9.1 Testing Hypotheses

Example: National Assessment of Adult Literacy (NAAL) survey scores (0-500)

- 233: can add numbers of 2 checks on a bank deposit slip
- 325: can price a meal from a menu
- 375: can trasnform from cents/oz to dollar/lb

275: can Balance a check book

We collect data from $n = 840$ men ages 21-25 (assume $\sigma = 59, \bar{X}_n = 272$)

Hypothesis Test Assumptions

$$X \sim N(275, 59^2)$$

pvalue$= Prob(\bar{X}_n \leq 272 | X \sim N(275, 59^2))$

$$H_0 : \mu \geq 275$$

$$H_A : \mu < 275$$

our test statistic under the H_0: $\bar{X}_n \sim N(275, 59^2/840)$

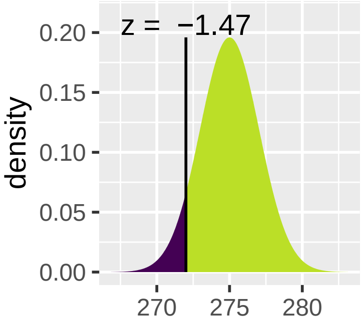
$z = \frac{\bar{x} - \mu}{\sigma / n} \sim N(0, 1)$ under H_0

$z_{obs} = -1.474$ and the pvalue (one sided) is 0.0708.

Using R

```
xpnorm(272,275,59/sqrt(840))
```

```
##
## If X ~ N(275, 2.036), then
##  P(X <= 272) = P(Z <= -1.474) = 0.07028
##  P(X >  272) = P(Z >  -1.474) = 0.9297
##
```



Applied to 9.1

$$H_0 = \theta \in \Omega_0$$
$$H_1 = \theta \in \Omega_1$$

Where $\Omega_0 \cup \Omega_1 = \Omega$: the entire sample space for θ

Our case the context of 9.1

$$\Omega_0 = [275, 500]$$
$$\Omega_1 = [0, 275)$$

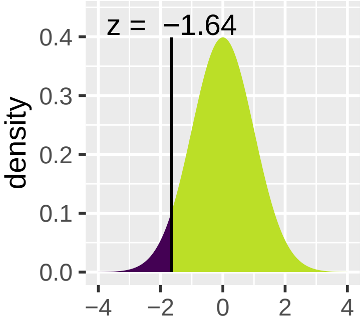
Test statistic: $T = r(x)$

Critical region: Reject H_0 if $T \leq -1.645$

R

```r
xqnorm(.05)
```

```
##
## If X ~ N(0, 1), then
##  P(X <= -1.644854) = 0.05
##  P(X >  -1.644854) = 0.95
##
```



Power Function

Def: The probability that a test procedure δ rejects H_0 for each value of $\theta \in \Omega$

$\pi(\theta|\delta) = Pr(x \in C|\theta)$ for $\theta \in \Omega$

An ideal power function:

$$\pi(\theta|\delta) = 0 \text{ for } \theta \in \Omega_0$$
$$\pi(\theta|\delta) = 1 \text{ for } \theta \in \Omega_1$$

BUT probabilities of Error exist.

Errors In Hypothese thests

	H_0 True	H_0 False
Accept H_0	OK	Type II Error
Reject H_0	Type I Error	OK

Prob(Type I Error)= Prob(Reject H_0 when H_0 is true)

$$= \pi(\theta|\delta) \text{ for } \theta \in \Omega_0$$

Prob(Type II Error)= Prob(Do not reject H_0 when H_0 is false)

$$= 1 - \pi(\theta|\delta) \text{ for } \theta \in \Omega_1$$

Scenarios

	Innocent	Gulity
Acquit	OK	Type II Error
Convict	Type I Error	OK

Which error is worse?

Convict an innocent person? Or let a criminal run free?

	Drug does not work	Drug works
Don't Approve	OK	Type II Error
Approve	Type I Error	OK

Which error is worse?

Type I Error: Drug is marketed but doesn't really work.
Type II Error: Drug that could save lives is not available

Hypothesis Testing Strategy

Set upper bound for Type I error. Then among those tests, pick one that maximizes Power (minimizes Type II Error)

Require:

$$\pi(\theta|\delta) \leq \alpha_0 \text{ for } \theta \in \Omega_0$$

Size $\alpha(\delta)$ of a test δ

$$\alpha(\delta) = \sup_{\theta \in \Omega_0} \pi(\theta|\delta)$$

Checkbook example

- Find P(Type I Error)
- Find Size of the test if $\mu = 275$
- Find power of the test if $\mu = 270$

Pr(Type I Error)

Pr(Type I Error)=Pr(Reject $H_0|H_0$ is tre)

$$
\begin{aligned}
&= Pr(z \leq 1.645|\mu = 275) \\
&= Pr(\frac{\bar{X}_n - 275}{59/\sqrt{840}} < -1.645) \\
&= Pr(\bar{X}_n < 271.65)
\end{aligned}
$$

So $\delta : \{x : \bar{(}x)_n < 271.65\} = \{x : z = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}}\}$

Size of the test

$$\begin{aligned}\alpha(\delta) &= \sup_{\mu \geq 275} \pi(\mu|\delta) \\ &= \sup_{\mu} Pr(\bar{X}_n < 271.65|\mu \geq 275) \\ &= Pr(\bar{X}_n < 271.65|\mu = 275) \\ &= Pr(Z < \frac{271.65 - 275}{59/\sqrt{840}}) \\ &= Pr(Z < -1.645) = 0.05\end{aligned}$$

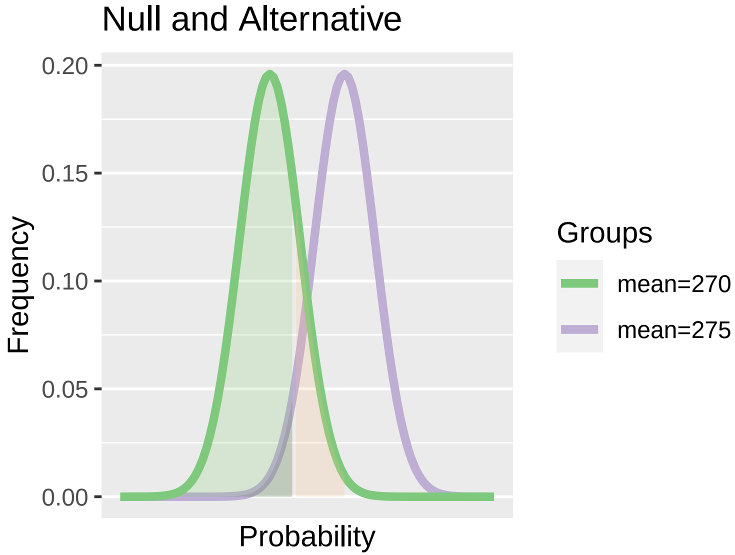
Power of the test for μ=270

$$
\begin{aligned}
\pi(\mu|\delta) &= Pr(\bar{X}_n < 271.65|\mu = 270) \\
&= Pr(Z < \frac{271.65 - 270}{59/\sqrt{840}}) \\
&= Pr(Z < 0.81) = 0.791
\end{aligned}
$$

```r
pnorm(0.81)
```

```
## [1] 0.7910299
```

Plot of Type I error, Type II Error, and Power



How do the following effect Power

Power=1-Pr(Type II Error) = Probability of corretly rejeting H_0

- α: as α increases, Power increases
- σ: as σ decreases, Power increases
- n: as n increases, Power increases
- μ: as μ decreases(futher away from μ_0), Power increases