

$$5.14 \text{ 解: } ① G_1(s) = \frac{50}{(2s+1)(8s+5)} = \frac{10}{(2s+1)(\frac{1}{4}s+1)}$$

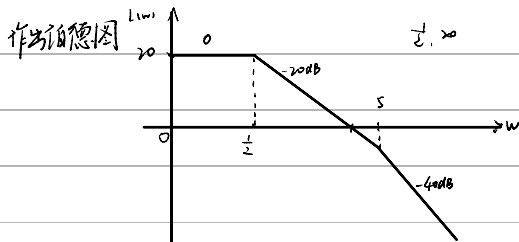
$$\text{转折频率 } \omega_1 = \frac{1}{2} \quad \omega_2 = 5$$

环节                  斜率                  累积斜率

$$10 \quad 0 \quad 0$$

$$\frac{1}{2s+1} \quad -20 \text{ dB/dec} \quad -20 \text{ dB/dec}$$

$$\frac{1}{\frac{1}{4}s+1} \quad -20 \text{ dB/dec} \quad -40 \text{ dB/dec}$$



$$A(\omega) = \frac{50}{\sqrt{4\omega^2+1} \sqrt{\omega^2+25}} \quad \text{当 } A(\omega_g) = 1 \text{ 时 } \omega_g = 3.91 \text{ rad/s}$$

$$\varphi(\omega_1) = -\arctan 2\omega - \arctan 0.2\omega$$

$$\varphi(\omega_c) = \varphi(3.91) = -121^\circ$$

$$\text{故相位裕度 } \gamma = 180^\circ + \varphi(\omega_c) = 59^\circ$$

由于存在  $59^\circ$  相位裕度, 故是稳定的。

$$③ G_2(s) = \frac{40(s^2+s+1)}{s(2s+1)(0.2s+1)(0.05s+1)}$$

环节                  转折  $\omega$                   斜率                  累积斜率

$$40 \quad / \quad 0 \quad 0$$

$$\frac{1}{s} \quad / \quad -20 \text{ dB} \quad -20 \text{ dB}$$

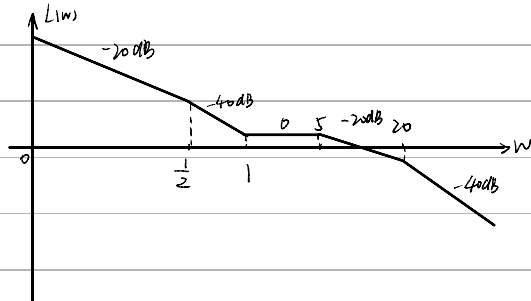
$$\frac{1}{2s+1} \quad \frac{1}{2} \quad -20 \text{ dB} \quad -40 \text{ dB}$$

$$s^2+s+1 \quad 1 \quad 40 \text{ dB} \quad 0$$

$$\frac{1}{0.2s+1} \quad 5 \quad -20 \text{ dB} \quad -20 \text{ dB}$$

$$\frac{1}{0.05s+1} \quad 20 \quad -20 \text{ dB} \quad -40 \text{ dB}$$

故幅频特性为



初用斜线近似  $L(w_g)=0$  时  $w_g=42.3$

$$\varphi(w) = \arctan \frac{w}{1-w} - \frac{\pi}{2} - \arctan 2w - \arctan 0.2w - \arctan 0.05w$$

$$\varphi(w_c) = -148.7^\circ \quad \gamma = 180^\circ + \varphi(w_c) = 31.3^\circ$$

存在  $31.3^\circ$  的相位裕度 因此稳定

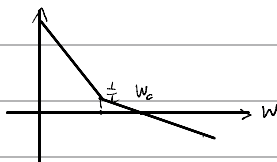
$$5.15 \text{ 解: } G(s) = \frac{Ts+1}{s^2} \quad G(jw) = \frac{j\omega T+1}{-w^2}$$

$$A(w) = \frac{\sqrt{1+(T\omega)^2}}{w^2} \quad \varphi(w) = \arctan T\omega - 180^\circ$$

环节      转折频率      斜率      累计斜率

$\frac{1}{s^2}$       /      -40dB      -40dB

$Ts+1$        $\frac{1}{T}$       20dB      -20dB



作为伯德图, 要存在  $45^\circ$  的相位裕度  $\varphi(w_c) = -180^\circ + 45^\circ = -135^\circ$

$$\text{故 } \arctan w_c T = 45^\circ \quad w_c T = 1$$

$$\text{当 } |G(w)| = 1 \text{ 时 } \frac{\sqrt{1+(w_c T)^2}}{w_c^2} = 1 \quad 1+(w_c T)^2 = w_c^4$$

$$w_c = \sqrt[4]{2} \quad \text{则 } T = \frac{1}{\sqrt[4]{2}} \approx 0.84$$