

3.1 解: 对微分方程进行拉氏变换有 $TS Y(s) + Y(s) = TS R(s) + R(s)$

$$\text{故 } H(s) = \frac{Y(s)}{R(s)} = \frac{TS+1}{TS+1} \quad \text{当 } r(t) = u(t), R(s) = \frac{1}{s} \text{ 时}$$

$$Y(s) = H(s) \cdot R(s) = \frac{TS+1}{TS+1} \cdot \frac{1}{s} = \frac{1}{s} - \frac{T-T}{TS+1}$$

$$\text{作拉氏反变换 } y(t) = u(t) - \frac{T-T}{T} e^{-\frac{t}{T}}$$

$$\textcircled{1} \text{ 当 } y(t) = 0.5 y(\infty) \text{ 时 } t = t_d \text{ 则有 } \frac{T-T}{T} e^{-\frac{t}{T}} = 0.5 \quad t = [0.693 + \ln(\frac{T-T}{T})]T$$

$\textcircled{2}$ t_r 为 $y(t)$ 从 $0.1 y(\infty)$ 上升至 $0.9 y(\infty)$ 所用时间, 设分别为 t_1, t_2

$$y(t_1) = 1 - \frac{T-T}{T} e^{-\frac{t_1}{T}} = 0.1 \Rightarrow t_1 = T[\ln(\frac{T-T}{T}) - \ln 0.9] \quad \text{故 } t_r = t_2 - t_1 = T \cdot \ln 9 = 2.2T$$

$$y(t_2) = 1 - \frac{T-T}{T} e^{-\frac{t_2}{T}} = 0.9 \Rightarrow t_2 = T[\ln(\frac{T-T}{T}) - \ln 0.1]$$

$\textcircled{3}$ 设调整误差为 Δ 由定义

$$y(t_s) = (1 - \Delta\%) y(\infty) \quad \text{即 } 1 - \frac{T-T}{T} e^{-\frac{t_s}{T}} = 1 - \Delta\%$$

$$\Rightarrow t_s = T[-\ln \Delta\% + \ln \frac{T-T}{T}] \quad \text{当 } \Delta = 5 \text{ 时}$$

$$t_s = T[2.99 + \ln \frac{T-T}{T}]$$

综上所述 证毕

3.2 解: $G(s) = \frac{10}{0.25s+1} \quad T = 0.2 \quad \text{增益为 } 10$

$$\text{加入反馈和比例后 } G_u(s) = k_0 \cdot \frac{G(s)}{1 + k_h G(s)} = \frac{\frac{10k_0}{0.25s+1}}{1 + \frac{10k_0}{0.25s+1}}$$

$$\text{由题知 } \frac{0.2}{10k_h+1} = 0.1 \times 0.2 \quad ; \quad \frac{10k_0}{10k_h+1} = 10$$

$$\text{故解得 } k_h = 0.9 \quad k_0 = 10$$

