# Bayes Test Procedure in R

## TV Radiation Example

```
# The data - TV radiation example
x <- c(0.65,0.28,0.47,0.44,0.25,1.03,0.28,0.64,0.71,0.49)
n <- length(x)
xbar <- mean(x)
snsq <- (n-1)*var(x)
```

$H_0 : \mu \leq \mu_0$ vs $H_1 : \mu > \mu_0$

Consider $\Pr(H_0$ true | data) and Reject $H_0$ if small. $Pr(H_0\ true|x) = \int_{\Omega_0} \xi(\mu|x)d\mu$

## Non-informative Prior:

$\xi(\mu, \tau) = \xi(\mu|\tau)\xi(\tau) = N(\mu_0, \lambda_0\tau)gamma(\alpha_0, \beta_0)$

For what values of $\mu_0, \lambda_0, \alpha_0, \beta_0$ would $\xi(\mu, \tau) = 1/\tau$ (Jeffreys Prior)?

```
# Usual improper prior hyperparameters
mu0 <- 0
lambda0 <- 0
alpha0 <- -0.5
beta0 <- 0
```

2. Find the values of $\mu_1, \lambda_1, \alpha_1, \beta_1$ using the non informative prior.

```
mu1 <- (lambda0*mu0+n*xbar)/(lambda0+n)
lambda1 <- lambda0 + n
alpha1 <- alpha0 + (n/2)
beta1 <- beta0 + (snsq/2) + ((n*lambda0*(xbar-mu0)^2)/(2*(lambda0+n)))

mu1;lambda1;alpha1;beta1
```

```
## [1] 0.524

## [1] 10

## [1] 4.5

## [1] 0.26262
```

3. Using these posterior values show the equivalence of the Bayes Test Procedure using Jeffreys prior to the frequentist result ($U \sim t_{(n-1)}$)

```
# Marginal posterior for mu
mu <- seq(.20,.80,length=1000)
dmu <- mu[2]-mu[1]
U1 <- sqrt((lambda1*alpha1)/beta1)*(mu-mu1)
y1 <- dt(U1,2*alpha1)

# Posterior probability that Ho true
Uat5 <- sqrt((lambda1*alpha1)/beta1)*(0.50-mu1)
ht5 <- dt(Uat5,2*alpha1)/(sum(y1)*dmu)
```

```r
probHotrue <- pt(Uat5,2*alpha1)
probHotrue
```

```
## [1] 0.3802791
```

4. Compare the posterior probability to p-value (using R).

```r
t.test(x,mu=.5,var.equal=TRUE,alternative="greater")
```

```
##
##  One Sample t-test
##
## data:  x
## t = 0.31416, df = 9, p-value = 0.3803
## alternative hypothesis: true mean is greater than 0.5
## 95 percent confidence interval:
##  0.3839617       Inf
## sample estimates:
## mean of x
##     0.524
```

5. Repeat using informative hyper parameters for the conjugate prior relationship. ($\mu_0 = .25; \lambda_0 = 4; \alpha_0 = 2; \beta_0 = 0.5$)

```r
# Informative prior hyperparameters
mu0 <- .25
lambda0 <- 4
alpha0 <- 2
beta0 <- 0.5
```

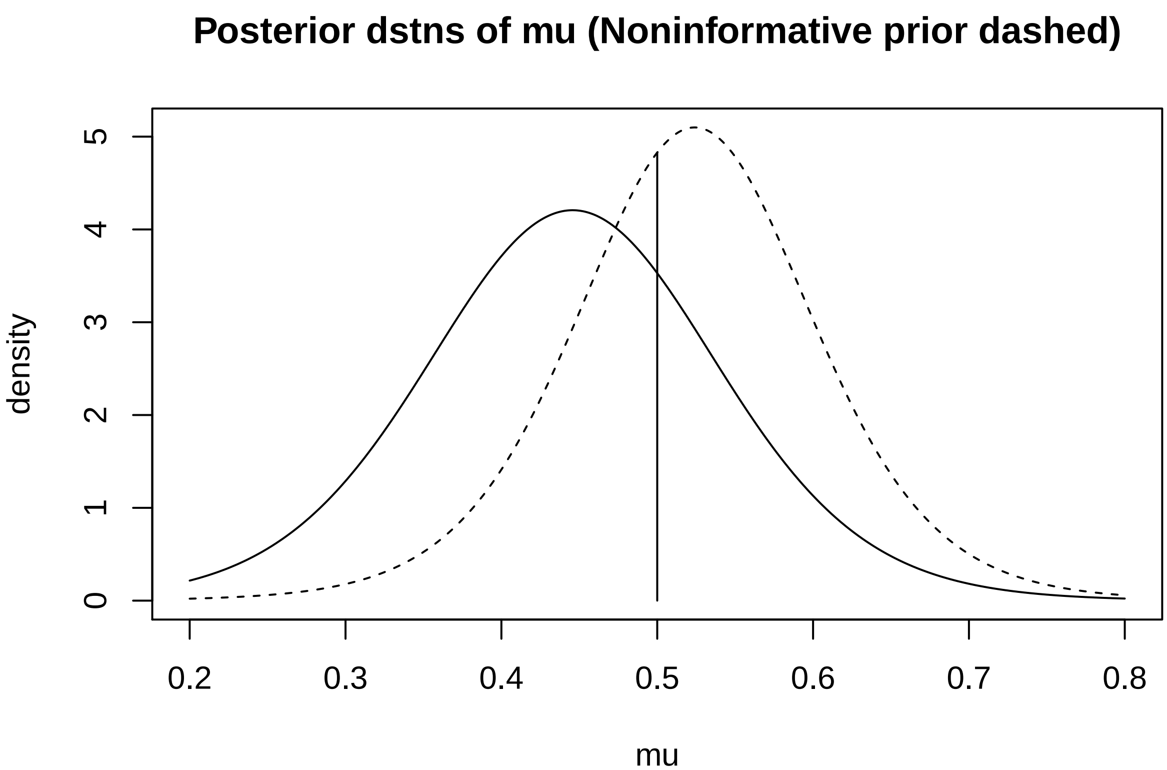
```r
# Posterior hyperparameters
mu1 <- (lambda0*mu0+n*xbar)/(lambda0+n)
lambda1 <- lambda0 + n
alpha1 <- alpha0 + (n/2)
beta1 <- beta0 + (snsq/2) + ((n*lambda0*(xbar-mu0)^2)/(2*(lambda0+n)))

# Marginal posterior for mu
U1b <- sqrt((lambda1*alpha1)/beta1)*(mu-mu1)
y1b <- dt(U1b,2*alpha1)

# Posterior probability that Ho true
Uat5b <- sqrt((lambda1*alpha1)/beta1)*(0.50-mu1)
ht5b <- dt(Uat5b,2*alpha1)/(sum(y1b)*dmu)
probHotrue <- pt(Uat5b,2*alpha1)
probHotrue
```

```
## [1] 0.7131854
```

```r
# Superimposed posteriors for mu
plot(mu,y1b/(sum(y1b)*dmu),type="l",xlab="mu",ylab="density",
  xlim=c(.2,.8),
  ylim=c(0,max(y1b/(sum(y1b)*dmu),y1/(sum(y1)*dmu))),
  main="Posterior dstns of mu (Noninformative prior dashed)")
lines(mu,y1/(sum(y1)*dmu),lty=2)
lines(c(.5,.5),c(0,max(ht5,ht5b)))
```

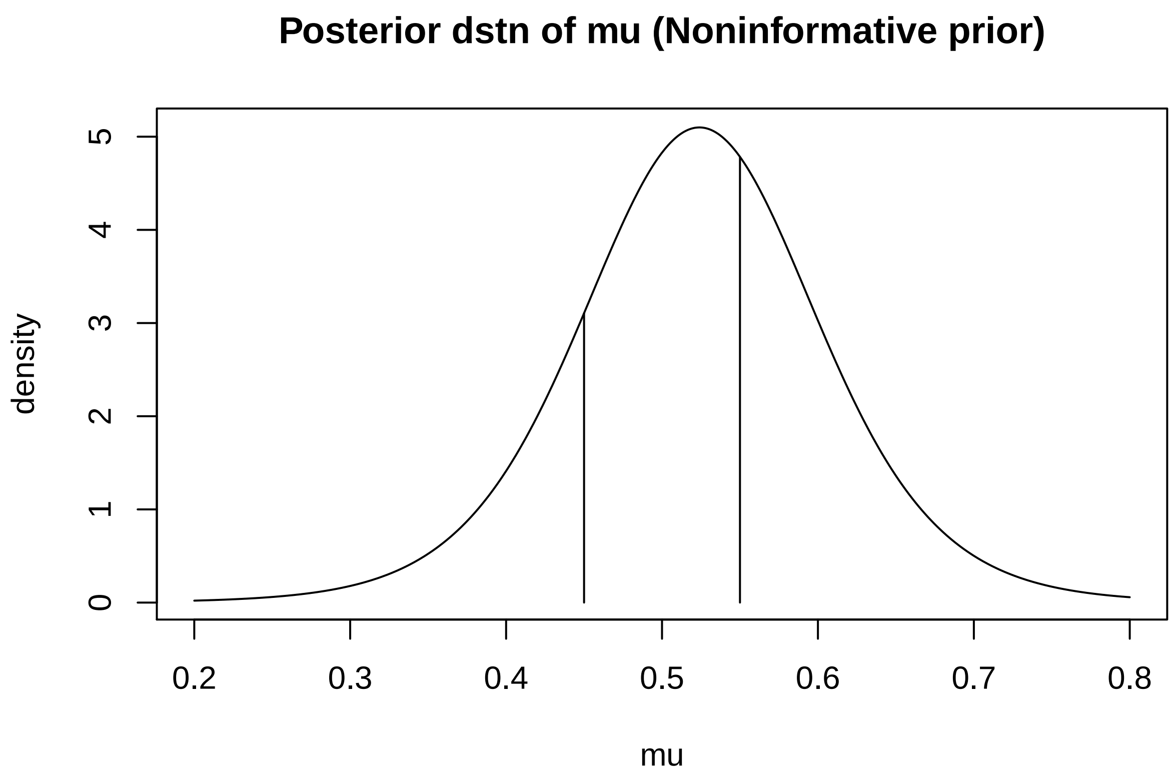



6. Show equivalence with non informative prior to the two-sided t-test.

```
# Usual improper prior hyperparameters
mu0 <- 0
lambda0 <- 0
alpha0 <- -0.5
beta0 <- 0

# Posterior hyperparameters
mu1 <- (lambda0*mu0+n*xbar)/(lambda0+n)
lambda1 <- lambda0 + n
alpha1 <- alpha0 + (n/2)
beta1 <- beta0 + (snsq/2) + ((n*lambda0*(xbar-mu0)^2)/(2*(lambda0+n)))

# Marginal posterior for mu
Uat45 <- sqrt((lambda1*alpha1)/beta1)*(0.45-mu1)
ht45 <- dt(Uat45,2*alpha1)/(sum(y1)*dmu)
Uat55 <- sqrt((lambda1*alpha1)/beta1)*(0.55-mu1)
ht55 <- dt(Uat55,2*alpha1)/(sum(y1)*dmu)
plot(mu,y1/(sum(y1)*dmu),type="l",xlab="mu",ylab="density",
  main="Posterior dstn of mu (Noninformative prior)")
lines(c(.45,.45),c(0,ht45))
lines(c(.55,.55),c(0,ht55))
```




```r
# Posterior probability that Ho true
probHotrue <- pt(Uat55,2*alpha1)-pt(Uat45,2*alpha1)
probHotrue
```

```
## [1] 0.4502856
```

```r
# Compare with frequentist t-test for mu
t.test(x,mu=0.50)
```

```
##
##  One Sample t-test
##
## data:  x
## t = 0.31416, df = 9, p-value = 0.7606
## alternative hypothesis: true mean is not equal to 0.5
## 95 percent confidence interval:
##  0.3511854 0.6968146
## sample estimates:
## mean of x
##     0.524
```