

3.5

解:  $x_2(t) = x_1(t-1) + x_1(-(t-1))$  显然  $x_2(t)$  为  $x_1(t)$  的翻转、移位线性组合而周期性不变, 故  $\omega_2 = \omega_1$

$$x_1(t) = \sum_{k=-\infty}^{+\infty} A_k e^{jk\omega_1 t} \quad \text{即 } x_1(t) \rightarrow A_k$$

$$x_1(t-1) \rightarrow e^{-jk\omega_1} A_k$$

$$x_1(1-t) = x_1(-(t-1)) \rightarrow e^{-jk\omega_1} A_{-k}$$

由线性性质  $x_2(t) \rightarrow e^{-jk\omega_1} (A_k + A_{-k})$

3.8

解:  $\because x(t)$  为实奇函数  $\therefore A_k^* = A_{-k} = -A_k$

$\therefore A_k$  为纯虚数

$$\omega_0 = \frac{2\pi}{T} = \pi$$

$$\because T=2 \quad |k|>1 \quad A_k=0$$

$$\therefore A_0=0 \quad \text{只有 } A_{-1} \text{ 和 } A_1 \text{ 不为 } 0$$

由 Parseval 定理  $\frac{1}{T} \int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |A_k|^2$

即  $|A_1|^2 + |A_{-1}|^2 = 1 \Rightarrow 2|A_1|^2 = 1 \quad A_1 = \frac{1}{\sqrt{2}}j \text{ 或 } A_1 = -\frac{1}{\sqrt{2}}j$

若  $A_1 = \frac{1}{\sqrt{2}}j \quad A_{-1} = -\frac{1}{\sqrt{2}}j \quad x(t) = \frac{1}{\sqrt{2}}j \cdot e^{j\pi t} - \frac{1}{\sqrt{2}}j e^{-j\pi t}$   
 $= \sqrt{2} \sin \pi t$

若  $A_1 = -\frac{1}{\sqrt{2}}j \quad A_{-1} = \frac{1}{\sqrt{2}}j \quad x(t) = -\frac{1}{\sqrt{2}}j e^{j\pi t} + \frac{1}{\sqrt{2}}j e^{-j\pi t}$   
 $= -\sqrt{2} \sin \pi t$

即  $x(t) = \sqrt{2} \sin \pi t \text{ 或 } -\sqrt{2} \sin \pi t$

No: \_\_\_\_\_

Date: \_\_\_\_\_

3.22

(a) 观察可知  $T=2$   $\omega_0 = \frac{2\pi}{T} = \pi$   $x(-t) = -x(t)$ .  $A_k$  为奇对称. 则  $A_0 = 0$

$$x(t) = \sum_{k=-\infty}^{\infty} A_k e^{-jk\pi t}$$

$$A_k = \frac{1}{2} \int_{-1}^1 t e^{-jk\pi t} dt = \frac{1}{2} \left( -\frac{1}{jk\pi} t e^{-jk\pi t} + \frac{1}{k^2\pi^2} e^{-jk\pi t} \right) \Big|_{-1}^1$$

$$= \frac{1}{2} \left( -\frac{1}{jk\pi} e^{-jk\pi} + \frac{1}{k^2\pi^2} e^{-jk\pi} - (-1) \frac{1}{jk\pi} (-1) e^{jk\pi} - \frac{1}{k^2\pi^2} e^{jk\pi} \right)$$

$$= \frac{1}{2} \left[ -\frac{1}{jk\pi} (e^{jk\pi} + e^{-jk\pi}) - \frac{1}{k^2\pi^2} (e^{jk\pi} - e^{-jk\pi}) \right]$$

$$= -\frac{1}{jk\pi} \cos k\pi - \frac{j}{k^2\pi^2} \sin k\pi$$

$$= \frac{j}{k\pi} (-1)^k \quad \text{即 } x(t) = \sum_{k=-\infty}^{\infty} \left( \frac{j}{k\pi} \right) (-1)^k e^{-jk\pi t}$$

(b)  $T=6$   $\omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$   $x(t) = x(-t)$   $A_k$  偶对称

$$x(t) = \begin{cases} t+2 & -2 < t < -1 \\ 1 & -1 < t < 1 \\ -t+2 & 1 < t < 2 \\ 0 & 2 < t < 4 \end{cases}$$

$$A_k = \frac{1}{6} \int_{-2}^4 x(t) e^{jk\omega_0 t} dt$$

$$= \frac{1}{6} \left( \int_{-2}^{-1} (t+2) e^{-jk\omega_0 t} dt + \int_{-1}^1 e^{-jk\omega_0 t} dt + \int_1^2 (-t+2) e^{-jk\omega_0 t} dt + 0 \right)$$

$$= \frac{1}{6} \left[ \frac{1}{jk\omega_0} (e^{jk\omega_0} - 2e^{2jk\omega_0}) + \frac{1}{k^2\omega_0^2} (e^{jk\omega_0} - e^{2jk\omega_0}) + \frac{2}{jk\omega_0} (e^{2jk\omega_0} - e^{jk\omega_0}) + \frac{1}{jk\omega_0} (e^{jk\omega_0} - e^{-jk\omega_0}) + \frac{1}{jk\omega_0} (2e^{-2jk\omega_0} - e^{-jk\omega_0}) + \frac{1}{k^2\omega_0^2} (e^{-jk\omega_0} - e^{-2jk\omega_0}) + \frac{2}{jk\omega_0} (e^{-jk\omega_0} - e^{-2jk\omega_0}) \right]$$

$$= \frac{1}{6} \left\{ \frac{1}{jk\omega_0} [0] + \frac{1}{k^2\omega_0^2} [(e^{jk\omega_0} - e^{-jk\omega_0}) - (e^{2jk\omega_0} - e^{-2jk\omega_0})] \right\}$$

$$= \frac{1}{6} \cdot \frac{9}{k^2\pi^2} \cdot (2 \cos \frac{\pi}{3} k - 2 \cos \frac{2\pi}{3} k) = \frac{3}{k^2\pi^2} (\cos \frac{\pi}{3} k - \cos \frac{2\pi}{3} k)$$



$$A_0 = \frac{1}{T} \int_{-2}^2 x(t) dt = \frac{1}{2}$$

$$\begin{aligned} \therefore x(t) &= \sum_{k=-\infty}^{\infty} \frac{1}{k^2 \pi^2} (\cos \frac{\pi}{3} k - \cos \frac{2\pi}{3} k) e^{j \frac{\pi}{3} k t} = \sum_{k=-\infty}^{\infty} \frac{1}{k^2 \pi^2} + A_0 + \sum_{k=1}^{\infty} \frac{1}{k^2 \pi^2} (1 - \cos \frac{2\pi}{3} k) \cos \frac{\pi}{3} k t \\ &= \frac{1}{2} + \sum_{k=1}^{\infty} \frac{1}{k^2 \pi^2} \sin^2 \frac{k\pi}{2} \cos \frac{\pi}{3} k t \end{aligned}$$

(10)  $T=3$   $\omega_0 = \frac{2\pi}{3}$   $A_0 = \frac{1}{3} \int_{-2}^1 x(t) dt = 1$

$$x(t) = \begin{cases} t+2 & -2 < t < 0 \\ -2t+2 & 0 < t < 1 \end{cases}$$

$$\begin{aligned} A_k &= \frac{1}{3} \int_{-2}^1 x(t) dt = \frac{1}{3} \left[ \int_{-2}^0 (t+2) e^{-jk\omega_0 t} dt + \int_0^1 (-2t+2) e^{-jk\omega_0 t} dt \right] \\ &= \frac{1}{3} \left[ \frac{1}{k^2 \omega_0^2} + \frac{-2}{jk\omega_0} e^{2jk\omega_0} - \frac{1}{k^2 \omega_0^2} e^{2jk\omega_0} - \frac{2}{jk\omega_0} (1 - e^{2jk\omega_0}) + \frac{2}{jk\omega_0} e^{-jk\omega_0} - \frac{2}{k^2 \omega_0^2} e^{-jk\omega_0} + \frac{2}{k^2 \omega_0^2} - \frac{2}{jk\omega_0} e^{-jk\omega_0} + \frac{2}{jk\omega_0} \right] \\ &= \frac{1}{3} \left[ \frac{1}{jk\omega_0} (0 + (-2+2) + (2-2) e^{-jk\omega_0}) + \frac{1}{k^2 \omega_0^2} [(1+2) e^{2jk\omega_0} - 2e^{-jk\omega_0}] \right] \\ &= \frac{1}{k^2 \omega_0^2} - \frac{1}{3k^2 \omega_0^2} [e^{2jk\omega_0} + e^{-jk\omega_0}] \\ &= \frac{9}{4k^2 \pi^2} - \frac{3}{2k^2 \pi^2} \cos k\pi \cdot e^{j \frac{k}{3} \pi} \\ \therefore x(t) &= \sum_{k=-\infty}^{\infty} A_k e^{j \frac{2\pi}{3} k t} \quad A_0 = 0 \quad A_k = \frac{9}{4k^2 \pi^2} - \frac{3}{2k^2 \pi^2} \cos k\pi \cdot e^{j \frac{k}{3} \pi} \quad k \neq 0 \end{aligned}$$

3.28

解: (1)  $N=6$   $\omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$   $x[n] = \begin{cases} 1 & 0 \leq n \leq 3 \\ 0 & 4 \leq n \leq 5 \end{cases}$

$$\begin{aligned} A_k &= \frac{1}{6} \sum_{n=0}^5 x[n] e^{-jk\omega_0 n} = \frac{1}{6} \sum_{n=0}^3 e^{-jk\omega_0 n} = \frac{1}{6} \cdot \frac{1 - (e^{-jk\omega_0})^4}{1 - e^{-jk\omega_0}} \\ &= \frac{1}{6} \cdot \frac{1 - e^{-j \frac{4\pi}{3} k}}{1 - e^{-j \frac{\pi}{3} k}} = \frac{1}{6} \frac{e^{-j \frac{\pi}{3} k} \sin \frac{4}{3} \pi k}{\sin \frac{\pi}{3} k} \end{aligned}$$

$$|A_k| = \frac{\sin^2 \frac{\pi}{3} k}{6 \sin^2 \frac{\pi}{6} k} \quad \angle A_k = -\frac{k}{2} \pi$$

No: \_\_\_\_\_

Date: \_\_\_\_\_

(c)  $N=6$   $\omega_0 = \frac{2\pi}{N} = \frac{\pi}{3}$  且  $x[n] = x[-n]$ . 则  $A_k = A_{-k}$

$$A_k = \frac{1}{6} \sum_{n=-\infty}^{\infty} x[n] e^{-jk\omega_0 n} = \frac{1}{6} [1 + 2(e^{-jk\omega_0} + e^{-j5k\omega_0}) - (e^{-2jk\omega_0} + e^{-4jk\omega_0})]$$

$$= \frac{1}{6} [1 + 2(e^{-jk\frac{\pi}{3}} + e^{jk\frac{\pi}{3}}) - (e^{-2jk\frac{\pi}{3}} + e^{2jk\frac{\pi}{3}})]$$

$$= \frac{1}{6} [1 + 4\cos\frac{k\pi}{3} - 4\cos\frac{2k\pi}{3}]$$

$$|A_k| = \frac{1}{6} |1 + 4\cos\frac{k\pi}{3} - 4\cos\frac{2k\pi}{3}| \quad \nabla A_k = 0$$

3.11

证明:  $\because x[n]$  是实偶信号.  $\therefore A_k = A_{-k}$  且  $A_k$  为实数

$$A_k = A_{k+10}$$

$$A_{11} = 5 \Rightarrow A_1 = A_{-1} = A_9 = 5$$

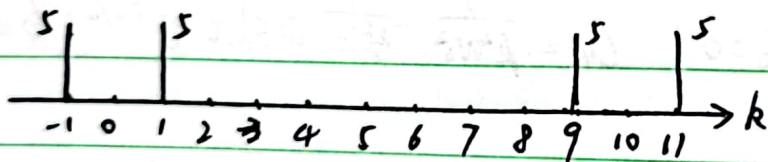
由 Parseval 定理

$$\frac{1}{10} \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{k=-\infty}^{\infty} A_k^2$$

$$\text{而 } A_1 = A_9 = 5$$

$$\therefore A_0, A_2, A_3, \dots, A_8 = 0$$

即  $x[n]$  的频谱为



$$\therefore x[n] = \sum_{k=-\infty}^{\infty} A_k e^{jk\omega_0 n}$$

$$\omega_0 = \frac{2\pi}{10}$$

$$\therefore x[n] = 5(e^{j\frac{2\pi}{10}n} + e^{j\frac{2\pi}{10}9n})$$

$$= 5(e^{-j\frac{2\pi}{10}n} + e^{j\frac{2\pi}{10}n})$$

$$= 5\cos(\frac{\pi}{5}n) = A\cos(Bn+C)$$

$$\therefore A=5 \quad B=\frac{\pi}{5} \quad C=0$$



3.30

$$(a) x[n] = 1 + \frac{1}{2}(e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}n}) \quad \therefore a_0 = 1 \quad a_1 = \frac{1}{2} \quad a_{-1} = \frac{1}{2} = a_5$$

$\therefore$  周期内的傅里叶系数为  $a_0 = 1 \quad a_1 = \frac{1}{2} \quad a_5 = \frac{1}{2}$

$$(b) y[n] = \sin(\frac{2\pi}{8}n + \frac{\pi}{4}) = \frac{1}{2j}(e^{j(\frac{2\pi}{8}n + \frac{\pi}{4})} - e^{-j(\frac{2\pi}{8}n + \frac{\pi}{4})})$$

$$= \frac{1}{2j} \cdot (e^{j\frac{\pi}{4}} \cdot e^{j\frac{2\pi}{8}n} - e^{-j\frac{\pi}{4}} \cdot e^{-j\frac{2\pi}{8}n})$$

$$= \frac{\sqrt{2}}{4}(1-j)e^{j\frac{2\pi}{8}n} + \frac{\sqrt{2}}{4}(1+j)e^{-j\frac{2\pi}{8}n}$$

$\therefore$  周期内的傅里叶系数  $b_1 = \frac{\sqrt{2}}{4}(1-j) \quad b_{-1} = b_5 = \frac{\sqrt{2}}{4}(1+j)$

$$(c) z[n] = x[n] \cdot y[n] \quad \text{则 } C_k = A_k \times b_k$$

$$\text{即 } C_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l} = \sum_{l=0}^5 a_l b_{k-l}$$

$$\therefore C_0 = \sum_{l=0}^5 a_l b_{0-l} = \frac{\sqrt{2}}{4} \quad C_1 = \sum_{l=0}^5 a_l b_{1-l} = \frac{\sqrt{2}}{4}(1-j)$$

$$C_2 = \sum_{l=0}^5 a_l b_{2-l} = \frac{\sqrt{2}}{8}(1-j) \quad C_3 = \sum_{l=0}^5 a_l b_{3-l} = 0$$

$$C_4 = \sum_{l=0}^5 a_l b_{4-l} = \frac{\sqrt{2}}{8}(1+j) \quad C_5 = \sum_{l=0}^5 a_l b_{5-l} = \frac{\sqrt{2}}{4}(1+j)$$

$$(d) z[n] = (1 + \cos \frac{2\pi}{8}n) \cdot \sin(\frac{2\pi}{8}n + \frac{\pi}{4})$$

$$= \sin(\frac{2\pi}{8}n + \frac{\pi}{4}) + \sin(\frac{2\pi}{8}n + \frac{\pi}{4}) \cdot \cos \frac{2\pi}{8}n$$

$$= \sin(\frac{2\pi}{8}n + \frac{\pi}{4}) + \frac{1}{2}[\sin(\frac{4\pi}{8}n + \frac{\pi}{4}) + \sin \frac{\pi}{4}]$$

$$= \frac{\sqrt{2}}{4} + \sin(\frac{2\pi}{8}n + \frac{\pi}{4}) + \frac{1}{2}\sin(\frac{4\pi}{8}n + \frac{\pi}{4})$$

$$= \frac{\sqrt{2}}{4} + \frac{1}{2j}(e^{j(\frac{2\pi}{8}n + \frac{\pi}{4})} - e^{-j(\frac{2\pi}{8}n + \frac{\pi}{4})}) + \frac{1}{4j}(e^{j(\frac{4\pi}{8}n + \frac{\pi}{4})} - e^{-j(\frac{4\pi}{8}n + \frac{\pi}{4})})$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}(1-j)e^{j\frac{2\pi}{8}n} + \frac{\sqrt{2}}{4}(1+j)e^{-j\frac{2\pi}{8}n} + \frac{\sqrt{2}}{8}(1-j)e^{j\frac{4\pi}{8}n} + \frac{\sqrt{2}}{8}(1+j)e^{-j\frac{4\pi}{8}n}$$

$$\therefore C_0 = \frac{\sqrt{2}}{4} \quad C_1 = \frac{\sqrt{2}}{4}(1-j) \quad C_{-1} = C_5 = \frac{\sqrt{2}}{4}(1+j) \quad C_2 = \frac{\sqrt{2}}{8}(1-j) \quad C_4 = \frac{\sqrt{2}}{8}(1+j)$$

和(3)相符  $C_3 = 0$

Date: \_\_\_\_\_

3.34

$$H(jk\omega_0) = \int_{-\infty}^{\infty} h(t) e^{-jk\omega_0 t} dt \quad T=2 \quad \omega_0 = \frac{2\pi}{T} = \pi$$

$$\therefore H(jk\omega_0) = \int_{-\infty}^0 e^{4t} \cdot e^{-jk\omega_0 t} dt + \int_0^{\infty} e^{-4t} e^{-jk\omega_0 t} dt$$

$$= \int_{-\infty}^0 e^{-(jk\omega_0 - 4)t} dt + \int_0^{\infty} e^{-(jk\omega_0 + 4)t} dt$$

$$= \frac{1}{4 - jk\omega_0} + \frac{1}{4 + jk\omega_0} = \frac{1}{4 - jk\pi} + \frac{1}{4 + jk\pi} = \frac{8}{16 + k^2\pi^2}$$

$$A_k = \frac{1}{T} \int_T x(t) e^{jk\omega_0 t} dt$$

$$= \frac{1}{2} \int_0^2 (\delta(t) - \delta(t-1)) e^{jk\omega_0 t} dt$$

$$= \frac{1}{2} \left( \int_0^2 \delta(t) e^{jk\omega_0 t} dt - \int_0^2 \delta(t-1) e^{jk\omega_0 t} dt \right)$$

$$= \frac{1}{2} (1 - e^{-jk\pi}) = \frac{1}{2} (1 - \cos k\pi) = \begin{cases} 0 & k \text{ 为偶} \\ 1 & k \text{ 为奇} \end{cases}$$

$$\therefore y(t) = \sum_{k=-\infty}^{\infty} A_k H(jk\omega_0) e^{jk\omega_0 t} = \begin{cases} 0 & k \text{ 为偶} \\ \frac{8}{16 + \pi^2 k^2} e^{jk\pi t} & k \text{ 为奇} \end{cases}$$

$$(c) x(t) = \begin{cases} 1 & -\frac{1}{4} < t < \frac{1}{4} \\ 0 & \frac{1}{4} < t < \frac{3}{4} \end{cases}$$

$$T=1 \quad \omega_0 = \frac{2\pi}{T} = 2\pi$$

$$A_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

$$= \int_{-\frac{1}{4}}^{\frac{1}{4}} x(t) e^{-jk\omega_0 t} dt$$

$$= \int_{-\frac{1}{4}}^{\frac{1}{4}} e^{-jk\omega_0 t} dt + \int_{\frac{1}{4}}^{\frac{3}{4}} 0 dt = \frac{\sin(\frac{1}{2}k\pi)}{k\pi}$$

$$\therefore y(t) = \sum_{k=-\infty}^{\infty} A_k H(jk\omega_0) e^{jk\omega_0 t}$$

$$= \frac{8}{16 + 4k^2\pi^2} \cdot \frac{\sin(\frac{1}{2}k\pi)}{k\pi} \cdot e^{jk2\pi t}$$



3.35

$$\text{解: } T = \frac{\pi}{7} \quad \omega_0 = \frac{2\pi}{T} = 14.$$

$$\text{由于 } y(t) = x(t) \Rightarrow b_k = a_k.$$

$$\text{又 } b_k = |d(j\omega)| \cdot a_k. \quad \therefore \text{当 } |d(j\omega)| = 1 \text{ 时才有 } y(t) = x(t)$$

$$\text{即 } |14k| \leq 250. \quad \therefore |k| \leq 7 \text{ 时 } a_k = 0.$$

3.36

$$\text{解: 由差分方程改写 } h[n] - \frac{1}{4}h[n-1] = \delta[n]$$

$$\therefore h[n] = \left(\frac{1}{4}\right)^n u[n].$$

$$\therefore H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k} = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$$

$$(a) x[n] = \sin \frac{3\pi}{4}n$$

$$= \frac{1}{2j} (e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}) \quad \frac{3\pi}{4 \cdot 2\pi} = \frac{3}{8} \quad \therefore N=8, \quad \omega_0 = \frac{2\pi}{N} = \frac{\pi}{4}$$

$$= \frac{1}{2j} (e^{j\frac{3\pi}{8} \cdot 3n} - e^{-j\frac{3\pi}{8} \cdot 3n})$$

$$\therefore a_3 = \frac{1}{2j} \quad a_{-3} = -\frac{1}{2j}$$

bkm. 8 为周期

$$\therefore b_3 = a_3 \cdot H(e^{j3\omega_0}) = \frac{1}{2j} \cdot \frac{1}{1 - \frac{1}{4}e^{-j\frac{3\pi}{4}}}$$

$$b_3 = a_{-3} \cdot H(e^{j3\omega_0}) = \frac{1}{2j} \cdot \frac{1}{1 - \frac{1}{4}e^{j\frac{3\pi}{4}}}$$

$$(b) x[n] = \cos \frac{\pi}{4}n + 2\cos \frac{\pi}{2}n$$

(其余为0)

$$= \frac{1}{2} (e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}) + (e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n})$$

$$N_1=8 \quad N_2=4 \quad \therefore N=8$$

$$= \frac{1}{2} (e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}n}) + (e^{j\frac{4\pi}{8} \cdot 2n} + e^{-j\frac{4\pi}{8} \cdot 2n})$$

$$\omega_0 = \frac{2\pi}{8}$$

$$\therefore a_1 = a_{-1} = \frac{1}{2} \quad a_2 = a_{-2} = 1$$

$$\therefore b_1 = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{4}e^{-j\frac{\pi}{4}}}$$

$$b_{-1} = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{4}e^{j\frac{\pi}{4}}}$$

$$b_2 = \frac{1}{1 - \frac{1}{4}e^{-j\frac{\pi}{2}}}$$

$$b_{-2} = \frac{1}{1 - \frac{1}{4}e^{j\frac{\pi}{2}}}$$

$$= b_7$$

其余为0

$$= b_6$$

西安交通大学

教材供应站

bkm. 8 为周期

电话: 029-82668318 (东区)

82655434 (西区)

86652028 (城市学院)

3.43

$$x(t) = \sum_{k=-\infty}^{+\infty} A_k e^{jk \frac{2\pi}{T} t}$$

$$x(t + \frac{T}{2}) = \sum_{k=-\infty}^{+\infty} A_k e^{jk \frac{2\pi}{T} (t + \frac{T}{2})} = \sum_{k=-\infty}^{+\infty} A_k e^{jk \frac{2\pi}{T} t} \cdot e^{jk\pi}$$

$$e^{jk\pi} = (-1)^k \quad \text{又: } k \text{ 为奇数才不为 } 0$$

$$\therefore x(t + \frac{T}{2}) = -x(t)$$

$$ii) A_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

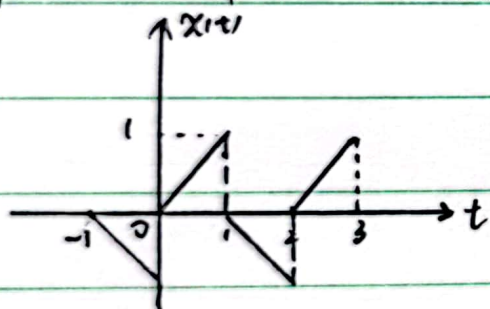
$$= \frac{1}{T} \int_0^{\frac{T}{2}} x(t) e^{-jk\omega_0 t} dt + \int_{\frac{T}{2}}^T x(t) e^{-jk\omega_0 t} dt \quad \text{令 } t = t - \frac{T}{2}$$

$$= \frac{1}{T} \int_0^{\frac{T}{2}} [x(t) + x(t + \frac{T}{2}) e^{-jk\pi}] e^{-jk\omega_0 t} dt$$

若  $x(t) = -x(t + \frac{T}{2})$  则  $A_k = 0$  当  $k$  为偶数时  $e^{-jk\pi} = 1$   $A_k = 0$

$\therefore$  若  $x(t) = -x(t + \frac{T}{2})$  则  $x(t)$  是奇谐的

ub)  $T=2$   $\omega_0 = \frac{2\pi}{T} = \pi$   $x(t) = -x(t+1)$  则有  $x(t)$  为



$$x(t) = \begin{cases} t & 0 < t < 1 \\ -t+1 & 1 < t < 2 \end{cases}$$

$$A_k = \frac{1}{2} \int_T x(t) e^{-jk\pi t} dt$$

$$= \frac{1}{2} \left( \int_0^1 t e^{-jk\pi t} dt + \int_1^2 (-t+1) e^{-jk\pi t} dt \right)$$

$$= \begin{cases} 0 & k \text{ 为偶} \\ \frac{1}{jk\pi} + \frac{2}{k^2\pi^2} & k \text{ 为奇} \end{cases}$$



No: \_\_\_\_\_

Date: \_\_\_\_\_

3.45

解: (a) 由  $x(t) = x_e(t) + x_o(t)$      $x_e(t) = \frac{x(t) + x(-t)}{2}$      $x_o(t) = \frac{x(t) - x(-t)}{2}$

$x_e(t) \xrightarrow{F} \text{Re}[A_k]$      $x_o(t) \xrightarrow{F} j \text{Im}[A_k]$

$x_e(t) = a_0 + 2 \sum_{k=1}^{\infty} B_k \cos k\omega_0 t$      $x_o(t) = 2 \sum_{k=1}^{\infty} [-C_k \sin k\omega_0 t]$

$\therefore \alpha_0 = 0$      $\alpha_k = B_k$   
 $\beta_0 = 0$      $\beta_k = \begin{cases} jC_k & k > 0 \\ -jC_k & k < 0 \end{cases}$

(b)  $\because x_e(t)$  为偶函数 实函数     $\therefore \alpha_k = \alpha_{-k}$

$\because x_o(t)$  为奇函数     $\therefore \beta_k = -\beta_{-k}$

(c)  $y_e(t) = 4(a_0 + d_0) + 2 \sum_{k=1}^{\infty} (B_k + \frac{1}{2} E_k) \cos \frac{2k\pi}{3} t$

$y_o(t) = 2 \sum_{k=1}^{\infty} (F_k \sin \frac{2k\pi}{3} t)$

3.48

$$x[n] = \sum_{k \in \langle N \rangle} a_k e^{jk \frac{2\pi}{N} n} \quad \frac{2\pi}{N} = \omega_0$$

$$(a) \quad x[n-n_0] = \sum_{k \in \langle N \rangle} e^{-jk \frac{2\pi}{N} n_0} \cdot a_k e^{jk \frac{2\pi}{N} n}$$

$$(b) \quad x[n] - x[n-1] = \sum_{k \in \langle N \rangle} a_k e^{jk \frac{2\pi}{N} n} - e^{-jk \frac{2\pi}{N}} \cdot a_k e^{jk \frac{2\pi}{N} n} \\ = \sum_{k \in \langle N \rangle} (1 - e^{-jk \frac{2\pi}{N}}) a_k e^{jk \frac{2\pi}{N} n}$$

$$(c) \quad x[n] - x[n - \frac{N}{2}] = \sum_{k \in \langle N \rangle} a_k e^{jk \frac{2\pi}{N} n} - e^{jk \frac{2\pi}{N} \frac{N}{2}} a_k e^{jk \frac{2\pi}{N} n} \\ = \sum_{k \in \langle N \rangle} [1 - (-1)^k] a_k e^{jk \frac{2\pi}{N} n}$$

$$(d) \quad x[n] + x[n + \frac{N}{2}] = \sum_{k \in \langle N \rangle} a_k e^{jk \frac{2\pi}{N} n} + e^{jk \pi} a_k e^{jk \frac{2\pi}{N} n} \\ = \sum_{k \in \langle N \rangle} [1 + (-1)^k] a_k e^{jk \frac{2\pi}{N} n}$$