

Introduction

Under sustained current stress, EM occurs and voids can form in the metal wires, leading to reliability problems. Therefore, it is important to estimate the EM void nucleation time (denoted as t_{nuc}). However, **PDN fabrication has stochasticity** and the number of interconnect tree nodes of very largescale integration (VLSI) **circuits are enormous**, resulting in computational difficulties in EM simulation.

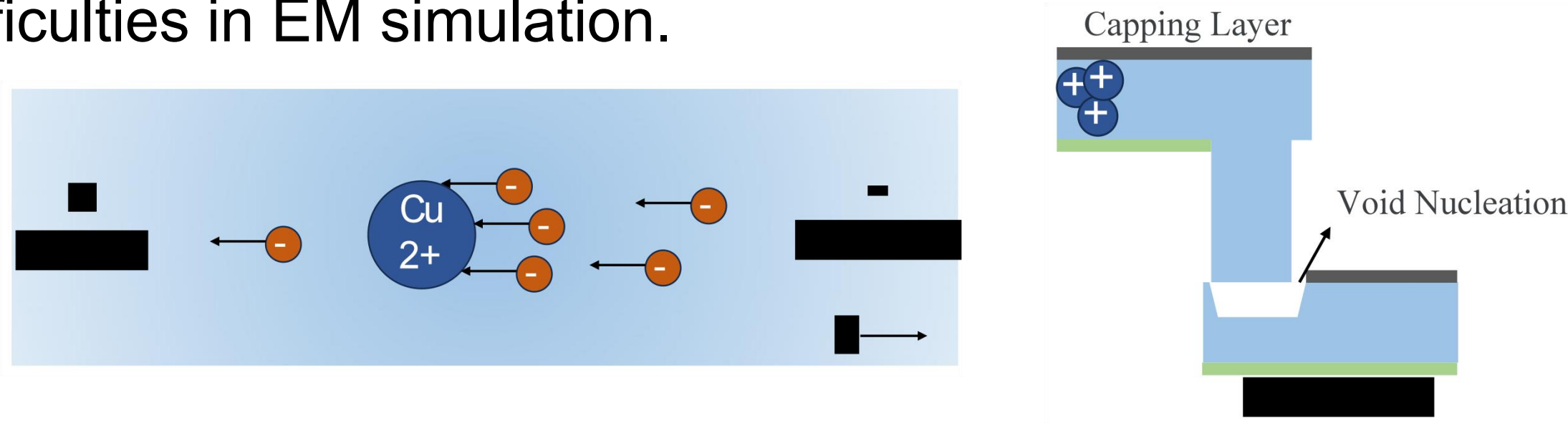


Fig.1 EM Phenomenon.

Wang et al. [1] present an efficient eigenfunction based method, but still lacked a more accurate solution for the random EM phenomenon. Previous works [2], [3] consider the random factors of EM and solve it using numerical methods, i.e., linear time-invariant system and FDM, which are **less efficient**. To solve these problems, we propose a fast estimation method for EM nucleation time based on random activation energy model.

Random Activation Energy Model

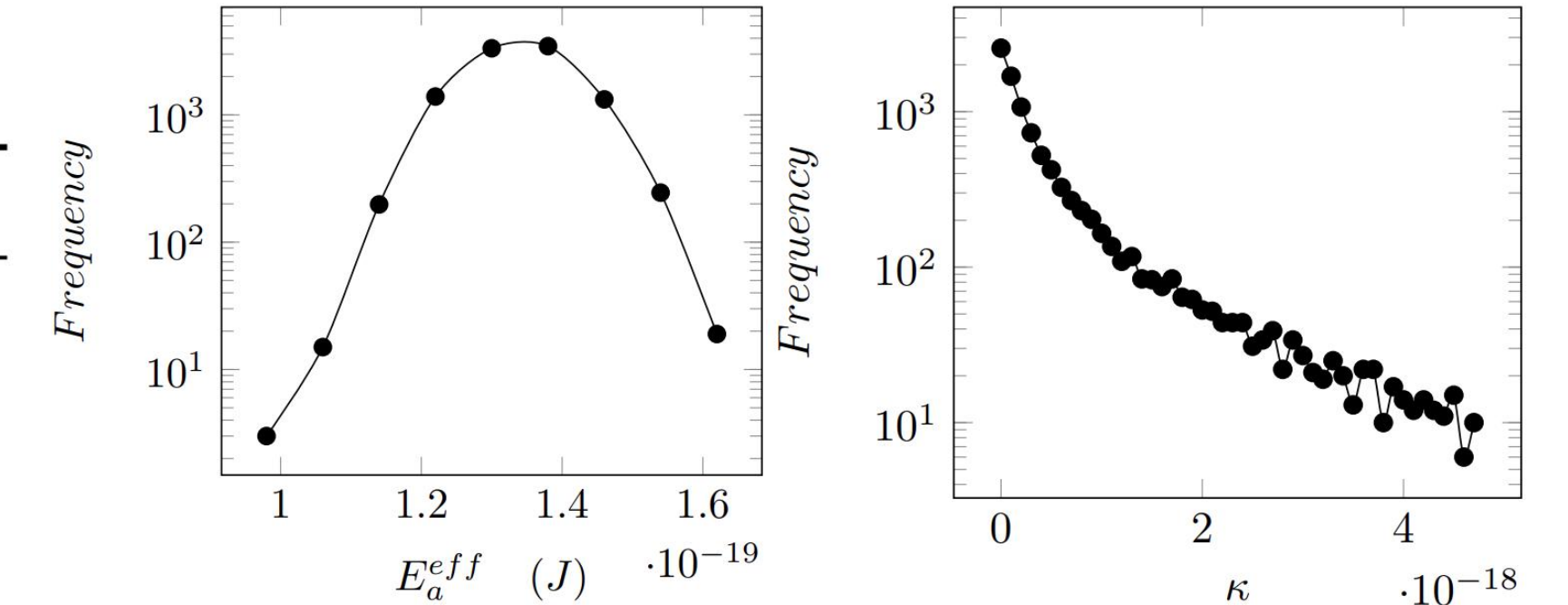
The randomness of the grain microstructure such as grain sizes leads to a random distribution of the metal atom activation energy E_a^{eff} , which is one of the key factors affecting EM. Previous work in [4] explored the impact of random activation energy on EM, but it is inefficient. **The numerical value of E_a^{eff} follows a normal distribution**:

$$f(E_a^{eff}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{E_a^{eff} - \overline{E_a^{eff}}}{\sigma}\right)^2\right]$$

Our method relies on modeling random activation energy E_a^{eff} that varies across different parts of the interconnect tree. By the central limit theorem, E_a^{eff} in each interconnect branch is assumed to follow a normal distribution, and **its mean value is taken as the representative value in that branch**.

Table.1 Physical Parameters.

Parameters	Value
Atomic Diffusion Coefficient, D_0	1.3e-9
Effective Bulk Elasticity Modulus, B	2.8e10 Pa
Boltzmann's Constant, k_B	1.38e-23 J/K
Temperature, T	378 K
Activation Energy Standard Deviation, σ	8e-21
Mean Activation Energy, $\overline{E_a^{eff}}$	1.38e-19 J

Fig.2 The distribution of E_a^{eff} and κ .

Fast Estimation

Filtering and Selecting

We can filter out the immortal trees by pre-computing the steady-state stress using the method in [5]. We use the eigenfunction-based method [1] and binary search to obtain typical t_{nuc} for each interconnect tree at this typical value of E_a^{eff} , which is corresponding to the probability peak. Then, we sort all interconnect trees according to typical t_{nuc} and **select the partial of trees with smaller t_{nuc} as the key interconnect trees** of the whole PDN.

Monte Carlo Sampling

We propose to conduct statistical t_{nuc} estimation based on Monte Carlo sampling of E_a^{eff} . In each sampling, **the internal nodes of a single branch are assigned randomly sampled E_a^{eff}** . The E_a^{eff} of nodes connecting different branches needs to be expressed according to the boundary conditions of "extended Korhonen's model" (EKM) proposed in [6]. After Monte Carlo sampling, the eigenfunction-based method [3] is utilized to solve for the transient stress in the interconnect tree. We perform a binary search on the time interval to obtain the moment when the maximum value of transient stress reaches the critical stress σ_{crit} , which is t_{nuc} . When repeated sampling multiple times, we can get the t_{nuc} distribution.

Parallel Acceleration

The t_{nuc} estimation is performed based on each single interconnect tree. **We use parallel computing**, i.e., OpenMP, for multi-core CPU parallel programming, to calculate the t_{nuc} of multiple interconnect trees at the same time.

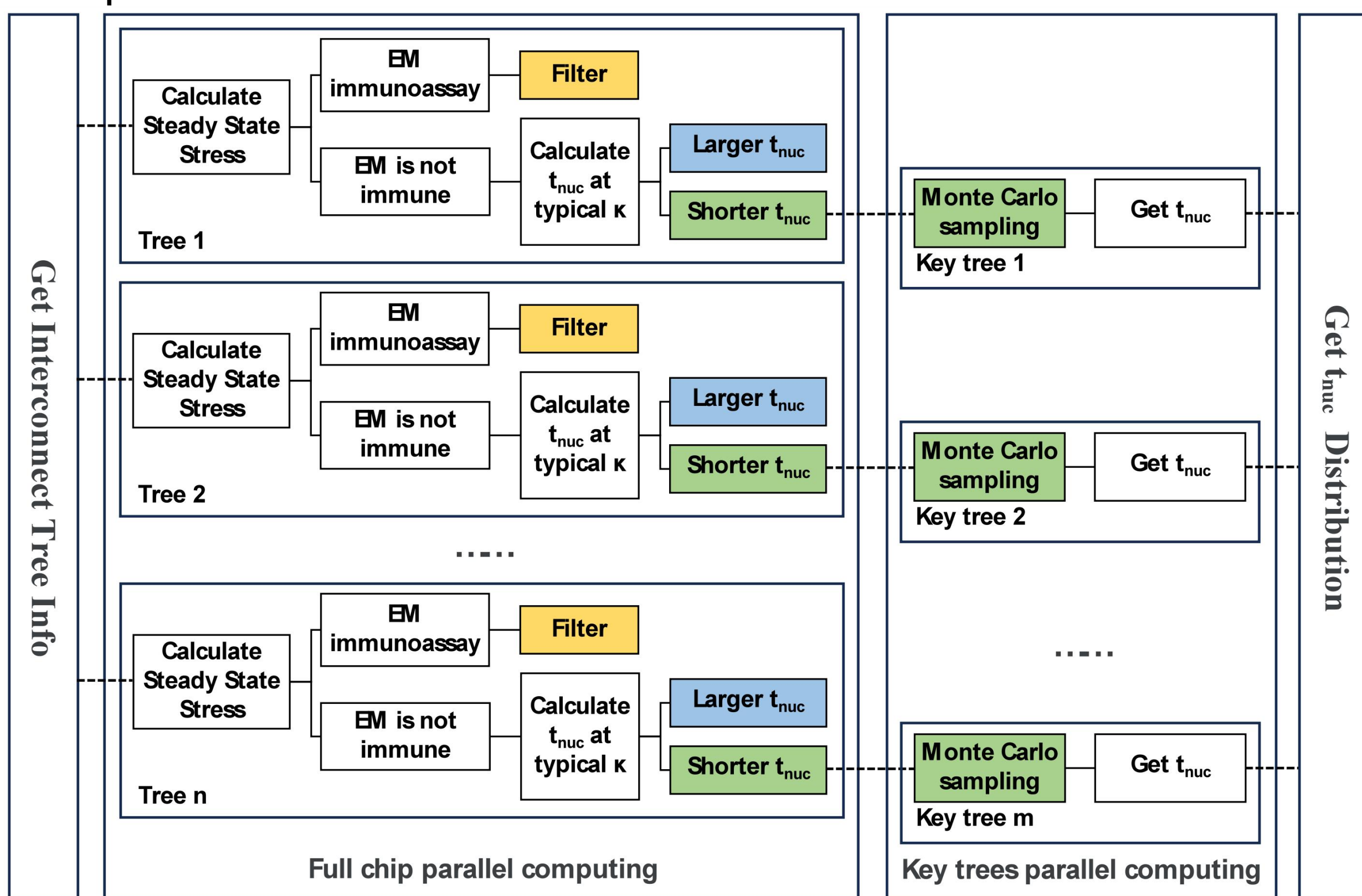


Fig.3 Illustration of the proposed EM estimation method.

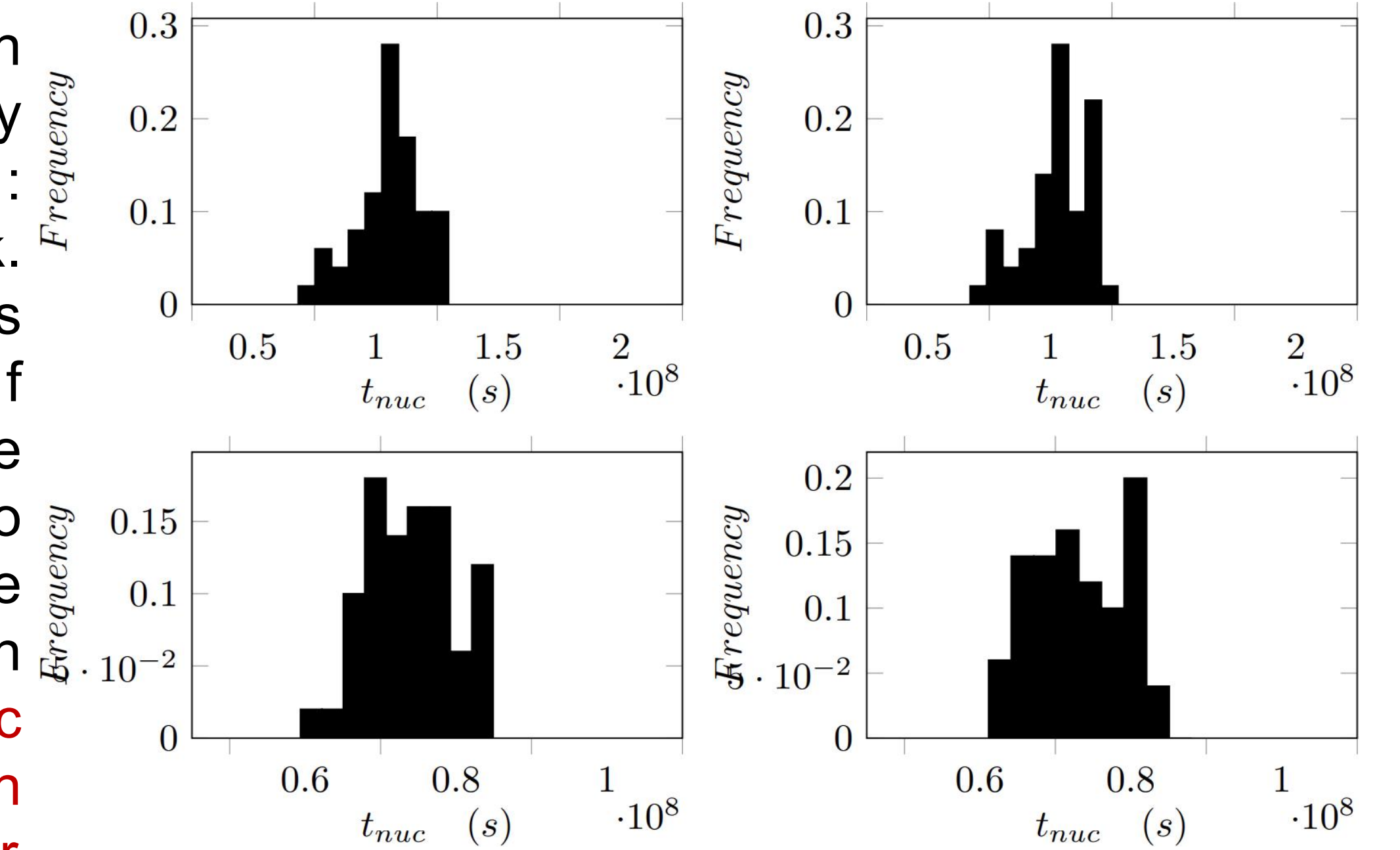
Evaluation

Experimental Setting

Our method is implemented using C++ and Python, and tested on a Linux platform with an 8-core CPU @3.20GHz and 16GB memory. The **IBMPG dataset** [7] is used as the PDN benchmark. **ISPD'23** [3] investigates the effects of random factors such as activation energy and critical stress.

Accuracy of t_{nuc} Estimation

Fig.4 visualizes the distribution of the nucleation time after fifty samplings on the datasets: IBMPG3 and IBMPG4 benchmark. Because each sampling has randomness, the results of ISPD'23 [3] and our method have random differences, thus the two distributions are not exactly the same. However, it still can be seen that **the shapes of the t_{nuc} distribution and the concentration area of the two methods are similar**.

Fig.4 The distribution of t_{nuc} .Efficiency of t_{nuc} Estimation

The average KL divergence is 0.0128, demonstrating that our method and ISPD'23 [3] yield highly similar distribution results. **Our method improves the estimation speed by 39.1%** on average in EM estimation compared with the baseline. Since the number, shape, and physical parameters of the interconnect trees are different, the improvement on each PDN is also different. Overall, Our method is more efficient and can be extended for different large-scale PDNs with better scalability and generalizability.

Table.2 Statistical Results for Nucleation Time Estimation.

Benchmark	# Nodes	# Trees	ISPD'23 (s)	Ours (s)	KL divergence
IBMPG2	127238	462	2868	1710	0.0310
IBMPG3	851584	8189	23245	4856	0.0118
IBMPG4	953583	9641	27496	24379	0.0072
IBMPG5	1079310	1982	9197	7329	0.0012
Avg.	-	-	15701.5	9568.5	0.0128

Conclusion

This paper proposes a fast estimation for EM nucleation time based on the random activation energy model, which provides an effective solution for the random problem of EM analysis. Experiments demonstrate that our method can accurately and efficiently analyze the nucleation time distribution under random processes, and **achieve 10% ~ 80% improvement in estimation efficiency** compared with the previous work.

References

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