1 *Differential entropy.* Evaluate the differential entropy $h(X) = -\int f \ln f \, dx$ for the following:

(a) The exponential density, $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$.

(b) The Laplace density, $f(x) = \frac{1}{2}\lambda e^{-\lambda|x|}$.

2 A Channel has an input ensemble X conisisting of numbers +1 and -1 used with the probabilities $P_X(+1)=P_X(-1)=\frac{1}{2}$. The output y is the sum of the input x and an independent noise random variable Z with the probability density $P_Z(z)=1/4$ for $-2<z\leq 2$ and $P_Z(z)=0$ eslewhere. In other words, the conditional probability density of y conditional on x is given $P_{Y|X}(y|x)=1/4$ by for $-2<y-x\leq 2$ and $P_{Y|X}(y|x)=0$ elsewhere.

(a) Find and sketch the output probability density for the channel.

(b) Find $I(X;Y)$.

(c) Suppose the output is transformed into a discrete processed output u defined by $u=1$ for $y>1$; $u=0$ for $-1<y\leq 1$; $u=-1$ for $y\leq -1$. Find I(X;U).