第四章 单端口网络与多端口网络

- 4.1 基本定义
- 4.2 互联网络
- 4.3 网络特性及其应用
- 4.4 散射参量 （S参数）

4 单端口和多端口网络

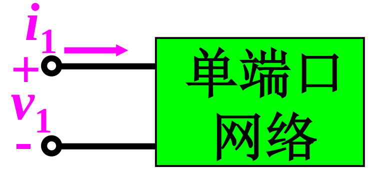
网络模型的作用

- ✓ 网络模型可以大量减少无源和有源器件数目；
- ✓ 避开电路的复杂性和非线性效应；
- ✓ 简化网络输入和输出特性的关系；
- ✓ 可以通过实验确定网络输入和输出参数，而不必了解系统内部的结构。

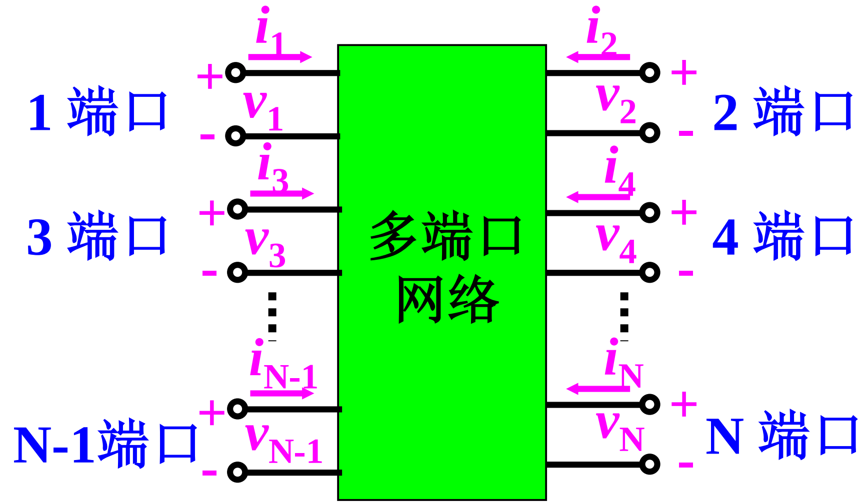
典型的网络

单口网络	负载，振荡器…
双口网络	滤波器、放大器、衰减器、隔离器…
多口网络	混频器、功分器、环行器、合成器…

4.1 基本定义



$$v_1 = Z_{11}i_1 + Z_{12}i_2 + \cdots + Z_{1N}i_N$$
$$v_2 = Z_{21}i_1 + Z_{22}i_2 + \cdots + Z_{2N}i_N$$
$$\vdots$$
$$v_N = Z_{N1}i_1 + Z_{N2}i_2 + \cdots + Z_{NN}i_N$$



$$\begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{Bmatrix} \Rightarrow \begin{Bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{Bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{Bmatrix}$$

$$Z_{nm} = \frac{v_n}{i_m}\bigg|_{i_k=0(for\ k\neq m)}$$

端口参量：[V]、[I]

网络参量：[Z]、[Y]

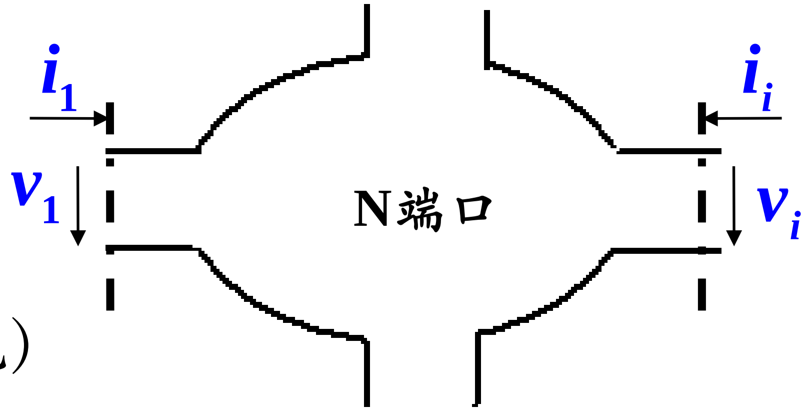
$$Y_{nm} = \frac{i_n}{v_m}\bigg|_{v_k=0(k\neq m)}$$

4.1 基本定义

$$Z_{nm}=\frac{v_n}{i_m}\bigg|_{i_k=0\,(\,for\ k\neq m)}$$

其它端口开路

m端口到n端口的转移阻抗（互阻抗）



$$Z_{ii}=\frac{v_i}{i_i}\bigg|_{i_k=0,\,k\neq i}$$

其它端口开路，

i端口的输入阻抗（自阻抗）

$$[Z]=[Y]^{-1}$$

$$Y_{nm}=\frac{i_n}{v_m}\bigg|_{v_k=0,\,k\neq m}$$

其它端口短路，

m端口到n端口的转移导纳（互导纳）

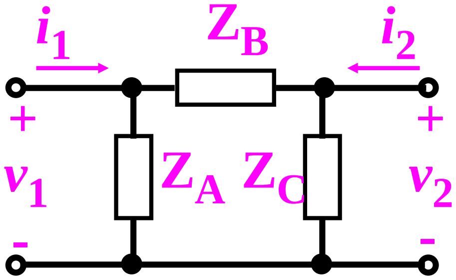
$$Y_{ii}=\frac{i_i}{v_i}\bigg|_{v_k=0,\,k\neq i}$$

其它端口短路，

i端口的输入导纳（自导纳）

4.1 基本定义

例4.1 求π形网络的阻抗矩阵和导纳矩阵。



解：

$$Z_{11}=\frac{v_1}{i_1}\bigg|_{i_2=0}=\frac{Z_A(Z_B+Z_C)}{Z_A+Z_B+Z_C}\qquad Z_{12}=\frac{v_1}{i_2}\bigg|_{i_1=0}=\frac{Z_AZ_C}{Z_A+Z_B+Z_C}$$

$$Z_{21}=\frac{v_2}{i_1}\bigg|_{i_2=0}=\frac{Z_AZ_C}{Z_A+Z_B+Z_C}\qquad Z_{22}=\frac{v_2}{i_2}\bigg|_{i_1=0}=\frac{Z_C(Z_A+Z_B)}{Z_A+Z_B+Z_C}$$

$$Y_{11}=\frac{i_1}{v_1}\bigg|_{v_2=0}=\frac{1}{Z_A}+\frac{1}{Z_B}\qquad Y_{12}=\frac{i_1}{v_2}\bigg|_{v_1=0}=-\frac{1}{Z_B}$$

$$Y_{21}=\frac{i_2}{v_1}\bigg|_{v_2=0}=-\frac{1}{Z_B}\qquad Y_{22}=\frac{i_2}{v_2}\bigg|_{v_1=0}=\frac{1}{Z_B}+\frac{1}{Z_C}$$

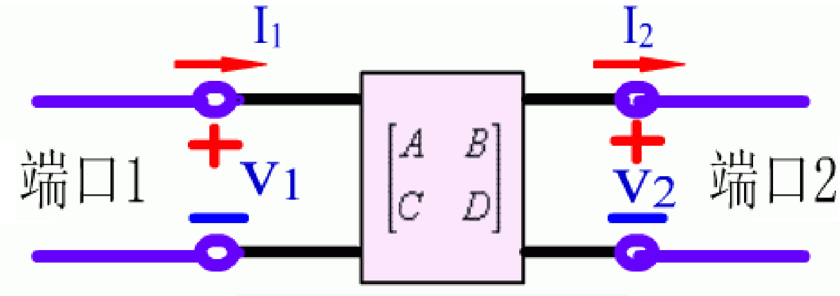
结论：通过假设网络端口为开路或短路状态，容易测得全部参数，且互易。

4.1 基本定义

转移参量 [A]

A矩阵

$$\begin{cases} v_1 = Av_2 + Bi_2 \\ i_1 = Cv_2 + Di_2 \end{cases} \qquad \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix}} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$



电流i_2的方向?

$A = \left.\dfrac{v_1}{v_2}\right|_{i_2=0}$ 端口2开路，2端口到1端口的电压转移系数

$D = \left.\dfrac{i_1}{i_2}\right|_{v_2=0}$ 端口2短路，2端口到1端口的电流转移系数

$B = \left.\dfrac{v_1}{i_2}\right|_{v_2=0}$ 端口2短路，2端口到1端口的转移阻抗

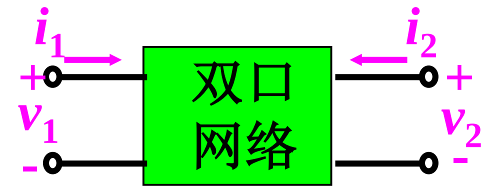
$C = \left.\dfrac{i_1}{v_2}\right|_{i_2=0}$ 端口2开路，2端口到1端口的转移导纳

4.1 基本定义

混合参量 [H]

H矩阵

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \boxed{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$



$$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0}$$ 端口2短路，端口1的输入阻抗

$$h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1=0}$$ 端口1开路，端口2的输入导纳

$$h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0}$$ 端口1开路，2端口到1端口的电压传输系数（反馈）

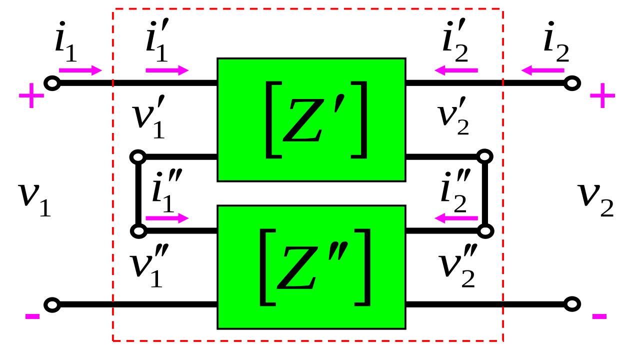
$$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0}$$ 端口2短路，1端口到2端口的电流传输系数（放大）

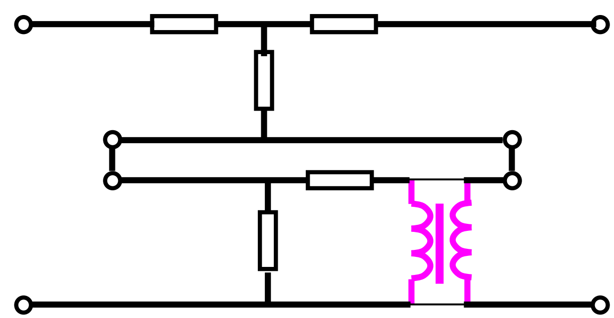
4.2.1 网络的串联

每个电压相互叠加而电流不变则用Z参数:

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{bmatrix} v_1' + v_1'' \\ v_2' + v_2'' \end{bmatrix} = [Z]\begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix}$$

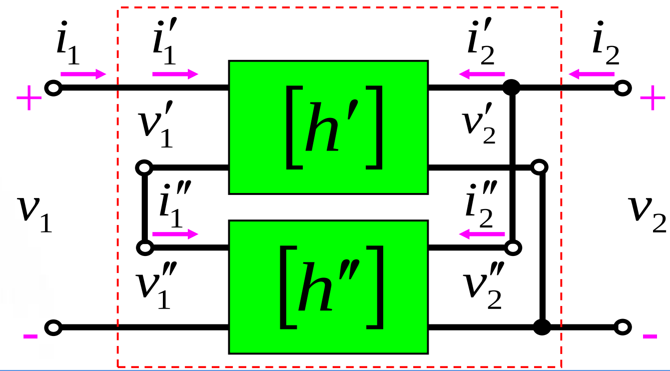
$$[Z] = [Z'] + [Z''] = \begin{bmatrix} Z_{11}' + Z_{11}'' & Z_{12}' + Z_{12}'' \\ Z_{21}' + Z_{21}'' & Z_{22}' + Z_{22}'' \end{bmatrix}$$





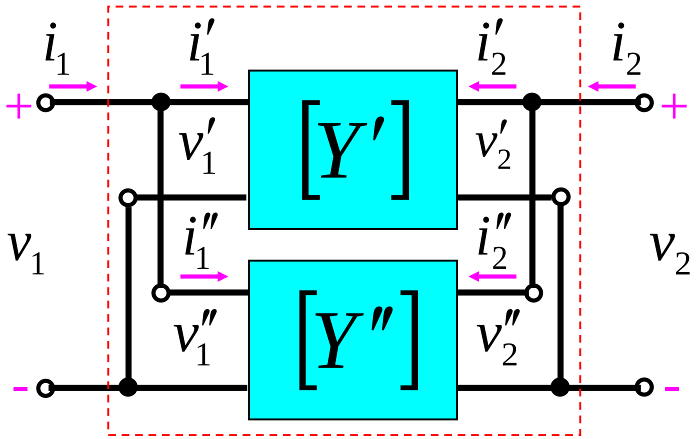
必须注意防止不加选择地将不同网络相连。

若输入电压及输出电流叠加，而输入电流及输出电压不变则用 h 参数:



$$\begin{Bmatrix} v_1 \\ i_2 \end{Bmatrix} = \begin{Bmatrix} v_1' + v_1'' \\ i_2' + i_2'' \end{Bmatrix} = \begin{bmatrix} h_{11}' + h_{11}'' & h_{12}' + h_{12}'' \\ h_{21}' + h_{21}'' & h_{22}' + h_{22}'' \end{bmatrix}\begin{Bmatrix} i_1 \\ v_2 \end{Bmatrix}$$

4.2.2 网络的并联

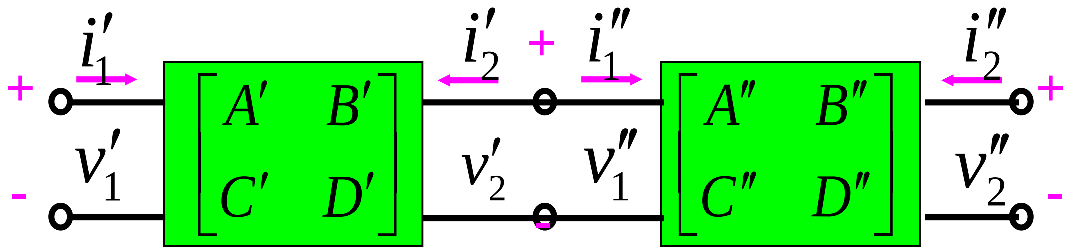


每个电流相互叠加而电压不变
则用Y参数:

$$\begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} = \begin{Bmatrix} i_1' + i_1'' \\ i_2' + i_2'' \end{Bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

$$[Y] = [Y'] + [Y'']$$

4.2.3 级联网络

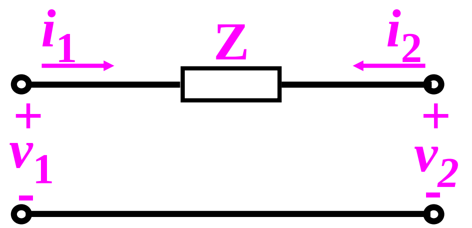


$$\begin{Bmatrix} v_1 \\ i_1 \end{Bmatrix} = \begin{Bmatrix} v_1' \\ i_1' \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} v_2' \\ -i_2' \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} v_1'' \\ i_1'' \end{Bmatrix}$$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{Bmatrix} v_1'' \\ -i_2'' \end{Bmatrix}$$

A参数特别适合级连网络

例4.2 求阻抗元件的ABCD参量。P105

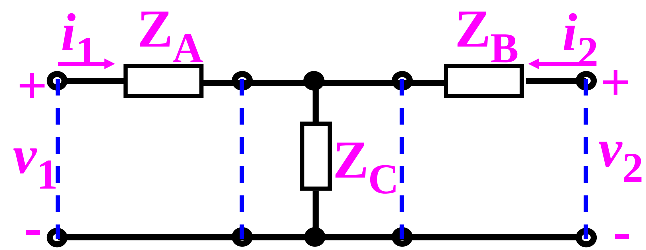


解：

$$A=\frac{v_1}{v_2}\bigg|_{i_2=0}=1 \qquad B=\frac{v_1}{-i_2}\bigg|_{v_2=0}=Z$$

$$C=\frac{i_1}{v_2}\bigg|_{i_2=0}=0 \qquad D=\frac{i_1}{-i_2}\bigg|_{v_2=0}=1$$

例4.3 求 T 形网络的ABCD参量。

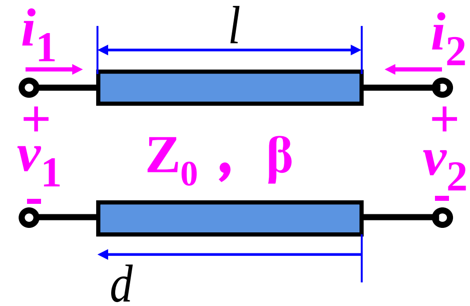


解：

$$[ABCD]=\boxed{\begin{bmatrix}1 & Z_A\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ Z_C^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & Z_B\\ 0 & 1\end{bmatrix}}$$

$$=\begin{bmatrix}1+Z_A/Z_C & Z_A+Z_B+Z_AZ_B/Z_C\\ 1/Z_C & 1+Z_B/Z_C\end{bmatrix}$$

例4.4 求传输线段的A参量。P106



解： 当端口2 开路时，在端口1有

$$V(d)=2V^{+}\cos(\beta d),\ I(d)=\frac{2jV^{+}}{Z_0}\sin(\beta d)$$

当端口2 短路时，在端口1有

$$V(d)=2jV^{+}\sin(\beta d),\ I(d)=\frac{2V^{+}}{Z_0}\cos(\beta d)$$

$I(d)$流向负载，

$i_1= I(d),\ i_2=-I(d)$

$$A=\frac{v_1}{v_2}\bigg|_{i_2=0}=\frac{2V^{+}\cos(\beta l)}{2V^{+}}=\cos(\beta l)$$

$$C=\frac{i_1}{v_2}\bigg|_{i_2=0}=\frac{2jV^{+}\sin(\beta l)/Z_0}{2V^{+}}=j\sin(\beta l)/Z_0$$

$$B=\frac{v_1}{-i_2}\bigg|_{v_2=0}=\frac{2jV^{+}\sin(\beta l)}{2V^{+}/Z_0}=jZ_0\sin(\beta l)$$

$$D=\frac{i_1}{-i_2}\bigg|_{v_2=0}=\frac{2V^{+}\cos(\beta l)/Z_0}{2V^{+}/Z}=\cos(\beta l)$$

4.3.1

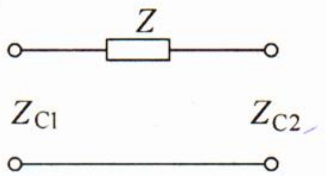
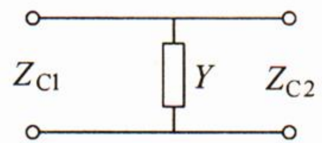
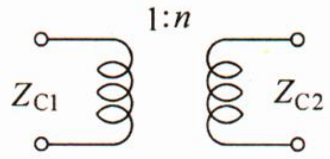
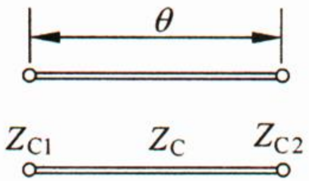
P109

表 4.2　不同网络参量之间的变换关系

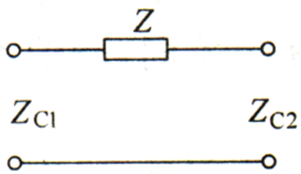
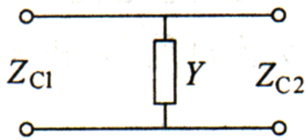
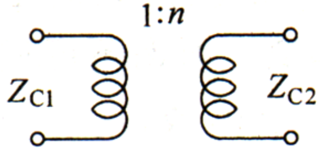
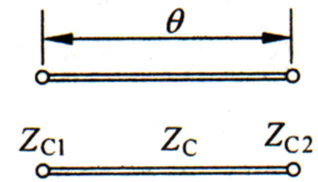
	[Z]	[Y]	[h]	[A]
[Z]	$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$
[Y]	$\begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$	$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$	$\begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$
[A]	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta ABCD}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & -\frac{\Delta ABCD}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta ABCD}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

4.3 网络特性及其应用

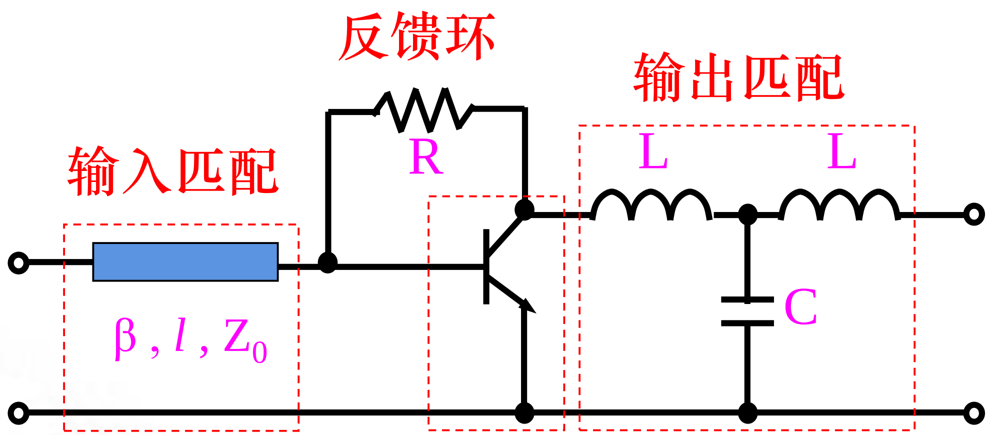
表 4.2　二端口等效单元电路的阻抗矩阵、导纳矩阵和转移矩阵

单元电路				
[Z]		$\begin{bmatrix} \frac{1}{Y} & \frac{1}{Y} \\ \frac{1}{Y} & \frac{1}{Y} \end{bmatrix}$		$\begin{bmatrix} -\mathrm{j}Z_C\cot\theta & \mathrm{j}Z_C\csc\theta \\ -\mathrm{j}Z_C\csc\theta & -\mathrm{j}Z_C\cot\theta \end{bmatrix}$
[z]		$\begin{bmatrix} \frac{1}{y} & \frac{1}{y\sqrt{r}} \\ \frac{1}{y\sqrt{r}} & \frac{1}{yr} \end{bmatrix}$		$\begin{bmatrix} -\mathrm{j}\cot\theta & -\mathrm{j}\csc\theta \\ -\mathrm{j}\csc\theta & -\mathrm{j}\cot\theta \end{bmatrix}$
[Y]	$\begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix}$			$\begin{bmatrix} -\frac{\mathrm{j}}{Z_C}\cot\theta & \frac{\mathrm{j}}{Z_C}\csc\theta \\ \frac{\mathrm{j}}{Z_C}\csc\theta & -\frac{\mathrm{j}}{Z_C}\cot\theta \end{bmatrix}$

续表

单元电路				
[y]	$\begin{bmatrix} \frac{1}{z} & -\frac{\sqrt{r}}{z} \\ -\frac{\sqrt{r}}{z} & \frac{r}{z} \end{bmatrix}$			$\begin{bmatrix} -\mathrm{j}\cot\theta & \mathrm{j}\csc\theta \\ \mathrm{j}\csc\theta & -\mathrm{j}\cot\theta \end{bmatrix}$
[A]	$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$	$\begin{bmatrix} \pm\frac{1}{n} & 0 \\ 0 & \pm n \end{bmatrix}$	$\begin{bmatrix} \cos\theta & \mathrm{j}Z_C\sin\theta \\ \frac{\mathrm{j}}{Z_C}\sin\theta & \cos\theta \end{bmatrix}$
[$\bar{a}$]	$\begin{bmatrix} \sqrt{r} & \frac{z}{\sqrt{r}} \\ 0 & \frac{1}{\sqrt{r}} \end{bmatrix}$	$\begin{bmatrix} \sqrt{r} & 0 \\ y\sqrt{r} & \frac{1}{\sqrt{r}} \end{bmatrix}$	$\begin{bmatrix} \pm\frac{\sqrt{r}}{n} & 0 \\ 0 & \pm\frac{n}{\sqrt{r}} \end{bmatrix}$	$\begin{bmatrix} \cos\theta & \mathrm{j}\sin\theta \\ \mathrm{j}\sin\theta & \cos\theta \end{bmatrix}$
说明：	$z=Z/Z_{C1}$ $r=Z_{C2}/Z_{C1}$	$y=YZ_{C1}$ $r=Z_{C2}/Z_{C1}$	$r=Z_{C2}/Z_{C1}$	$Z_{C1}=Z_{C2}=Z_C$

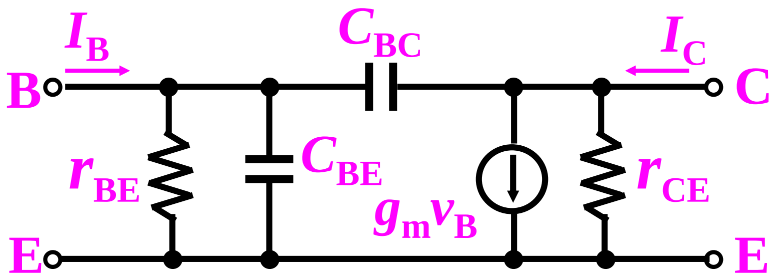
4.3.2 微波放大器分析



分析思路：

- 将h 参量变换为Y参量与反馈环并联
- 变换为A参量与匹配网络级连。

4.3 网络特性及其应用



$$h_{11}=\left.\frac{v_1}{i_1}\right|_{v_2=0}=\frac{v_1}{v_1\left(1/r_{BE}+j\omega C_{BE}+j\omega C_{BC}\right)}=\frac{r_{BE}}{1+j\omega\left(C_{BE}+C_{BC}\right)r_{BE}}$$

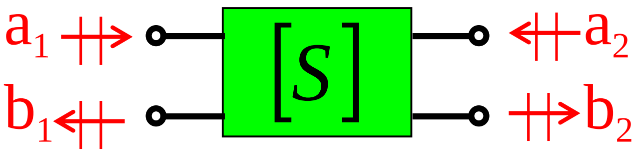
$$h_{12}=\left.\frac{v_1}{v_2}\right|_{i_1=0}=\frac{r_{BE}/\left(1+j\omega r_{BE}C_{BE}\right)}{1/j\omega C_{BC}+r_{BE}/\left(1+j\omega r_{BE}C_{BE}\right)}=\frac{j\omega C_{BC}r_{BE}}{1+j\omega\left(C_{BE}+C_{BC}\right)r_{BE}}$$

$$h_{21}=\left.\frac{i_2}{i_1}\right|_{v_2=0}=\frac{g_m v_1-j\omega C_{BC}v_1}{v_1\left(1/r_{BE}+j\omega C_{BE}+j\omega C_{BC}\right)}=\frac{r_{BE}\left(g_m-j\omega C_{BC}\right)}{1+j\omega\left(C_{BE}+C_{BC}\right)r_{BE}}$$

$$h_{22}=\left.\frac{i_2}{v_2}\right|_{i_1=0}=\frac{1}{r_{CE}}+\frac{g_m j\omega C_{BC}r_{BE}}{1+j\omega\left(C_{BE}+C_{BC}\right)r_{BE}}+\frac{j\omega C_{BC}\left(1+j\omega C_{BE}r_{BE}\right)}{1+j\omega\left(C_{BE}+C_{BC}\right)r_{BE}}$$

4.4 散射参量 [S]

4.4.1 S 参量的定义



定义归一化入射电压波：$a_n = \dfrac{V_n + Z_0 I_n}{2\sqrt{Z_0}}$

定义归一化反射电压波：$b_n = \dfrac{V_n - Z_0 I_n}{2\sqrt{Z_0}}$

相加：$V_n = (a_n + b_n)\sqrt{Z_0}$

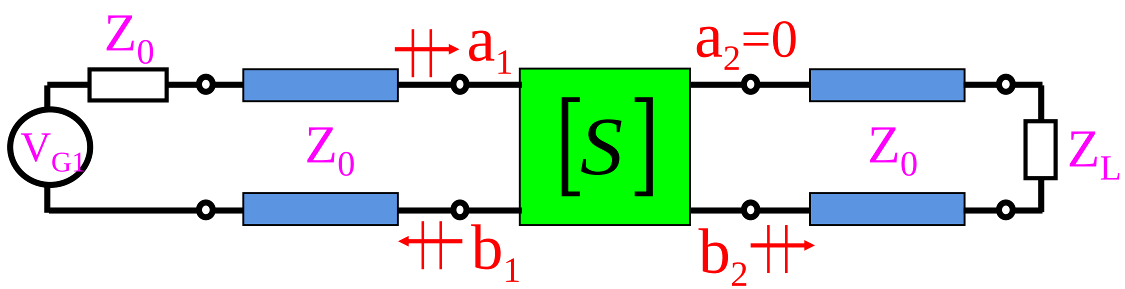
相减：$I_n = (a_n - b_n)/\sqrt{Z_0}$

$$P_n = \frac{1}{2}\mathrm{Re}\{V_n I_n^*\} = \frac{1}{2}\left(|a_n|^2 - |b_n|^2\right)$$

所以：$a_n = V_n^+/\sqrt{Z_0} = I_n^+\sqrt{Z_0}$　　$b_n = V_n^-/\sqrt{Z_0} = -I_n^-\sqrt{Z_0}$

定义S 参量：$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$ 其中：$S_{ij} = \dfrac{b_i}{a_j}\Big|_{a_n=0(n\neq j)}$

4.4.2 S 参量的物理意义

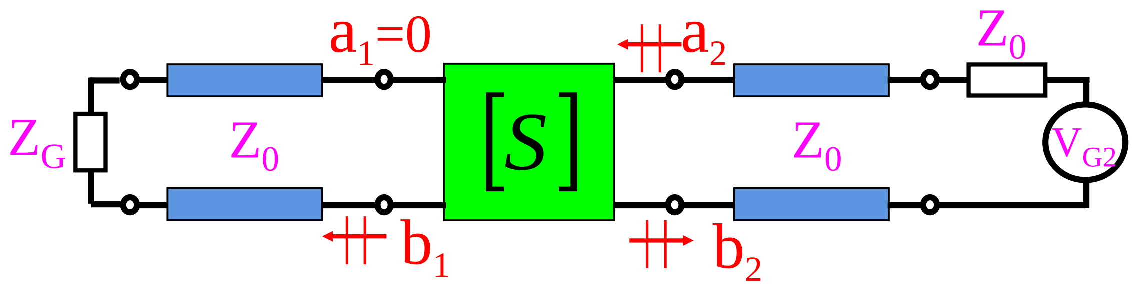


测量S_{11}和S_{21}, 为保证$a_2= 0$，必须使 $Z_L=Z_0$

则：$$S_{11}=\left.\frac{b_1}{a_1}\right|_{a_2=0}=\frac{V_1^-}{V_1^+}=\Gamma_{in}=\frac{Z_{in}-Z_0}{Z_{in}+Z_0}$$

$$S_{21}=\left.\frac{b_2}{a_1}\right|_{a_2=0}=\left.\frac{V_2^-/\sqrt{Z_0}}{\left(V_1+Z_0I_1\right)/\left(2\sqrt{Z_0}\right)}\right|_{I_2^+=V_2^+=0}=\frac{2V_2^-}{V_{G1}}=\frac{2V_2}{V_{G1}}$$ 正向电压增益

4.4.2 S 参量的物理意义

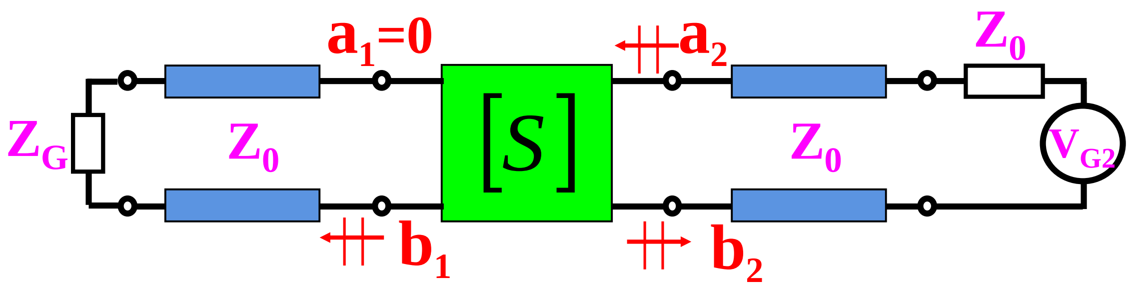


测量S_{22}和S_{12}, 为保证a_1= 0，必须使 $Z_G=Z_0$

则：$$S_{22}=\frac{b_2}{a_2}\bigg|_{a_1=0}=\frac{V_2^-}{V_2^+}=\Gamma_{out}=\frac{Z_{out}-Z_0}{Z_{out}+Z_0}$$

$$S_{12}=\frac{b_1}{a_2}\bigg|_{a_1=0}=\frac{V_1^-/\sqrt{Z_0}}{\left(V_2+Z_0I_2\right)/\left(2\sqrt{Z_0}\right)}\Bigg|_{I_1^+=V_1^+=0}=\frac{2V_1^-}{V_{G2}}=\frac{2V_1}{V_{G2}}$$ 反向电压增益

4.4.2 S 参量的物理意义



常用S参数名称

S11

前向反射系数

- 输入回波损耗
- 输入匹配
- VSWR

S21

前向传输系数

- 增益
- 损耗

S12

反向传输系数

- 反向隔离度

S22

反向反射系数

- 输出回波损耗
- 输出匹配

VSWR

4.4.2 S 参量的物理意义

电压驻波比（VSWR）与S参数关系

电压驻波比表示在端接任意负载的情况下，传输线Z0上可以测量到的最大电压与最小电压之比（驻波波峰电压与波谷电压的比值）。

对于输入端口 VSWR (s_{in}) 表示为

$$s_{\text{in}} = \frac{1 + |S_{11}|}{1 - |S_{11}|}$$

对于输出端口VSWR (s_{out})表示为

$$s_{\text{out}} = \frac{1 + |S_{22}|}{1 - |S_{22}|}$$

增益与S参数关系

网络增益与S参数之间关系如下

$$G = S_{21} = \frac{b_2}{a_1}$$

取标量可得

$$|G| = |S_{21}|$$

取对数表示则如下

$$g = 20\log_{10}|S_{21}| \ \text{dB}$$

4.4.2 S 参量的物理意义

回波损耗（Return Loss）与S参数关系

$$RL(\mathrm{dB}) = 10\log_{10}\frac{P_\mathrm{i}}{P_\mathrm{r}}$$

其中P_i 为输入功率，P_r 为反射功率。

对于二端口网络输入端口回波损耗为

$$RL_{\mathrm{in}} = 10\log_{10}\left|\frac{1}{S_{11}^2}\right| = -20\log_{10}|S_{11}| \quad \mathrm{dB}$$

输出端口回波损耗为

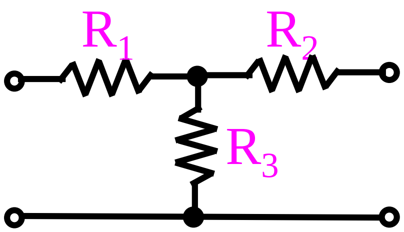
$$RL_{\mathrm{out}} = -20\log_{10}|S_{22}| \quad \mathrm{dB}$$

插入损耗（Insert Loss）与S参数关系

$$IL = -20\log_{10}|S_{21}| \quad \mathrm{dB}.$$

4.4.2 S 参量的物理意义

例4.5 假设一3dB衰减网络插入到 $Z_0 = 50\Omega$的传输线中，求该网络的S 参量和电阻。**P115**

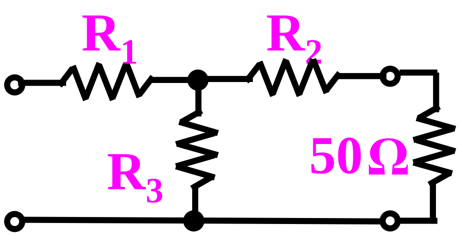


解：网络匹配、对称： $S_{11}= S_{22}= 0$

$R_1=R_2 \quad V_1=V_1^+ \quad V_2=V_2^-$

$$Z_{in} = R_1 + R_3 \| (R_2 + 50) = R_1 + \frac{R_3 (R_2 + 50)}{R_3 + R_2 + 50} = 50\Omega$$

$$V_2 = \left(\frac{R_3 \| (R_2 + 50)}{R_1 + R_3 \| (R_2 + 50)} \right) \frac{50}{R_2 + 50} V_1$$

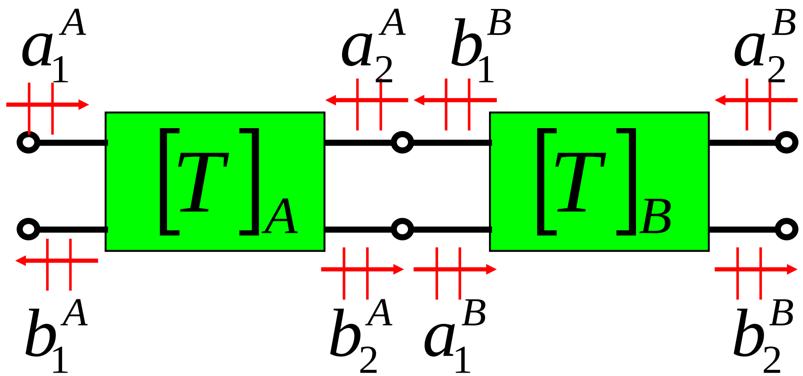


S_{11}和S_{21}测试电路

3dB衰减 $\quad S_{21} = \frac{V_2^-}{V_1^+} = \frac{V_2}{V_1} = \frac{1}{\sqrt{2}}$

得：$R_1= R_2= 8.58\ \Omega$， $R_3=141.4\ \Omega$

4.4.3 链式散射参量矩阵 [T]



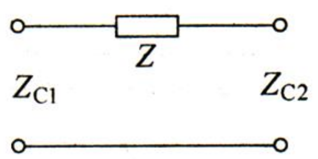
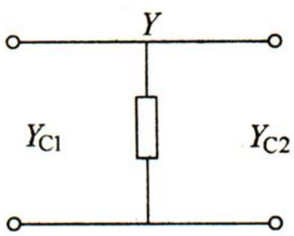
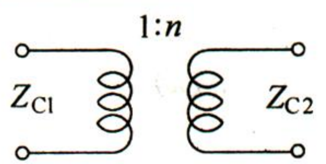
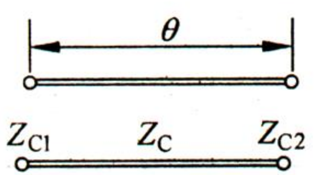
按输入输出口分类重写电压波关系式:

$$\begin{Bmatrix} a_1^A \\ b_1^A \end{Bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix} \begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix} \begin{Bmatrix} b_2^B \\ a_2^B \end{Bmatrix}$$

链形散射矩阵与A矩阵作用相同

4.4.4 S参量与其它网络参量的转换

表 4.3　二端口等效单元电路的散射矩阵和传输矩阵

单元电路	[s]	[t]	说明
	$\begin{bmatrix} \frac{z+r-1}{z+r+1} & \frac{2\sqrt{r}}{z+r+1} \\ \frac{2\sqrt{r}}{z+r+1} & \frac{z-r+1}{z+r+1} \end{bmatrix}$	$\begin{bmatrix} \frac{r+z+1}{2\sqrt{r}} & \frac{r-z-1}{2\sqrt{r}} \\ \frac{r+z-1}{2\sqrt{r}} & \frac{r-z+1}{2\sqrt{r}} \end{bmatrix}$	$z=Z/Z_{C1}$ $r=Z_{C2}/Z_{C1}$
	$\begin{bmatrix} \frac{1-y-1/r}{1+y+1/r} & \frac{2/\sqrt{r}}{1+y+1/r} \\ \frac{2/\sqrt{r}}{1+y+1/r} & -\frac{1+y-1/r}{1+y+1/r} \end{bmatrix}$	$\begin{bmatrix} \frac{1+y+1/r}{2/\sqrt{r}} & \frac{1+y-1/r}{2/\sqrt{r}} \\ \frac{1-y-1/r}{2/\sqrt{r}} & \frac{1-y+1/r}{2/\sqrt{r}} \end{bmatrix}$	$y=Y/Y_{C1}$ $r=Y_{C1}/Y_{C2}$
	$\begin{bmatrix} \frac{r-n^2}{r+n^2} & \frac{\pm 2n\sqrt{r}}{r+n^2} \\ \frac{\pm 2n\sqrt{r}}{r+n^2} & \frac{n^2-r}{r+n^2} \end{bmatrix}$	$\begin{bmatrix} \pm\frac{r+n^2}{2n\sqrt{r}} & \pm\frac{r-n^2}{2n\sqrt{r}} \\ \pm\frac{r-n^2}{2n\sqrt{r}} & \pm\frac{r+n^2}{2n\sqrt{r}} \end{bmatrix}$	$r=Z_{C2}/Z_{C1}$
	$\begin{bmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix}$	$Z_{C1}=Z_{C2}=Z_C$

4.4.4 S参量与其它网络参量的转换

矩阵参量	用[s]表示	用[z]表示	用[y]表示	用[ā]表示
$[s]$	$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$	$[s]=([z]+[1])^{-1}\times([z]-[1])$ $s_{11}=\dfrac{\lvert z\rvert+z_{11}-z_{22}-1}{\lvert z\rvert+z_{11}+z_{22}+1}$ $s_{12}=\dfrac{2z_{12}}{\lvert z\rvert+z_{11}+z_{22}+1}$ $s_{21}=\dfrac{2z_{21}}{\lvert z\rvert+z_{11}+z_{22}+1}$ $s_{22}=\dfrac{\lvert z\rvert-z_{11}+z_{22}-1}{\lvert z\rvert+z_{11}+z_{22}+1}$	$[s]=([1]+[y])^{-1}\times([1]-[y])$ $s_{11}=\dfrac{1-y_{11}+y_{22}-\lvert y\rvert}{1+y_{11}+y_{22}+\lvert y\rvert}$ $s_{12}=\dfrac{-2y_{12}}{1+y_{11}+y_{22}+\lvert y\rvert}$ $s_{21}=\dfrac{-2y_{21}}{1+y_{11}+y_{22}+\lvert y\rvert}$ $s_{22}=\dfrac{1+y_{11}-y_{22}-\lvert y\rvert}{1+y_{11}+y_{22}+\lvert y\rvert}$	$s_{11}=\dfrac{\bar{a}+\bar{b}-\bar{c}-\bar{d}}{\bar{a}+\bar{b}+\bar{c}+\bar{d}}$ $s_{12}=\dfrac{2\lvert\bar{a}\rvert}{\bar{a}+\bar{b}+\bar{c}+\bar{d}}$ $s_{21}=\dfrac{2}{\bar{a}+\bar{b}+\bar{c}+\bar{d}}$ $s_{22}=\dfrac{-\bar{a}+\bar{b}-\bar{c}+\bar{d}}{\bar{a}+\bar{b}+\bar{c}+\bar{d}}$
$[z]$	$[z]=([1]-[s])^{-1}\times([1]+[s])$ $z_{11}=\dfrac{1+s_{11}-s_{22}-\lvert s\rvert}{1-s_{11}-s_{22}+\lvert s\rvert}$ $z_{12}=\dfrac{2s_{12}}{1-s_{11}-s_{22}+\lvert s\rvert}$ $z_{21}=\dfrac{2s_{21}}{1-s_{11}-s_{22}+\lvert s\rvert}$ $z_{22}=\dfrac{1-s_{11}+s_{22}-\lvert s\rvert}{1-s_{11}-s_{22}+\lvert s\rvert}$	$[z]=\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$[z]=[y]^{-1}=\dfrac{1}{\lvert y\rvert}\times\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$	$[z]=\dfrac{1}{\bar{c}}\begin{bmatrix} \bar{a} & \lvert\bar{a}\rvert \\ 1 & \bar{d} \end{bmatrix}$

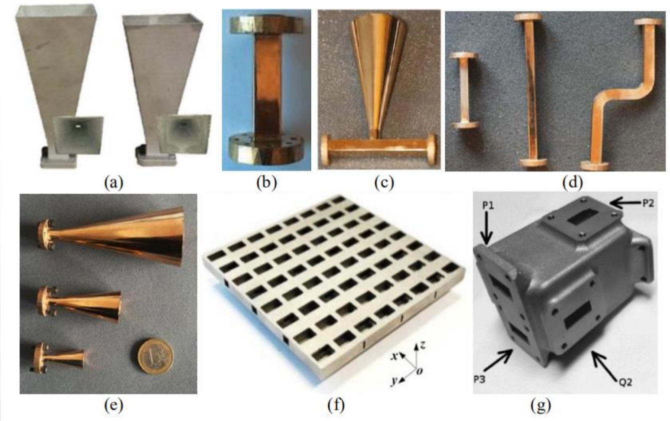
4.4.4 S参量与其它网络参量的转换

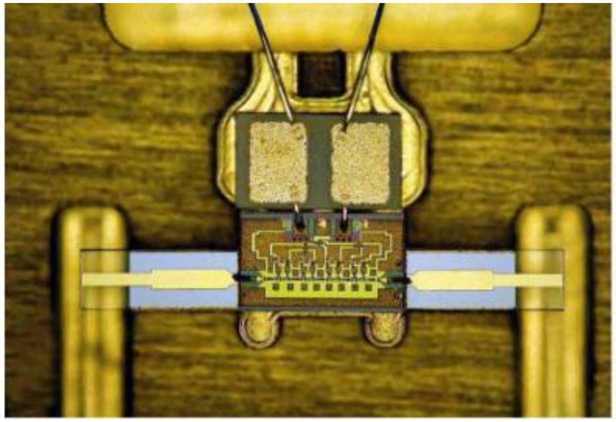
续表

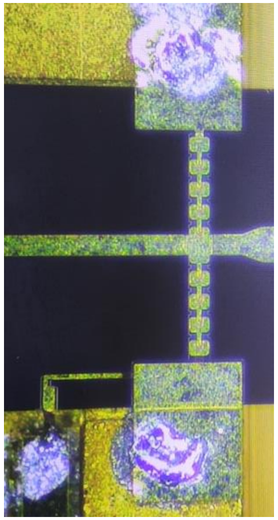
矩阵参量	用[s]表示	用[z]表示	用[y]表示	用[$\bar{a}$]表示
[y]	$[y]=([1]-[s])\times([1]+[s])^{-1}$ $y_{11}=\dfrac{1-s_{11}+s_{22}-\lvert s\rvert}{1+s_{11}+s_{22}+\lvert s\rvert}$ $y_{12}=\dfrac{-2s_{12}}{1+s_{11}+s_{22}+\lvert s\rvert}$ $y_{21}=\dfrac{-2s_{21}}{1+s_{11}+s_{22}+\lvert s\rvert}$ $y_{22}=\dfrac{1+s_{11}-s_{22}-\lvert s\rvert}{1+s_{11}+s_{22}+\lvert s\rvert}$	$[y]=[z]^{-1}=\dfrac{1}{\lvert z\rvert}\times\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$	$[y]=\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$[y]=\dfrac{1}{\bar{b}}\times\begin{bmatrix} \bar{d} & -\lvert\bar{a}\rvert \\ -1 & \bar{a} \end{bmatrix}$
[$\bar{a}$]	$\bar{a}=\dfrac{1}{2s_{21}}(1+s_{11}-s_{22}-\lvert s\rvert)$ $\bar{b}=\dfrac{1}{2s_{21}}(1+s_{11}+s_{22}+\lvert s\rvert)$ $\bar{c}=\dfrac{1}{2s_{21}}(1-s_{11}-s_{22}+\lvert s\rvert)$ $\bar{d}=\dfrac{1}{2s_{21}}(1-s_{11}+s_{22}-\lvert s\rvert)$	$[\bar{a}]=\dfrac{1}{z_{21}}\begin{bmatrix} z_{11} & \lvert z\rvert \\ 1 & z_{22} \end{bmatrix}$	$[\bar{a}]=\dfrac{-1}{y_{21}}\begin{bmatrix} y_{22} & 1 \\ \lvert y\rvert & y_{11} \end{bmatrix}$	$[\bar{a}]=\begin{bmatrix} \bar{a} & \bar{b} \\ \bar{c} & \bar{d} \end{bmatrix}$

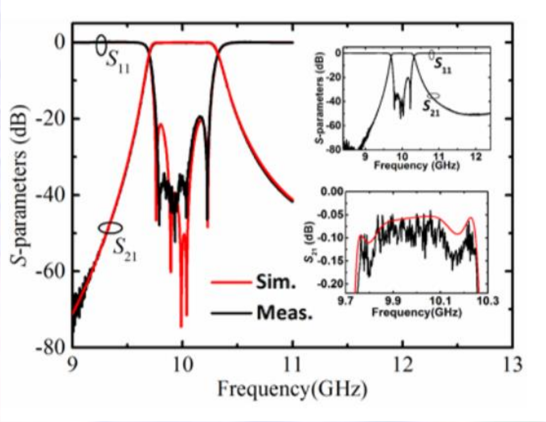
小结

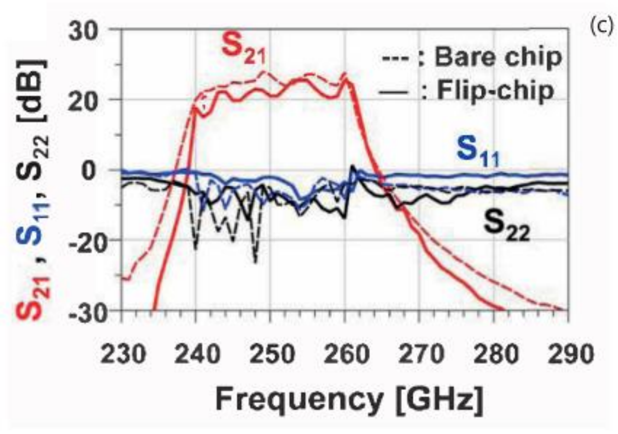
- 网络分析法的意义
  - 可以几乎什么细节也不懂，还看得懂结果

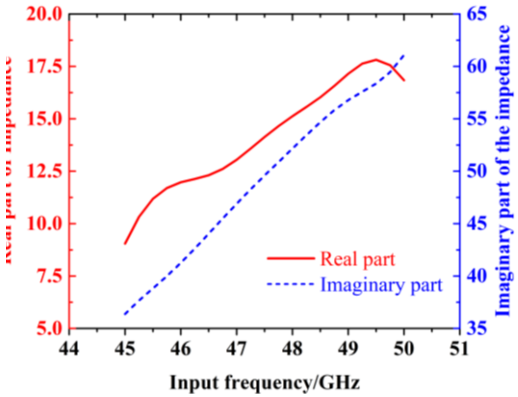












小结

- 最常见的三种网络参数
  - 阻抗/导纳网络：关心电流电压特性
  - A网络：关心一个端口上的电流和电压和另一个端口上的电流和电压关系
  - S参数网络，关心各个端口间的能量输入输出关系

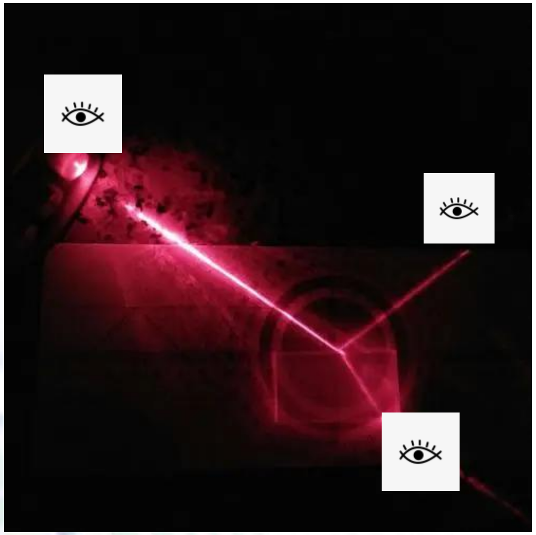
- 网络参量的意义在于把一个系统的内部结构和细节忽略掉，只关心端口上的特性。
- 根据输入输出的不同，定义了Z矩阵（端口电压用电流表示），Y矩阵，A矩阵（一个端口的电流电压用另一个端口的电流电压表示）
- 网络表现出的性能和网络参数及外部激励都有关
- 各类网络之间可以通过公式进行转换

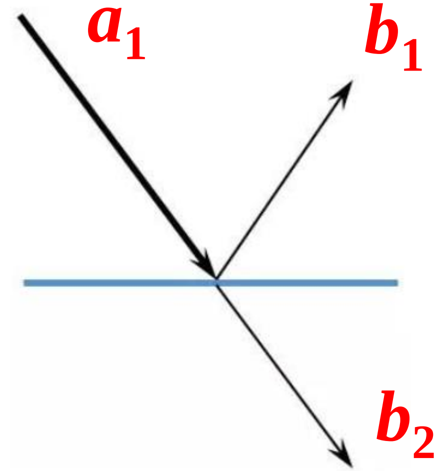
参数测量

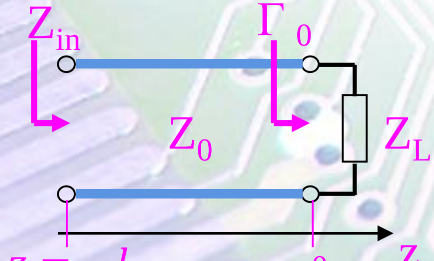
高频下出现的问题

- 高频时网络参数测量遇到的问题
  - Z、Y、H、ABCD参数都可以在某个端口开路或短路的条件下通过测量端口电压电流的方法获得，但是当信号频率很高时，这种测量方法变得很不实际
    - 由于寄生元件的存在，理想的开路和短路很难实现
    - 即使可以做到接近理想的开路和短路，电路也很有可能因此而不稳定
    - 由于信号以波的形式传播，在不同测量点上幅度和相位都可能不同，这也使得基于电压和电流的测量方法很不准确，难以应用

如何测量反射





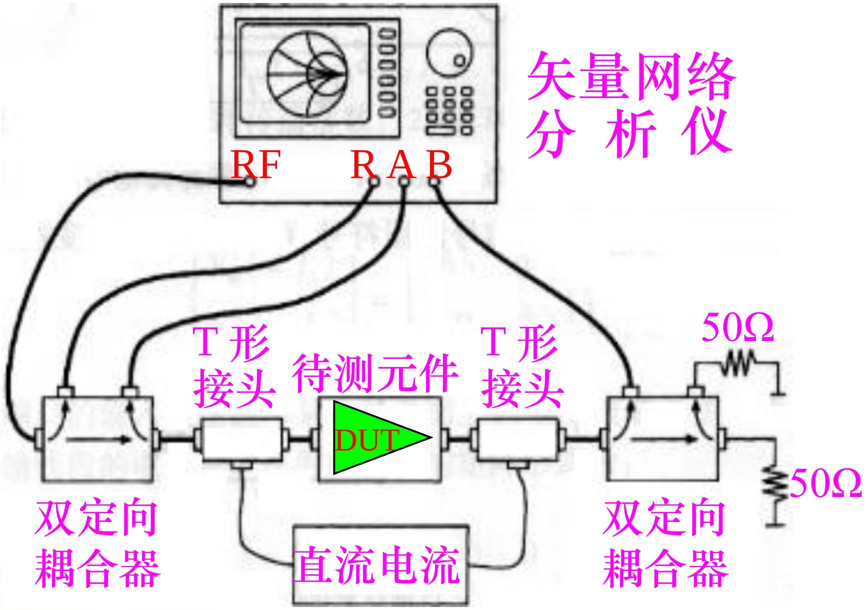


$$\Gamma_0 = \frac{V^-}{V^+}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0},\ S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

4.4.7 S 参量的测量

- 射频源RF输出射频信号
- 测量通道R用于测量入射波，同时也作为参考端口。
- 通道A和B：测量反射波和传输波(S_{11}=A/R，S_{21}=B/R)。
- 若要测量S_{12}和S_{22}，将待测元件反接。



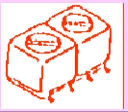
测量S_{11}和S_{21}的实验系统

4.4.7 S 参量的测量

为什么要进行校正测量

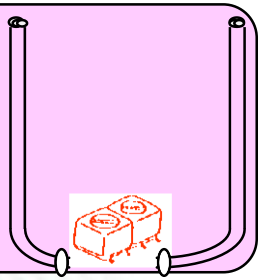
A =

待测元件特性



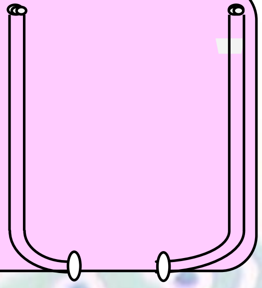
B —

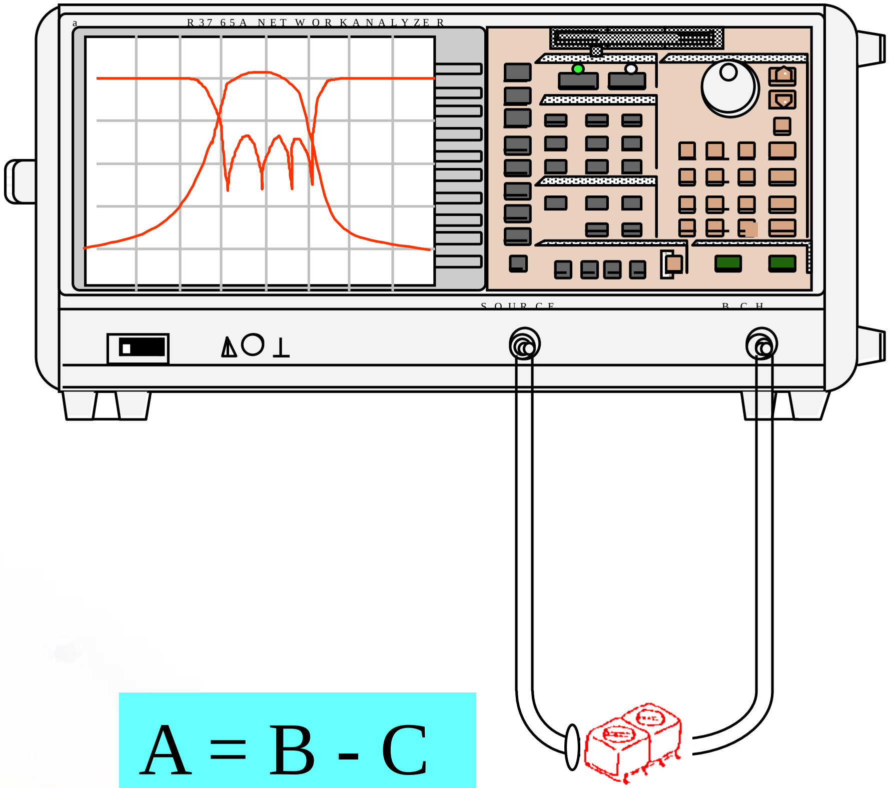
全体的特性



C

测定系统的特性





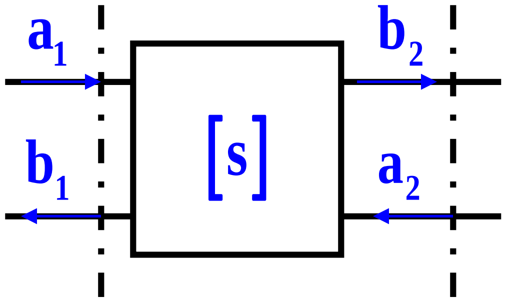
A = B - C

4.4.5 信号流图模型

一、微波网络的信号流图

$$b_1 = s_{11}a_1 + s_{12}a_2$$

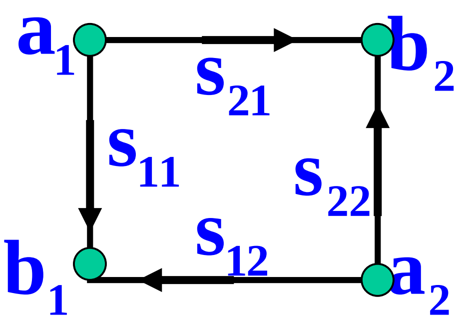
$$b_2 = s_{21}a_1 + s_{22}a_2$$



信号流图：线性方程组对应的拓扑图（节点与方向支线）

节点：

支线系数：

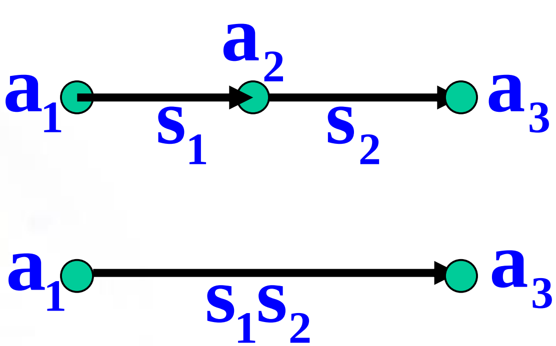


二、信号流图简化法则

1. 串联支线合并法则

$$a_2 = s_1 a_1 \quad a_3 = s_2 a_2$$

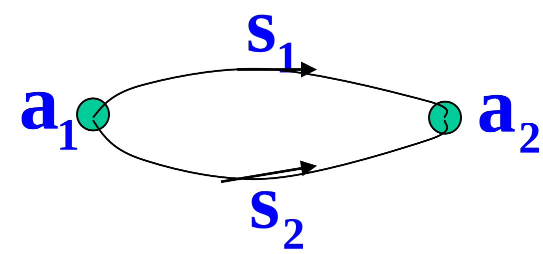
$$a_3 = s_1 s_2 a_1$$

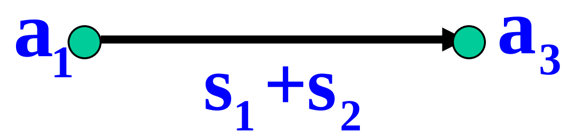


4.4.5 信号流图模型

2. 并联支线合并法则

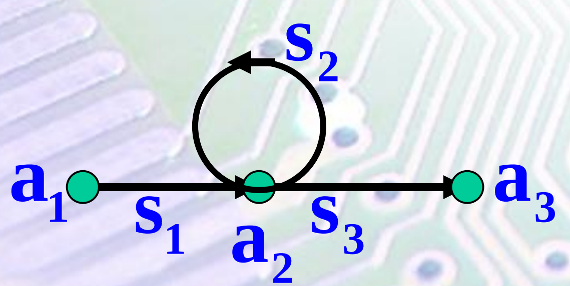
$$a_2 = s_1 a_1 + s_2 a_1 = (s_1 + s_2) a_1$$

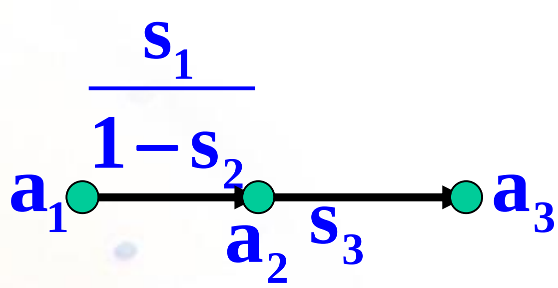




3. 自闭环消除法则

$$a_2 = s_1 a_1 + s_2 a_2 \qquad a_2 = \frac{s_1}{1 - s_2} a_1$$

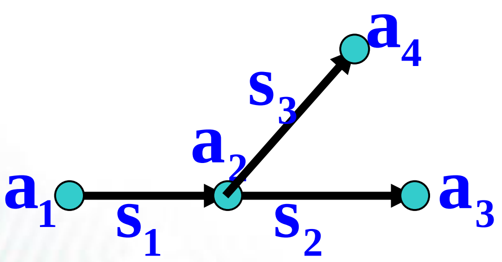


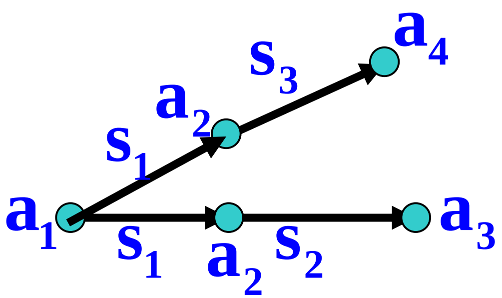


4. 结点分裂法则

$$a_2 = s_1 a_1 \quad a_3 = s_2 a_2 \quad a_4 = s_3 a_2$$

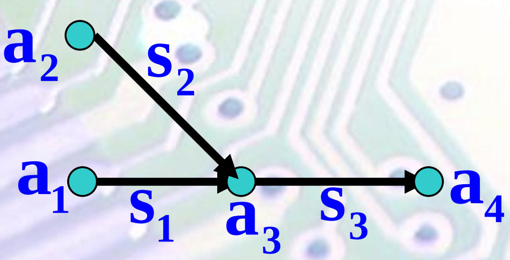
$$a_3 = s_1 s_2 a_1 \quad a_4 = s_1 s_3 a_1$$

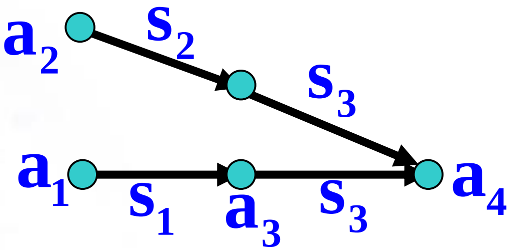




$$a_3 = s_1 a_1 + s_2 a_2 \quad a_4 = s_3 a_3$$

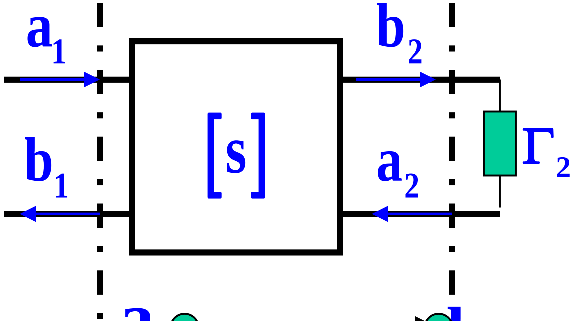
$$a_4 = s_1 s_3 a_1 + s_2 s_3 a_2$$

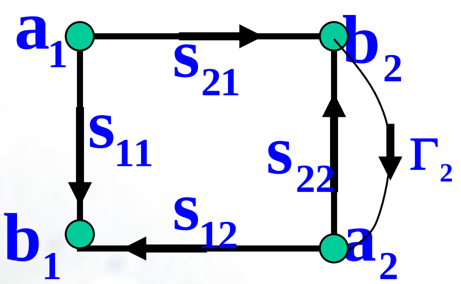




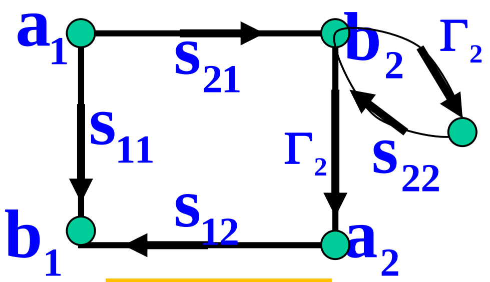
4.4.5 信号流图模型

例4.6 求：$\Gamma_{in} = \dfrac{b_1}{a_1}$

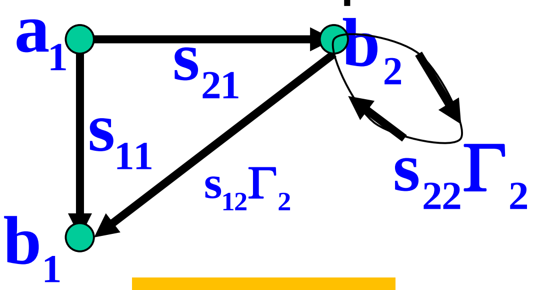




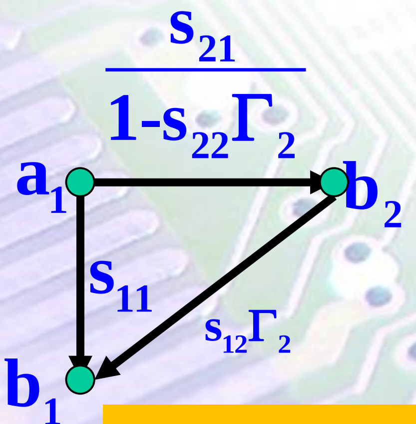
信号流图



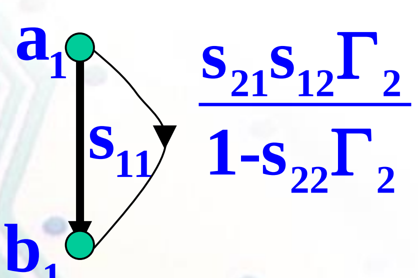
分裂b2



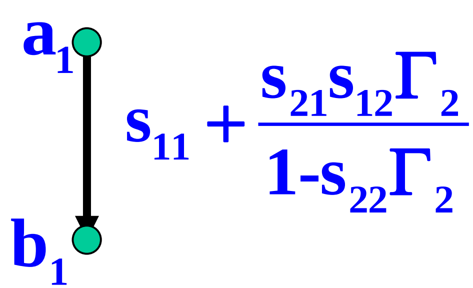
串联消除



消除自闭环



消除串联



消除并联

4.4.5 信号流图模型

端接短路、开路和匹配负载（$\Gamma_L=-1$、1、0），测得的输入端反射系数分别为Γ_s、Γ_o和Γ_m，代入公式可得：

$$\begin{cases} \Gamma_s = s_{11} - \dfrac{s_{12}^2}{1+s_{22}} \\ \Gamma_m = s_{11} \\ \Gamma_o = s_{11} + \dfrac{s_{12}^2}{1-s_{22}} \end{cases} \Rightarrow \begin{cases} s_{12}^2 = \dfrac{2(\Gamma_0-\Gamma_m)(\Gamma_s-\Gamma_m)}{\Gamma_s-\Gamma_0} \\ s_{11} = \Gamma_m \\ s_{22} = \dfrac{2\Gamma_m-\Gamma_0-\Gamma_s}{\Gamma_s-\Gamma_0} \end{cases}$$

$$\Gamma_{in} = s_{11} + \frac{s_{21}s_{12}\Gamma_L}{1-s_{22}\Gamma_L}$$

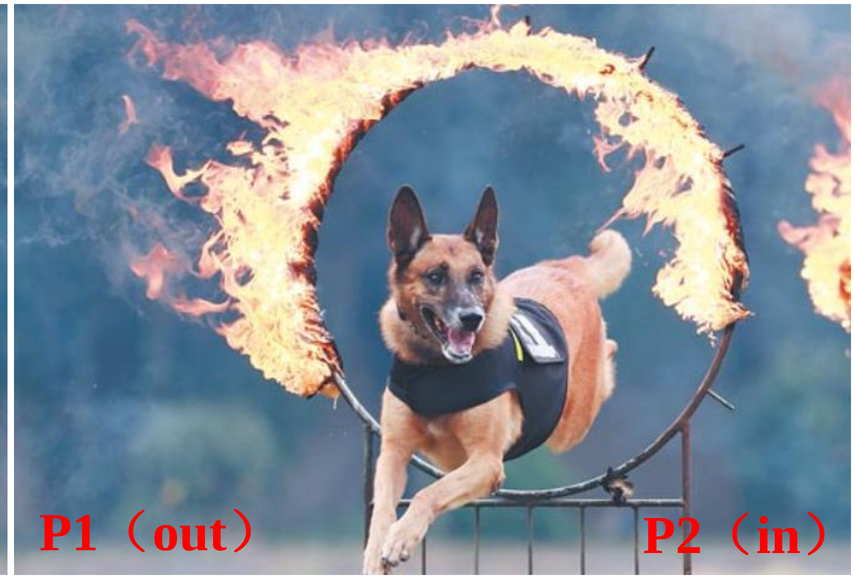
若进行直通测量可直接测得S_{21}

该测量方法叫：SOLT，是常用的校准法

矢量网络分析仪的测量精度很大程度上依赖于校准原始误差（校准前）2%~80% 剩余误差（校准后）0.1%~2%

接头是一个重要的误差来源，特别在较高的频率

S网络参量的互易和对称

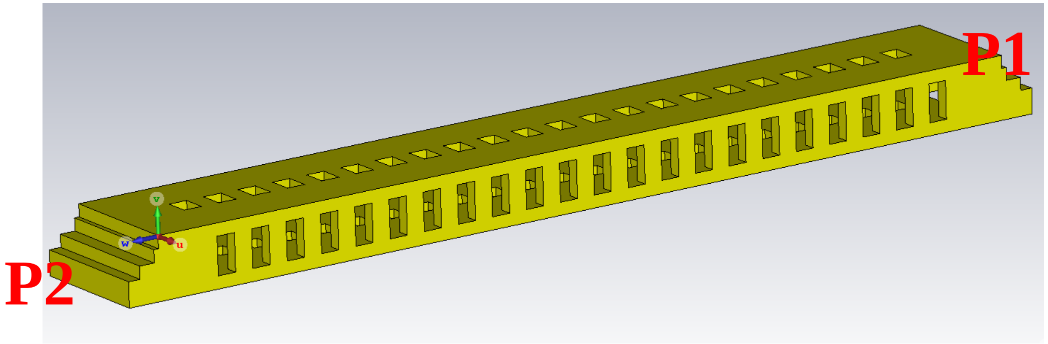


$$S_{12} = S_{21}, \quad S_{11} = S_{22}$$

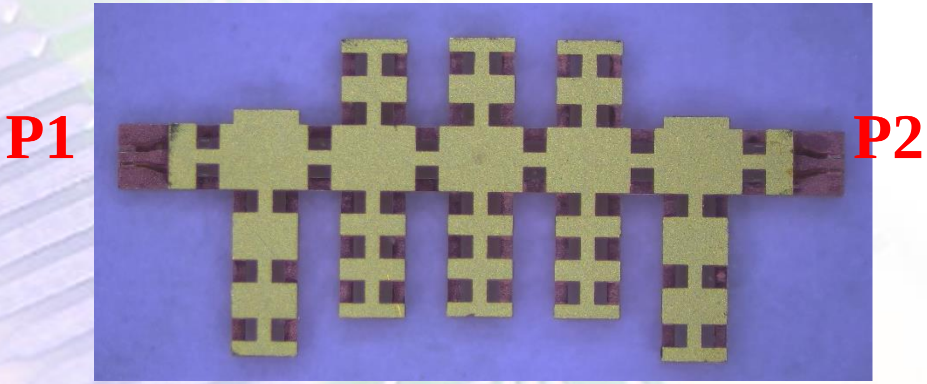
互易且对称

例：互易且对称

- 传输线的S参数

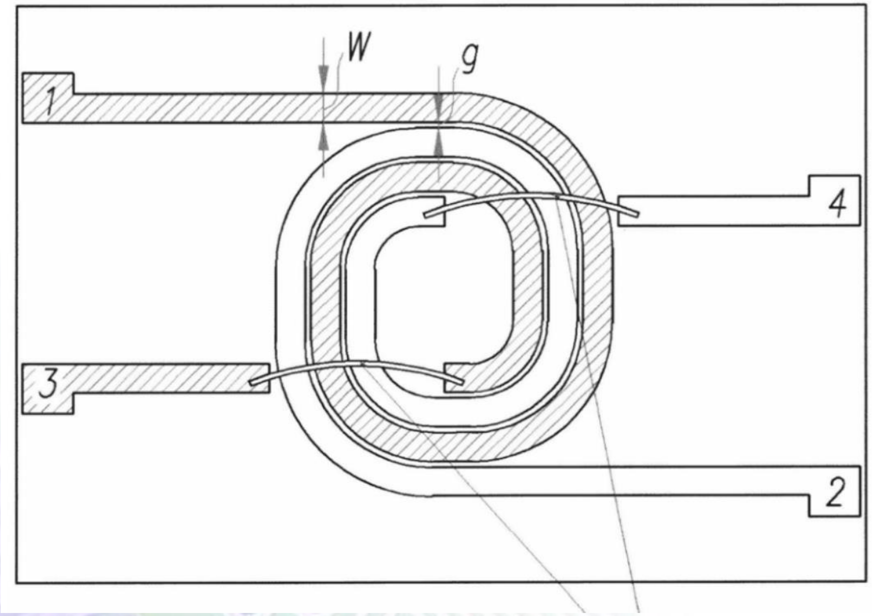


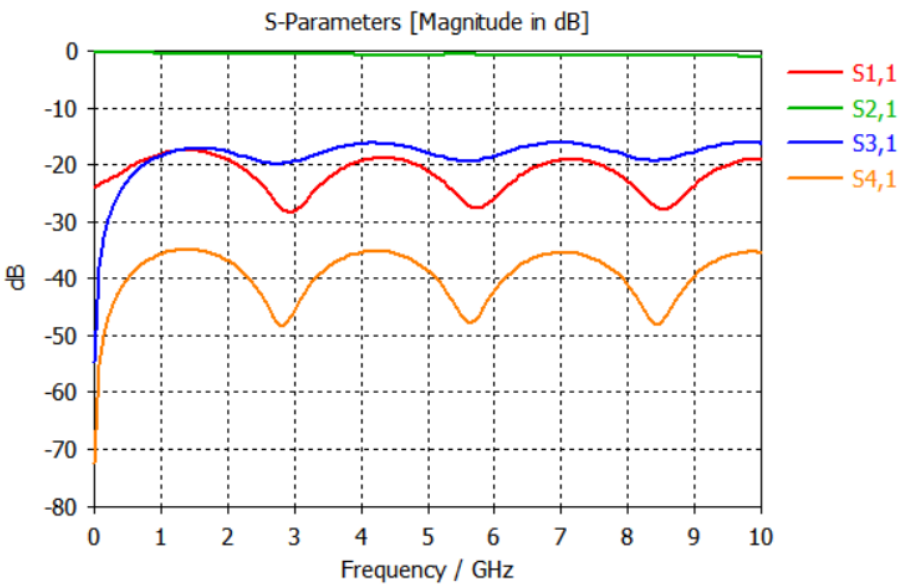
- 滤波器的S参数



例：互易且对称

- 定向耦合器





$$S_{11} = S_{22} = S_{33} = S_{44}$$

$$S_{13} = S_{31} = S_{24} = S_{42}$$

$$S_{12} = S_{21} = S_{34} = S_{43}$$

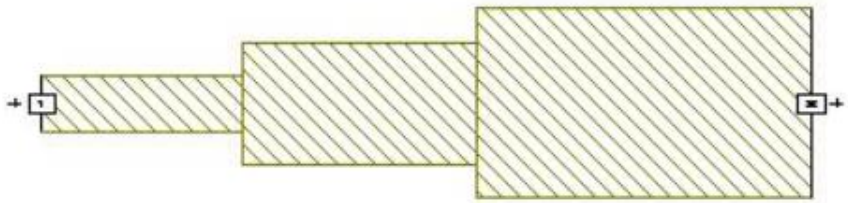
例：互易不对称

P1

P2

$$S_{12} = S_{21}, \quad S_{11} \neq S_{22}$$

互易代表“可逆”



$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

P1

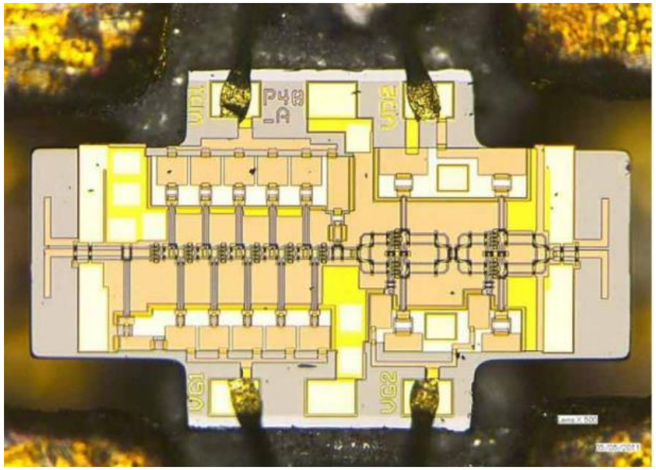
P2

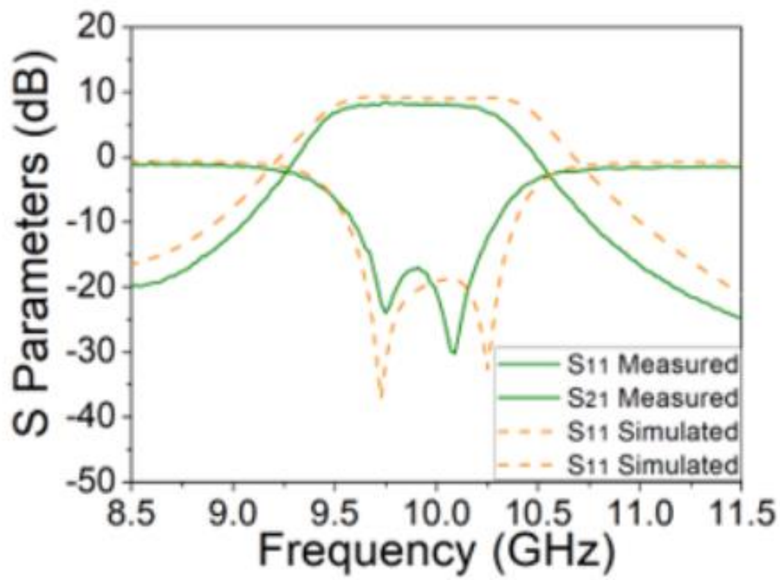
例：非互易非对称

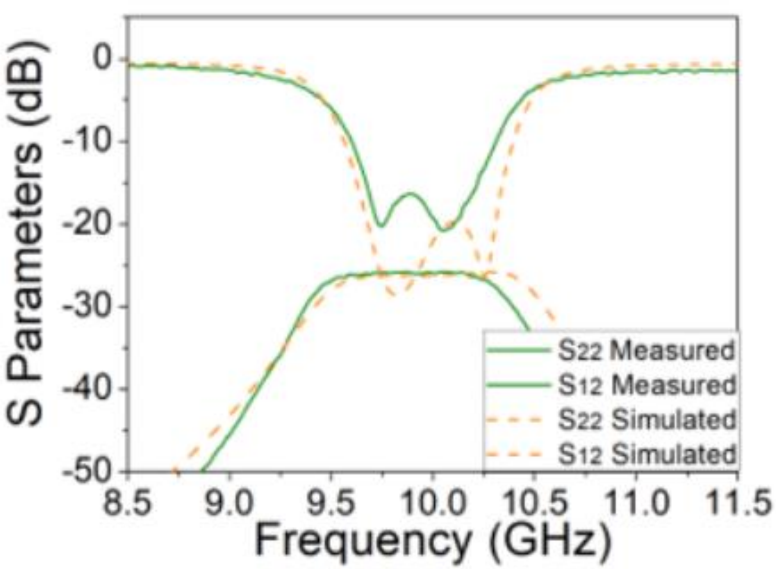
- 放大器的S参数

$$S_{12} \neq S_{21}$$

$$S_{11} \neq S_{22}$$







S参数与反射系数

$$S_{11}=\frac{b_1}{a_1}\bigg|_{a_2=0},\ S_{21}=\frac{b_2}{a_1}\bigg|_{a_2=0},\ S_{12}=\frac{b_1}{a_2}\bigg|_{a_1=0},\ S_{22}=\frac{b_2}{a_2}\bigg|_{a_1=0}$$

$$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix}=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

$$\Gamma_0=\frac{Z_L-Z_0}{Z_L+Z_0}$$

$$a_n=V_n^+/\sqrt{Z_0}=I_n^+\sqrt{Z_0}$$

$$\Gamma_0=\frac{V^-}{V^+}$$

$$b_n=V_n^-/\sqrt{Z_0}=-I_n^-\sqrt{Z_0}$$

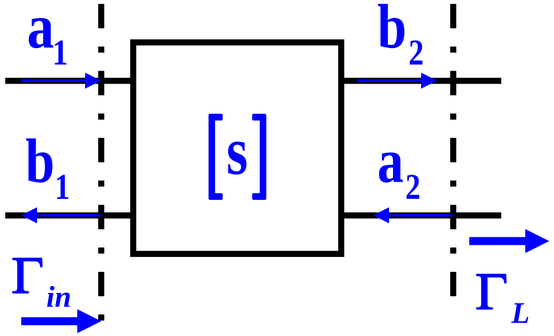
S11和反射系数有紧密的联系，但不是完全一样的。

本章作业

附加题1：已知放大器输入、输出端口的驻波系数分别为 VSWR=2 和 VSWR=3，求输入、输出端口反射系数的模。若采用 S_{11}和S_{22}表示计算结果，其物理含义是什么？

附加题2：利用散射参量方程推导（互易）

$$\Gamma_{in} = s_{11} + \frac{s_{12}^2 \Gamma_L}{1 - s_{22}\Gamma_L}$$



习题4.2、4.13、4.32

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