HW9 R

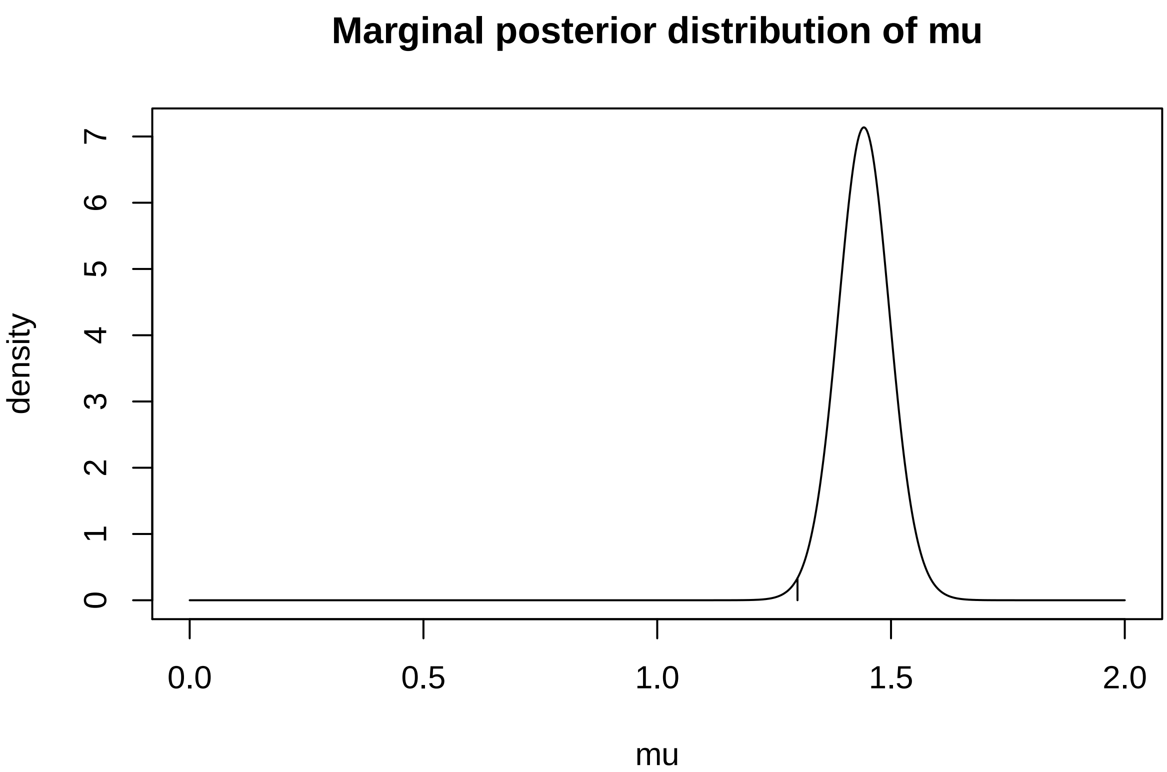
Stat 322 HW9 R problems

R Problem

1. Return to the cheese data from Exercise 8.6.16 on p.506 (consider all 30 observations). Traditionally the mean lactic acid concentration is 1.30, but we wish to test whether or not we have significant evidence that the average has drifted above this mark.

a.Conduct a Bayes test using the usual improper prior distributions. Interpret your results carefully; compare and contrast the Bayes test with the frequentist one-sample t-test.

```r
# The data - lactic acid concentrations in cheese
x10 <- c(0.86,1.53,1.57,1.81,0.99,1.09,1.29,1.78,1.29,1.58)
x20 <- c(1.68,1.9,1.06,1.3,1.52,1.74,1.16,1.49,1.63,1.99,
         1.15,1.33,1.44,2.01,1.31,1.46,1.72,1.25,1.08,1.25)
x <- c(x10,x20)
n <- length(x)
xbar <- mean(x)
snsq <- (n-1)*var(x)

# Test Ho: mu<=1.30 vs. H1: mu>1.30

# a) Usual improper prior hyperparameters
mu0 <- 0
lambda0 <- 0
alpha0 <- -0.5
beta0 <- 0

# Posterior hyperparameters
mu1 <- (lambda0*mu0+n*xbar)/(lambda0+n)
lambda1 <- lambda0 + n
alpha1 <- alpha0 + (n/2)
beta1 <- beta0 + (snsq/2) + ((n*lambda0*(xbar-mu0)^2)/(2*(lambda0+n)))

# Marginal posterior for mu
x5 <- seq(0,2,length=1000)
dx5 <- x5[2]-x5[1]
Ua <- sqrt((lambda1*alpha1)/beta1)*(x5-mu1)
y5a <- dt(Ua,2*alpha1)
plot(x5,y5a/(sum(y5a)*dx5),type="l",xlab="mu",ylab="density",
  main="Marginal posterior distribution of mu")
Uat13a <- sqrt((lambda1*alpha1)/beta1)*(1.3-mu1)
ht13a <- dt(Uat13a,2*alpha1)/(sum(y5a)*dx5)
lines(c(1.3,1.3),c(0,ht13a))
```




```r
# Posterior probability that Ho true
probHotrue <- pt(Uat13a,2*alpha1)
probHotrue
```

```
## [1] 0.007919649
```

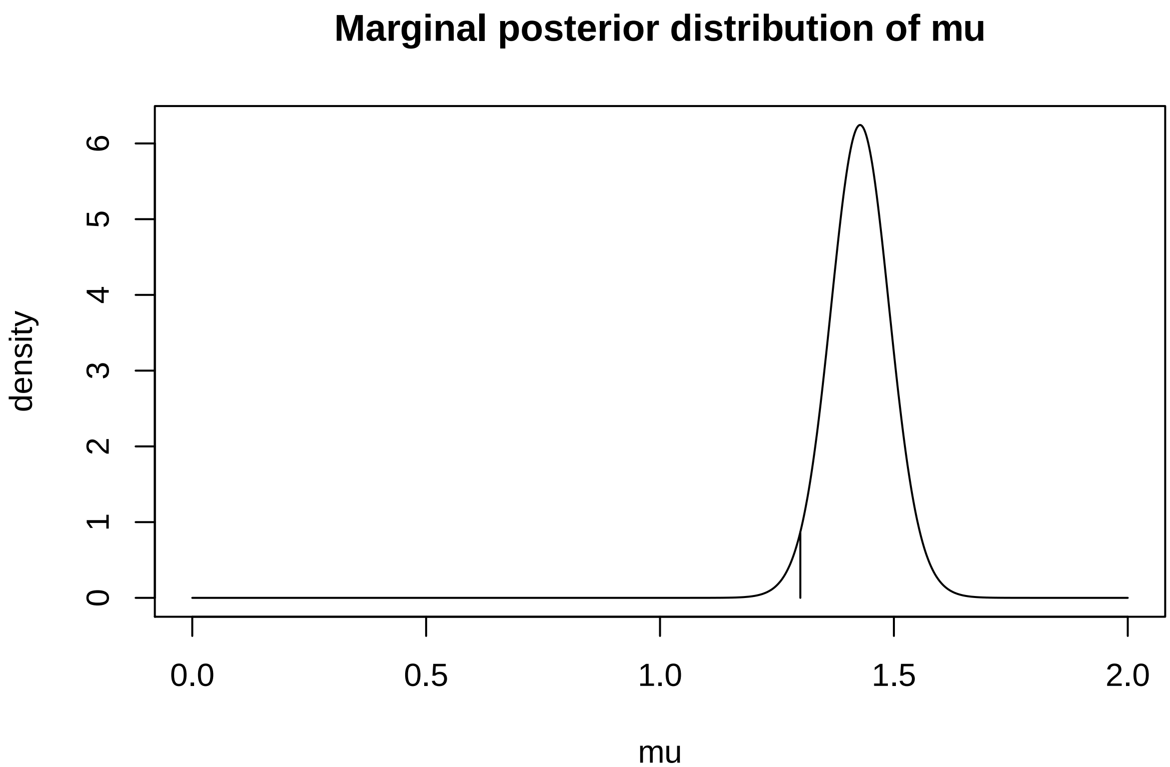
```r
# Compare with frequentist t-test for mu
t.test(x,mu=1.3,alternative="greater")
```

```
##
##  One Sample t-test
##
## data:  x
## t = 2.5627, df = 29, p-value = 0.00792
## alternative hypothesis: true mean is greater than 1.3
## 95 percent confidence interval:
##  1.347852      Inf
## sample estimates:
## mean of x
##     1.442
```

b. Conduct a Bayes test using the prior distribution from Example 8.6.2, and compare your results (both verbally and graphically) with the Bayes test in (a).

```r
mu0 <- 1
lambda0 <- 1
alpha0 <- 0.5
beta0 <- 0.5

# Posterior hyperparameters
mu1 <- (lambda0*mu0+n*xbar)/(lambda0+n)
lambda1 <- lambda0 + n
```

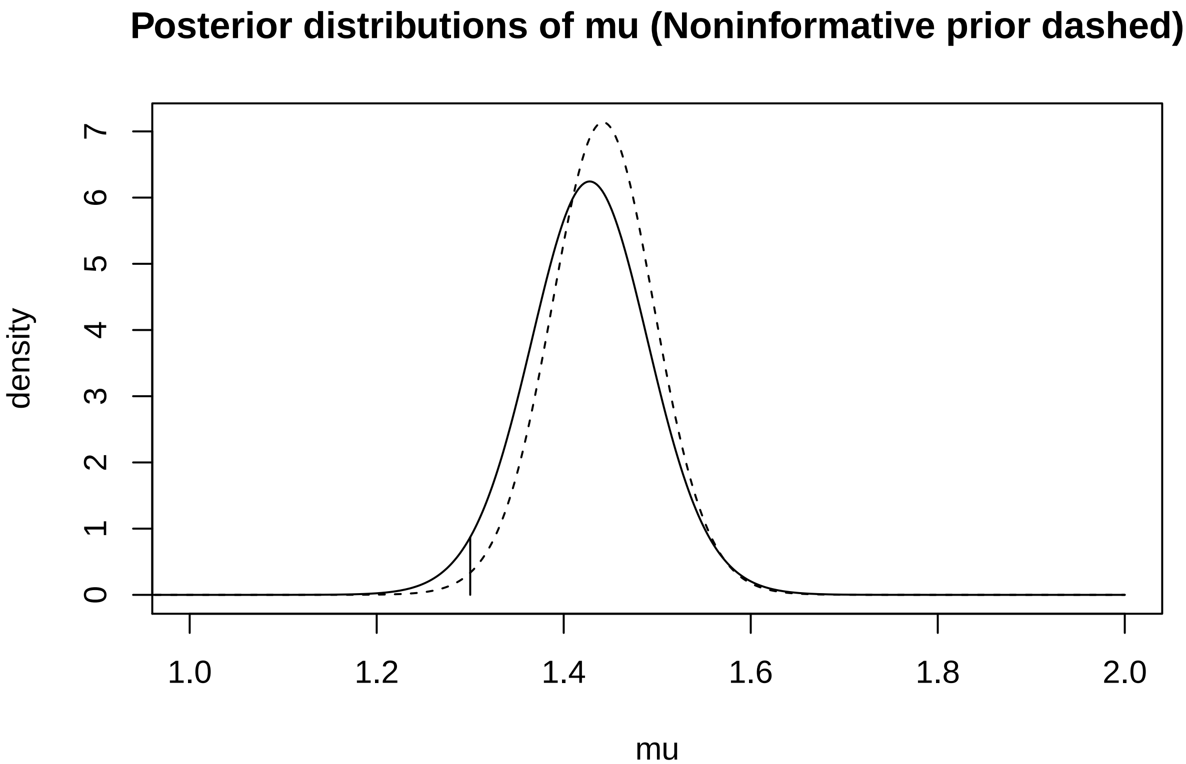
```r
alpha1 <- alpha0 + (n/2)
beta1 <- beta0 + (snsq/2) + ((n*lambda0*(xbar-mu0)^2)/(2*(lambda0+n)))

# Marginal posterior for mu
Ub <- sqrt((lambda1*alpha1)/beta1)*(x5-mu1)
y5b <- dt(Ub,2*alpha1)
plot(x5,y5b/(sum(y5b)*dx5),type="l",xlab="mu",ylab="density",
  main="Marginal posterior distribution of mu")
Uat13b <- sqrt((lambda1*alpha1)/beta1)*(1.3-mu1)
ht13b <- dt(Uat13b,2*alpha1)/(sum(y5b)*dx5)
lines(c(1.3,1.3),c(0,ht13b))
```




```r
# Posterior probability that Ho true
probHotrue <- pt(Uat13b,2*alpha1)
probHotrue
```

```
## [1] 0.02629332
```

```r
# Superimposed posteriors for mu
plot(x5,y5b/(sum(y5b)*dx5),type="l",xlab="mu",ylab="density",
  xlim=c(1,2),ylim=c(0,max(y5b/(sum(y5b)*dx5),y5a/(sum(y5a)*dx5))),
  main="Posterior distributions of mu (Noninformative prior dashed)")
lines(x5,y5a/(sum(y5a)*dx5),lty=2)
lines(c(1.3,1.3),c(0,max(ht13a,ht13b)))
```



c. Suppose we wish to consider a two-sided alternative hypothesis. Create a plot as in Figure 9.16 (p. 610) showing $Pr(|\mu - \mu_0| \leq d|\mathbf{x})$ versus d using the priors from (b).

```
d <- seq(0,0.5,length=1000)
Ulower <- sqrt((lambda1*alpha1)/beta1)*(1.3-d-mu1)
Uupper <- sqrt((lambda1*alpha1)/beta1)*(1.3+d-mu1)
probHotrue <- pt(Uupper,2*alpha1) - pt(Ulower,2*alpha1)
plot(d,probHotrue,type="l")
```