

5.1 解: 1. $r(t) = \sin 2t$ $R(s) = \frac{2}{s^2 + 4}$

系统闭环传递函数 $G(s) = \frac{\frac{1}{s+1}}{1 + \frac{1}{s+1}} = \frac{1}{s+2}$

频率响应 $G(j\omega) = G(s)|_{s=j\omega} = \frac{1}{j\omega+2}$ 当 $r(t) = \sin 2t$ 时 $\omega = 2$ rad

$A(\omega) = \frac{\sqrt{2}}{4}$ $\phi(\omega) = -\arctan 1 = -\frac{\pi}{4}$

由正弦信号输入的频响规则 $C_0(t) = \frac{\sqrt{2}}{4} \sin(2t - \frac{\pi}{4})$

稳态误差 $C_0(t) = \sin 2t - \frac{\sqrt{2}}{4} \sin(2t - \frac{\pi}{4})$

5.2 解: $h(t) = 1 - 1.8e^{-4t} + 0.8e^{-9t}$ $t \geq 0$

作拉氏变换 $H(s) = \frac{1}{s} - \frac{1.8}{s+4} + \frac{0.8}{s+9} = \frac{36}{s(s+4)(s+9)}$

单位阶跃响应下 $R(s) = \frac{1}{s}$ $G(s) = \frac{H(s)}{R(s)} = \frac{36}{(s+4)(s+9)}$

即 $G(j\omega) = \frac{36}{(j\omega+4)(j\omega+9)}$ $A(\omega) = |G(j\omega)| = \frac{36}{\sqrt{(16+\omega^2)(81+\omega^2)}}$

$\phi(\omega) = -\arctan \frac{13\omega}{36-\omega^2}$

5.3 解: 系统的闭环传递函数 $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + 1}$ $G(j\omega) = \frac{\omega_n^2}{2j\zeta\omega_n\omega + \omega_n^2 - \omega^2}$

当 $r(t) = 2 \sin t$ 时 $C(t) = 4 \sin(t - 45^\circ)$ 则 $A(\omega)|_{\omega=1} = |G(j\omega)|_{\omega=1} = 2$

$\phi(\omega)|_{\omega=1} = \angle G(j\omega)|_{\omega=1} = -45^\circ$

即: $\begin{cases} \frac{\omega_n^2}{\sqrt{(\omega_n^2-1)^2 + 4\zeta^2\omega_n^2}} = 2 \\ -\arctan \frac{2\zeta\omega_n}{\omega_n^2-1} = -45^\circ \end{cases} \Rightarrow \begin{cases} \omega_n = 1.24 \\ \zeta = 0.22 \end{cases}$