Date:

5.9

解: $x[n]=0,\ n>0$ $\qquad x[0]>0$

$x_o[n]=\frac{1}{2}(x[n]-x[-n])$ $\qquad x_o[n]\xrightarrow{F} j\,Im\{X(e^{jw})\}=(\sin w-\sin 2w)j$

即 $x_o[n]\xleftrightarrow{F}\frac{1}{2}(e^{jw}-e^{-jw}-(e^{j2w}-e^{-j2w}))$

$\therefore x_o[n]=\frac{1}{2}(\delta[n+1]-\delta[n-1]-\delta[n+2]+\delta[n-2])$

$\therefore x[n]=\delta[n+1]-\delta[n+2]+x[0]$

由帕斯瓦尔定理 $\sum_{n=-\infty}^{0}|x[n]|^2=3$ 得 $x[0]=1$.

$\therefore x[n]=\delta[n]+\delta[n+1]-\delta[n+2]$

5.21

解:(a) $x[n]=u[n-2]-u[n-6]$

$x[n]$为中心在原点宽为4的门信号向右平移

$x_1[n]=\delta[n+2]-\delta[n-2]$ $\qquad x[n]=x_1[n-4]$

$x_1[n]\xrightarrow{F}\frac{\sin\frac{5}{2}w}{\sin\frac{1}{2}w}$ $\qquad \therefore x[n]\xrightarrow{F}e^{-j\frac{7}{2}w}\cdot\frac{\sin\frac{5}{2}w}{\sin\frac{1}{2}w}$

(c) $x[n]=(\frac{1}{3})^{|n|}u[-n-2]$.

5.21

解:(a) $x[n]=u[n-2]-u[n-6]=\delta[n-2]+\delta[n-3]+\delta[n-4]+\delta[n-5]$

$\therefore X(e^{jw})=e^{-j2w}+e^{-j3w}+e^{-j4w}+e^{-j5w}$

$=e^{-j2w}\cdot\frac{1-e^{-j4w}}{1-e^{-jw}}$

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(c) $x[n]=(\frac{1}{3})^{|n|}u[-n-2]$. $=(\frac{1}{3})^{(-n)}u[-n-2]$

$=(\frac{1}{3})^{2}\cdot(\frac{1}{3})^{(-n-2)}u[-n-2]$ $=\frac{1}{9}(\frac{1}{3})^{(-n-2)}u[n-2]$.

$x_1[n]=(\frac{1}{3})^{n}u[n]$ $X_1(e^{j\omega})=\frac{1}{1-\frac{1}{3}e^{-j\omega}}$

$x[n]=\frac{1}{9}x[-n-2]$ $x_1[-n]\xrightarrow{F}\frac{1}{1-\frac{1}{3}e^{j\omega}}$ $x_1[-(n+2)]=e^{j2\omega}\cdot\frac{1}{1-\frac{1}{3}e^{j\omega}}$

$\therefore X(e^{j\omega})=\frac{1}{9}e^{j2\omega}\cdot\frac{1}{1-\frac{1}{3}e^{j\omega}}$

(h) $x[n]=\sin\frac{5}{3}\pi n+\cos\frac{7}{3}\pi n$ $N=6$.

$=\frac{1}{2j}(e^{j\frac{5}{3}\pi n}-e^{-j\frac{5}{3}\pi n})+\frac{1}{2}(e^{j\frac{7}{3}\pi n}+e^{j\frac{7}{3}\pi n})$

$\therefore X(e^{j\omega})=\frac{\pi}{j}\sum_{l=-\infty}^{+\infty}[\delta(w-\frac{5}{3}\pi-2\pi l)-\delta(w+\frac{5}{3}\pi-2\pi l)]+\pi\sum_{l=-\infty}^{+\infty}[\delta(w-\frac{7}{3}\pi-2\pi l)+$
$\delta(w+\frac{7}{3}\pi-2\pi l)]$

$=j\pi[\delta(w-\frac{\pi}{3})-\delta(w+\frac{\pi}{3})]+\pi[\delta(w+\frac{\pi}{3})+\delta(w-\frac{\pi}{3})]$

22.

解: (b) $X(e^{j\omega})=1+3e^{-j\omega}+2e^{-j2\omega}-4e^{-j3\omega}+e^{-j10\omega}$.

由线性.时移性质 $\delta[n]\longrightarrow 1$

$\therefore x(t)=\delta[n]+3\delta[n-1]+2\delta[n-2]-4\delta[n-3]+\delta[n-10]$

(e) $X(e^{j\omega})=\sum_{k=-\infty}^{+\infty}(-1)^k\delta(w-\frac{\pi}{2}k)$

$=\sum_{k=-\infty}^{+\infty}2\pi\cdot\frac{(-1)^k}{2\pi}\cdot\delta(w-\frac{\pi}{2}k)$

$=2\pi\sum_{k=-\infty}^{\infty}\frac{(-1)^k}{2\pi}\delta(w-\frac{2k\pi}{4})$

$\therefore N=4$ $a_k=\frac{(-1)^k}{2\pi}$

$x[n]=\sum_{k=\langle 4\rangle}\frac{(-1)^k}{2\pi}e^{jk\cdot\frac{2\pi}{4}n}=\frac{1}{2\pi}(1-e^{j\frac{\pi}{2}n}+e^{j\pi n}-e^{j\frac{3}{2}\pi n})$

$=\frac{1}{2\pi}(1-(-1)^n-2\cos\frac{\pi}{2}n)=\frac{2}{\pi}$ $n=4m+2$. $m=0,1,2,\cdots$

(g) $X(e^{j\omega})=\dfrac{1-\frac{1}{3}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-j2\omega}}=\dfrac{1-\frac{1}{3}e^{-j\omega}}{(1-\frac{1}{2}e^{-j\omega})(1+\frac{1}{4}e^{-j\omega})}$

$$=\frac{\frac{2}{9}}{1-\frac{1}{2}e^{-j\omega}}+\frac{\frac{7}{9}}{1+\frac{1}{4}e^{-j\omega}}$$

$\therefore x[n]=\frac{2}{9}\left(\frac{1}{2}\right)^n u[n]+\frac{7}{9}\cdot\left(\frac{1}{4}\right)^n u[n]$

23.

解:(a) $x[n]=\frac{1}{2\pi}\int_{2\pi}X(e^{j\omega})e^{j\omega n}d\omega \quad X(e^{j\omega})=\sum_{n=-\infty}^{+\infty}x[n]e^{-j\omega n}$

$X(e^{j0})=\sum_{n=-\infty}^{+\infty}x[n]=6$

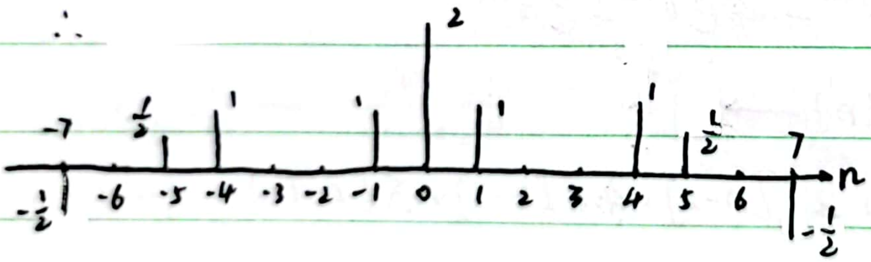
(c) $x[0]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})e^{j0}d\omega=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})d\omega$

$\therefore \int_{-\pi}^{\pi}X(e^{j\omega})d\omega=2\pi x[0]=4\pi$

(e) $x_e(t)\xrightarrow{F}Re[X(e^{j\omega})]$

$x_e(t)=\frac{1}{2}(x(t)+x(-t))$ 作图如下

∴



24

解: 1. $Re[X(e^{j\omega})]=0$ 又有$x[n]$为实信号 $\therefore X(e^{j\omega})$为纯虚数

$\therefore x[n]$奇对称.

2. $Im[X(e^{j\omega})]=0$ $\therefore X(e^{j\omega})$为实数 $x[n]$偶对称.

3. $e^{j\alpha\omega}X(e^{j\omega})$为实. 由时移性质 $x[n-\alpha]$为偶对称.

$\exists\alpha\in R$

4. $x[0]=\frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})d\omega=0$ 即 $x[0]=0$

5 $X(e^{j\omega})$是周期的 $\Rightarrow$ $x[n]$是离散的

6 $X(e^{j0})=\sum_{n=-\infty}^{+\infty} x[n]=0$

综上：(a)满足 3.5

(b)满足 1.3.4.5.6

(c)满足 5.6

(d)满足 2.3.4.5.

5.12

解：$y[n]=\left(\frac{\sin\frac{\pi}{4}n}{\pi n}\right)^2 * \left(\frac{\sin W_c n}{\pi n}\right)$

$y_1[n]=\frac{\sin\frac{\pi}{4}n}{\pi n}$ $\quad Y_1(e^{j\omega})=\begin{cases}1 & |\omega|<\frac{\pi}{4}\\ 0 & |\omega|>\frac{\pi}{4} 但 |\omega|<\pi\end{cases}$

$y_1[n]\cdot y_1[n] \xleftrightarrow{F} Y_1(e^{j\omega})*Y_1(e^{j\omega})=\begin{cases}\frac{2}{\pi}\omega+1 & -\frac{\pi}{2}<\omega<0\\ -\frac{2}{\pi}\omega+1 & 0<\omega<\frac{\pi}{2}\\ 0 & |\omega|<\pi 但为其他.\end{cases}$

$\because y[n]=\left(\frac{\sin\frac{\pi}{4}n}{\pi n}\right)^2 \Rightarrow Y(e^{j\omega})=Y_1(e^{j\omega})*Y_1(e^{j\omega})$

$\frac{\sin W_c n}{\pi n} \xrightarrow{F} Y_2(e^{j\omega})=\begin{cases}1 & |\omega|<W_c\\ 0 & W_c<|\omega|\le\pi\end{cases}$

$\because [Y_1(e^{j\omega})*Y_1(e^{j\omega})]\cdot Y_2(e^{j\omega})=[Y_1(e^{j\omega})*Y_1(e^{j\omega})]$

故 $W_c>\frac{\pi}{2}$

$\therefore \frac{\pi}{2}<|W_c|\le\pi$

Date:

5.19

解：(a) 作傅里叶变换，有

$$Y(e^{j\omega})-\frac{1}{6}e^{-j\omega}Y(e^{j\omega})-\frac{1}{6}e^{-j2\omega}Y(e^{j\omega})=X(e^{j\omega})$$

$$\therefore H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-j2\omega}}=\frac{1}{(1-\frac{1}{2}e^{-j\omega})(1+\frac{1}{3}e^{-j\omega})}$$

$$=\frac{\frac{3}{5}}{1-\frac{1}{2}e^{-j\omega}}+\frac{\frac{2}{5}}{1+\frac{1}{3}e^{-j\omega}}$$

(b) 由变换对和线性性质

$$h[n]=\frac{3}{5}\left(\frac{1}{2}\right)^n u[n]+\frac{2}{5}\left(-\frac{1}{3}\right)^n u[n]$$

5.26

解：(a) $X_2(e^{j\omega})=Re\{X_1(e^{j\omega})\}+Re\{X_1(e^{j(\omega+\frac{2}{3}\pi)})\}+Re\{X_1(e^{j(\omega-\frac{2}{3}\pi)})\}$

由频移性质 $x_2[n]=x_{1e}[n]+e^{j\frac{2}{3}\pi n}x_{1e}[n]+e^{-j\frac{2}{3}\pi n}x_{1e}[n]$

$$=x_{1e}[n]+2\cos\frac{2}{3}\pi n\, x_{1e}[n]$$

$$=\left(1+2\cos\frac{2}{3}\pi n\right)x_{1e}[n] \qquad x_{1e}[n]=Ev\{x_1[n]\}$$

(b) $X_3(e^{j\omega})=Im\{X_1(e^{j(\omega-\pi)})\}$

由频移性质 $x_3[n]=e^{j\pi n}x_{1o}[n]\cdot(-j)$

$$=-j(-1)^n x_{1o}[n] \qquad x_{1o}[n]=Od\{x_1[n]\}$$

(c) $\alpha=\dfrac{\sum_{n=-\infty}^{+\infty}nx_1[n]}{\sum_{n=-\infty}^{+\infty}x_1[n]}$ $\xrightarrow{\text{每一项同乘 } e^{-j\omega n}}$ $\alpha=\dfrac{\sum_{n=-\infty}^{+\infty}nx_1[n]e^{-j\omega n}}{\sum_{n=-\infty}^{+\infty}x_1[n]e^{-j\omega n}}$

则 $\alpha=\left.\dfrac{X_2(e^{j\omega})}{X_1(e^{j\omega})}\right|_{\omega=0}$ $\qquad X_1(e^{j\omega})\overset{F}{\longleftrightarrow}x_1[n]$

$X_2(e^{j\omega})\overset{F}{\longleftrightarrow}x_2[n]$.

$x_2[n] = n x_1[n]$ 则由频域微分性质 $X_2(e^{j\omega}) = j\cdot\frac{dX_1(e^{j\omega})}{d\omega}$

$$X_1(e^{j\omega}) = \begin{cases} 0 & \text{其他} \\ \frac{6}{\pi}\omega + 2 + j & -\frac{\pi}{3} < \omega < -\frac{\pi}{6} \\ 1 - \frac{6}{\pi}j\omega & -\frac{\pi}{6} < \omega < \frac{\pi}{6} \\ -\frac{6}{\pi}\omega + 2 - j & \frac{\pi}{6} < \omega < \frac{\pi}{3} \end{cases} \qquad X_2(e^{j\omega}) = \begin{cases} 0 & \text{其他} \\ \frac{6}{\pi}j & -\frac{\pi}{3} < \omega < -\frac{\pi}{6} \\ \frac{6}{\pi} & -\frac{\pi}{6} < \omega < \frac{\pi}{6} \\ -\frac{6}{\pi}j & \frac{\pi}{6} < \omega < \frac{\pi}{3} \end{cases}$$

$$\therefore \alpha = \left.\frac{X_2(e^{j\omega})}{X_1(e^{j\omega})}\right|_{\omega=0} = \frac{6}{\pi}$$

5.35

解: (a) 作傅里叶变换,有

$$Y(e^{j\omega}) - ae^{-j\omega}Y(e^{-j\omega}) = bX(e^{j\omega}) + e^{-j\omega}X(e^{-j\omega})$$

$$\therefore H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{b + e^{-j\omega}}{1 - ae^{-j\omega}}$$

$$|H(e^{j\omega})| = \left|\frac{1 + 2b\cos\omega + b^2}{1 + a^2 - 2a\cos\omega}\right| = 1$$

则 $b = -a$ 时 $|H(e^{j\omega})| = 1$ 对任意 ω 均成立

5.50

解: (a) $X(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{4}\cdot\frac{e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}} = \frac{1-\frac{1}{4}e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}$

$$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$\therefore H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1-\frac{1}{2}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})} = \frac{1-\frac{1}{2}e^{-j\omega}}{1-\frac{7}{12}e^{-j\omega}+\frac{1}{12}e^{-j2\omega}}$$

$$= \frac{-2}{1-\frac{1}{3}e^{-j\omega}} + \frac{3}{1-\frac{1}{4}e^{-j\omega}}$$

$h(t) = -2\cdot\left(\frac{1}{3}\right)^n u[n] + 3\left(\frac{1}{4}\right)^n u[n]$

相乘并且傅里叶反变换得

$$y[n] - \frac{7}{12}y[n-1] + \frac{1}{12}y[n-2] = x[n] - \frac{1}{2}x[n-1]$$

(b) $x_1[n] = (n+2)\left(\frac{1}{2}\right)^n u[n]$ $\quad y_1[n] = \left(\frac{1}{4}\right)^n u[n]$

$x_2[n] = \delta[n] - \left(-\frac{1}{2}\right)^n u[n]$

$$x_1[n] - x_1[n-1] = (n+2)\left(\frac{1}{2}\right)^n u[n] - (n+1)\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

$$= (n+2)\left(\frac{1}{2}\right)^n u[n] - 2(n+1)\left(\frac{1}{2}\right)^n u[n-1]$$

$$= \left(\frac{1}{2}\right)^n \{u[n]\cdot(n+2) - u[n-1]\cdot 2(n+1)\}$$

$$= 2\delta[n] - n u[n-1]$$

$x_1[n] = n\cdot\left(\frac{1}{2}\right)^n u[n] + 2\cdot\left(\frac{1}{2}\right)^n u[n].$

$$x_1'[n] = \left(\frac{1}{2}\right)^n u[n] \xrightarrow{F} X_1'(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$$x_1[n] \xrightarrow{F} j\cdot\frac{dX_1'(e^{j\omega})}{d\omega} + 2X_1'(e^{j\omega}) = \frac{2-\frac{1}{2}e^{-j\omega}}{(1-\frac{1}{2}e^{-j\omega})^2}$$

$$y_1[n] = \left(\frac{1}{4}\right)^n u[n] \qquad Y_1(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$\therefore H(e^{j\omega}) = \frac{Y_1(e^{j\omega})}{X_1(e^{j\omega})} = \frac{(1-\frac{1}{2}e^{-j\omega})^2}{2(1-\frac{1}{4}e^{-j\omega})^2}$$

$$y_2[n] = \delta[n] - \left(-\frac{1}{2}\right)^n u[n] \qquad Y_2(e^{j\omega}) = 1 - \frac{1}{1+\frac{1}{2}e^{-j\omega}} = \frac{\frac{1}{2}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}$$

$$\therefore X_2(e^{j\omega}) = \frac{Y_2(e^{j\omega})}{H(e^{j\omega})} = \frac{e^{-j\omega}\cdot(1-\frac{1}{4}e^{-j\omega})^2}{(1+\frac{1}{2}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})^2}$$

$$= \left[\frac{\frac{3}{8}}{1+\frac{1}{2}e^{-j\omega}} + \frac{\frac{3}{8}}{1-\frac{1}{2}e^{-j\omega}} + \frac{\frac{1}{8}}{(1-\frac{1}{2}e^{-j\omega})^2}\right]e^{-j\omega}$$

$$\therefore x_2[n] = \frac{3}{8}\left(-\frac{1}{2}\right)^{n-1}u[n-1] + \frac{3}{8}\left(\frac{1}{2}\right)^{n-1}u[n-1] + \frac{1}{8}\,n\left(\frac{1}{2}\right)^{n-1}u[n-1]$$