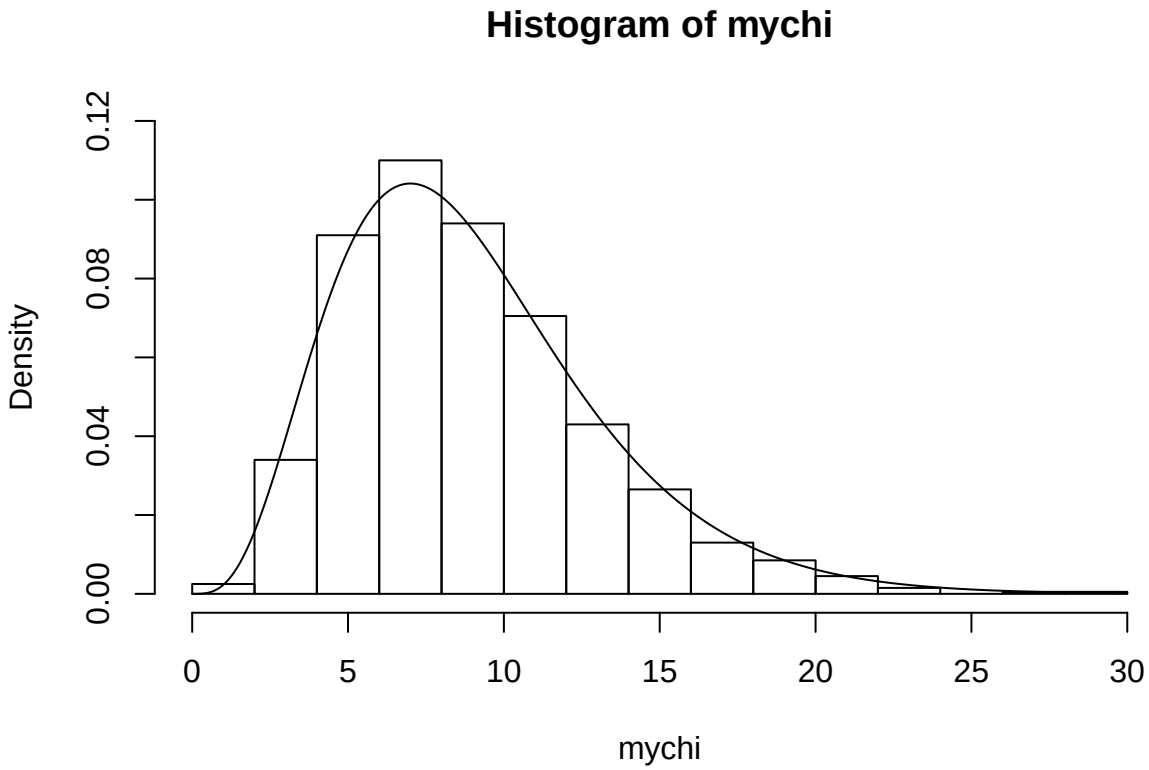
HW 6 R

Stat 322 HW6 R problems

Use R to numerically validate the following theorems and results. For each problem, be sure to hand in your complete R code, any useful numerical results, and well-labeled plots.

1. If $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, then $\frac{\sum(X_i - \bar{x}_n)^2}{\sigma^2} \sim \chi^2_{n-1}$

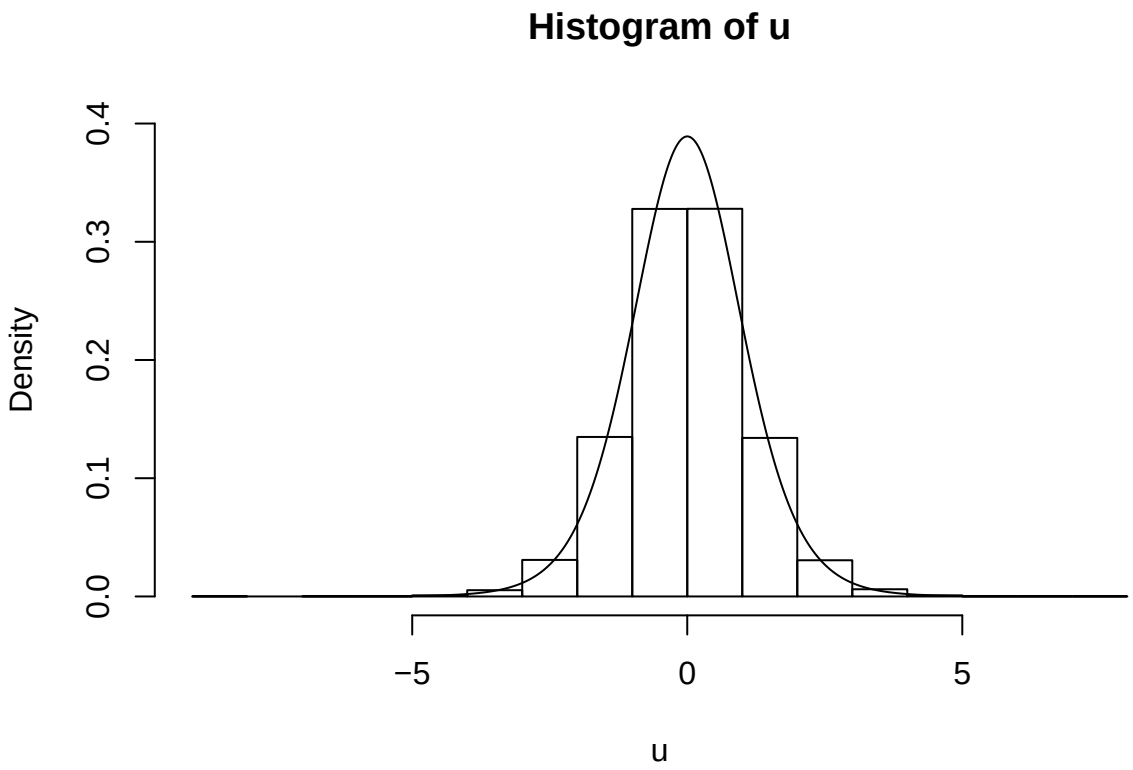
```
y <- rnorm(10000,mean=4,sd=2)
ymat <- matrix(y,ncol=10)
ymean <- apply(ymat,1,mean)
ydiff <- ymat-ymean
ydiff2 <- ydiff^2
ydiffsum <-apply(ydiff2,1,sum)
mychi <- ydiffsum/4

hist(mychi,probability=TRUE,ylim=c(0,.12))
x <- seq(0,30,length=1000)
chi9 <- dchisq(x,df=9)
lines(x,chi9)
```



2. If $Z \sim N(0,1), Y \sim \chi^2_n$ and Z and Y are independent, then $U \sim \frac{Z}{\sqrt{Y/n}} \sim t_n$.

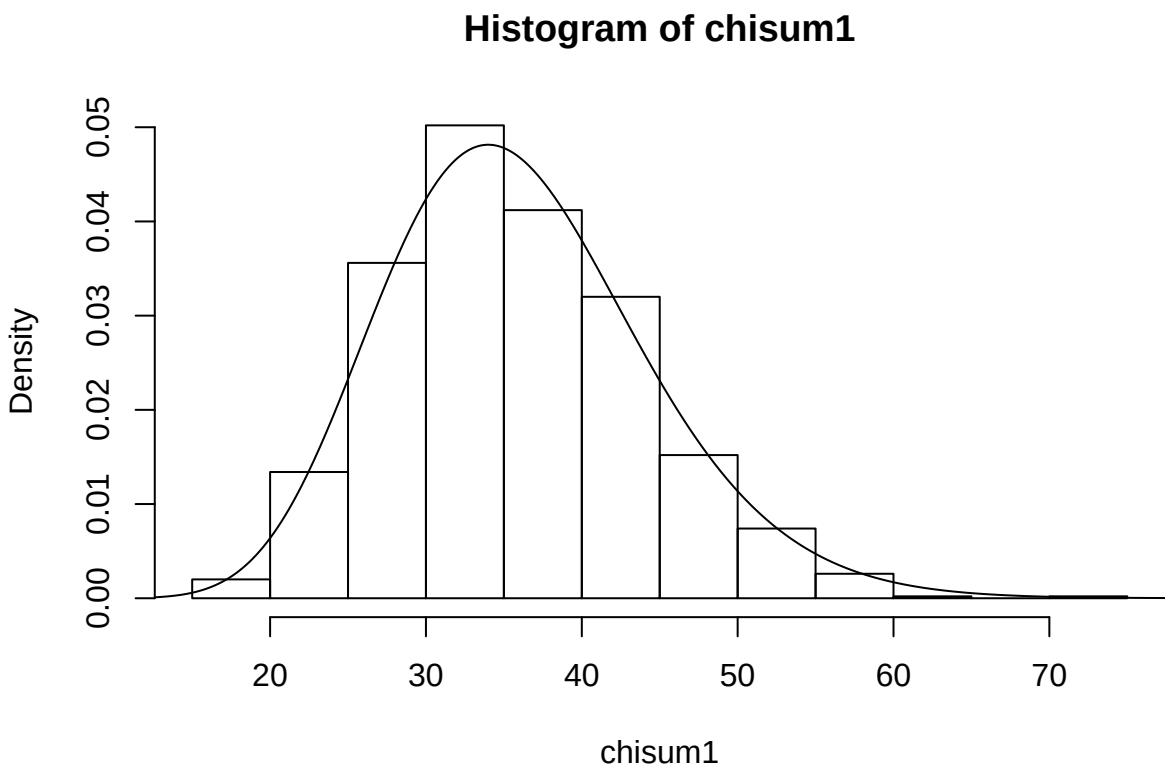
```
z <- rnorm(10000,mean=0,sd=1)
y <- rchisq(10000,df=10)
u <- z / sqrt(y/10)
x <- seq(-5,5,length=1000)
tn <- dt(x,df=10)
```

```r
hist(u,probability=TRUE,ylim=c(0,.4))
lines(x,tn)
```



3. If $X_1 \sim \chi^2_{n_1}, X_2 \sim \chi^2_{n_2}, ...X_k \sim \chi^2_{n_k}$, and $X_1, X_2, ..., X_k$ are independent,\ then $\sum_{i=1}^{k} X_i \sim \chi^2_{\Sigma n_i}$

```r
chi1 <- rchisq(1000,df=1)
chi2 <- rchisq(1000,df=3)
chi3 <- rchisq(1000,df=5)
chi4 <- rchisq(1000,df=7)
chi5 <- rchisq(1000,df=9)
chi6 <- rchisq(1000,df=11)


tot=1+3+5+7+9+11

chisum1 <- chi1+chi2+chi3+chi4+chi5+chi6
x <- seq(0,80,length=1000)
chi_tot <- dchisq(x,df=tot)
hist(chisum1,probability=TRUE)
lines(x,chi_tot)
```



4. From Example 8.5.11, if $X_1, X_2 \sim Uniform(\theta - \frac{1}{2}, \theta + \frac{1}{2})$, then the interval between $y_1 = \min(x_1, x_2)$ and $y_2 = \max(x_1, x_2)$ is a 50% confidence interval for θ.

```
theta <- 5
x1 <- runif(10000,theta-.5,theta+.5)
x2 <- runif(10000,theta-.5,theta+.5)
xmat <- cbind(x1,x2)
y1 <- apply(xmat,1,min)
y2 <- apply(xmat,1,max)
sum(y1<theta & theta<y2)/10000
```

```
## [1] 0.5002
```