第四章

4.13

解：(a) $X(jw)=\delta(w)+\delta(w-\pi)+\delta(w-5) \xrightarrow{F^{-1}} x(t)=\frac{1}{2\pi}+\frac{1}{2\pi}e^{j\pi t}+\frac{1}{2\pi}e^{j5t}$

$\frac{\pi}{2\pi}=\frac{1}{2}$　$\frac{5}{2\pi}=\frac{5}{2\pi}$　公因数不为有理数　∴不是周期的

(b) $Y(jw)=\frac{2\sin w}{w}e^{-jw}[\delta(w)+\delta(w-\pi)+\delta(w-5)]$

$=2\delta(w)+\frac{2\sin 5}{5}e^{-j5}\delta(w-5)$

$\therefore y(t)=\frac{1}{\pi}+\frac{2\sin 5}{5\times 2\pi}e^{-j5}\cdot e^{j5t}=\frac{1}{\pi}+\frac{\sin 5}{5\pi}\cdot e^{j5(t-1)}$

$T=\frac{2\pi}{w}=\frac{2}{5}\pi$ 是周期的

(c) 可能. $x(t)=\frac{1}{2\pi}+\frac{1}{2\pi}e^{j\pi t}+\frac{1}{2\pi}e^{j5t}$

$y(t)=u(t)-u(t-2)$　卷积是周期的

4.14

解. $x(t)$为实函数　则 $X(jw)$模偶对称、相位奇对称、

$Ae^{-2t}u(t) \xrightarrow{F} A\cdot\frac{1}{2+jw}=(1+jw)X(jw)$

$\therefore X(jw)=\frac{A}{(2+jw)(1+jw)}=A\left(\frac{1}{1+jw}-\frac{1}{2+jw}\right)$

则 $x(t)=Ae^{-t}u(t)-Ae^{-2t}u(t)$

由帕斯瓦尔定理　$\int_{-\infty}^{+\infty}|x(t)|^2dt=1$

$\therefore A^2\int_0^{+\infty}e^{-2t}+e^{-4t}-2e^{-3t}\,dt=1$　即 $\frac{1}{12}A^2=1$

$\therefore A=\pm 2\sqrt{3}$

$\therefore x(t)=2\sqrt{3}\,(e^{-t}u(t)-e^{-2t}u(t))$

或 $x(t)=-2\sqrt{3}\,(e^{-t}u(t)-e^{-2t}u(t))$

4.24

解: (a)

(1) $\because Re[X(jw)]=0$ $\therefore x(t)$为实信号且奇对称.

(2) $\because Im[X(jw)]=0$ $\therefore x(t)$为实信号且偶对称.

(3) $e^{jaw}X(jw)$为实函数 $x(t-a)$为实偶信号

(4) $x(0)=0$ (5) $x'(0)=0$

(6) $X(jw)$为周期的 则$x(t)$为离散信号.

观察时域特性: (a)满足条件 (1)、(3)、(4)

(b)满足 (3) (4) (5) (6) (c)满足 (1) (4) (5)

(d)满足 (1) (4) (e)满足 (2) (5) (3)

(f)满足 (2) (4) (5) (3)

(b) $x(t)=t^3$ 为奇对称. $x(0)=0$ $x'(0)=3t^2|_{t=0}=0$.

4.25

解: 观察到 $x(t+1)$为实偶信号 则对应的$X(jw)$为偶函数

(a) $x(t)\xrightarrow{F}X(jw)$ $x(t+1)\xrightarrow{F}e^{jw}X(jw)$

由于 $e^{jw}X(jw)$为实偶函数 $\therefore \measuredangle e^{jw}X(jw)=0$

$\therefore \measuredangle X(jw)=-w$

(b) $X(j0)=\int_{-\infty}^{+\infty}x(t)e^{-jwt}dt\Big|_{w=0}=\int_{-\infty}^{+\infty}x(t)dt=7$

(c) $\int_{-\infty}^{+\infty}X(jw)dw\overset{2\pi}{=}x(t)\Big|_{t=0}=2\pi x(0)=4\pi$

(d) $X(jw)\frac{2\sin w}{w}\xrightarrow{F^{-1}}x(t)*y(t)$ $y(t)=\begin{cases}1 & |t|<1\\0 & |t|>1\end{cases}$

$\int_{-\infty}^{+\infty}X(jw)\cdot\frac{2\sin w}{w}\cdot e^{j2w}dw=2\pi\ x(t)*y(t)\Big|_{t=2}$

$=2\pi\cdot\frac{7}{2}=7\pi$

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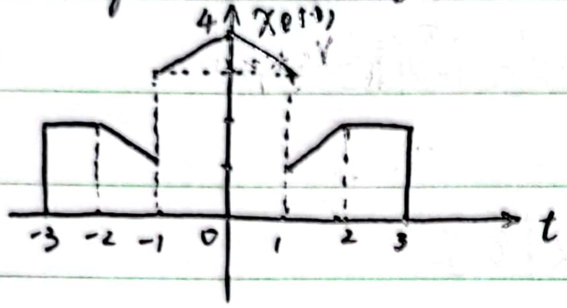
(e) $\int_{-\infty}^{+\infty}|X(j\omega)|^2 d\omega=\int_{-\infty}^{+\infty}|x(t)|^2 dt\cdot 2\pi=2\pi\int_{-\infty}^{+\infty}|x(t+1)|^2 dt$

$=4\pi\int_{0}^{+\infty}|x(t+1)|^2 dt=\frac{25}{3}\pi$

(f) $Re[X(j\omega)]\xrightarrow{F^{-1}}x_e(t)=\frac{1}{2}(x(t)+x(-t))$

则为



4.27

解: (a) $x(t)=u(t-1)-2u(t-2)+u(t-3)$

$X(j\omega)=e^{-j\omega}\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)-2e^{-j2\omega}\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)+e^{-j3\omega}\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)$

$=\left(e^{-j\omega}-2e^{-j2\omega}+e^{-j3\omega}\right)\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)$

(b) $a_k=\frac{1}{T}\int_T \tilde{x}(t)e^{-jk\omega_0 t}dt=\frac{1}{T}\int_T \tilde{x}(t)e^{-jk\frac{2\pi}{T}t}dt$

$=\frac{1}{T}\left(\int_1^2 e^{-jk\frac{2\pi}{T}t}dt-\int_2^3 e^{-jk\frac{2\pi}{T}t}dt\right)$

$=\frac{1}{T}X(jk\omega_0)\quad \omega_0=\frac{2\pi}{T}$

$X(j\omega)=F(u(t-1)-u(t-2))-F(u(t-2)-u(t-3))$

$=\int_1^2 e^{-j\omega t}dt-\int_2^3 e^{-j\omega t}dt$

$=e^{-\frac{3}{2}jt}\frac{2\sin\frac{1}{2}\omega}{\omega}-e^{-\frac{5}{2}jt}\frac{2\sin\frac{1}{2}\omega}{\omega}$

$=\left(e^{-\frac{3}{2}jt}-e^{-\frac{5}{2}jt}\right)\frac{2\sin\frac{1}{2}\omega}{\omega}$

4.11

解：作傅里叶变换 $Y(j\omega)=X(j\omega)\cdot H(j\omega)$　　$G(j\omega)=\frac{1}{3}X(\frac{j\omega}{3})\cdot\frac{1}{3}H(\frac{j\omega}{3})$

$\therefore G(j\omega)=\frac{1}{9}X(\frac{j\omega}{3})H(\frac{j\omega}{3})=\frac{1}{9}Y(\frac{j\omega}{3})$

$Y(j\omega)\longrightarrow\frac{1}{3}Y(\frac{1}{3}j\omega)$　　则 $G(j\omega)=\frac{1}{3}Y(3j\omega)$

$\therefore g(t)=\frac{1}{3}y(3t)$　　即 $A=\frac{1}{3}$　$B=3$.

4.22.

解：(a) $X(j\omega)=\frac{2\sin[3(\omega-2\pi)]}{\omega-2\pi}$　　$X(j(\omega+2\pi))=\frac{2\sin 3\omega}{\omega}$.

由变换对 $X(j(\omega+2\pi))\xleftrightarrow{F}x(t)=\begin{cases}1 & |t|<3\\ 0 & |t|>3\end{cases}$

由频移性质 $X(j\omega)\xleftrightarrow{F}y(t)=\begin{cases}e^{j2\pi t}, & |t|<3\\ 0, & |t|>3\end{cases}$

(c) $X(j\omega)$模偶对称、相位奇对称　　则 $x(t)$为实信号

$|X(j\omega)|=\omega$　　$\measuredangle X(j\omega)=-3\omega$.

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|\cdot e^{j\measuredangle X(j\omega)}e^{j\omega t}d\omega$$

$$=\frac{1}{2\pi}\left[\int_{-1}^{0}(-\omega)\cdot e^{-j3\omega}\cdot e^{j\omega t}d\omega+\int_{0}^{1}\omega\cdot e^{-j3\omega}e^{j\omega t}d\omega\right]$$

$$=\frac{1}{2\pi}\left[\int_{-1}^{0}(-\omega)e^{j\omega(t-3)}d\omega+\int_{0}^{1}\omega e^{j\omega(t-3)}d\omega\right]$$

$$=\frac{1}{2\pi}\left\{-\frac{e^{-j(t-3)}}{j(t-3)}-\frac{1}{(t-3)^2}[1-e^{-j(t-3)}]+\frac{e^{j(t-3)}}{j(t-3)}-\frac{1}{(t-3)^2}[1-e^{j(t-3)}]\right\}$$

$$=\frac{1}{\pi}\left\{\frac{e^{j(t-3)}-e^{-j(t-3)}}{j(t-3)}-\frac{1}{2(t-3)^2}[2-e^{j(t-3)}-e^{-j(t-3)}]\right\}$$

$$=\frac{1}{\pi}\left[\frac{\sin(t-3)}{t-3}+\frac{\cos(t-3)-1}{(t-3)^2}\right]$$

(e) $X(j\omega)=\begin{cases}-1 & -3<\omega<-2\\ \omega+1 & -2<\omega<-1\\ 0 & -1<\omega<1\\ \omega-1 & 1<\omega<2\\ & 2<\omega<3\end{cases}$

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$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)\,e^{j\omega t}d\omega=\frac{1}{2\pi}\Big[\int_{-3}^{-2}-e^{j\omega t}d\omega+\int_{-2}^{-1}(\omega+1)e^{j\omega t}d\omega+\int_{1}^{2}(\omega-1)e^{j\omega t}d\omega+\int_{2}^{3}e^{j\omega t}d\omega\Big]$$

$$=\frac{1}{2\pi}\Big\{\frac{1}{jt}(e^{-j3t}-e^{-j2t})+\frac{e^{-j2t}}{jt}+\frac{1}{t^2}(e^{-jt}-e^{-j2t})+\frac{e^{j2t}}{jt}+\frac{1}{t^2}(e^{j2t}-e^{jt})+\frac{1}{jt}(e^{j3t}-e^{j2t})\Big\}$$

$$=\frac{\cos 3t}{j\pi t}+\frac{\sin t-\sin 2t}{j\pi t^2}$$

4.31

解 (a) $H_1(j\omega)=\frac{1}{j\omega}+\pi\delta(\omega)$

$H_2(j\omega)=-2+5\cdot\frac{1}{2+j\omega}$

$H_3(j\omega)=2j\cdot\left(\frac{1}{1+j\omega}\right)'=\frac{2}{(j\omega+1)^2}$

$x(t)=\cos t\xleftrightarrow{F}X(j\omega)=\pi[\delta(\omega+1)+\delta(\omega-1)]$

$Y_1(j\omega)=X(j\omega)\cdot H_1(j\omega)=\left[\frac{1}{j\omega}+\pi\delta(\omega)\right][\delta(\omega+1)+\delta(\omega-1)]\cdot\pi$

$=\pi\cdot\left[\frac{1}{j\omega}\Big|_{\omega=-1}\cdot\delta(\omega+1)+\frac{1}{j\omega}\Big|_{\omega=1}\cdot\delta(\omega-1)\right]$

$=\frac{\pi}{j}\cdot[-\delta(\omega+1)+\delta(\omega-1)]$.

$\therefore y_1(t)=\sin t$

$Y_2(j\omega)=X(j\omega)\cdot H_2(j\omega)=\left(-2+\frac{5}{2+j\omega}\right)\cdot\pi\cdot[\delta(\omega+1)+\delta(\omega-1)]$

$=\pi\cdot\left[-2\delta(\omega+1)-2\delta(\omega-1)+\frac{5}{2+j\omega}\Big|_{\omega=-1}\cdot\delta(\omega+1)+\frac{5}{2+j\omega}\Big|_{\omega=1}\cdot\delta(\omega-1)\right]$

$=\pi\cdot\left[-2\delta(\omega+1)-2\delta(\omega-1)+\frac{5}{2-j}\delta(\omega+1)+\frac{5}{2+j}\cdot\delta(\omega-1)\right]$

$=\pi j[\delta(\omega+1)-\delta(\omega-1)]$

$y_2(t)=\sin t$

$Y_3(j\omega)=H_3(j\omega)X(j\omega)=\frac{2}{(j\omega+1)^2}\cdot\pi[\delta(\omega+1)+\delta(\omega-1)]$

$=\pi\cdot\left[\frac{2}{(1-j)^2}\delta(w+1)+\frac{2}{(1+j)^2}\delta(w-1)\right]$

$=\frac{\pi}{j}[\delta(w-1)-\delta(w+1)]$

$y_1(t)=\sin t$　　综上 对 $x_1(t)=\cos t$ 的响应均为 $\sin t$

(b) $X(jw)=\pi[\delta(w+1)+\delta(w-1)]$　　$Y(jw)=\pi j[\delta(w+1)-\delta(w-1)]$

令 $H_4(jw)=\frac{1}{2}[H_1(jw)+H_3(jw)]$

$Y_4(jw)=\frac{1}{2}H_1(jw)X(jw)+\frac{1}{2}X(jw)H_3(jw)$

$=\frac{1}{2}\{\frac{\pi}{j}(-\delta(w+1)+\delta(w-1)]+\frac{\pi}{j}[\delta(w-1)-\delta(w+1)]\}$

$=\pi j[\delta(w+1)-\delta(w-1)]$

$\therefore h_4(t)=\frac{1}{2}[u(t)+2te^{-t}u(t)]$

4.32

解: $h(t)=\frac{\sin 4(t-1)}{\pi(t-1)}$　　$h(t+1)=\frac{\sin 4t}{\pi t}\xrightarrow{F}\begin{cases}1 & |w|<4\\ 0 & |w|>4\end{cases}$

$\therefore H(jw)=\begin{cases}e^{-jw} & |w|>4\\ 0 & |w|<4\end{cases}$

(a) $x_1(t)=\cos(6(t+\frac{\pi}{12}))$　　$x_1(t)=\cos t\xrightarrow{F}\pi[\delta(w+1)+\delta(w-1)]$

$X_1(jw)=\frac{1}{6}e^{j\frac{\pi}{12}w}\cdot\pi[\delta(\frac{w}{6}+1)+\delta(\frac{w}{6}-1)]$

当 $w=\pm6$ 时 $X(jw)\neq0$ 但 $6>4$ 故不能通过低频　$Y_1(jw)=0$

$\therefore y_1(t)=0$

(b) $x_2(t)=\sum_{k=0}^{\infty}(\frac{1}{2})^k\sin 3kt$　　$X_2(jw)=\sum_{k=1}^{\infty}(\frac{1}{2})^k\cdot\frac{1}{3k}\pi j[\delta(\frac{w}{3k}+1)-\delta(\frac{w}{3k}-1)]$

当 $w=\pm3k$ 时　$X(jw)\neq0$

$Y_2(jw)=e^{-jw}\cdot\{\quad\frac{1}{2}\cdot\frac{1}{3}\pi j[\delta(\frac{w}{3}+1)-\delta(\frac{w}{3}-1)]$

$y_2(t)=\frac{1}{2}\sin(3(t-1))$

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(c) $x_3(t)=\frac{\sin(4(t+1))}{\pi(t+1)}$ $x_3(t-1)=\frac{\sin 4t}{\pi t}$

$X_3(jw)=\begin{cases} e^{jw} & |w|<4 \\ 0 & |w|>4 \end{cases}$

$Y_3(jw)=X_3(jw)H(jw)=\begin{cases} 1 & |w|<4 \\ 0 & |w|>0 \end{cases}$

$\therefore y_3(t)=\frac{\sin 4t}{\pi t}$

(d) $x_4(t)=\left(\frac{\sin(2t)}{\pi t}\right)^2$ $x_4(t)=\frac{\sin(2t)}{\pi t}\cdot\frac{\sin(2t)}{\pi t}$

$X_4(jw)=\frac{1}{2\pi}X(jw)*X(jw)$ $X(jw)=\begin{cases} 1 & |w|<2 \\ 0 & |w|>2 \end{cases}$

$=\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\Omega)\cdot X(j(w-\Omega))\,d\Omega$

$=\begin{cases} \frac{1}{2\pi}(w+4) & -4<w<0 \\ -\frac{1}{2\pi}(w-4) & 0<w<4 \\ 0 & \text{其他} \end{cases}$

$\therefore Y_4(jw)=X_4(jw)\cdot H(jw)=X_4(jw)e^{-jw}$

$\therefore y_4(t)=x_4(t-1)=\left[\frac{\sin(2(t-1))}{\pi(t-1)}\right]^2$

4.10

解: (a) $x(t)=t\left(\frac{\sin t}{\pi t}\right)^2$ $x_1(t)=\frac{\sin t}{\pi t}\xrightarrow{F}X_1(jw)=\begin{cases} 1 & |w|<1 \\ 0 & |w|>1 \end{cases}$

$x_1(t)\cdot x_1(t)\xrightarrow{F}X_1(jw)*X_1(jw)$

$=\begin{cases} \frac{1}{2\pi}(w+2) & -2<w<0 \\ \frac{1}{2\pi}(-w+2) & 0<w<2 \\ 0 & \text{其他} \end{cases}$

$x(t)=t\,x(t)\xrightarrow{F}j[X_1(jw)*X_1(jw)]'=\begin{cases} j\frac{1}{2\pi} & -2<w<0 \\ -j\frac{1}{2\pi} & 0<w<2 \\ 0 & \text{其他} \end{cases}$

(b) $A=\int_{-\infty}^{+\infty} t^2\cdot\left(\frac{\sin\pi t}{\pi t}\right)^4\cdot dt = \int_{-\infty}^{+\infty} x^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(jw)|^2 dw$

$= \frac{1}{2\pi}\cdot\left(\frac{2}{4\pi^2}+\frac{2}{4\pi^2}\right) = \frac{1}{2\pi^3}$

4.21

解: (a) $[e^{-at}\cos w_0 t]\cdot u(t),\ a>0 \qquad x_1(t)=e^{-at}u(t) \qquad x_2(t)=\cos w_0 t$

$x_1(t) \xrightarrow{F} X_1(jw)=\frac{1}{a+jw} \qquad x_2(t)\xrightarrow{F} X_2(jw)=\pi[\delta(w+w_0)+\delta(w-w_0)]$

$X(jw)=X_1(jw)*X_2(jw)$

$=\int_{-\infty}^{+\infty}\frac{1}{a+j\Omega}\cdot\pi[\delta(w-\Omega+w_0)+\delta(w-\Omega-w_0)]\,d\Omega$

$=\frac{1}{2}\cdot\frac{1}{jw+a-jw_0}+\frac{1}{2}\frac{1}{jw+a+jw_0}$

$=\frac{jw+a}{(jw+a)^2+w_0^2}$

(c) $x(t)=\begin{cases}1+\cos\pi t & |t|<1\\ 0 & |t|>1\end{cases}$

$X(jw)=\int_{-\infty}^{+\infty}(1+\cos\pi t)e^{-jwt}dt$

$=\int_{-\infty}^{+\infty}\left[1+\frac{1}{2}(e^{j\pi t}+e^{-j\pi t})\right]e^{-jwt}dt$

$=\int_{-1}^{+1}e^{-jwt}dt+\frac{1}{2}\left(\int_{-1}^{+1}e^{j\pi t}e^{-jwt}dt+\int_{-1}^{1}e^{j\pi t}e^{-jwt}dt\right)$

$=\frac{1}{jw}(e^{jw}-e^{-jw})+\frac{1}{2j(\pi-w)}[e^{j(\pi-w)}-e^{-j(\pi-w)}]+\frac{1}{2j(\pi+w)}[e^{j(\pi+w)}-e^{-j(\pi+w)}]$

$=\frac{2\sin w}{w}+\frac{\sin(\pi-w)}{\pi-w}+\frac{\sin(\pi+w)}{\pi+w}$

$=\frac{2\sin w}{w}+\frac{\sin w}{\pi-w}-\frac{\sin w}{\pi+w}.$

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(e) $[te^{-2t}\sin 4t]u(t)$

$x(t)=[te^{-2t}\sin 4t]u(t)=[te^{-2t}\cdot\frac{1}{2j}(e^{j4t}-e^{-j4t})]u(t)$

$=\frac{1}{2j}\cdot[te^{-2t}u(t)\cdot e^{j4t}-te^{-2t}u(t)\cdot e^{-j4t}]$

$=\frac{1}{2j}j[\frac{d}{dw}\cdot\frac{1}{2+j(w-4)}-\frac{d}{dw}\cdot\frac{1}{2+j(w+4)}]$

$=\frac{j}{2}\{\frac{-1}{[2+j(w-4)]^2}+\frac{1}{[2+j(w+4)]^2}\}$

$=\frac{j}{2}\{\frac{1}{[2+j(w+4)]^2}-\frac{1}{[2+j(w-4)]^2}\}$

(g) $x(t)=\begin{cases}-1 & -2<t<-1\\ t & -1<t<1\\ 1 & 1<t<2\end{cases}$ 其余为0

$X(jw)=\int_{-\infty}^{+\infty}x(t)e^{-jwt}dt$

$=\int_{-2}^{-1}-e^{-jwt}dt+\int_{-1}^{1}te^{-jwt}dt+\int_{1}^{2}e^{-jwt}dt$

$=\frac{1}{jw}e^{-jwt}\Big|_{-2}^{-1}+(-\frac{1}{jw}te^{-jwt}+\frac{1}{w^2}e^{-jwt})\Big|_{-1}^{1}-\frac{1}{jw}e^{-jwt}\Big|_{1}^{2}$

$=\frac{1}{jw}(-e^{j2w}-e^{-j2w})-\frac{1}{w^2}(e^{jw}-e^{-jw})$

$=\frac{2j\cos 2w}{w}-\frac{2j\sin w}{w^2}$

$=\frac{2j}{w}(\cos 2w-\frac{\sin w}{w})$

4.34

解:(a) $H(jw)=\frac{jw+4}{6-w^2+5jw}=\frac{jw+4}{(jw)^2+5jw+6}=\frac{jw+4}{(jw+2)(jw+3)}$

$=\frac{2}{jw+2}-\frac{1}{jw+3}$

$\therefore h(t)=2\cdot e^{-2t}u(t)-e^{-3t}u(t)$

由题知 $\frac{Y(j\omega)}{X(j\omega)}=H(j\omega)$ 则 $[(j\omega)^2+5(j\omega)+6]Y(j\omega)=(j\omega+4)X(j\omega)$

∴微分方程为 $y''(t)+5y'(t)+6y(t)=x'(t)+4x(t)$

(b) $h(t)=2e^{-2t}u(t)-e^{-3t}u(t)$

(c) $x(t)=e^{-4t}u(t)-te^{-4t}u(t)$

$X(j\omega)=\frac{1}{4+j\omega}-j\cdot\left(\frac{1}{4+j\omega}\right)'=\frac{1}{4+j\omega}-\frac{1}{(4+j\omega)^2}$

$Y(j\omega)=H(j\omega)\cdot X(j\omega)=\left(\frac{2}{j\omega+2}-\frac{1}{j\omega+3}\right)\left(\frac{1}{4+j\omega}-\frac{1}{(4+j\omega)^2}\right)$

$=\frac{\frac{1}{2}}{j\omega+2}-\frac{\frac{1}{2}}{j\omega+4}$ ∴ $y(t)=\frac{1}{2}e^{-2t}u(t)-\frac{1}{2}e^{-4t}u(t)$

4.35

解:(a) $H(j\omega)=\frac{a-j\omega}{a+j\omega}$ $|H(j\omega)|=\sqrt{X(j\omega)\cdot X^*(j\omega)}=1$

$\measuredangle H(j\omega)=-2\arctan\frac{\omega}{a}$

$H(j\omega)=\frac{-(a+j\omega)+2a}{a+j\omega}=-1+\frac{2a}{a+j\omega}$ $X(j\omega)=1$

∴ $Y(j\omega)=-1+\frac{2a}{a+j\omega}$ $y(t)=-\delta(t)+2ae^{-at}u(t)$

(b) $a=1$ $H(j\omega)=\frac{1-j\omega}{1+j\omega}=-1+\frac{2}{1+j\omega}$

$x(t)=\cos\frac{t}{\sqrt{3}}+\cos t+\cos\sqrt{3}t$

$x_1(t)=\cos t\xrightarrow{F}X_1(j\omega)=\pi[\delta(\omega-1)+\delta(\omega+1)]$

$\cos\left(\frac{t}{\sqrt{3}}\right)\xrightarrow{F}\sqrt{3}\pi[\delta(\sqrt{3}\omega-1)+\delta(\sqrt{3}\omega+1)]$

$\cos(\sqrt{3}t)\xrightarrow{F}\frac{1}{\sqrt{3}}\pi\left[\delta\left(\frac{\omega}{\sqrt{3}}-1\right)+\delta\left(\frac{\omega}{\sqrt{3}}+1\right)\right]$

∴ $Y(j\omega)=H(j\omega)\cdot\pi\left\{\delta(\omega-1)+\delta(\omega+1)+\sqrt{3}[\delta(\sqrt{3}\omega-1)+\delta(\sqrt{3}\omega+1)]+\frac{1}{\sqrt{3}}\left[\delta\left(\frac{\omega}{\sqrt{3}}-1\right)+\delta\left(\frac{\omega}{\sqrt{3}}+1\right)\right]\right\}$

$=\frac{1-j}{1+j}\pi\delta(\omega-1)+\frac{1+j}{1-j}\pi\delta(\omega+1)+\sqrt{3}\cdot\frac{1-j\frac{1}{\sqrt{3}}}{1+j\frac{1}{\sqrt{3}}}\pi\delta(\sqrt{3}\omega-1)+\cdots\cdots+\frac{1}{\sqrt{3}}\frac{1+\sqrt{3}j}{1-\sqrt{3}j}\pi\delta\left(\frac{\omega}{\sqrt{3}}+1\right)$