7.6

解: $x(t)=x_1(t)\cdot x_2(t)$　　则 $X(jw)=X_1(jw)*X_2(jw)$

∵ $X_1(jw)$带限于w_1、$X_2(jw)$带限于w_2　　∴ $X(jw)=X_1(jw)*X_2(jw)$带限于w_1+w_2

由采样定理　当 $w_s>2(w_1+w_2)$ 时 可以内插恢复

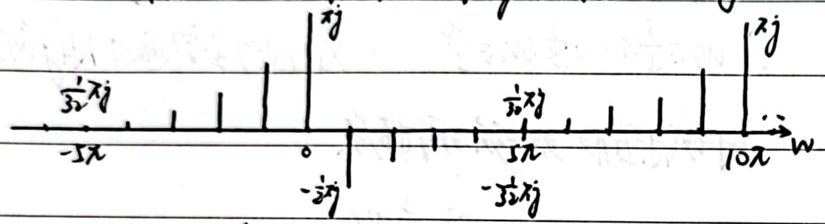
∴ $T<\frac{\pi}{w_1+w_2}$

7.8 解: (a) $x(t)=\sum_{k=0}^{5}(\frac{1}{2})^k\sin(k\pi t)$　　显然 $x(t)$在 $0、\pi、2\pi、3\pi、4\pi、5\pi$ 频点处的频谱不为0

∴ $w_m=5\pi$　　∵ $T=0.2$　$w_s=\frac{2\pi}{T}=10\pi$

由于 $w_s=2w_m$ 且在 $w_m=-5\pi$ 处有值　∴会发生频谱混叠

(b) $x(t)=\sum_{k=0}^{5}(\frac{1}{2})^k\sin(k\pi t)$　　$X(jw)=\sum_{k=0}^{5}(\frac{1}{2})^k\cdot\pi j[\delta(w+k\pi)-\delta(w-k\pi)]$　如下所示:



$W=5k\pi$处的 $X(jw)=0$

$W_c=\frac{\pi}{T}=5\pi$

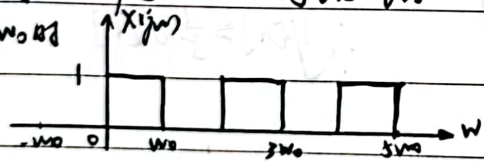
∴ $G(jw)=T\cdot\sum_{k=0}^{4}(\frac{1}{2})^k\pi j[\delta(w+k\pi)-\delta(w-k\pi)]$ ⇒ $g(t)=\sum_{k=0}^{4}\frac{1}{5}\cdot(\frac{1}{2})^k\sin(k\pi t)$

7.10 解: (a) 错　　$x(t)$时域为宽为$2T$的门信号　$X(jw)$非带限　　故总会出现混叠

(b) 对　　$X(jw)$带限于w_0　　$w_s>2w_0$　满足采样定理的条件　故不会混叠

(c) 对　　$T<\frac{2\pi}{w_0}$　则 $w_s>w_0$　　$X(jw)$为$(0,w_0)$之间的门信号,周期延拓后如下:

当$w_s=w_0$时



∴不会出现混叠

7.21 解: $T=10^{-4}s$　　$w_s=\frac{2\pi}{T}=2\times10^4\pi$

(a) $w_m=5000\pi$　　$w_s>2w_m$　　不会发生混叠 故可以完全恢复

(b) $w_m=15000\pi$　　$w_s<2w_m$　　出现混叠　不能完全恢复

(c) $X(jw)$不一定带限　　不能保证完全恢复

(d) $x(t)$为实　$|X(jw)|$偶对称,　则 $w_m=5000\pi$　　$w_s>2w_m$　可以保证完全恢复

(e) 和(d)类似,　$w_m=1.5\times10^4\pi$　　$w_s<2w_m$　不能完全恢复

(f) $X(jw)*X(jw)=0$　　$w_m=7500\pi$　　$w_s>2w_m$　可以完全恢复

(g) 当 $w<-5000\pi$时　$|X(jw)|$未知　　　不能完全恢复

7.22

解: $y(t)=x_1(t)*x_2(t)$　　$Y(j\omega)=X_1(j\omega)\cdot X_2(j\omega)$

$\therefore \omega_m=1000\pi$　　由采样定理　$\omega_s>2\omega_m$　即　$\omega_s>2000\pi$

$\therefore T=\frac{2\pi}{\omega_s}<1ms$

7.15 解: $\omega_M=\frac{3}{7}\pi$　　$\therefore \omega_s>2\omega_M$　　即 $\omega_s>\frac{6}{7}\pi$

$\therefore N=\frac{2\pi}{\omega_s}<\frac{7}{3}$　　$\therefore N_{max}=2$

7.19 解 (a) $\omega_1\le\frac{3}{5}\pi$　设 $x[n]\xrightarrow{F}X(e^{j\omega})$ 则有 $X(e^{j\omega})=\begin{cases}1 & |\omega|<\omega_1\\ 0 & \omega_1<|\omega|<\pi\end{cases}$

经过零值插入后　$x[n]$ 扩展3倍　$X(e^{j\omega})$ 压缩 $\frac{1}{3}$　即有

$X_1(e^{j\omega})=\begin{cases}1 & |\omega|<\frac{1}{3}\omega_1\\ 0 & \frac{1}{3}\omega_1<|\omega|<\pi\end{cases}$　　$\because \omega_1\le\frac{3}{5}\pi$　$\frac{1}{3}\omega_1\le\frac{\pi}{5}$　$\therefore X_1(e^{j\omega})$全部通过$H(e^{j\omega})$

$\therefore W(e^{j\omega})=\begin{cases}1 & |\omega|<\frac{1}{3}\omega_1\\ 0 & \frac{1}{3}\omega_1<|\omega|<\pi\end{cases}$　　时域抽取 则频域扩展

$Y(e^{j\omega})=\begin{cases}\frac{1}{5} & |\omega|<\frac{5}{3}\omega_1\\ 0 & \frac{5}{3}\omega_1<|\omega|<\pi\end{cases}$　　$\therefore y[n]=\frac{\sin\frac{5}{3}\omega_1 n}{5\pi n}$

(b) $X(e^{j\omega})=\begin{cases}1 & |\omega|<\omega_1\\ 0 & \omega_1<|\omega|<\pi\end{cases}$　　$X_1(e^{j\omega})=\begin{cases}1 & |\omega|<\frac{1}{3}\omega_1\\ 0 & \frac{1}{3}\omega_1<|\omega|<\pi\end{cases}$

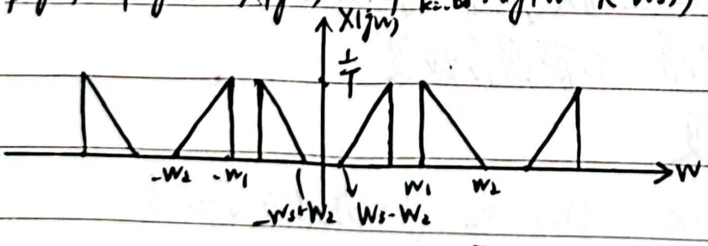
$\because \frac{1}{3}\omega_1>\frac{\pi}{5}$　$\therefore W(e^{j\omega})$带限于$\frac{\pi}{5}$　　$W(e^{j\omega})=\begin{cases}1 & |\omega|<\frac{\pi}{5}\\ 0 & \frac{\pi}{5}<|\omega|<\pi\end{cases}$

$\therefore Y(e^{j\omega})=\begin{cases}\frac{1}{5} & |\omega|<\pi\\ 0 & \pi<|\omega|<\pi\end{cases}$　即 $Y(e^{j\omega})=\frac{1}{5}$　　$\therefore y[n]=\frac{1}{5}\delta[n]$

7.26

解: $P(j\omega)=\sum_{k=-\infty}^{\infty}\frac{2\pi}{T}\cdot\delta(\omega-k\cdot\omega_s)$

$X_p(j\omega)=P(j\omega)*X(j\omega)=\frac{1}{T}\sum_{k=-\infty}^{+\infty}X(j(\omega-k\cdot\omega_s))$　即以ω_s进行周期化延拓



当$\omega_s>2\omega_2$　满足采样定理,不混叠

当$2\omega_1<\omega_s<2\omega_2$. 有混叠

当$2\omega_1<\omega_s=\omega_2$　无混叠

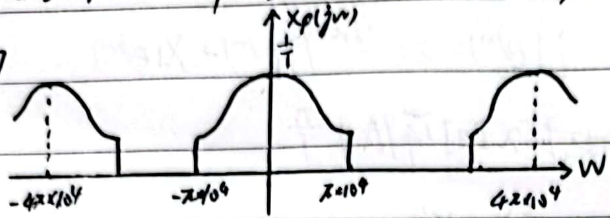
$\therefore \omega_{s\,max}=\omega_2$　　$T=\frac{2\pi}{\omega_s}=\frac{2\pi}{\omega_2}$

恢复时　需要 $A\cdot\frac{1}{T}=1$　故 $A=T=\frac{2\pi}{\omega_2}$

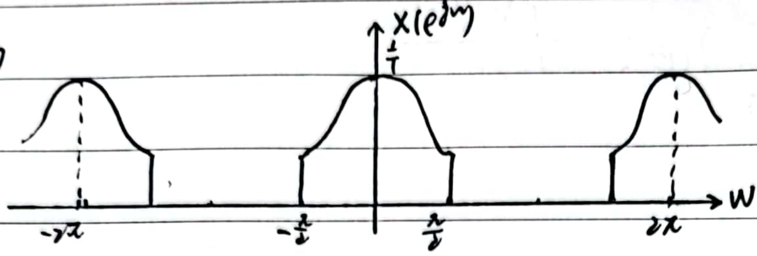
$\omega_a=\frac{2\pi}{T}-\omega_1$　　$\omega_b=\frac{2\pi}{T}$

7.29 解: $\frac{1}{T}=20kHz$ $T=5\times10^{-5}s$. $W_s=\frac{2\pi}{T}=4\times10^4\pi$

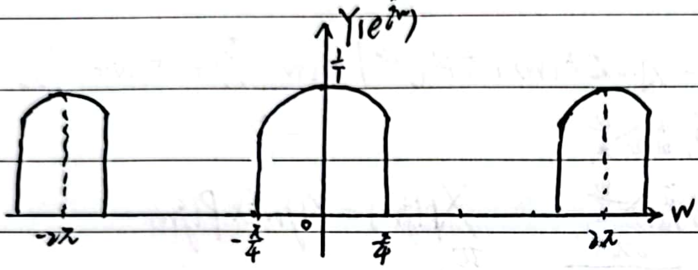
$X_p(jw)$为



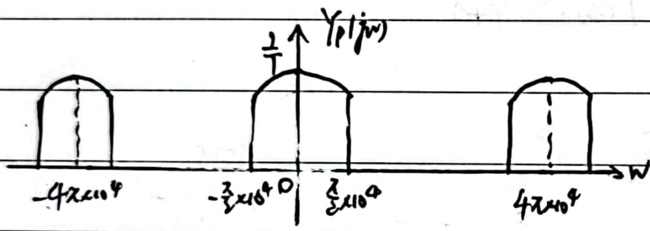
$X(e^{jw})$为



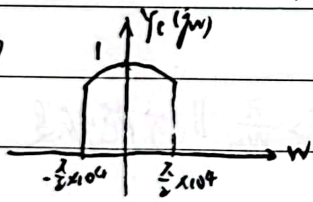
$Y(e^{jw})$为



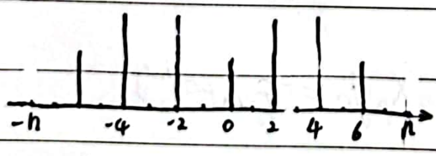
$Y_p(jw)$为



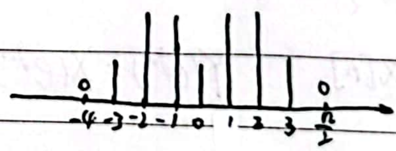
$Y_c(jw)$为



7.35 解 (a) $x_p[n]$为

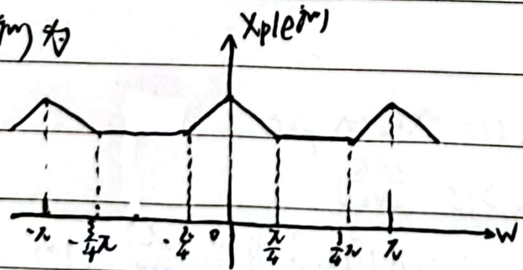


$x_d[n]$为

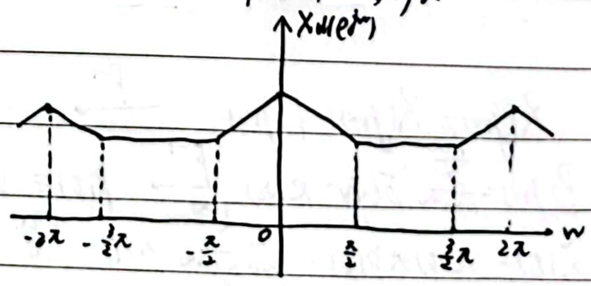


(b) $N=2$、为混叠 $W_s=\frac{2\pi}{N}=\pi$

$Y_p(e^{jw})$为



$X_d(e^{jw})$为$X_p(e^{jw})$的扩展



7.31

解：$y[n]=\frac{1}{2}y[n-1]+x[n]$. 作傅里叶变换 $Y(e^{j\omega})=\frac{1}{2}e^{-j\omega}\cdot Y(e^{j\omega})+X(e^{j\omega})$

$\therefore H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{1-\frac{1}{2}e^{-j\omega}}$ 由于 $x[n]$ 带限于 $\frac{\pi}{T}$.

$\therefore X_d(e^{j\omega})=\frac{1}{T}X_c(j\frac{\omega}{T})$ $\Rightarrow Y(e^{j\omega})=\frac{\frac{1}{T}X_c(j\frac{\omega}{T})}{1-\frac{1}{2}e^{-j\omega}}$ $|\omega|<\pi$

$\therefore H_c(j\omega)=\frac{1}{T}\cdot\frac{1}{1-\frac{1}{2}e^{-j\omega T}}$, $|\omega|<\frac{\pi}{T}$

7.38

解：$x(t)=A+B\cos(\frac{2\pi}{T}t+\theta)$ $X(j\omega)=A\cdot 2\pi\delta(\omega)+\pi Be^{j\theta}[\delta(\omega-\frac{2\pi}{T})+\delta(\omega+\frac{2\pi}{T})]$

即 $X(j\omega)$ 在 0、$\frac{2\pi}{T}$、$-\frac{2\pi}{T}$ 处存在频谱分量

$P(j\omega)$ 在 $\omega=0$. $\omega=\pm\frac{2\pi n}{T+\Delta}$ 处存在频谱分量 $X_p(j\omega)=X(j\omega)*P(j\omega)$

作周期延拓有 $\frac{2\pi}{T}-\frac{2\pi}{T+\Delta}=\frac{2\pi\Delta}{T(T+\Delta)}<\frac{\pi}{T+\Delta}$

$\therefore \Delta<\frac{\pi T}{2}$

且由频谱得 $a=\frac{\frac{2\pi\Delta}{T(T+\Delta)}}{\frac{2\pi}{T}}=\frac{\Delta}{T+\Delta}$

7.41

解：(a) $x(t)$ 带限于 ω_m. 由采样定理 只有当 $\omega_s>2\omega_m$ 时 即 $T>\frac{\pi}{\omega_m}$ 时才能恢复

$\because T=T_0<\frac{\pi}{\omega_m}$ $\therefore y[n]=x[n]$. 有 $y_c(t)=x(t)$

$\therefore H_d(j\Omega)=\frac{Y(e^{j\Omega})}{S(e^{j\Omega})}=\frac{X(e^{j\Omega})}{X(e^{j\Omega})+\alpha e^{-j\Omega}X(e^{j\Omega})}=\frac{1}{1+\alpha e^{-j\Omega}}$

$\therefore y[n]+\alpha y[n-1]=s[n]$

(b) $y[n]=x[n]$ $Y(e^{j\omega})=X(e^{j\omega})$ $\therefore Y_p(j\omega)=\frac{1}{T_0}X(e^{j\frac{\omega}{T_0}})$ $\therefore A=T_0$

7.52

解：(a) $\because \hat{X}(j\omega)=X(j\omega)\cdot P(j\omega)$ $\xrightarrow{F^{-1}}$ $\therefore \hat{x}(t)=x(t)*p(t)$

(b) $P(j\omega)=\sum_{k=-\infty}^{+\infty}\delta(\omega-k\omega_0)$ $\xrightarrow{F^{-1}}$ $p(t)=\frac{1}{\omega_0}\sum_{k=-\infty}^{+\infty}\delta(t-\frac{2\pi k}{\omega_0})$

$\hat{x}(t)=x(t)*p(t)=\frac{1}{\omega_0}\sum_{k=-\infty}^{+\infty}x(t-\frac{2\pi k}{\omega_0})$ $x(t)$ 时限. 即当 $|t|>\frac{\pi}{\omega_0}$ $x(t)=0$

$\therefore w(t)=\begin{cases}1 & |t|<\frac{\pi}{\omega_0}\\ 0 & 其他\end{cases}$

(c) 当 $|t|\geq\frac{\pi}{\omega_0}$ 时 $x(t)$ 不限为 0 $\hat{x}(t)$ 出现混叠. 必然不能从 $\hat{x}(t)$ 中恢复 $x(t)$