深蓝学院
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Autonomous Valet Parking（AVP） Theory and Practice
自主代客泊车理论与实践

Lecture 3： Camera Model

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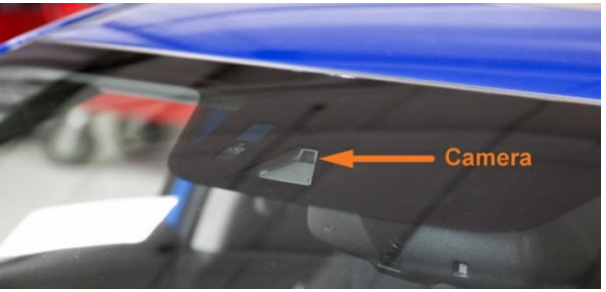
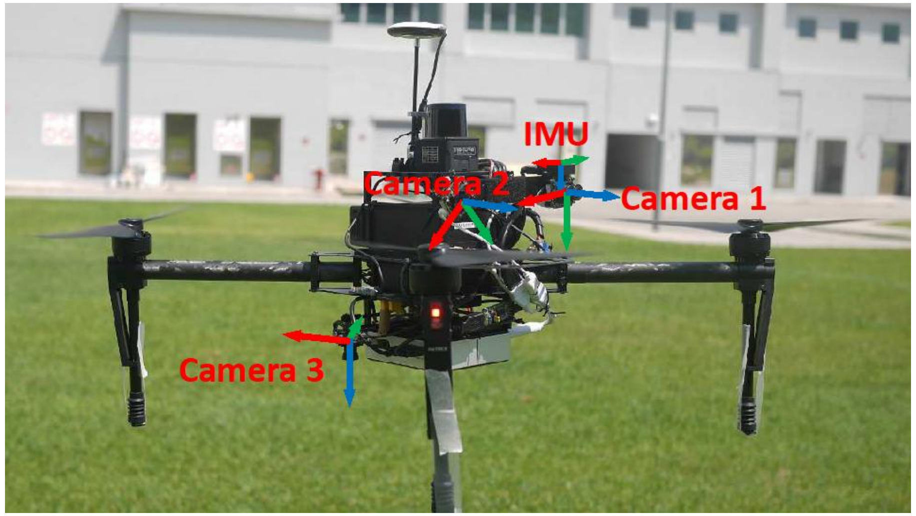
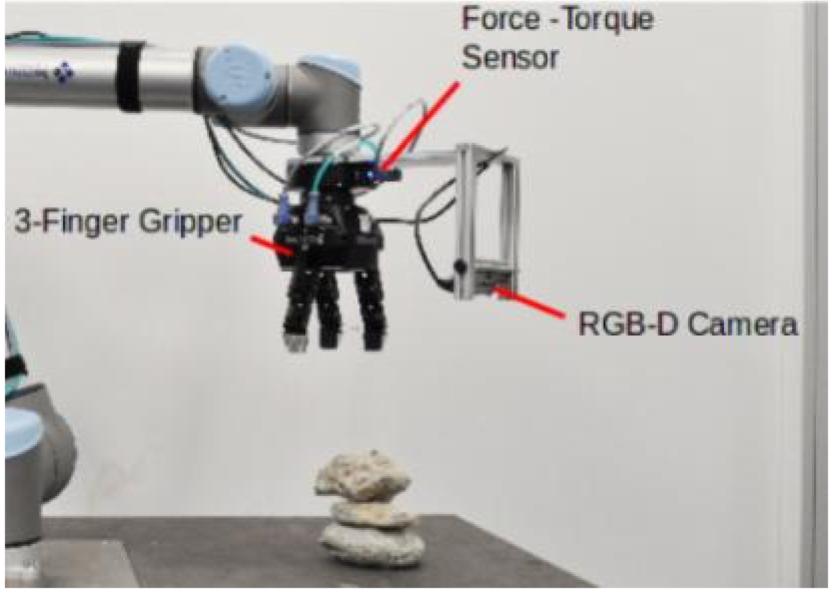
Content

1. Camera Background
2. Pinhole Camera Model
3. Distortion
4. Fisheye Camera Model (MEI)
5. IPM (Inverse Projective Mapping)
6. Assignment

Robot Sees with Cameras

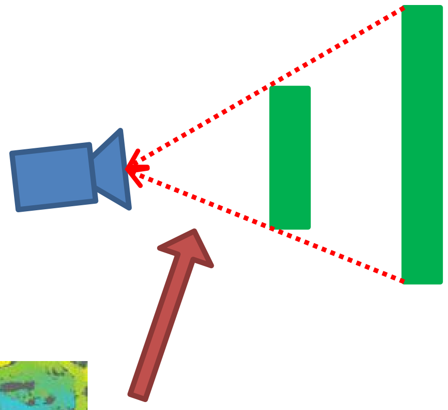






Cameras

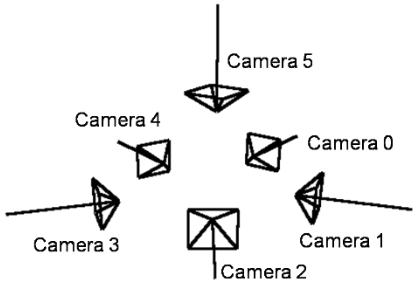
- Monocular
  - Simplest setup
  - Depth unknown



- Stereo
  - Able to compute depth
  - Depth accuracy affected by baseline, resolution, and calibration

- Multi-Camera
  - Overlapping / Non-overlapping field-of-view

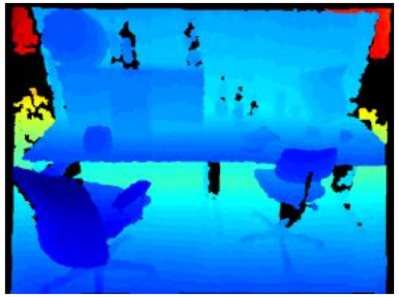
Omnidirectional multi-camera system
Ladybug



Relative position and posture of each camera

Cameras

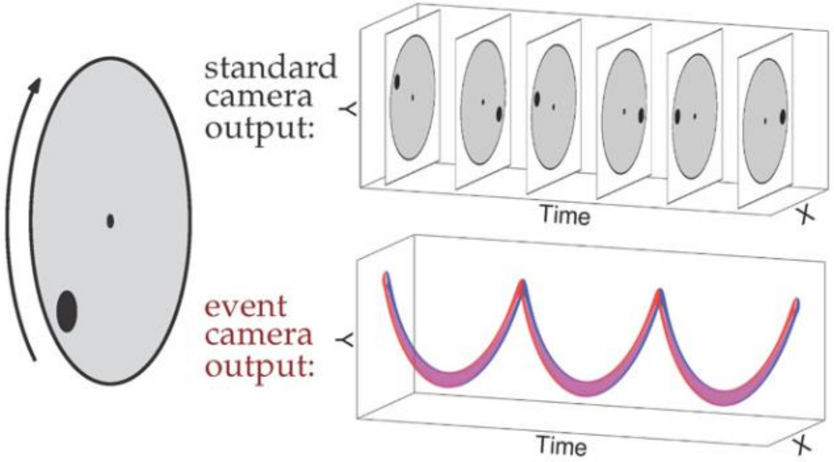
- RGB-D Sensor
  - Great depth in limited range
  - Does not work outdoors

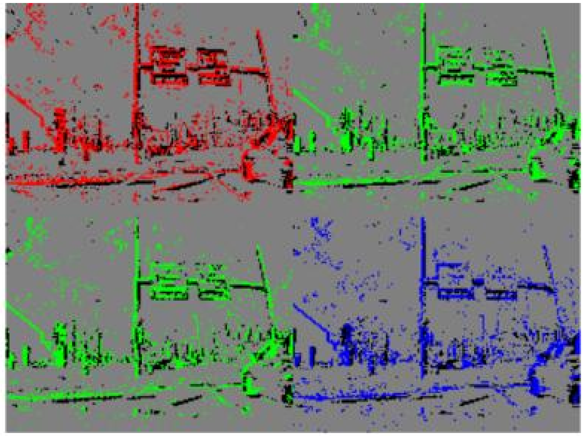


- Omnidirectional camera
  - 360 capture
  - Strong distortion

Cameras

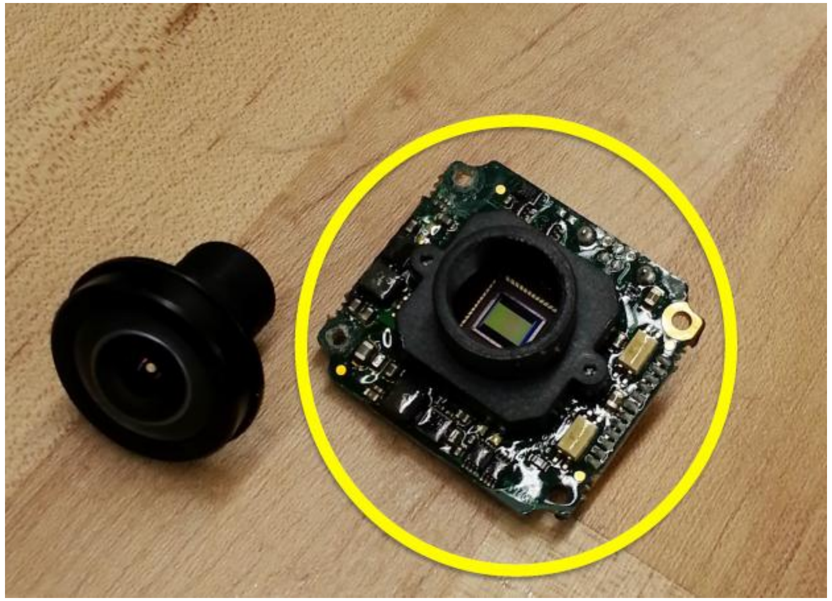
- Event Camera
  - High Temporal Resolution
  - Lack of Rich Visual Information

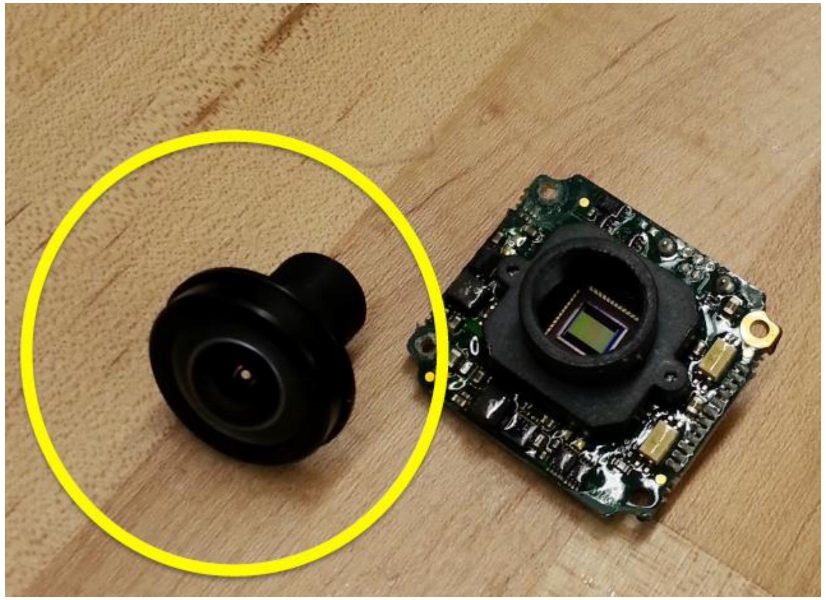




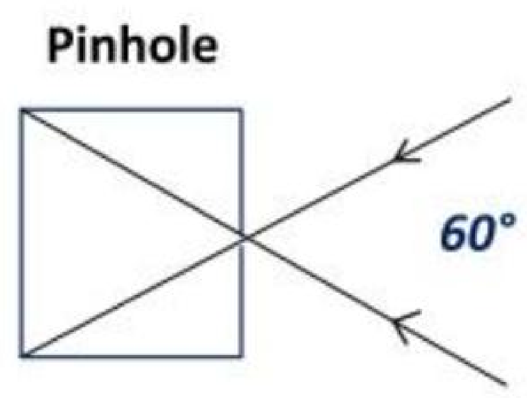
Cameras

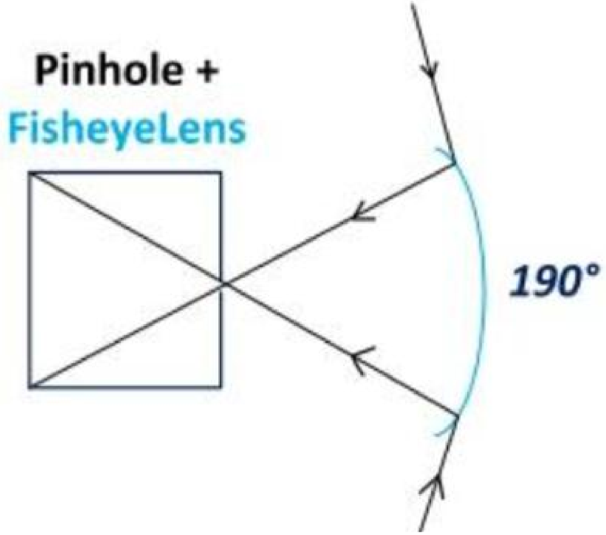
- Sensor
- Lens

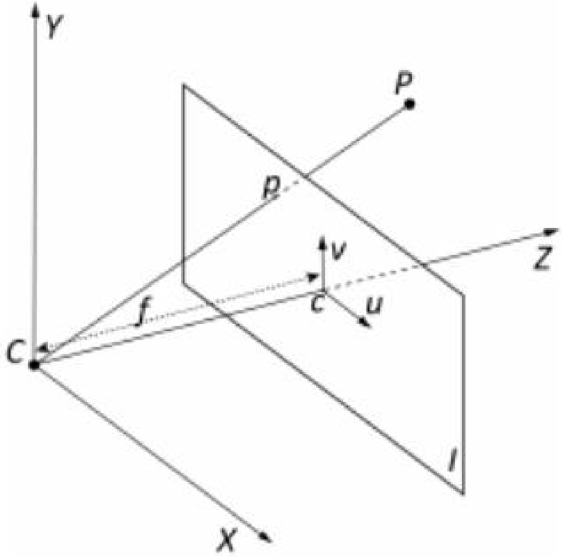




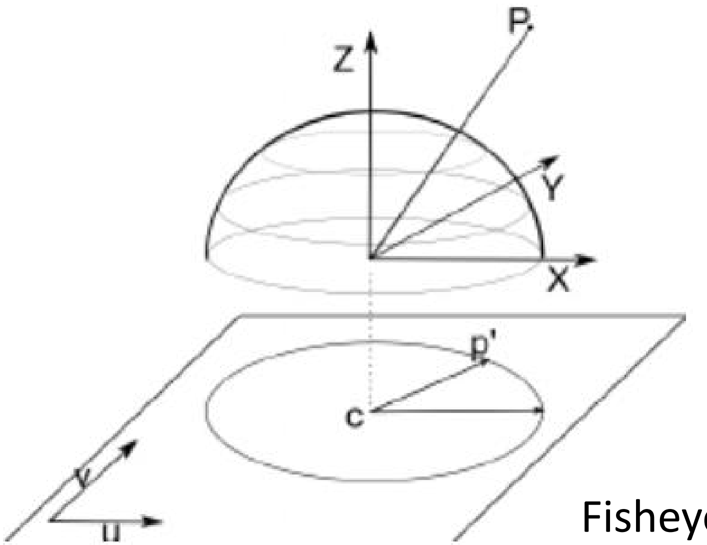
Cameras







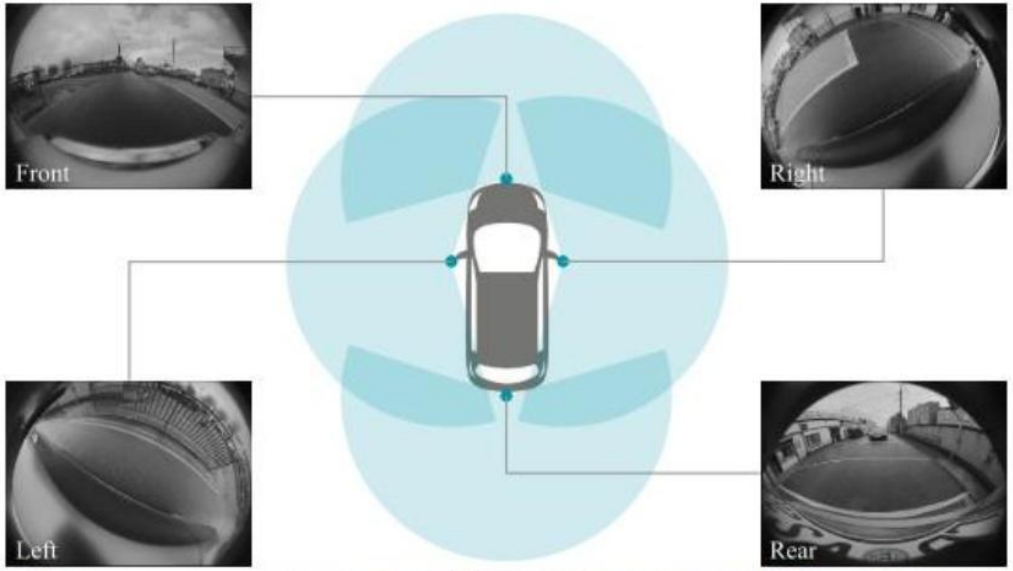
Pinhole model



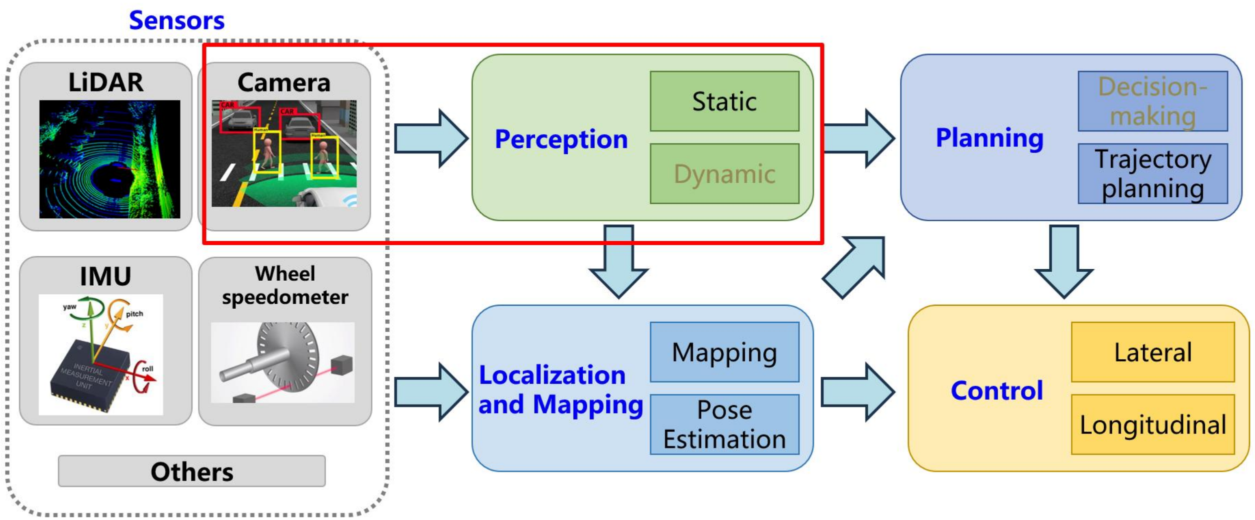
Fisheye model

Fisheye model

Cameras for AVP



AVP Architecture



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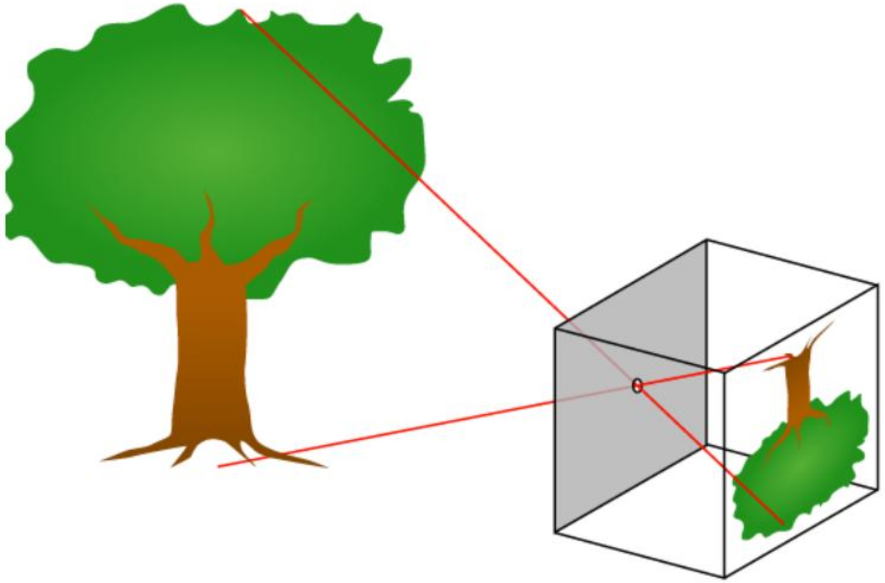
Pin-hole Camera Model

Pinhole imaging

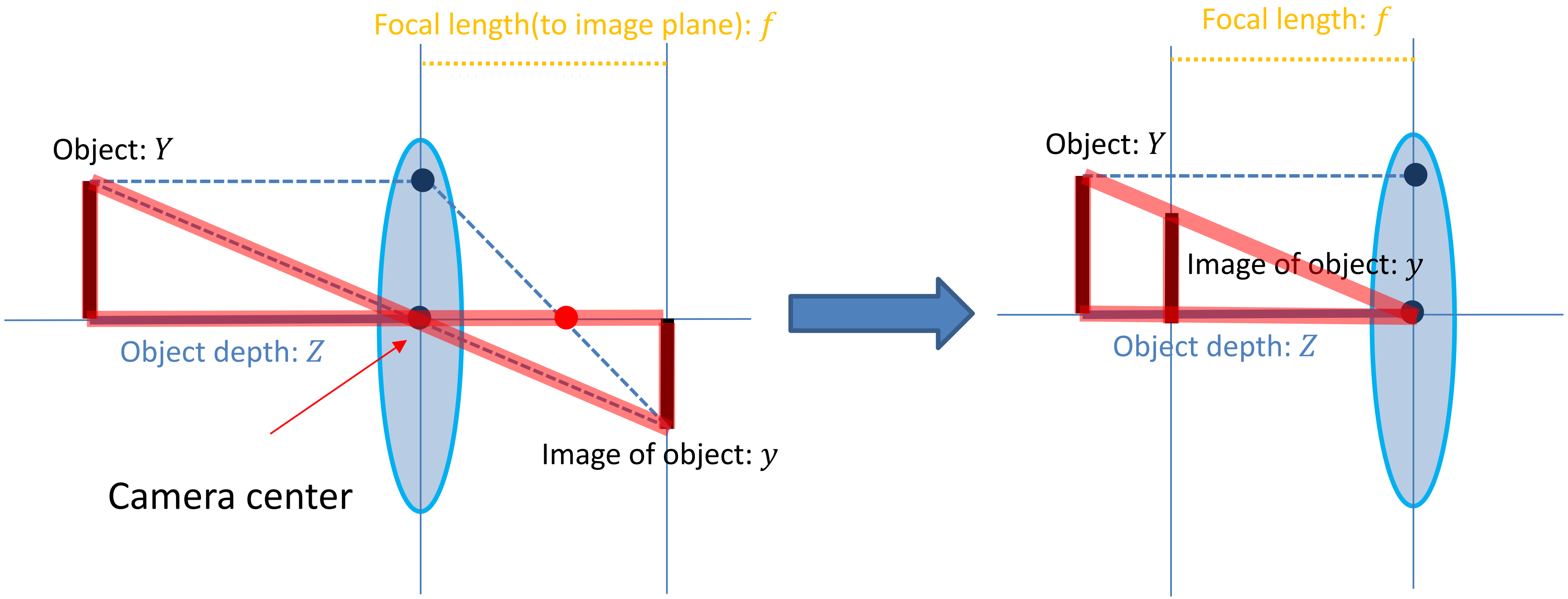


Light from an object passes through a small hole (the pinhole) and projects an inverted image onto a surface or screen on the other side.

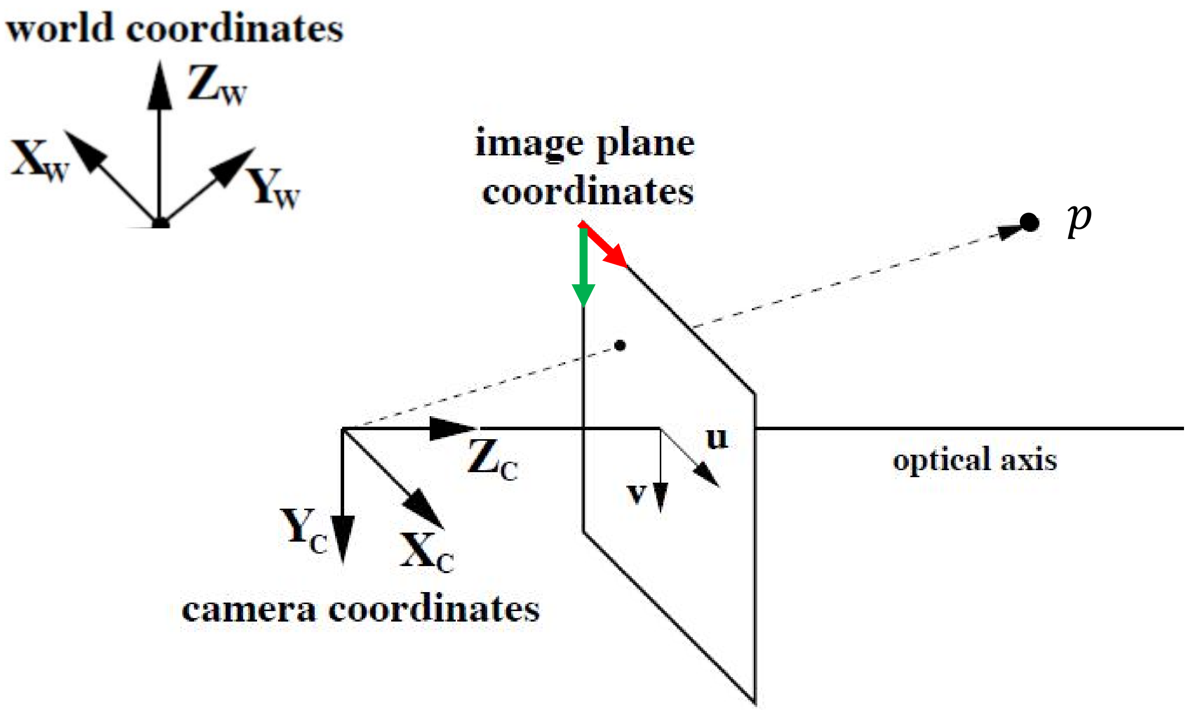
Pin-hole Camera Model

- Assume that the image plane is in front of the lens:

$$-y = f\frac{Y}{Z}$$



Camera Coordinate System



Coordinates：

- World ${}^{w}p = [x_w, y_w, z_w]$
- Camera ${}^{c}p = [x_c, y_c, z_c]$
- Image plane ${}^{I}p = [u, v]$

What's the relationship between ${}^{w}p$, ${}^{C}p$, ${}^{I}p$?

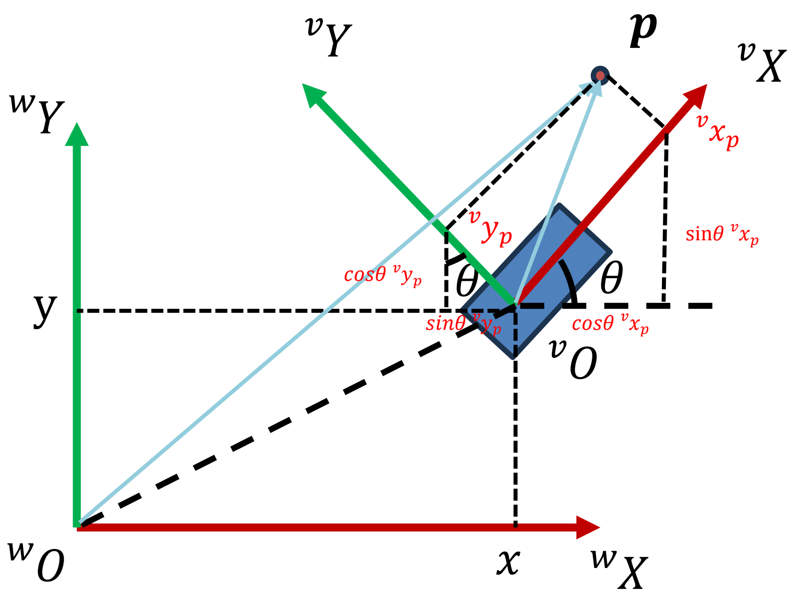
3D Coordinate Transformation

- Recall： 2D Transformations
  - World (Global) Coordinate
  - Vehicle (Local) Coordinate

Vehicle 2D State: $s = [x,\ y,\ \theta]$

$$^{w}\boldsymbol{p} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{^{w}R_{v}} {}^{v}\boldsymbol{p} + \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{^{w}t_{v}}$$

$$^{v}\boldsymbol{p} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}^{T}}_{^{v}R_{w}} {}^{w}\boldsymbol{p} \underbrace{- \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}^{T} \begin{bmatrix} x \\ y \end{bmatrix}}_{^{v}t_{w}}$$



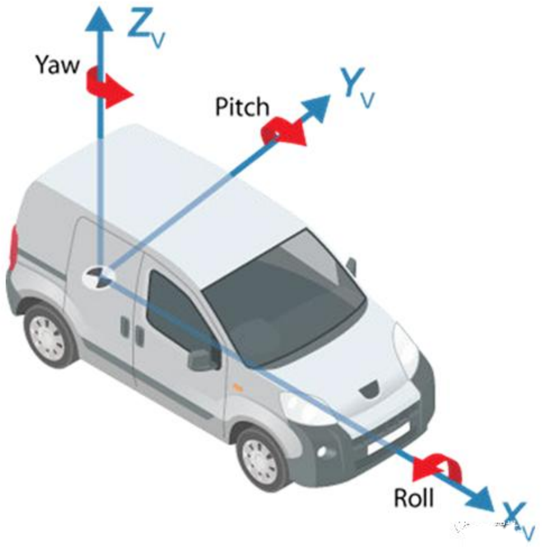
$^{w}R_{v}$, $^{w}t_{v}$: rotation matrix and translation vector from vehicle frame to world frame

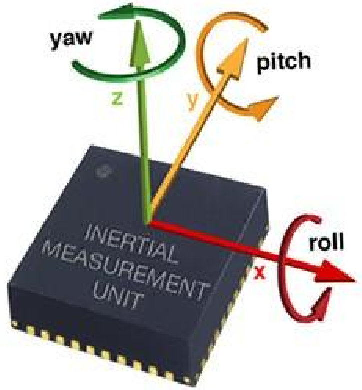
$^{v}R_{w}$ $^{v}t_{w}$: rotation matrix and translation vector from world frame to vehicle frame

3D Coordinate Transformation

Rotation Presentation

- ➢ Euler Angle
  - Roll(横滚) /Pitch(俯仰) / Yaw(偏航)





- ➢ Rotation Matrix

$$R = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix}_{3\times3}$$

- ➢ Quaternion

$$q = [w, x, y, z]^{\top}$$

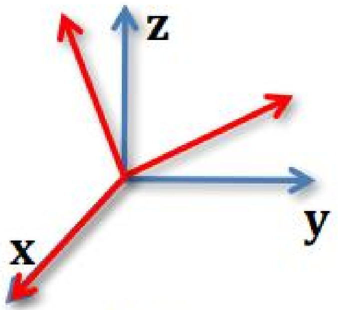
Quaternion -> R

$$\mathbf{R} = \begin{bmatrix} q_w^2 + q_x^2 - q_y^2 - q_z^2 & 2(q_x q_y - q_w q_z) & 2(q_x q_z + q_w q_y) \\ 2(q_x q_y + q_w q_z) & q_w^2 - q_x^2 + q_y^2 - q_z^2 & 2(q_y q_z - q_w q_x) \\ 2(q_x q_z - q_w q_y) & 2(q_y q_z + q_w q_x) & q_w^2 - q_x^2 - q_y^2 + q_z^2 \end{bmatrix}$$

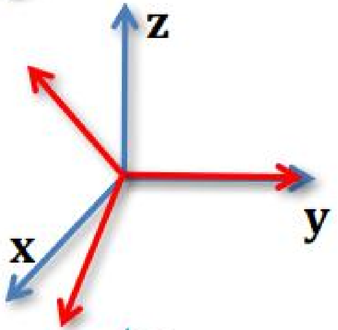
3D Coordinate Transformation

Correspondence: Rotation Matrix - Euler Angles

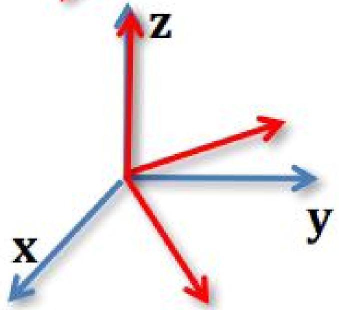
- $\boldsymbol{R}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$



- $\boldsymbol{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

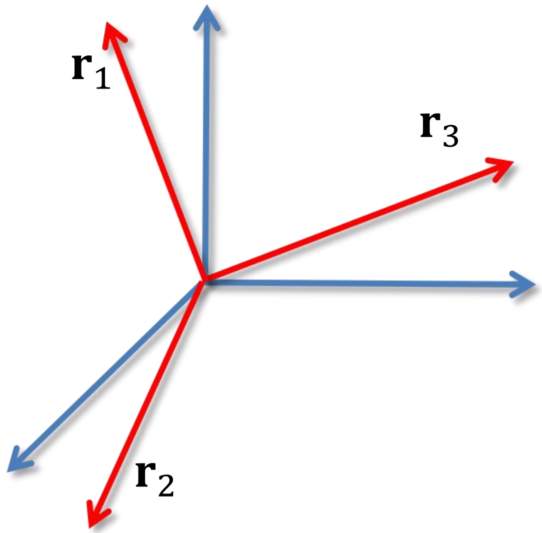


- $\boldsymbol{R}_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$



3D Coordinate Transformation

- Let $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3]$ be a rotation matrix
- Orthogonal:
  - $\mathbf{r}_i{}^T \cdot \mathbf{r}_j = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$
  - $\mathbf{R} \cdot \mathbf{R}^T = \mathbf{I}$
- Special orthogonal:
  - $\det\mathbf{R} = \mathbf{r}_1{}^T \cdot (\mathbf{r}_2 \times \mathbf{r}_3) = \mathbf{r}_1{}^T \cdot \mathbf{r}_1 = 1$
- The set of all rotations forms the Special Orthogonal Group
  - Special orthogonal group
  - 3D rotations: $SO(3)$
  - 2D rotations: $SO(2)$
  - $SO(n) = \{\mathbf{R} \in \mathbb{R}^{n\times n} | \mathbf{R} \cdot \mathbf{R}^T = \mathbf{I}, \det\mathbf{R} = 1\}$

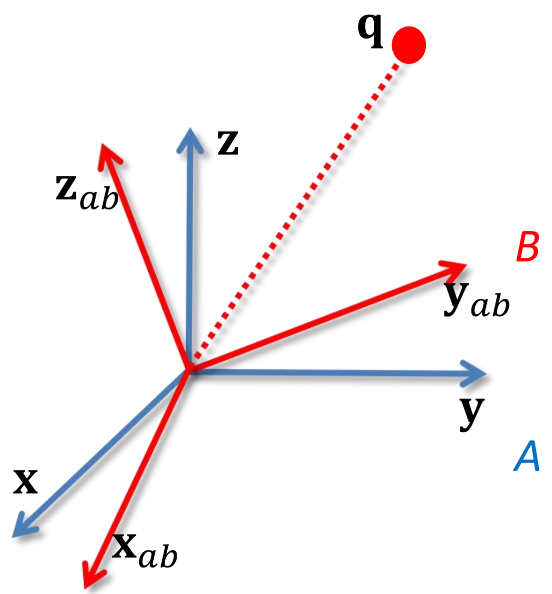


3D Coordinate Transformation

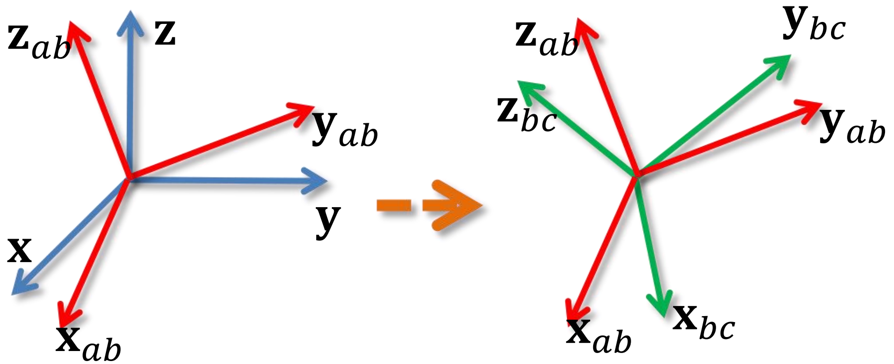
- A transformation that rotates the coordinates of a point from frame *B* to frame *A*
  - Let $\mathbf{q}_b = [x_b,\ y_b,\ z_b]^T \in \mathbb{R}^3$ be coordinate of point $\mathbf{q}$ in frame *B*
  - Let $\mathbf{q}_a$ be coordinate of point $\mathbf{q}$ in frame *A*
  - $\mathbf{q}_a = x_b \cdot \mathbf{x}_{ab} + y_b \cdot \mathbf{y}_{ab} + z_b \cdot \mathbf{z}_{ab}$

$$= [\mathbf{x}_{ab}, \mathbf{y}_{ab}, \mathbf{z}_{ab}] \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = {}^a\mathbf{R}_b \cdot \mathbf{q}_b$$

Rotation matrix from B to A

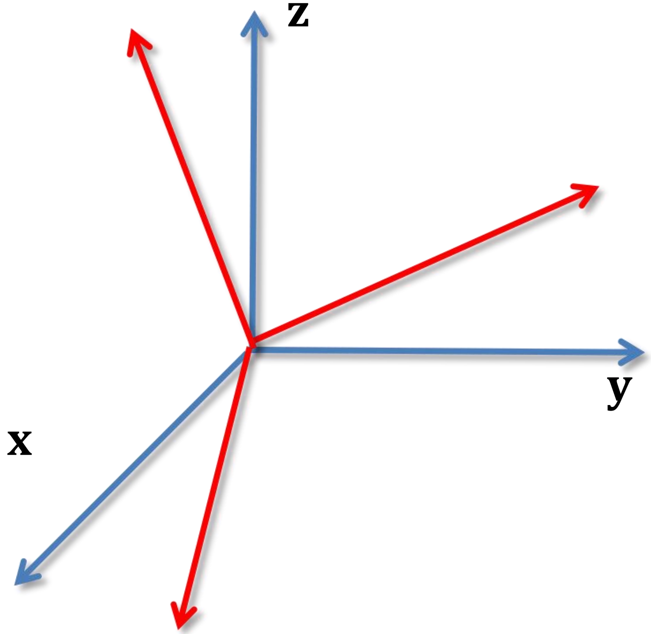


- Composition Rule
  - ${}^a\mathbf{R}_c = {}^a\mathbf{R}_b \cdot {}^b\mathbf{R}_c$



3D Coordinate Transformation

- Any rotation can be described by three successive rotations about linear independent axes
- However, this is an almost 1-1 transform with singularities:
  - $\boldsymbol{R}_z(\psi) \cdot \boldsymbol{R}_x(\phi) \cdot \boldsymbol{R}_y(\theta) \Rightarrow \boldsymbol{R}$
  - $\boldsymbol{R}_z(\psi) \cdot \boldsymbol{R}_x(\phi) \cdot \boldsymbol{R}_y(\theta) \nLeftarrow \boldsymbol{R}$



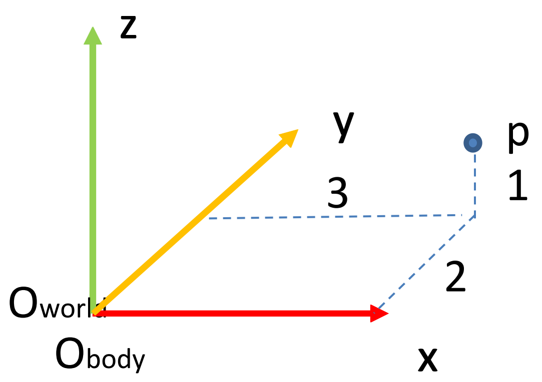
3D Coordinate Transformation

- Different Euler angle conversions:

Proper Euler angles	Tait-Bryan angles
$X_1Z_2X_3 = \begin{bmatrix} c_2 & -c_3s_2 & s_2s_3 \\ c_1s_2 & c_1c_2c_3 - s_1s_3 & -c_3s_1 - c_1c_2s_3 \\ s_1s_2 & c_1s_3 + c_2c_3s_1 & c_1c_3 - c_2s_1s_3 \end{bmatrix}$	$X_1Z_2Y_3 = \begin{bmatrix} c_2c_3 & -s_2 & c_2s_3 \\ s_1s_3 + c_1c_3s_2 & c_1c_2 & c_1s_2s_3 - c_3s_1 \\ c_3s_1s_2 - c_1s_3 & c_2s_1 & c_1c_3 + s_1s_2s_3 \end{bmatrix}$
$X_1Y_2X_3 = \begin{bmatrix} c_2 & s_2s_3 & c_3s_2 \\ s_1s_2 & c_1c_3 - c_2s_1s_3 & -c_1s_3 - c_2c_3s_1 \\ -c_1s_2 & c_3s_1 + c_1c_2s_3 & c_1c_2c_3 - s_1s_3 \end{bmatrix}$	$X_1Y_2Z_3 = \begin{bmatrix} c_2c_3 & -c_2s_3 & s_2 \\ c_1s_3 + c_3s_1s_2 & c_1c_3 - s_1s_2s_3 & -c_2s_1 \\ s_1s_3 - c_1c_3s_2 & c_3s_1 + c_1s_2s_3 & c_1c_2 \end{bmatrix}$
$Y_1X_2Y_3 = \begin{bmatrix} c_1c_3 - c_2s_1s_3 & s_1s_2 & c_1s_3 + c_2c_3s_1 \\ s_2s_3 & c_2 & -c_3s_2 \\ -c_3s_1 - c_1c_2s_3 & c_1s_2 & c_1c_2c_3 - s_1s_3 \end{bmatrix}$	$Y_1X_2Z_3 = \begin{bmatrix} c_1c_3 + s_1s_2s_3 & c_3s_1s_2 - c_1s_3 & c_2s_1 \\ c_2s_3 & c_2c_3 & -s_2 \\ c_1s_2s_3 - c_3s_1 & c_1c_3s_2 + s_1s_3 & c_1c_2 \end{bmatrix}$
$Y_1Z_2Y_3 = \begin{bmatrix} c_1c_2c_3 - s_1s_3 & -c_1s_2 & c_3s_1 + c_1c_2s_3 \\ c_3s_2 & c_2 & s_2s_3 \\ -c_1s_3 - c_2c_3s_1 & s_1s_2 & c_1c_3 - c_2s_1s_3 \end{bmatrix}$	$Y_1Z_2X_3 = \begin{bmatrix} c_1c_2 & s_1s_3 - c_1c_3s_2 & c_3s_1 + c_1s_2s_3 \\ s_2 & c_2c_3 & -c_2s_3 \\ -c_2s_1 & c_1s_3 + c_3s_1s_2 & c_1c_3 - s_1s_2s_3 \end{bmatrix}$
$Z_1Y_2Z_3 = \begin{bmatrix} c_1c_2c_3 - s_1s_3 & -c_3s_1 - c_1c_2s_3 & c_1s_2 \\ c_1s_3 + c_2c_3s_1 & c_1c_3 - c_2s_1s_3 & s_1s_2 \\ -c_3s_2 & s_2s_3 & c_2 \end{bmatrix}$	$Z_1Y_2X_3 = \begin{bmatrix} c_1c_2 & c_1s_2s_3 - c_3s_1 & s_1s_3 + c_1c_3s_2 \\ c_2s_1 & c_1c_3 + s_1s_2s_3 & c_3s_1s_2 - c_1s_3 \\ -s_2 & c_2s_3 & c_2c_3 \end{bmatrix}$
$Z_1X_2Z_3 = \begin{bmatrix} c_1c_3 - c_2s_1s_3 & -c_1s_3 - c_2c_3s_1 & s_1s_2 \\ c_3s_1 + c_1c_2s_3 & c_1c_2c_3 - s_1s_3 & -c_1s_2 \\ s_2s_3 & c_3s_2 & c_2 \end{bmatrix}$	$Z_1X_2Y_3 = \begin{bmatrix} c_1c_3 - s_1s_2s_3 & -c_2s_1 & c_1s_3 + c_3s_1s_2 \\ c_3s_1 + c_1s_2s_3 & c_1c_2 & s_1s_3 - c_1c_3s_2 \\ -c_2s_3 & s_2 & c_2c_3 \end{bmatrix}$

3D Coordinate Transformation

- World frame (fixed)
- Body frame (moving)



The two coordinate systems coincide.

P is a fixed point in body frame.

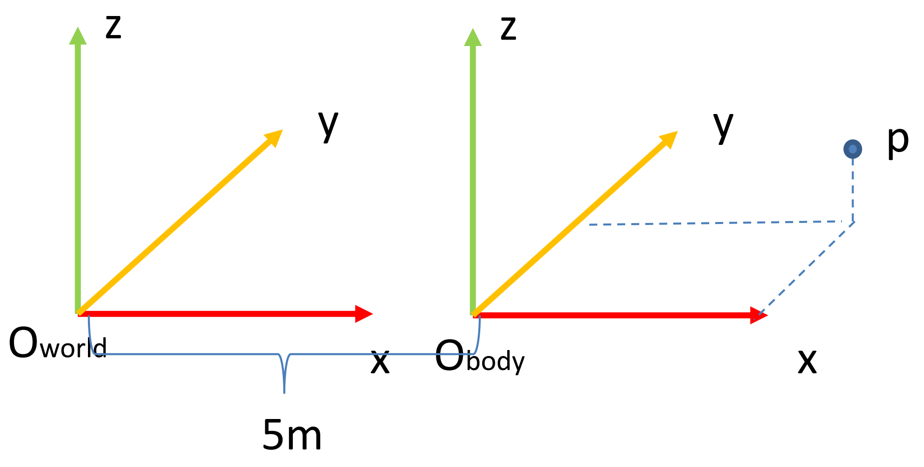
$$ {}^{b}p = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} $$

$$ {}^{w}p = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}^{w}R_b} {}^{b}p + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{{}^{w}t_b} \qquad {}^{w}p = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} $$

3D Coordinate Transformation

- World frame (fixed)
- Body frame (moving)

P is a fixed point in body frame.



$$^{b}p = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

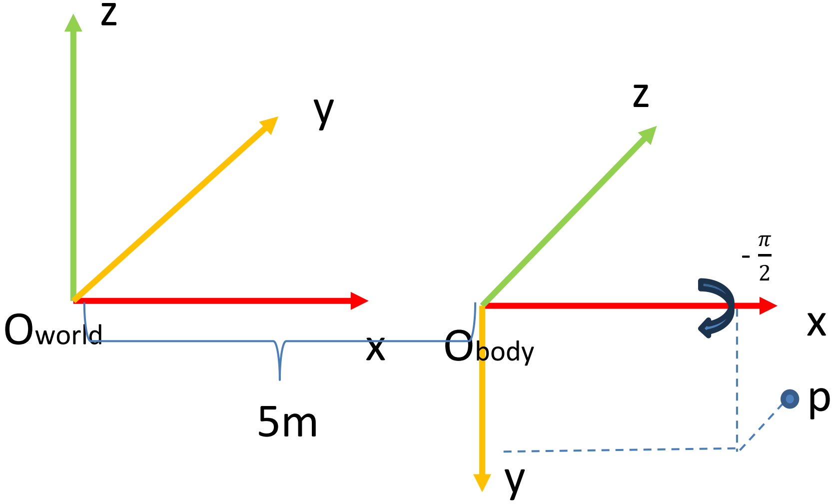
$$^{w}p = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{^{w}R_b} {}^{b}p + \underbrace{\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}}_{^{w}t_b} \qquad {}^{w}p = \begin{bmatrix} 8 \\ 2 \\ 1 \end{bmatrix}$$

3D Coordinate Transformation

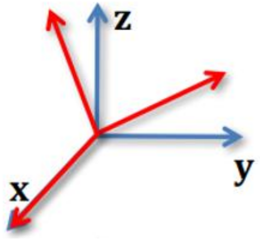
- World frame (fixed)
- Body frame (moving)

P is a fixed point in body frame.

$$^{b}p = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$



$$\boldsymbol{R}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

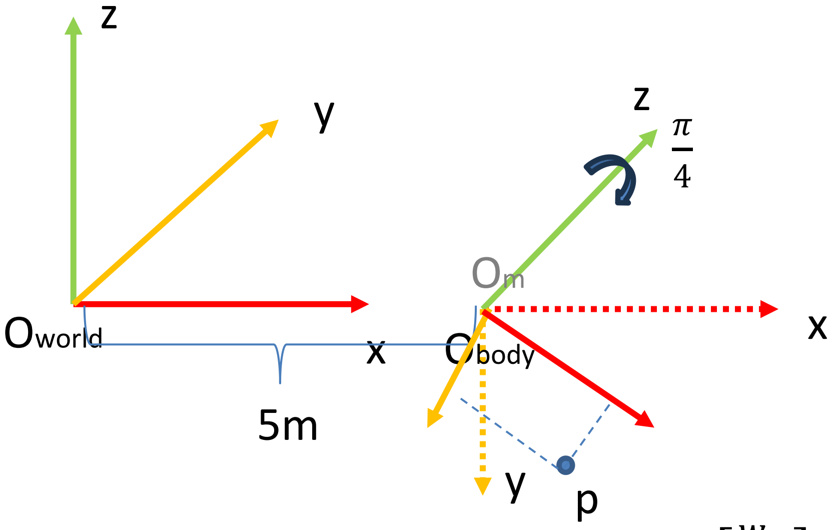


$$^{w}p = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}}_{^{w}R_b} {}^{b}p + \underbrace{\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}}_{^{w}t_b} \qquad {}^{w}p = \begin{bmatrix} 8 \\ 1 \\ -2 \end{bmatrix}$$

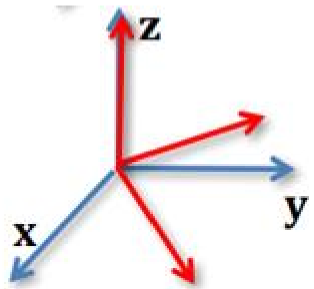
3D Coordinate Transformation

- World frame (fixed)
- Body frame (moving)

P is a fixed point in body frame.



$$\boldsymbol{R}_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$${}^{b}p = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

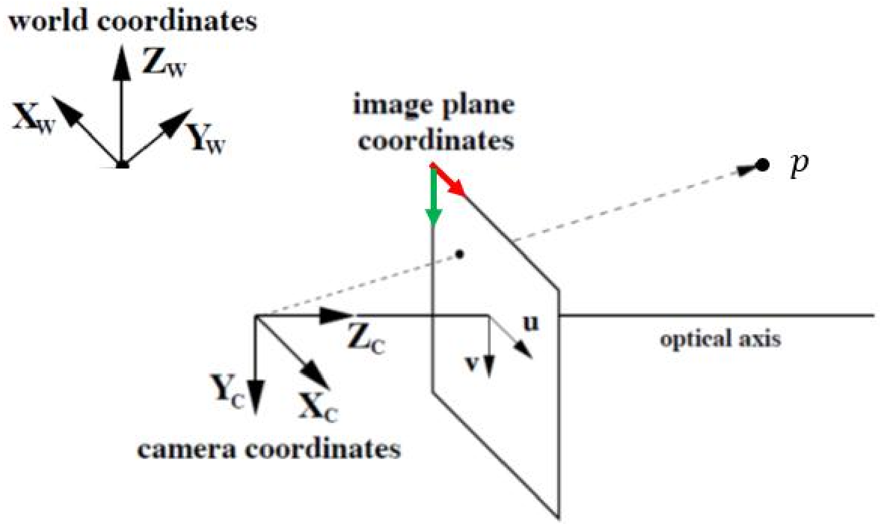
$$\begin{bmatrix} {}^{w}p \\ 1 \end{bmatrix} = \overbrace{\begin{bmatrix} {}^{w}\boldsymbol{R}_{b_m} & {}^{w}t_{b_m} \\ \boldsymbol{0} & 1 \end{bmatrix}}^{{}^{w}\boldsymbol{T}_{b_m}} \overbrace{\begin{bmatrix} {}^{b_m}\boldsymbol{R}_{b} & {}^{b_m}t_{b} \\ \boldsymbol{0} & 1 \end{bmatrix}}^{{}^{b_m}\boldsymbol{T}_{b}} \begin{bmatrix} {}^{b}p \\ 1 \end{bmatrix}$$

$${}^{b_m}p = \underbrace{\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}^{b_m}\boldsymbol{R}_{b}} {}^{b}p + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{{}^{b_m}t_{b}}$$

$${}^{w}p = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}}_{{}^{w}\boldsymbol{R}_{b_m}} {}^{b_m}p + \underbrace{\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}}_{{}^{w}t_{b_m}} = \begin{bmatrix} 5 + \frac{\sqrt{2}}{2} \\ 1 \\ -\frac{5\sqrt{2}}{2} \end{bmatrix}$$

Perspective Projection

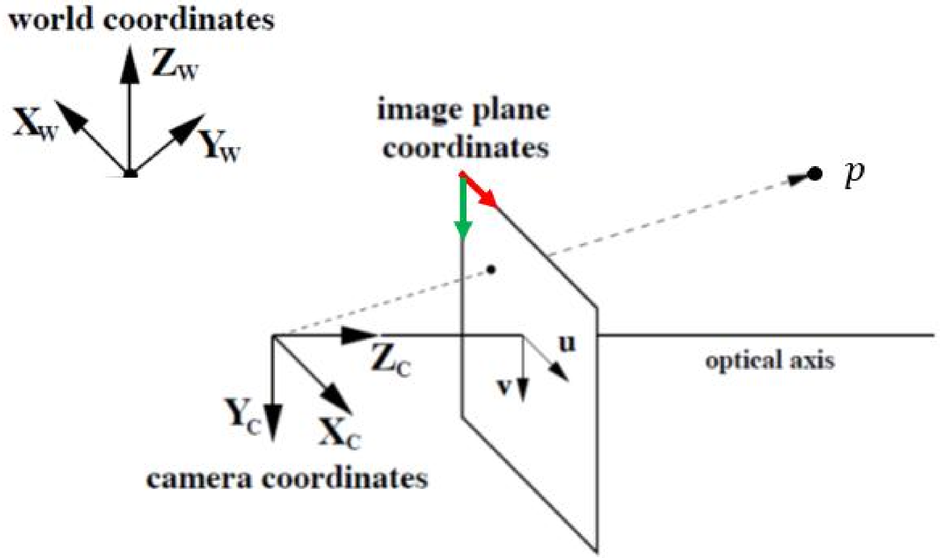
- Optical axis is the z-axis
- The image plane (u, v) is perpendicular to the optical axis
- Intersection of the image plane and the optical axis is the image center (u_0, v_0)
- f is the distance of the image plane from the origin (in pixels)
- Point in Camera Coordinate: ${}^{c}p = [x_c, y_c, z_c]$
- Formulation:
  - $u = \frac{f x_c}{z_c} + u_0$
  - $v = \frac{f y_c}{z_c} + v_0$



Perspective Projection

- Matrix form:

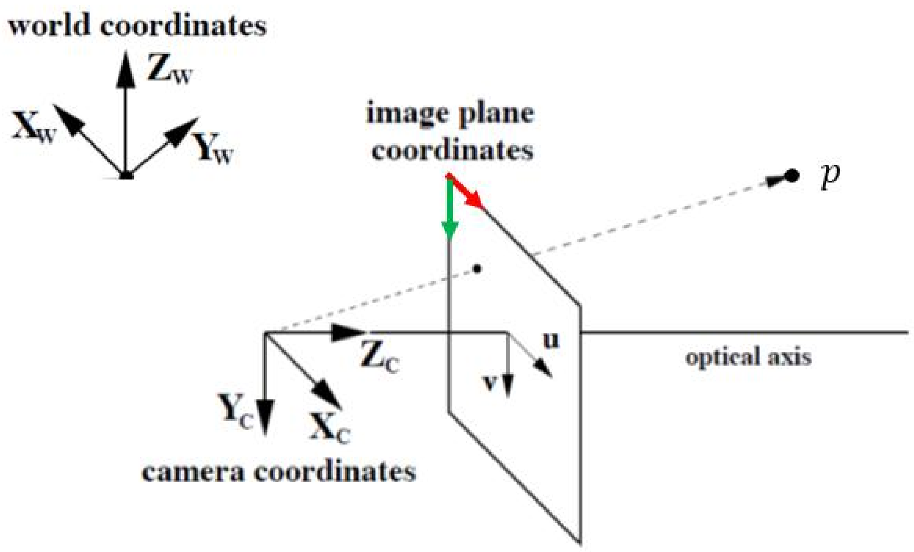
$$\lambda\begin{pmatrix}u\\v\\1\end{pmatrix}=\begin{pmatrix}f & 0 & u_o\\0 & f & v_o\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_c\\y_c\\z_c\end{pmatrix}$$



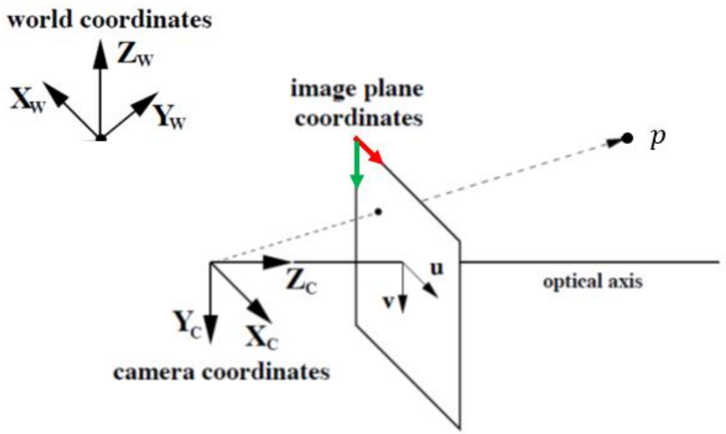
Perspective Projection

- From camera to world:

$$\begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = \begin{pmatrix} {}^c\boldsymbol{R}_w & {}^c\boldsymbol{t}_w \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_w \\ y_w \\ z_w \\ 1 \end{pmatrix}$$



Perspective Projection

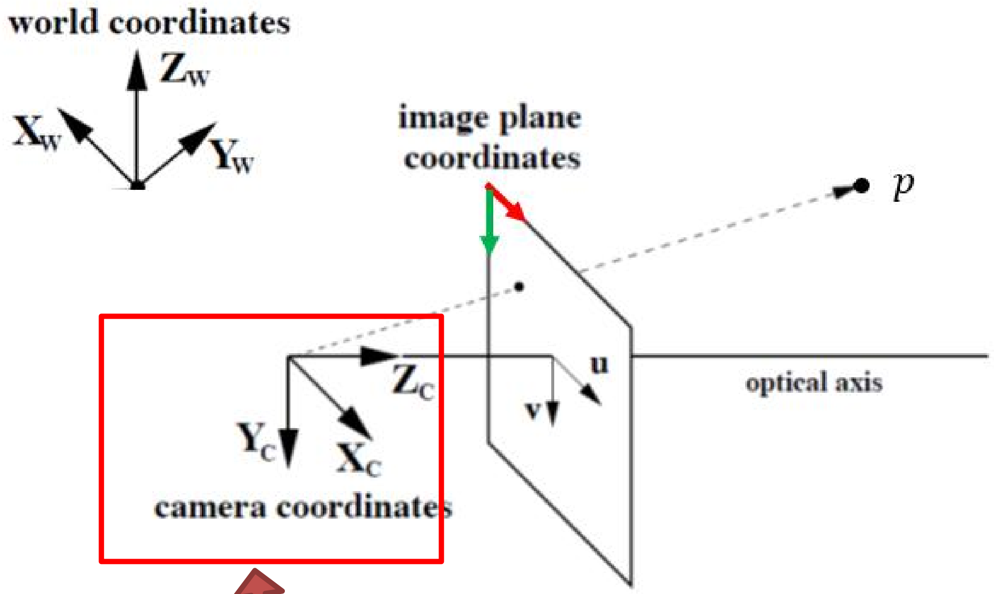


$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & u_o \\ 0 & f & v_o \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix}$$

- From camera to world:

$$\begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} {}^c\boldsymbol{R}_w & {}^c\boldsymbol{t}_w \\ 0 & 1 \end{pmatrix}} \begin{pmatrix} x_w \\ y_w \\ z_w \\ 1 \end{pmatrix}$$

Pin-hole Camera Model



$$\boxed{\lambda} \boxed{\begin{pmatrix} u \\ v \\ 1 \end{pmatrix}} = \boxed{\begin{pmatrix} f & 0 & u_o \\ 0 & f & v_o \\ 0 & 0 & 1 \end{pmatrix}} \boxed{\begin{pmatrix} {}^{c}\boldsymbol{R}_{w} & {}^{c}\boldsymbol{t}_{w} \end{pmatrix}} \boxed{\begin{pmatrix} x_w \\ y_w \\ z_w \\ 1 \end{pmatrix}} = P \begin{pmatrix} x_w \\ y_w \\ z_w \\ 1 \end{pmatrix}$$

Depth

Pixel Values

Intrinsic Calibration Matrix

(Inverse) Camera Pose

${}^{w}\boldsymbol{R}_{c}$ ${}^{w}\boldsymbol{t}_{c}$

World Point

Content

1. Camera Background
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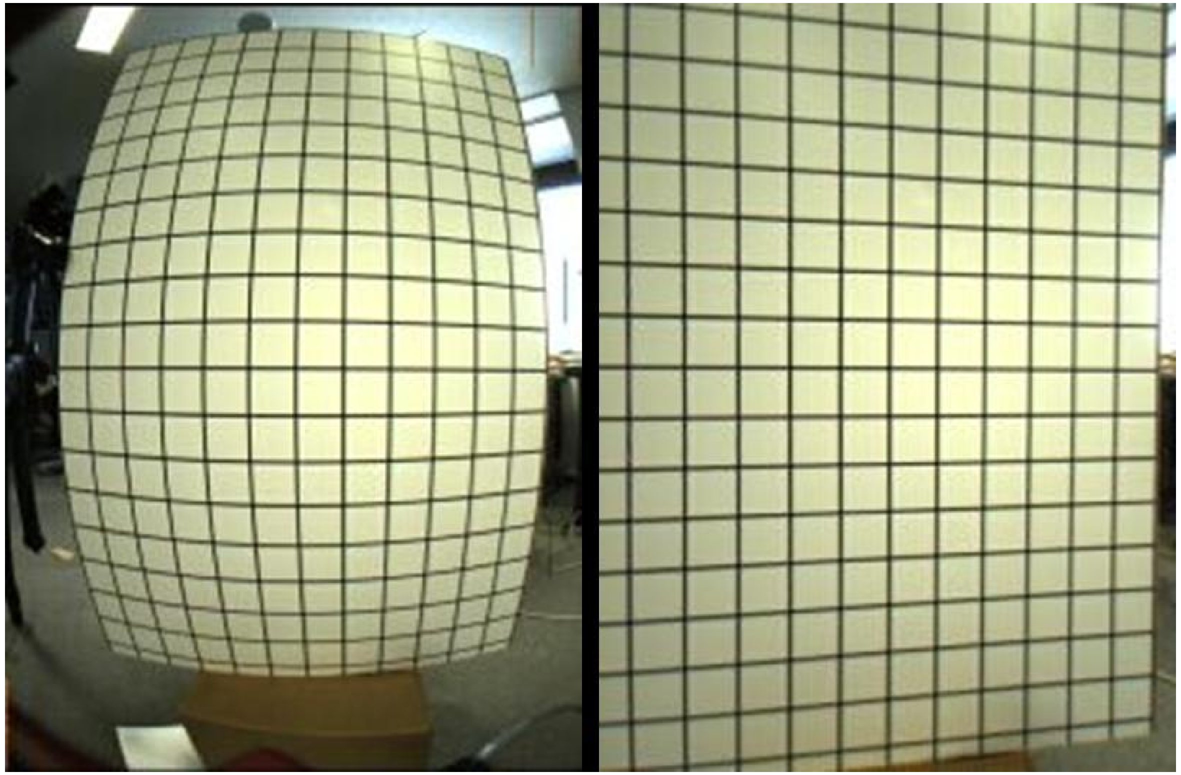
6. Assignment

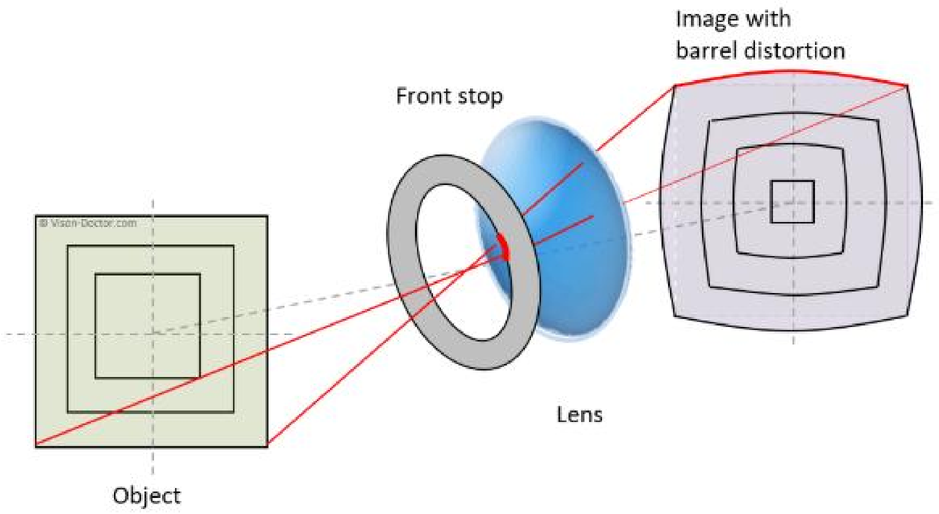
Camera Calibration

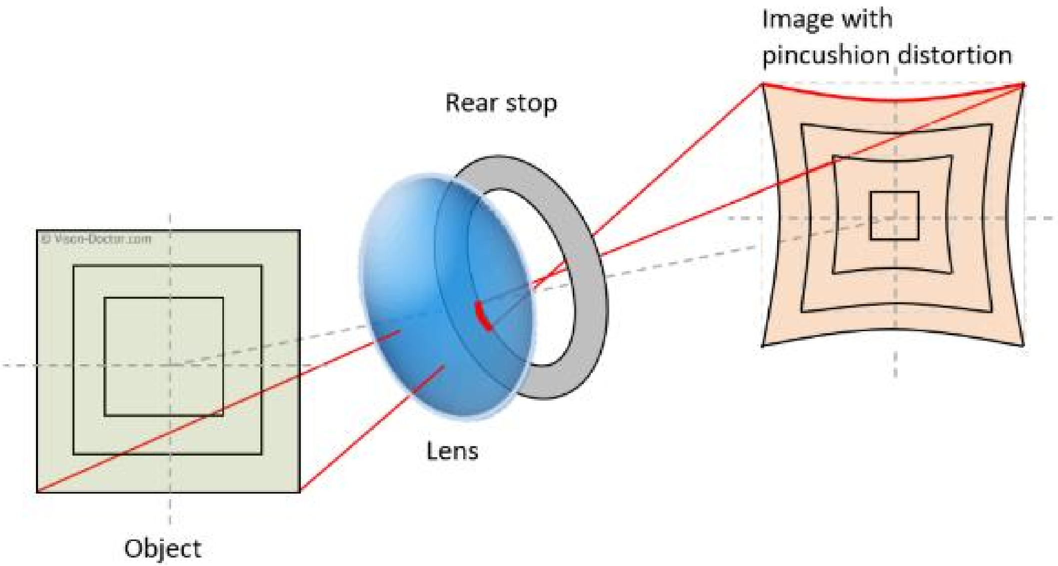
- Straight lines should be straight



Distortion model

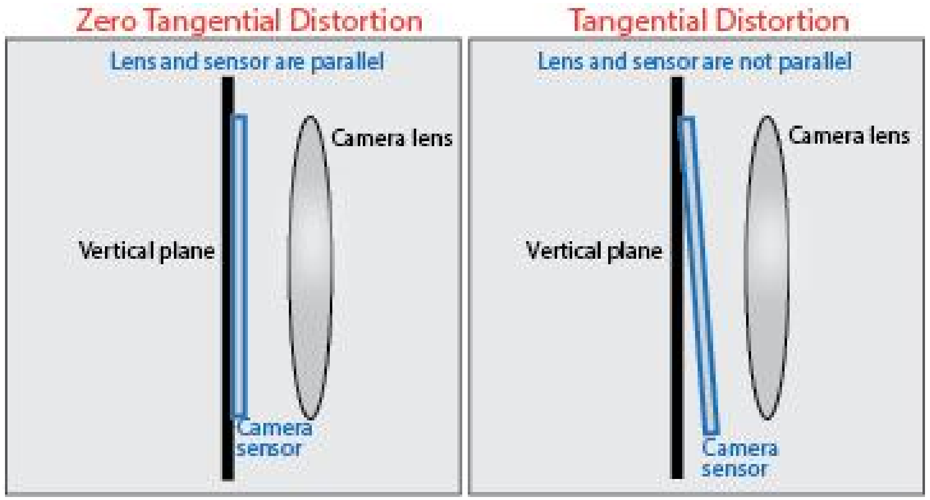
- Tangential and radial distortion model
  - Radial distortion

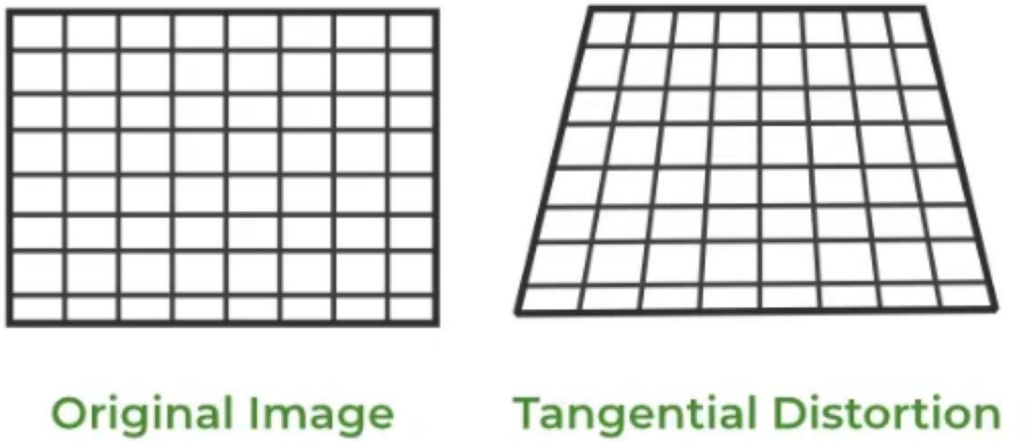




Distortion model

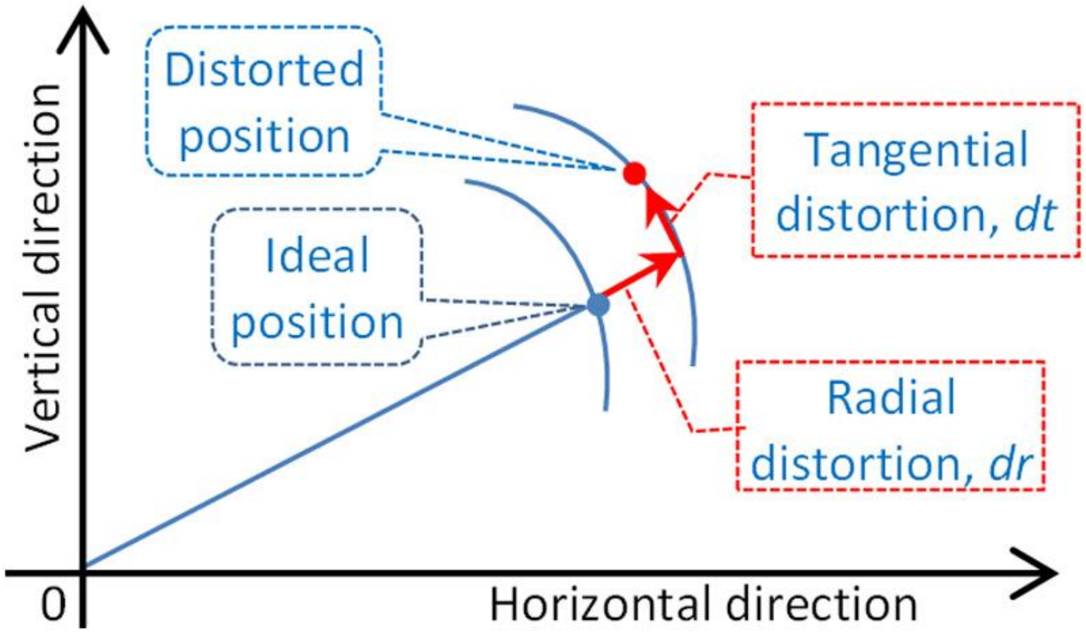
- Tangential and radial distortion model
  - Tangential distortion





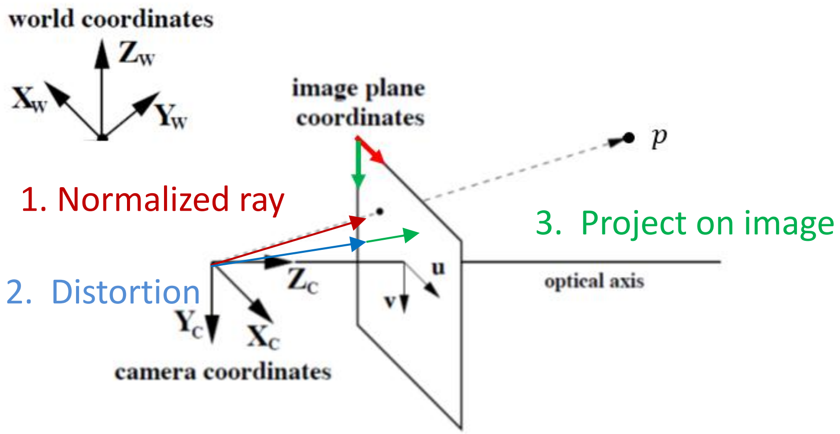
Camera Calibration

- Tangential and radial distortion model



Distortion model

- Tangential and radial distortion model
  - $k1,\ k2,\ [k3,\ [k4,\ [k5,\ [k6]]]]$ are the radial coefficients
  - $p1$ and $p2$ are the tangential distortion coefficients



1. Project to unit plane (z = 1)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x_c/z_c \\ y_c/z_c \end{bmatrix}$$

2. Distortion:
(Distortion coefficient D)

$$r^2 = x'^2 + y'^2$$

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x' \frac{1+k_1 r^2 + k_2 r^4 + k_3 r^6}{1+k_4 r^2 + k_5 r^4 + k_6 r^6} + 2p_1 x' y' + p_2(r^2 + 2x'^2) \\ y' \frac{1+k_1 r^2 + k_2 r^4 + k_3 r^6}{1+k_4 r^2 + k_5 r^4 + k_6 r^6} + p_1(r^2 + 2y'^2) + 2p_2 x' y' \end{bmatrix}$$

3. Image Projection:
(Camera matrix K)

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f_x x'' + c_x \\ f_y y'' + c_y \end{bmatrix}$$

Projection Example

- Pinhole + Tangential and radial Distortion

$$ {}^{c}p = [6,\ 4,\ 2] $$


```yaml
%YAML:1.0
---
calibration_Time: "2024年12月12日 星期四 16时32分21秒"
nrOfFrames: 57
image_Width: 640
image_Height: 480
board_Width: 7
board_Height: 5
square_Size: 1.0000000149011612e-01
Camera_Matrix: !!opencv-matrix
   rows: 3
   cols: 3
   dt: d
   data: [ 5.6865194941315929e+02, 0., 3.2513762614210782e+02, 0.,
       7.6375465300688415e+02, 2.3177702561676526e+02, 0., 0., 1. ]
Distortion_Coefficients: !!opencv-matrix
   rows: 1
   cols: 5
   dt: d
   data: [ 1.1726086573014034e-02, -5.2021897536855632e-01,
       -4.8681362994895585e-03, 9.6693728067371231e-04,
       2.2217257704057318e+00 ]
```


camera_calib.yaml

$$ \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} $$

$$ (k_1 \quad k_2 \quad p_1 \quad p_2 \quad k_3) $$

$$ \begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x' \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6} + 2p_1x'y' + p_2(r^2+2x'^2) \\ y' \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6} + p_1(r^2+2y'^2) + 2p_2x'y' \end{bmatrix} $$

1. Project to unit plane : $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x_c/z_c \\ y_c/z_c \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

2. Distortion:$r^2 = 9 + 4 = 13$

$$ \begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x'(1 + k_1r^2 + k_2r^4 + k_3r^6) + 2p_1x'y' + p_2(r^2 + 2x'^2) \\ y'(1 + k_1r^2 + k_2r^4 + k_3r^6) + p_1\left(r^2 + 2y'^2\right) + 2p_2x'y' \end{bmatrix} $$

3. Image Projection:

$$ \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f_x x'' + c_x \\ f_y y'' + c_y \end{bmatrix} = \begin{bmatrix} 568.65\, x'' + 325.13 \\ 763.75 y'' + 231.77 \end{bmatrix} $$

Content

1. Camera Background
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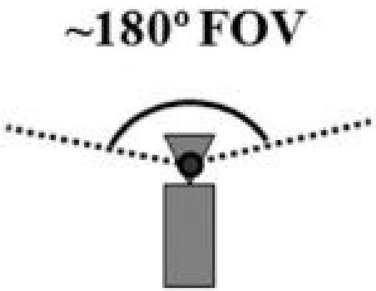
6. Assignment

Fisheye Camera

- Dioptric camera

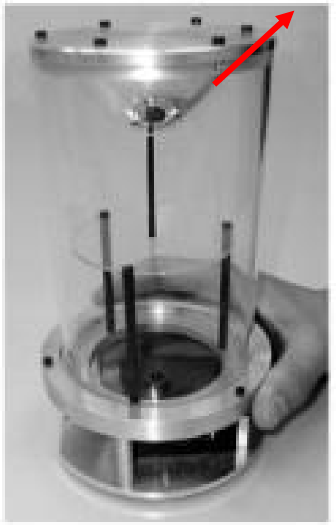
  折射

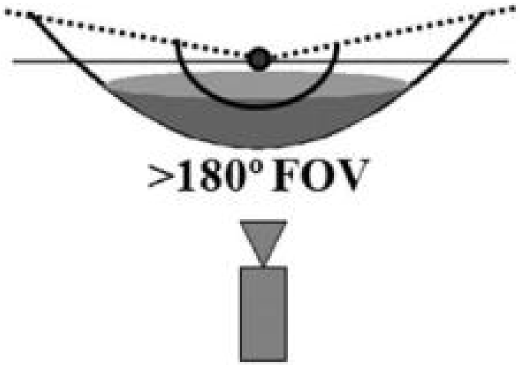


- Catadioptric camera

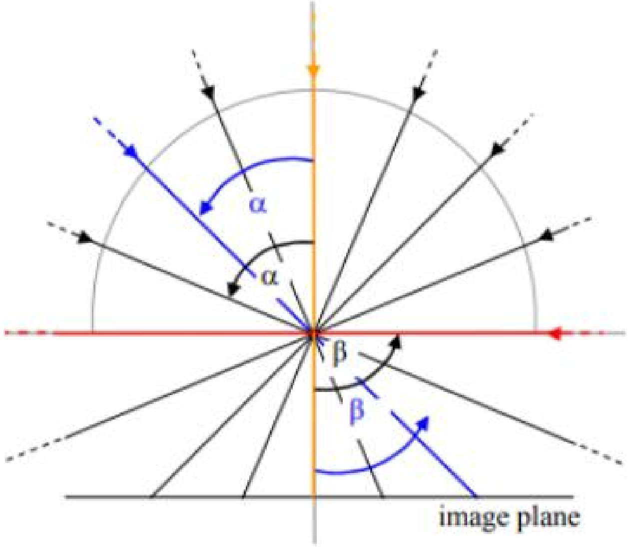
  反射折射

mirror

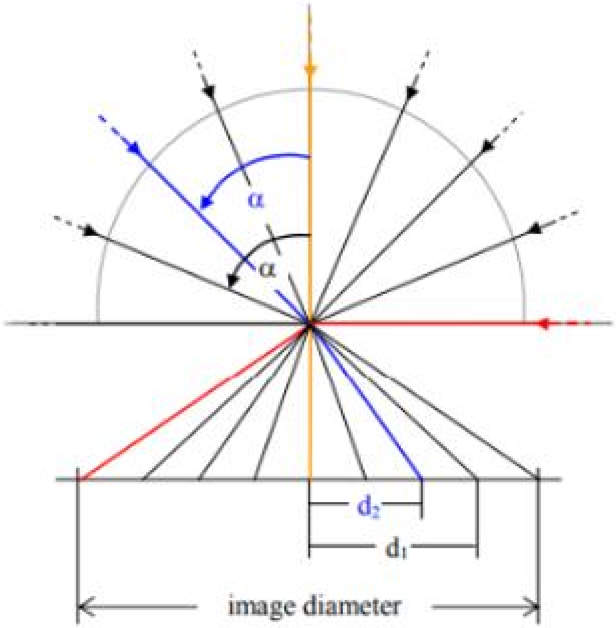




Fisheye Camera Model

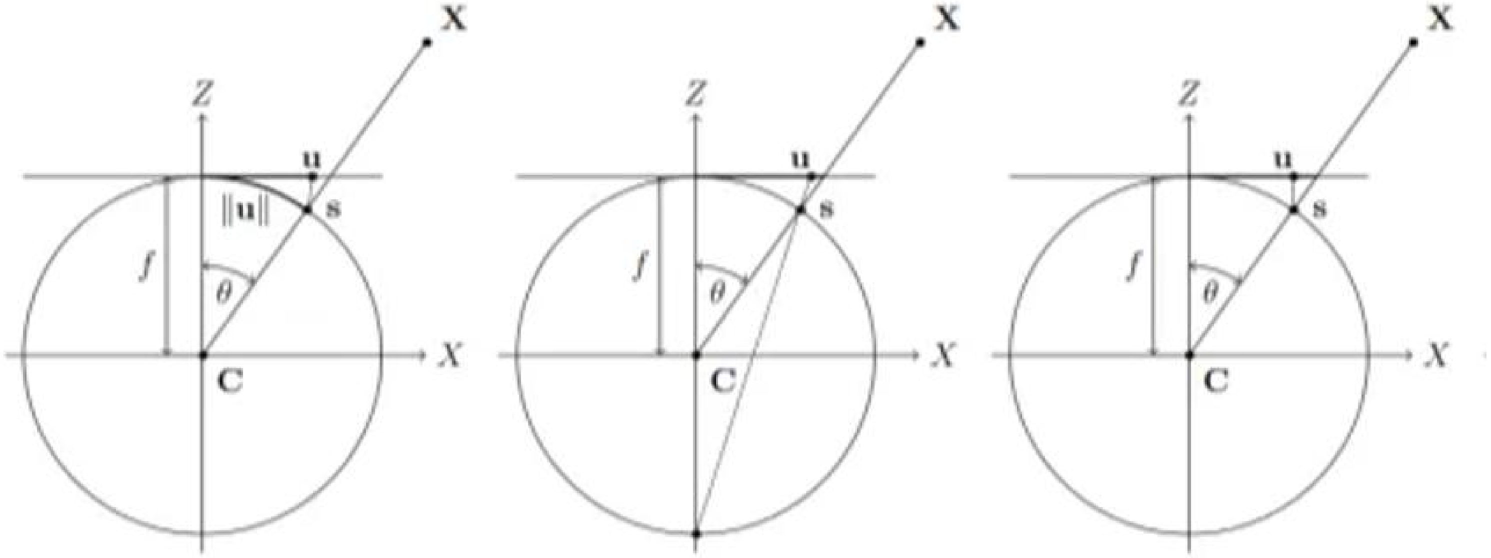


Pinhole perspective model



Fisheye perspective model

Classical Fisheye Camera Models



... ...

(a) Equidistant
等距

$$u = f\theta$$

(b) Stereographic
体视

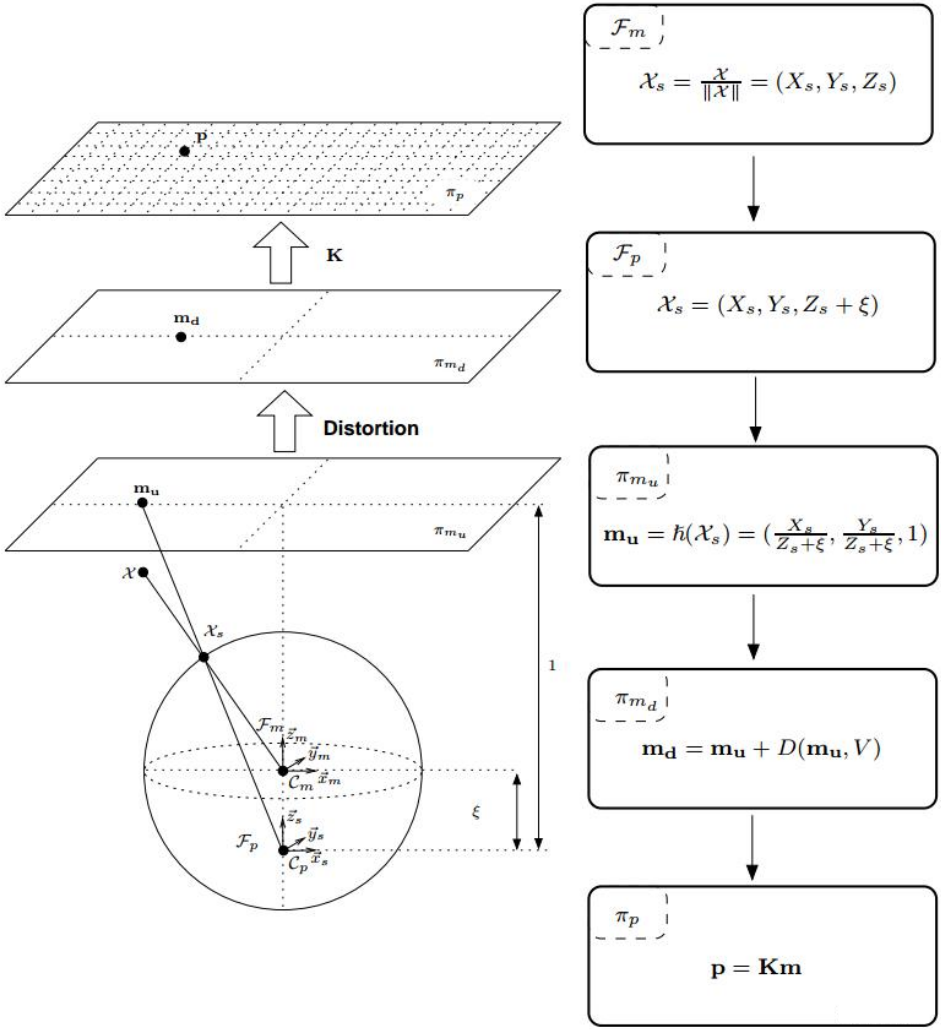
$$u = 2f\tan\frac{\theta}{2}$$

(c) Orthographic
正交

$$u = f\sin\theta$$

OpenCV::Fisheye

MEI [1] Camera Model



1. 在 mirror 坐标系上的世界点映射到单位圆上：

$$(\mathcal{X})_{\mathcal{F}_m} \rightarrow (\mathcal{X}_s)_{\mathcal{F}_m} = \frac{\mathcal{X}}{\|\mathcal{X}\|} = (X_s, Y_s, Z_s)$$

2. 将这些点转换为以 $\mathcal{C}_p = (0, 0, \xi)$ 为中心的新参考系上：

$$(\mathcal{X}_s)_{\mathcal{F}_m} \rightarrow (\mathcal{X}_s)_{\mathcal{F}_p} = (X_s, Y_s, Z_s + \xi)$$

3. 将它们投影到归一化图像平面上：

$$\mathbf{m}_u = \left(\frac{X_s}{Z_s + \xi}, \frac{Y_s}{Z_s + \xi}, 1\right) = \hbar(\mathcal{X}_s)$$

4. 添加上径向和切向畸变：

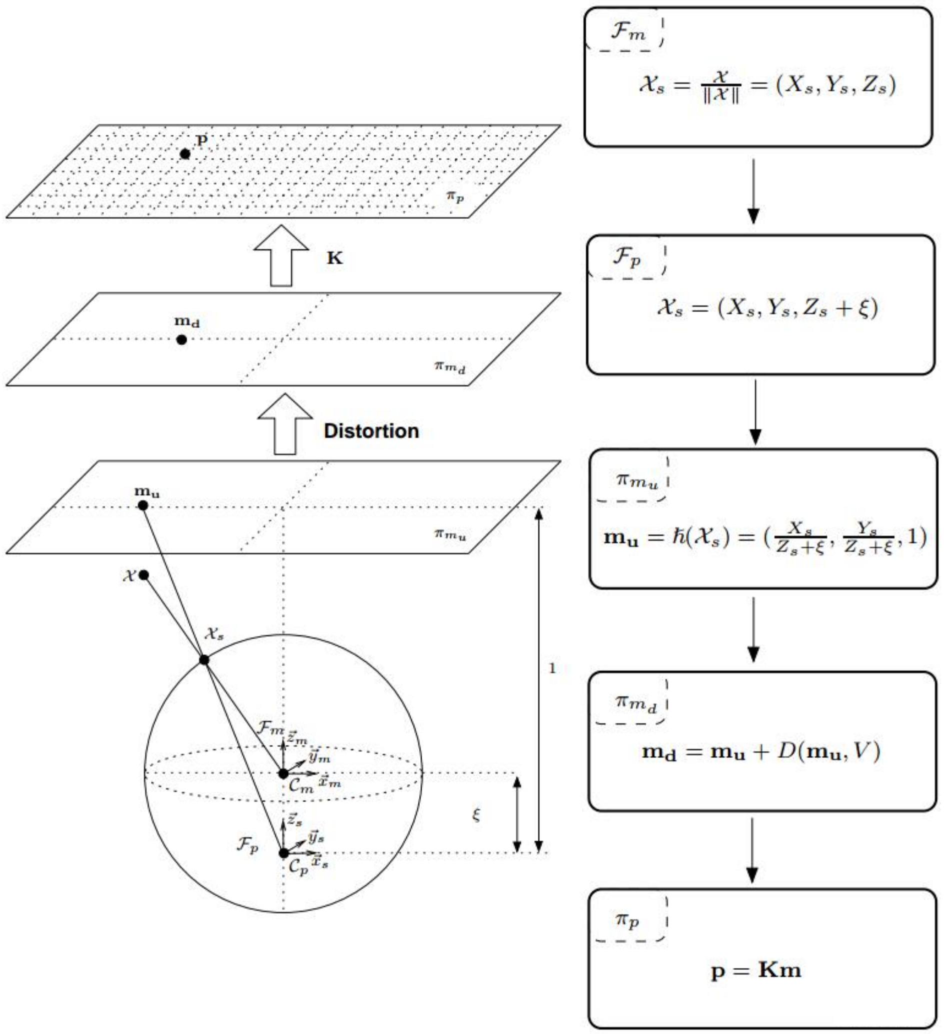
$$\mathbf{m}_d = \mathbf{m}_u + D(\mathbf{m}_u, V)$$

5. 最终投影涉及一个广义相机投影矩阵 $\mathbf{K}$(其中 γ 为广义焦距，(u_0, v_0) 为主点，s 斜度)：

$$\mathbf{p} = \mathbf{K}\mathbf{m} = \begin{bmatrix} \gamma & \gamma s & u_0 \\ 0 & \gamma r & v_0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{m} = k(\mathbf{m})$$

[1] C. Mei and P. Rives, "Single View Point Omnidirectional Camera Calibration from Planar Grids," *Proceedings 2007 IEEE International Conference on Robotics and Automation*, Rome, Italy, 2007, pp. 3945-3950, doi: 10.1109/ROBOT.2007.364084.

Example: MEI Camera Model



0_intrinsic.yaml


```yaml
%YAML:1.0
---
model_type: MEI
camera_name: camera
image_width: 1920
image_height: 1080
mirror_parameters:
   xi: 1.0
distortion_parameters:
   k1: 0.0
   k2: 0.0
   p1: 0.0
   p2: 0.0
projection_parameters:
   gamma1: 935.307436087
   gamma2: 935.307436087
   u0: 960
   v0: 540
```


$$^{c}p = [2,\ 2,\ 1]$$

1. Normalized ray vector:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x_c/\|p\| \\ y_c/\|p\| \\ z_c/\|p\| \end{bmatrix} = \begin{bmatrix} 0.67 \\ 0.67 \\ 0.33 \end{bmatrix}$$

2. Change to C_p cooridinate:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' + \xi \end{bmatrix} = \begin{bmatrix} 0.67 \\ 0.67 \\ 1.33 \end{bmatrix}$$

3. Project to unit plane:

$$\begin{bmatrix} x''' \\ y''' \end{bmatrix} = \begin{bmatrix} x''/z'' \\ y''/z'' \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

4. Distortion:

5. Image projection:

$$\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} f_x x''' + c_x \\ f_y y''' + c_y \end{bmatrix} = \begin{bmatrix} 935.3\ x''' + 960 \\ 935.3\ y''' + 540 \end{bmatrix} = \begin{bmatrix} 1427.66 \\ 1007.66 \end{bmatrix}$$

Content

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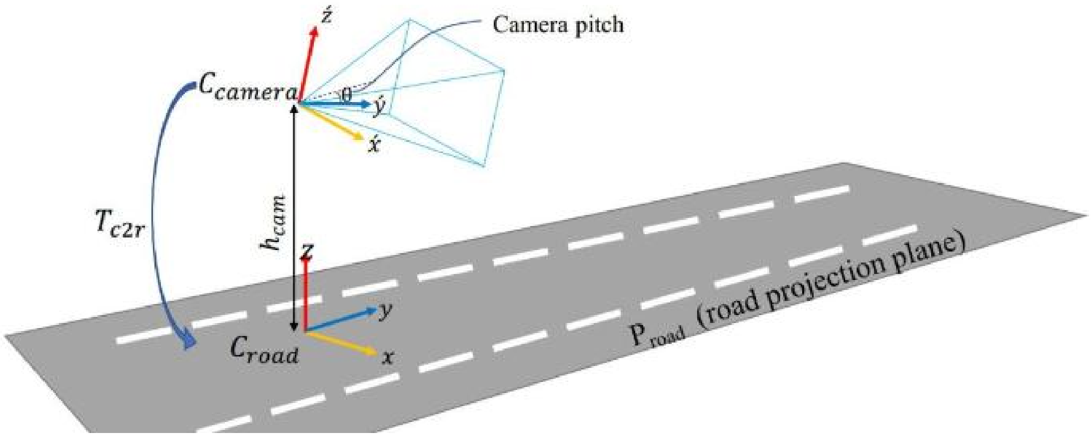
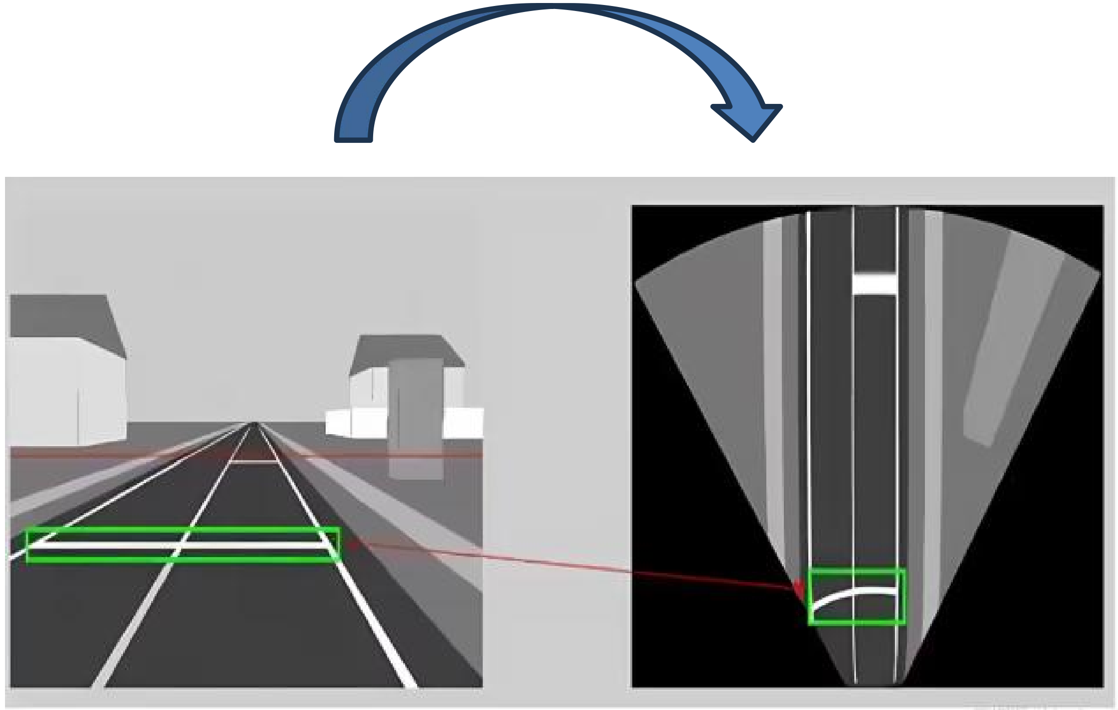
IPM Image

- IPM (Inverse Projective Mapping)



Inverse Projective



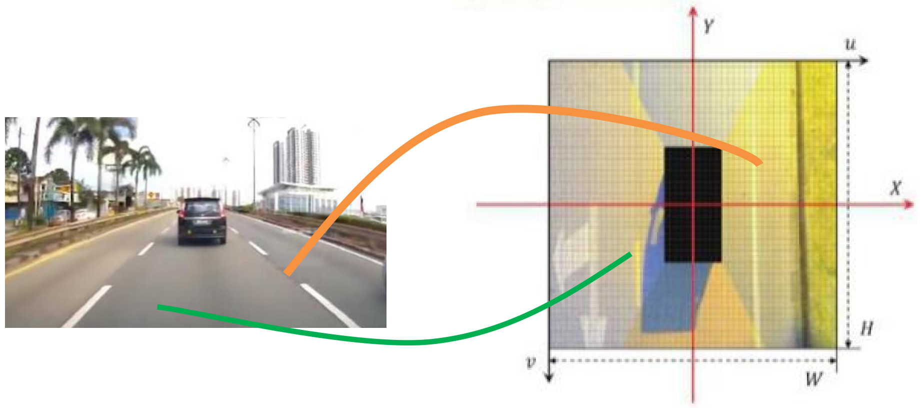
IPM Image

Method I:

r, g, b →

$(u,\ v)$ $\xrightarrow[\text{Inverse projection}]{\text{Camera model}}$ $\lambda(x_c, y_c, 1)$ $\xrightarrow{{}^{v}R_c\ ,\ {}^{v}t_c}$ $(x_v, y_v, 0)$ → (u_{ipm}, v_{ipm})

Raw image plane | Camera coordinate | Vehicle coordinate | IPM plane



Method II:

← r, g, b

(u_{ipm}, v_{ipm}) → $(x_v, y_v, 0)$ $\xrightarrow{{}^{c}R_v\ ,\ {}^{c}t_v}$ (x_c, y_c, z_c) $\xrightarrow[\text{projection}]{\text{Camera model}}$ $(u,\ v)$

IPM plane | Vehicle coordinate | Camera coordinate | Raw image plane

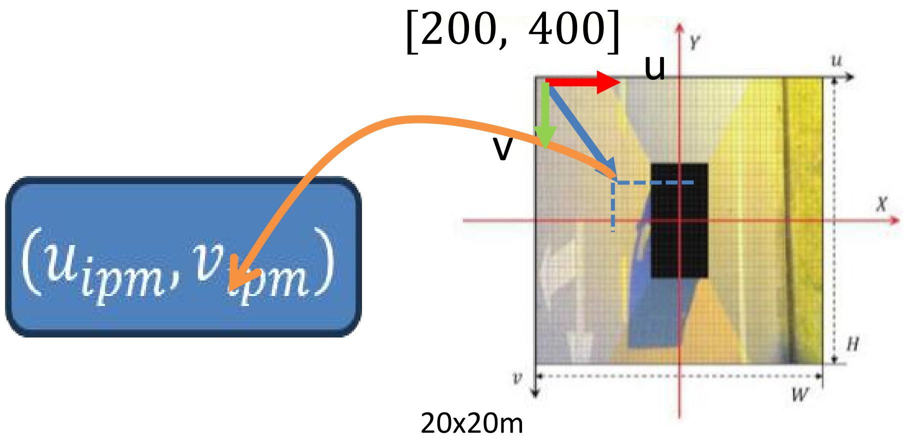
IPM Image

Method II:

$$(u_{ipm}, v_{ipm}) \longrightarrow (x_v, y_v, 0) \xrightarrow{{}^cR_v,\ {}^ct_v} (x_c, y_c, z_c) \xrightarrow[\text{projection}]{\text{Camera model}} (u, v)$$

r, g, b

IPM plane | Vehicle coordinate | Camera coordinate | Raw image plane



20x20m
1000x1000 pixel
0.02m/pixel

1. Choose one point on IPM plane, for example: $\begin{bmatrix} u_{ipm} \\ v_{ipm} \end{bmatrix} = \begin{bmatrix} 200 \\ 400 \end{bmatrix}$

2. Vehicle coordinate: $\begin{bmatrix} x_v \\ y_v \\ z_v \end{bmatrix} = \begin{bmatrix} -(500 - u_{ipm}) * 0.02 \\ (500 - v_{ipm}) * 0.02 \\ 0 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \\ 0 \end{bmatrix}$

3. Camera coordinate: $\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} {}^cR_v & {}^ct_v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} = \begin{bmatrix} {}^vR_c & {}^vt_c \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$

4. Project to image plane: $\begin{bmatrix} u \\ v \end{bmatrix} = \pi(p_c, K, D)$

0_extrinsic.yaml

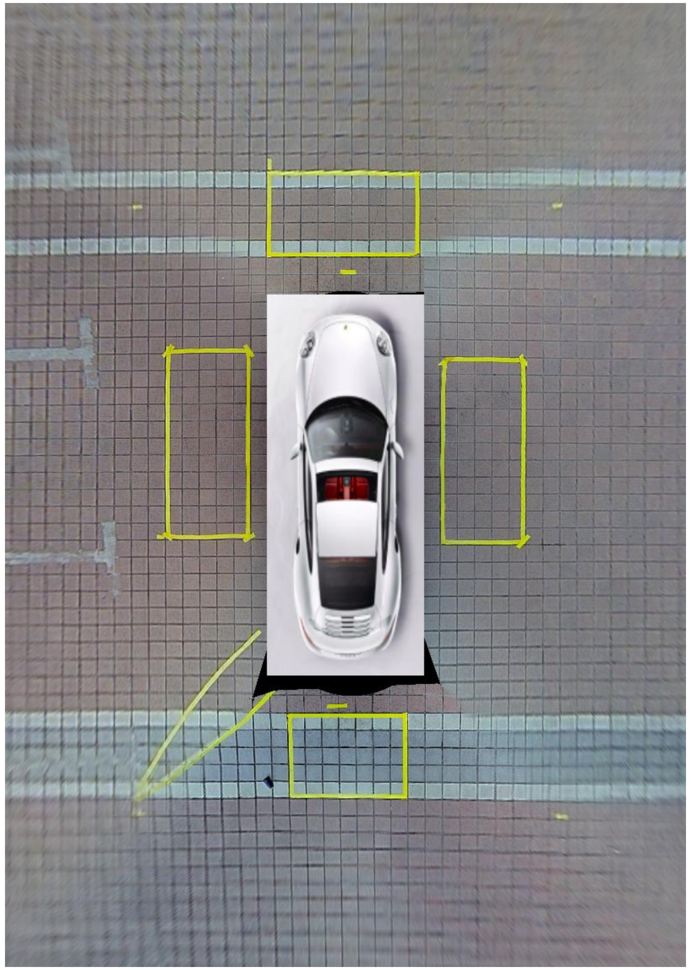

```yaml
%YAML:1.0
---
frame_id: cam0
transform:
  translation:
    x: 0.0
    y: 2.5
    z: 1.05
  rotation:
    x: -0.866
    y: 0.0
    z: 0.0
    w: 0.5
```


${}^vt_c$

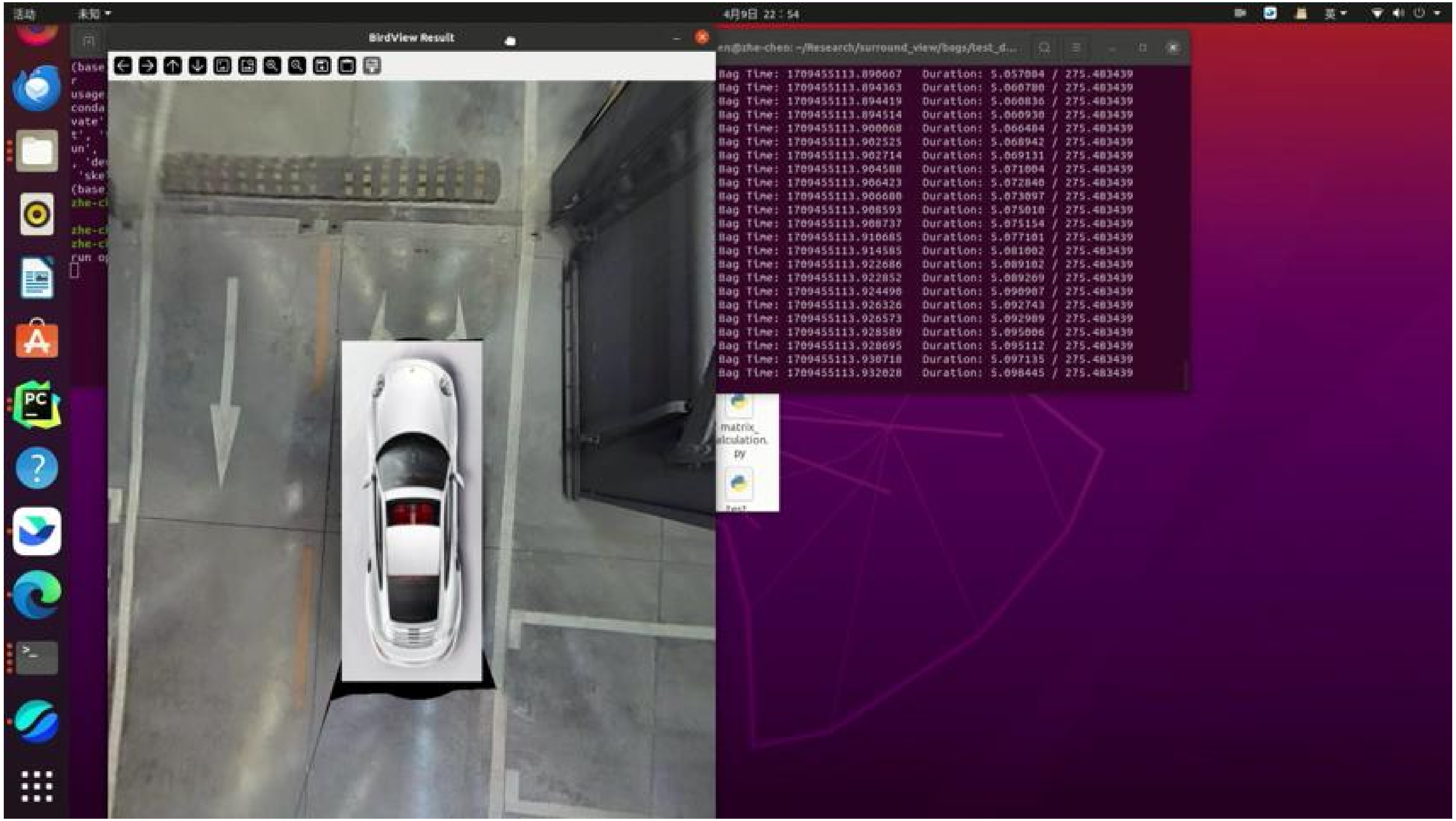
${}^vR_c$

IPM Image Stitching

Weight fusion



IPM Image Stitching



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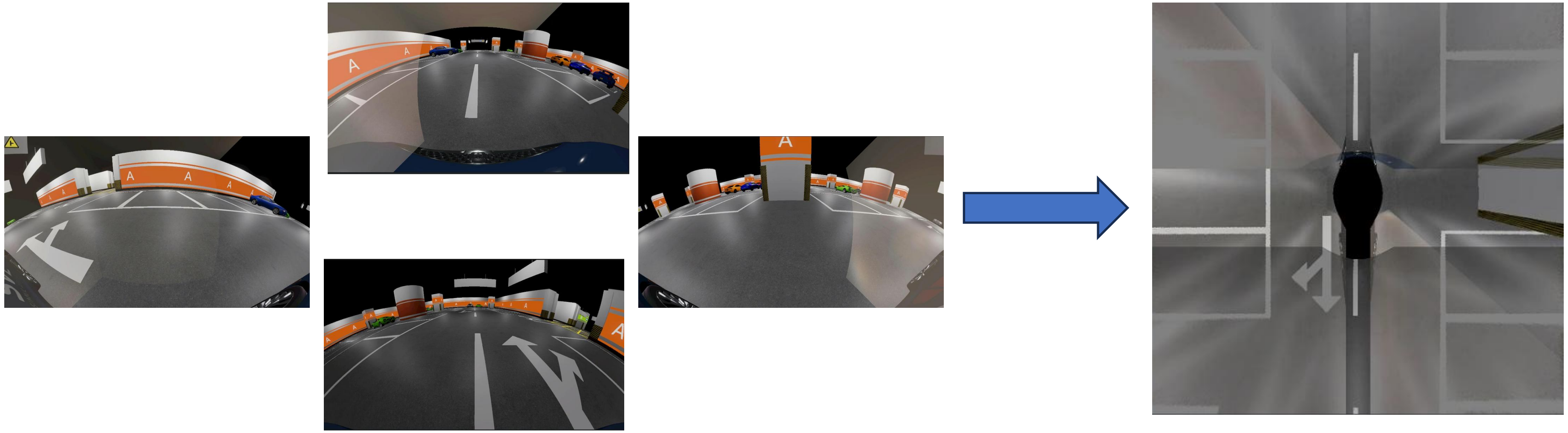
Assignment

Implement the IPM generation:

- Complete the function *GenerateIPMImage()*

```
cv::Mat IPM::GenerateIPMImage(const std::vector<cv::Mat>& images) const {
```

- Expected result:



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shenlanxueyuan.com

感谢聆听!

Thanks for Listening