

6.3

$$\text{解: (a) } |H(j\omega)| = \frac{|1-j\omega|}{|1+j\omega|} = 1$$

$$\therefore A=1$$

$$\text{(b) } \angle H(j\omega) = -2 \arctan \omega$$

$$\frac{d\angle H(j\omega)}{d\omega} = \frac{-2}{1+\omega^2}$$

$$\therefore \text{当 } \omega > 0 \text{ 时 } \tau(\omega) = -\frac{2}{1+\omega^2} < 0 \quad \text{即 3 说法正确。}$$

6.6

$$\text{解: } H(e^{j\omega}) = \begin{cases} 1 & \pi - \omega_c \leq |\omega| \leq \pi \\ 0 & |\omega| > \pi - \omega_c \end{cases}$$

$$\therefore h[n] = \left(\frac{\sin \omega_c n}{\pi n} \right) g[n] \quad \text{作傅里叶变换有}$$

$$H(e^{j\omega}) = F(e^{j\omega}) * G(e^{j\omega}) \quad F(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$$

$$\text{对 } H(e^{j\omega}) \text{ 作傅里叶反变换 由变换对 } \frac{\sin Wn}{\pi n} \xrightarrow{F} F(e^{j\omega}) \begin{cases} 1 & |\omega| < W \\ 0 & W < |\omega| < \pi \end{cases}$$

$$\text{得到 } H(e^{j\omega}) = F(e^{j(\omega - (\pi - \frac{\omega_c}{2}))}) + F(e^{j(\omega + (\pi - \frac{\omega_c}{2}))}) \quad W = \frac{\omega_c}{2}$$

由频移和线性性质

$$h[n] = e^{j(\pi - \frac{\omega_c}{2})n} \frac{\sin \frac{\omega_c}{2} n}{\pi n} + e^{-j(\pi - \frac{\omega_c}{2})n} \frac{\sin \frac{\omega_c}{2} n}{\pi n}$$

$$= (-1)^n \cdot 2 \frac{\cos \frac{\omega_c}{2} n \cdot \sin \frac{\omega_c}{2} n}{\pi n}$$

$$= (-1)^n \frac{\sin \omega_c n}{\pi n}$$

$$\therefore g[n] = (-1)^n \quad \omega_c > \frac{\omega_c}{2}$$

6.22

$$\text{解: (a) } |H(j\omega)| = \begin{cases} \frac{\omega}{3\pi} & 0 < \omega < 3\pi \\ -\frac{\omega}{3\pi} & -3\pi < \omega < 0 \end{cases}$$

$$\angle H(j\omega) = \begin{cases} \frac{\pi}{2} & 0 < \omega < 3\pi \\ -\frac{\pi}{2} & -3\pi < \omega < 0 \end{cases}$$

$$\therefore x(t) = \cos(2\pi t + 0)$$

$$\omega_0 = 2\pi$$

$$\therefore y(t) = \frac{2\pi}{3\pi} \cos(2\pi t + 0 + \frac{\pi}{2})$$

$$= -\frac{2}{3} \sin(2\pi t + 0)$$

(b) $x(t) = \cos(4\pi t + \theta)$ $\omega_0 = 4\pi$ 不在低通微分器的频率范围内

$$\therefore y(t) = 0$$

6.28

$$\text{解: (iv)} \quad 20 \lg \left| \frac{1 - \frac{1}{10}j\omega}{1 + j\omega} \right| = 20 \lg \left(\frac{1 + \frac{1}{100}\omega^2}{1 + \omega^2} \right)$$

若 $\omega \ll 1$ $20 \lg \left(\frac{1 + \frac{1}{100}\omega^2}{1 + \omega^2} \right) = 20 \lg 1 = 0 \text{ dB}$ $\omega = 10$ 和 $\omega = 1$ 为折断频率

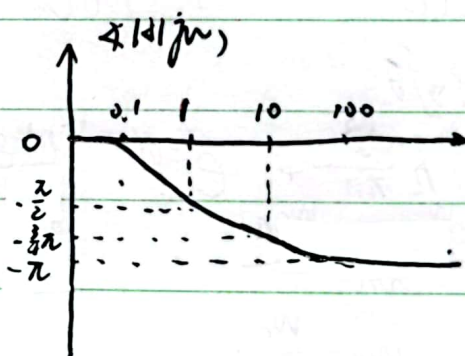
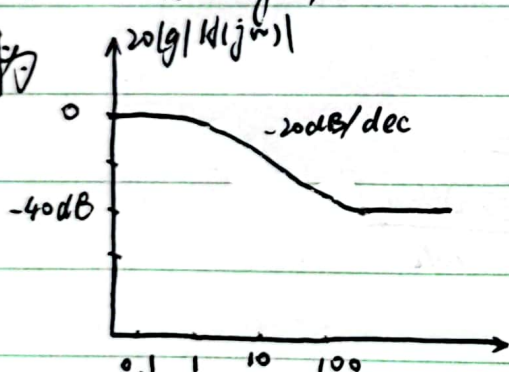
若 $\omega \gg 10$ $20 \lg \left(\frac{1 + \frac{1}{100}\omega^2}{1 + \omega^2} \right) = 20 \lg \left(\frac{1}{\omega^2} \right) = -40 \text{ dB}$

相频特性 $\angle H(j\omega) = -\arctan \frac{\omega}{10} - \arctan \omega$

$\omega \gg 10$ $\angle H(j\omega) = -\pi$ $\omega = 1$ $\angle H(j\omega) = -\frac{\pi}{4}$ $\omega = 10$ $\angle H(j\omega) = -\frac{3\pi}{4}$

$\omega \ll 1$ $\angle H(j\omega) = 0$

作图



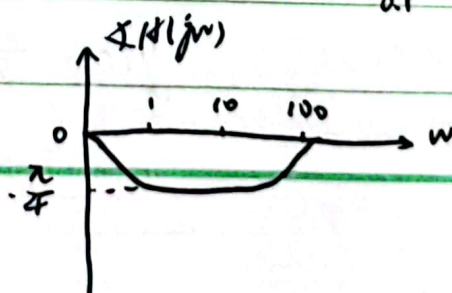
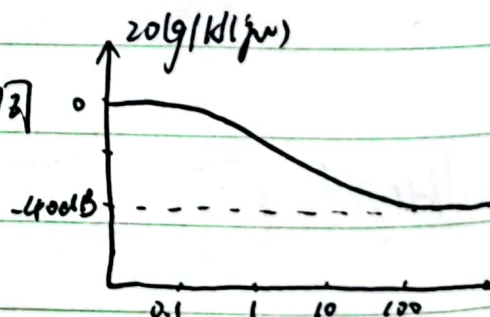
(vi) 类似 (v) $20 \lg \left| \frac{1 + \frac{1}{10}j\omega}{1 + j\omega} \right| = 20 \lg \left(\frac{1 + \frac{1}{100}\omega^2}{1 + \omega^2} \right)$ 类似 (v)

$\angle H(j\omega) = \arctan \frac{\omega}{10} - \arctan \omega$

$\omega \ll 1$ $\angle H(j\omega) = 0$

$\omega \gg 10$ $\angle H(j\omega) = 0$

$\omega = 1$ $\angle H(j\omega) = -\frac{\pi}{4}$ $\omega = 10$ $\angle H(j\omega) = \frac{\pi}{4}$



$$(b) H_1(j\omega) = \frac{1 - \frac{1}{10}j\omega}{1 + j\omega} \quad H_2(j\omega) = \frac{1 + \frac{1}{10}j\omega}{1 + j\omega}$$

$$\delta(t) \xrightarrow{F} 1 \quad u(t) \xrightarrow{F} \frac{1}{j\omega} + \pi\delta(\omega)$$

$$\therefore \text{冲激响应: } Y_1(j\omega) = \frac{1 - \frac{1}{10}j\omega}{1 + j\omega} = \frac{11}{10} \frac{1}{1 + j\omega} - \frac{1}{10}$$

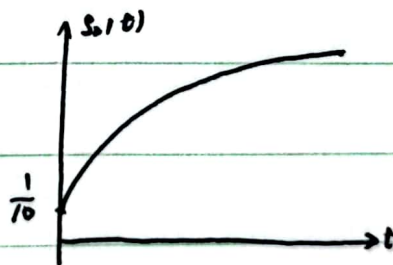
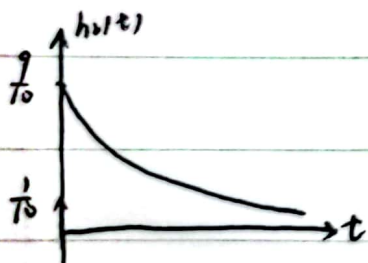
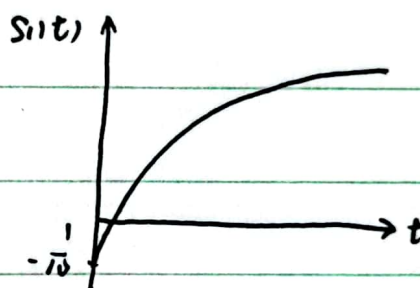
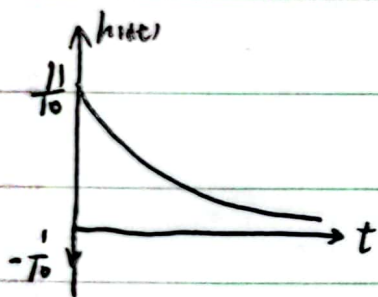
$$h_1(t) = \frac{11}{10} e^{-t} u(t) - \frac{1}{10} \delta(t)$$

$$Y_2(j\omega) = \frac{1 + \frac{1}{10}j\omega}{1 + j\omega} = \frac{9}{10} \frac{1}{1 + j\omega} + \frac{1}{10}$$

$$h_2(t) = \frac{9}{10} e^{-t} u(t) + \frac{1}{10} \delta(t)$$

$$\text{阶跃响应: } S_1(t) = \int_{-\infty}^t h_1(\tau) d\tau = \frac{11}{10} (1 - e^{-t}) u(t) - \frac{1}{10} u(t)$$

$$S_2(t) = \int_{-\infty}^t h_2(\tau) d\tau = \frac{9}{10} (1 - e^{-t}) u(t) + \frac{1}{10} u(t)$$

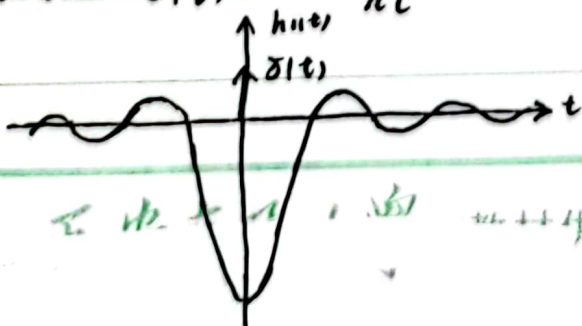


6.33.

$$\text{解: (a) } K(j\omega) = \begin{cases} 1 & |\omega| < \omega_{lp} \\ 0 & |\omega| > \omega_{lp} \end{cases}$$

$$\text{单位冲激响应 } h(t) = 1 - K(j\omega) = \begin{cases} 0 & |\omega| < \omega_{lp} \\ 1 & |\omega| > \omega_{lp} \end{cases} \quad \text{截止频率 } \omega_{lp}$$

$$h(t) = \delta(t) - \frac{\sin \omega_{lp} t}{\pi t}$$



No: _____

Date: _____

$$(b) |H_1(j\omega)| = \begin{cases} 1 & |\omega| > W_{hp} \\ 0 & |\omega| < W_{hp} \end{cases}$$

互补滤波器 $|H_2(j\omega)| = 1 - |H_1(j\omega)| = \begin{cases} 1 & |\omega| < W_{hp} \\ 0 & |\omega| > W_{hp} \end{cases}$ 截止在 W_{hp} 低通

$$h(t) = \frac{\sin W_{hp} t}{\pi t}$$

$$(c) |H_1(e^{j\omega})| = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \omega_0 < |\omega| < \pi \end{cases}$$

$$|H_3(e^{j\omega})| = 1 - |H_1(e^{j\omega})| = \begin{cases} 0 & |\omega| < \omega_0 \\ 1 & \omega_0 < |\omega| < \pi \end{cases}$$

是高通滤波器

6.15

解: (a) 作傅里叶变换 $(j\omega)^2 Y(j\omega) + 4j\omega Y(j\omega) + 4Y(j\omega) = X(j\omega)$

$$\therefore H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{(j\omega)^2 + 4j\omega + 4} = \frac{1}{(j\omega + 2)^2} = \frac{\frac{1}{4}}{(\frac{j\omega}{2} + j\omega + 1)} \quad \zeta = 1 \quad \text{临界阻尼}$$

$$(b) \quad 5(j\omega)^2 Y(j\omega) + 4j\omega Y(j\omega) + 5Y(j\omega) = 7X(j\omega)$$

$$\therefore H(j\omega) = \frac{7}{5(j\omega)^2 + 4j\omega + 5} = \frac{\frac{7}{5}}{(j\omega)^2 + 2(\frac{2}{5})j\omega + 1} \quad \zeta = \frac{2}{5} \quad \text{欠阻尼}$$

$$(c) \quad (j\omega)^2 Y(j\omega) + 20(j\omega) Y(j\omega) + Y(j\omega) = X(j\omega)$$

$$H(j\omega) = \frac{1}{(j\omega)^2 + 20j\omega + 1} = \frac{1}{(j\omega)^2 + 2(10)j\omega + 1} \quad \zeta = 10 \quad \text{过阻尼}$$

$$(d) \quad 5(j\omega)^2 Y(j\omega) + 4(j\omega) Y(j\omega) + 5Y(j\omega) = 7X(j\omega) + \frac{1}{3}(j\omega) X(j\omega)$$

$$H(j\omega) = \frac{7 + \frac{1}{3}j\omega}{5(j\omega)^2 + 4j\omega + 5} \quad \text{分子无影响} \quad \text{故仍为欠阻尼}$$

6.17

解: (a) 作F变换为

$$Y(e^{j\omega}) + e^{j\omega} Y(e^{j\omega}) + \frac{1}{4} e^{-j\omega} Y(e^{j\omega}) = X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{1}{\frac{1}{4} e^{j\omega} + e^{j\omega} + 1} = \frac{1}{(\frac{1}{2} e^{j\omega})^2 - 2 \times \frac{1}{2} \times (-1) e^{j\omega} + 1} \quad \cos \theta = -1 \quad \theta = \pi \quad \text{是振荡}$$

$$(b) \quad H(e^{j\omega}) = \frac{1}{\frac{1}{4} e^{j\omega} - e^{j\omega} + 1} = \frac{1}{(\frac{1}{2} e^{j\omega})^2 - 2 \times \frac{1}{2} \times 1 e^{j\omega} + 1} \quad \cos \theta = 1 \quad \theta = 0 \quad \text{不是振荡}$$

6.27

$$\text{解: 作傅里叶变换有 } (j\omega) \cdot Y(j\omega) + 2Y(j\omega) = X(j\omega) \Rightarrow H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{2 + j\omega}$$

$$(a) \quad 20 \lg |H(j\omega)| = 10 \lg \frac{1}{4 + \omega^2}$$

$$\angle H(j\omega) = -\arctan \frac{\omega}{2}$$

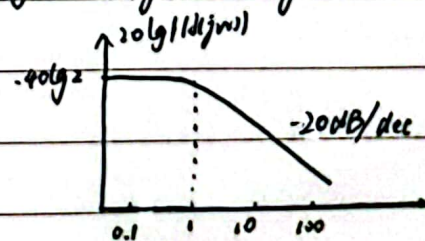
$$\omega \ll 1 \quad 20 \lg |H(j\omega)| = -20 \lg 2 \text{ dB}$$

$$\omega \gg 1 \quad 20 \lg |H(j\omega)| = -10 \lg (4 + \omega^2) \approx -20 \lg \omega$$

$$(b) \quad \tau(\omega) = \frac{d \angle H(j\omega)}{d\omega} = + \frac{1}{2} \cdot \frac{1}{1 + \frac{\omega^2}{4}} = \frac{2}{4 + \omega^2}$$

$$(c) \quad X(j\omega) = \frac{1}{1 + j\omega} \quad Y(j\omega) = X(j\omega) \cdot H(j\omega) = \frac{1}{(1 + j\omega)(2 + j\omega)} = \frac{1}{1 + j\omega} - \frac{1}{2 + j\omega}$$

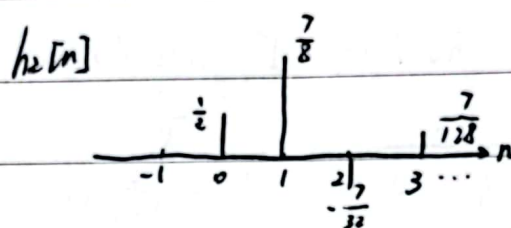
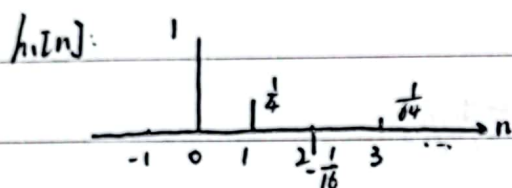
$$(d) \quad y(t) = e^{-t} u(t) - e^{-2t} u(t)$$



02

$$b) \quad H_1(e^{j\omega}) = \frac{1 + \frac{1}{2}e^{-j\omega}}{1 + \frac{1}{4}e^{-j\omega}} = -\frac{1}{1 + \frac{1}{4}e^{-j\omega}} + 2 \quad h_1[n] = -(-\frac{1}{4})^n u[n] + 2\delta[n]$$

$$H_2(e^{j\omega}) = \frac{\frac{1}{2} + e^{-j\omega}}{1 + \frac{1}{4}e^{-j\omega}} = -\frac{\frac{7}{2}}{1 + \frac{1}{4}e^{-j\omega}} + 4 \quad h_2[n] = 4\delta[n] - \frac{7}{2}(-\frac{1}{4})^n u[n]$$



$$c) \quad G(e^{j\omega}) = \frac{H_2(e^{j\omega})}{H_1(e^{j\omega})} = \frac{\frac{1}{2} + e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}} \quad |G(e^{j\omega})| = \sqrt{\frac{\frac{1}{4} + 1}{1 + 1}} = 1$$

$\therefore \forall \omega$ 均有 $|G(e^{j\omega})| = 1$ 是全通系统