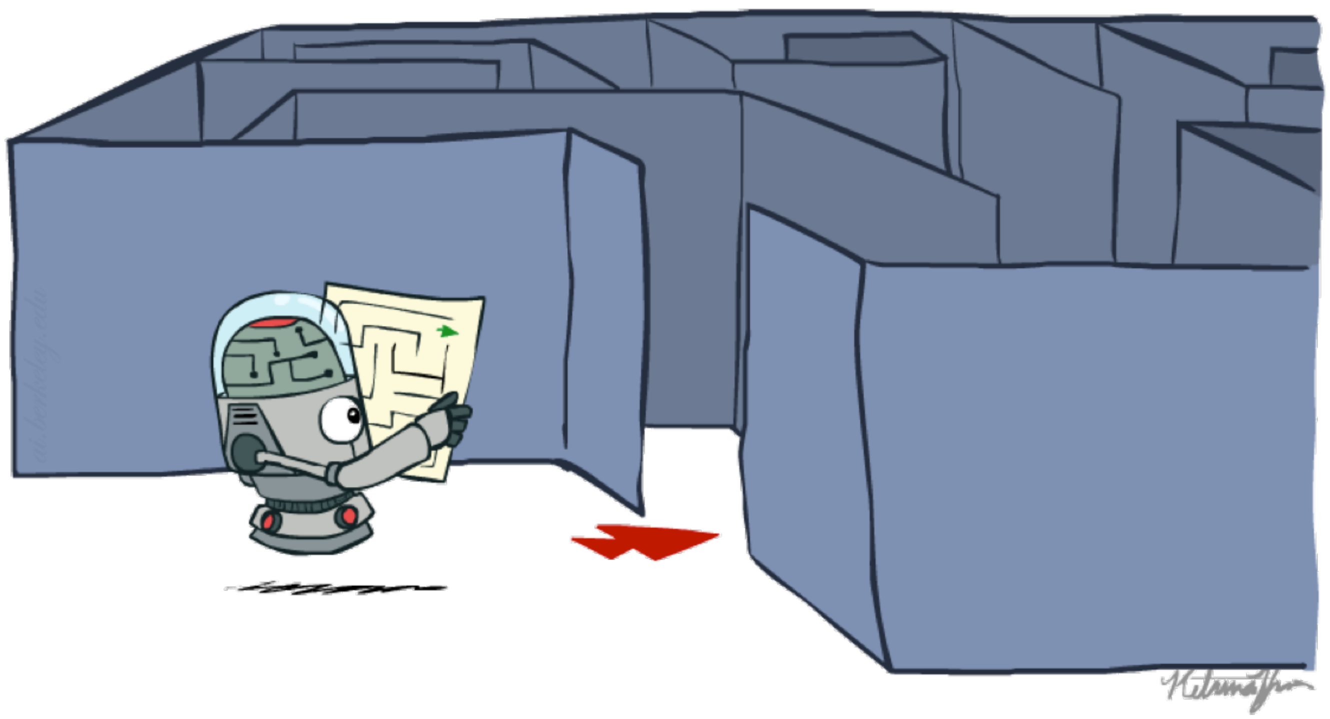
- Project 1 will also be released on next Monday

- HW 1 released today

- Survey for deciding OHs and Recitation times
  - Respond by the end of the week
  - OH and Recitation start next week

Ve492: Introduction to Artificial Intelligence

Search



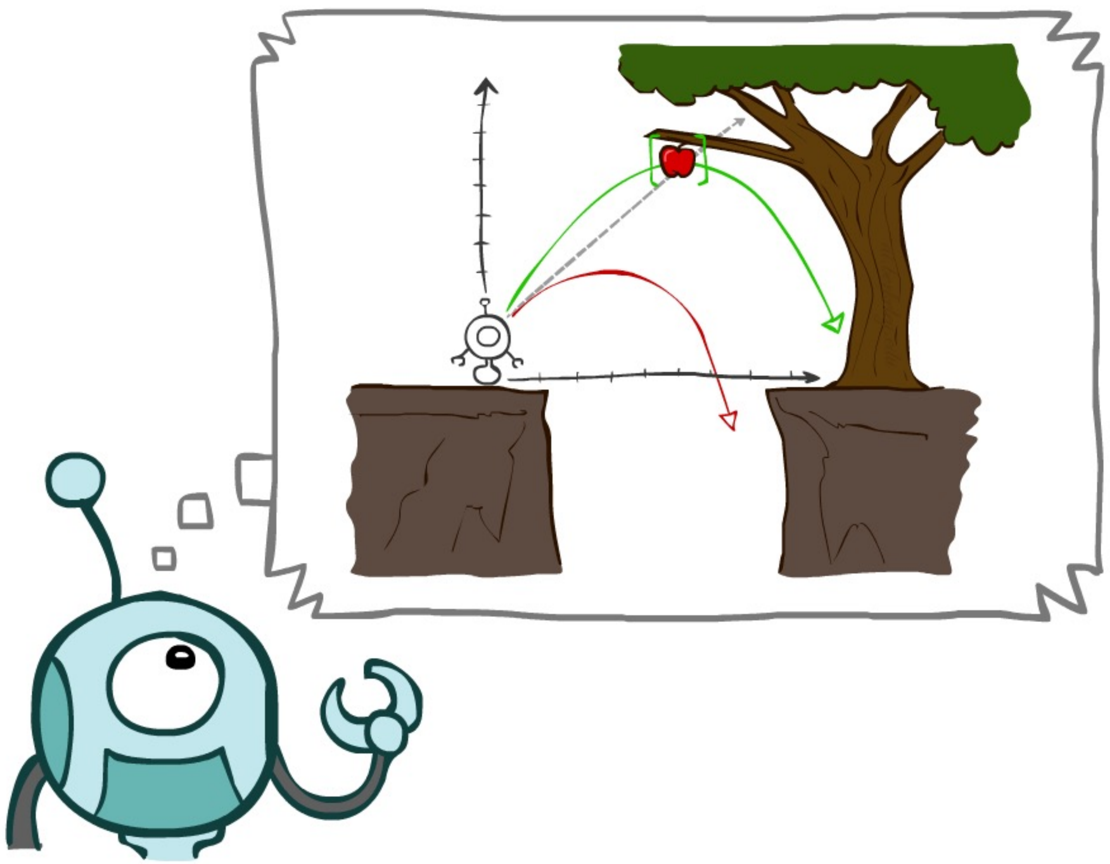
Paul Weng

UM-SJTU Joint Institute

Slides adapted from http://ai.berkeley.edu, AIMA, UM, CMU

Outline

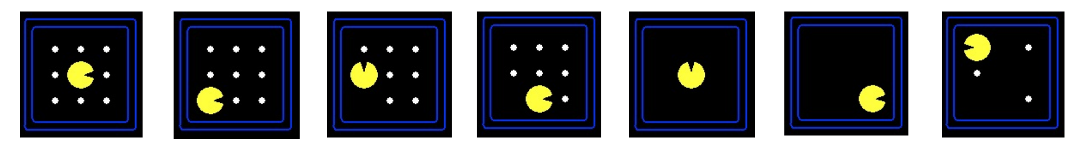
- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search



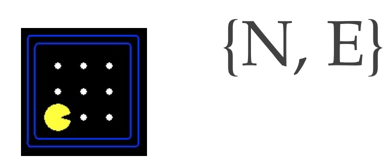
Search Problems

Search Problems

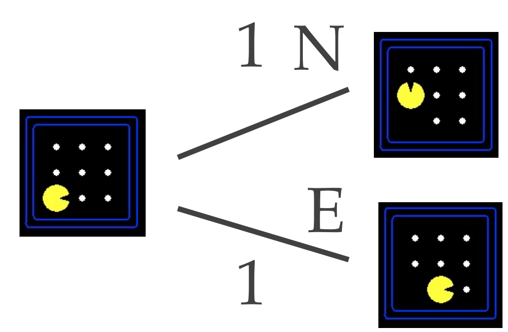
- A search problem consists of:
  - A state space



  - For each state s, an action set *Actions(s)* of allowable actions

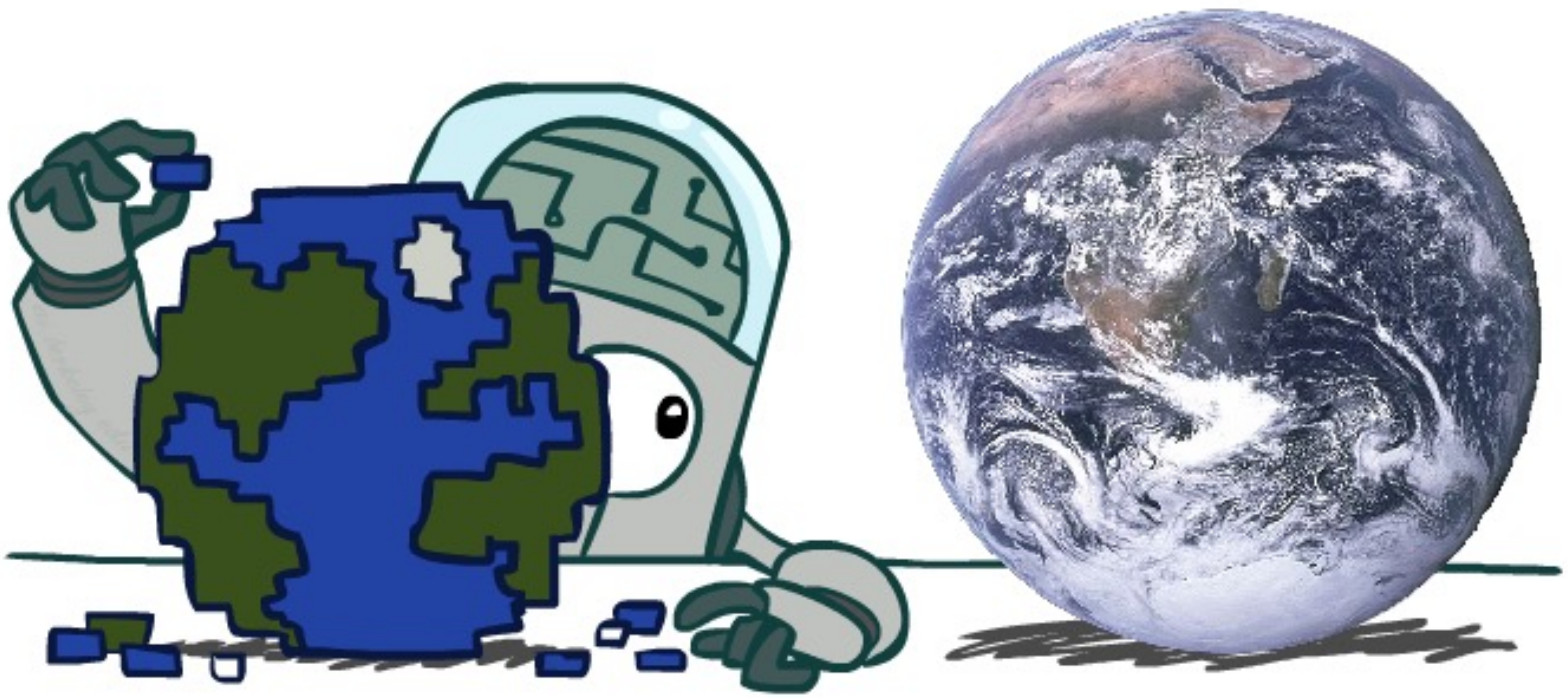


  - A transition model *Succ(s,a)*
  - A step cost function *c(s,a,s')*



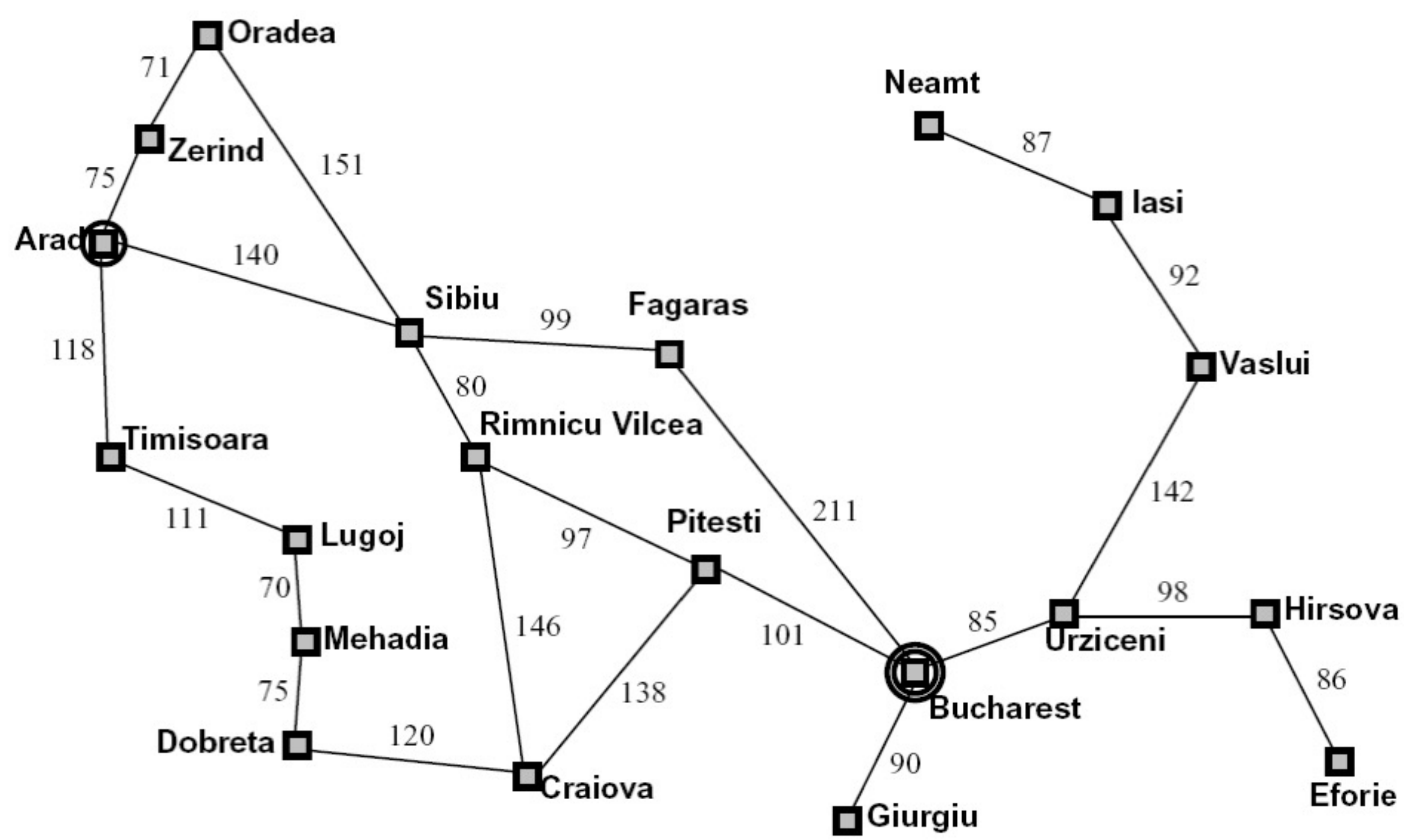
  - A start state and a goal test
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

Search Problems Are Models



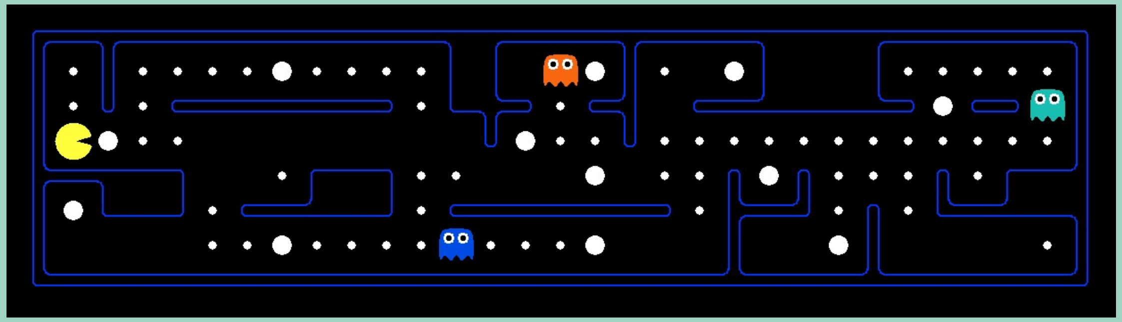
Example: Traveling in Romania

- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?



What's in a State Space?

The world state includes every last detail of the environment



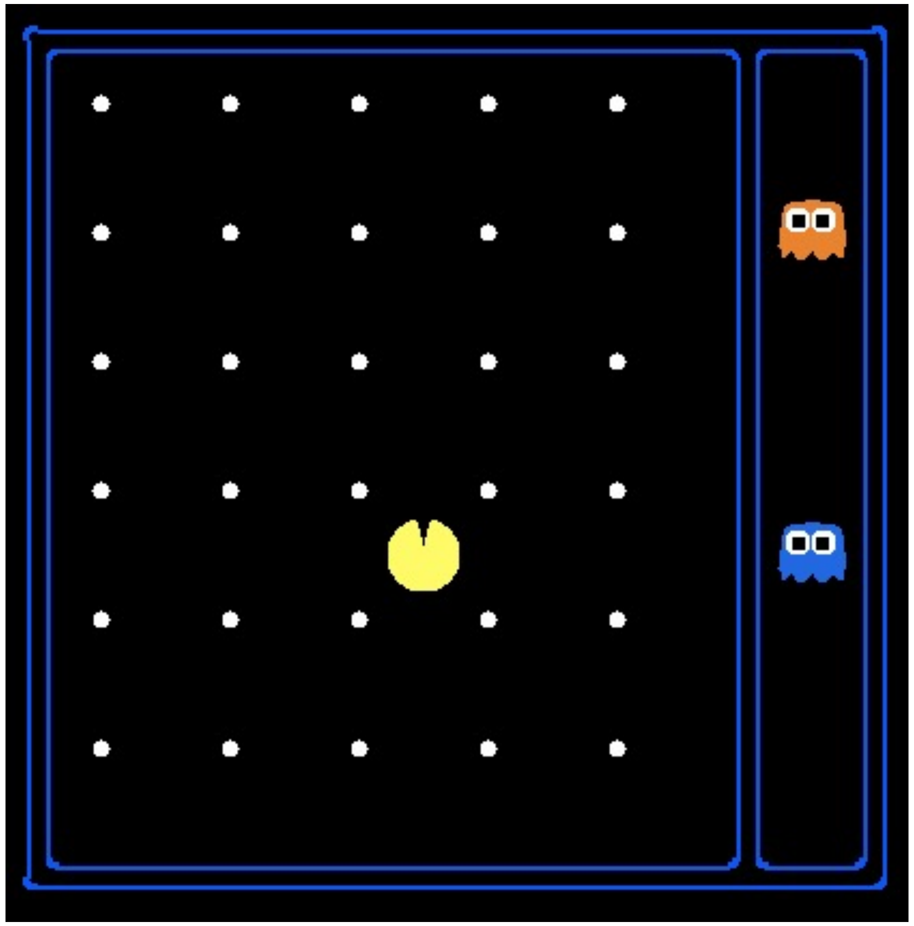
A search state keeps only the details needed for planning (abstraction)

- **Problem: Navigation**
  - **States**: (x,y) location
  - **Actions**: NSEW
  - **Successor**: update location only
  - **Goal test**: is (x,y)=goal position

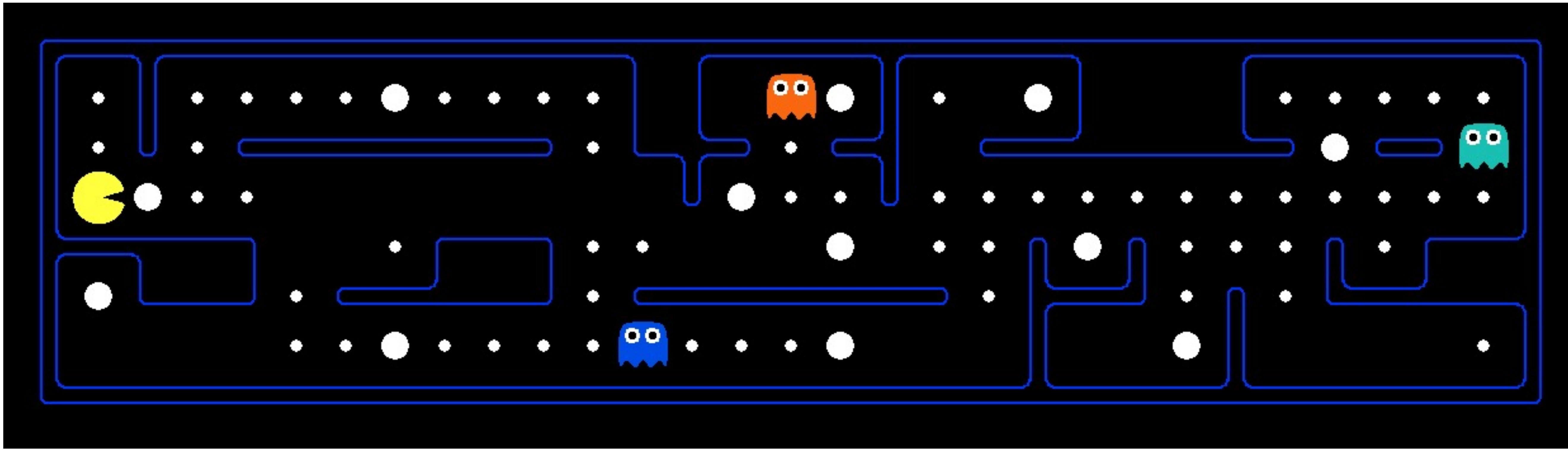
- **Problem: Eat-All-Dots**
  - **States**: {(x,y), dot booleans}
  - **Actions**: NSEW
  - **Successor**: update location and possibly a dot boolean
  - **Goal test**: dots all false

State Space Sizes?

- World state:
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW
- How many
  - World states?
  - $120 \times 2^{30} \times 12^2 \times 4$
  - States for navigation?
  - 120
  - States for eat-all-dots?
  - 120×2^{30}

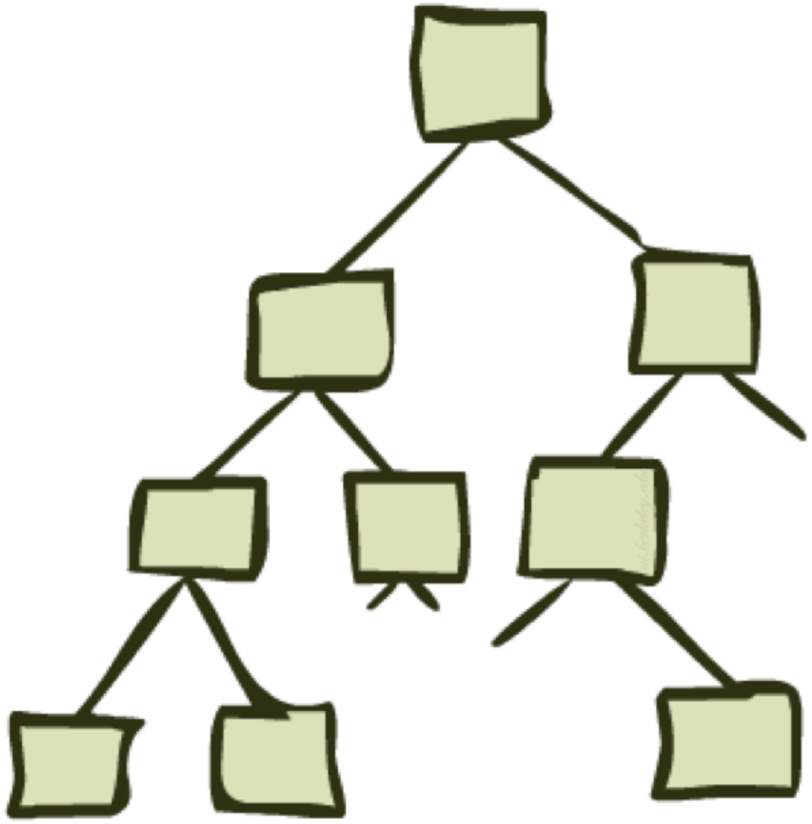


Example: Safe Passage



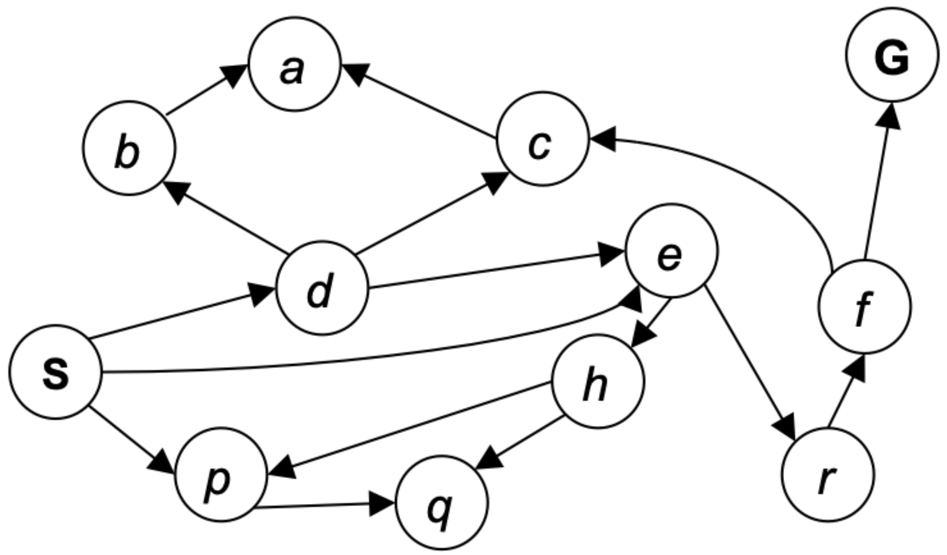
- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?
    - (agent position, dot booleans, power pellet booleans, remaining scared time)

State Space Graphs and Search Trees



State Space Graphs

- State space graph = mathematical representation of a search problem
  - Nodes = (abstracted) world states
  - Arcs represent successors (action results)
  - Possibly with arc costs
  - Goal test = set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea



Tiny search graph for a tiny search problem

State Space Graphs

- State space graph = mathematical representation of a search problem
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  - Possibly costs of arcs
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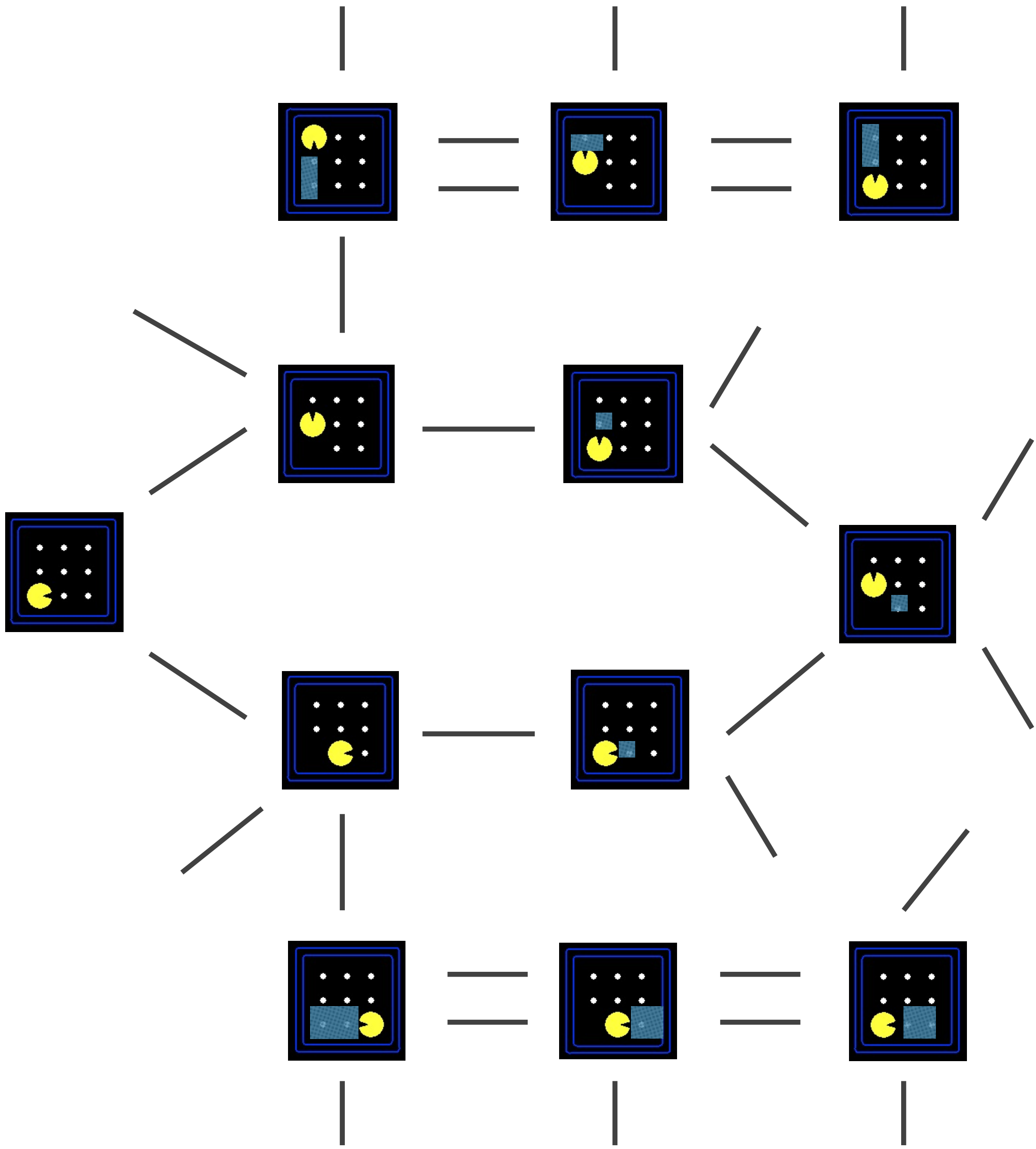
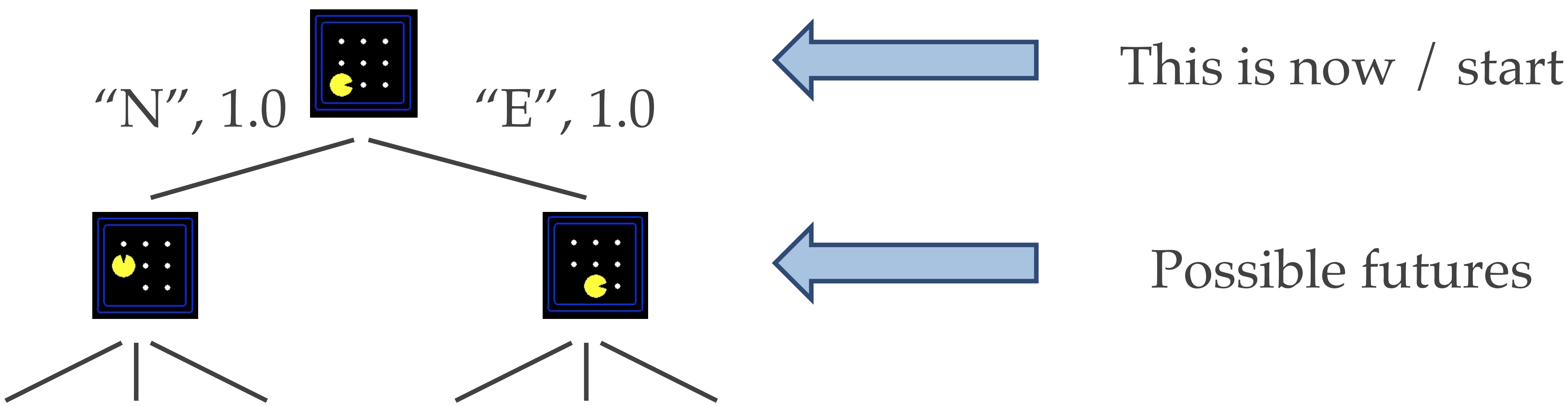
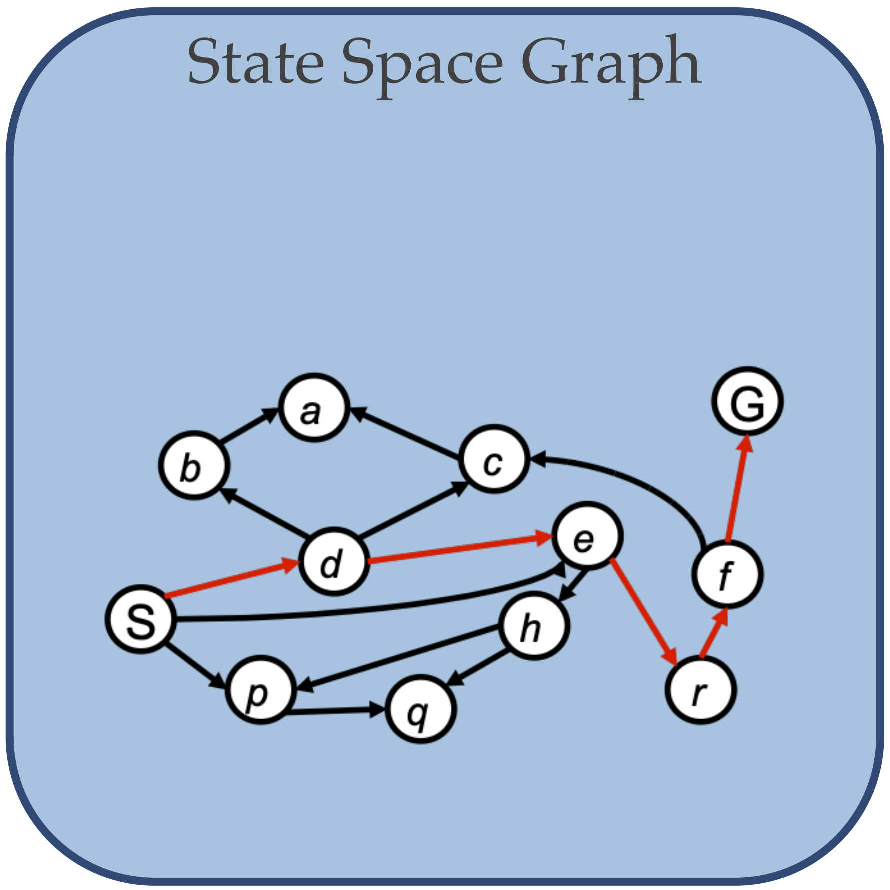


Search Trees



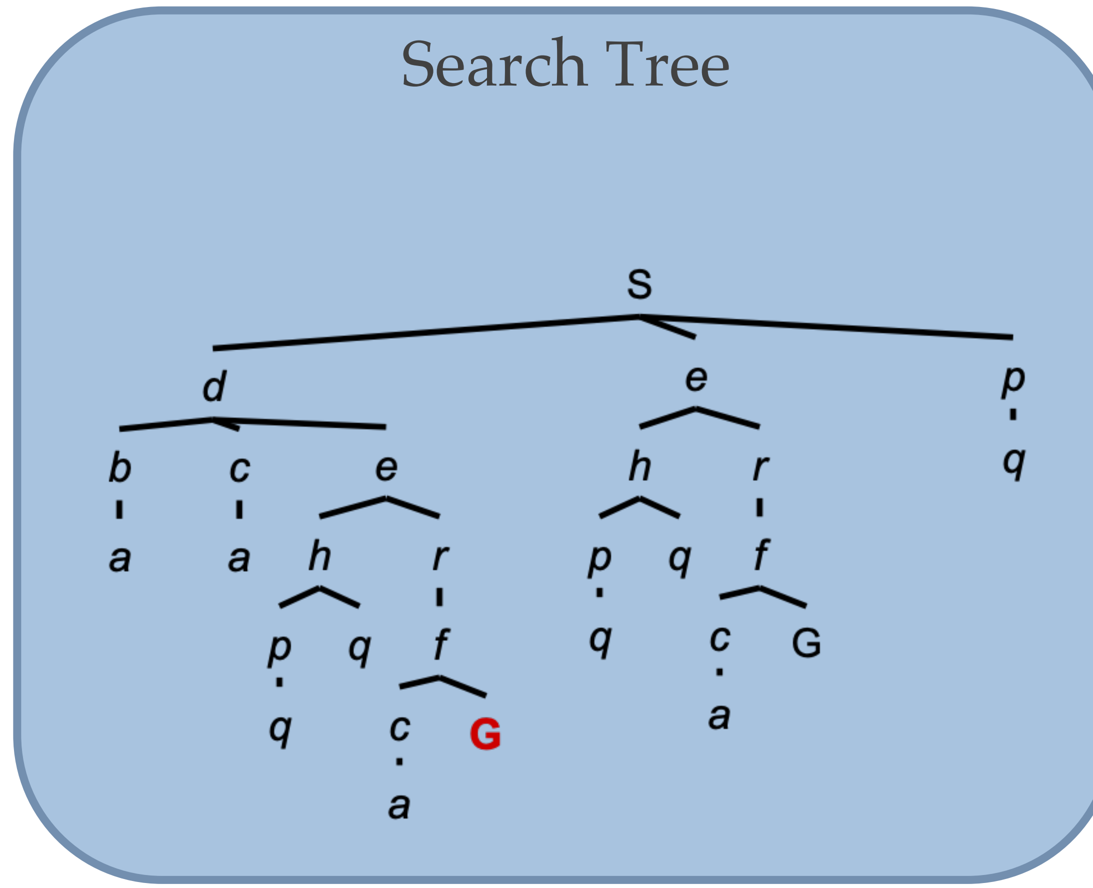
- A search tree:
  - A "what if" tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree

State Space Graphs vs. Search Trees



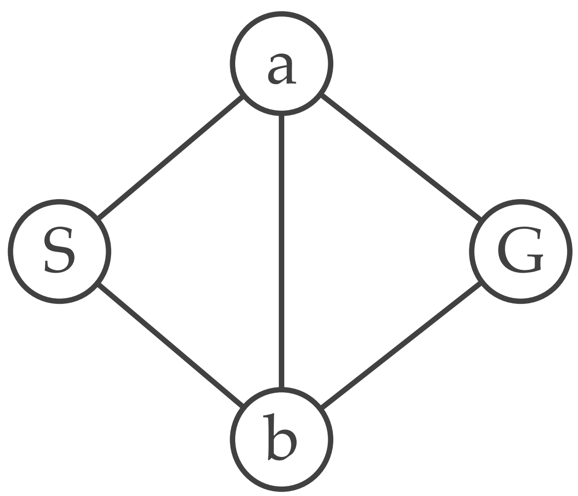
Each NODE in in the search tree is an entire PATH in the state space graph.

We construct both on demand – and we construct as little as possible.

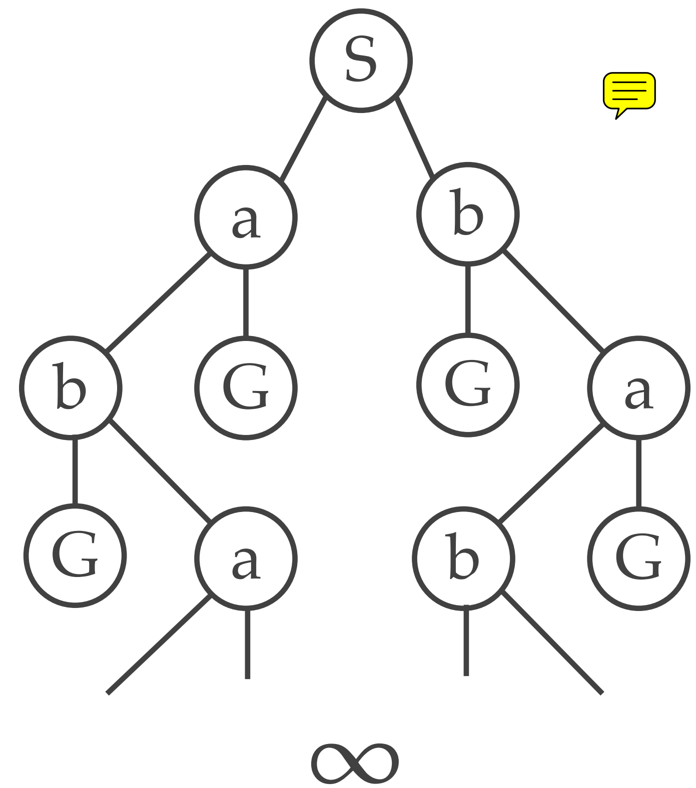


Quiz: State Space Graphs vs. Search Trees

Consider this 4-state graph:

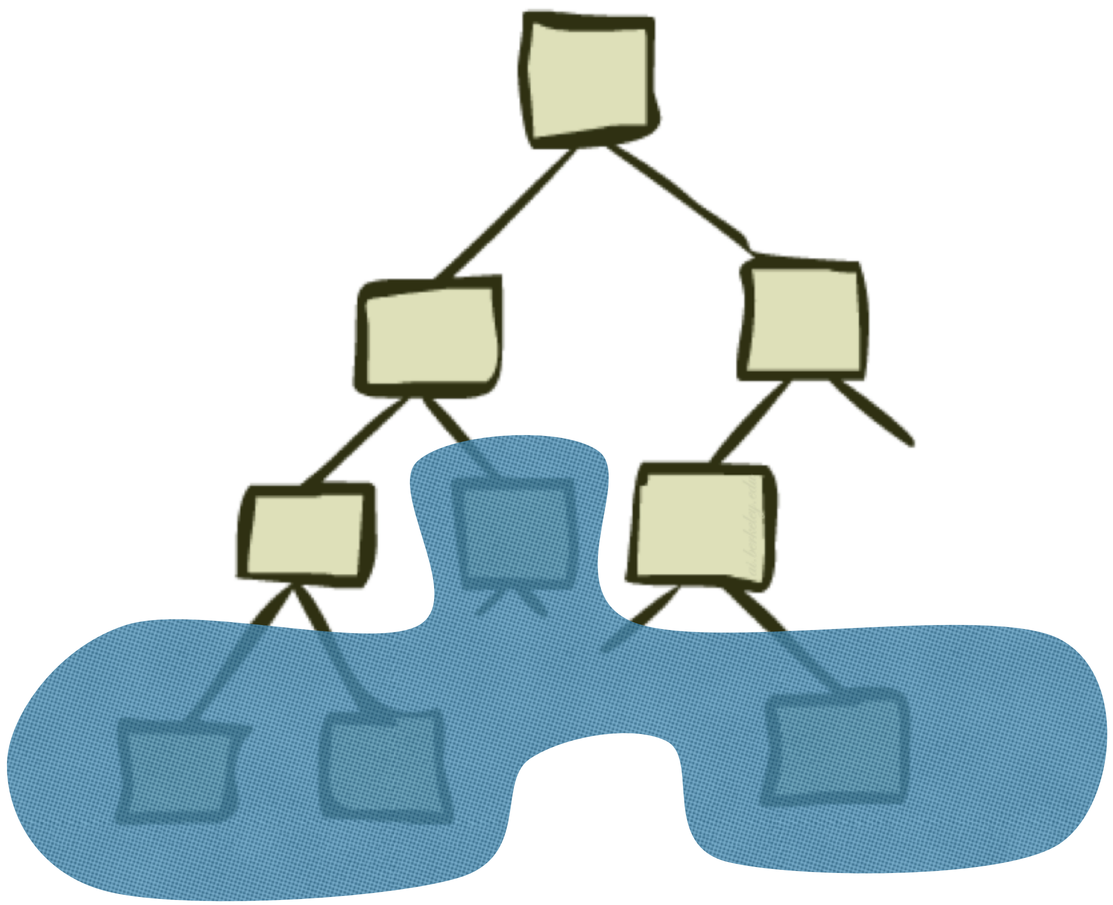


How big is its search tree (from S)?



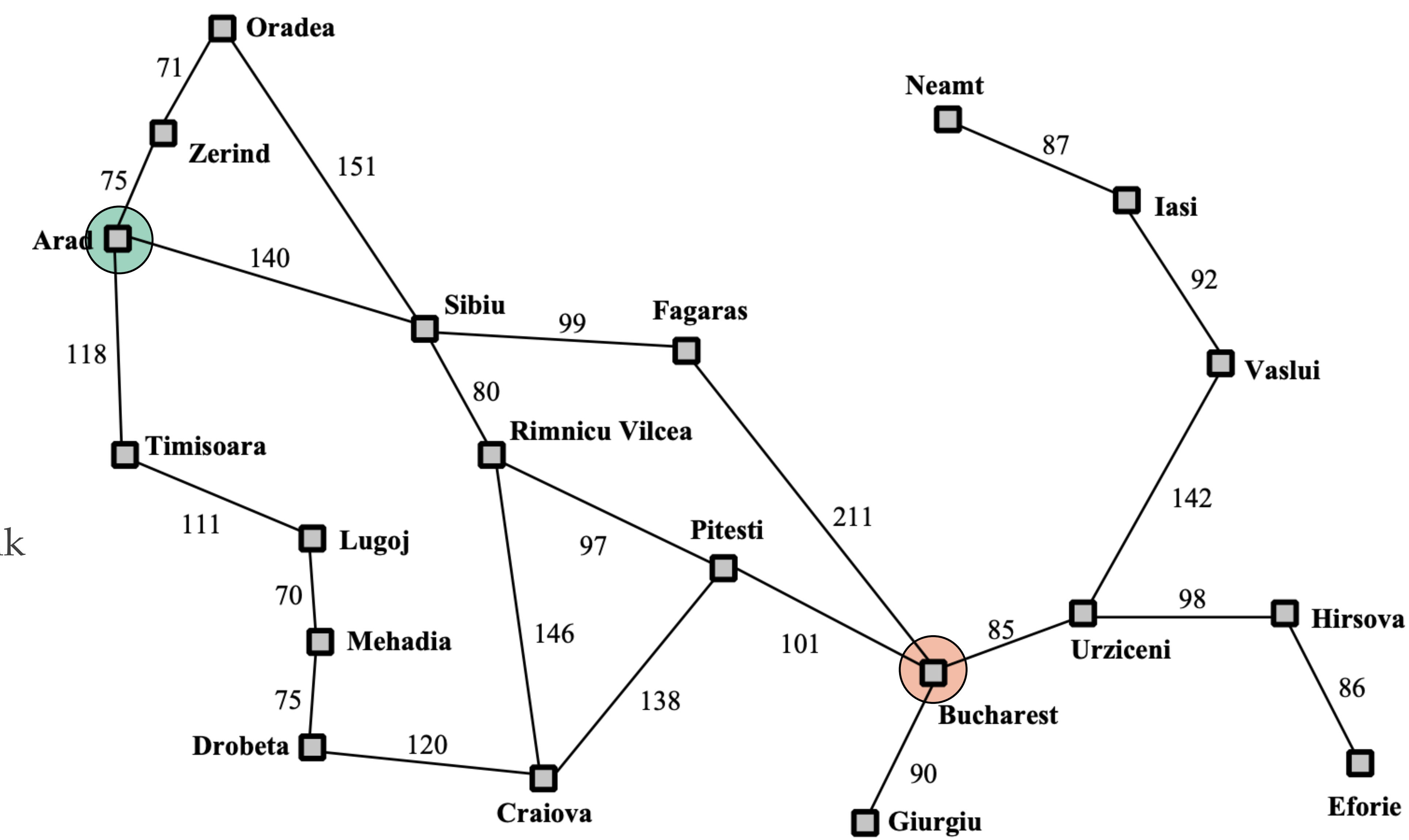
Important: Lots of repeated structure in the search tree!

Tree Search

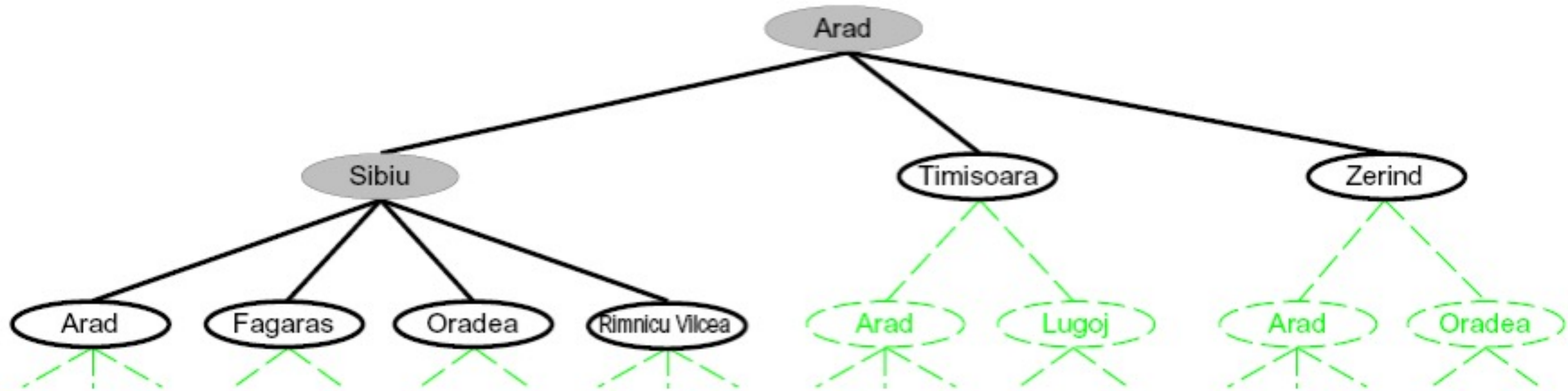


Search Example: Traveling in Romania

- State space:
  - Cities
- Actions:
  - Go to adjacent city
- Successor function:
  - Succ(A, Go(B)) = B
- Step cost:
  - Distance along road link
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?



Searching with a Search Tree



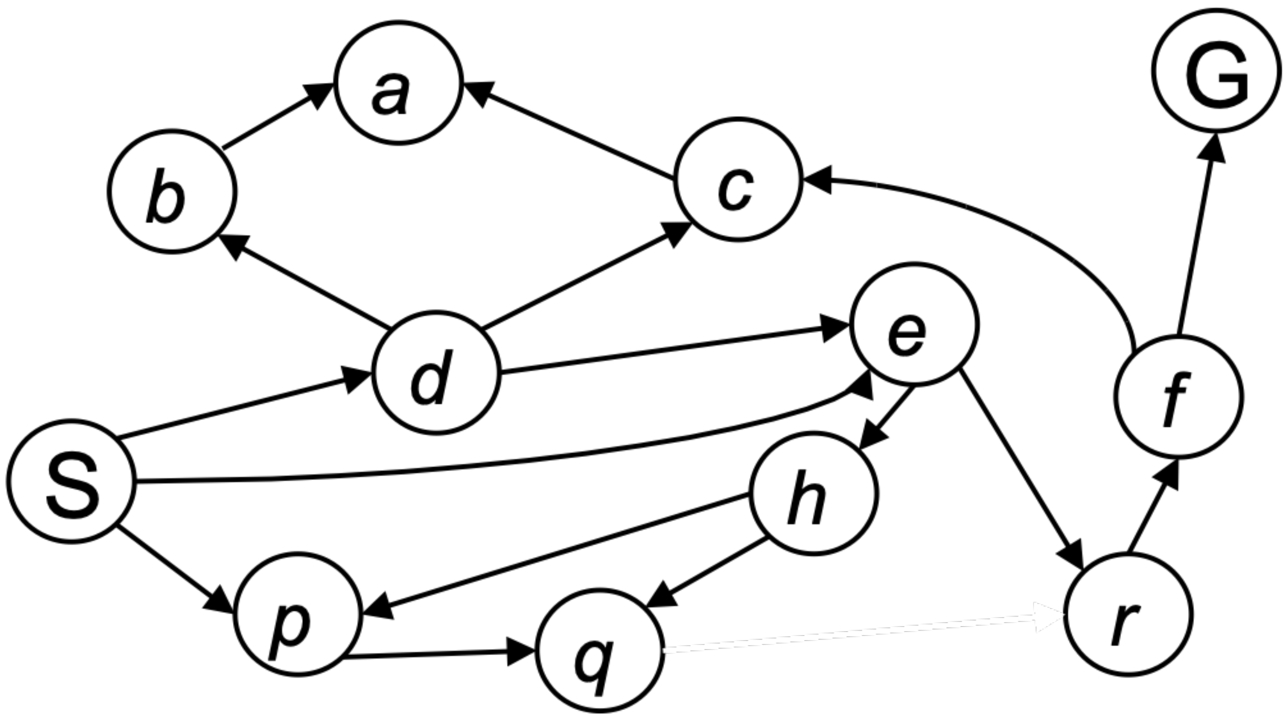
- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a fringe of partial plans under consideration
  - Try to expand as few tree nodes as possible

General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

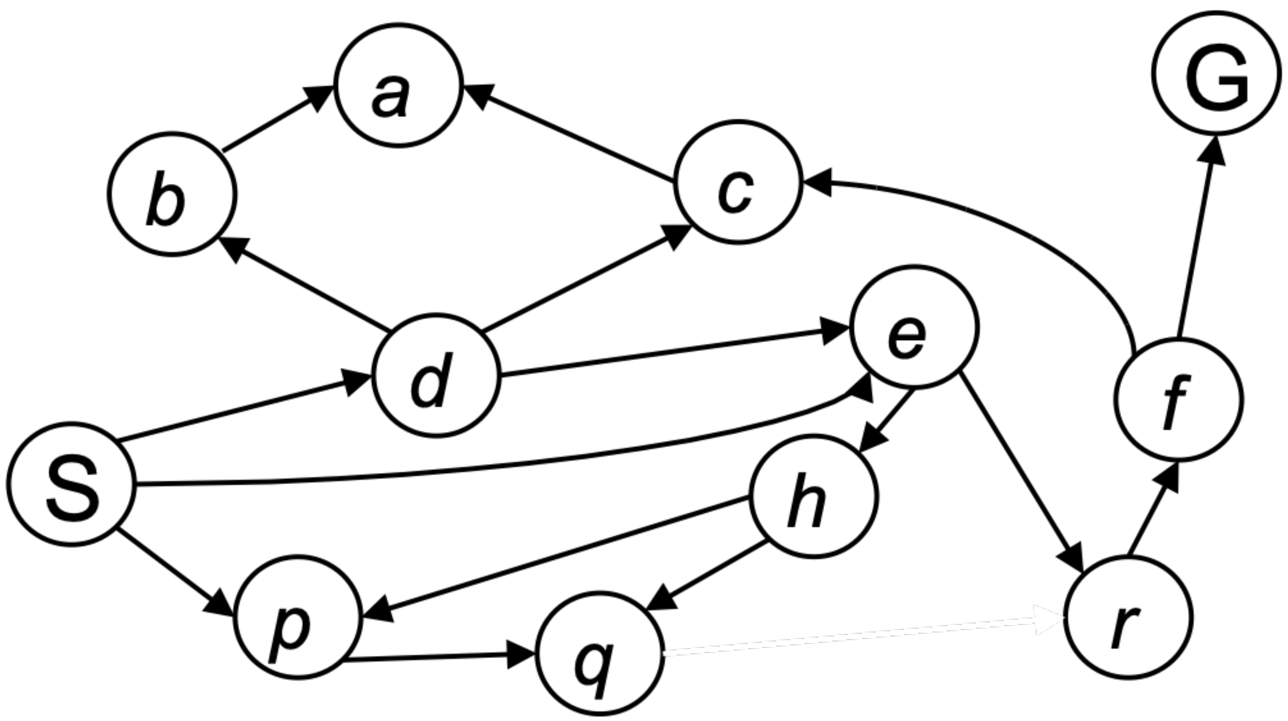
Example: Tree Search and Fringe



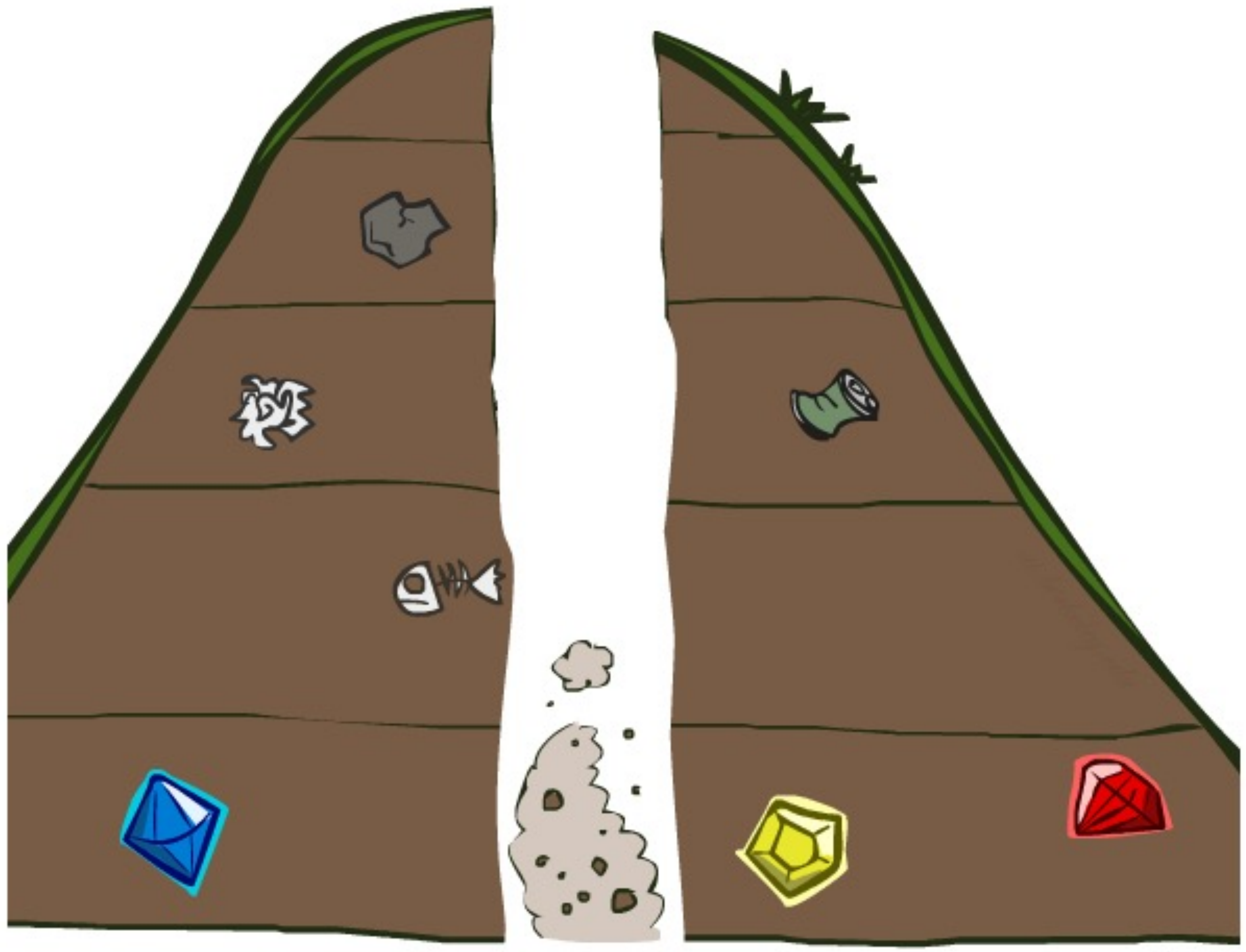
Depth-First Search

Example: Depth-First Search

Strategy: expand a deepest node first
Implementation: Fringe is a LIFO stack

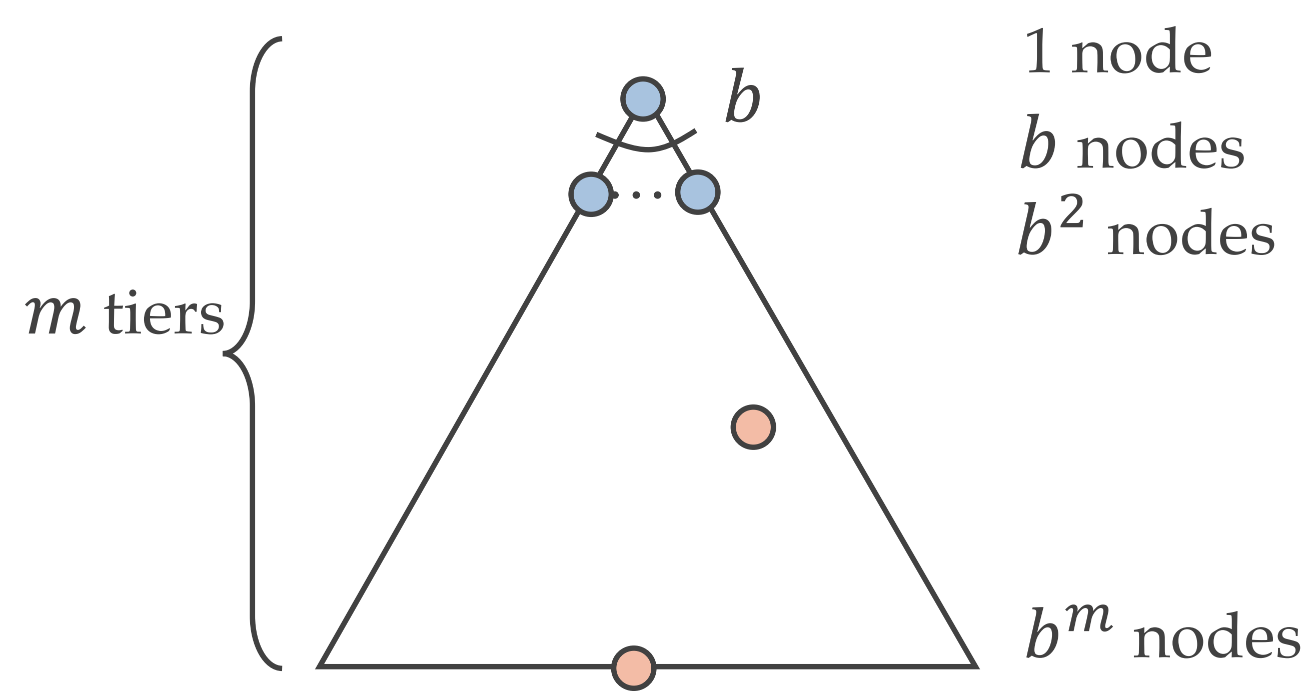


Search Algorithm Properties



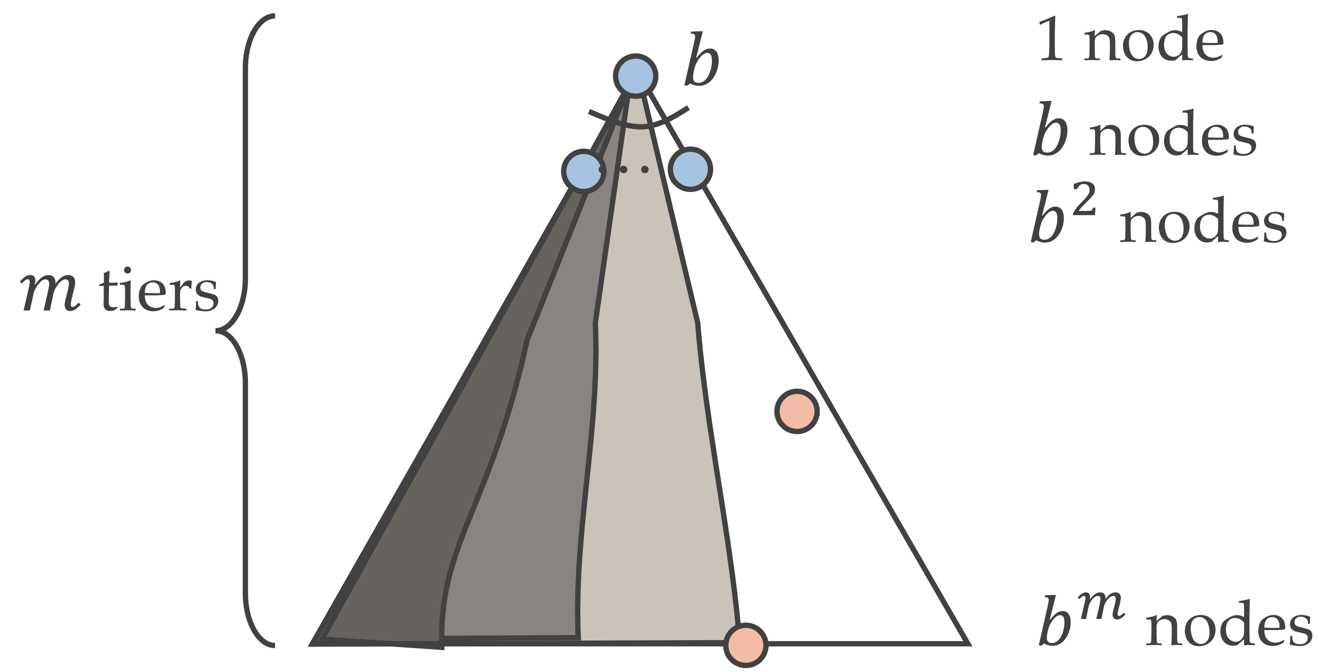
Search Algorithm Properties

- **Complete:** Guaranteed to find a solution if one exists?
- **Optimal:** Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths



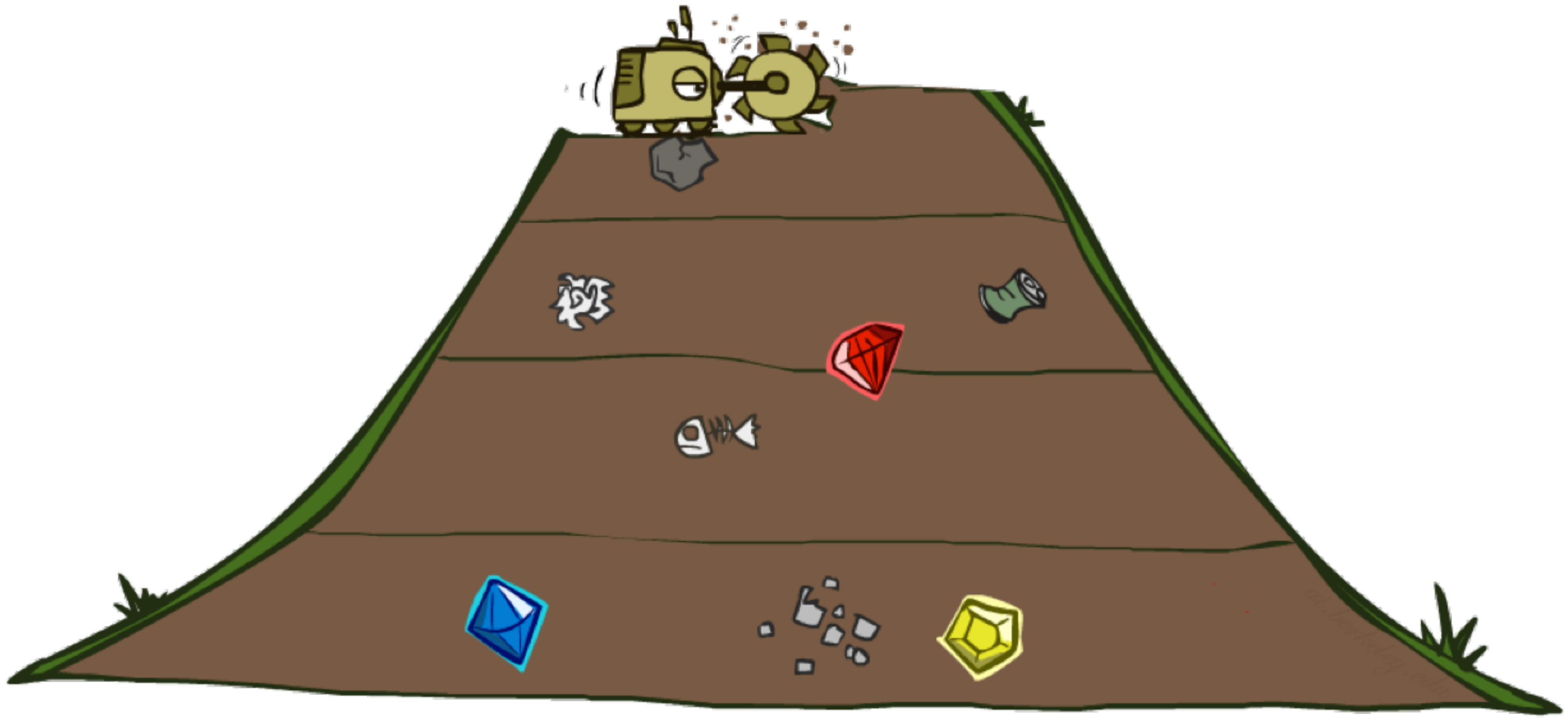
- Number of nodes in entire tree?
  - $1 + b + b^2 + \cdots + b^m = O(b^m)$

Depth-First Search (DFS) Properties



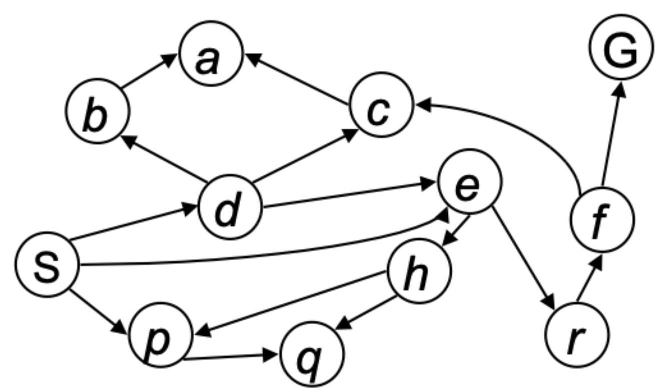
- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$
- How much space does the fringe take?
  - Only has siblings on path to root, so $O(bm)$
- Is it complete?
  - m could be infinite, so only if we prevent cycles (more later)
- Is it optimal?
  - No, it finds the "leftmost" solution, regardless of depth or cost

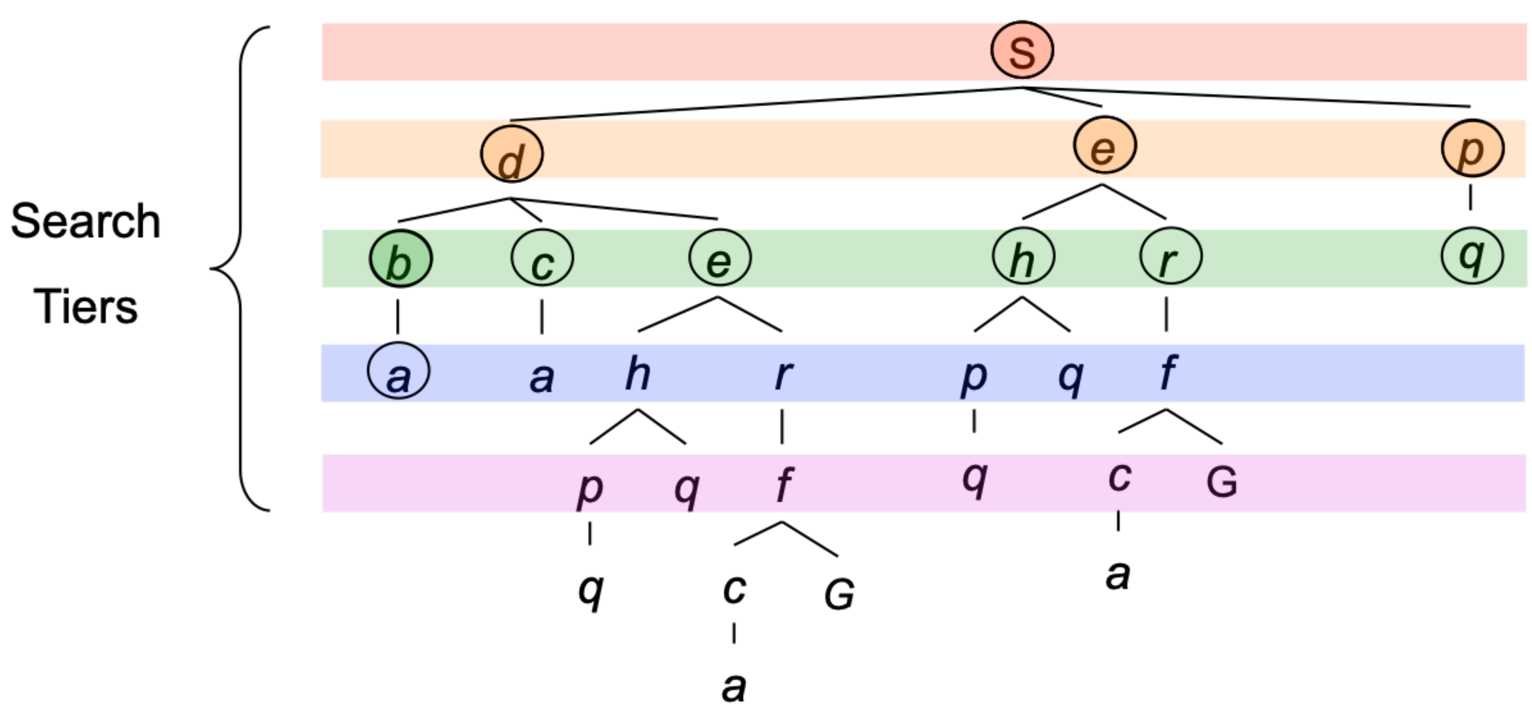
Breadth-First Search



Breadth-First Search

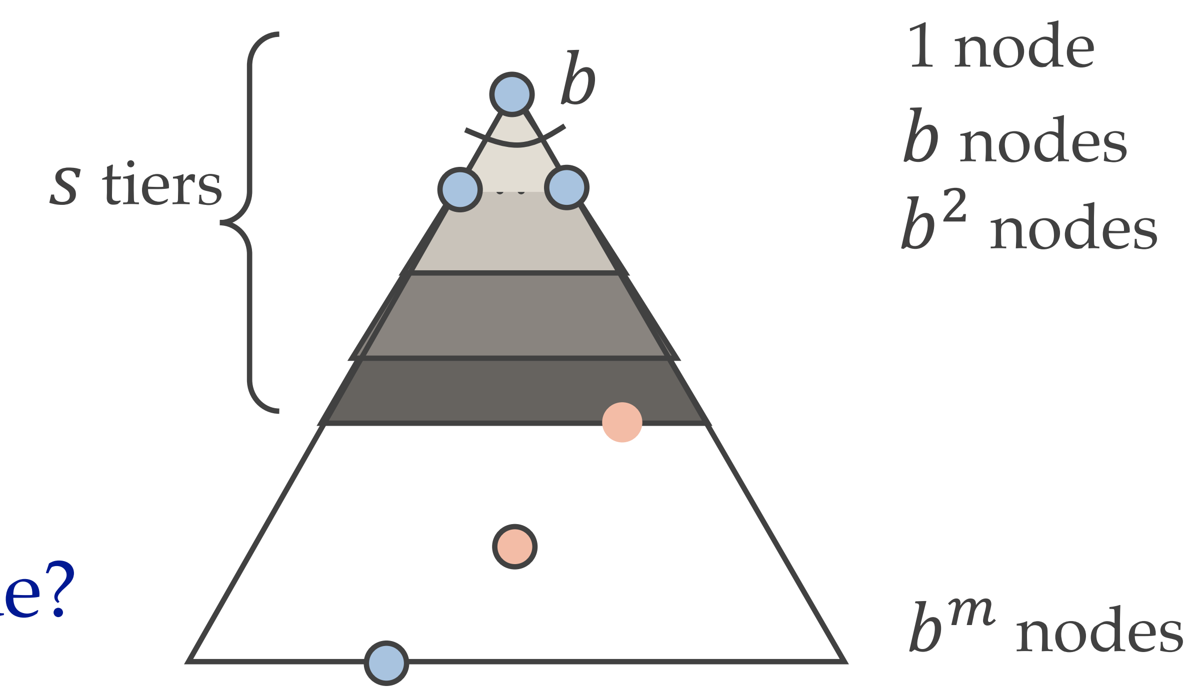
Strategy: expand a shallowest node first
Implementation: Fringe is a FIFO queue



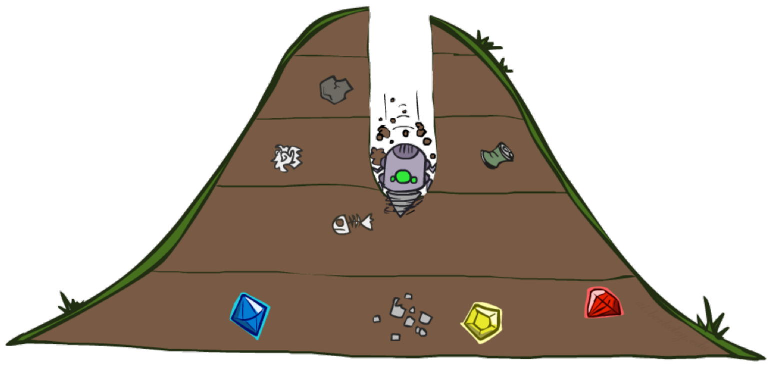


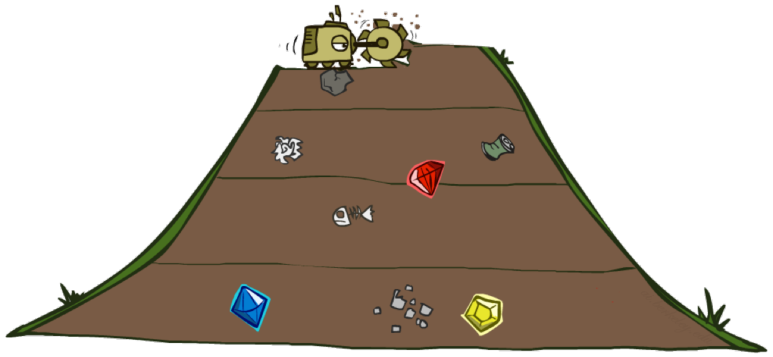
Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s
  - Search takes time $O(b^s)$
- How much space does the fringe take?
  - Has roughly the last tier, so (b^s)
- Is it complete?
  - s must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



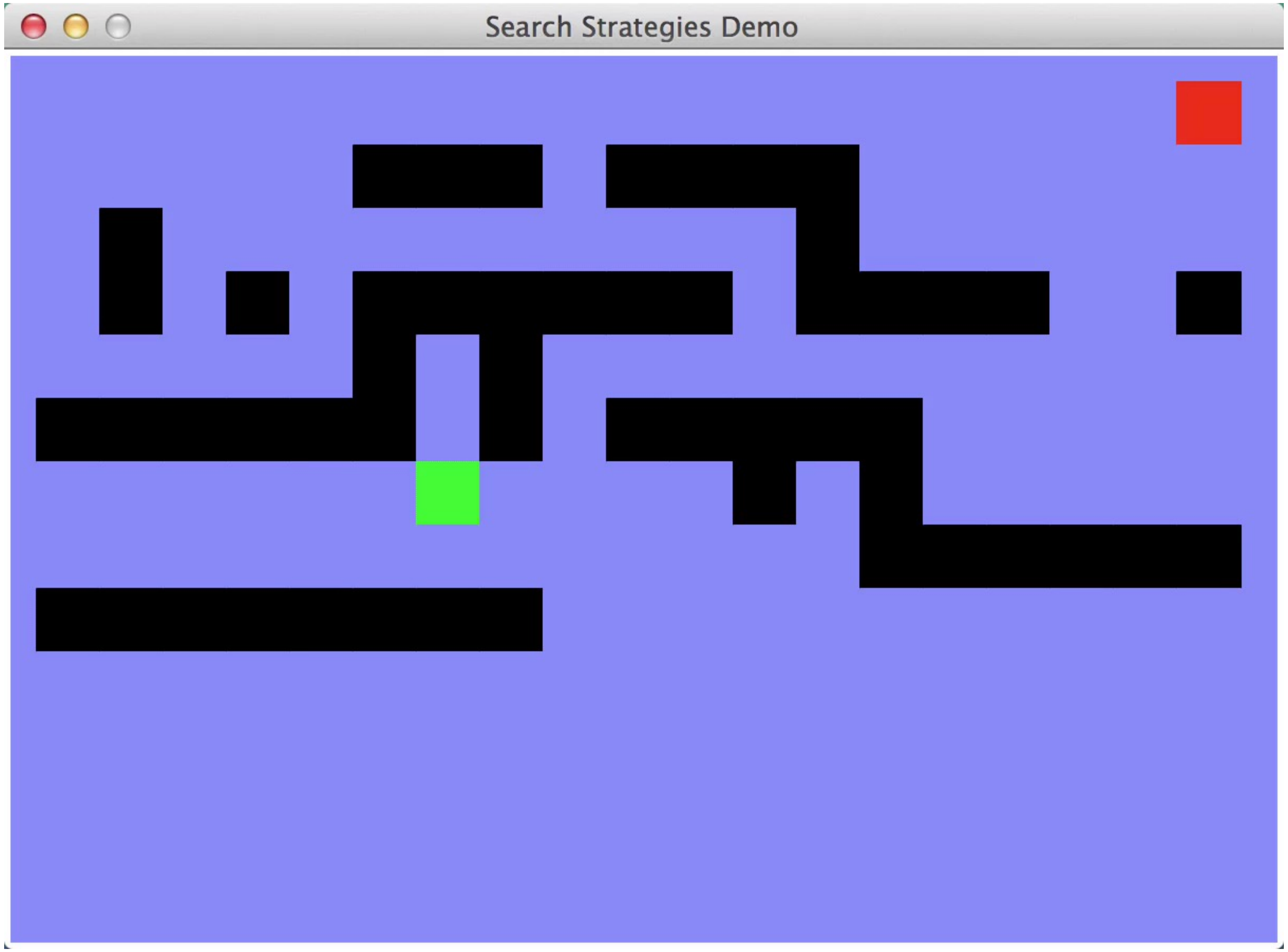
DFS vs BFS



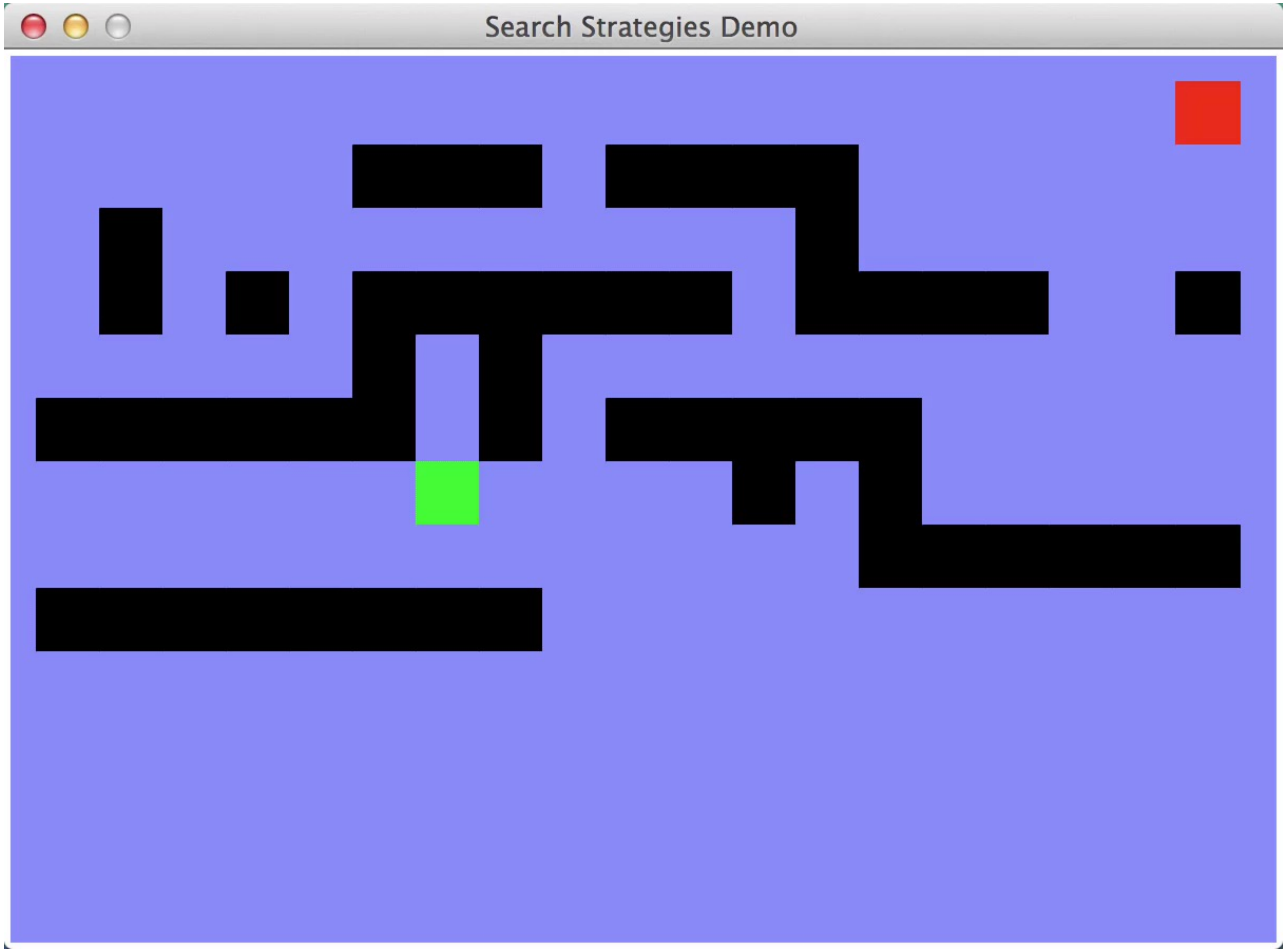


- When will BFS outperform DFS?
- When will DFS outperform BFS?

Video of Demo Maze Water DFS/BFS (part 1)

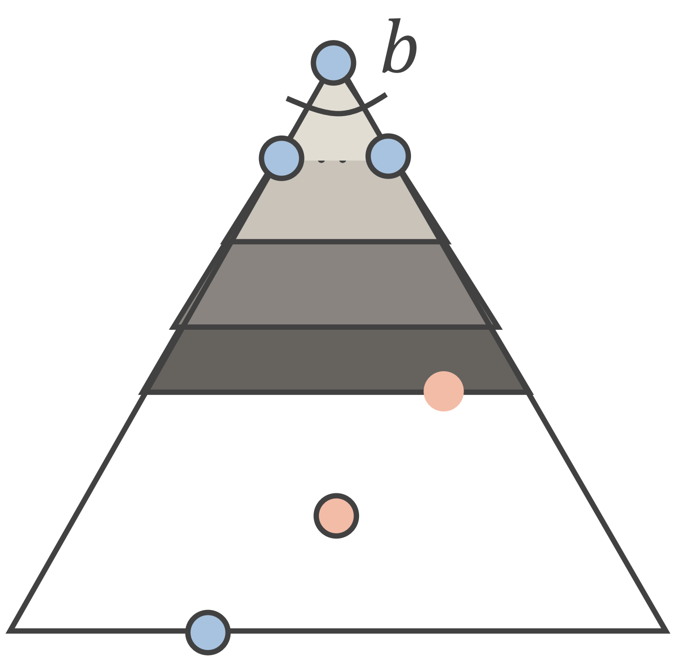


Video of Demo Maze Water DFS/BFS (part 2)



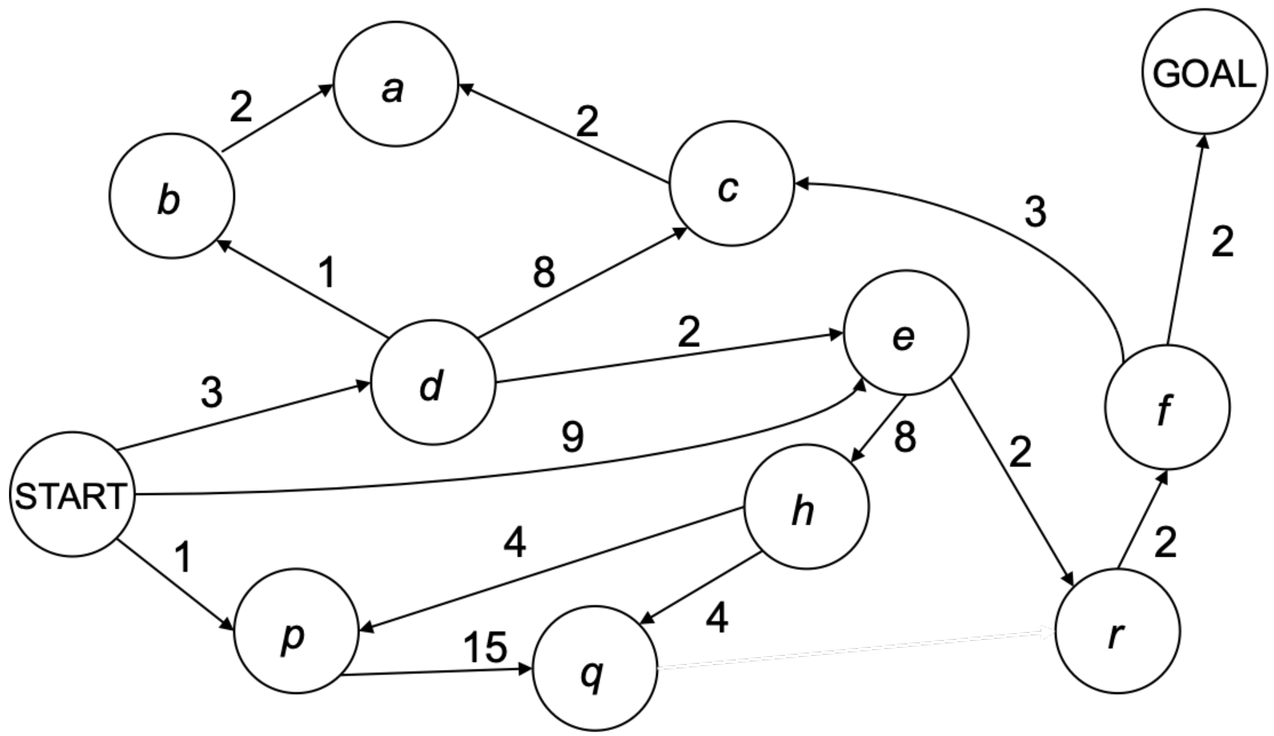
Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution…
  - Run a DFS with depth limit 2. If no solution…
  - Run a DFS with depth limit 3. …..



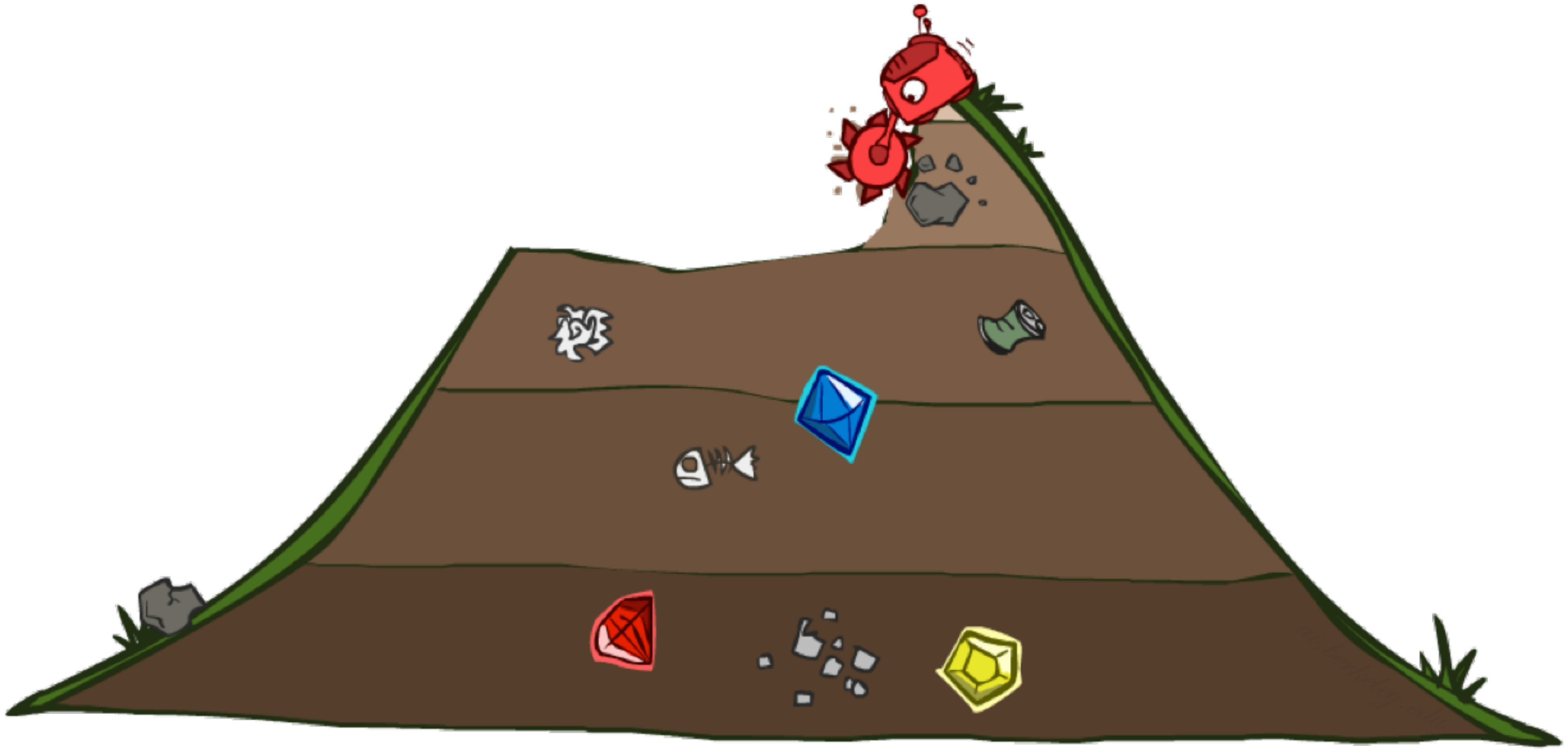
- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!

Cost-Sensitive Search



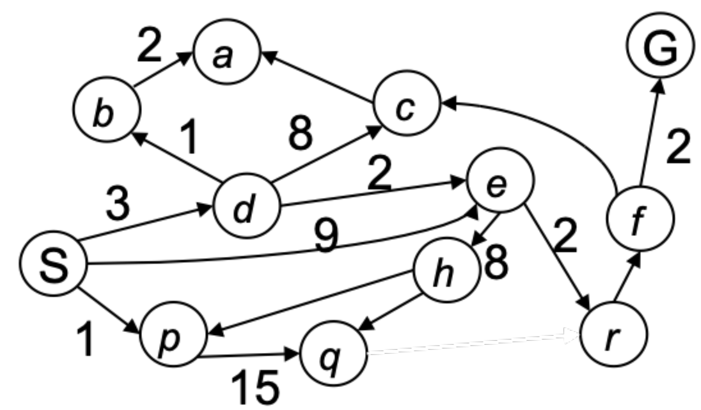
- BFS finds the shortest path in terms of number of actions.
- It does not find the least-cost path. We will now cover a similar algorithm which does find the least-cost path.

Uniform Cost Search

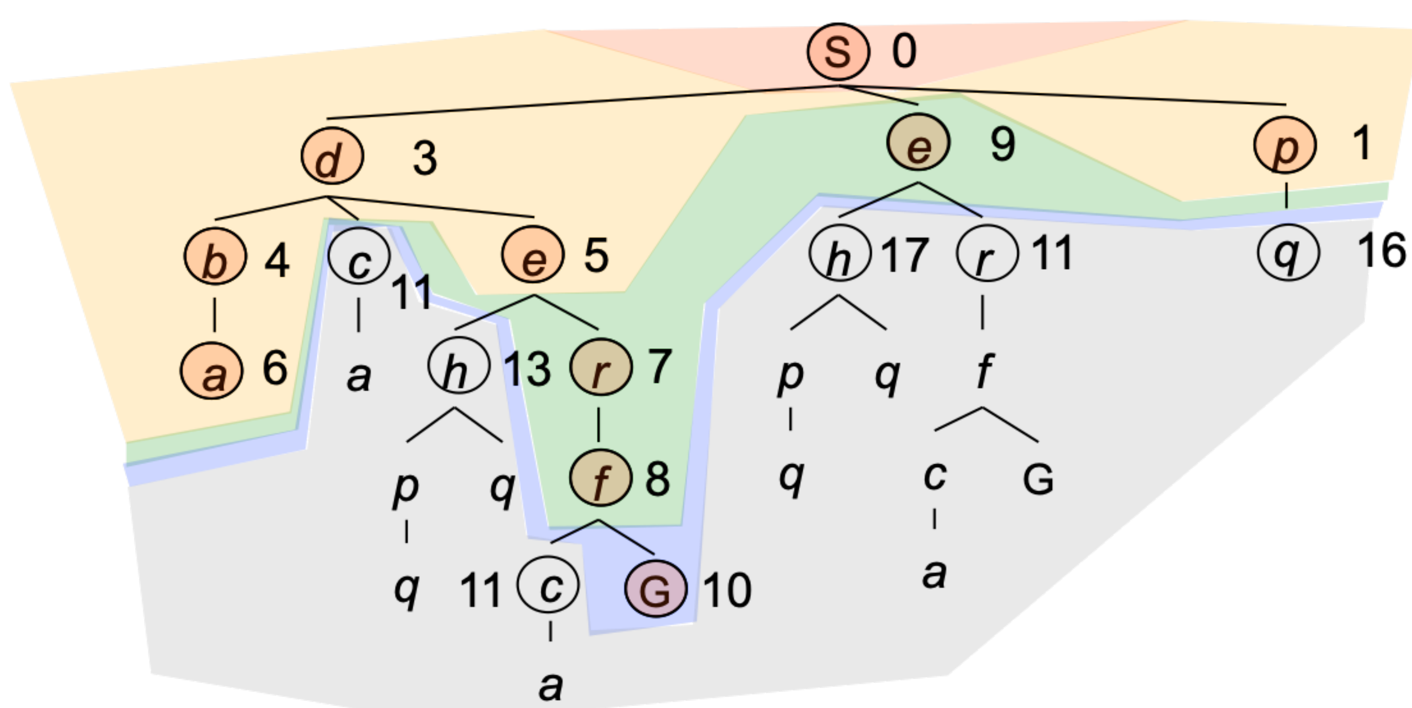


Uniform Cost Search

Strategy: expand a cheapest node first:
Fringe: a priority queue (priority: cumulative cost)

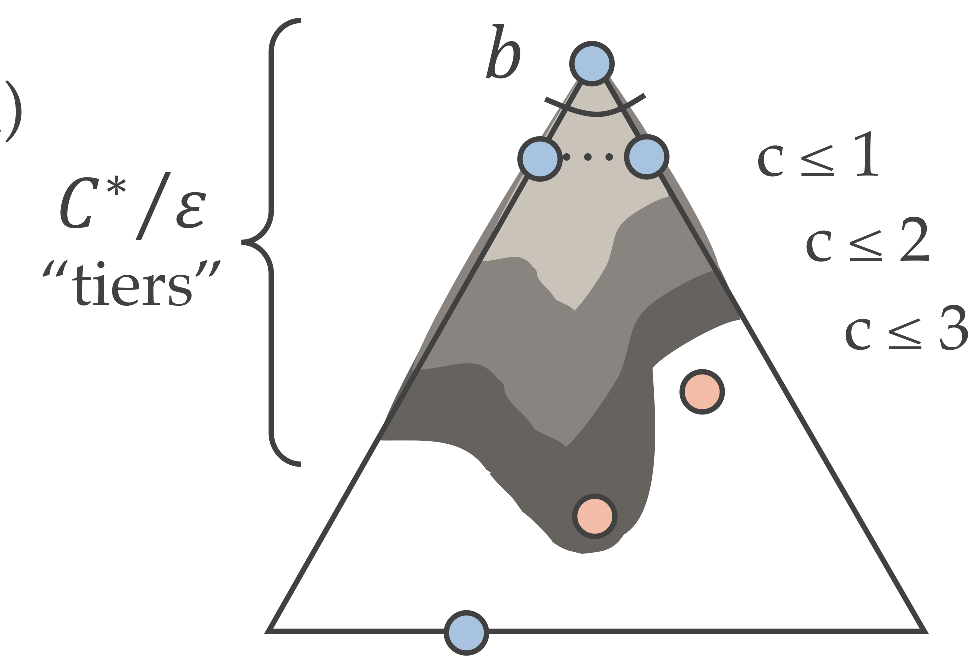


Cost contours



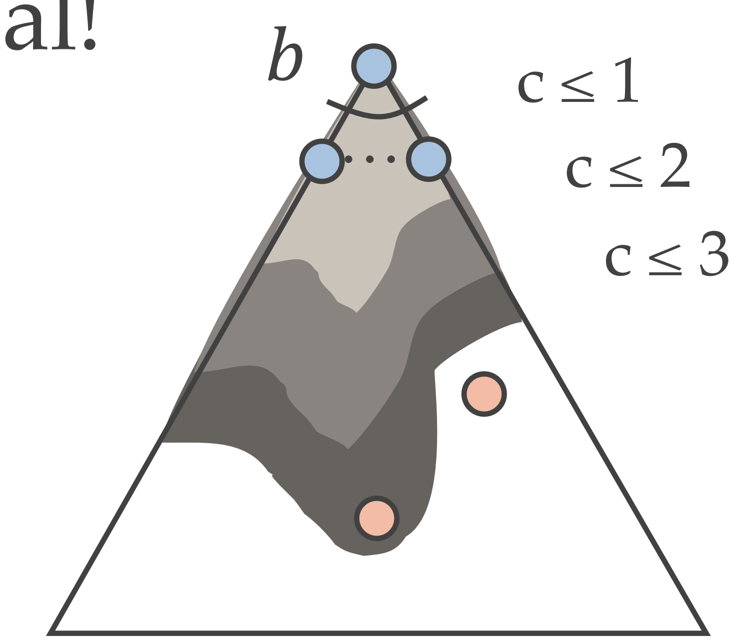
Uniform Cost Search (UCS) Properties

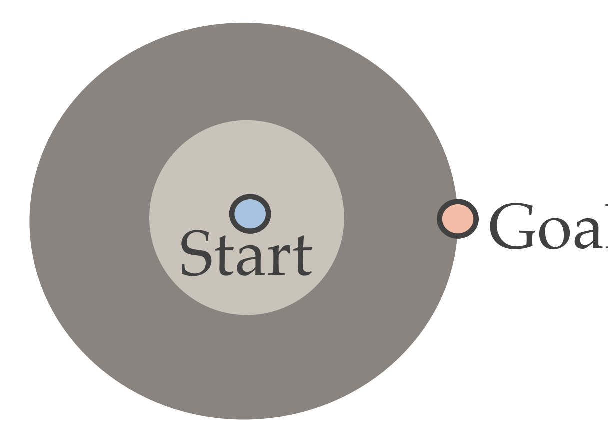
- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε , then the "effective depth" is roughly C^*/ε
  - Takes time $O(b^{\frac{C^*}{\varepsilon}})$ (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{\frac{C^*}{\varepsilon}})$
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes! (Proof next lecture via A*)



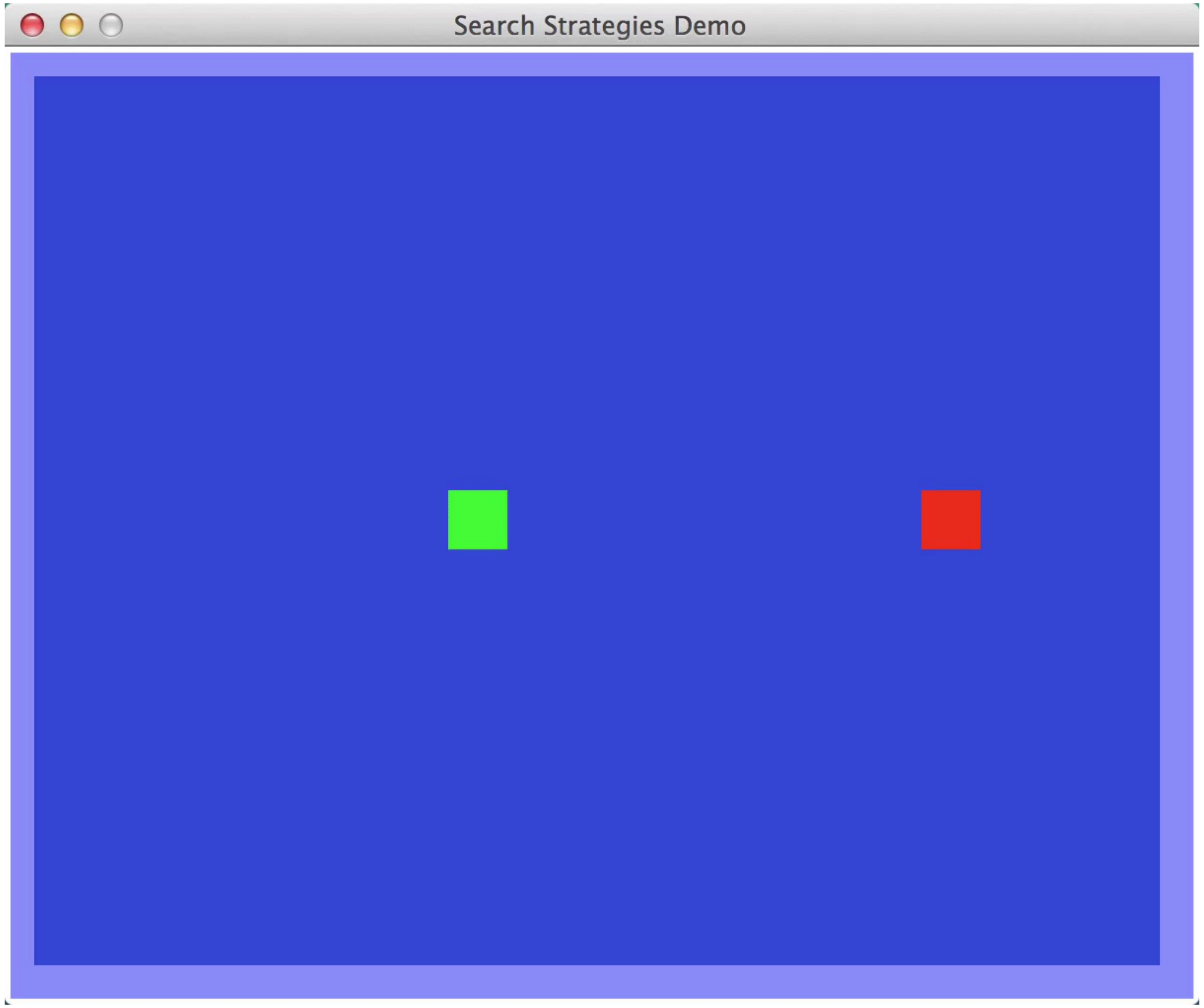
Uniform Cost Issues

- Remember: UCS explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location
- We'll fix that soon!

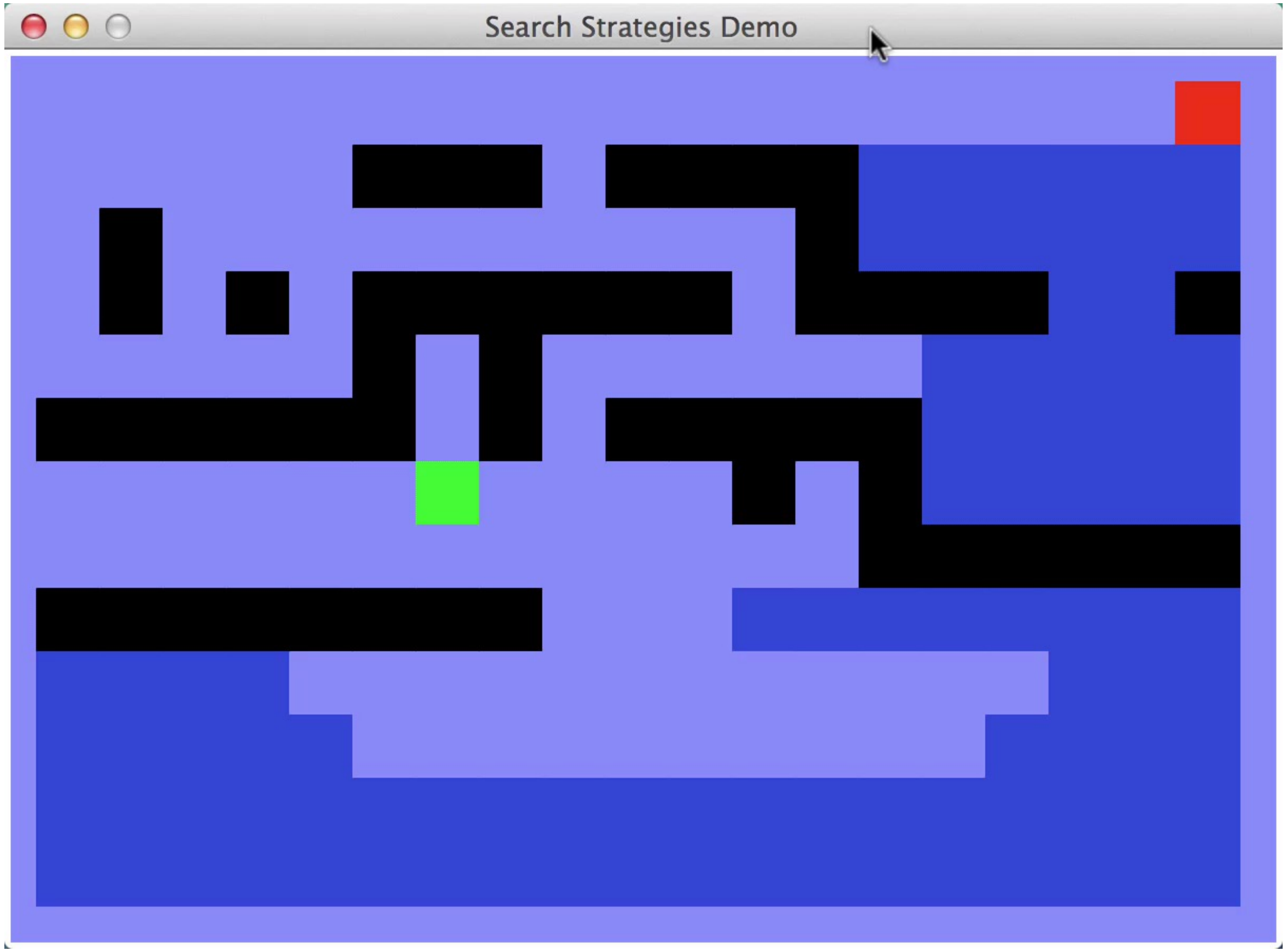




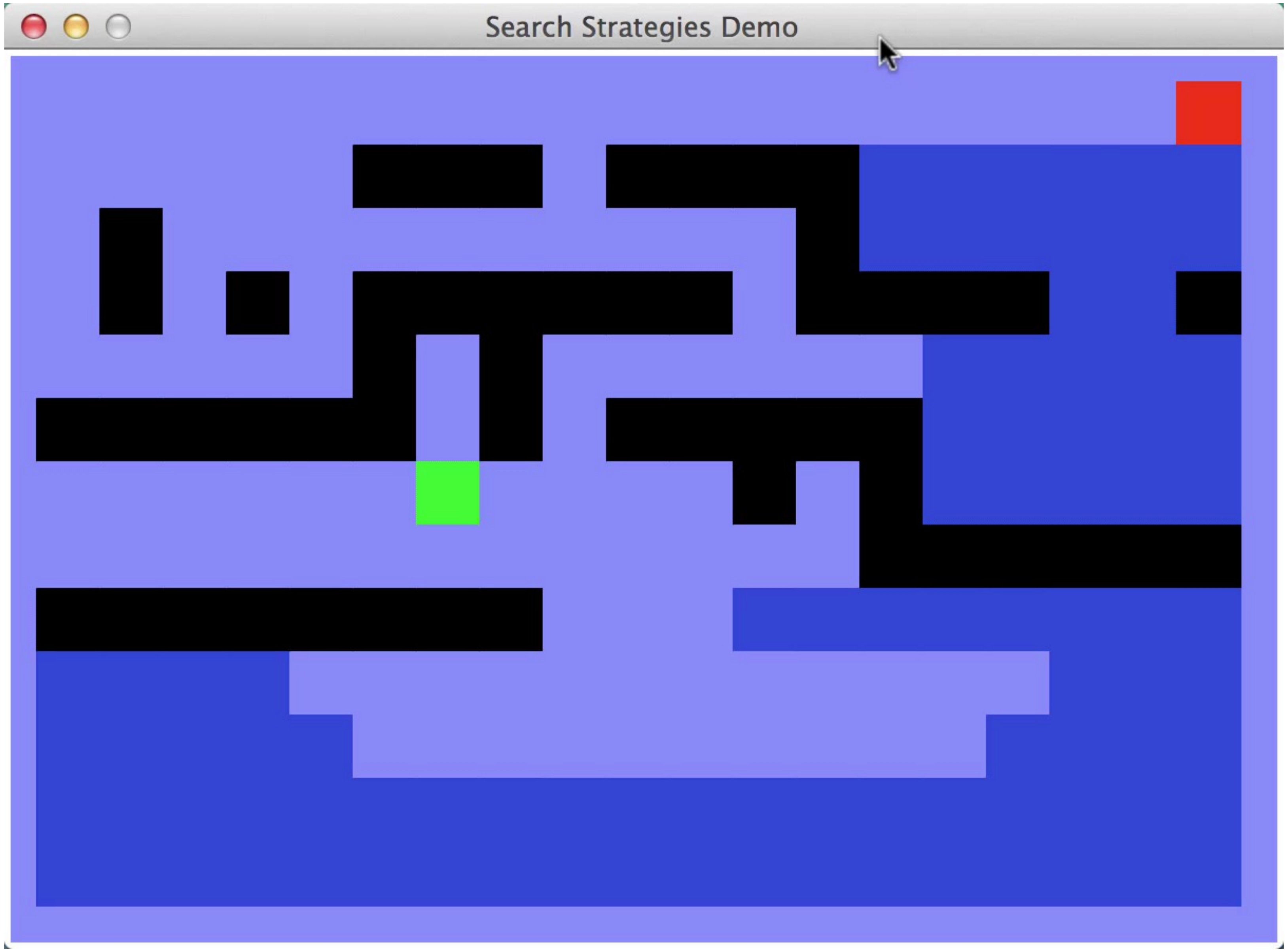
Video of Demo Empty UCS



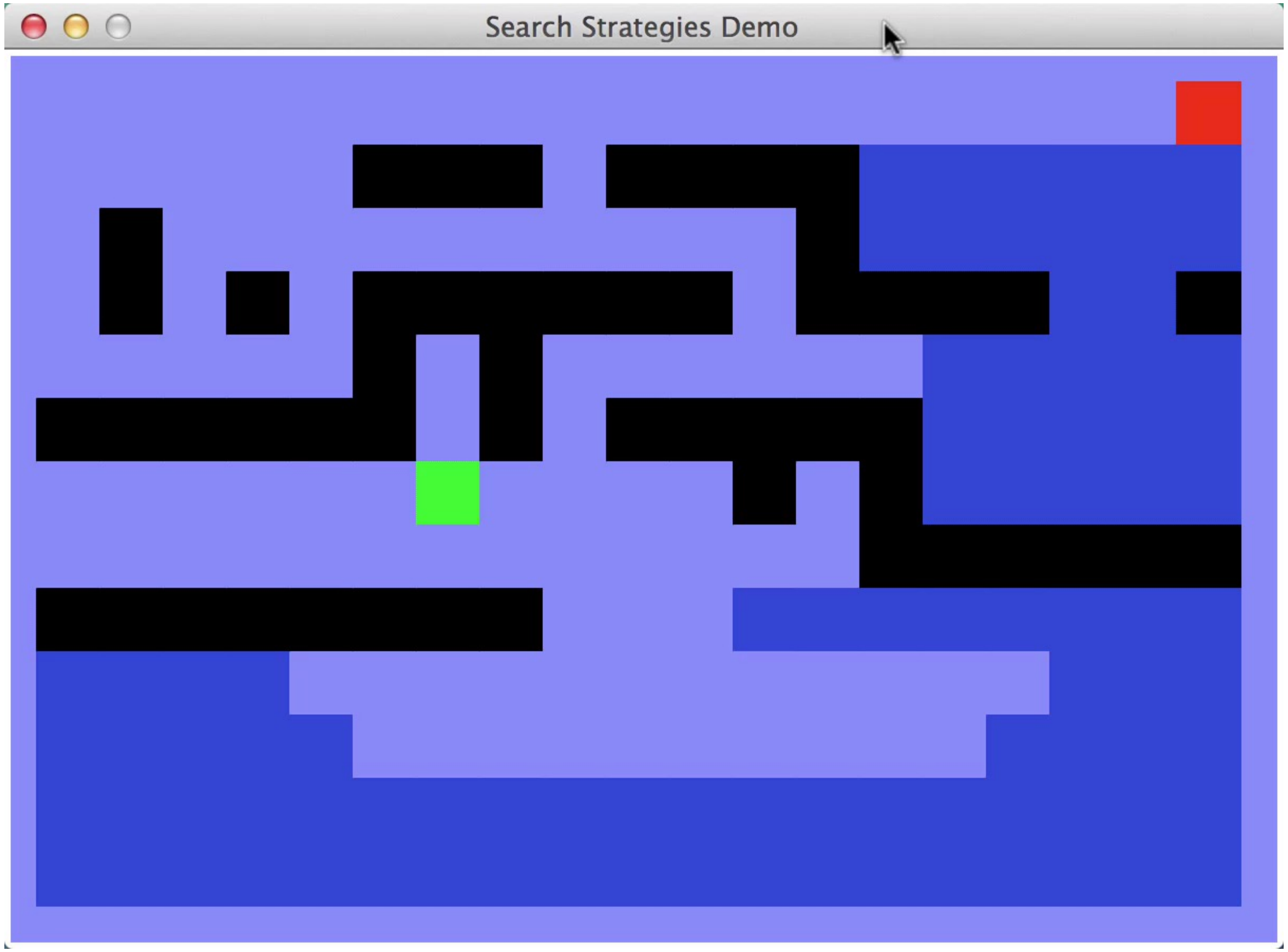
Quiz: Video of Demo Maze with Deep/Shallow Water DFS, BFS, or UCS? (part 1)



Quiz: Video of Demo Maze with Deep/Shallow Water DFS, BFS, or UCS? (part 2)

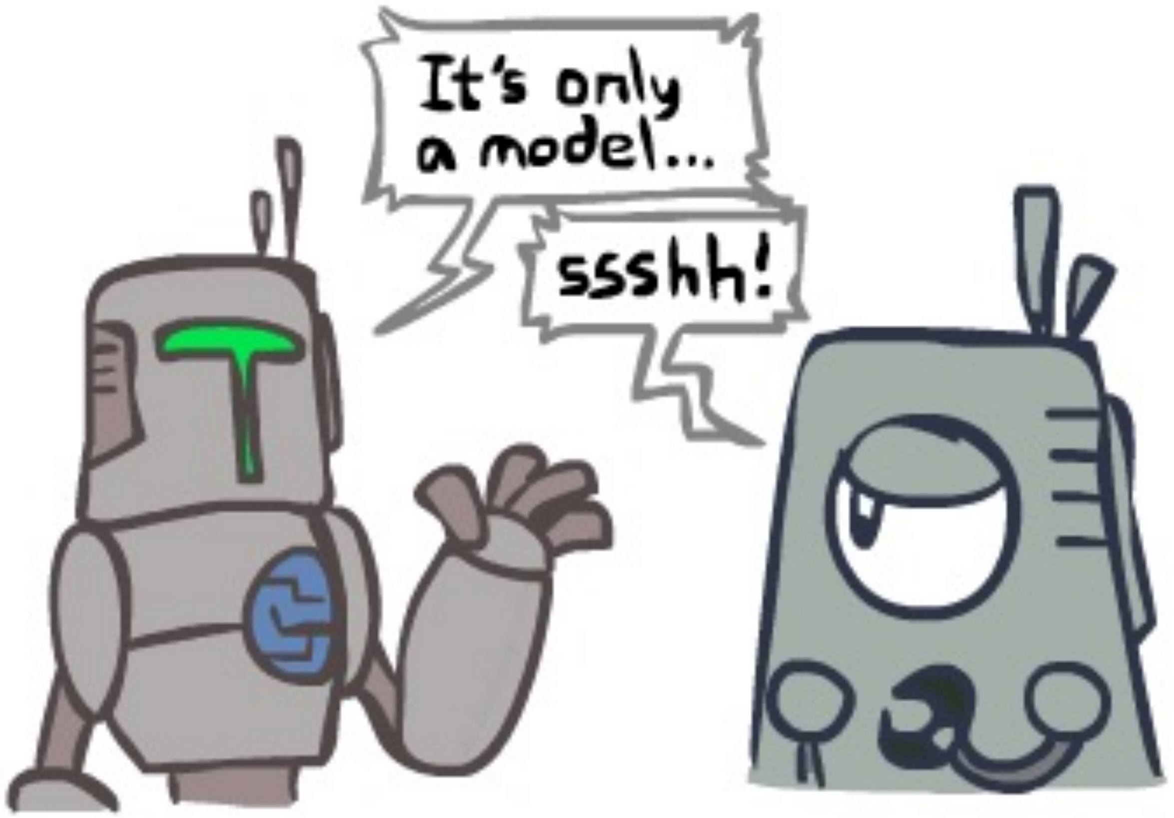


Quiz: Video of Demo Maze with Deep/Shallow Water
DFS, BFS, or UCS? (part 3)



Search and Models

- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all "in simulation"
  - Your search is only as good as your models…

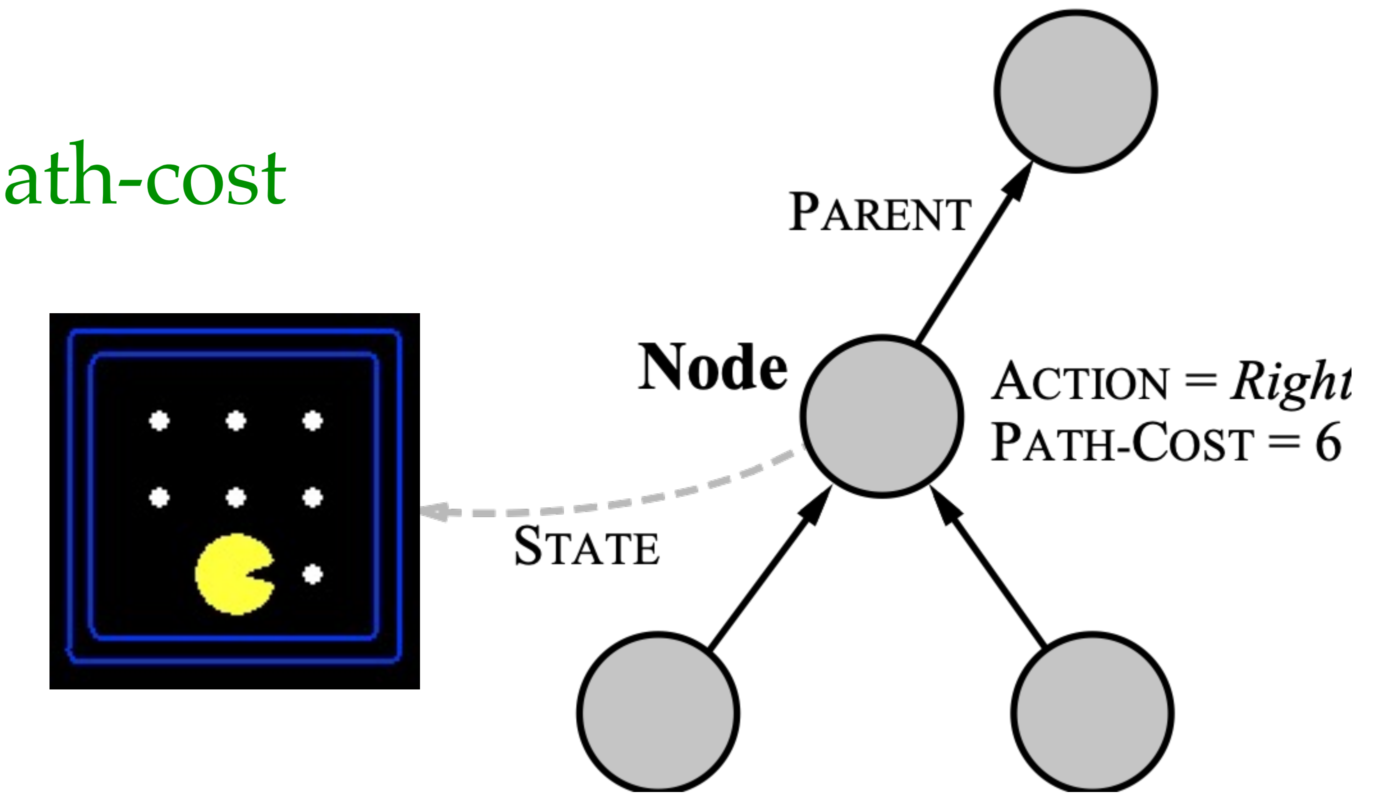


A Note on Implementation

Nodes have state, parent, action, path-cost

A child of node by action a has

- state = Succ(node.state,a)
- parent = node
- action = a
- path-cost = node.path_cost + step_cost(node.state, a, self.state)



Extract solution by tracing back parent pointers, collecting actions