Chapter 8 Simulations

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There are 5 properties from Chapter 8 that we can validate through simulation in R.

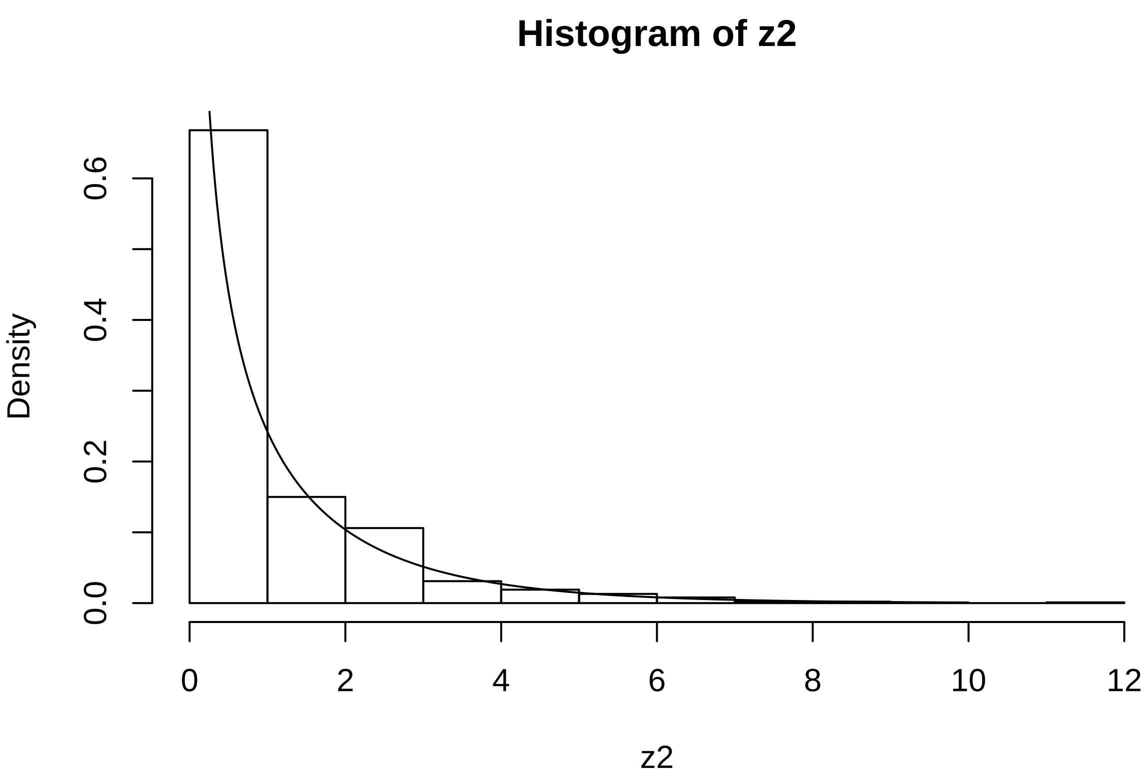
1. If $Z \sim N(0,1)$ then $Z^2 \sim \chi^2_{(1)}$
2. If $Z_1, Z_2, ..., Z_n \sim N(0,1)$ then $\sum_{i=1}^n Z_i^2 \sim \chi^2_{(n)}$
3. If $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$ then $\bar{X}_n$ and $\hat{sigma}^2$ are independent random variables.
4. Confirm the 95% Confidence interval for σ^2.

[In class we showed that a 95% CI for σ was $(\sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}}), (\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}})$ where s^2 is the usual variance cacluation dividing by $(n-1)$.]

5. Confirm that $\frac{\bar{X_n}-\mu}{s/\sqrt{n}} \sim t_(n-1)$. Note that $\hat{sigma}^2 = \frac{(n-1)}{n}s^2$

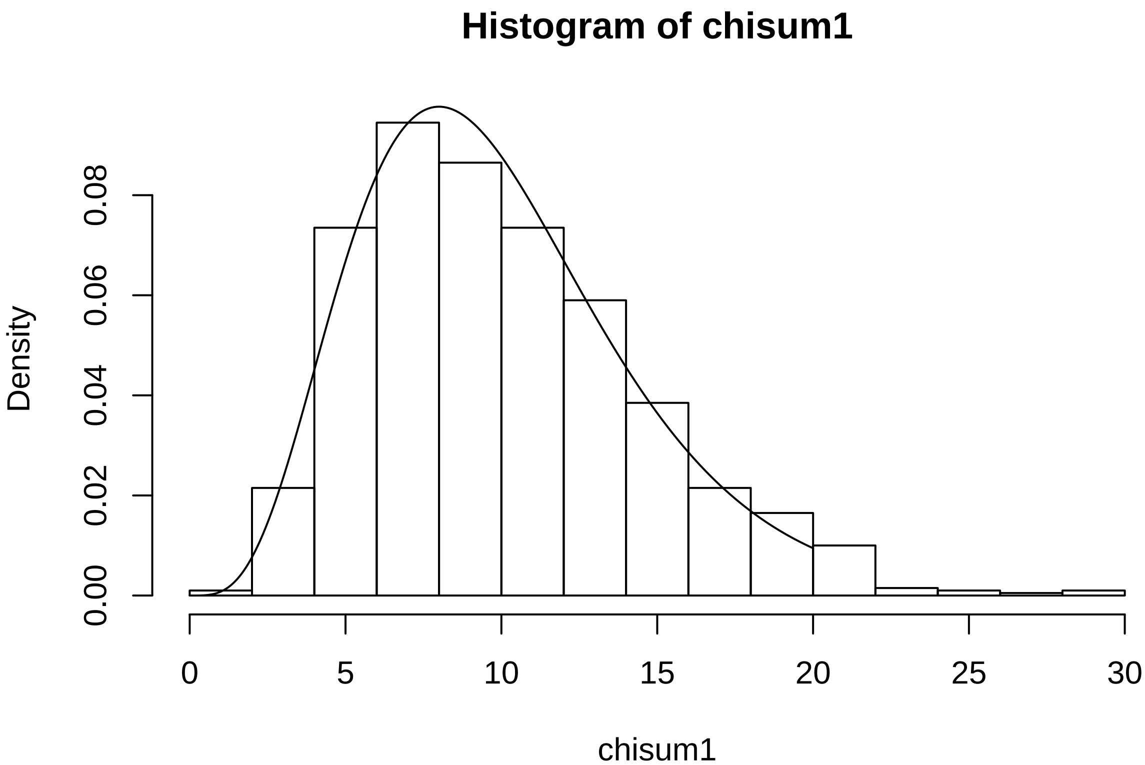
1.

```
z <- rnorm(1000,mean=0,sd=1) #generate 1000 normals
z2 <- z*z #square them
x <- seq(0,10,length=1000)
chi <- dchisq(x,df=1)
hist(z2,probability=TRUE) # comapare the historgram of the random normals
lines(x,chi) # to the chi-squre with 1 df distribution
```

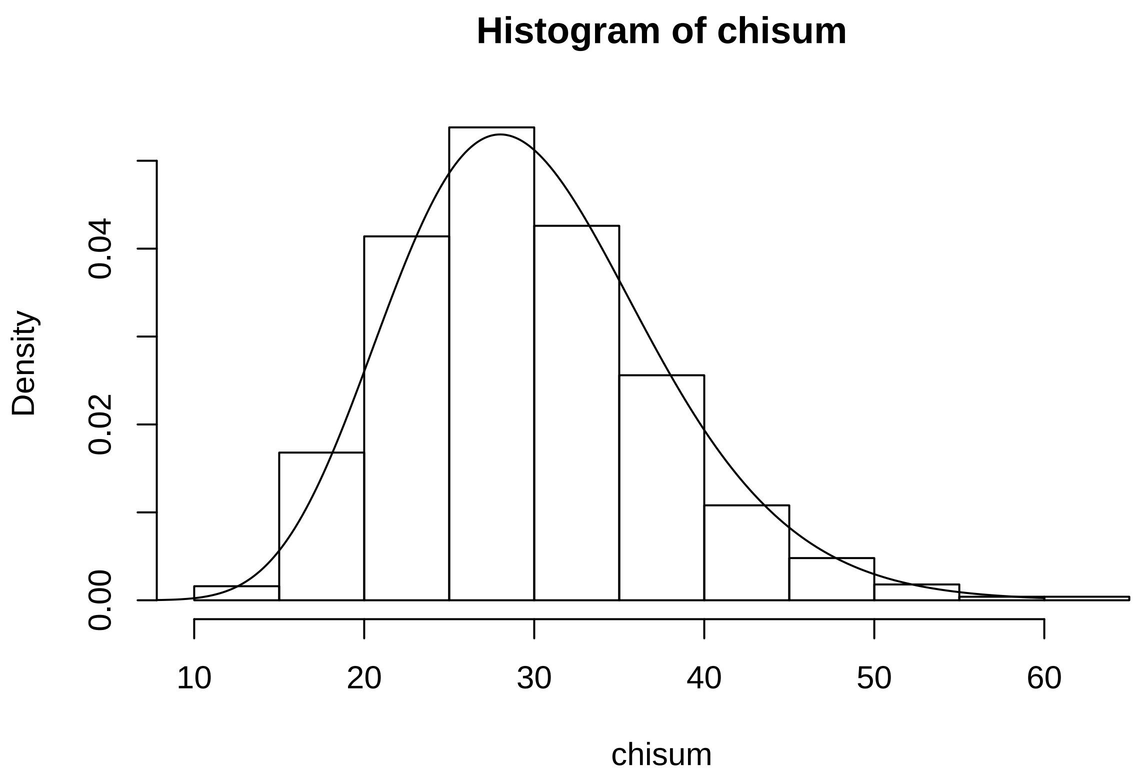


2.

```
chi1 <- rchisq(10000,df=1) #since a normal squared is chi-square we can jsut start there
chimat1 <- matrix(chi1,ncol=10) # since we want to sum a sequence of z^2 we can set this up in a matrix
chisum1 <- apply(chimat1,1,sum) # and then sum columns
x <- seq(0,20,length=1000)
chi10 <- dchisq(x,df=10) # the sum of 10 chi(1) will be chi(10)
hist(chisum1,probability=TRUE)
lines(x,chi10)
```




```
#a twist
chi <- rchisq(10000,df=3) #alternatively we can start with generating 10000 chi-squares with 3 df for f
chimat <- matrix(chi,ncol=10)
chisum <- apply(chimat,1,sum)
x <- seq(0,60,length=1000)
chi30 <- dchisq(x,df=30)
hist(chisum,probability=TRUE)
lines(x,chi30)
```



3.

```
z <- rnorm(10000,mean=0,sd=1)
zmat <- matrix(z,ncol=10)
zmean <- apply(zmat,1,mean)
zvar <- apply(zmat,1,var)
zsigma2 <- 9*zvar/10
cor(zmean,zsigma2)
```

```
## [1] -0.002553263
```

4.

```
x <- rnorm(10000,mean=20,sd=5)
xmat <- matrix(x,ncol=10)
xvar <- apply(xmat,1,var)
xsig2hat <- 9*xvar/10
lb <- 10*xsig2hat/qchisq(.975,df=9)
ub <- 10*xsig2hat/qchisq(.025,df=9)
sum(lb<25 & ub>25)/1000
```

```
## [1] 0.95
```

5.

```
x <- rnorm(5000,mean=20,sd=5)
xmat <- matrix(x,ncol=5)
```

```r
xmean <- apply(xmat,1,mean)
xsd <- apply(xmat,1,sd)
tstat <- (xmean-20)/(xsd/sqrt(5))
z <- seq(-3,3,length=1000)
znorm <- dnorm(z,mean=0,sd=1)
par(mfrow=c(1,2))
hist(tstat,probability=TRUE,ylim=c(0,max(znorm)))
lines(z,znorm)
qqnorm(tstat)
```