CSCE 221 Cover Page
Homework Assignment #2
Due March 27 at 23:59 pm to eCampus

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Please list all sources in the table below including web pages which you used to solve or implement the current homework. If you fail to cite sources you can get a lower number of points or even zero, read more Aggie Honor System Office http://aggiehonor.tamu.edu/

Type of sources	web sources	
People		
Web pages (provide URL)	https://www.cise.ufl.edu/class/cot3100fa07/quicksort_analysis.pdf	http://www.cs.princeto
Printed material		
Other Sources		

I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work.

"On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work."

Your Name Jialu Zhao Date 03/16/2017

Homework 2

due March 27 at 11:59 pm to eCampus.

1.(15 points) Write a recursive function that counts the number of nodes in a singly linked list. Write a recurrence relation that represents your algorithm. Solve the relation and obtain the running time of the algorithm in Big-O.

recursive function:

```
int count(node* Iter)
{
if(Iter == NULL)
return 0;
return 1+count(Iter->next);
}
```

recurrence relations:

$$F_n = F_{n-1} + 1$$

initial value:

$$F_0 = 0$$

solve the relation:

$$F_n = F_{n-1} + 1 = F_{n-2} + 1 + 1 = F_{n-3} + 1 + 1 + 1 = ... = F_0 + 1 + 1 + ... + 1 = F_0 + n = 0 + n = n$$

running time:

$$O(n)$$

2.(15 points) Write a recursive function that finds the maximum value in an array of int values without using any loops. Write a recurrence relation that represents your algorithm. Solve the relation and obtain the running time of the algorithm in Big-O.

recursive function:

```
int findMaximum(int array[], int low, int high)
{
if(low == high)
return array[low];
return max(array[low],findMaximum(array,low+1,high));
}
```

note:

low = 0;

high = the size of array - 1;

recurrence relations:

$$F_n = F_{n-1} + 1;$$

00

Note:F_n represents the times of do max function

initial value:

$$F_0 = 0$$

solve the relation:

$$F_n = F_{n-1} + 1 = F_{n-2} + 1 + 1 = F_{n-3} + 1 + 1 + 1 = ... = F_0 + 1 + 1 + ... + 1 = F_0 + n = 0 + n = n$$

running time:

$$O(n)$$

3.(10 points) What data structure is most suitable to determine if a string s is a palindrome, that is, it is equal to its reverse. For example, "racecar" and "gohan gasalamiimalasagnahog" are palindromes. Justify your answer. Use Big-O notation to represent the efficiency of your algorithm.

Stack is most suitable to determine if a string s is a palindrome.

Algorithm:

```
step 1:push every characters in the string to stack -- O(n)
step 2:pop every elements on the stack to a new string -- -- O(n)
step3: if new string is same with the old string, this string is palindrome -- O(n)
```

So the efficiency of this Algorithm:

$$O(n)$$

4.(10 points) Describe how to implement the stack ADT using two queues. What is the running time of the push and pop functions in this case?

Solution:

Use two queues Q1 and Q2, where Q1 stores elements and Q2 is used for auxiliary.

```
bool isEmpty() { return Q1.isEmpty();} --- O(1)
int size() { return Q1.size();} --- O(1)
T top()  --- O(n)
{
while(!Q1.isEmpty())
Q2.enqueue(Q1.dequeue()); // use another queue to help change the sequence
T elem = Q2.first();
while(!Q2.isEmpty())
Q1.enqueue(Q2.dequeue()); // change the sequence back
return elem;
}
T pop()  --- O(n)
{
while(!Q1.isEmpty())
Q2.enqueue(Q1.dequeue()); // use another queue to help change the sequence
T elem = Q2.dequeue();
while(!Q2.isEmpty())
Q1.enqueue(Q2.dequeue()); // change the sequence back
return elem;
}
void push(T elem) { Q1.enqueue(elem);}  --- O(1)
```

5.(10 points) What is the best, worst and average running time of quick sort algorithm? Provide arrangement of the input and the selection of the pivot point at every case. Provide a recursive relation and solution for each case.

(a)Best-case analysis:

$$O(nlogn)$$

The best case of quicksort occurs when the pivot we pick happens to divide the array into two exactly equal parts, in every step.Thus we have k=n/2 and n-k=n/2 for the original array of size n.

$$T(n) = 2T(\frac{n}{2}) + an$$

$$= 2(2T(\frac{n}{4}) + a\frac{n}{2}) + an$$

$$= 4T(\frac{n}{4}) + 2an$$

$$= 2^2T(\frac{n}{2^2}) + 2an$$

.....

$$= 2^iT(\frac{n}{2^i}) + ian$$

this recurrence will stop when i = logn:

$$= nT(1) + anlogn$$

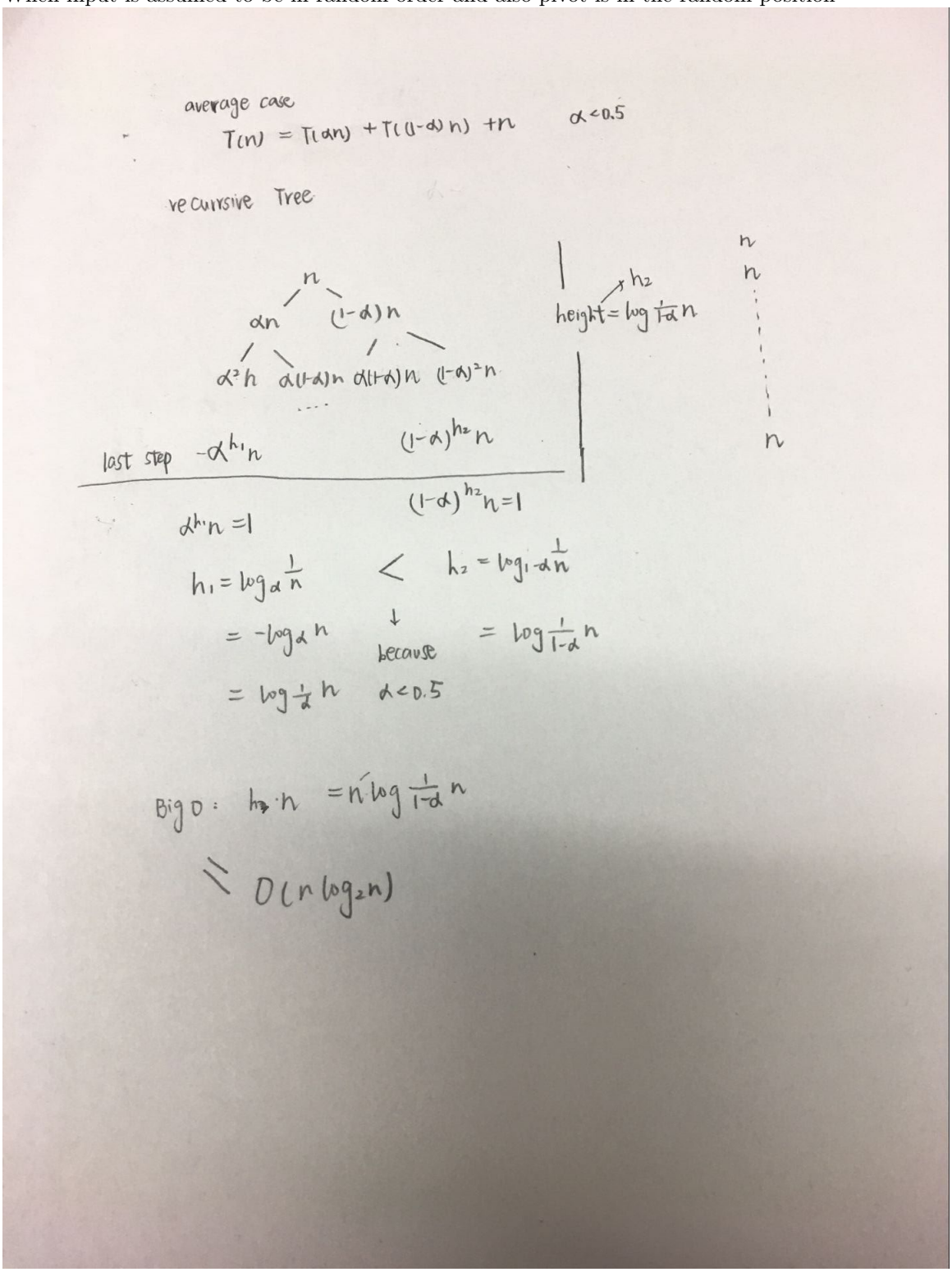
then we got the result:

$$O(nlogn)$$

(b)Average-case analysis

$O(nlogn)$

When input is assumed to be in random order and also pivot is in the random position



(c)Worst-case analysis

$O(n^2)$

When the pivot happened to be the least element of the array, so that we had k=1 and n-k=n-1. In such case, we have:

$$T(n) = T(1) + T(n-1) + an$$

Now let us anaysis the time complexity of quick sort in such a case:

$$T(n) = T(1) + T(n-1) + an$$

$$= T(1) + (T(n-2) + T(1) + a(n-1)) + an$$

$$= 2T(1) + T(n-2) + an + a(n-1)$$

$$= 3T(1) + T(n-3) + an + a(n-1) + a(n-2)$$

.....

$$= iT(1) + T(n-i) + a(n + (n-1) + (n-2) + ...(n-i+1))$$

this recurrence will stop when i = n:

$$= nT(1) + T(0) + a(n + (n-1) + (n-2) + ... + 2 + 1)$$

$$= nT(1) + T(0) + a\frac{n(n+1)}{2}$$

which we can got the result:

$$O(n^2)$$

6.(10 points) What is the best, worst and average running time of merge sort algorithm? Use two methods for solving a recurrence relation for the average case to justify your answer.

The merge sort requires $O(nlogn)$ comparisons, where n is the length of an array to be sorted. There are no best-case or worst-case for the merge sort. Because the merge sort needs a duplicate copy of the array being sorted therefore in many situcations it can be impractical.

two methods:

(a)Iterativing method:

$$T(n) = 2T(\frac{n}{2}) + an$$

$$= 2(2T(\frac{n}{4}) + a\frac{n}{2}) + an$$

$$= 4T(\frac{n}{4}) + 2an$$

$$= 2^2T(\frac{n}{2^2}) + 2an$$

.....

$$= 2^iT(\frac{n}{2^i}) + ian$$

this recurrence will stop when i = logn:

$$= nT(1) + anlogn$$

then we got the result:

$$O(nlogn)$$

(b)Master Theorem

$$T(n) = 2T(\frac{n}{2}) + an$$

$$a = 2, b = 2, f(n) = an;$$

according to master theorem:

$$T(n) = a(n/b) + f(n)$$

$$f(n) = \Theta(nlog_2 2) = \Theta(n)$$

so we can got the result:

$$O(nlogn)$$

7.(10 points) R-10.17 p. 493

For the following statements about red-black trees, provide a justification for each true statement and a counterexample for each false one.

(a)A subtree of a red-black tree is itself a red-black tree.

False:

counterexample:

A subtree with a red root is not a red black tree.

(b)The sibling of an external node is either external or it is red.

True:

Proof: For this question, we consider from the opposite way: we assume that there is an external node has a black internal sibling and let h be the black depth of the external node. Then the black depth of the external node's black, internal sibling would be at least h+1 which violate the depth rule of a red-black tree.

(c)There is a unique (2,4) tree associated with a given red-black tree.

True:

Proof:

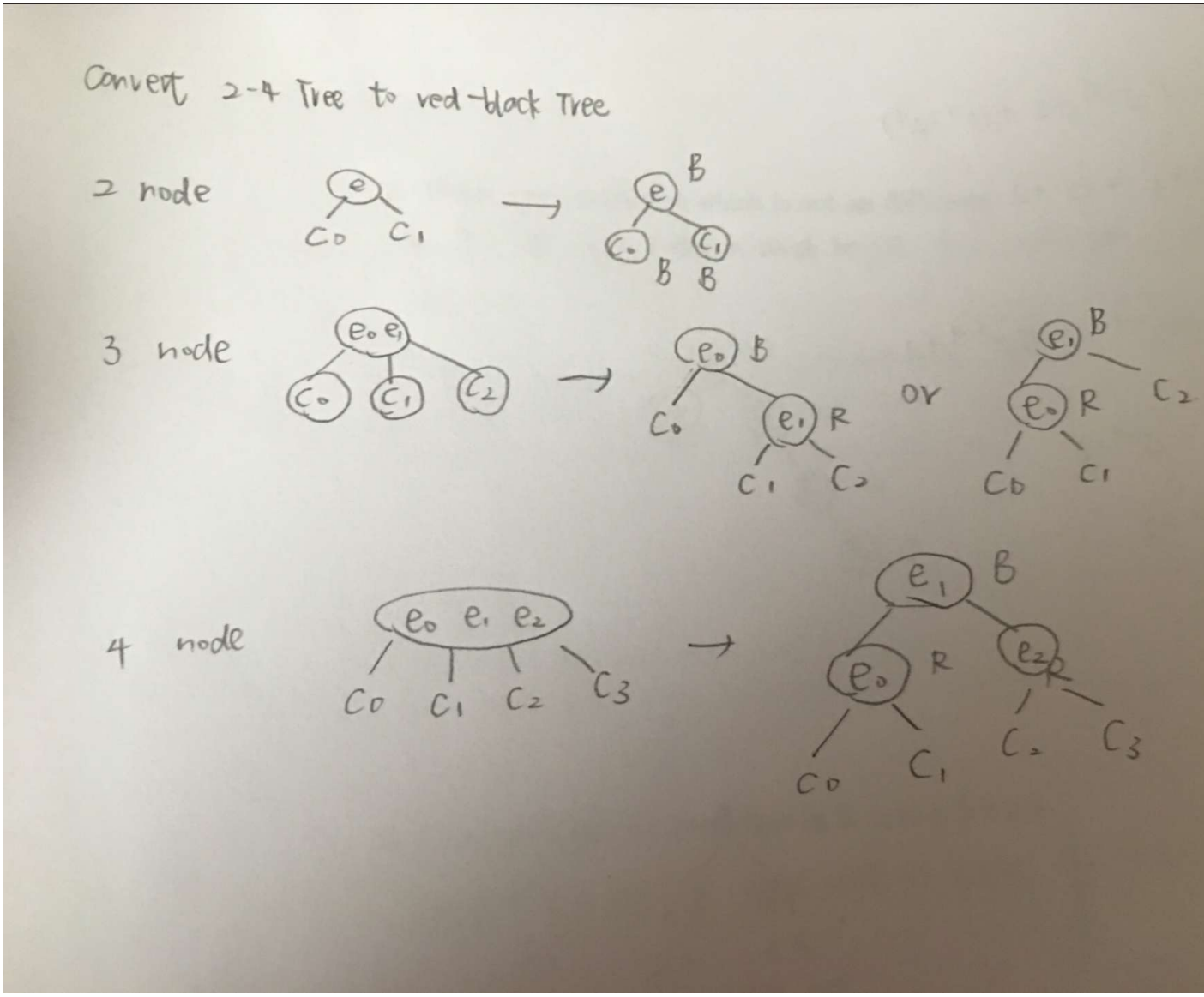
The given one means that given a red-black tree T, there is an unique 2-4 tree T' associated with T. Every node in the red-black tree with two red children is uniquely represented as a 4 -node in T', each node with one red child is uniquely represented as a 3-node in T', and a node with no red children is uniquely represented as a 2-node in T'.

(d)There is a unique red-black tree associated with a given (2,4) tree.

False:

counter example:

A 3-node has two different possible representations in a red-black tree.



8.(10 points) R-10.19 p. 493

Consider a tree T storing 100,000 entries. What is the worst-case height of T in the following cases?

(a)T is an AVL tree.

AVL tree:

$$h < 1.44log(n+1)$$

so the worst case height of T is 24.

(b)T is a (2,4) tree.

(2,4) tree:

$$h <= log(n+1)$$

so the worst case height of T is 17.

(c)T is a red-black tree.
red-black tree:

$$h <= 2log(n+1)$$

so the worst case height of T is 33.

(d)T is a binary search tree.
T is a binary search tree.
The worst case is linear so 100,000.

9.(10 points) R-9.16 p. 418

Draw an example skip list that results from performing the following series of operations on the skip list shown in Figure 9.12: `erase(38)`, `insert(48,x)`, `insert(24,y)`, `erase(55)`. Record your coin flips, as well.