# HW 7 R

## Stat 322 HW7 R problems

Section 8.6 # 16. Do (a) and (b) by hand – i.e. find requested posterior hyperparameters, compare, and comment on the implications. Then, present R code and plots to show the following. [You'll find a good chunk of code to get you started on the R server!]

```
# The data - lactic acid concentrations in cheese
x10 <- c(0.86,1.53,1.57,1.81,0.99,1.09,1.29,1.78,1.29,1.58)
x20 <- c(1.68,1.9,1.06,1.3,1.52,1.74,1.16,1.49,1.63,1.99,
         1.15,1.33,1.44,2.01,1.31,1.46,1.72,1.25,1.08,1.25)
x <- c(x10,x20)
n <- length(x); n
```

```
## [1] 30
```

```
xbar <- mean(x); xbar
```

```
## [1] 1.442
```

```
snsq <- (n-1)*var(x); snsq
```

```
## [1] 2.67108
```

1. prior and posterior marginal distributions of $\tau$ superimposed on a single plot

```
# Prior hyperparameters from Example 8.6.1
mu0 <- 1
lambda0 <- 1
alpha0 <- 0.5
beta0 <- 0.5

# Posterior hyperparameters
mu1 <- (lambda0*mu0+n*xbar)/(lambda0+n); mu1
```

```
## [1] 1.427742
```

```
lambda1 <- lambda0 + n ; lambda1
```

```
## [1] 31
```

```
alpha1 <- alpha0 + (n/2) ; alpha1
```

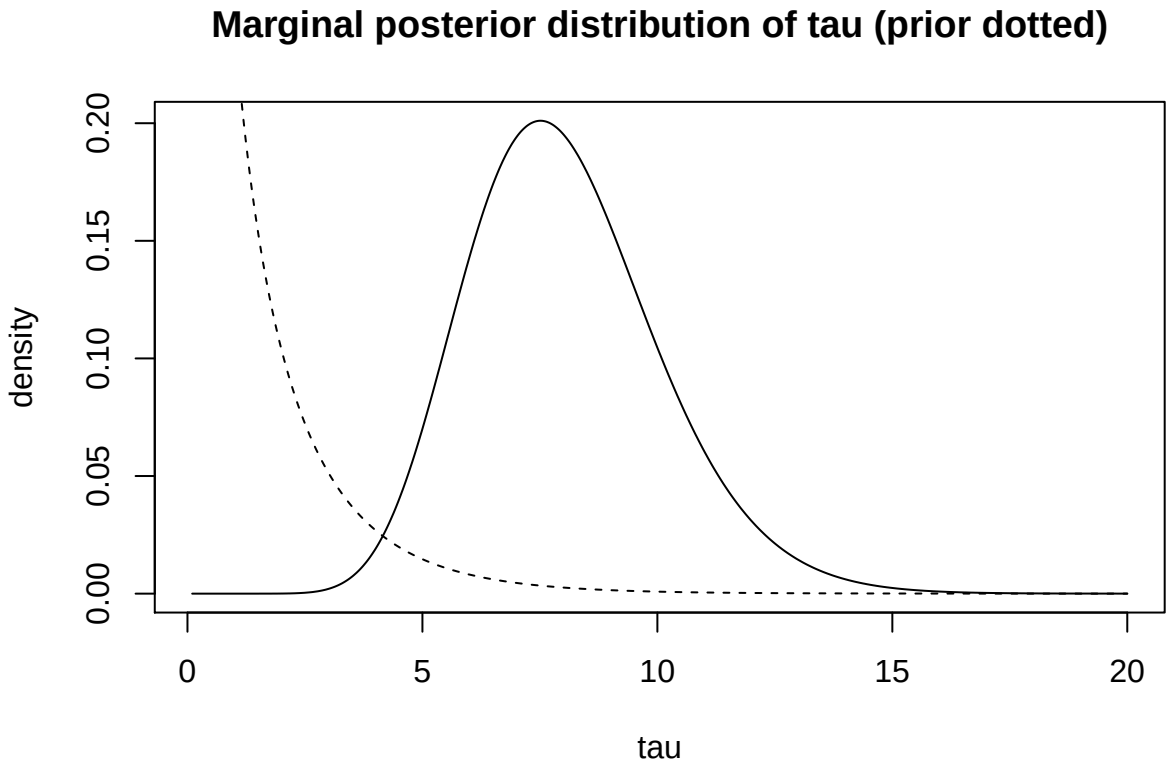
```
## [1] 15.5
```

```
beta1 <- beta0 + (snsq/2) + ((n*lambda0*(xbar-mu0)^2)/(2*(lambda0+n))); beta1
```

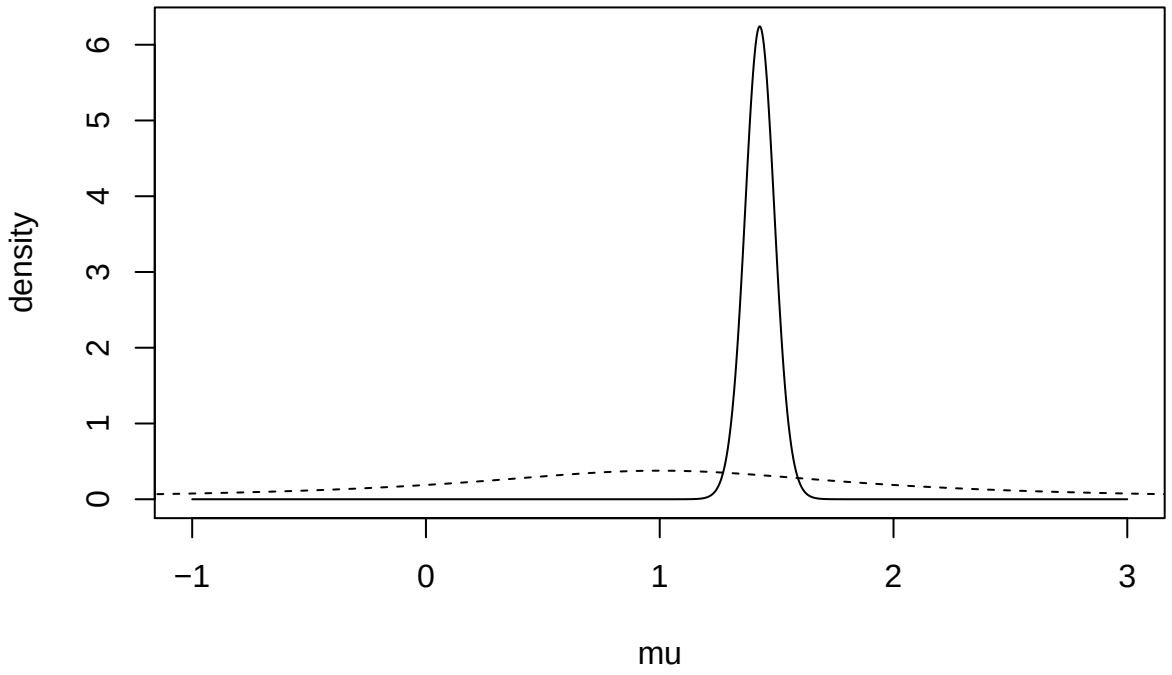
```
## [1] 1.930071
```

```
# Marginal prior for tau
x1 <- seq(0.1,20,length=1000)
y1 <- dgamma(x1,alpha0,beta0)

# Marginal posterior for tau
x4 <- seq(0.1,20,length=1000)
y4 <- dgamma(x4,alpha1,beta1)
plot(x4,y4,type="l",xlab="tau",ylab="density",
```

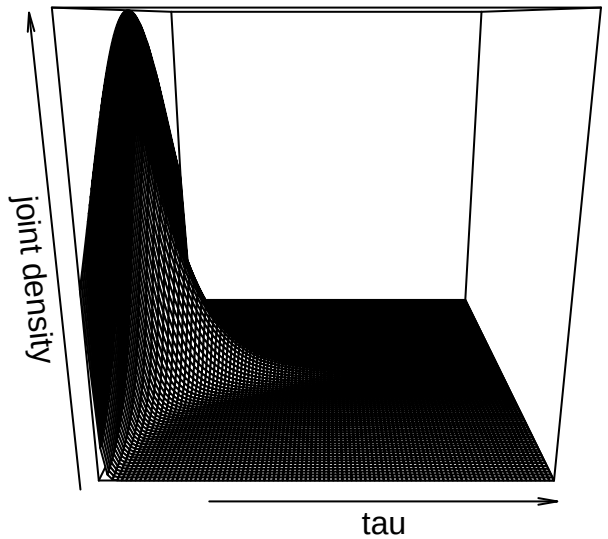
```
  main="Marginal posterior distribution of tau (prior dotted)")
lines(x1,y1,lty=2)
```



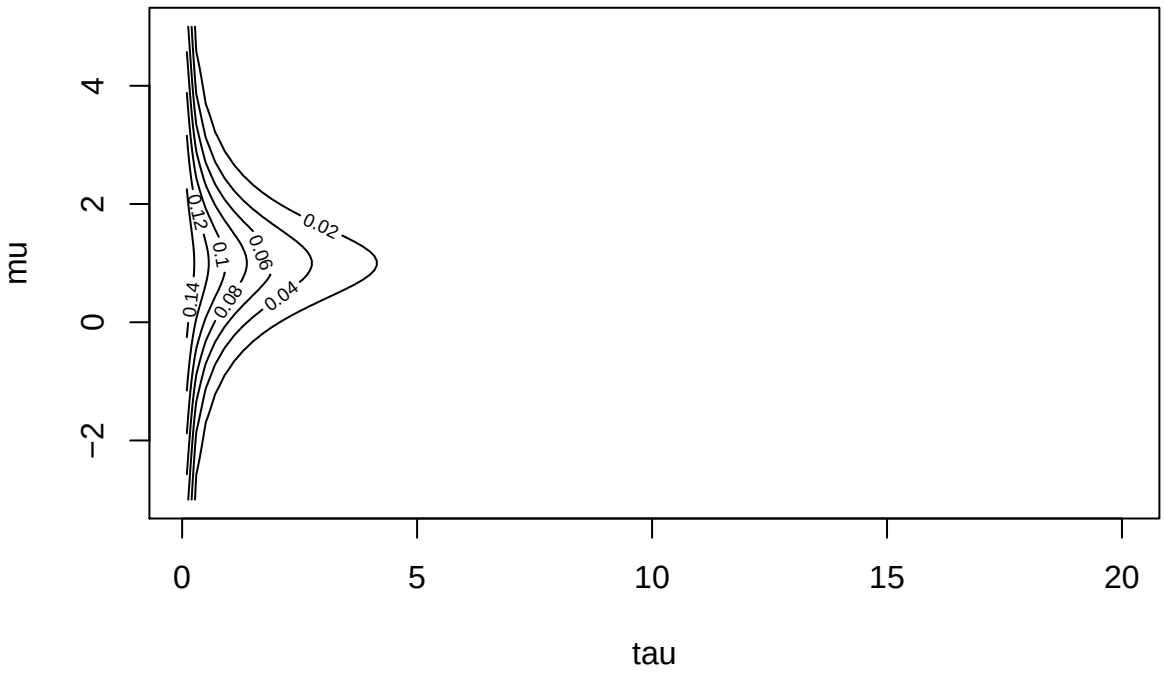

```
# Marginal prior for mu
x2 <- seq(-3,5,length=1000)
dx2 <- x2[2]-x2[1]
U <- sqrt((lambda0*alpha0)/beta0)*(x2-mu0)
y2 <- dt(U,2*alpha0)

# Marginal posterior for mu
x5 <- seq(-1,3,length=1000)
dx5 <- x5[2]-x5[1]
U <- sqrt((lambda1*alpha1)/beta1)*(x5-mu1)
y5 <- dt(U,2*alpha1)
#y5 is the marginal prior based on U
plot(x5,y5/(sum(y5)*dx5),type="l",xlab="mu",ylab="density",
  main="Marginal posterior distribution of mu (prior dotted)")
lines(x2,y2/(sum(y2)*dx2),lty=2)
```



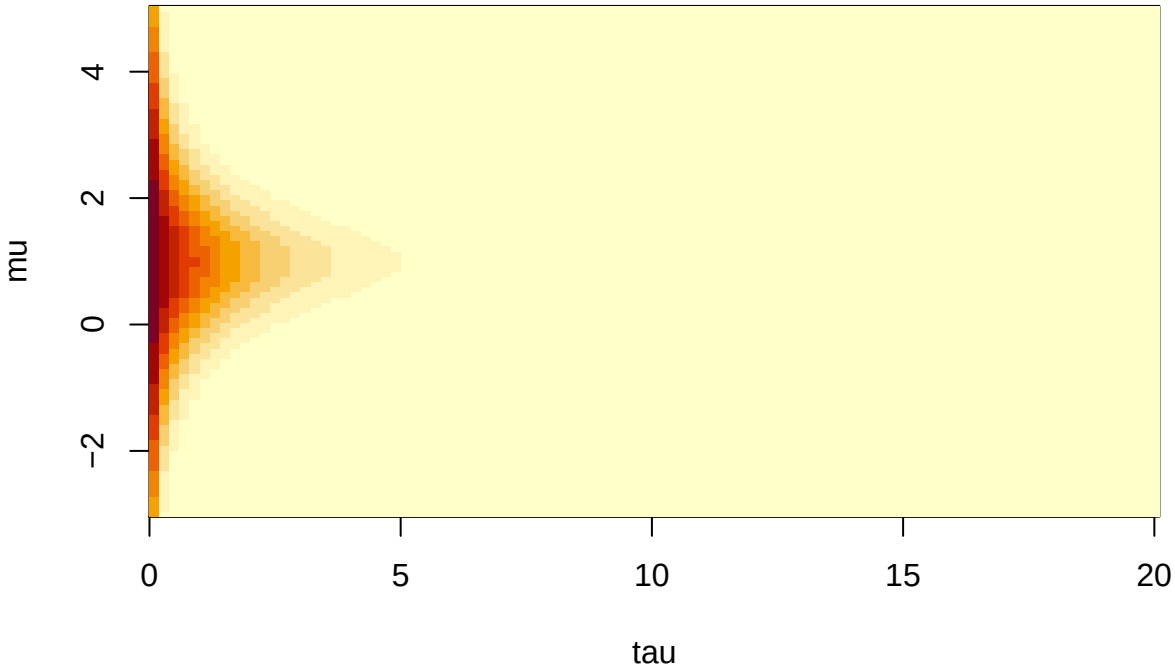

```
# Joint prior distribution of mu and tau
x3 <- seq(.1,20,length=100)
y3 <- seq(-3,5,length=100)
xy <- expand.grid(x3,y3)
z3 <- dgamma(xy[,1],alpha0,beta0)*dnorm(xy[,2],mu0,1/sqrt(lambda0*xy[,1]))
z3mat <- matrix(z3,ncol=100)
persp(x3,y3,z3mat,xlab="tau",ylab="mu",zlab="joint density")
```



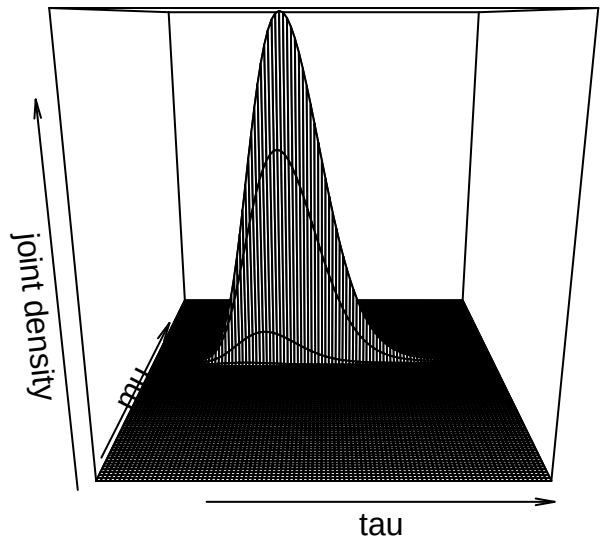

```
contour(x3,y3,z3mat,xlab="tau",ylab="mu")
```



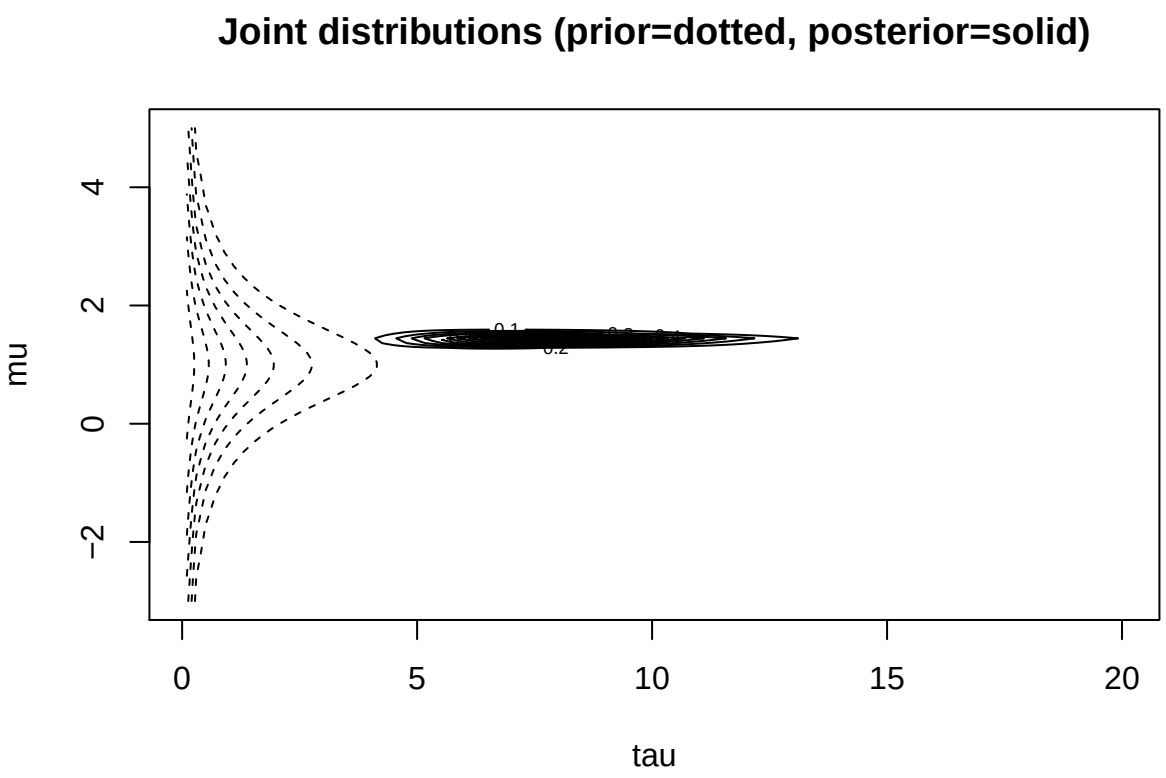

```
image(x3,y3,z3mat,xlab="tau",ylab="mu")
```



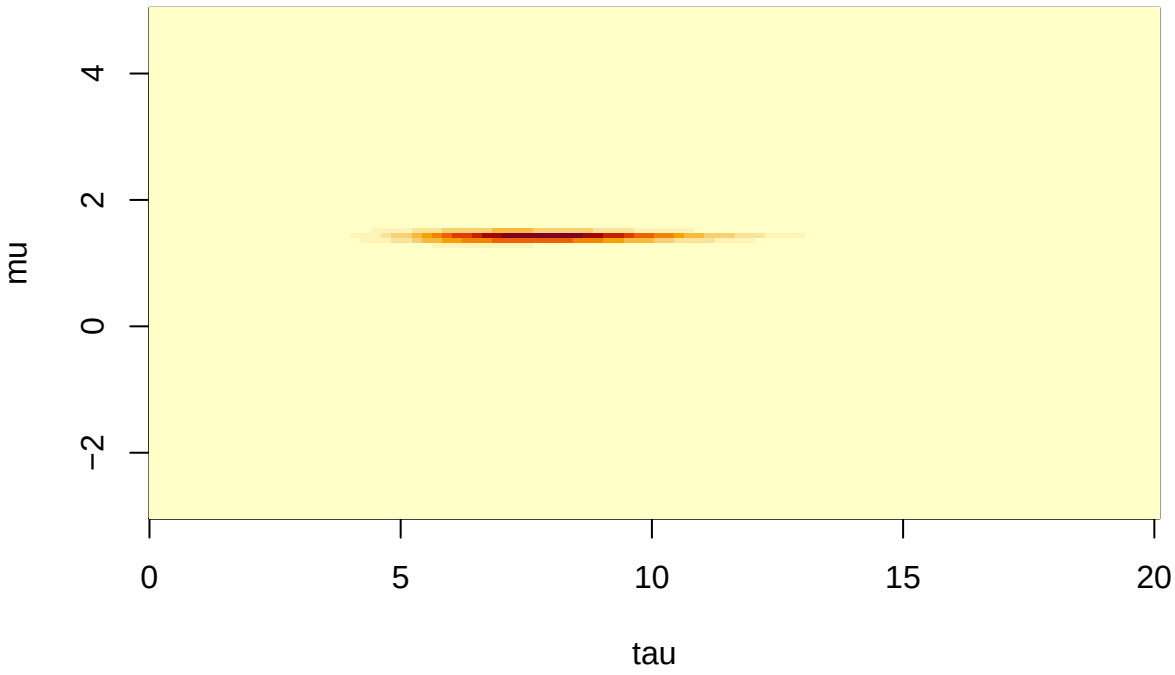

```
# Joint posterior distribution of mu and tau
x6 <- seq(.1,20,length=100)
y6 <- seq(-3,5,length=100)
xy <- expand.grid(x6,y6)
z6 <- dgamma(xy[,1],alpha1,beta1)*dnorm(xy[,2],mu1,1/sqrt(lambda1*xy[,1]))
z6mat <- matrix(z6,ncol=100)
persp(x6,y6,z6mat,xlab="tau",ylab="mu",zlab="joint density")
```



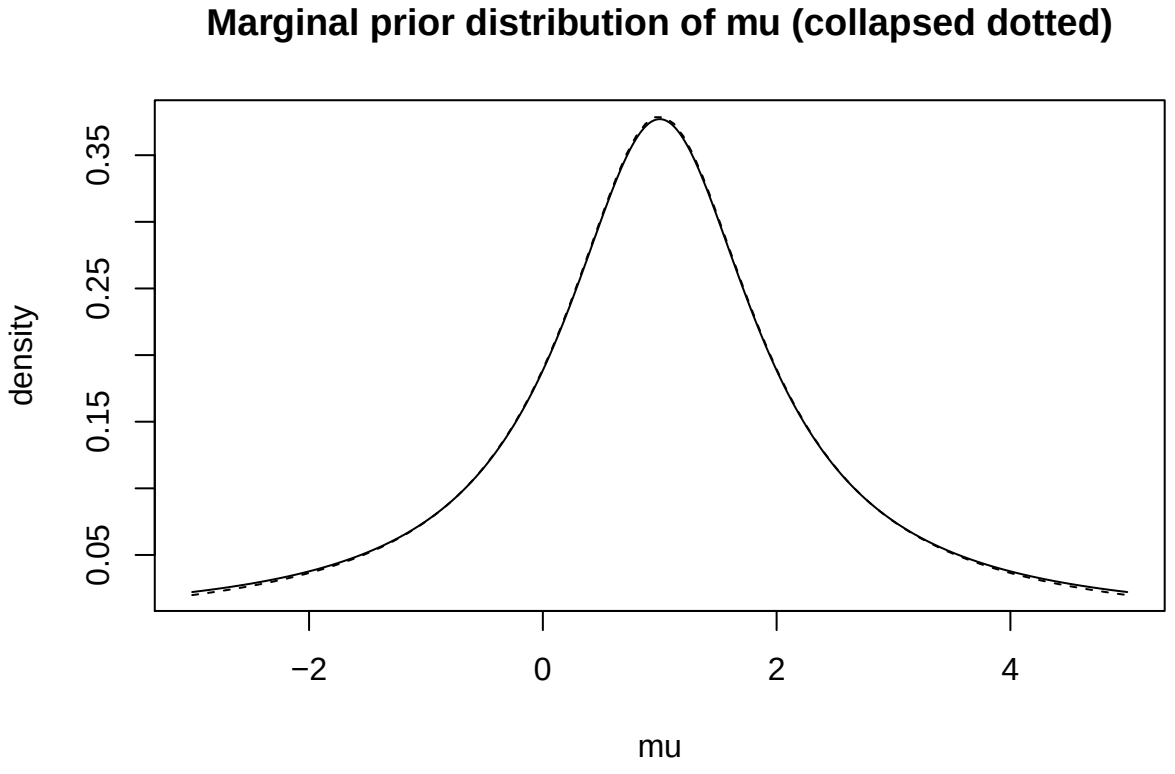

```
contour(x6,y6,z6mat,xlab="tau",ylab="mu",
  main="Joint distributions (prior=dotted, posterior=solid)")
     priorcontour <- contourLines(x3,y3,z3mat)
     templines <- function(clines) {
       lines(clines[[2]], clines[[3]],lty=2)
     }
     invisible(lapply(priorcontour, templines))
```



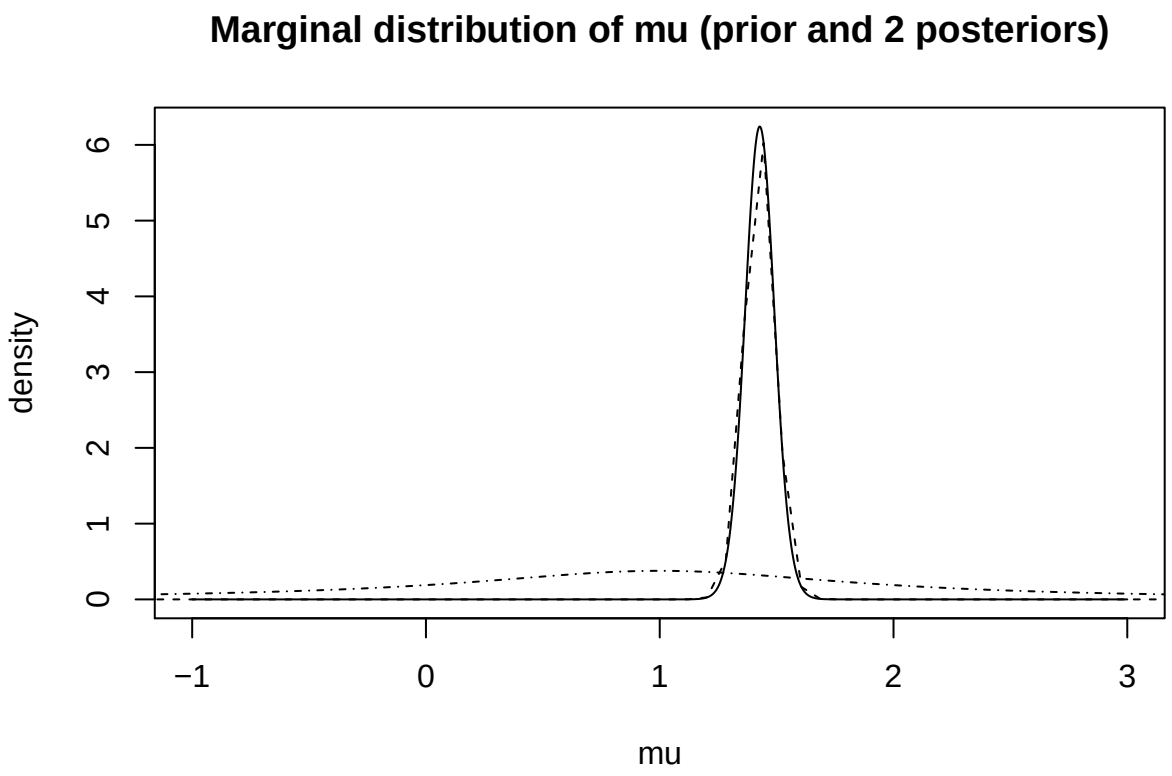

```
image(x6,y6,z6mat,xlab="tau",ylab="mu")
```




```
# Marginal prior for mu based on joint distribution above
marg.mu <- apply(z3mat,2,sum)
#sum across columns of grid/matrix
dy3 <- y3[2]-y3[1]
ht <- marg.mu/(sum(marg.mu)*dy3)
plot(x2,y2/(sum(y2)*dx2),type="l",xlab="mu",ylab="density",
  main="Marginal prior distribution of mu (collapsed dotted)")
lines(y3,ht,lty=2)
```



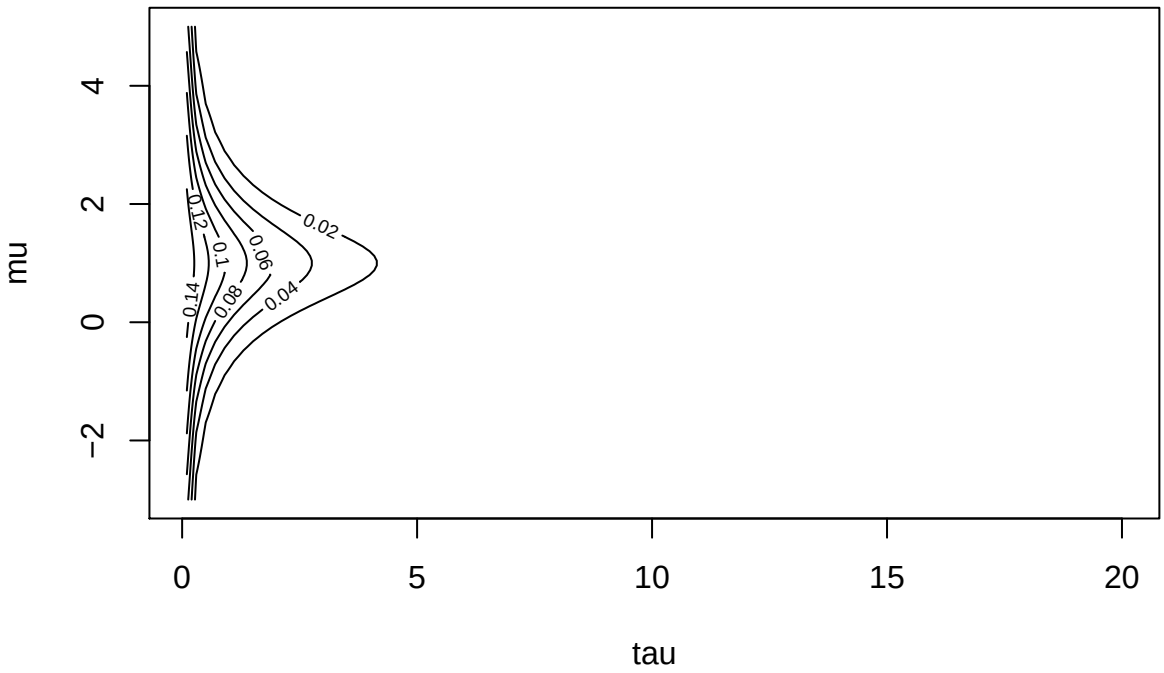
```r
# Marginal posterior for mu based on joint distribution above
marg.mu <- apply(z6mat,2,sum)
dy6 <- y6[2]-y6[1]
ht <- marg.mu/(sum(marg.mu)*dy6)
plot(x5,y5/(sum(y5)*dx5),type="l",xlab="mu",ylab="density",
  main="Marginal distribution of mu (prior and 2 posteriors)")
lines(y6,ht,lty=2)
lines(x2,y2/(sum(y2)*dx2),lty=4)
```



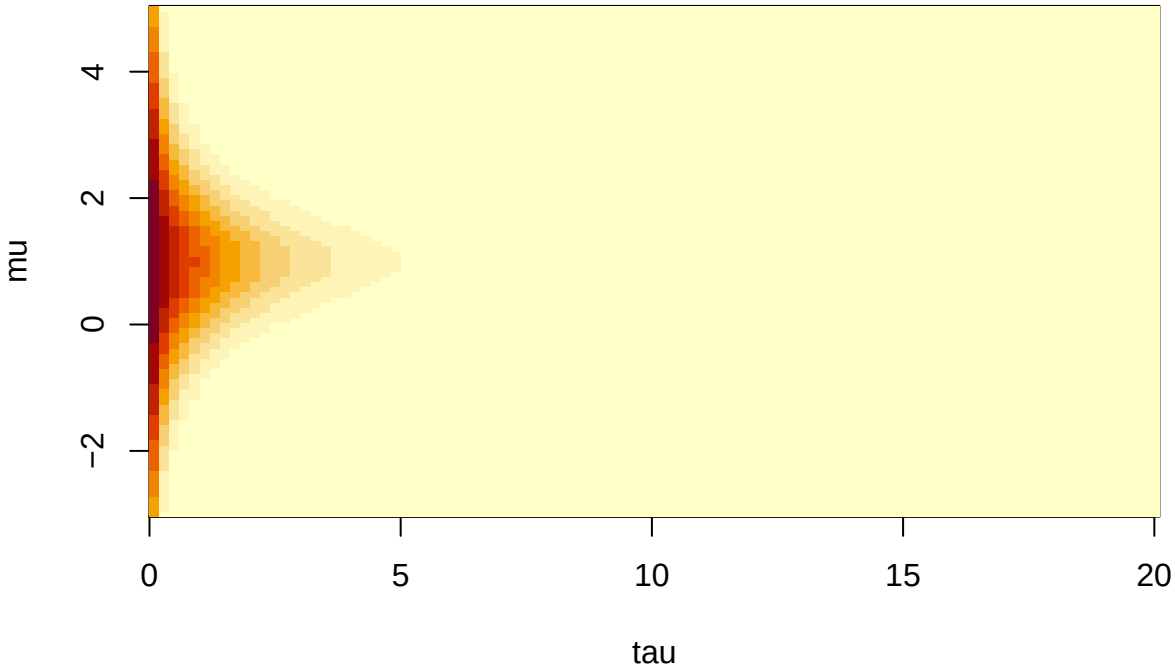

```r
#y2 is marginal prior, y5 is marginal posterior, ht is marginal based on joint)
```

2. a perspective plot (3D surface using `persp()` ) of the joint posterior distribution of $\mu$ and $\tau$
3. prior and posterior joint distributions of $\mu$ and $\tau$ superimposed as contour plots

```r
contour(x3,y3,z3mat,xlab="tau",ylab="mu")
```



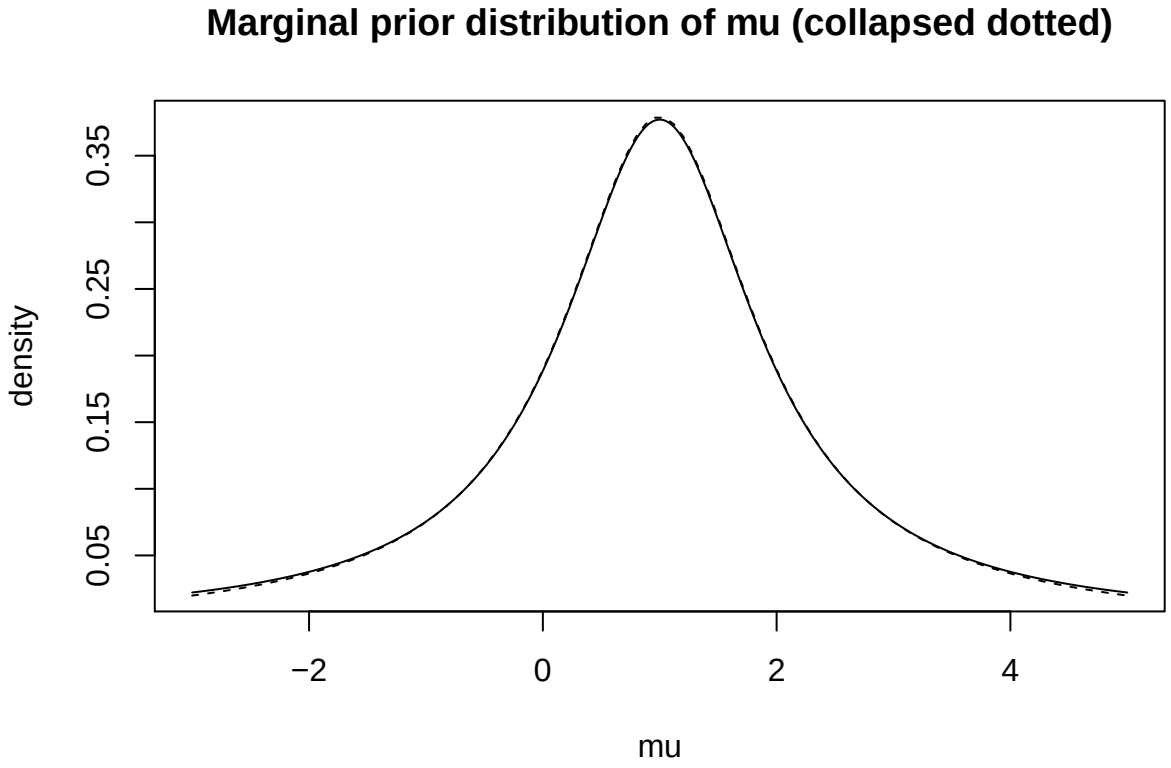

```
image(x3,y3,z3mat,xlab="tau",ylab="mu")
```




4. superimpose the following three plots onto a single set of axes:
    a. marginal prior distribution of $\mu$ based on the distribution of U
    b. marginal posterior distribution of $\mu$ based on distribution of U
    c. marginal posterior for mu based on joint distribution above

```
# Marginal prior for mu based on joint distribution above
marg.mu <- apply(z3mat,2,sum)
#sum across columns of grid/matrix
dy3 <- y3[2]-y3[1]
ht <- marg.mu/(sum(marg.mu)*dy3)
```

```r
plot(x2,y2/(sum(y2)*dx2),type="l",xlab="mu",ylab="density",
  main="Marginal prior distribution of mu (collapsed dotted)")
lines(y3,ht,lty=2)
```




5. 90% probability intervals for $\mu$ calculated:
   a. using known distribution of U
   b. computationally, using marginal posterior of $\mu$ from (4c)

```r
# Prior probability interval for mu
lb <- mu0 - sqrt(beta0/(lambda0*alpha0))*qt(.975,2*alpha0)
ub <- mu0 + sqrt(beta0/(lambda0*alpha0))*qt(.975,2*alpha0)

# Posterior probability interval for mu
lb <- mu1 - sqrt(beta1/(lambda1*alpha1))*qt(.95,2*alpha1)
ub <- mu1 + sqrt(beta1/(lambda1*alpha1))*qt(.95,2*alpha1)

# Posterior probability interval for mu based on marginal posterior
lbnum <- sum(cumsum(ht)/sum(ht) < .05)
lbemp <- y6[lbnum]
ubnum <- sum(cumsum(ht)/sum(ht) < .95)
ubemp <- y6[ubnum+1]

# Compare with 95% confidence interval for mu
t.test(x,conf.level=.90)$conf.int
```

```
## [1] 1.347852 1.536148
## attr(,"conf.level")
## [1] 0.9
```

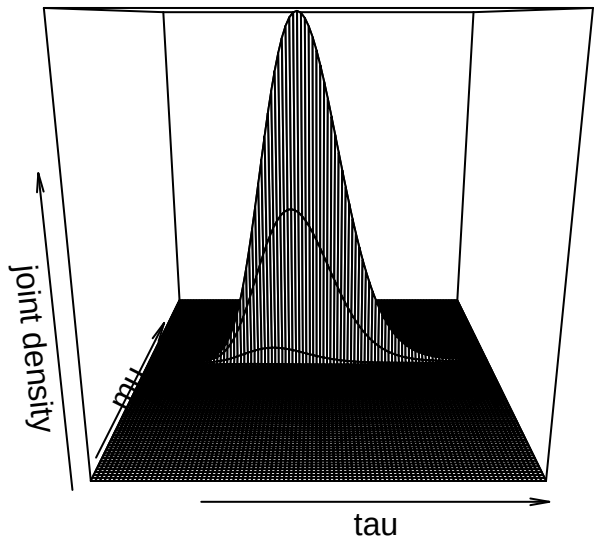
6. Assume that the prior distribution of $\tau$ is Weibull with shape parameter = 1.5 and scale parameter = 5, and the prior distribution of $\mu$ (independent of $\tau$) follows a $t$-distribution with df = 4 and ncp = 1

(ncp is the non-centrality parameter).

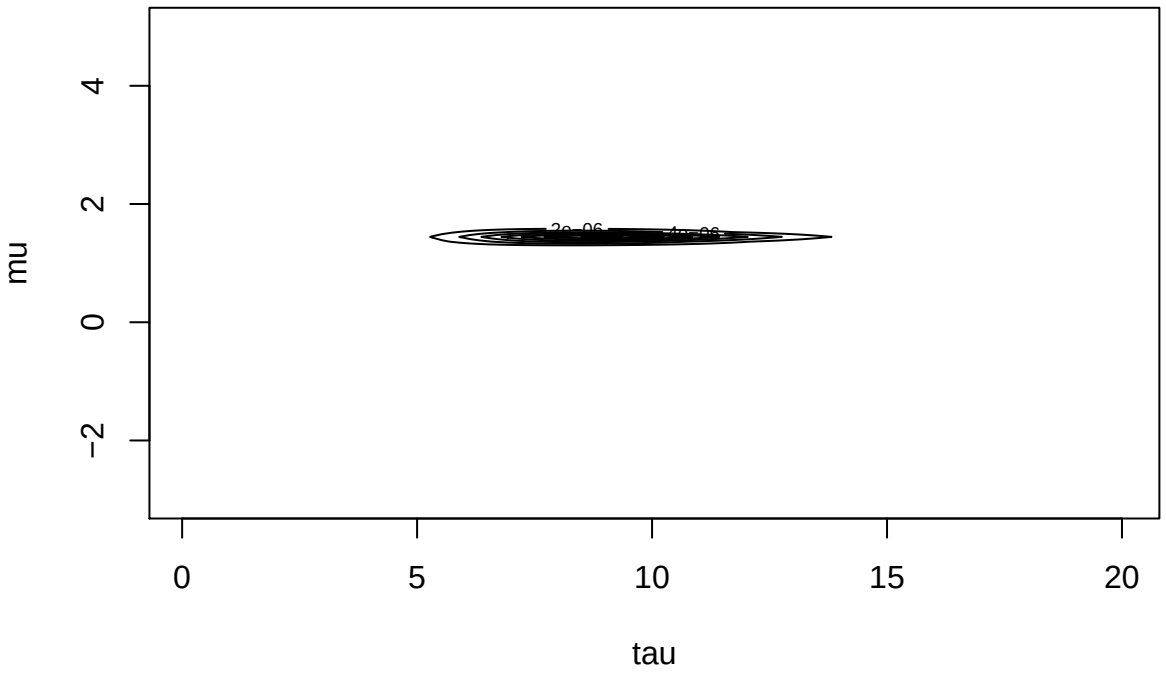
a. Superimpose the marginal posterior distribution of $\mu$ from these new priors along with your marginal posterior from (4b)

b. Superimpose the marginal posterior distribution of $\tau$ from these new priors along with your marginal posterior from (1)

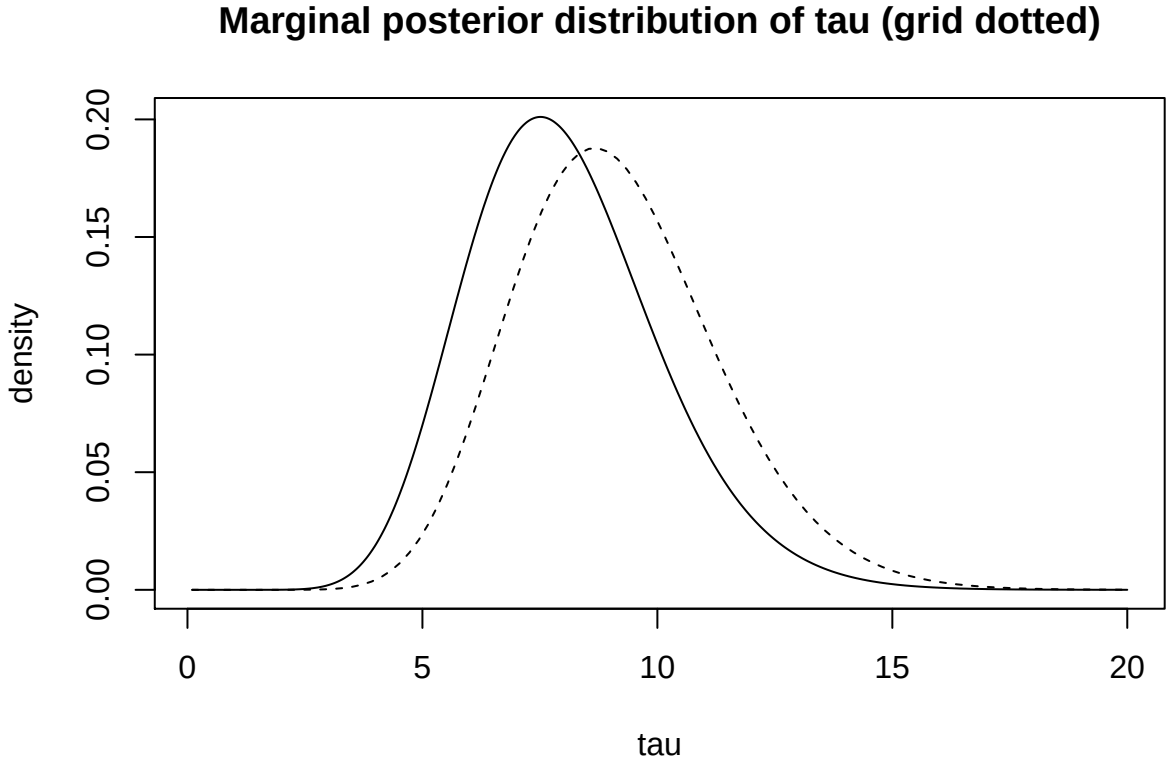
```
sumx <- sum(x)
sumx2 <- sum(x^2)
x7 <- seq(.1,20,length=100)
y7 <- seq(-3,5,length=100)
xy <- expand.grid(x7,y7)
z7 <- dweibull(xy[,1],shape=1.5,scale=5)*dt(xy[,2],df=4,ncp=1)*
  ((xy[,1]/(2*pi))^(n/2))*exp((-xy[,1]/2)*(sumx2-2*xy[,2]*sumx+n*xy[,2]^2))
z7mat <- matrix(z7,ncol=100)
persp(x7,y7,z7mat,xlab="tau",ylab="mu",zlab="joint density")
```




```
contour(x7,y7,z7mat,xlab="tau",ylab="mu")
```

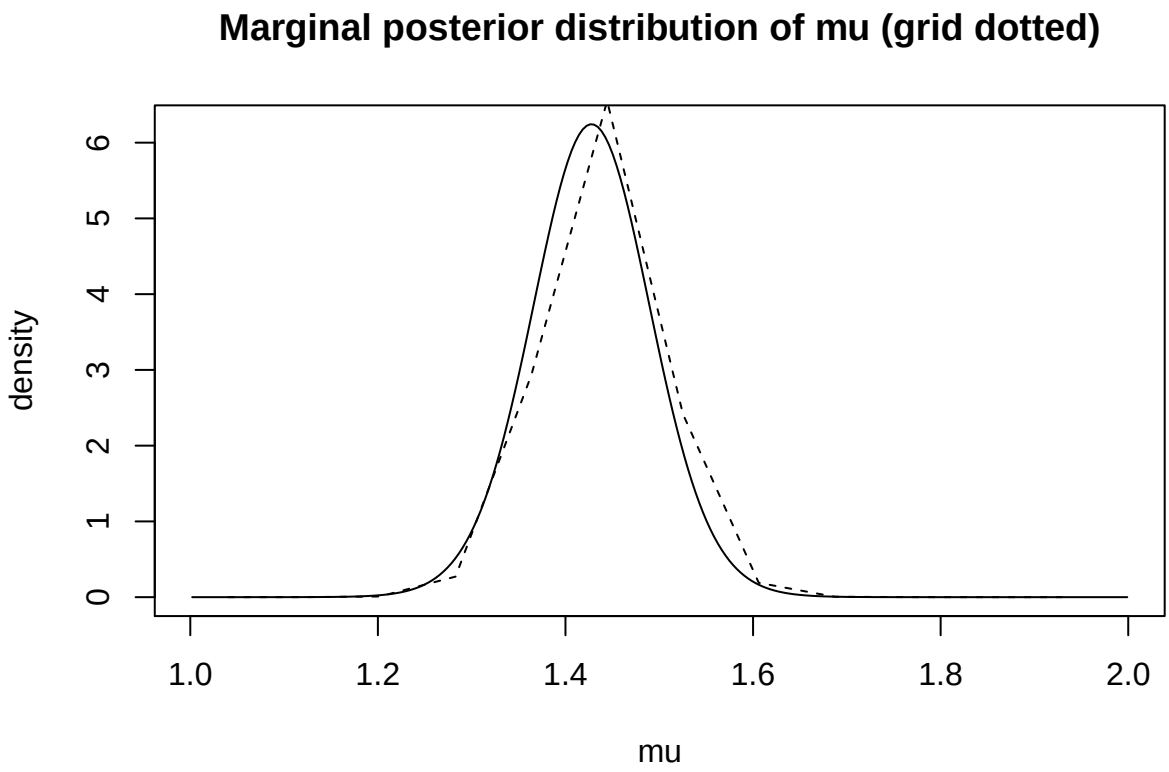



```
marg.tau <- apply(z7mat,1,sum)
dx7 <- x7[2]-x7[1]
ht <- marg.tau/(sum(marg.tau)*dx7)
plot(x4,y4,type="l",xlab="tau",ylab="density",
  main="Marginal posterior distribution of tau (grid dotted)")
lines(x7,ht,lty=2)
```



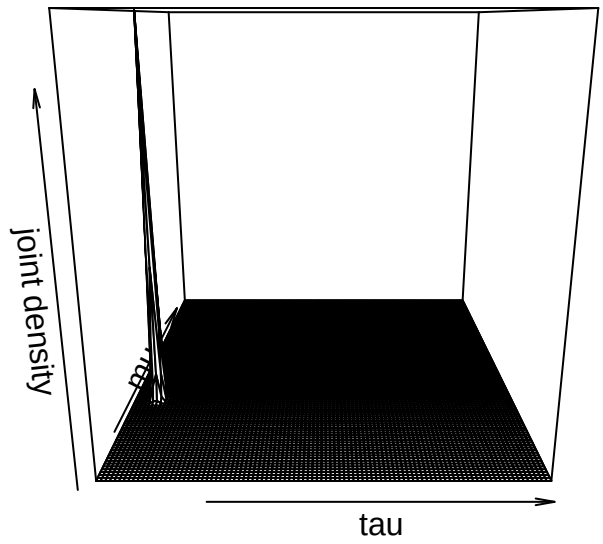

```
marg.mu <- apply(z7mat,2,sum)
```

```
dy7 <- y7[2]-y7[1]
ht <- marg.mu/(sum(marg.mu)*dy7)
plot(x5[x5>1 & x5<2],y5[x5>1 & x5<2]/(sum(y5)*dx5),type="l",
  xlab="mu",ylab="density",
  main="Marginal posterior distribution of mu (grid dotted)")
lines(y7[y7>1 & y7<2],ht[y7>1 & y7<2],lty=2)
```

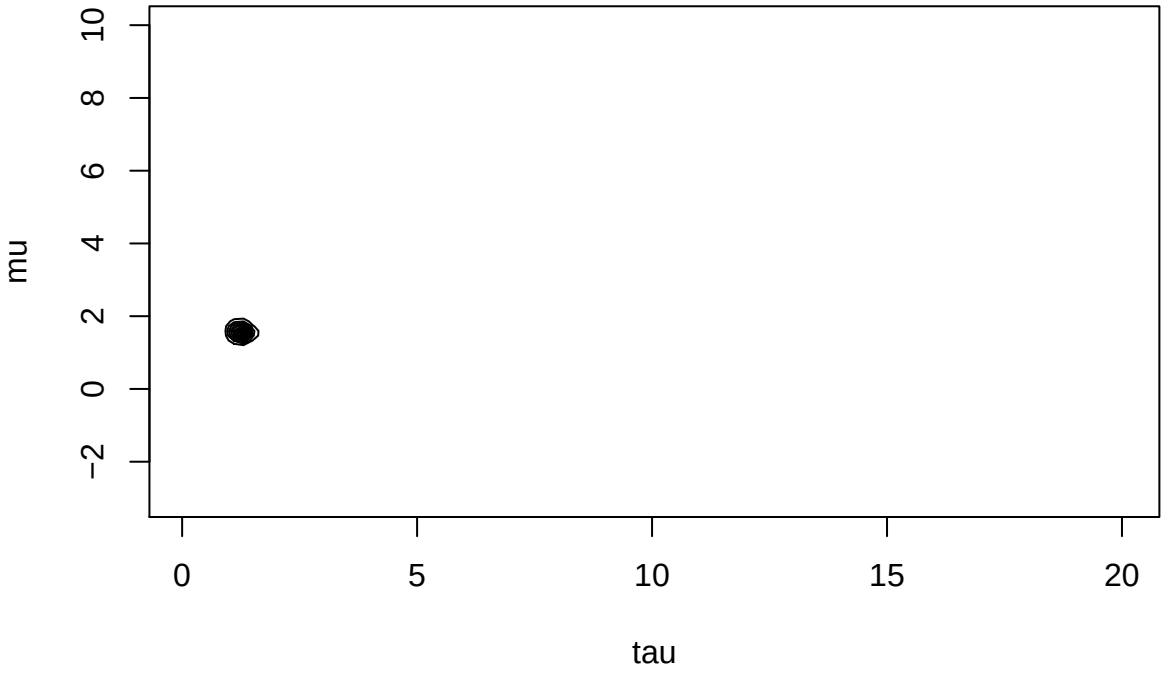



c. Repeat (6) with Weibull parameters (shape=5, scale=1) and $t$ parameters (df=100, ncp=6). Comment on your findings.

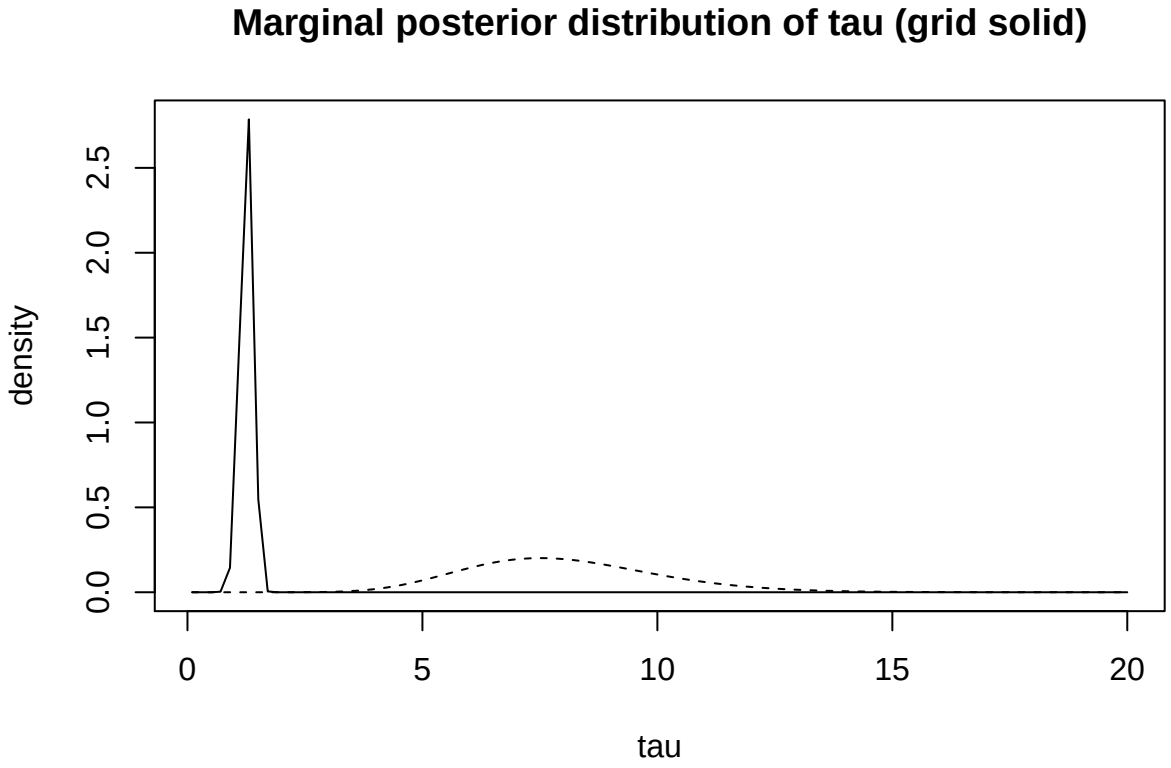
```
# What if use more extreme (more precise) priors
sumx <- sum(x)
sumx2 <- sum(x^2)
x7 <- seq(.1,20,length=100)
y7 <- seq(-3,10,length=100)
xy <- expand.grid(x7,y7)
z7 <- dweibull(xy[,1],shape=5,scale=1)*dt(xy[,2],df=100,ncp=6)*
  ((xy[,1]/(2*pi))^(n/2))*exp((-xy[,1]/2)*(sumx2-2*xy[,2]*sumx+n*xy[,2]^2))
z7mat <- matrix(z7,ncol=100)
persp(x7,y7,z7mat,xlab="tau",ylab="mu",zlab="joint density")
```



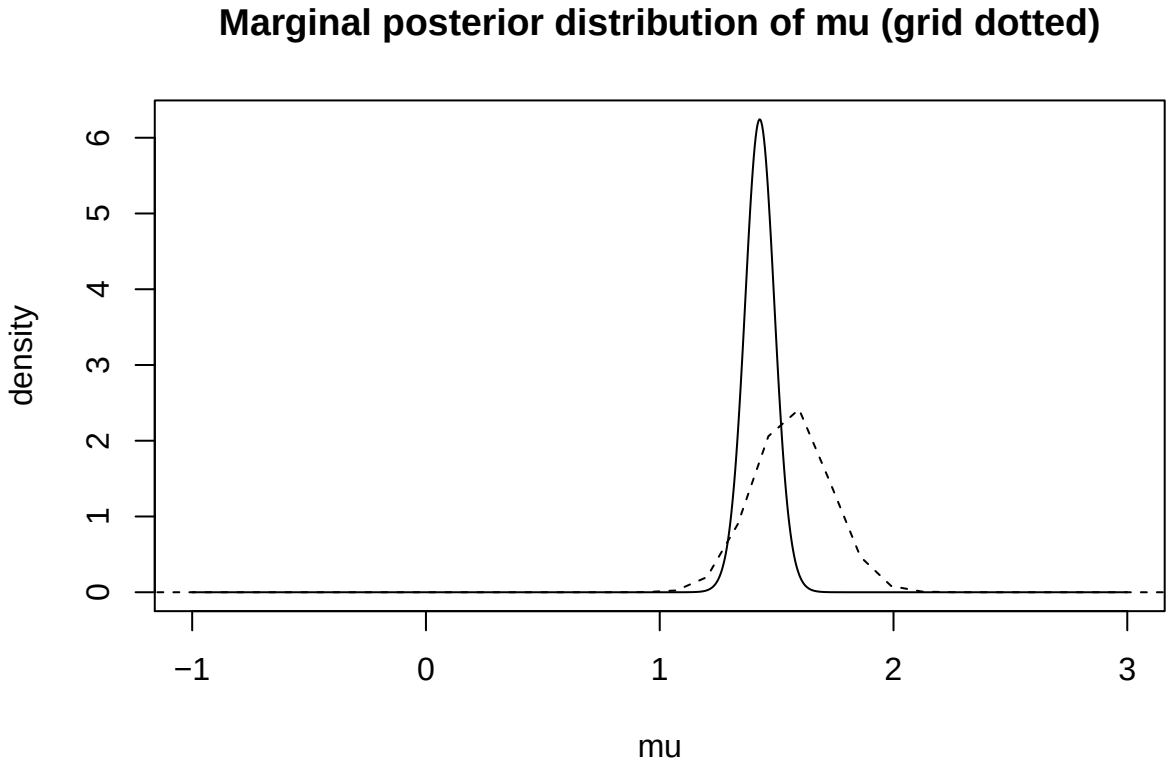

```
contour(x7,y7,z7mat,xlab="tau",ylab="mu")
```




```
marg.tau <- apply(z7mat,1,sum)
dx7 <- x7[2]-x7[1]
ht <- marg.tau/(sum(marg.tau)*dx7)
plot(x7,ht,type="l",xlab="tau",ylab="density",
  main="Marginal posterior distribution of tau (grid solid)")
lines(x4,y4,lty=2)
```




```
marg.mu <- apply(z7mat,2,sum)
dy7 <- y7[2]-y7[1]
ht <- marg.mu/(sum(marg.mu)*dy7)
plot(x5,y5/(sum(y5)*dx5),type="l",
  xlab="mu",ylab="density",
  main="Marginal posterior distribution of mu (grid dotted)")
lines(y7,ht,lty=2)
```


Marginal posterior distribution of mu (grid dotted)
density
0
1
2
3
4
5
6
−1
0
1
2
3
mu