1) Golden numbers are known as integers whose integer divisors sum up to the numbers themselves. For example, 28 is a golden number, because $1 + 2 + 4 + 7 + 14 = 28$. Find all golden numbers up to $10,000$.

2) Search the web for happy numbers. Write a program that finds happy numbers up to a specified integer.

3) An approximation to compute the square-root of a number is as follows. Suppose N is your number, and you want to approximate $\sqrt{N}$. A well-known approximation is $NG = \frac{1}{2}(FG + \frac{N}{FG})$, where FG: first guess, NG: next guess. Write a function that takes N as an argument, performs this approximation until 0.1% accuracy is achieved, and reports the number of iterations that are needed to achieve this accuracy, exact root and approximated root.

4) Let x and y denote the lengths of the legs of a right triangle and z the length of its hypotenuse. Then, by the Pythagorean theorem, x, y and z satisfy the diophantine equation $x^2 + y^2 = z^2$. The positive integer triplet $x - y - z$ is called a **Pythagorean triple**. Thus, the task of finding all Pythagorean triangles is the same as that of finding all Pythagorean triples. Find all unique Pythagorean triples such that $z \leq 100$.

5) Let n be a four-digit decimal integer, with not all digits the same. Let $n^{'}$ and $n^{''}$ be the integers obtained by arranging the digits of n in nondecreasing and nonincreasing orders, respectively. Define $K(n) = n^{'} - n^{''}$. For example, $K(1995) = 9951 - 1599 = 8352$. When one repeats this process, within a maximum of eight steps, a MAGIC number will be found. Find this totally and absolutely fantastic magical number which is known as Kaprekar's constant in mathematics community.

6) 2 and 3 are the only two consecutive integers that are primes. Are there any primes that differ by 2? Clearly, 3 and 5, and 5 and 7 are two such pairs. Such pairs are called **twin primes**. Find all twin primes that are less than 100.

7) A palindromic number is a symmetrical number like 16461, that remains the same when its digits are reversed. Find all **palindromic primes** up to 500.

8) A **Cunningham chain** of primes is a sequence of primes $2p + 1$ in which each element is one more than twice its predecessor. The smallest five-element chain is 2-5-11-23-47. What is the smallest six-element chain? (Hint: All last digits of the numbers in this chain are 9.)

9) A **cyclic prime** is a prime such that every cyclic permutation of its digits yields a prime. For example, 79 and 97 are cyclic primes. Find all cyclic primes up to 1000.

10) A **reversible prime** is a prime that yields a prime when read from right to left. For instance, 113 is a reversible prime. Find all reversible primes up to 1000.

11) The Chevalier de Mere was a seventeenth century French man who made money by betting –at even money– that he could throw at least one six in four die rolls. Eventually, he could not find anyone to bet against him so he changed the bet to at least one double six in twenty four rolls of two dice, and started losing money. Explain why by devising a simulation study. (Hint: Calculate the probability of at least one six in four rolls analytically, you will see that it is greater than 0.5. That is why he was making money in the first bet. Similarly, calculate the probability of at least one double six in twenty four rolls analytically. This time probability is less than 0.5).

12) Suppose that we toss a fair coin twenty times. What is the probability that we get a sequence of exactly four heads in a row at some point? Write an R function for this and report the result. Note that your function should be a general one that takes the number of tosses and the length of the sequence as input arguments. For example, you should be able to compute the probability that we get a sequence of exactly (say) six tails when we toss a coin (say) 50 times.

13) Suppose that a row of mailboxes are numbered 1 through 150 and that beginning with mailbox 2, we open the doors of all the even-numbered mailboxes. Next, beginning with mailbox 3, we go to every third mailbox, opening its door if it is closed and closing if it is open. We repeat this procedure with every fourth mailbox, then every fifth mailbox, and so on. Write a program to determine which mailboxes will be closed when this procedure is completed. If you did not say "aha, now I see what is going on" in the end, stop doing science and do something else.

14) The famous *Buffon Needle problem* is as follows: A board is ruled with equidistant parallel lines, and a needle whose length is equal to the distance between these lines is dropped at random on the board. What is the probability that it crosses one of these lines? The answer to this problem is a simple function of π. Write a program to simulate this experiment and obtain an estimate for π.

15) A magic square is an $n \times n$ array in which all integers 1, 2,..., n^2 appear exactly once, and all column sums, row sums, and diagonal sums are equal. The following is a procedure for constructing an $n \times n$ magic square for any odd integer n. Place 1 in the middle of the top row. Then after placing integer k, move up one row and one column to the right to place the next integer $k + 1$, unless one of the following occurs:

a) If a move takes you above the top row in the j^{th} column, move to the bottom of the j^{th} column and place the integer there.

b) If a move takes you outside to the right of the square in the i^{th} row, place the integer in the i^{th} row at the left side.

c) If a move takes you to an already filled square or if you move out of the square at the upper right-hand corner, place $k + 1$ immediately below k.

Write a program to construct a magic square for any odd value of n.

16) Given two positive integers (say A and B), write a program that finds the greatest common divisor (GCD) and least common multiple (LCM) of these two integers. Note that $GCD * LCM = A * B$. You are only allowed to use this fact for verification purposes. The short cut solution such as finding only one of these, and compute the other by the above formula is unacceptable.

17) Design a simulation study to investigate the *birthday problem*: If there are n persons in a room, what is the probability that two or more of them have the same birthday? Plot n versus corresponding probabilities for $8 \leq n \leq 50$. For simplicity, assume that February 29^{th} does not exist.

18) Design a simulation study for the famous *Monty Hall* problem. Internet is full of resources. For example, see $http://mathworld.wolfram.com/MontyHallProblem.html$.

19) In mathematics, sexy primes are prime numbers that differ from each other by six. For example, the numbers 5 and 11 are both sexy primes, because they differ by six. The term "sexy prime" stems from the Latin word for six: sex. There are seven sexy prime quadruplets below 1000. One of them is $(5, 11, 17, 23)$. Find the other six quadruplets.

20) A truck travels from X to Y. Going uphill, it goes at 56 mph. Going downhill, it goes at 72 mph. On level ground, it goes at 63 mph. If it takes 4 hours to travel from X to Y, and 5 hours to come back, what is the distance between X and Y?

21) In number theory, a sphenic number is a positive integer which is the product of three distinct prime numbers. Find all sphenic numbers that are less than 1000.

22) In mathematics, the n^{th} taxicab number, typically denoted $Ta(n)$, is defined as the smallest number that can be expressed as a sum of two positive algebraic cubes in n distinct ways. Clearly, $Ta(1) = 2 = 1^3 + 1^3$. Find $Ta(2)$ and $Ta(3)$. It is also acceptable for you to find all second order taxicab numbers that are less than $50,000$. Take your pick.

23) A father, in his will, left his money to his children in the following manner: 1000 USD to the first born and 1/10 of what then remains, then 2000 USD to the second born and 1/10 of what then remains, then 3000 USD to the third born and 1/10 of what then remains, and so on. When this was done each child had the same amount. How many children were there?

24) Arrange the digits $1 - 9$ into a 9-digit number such that for each N from 1 to 9, the first N digits of the number are divisible by N.

25) Leon decided to sell his collection of books. To Eric, he sold 2 books, and one fifth of what was left. Later to Anubhav he sold 6 books, and one fifth of what then remained. If he sold more books to Eric than to Anubhav, what was the least possible number of books in his original collection?

26) Within the realm of multiples of three we have the following: If you sum the cubes of the digits of such a number you get another number which is also a multiple of three. If you iterate the process of summing cube of digits, you ultimately arrive at a magical number, which equals the sum of the cubes of its own digits. What is this number?

27) The ratio of the number of apples to oranges to pears is $7 : 11 : 9$. Anubhav ate 21 fruits. As a result, the ratio of the number of apples to oranges to pears became $2 : 3 : 3$. How many fruits were left? How many apples did Anubhav eat?

28) Show that the ratio $\frac{a^2+b^2}{ab+1}$ is a square when it is an integer.

29) A gorilla harvests 3000 bananas and needs to carry them 1000 miles to the supermarket. He can only carry 1000 at a time. Since he is a gorilla he eats one banana every mile he goes in any direction. He can (and will have to) leave bananas anywhere along the way. Once all his bananas have reached the end he DOES NOT need any to eat to get back. Remember he eats one banana every mile he goes even if he is going back to pick up more bananas. What is the maximum number of bananas he can get to the market?

30) Lian added a double digit number and a single digit number for the heck of it. Donghong used the same numbers, but instead of adding them she multiplied them. During their discussion in the class break they shockingly realized that what they found was the same except that resulting numbers are reversed. They thought it might be a unique incidence. They asked Angle to take a picture of theirs that they would keep for eternity. Suddenly, Anubhav and Leon discovered another pair of numbers (one single digit, one double digit) that has the same property. The professor and everybody else had a big, fat grin on their faces. They all happily lived ever after. What are these pairs?

31) Find all Fibonacci numbers that are less than $1,000$ and find the golden ratio of two consecutive Fibonacci numbers.

32a) A weird number is a natural number that is abundant but not semiperfect. In other words, the sum of the proper divisors (divisors including 1 but not itself) of the number is greater than the number, but no subset of those divisors sums to the number itself. There are seven weird numbers less than $10,000$. Find them.

32b) An n-digit number that is the sum of the n^{th} powers of its digits is called a narcissistic number. For example, 153 is a narcissistic number, because $153 = 1^3 + 5^3 + 3^3$. There are six other non-trivial (non-single digit) narcissistic numbers less than $10,000$. Find them.

33) A Kaprekar number for a given base is a non-negative integer, the representation of whose square in that base can be split into two parts that add up to the original number again. For instance, 45 is a Kaprekar number, because $45^2 = 2025$ and $20 + 25 = 45$. More formally, let X be a non-negative integer. X is a Kaprekar number if there exist non-negative integers n, A, and positive number B satisfying $X^2 = A * 10^n + B$, where $0 < B < 10^n$, and $X = A + B$. There are 17 Kaprekar numbers less than $10,000$. Find them.

34) There are only two integers on the face of earth, whose square has no isolated digits (For example, 1255 has isolated digits, but 2255 does not). Only one of them is less than 1,000,000. Find this magical number. Make a connection to music. Good math is musical, good music is mathematical.

35) Imagine we have a $3 * 3$ square that has nine subsquares. The middle subsquare is unavailable. This means it has eight slots to put numbers. Our numbers are 3, 5, 7, 11, 13, 17, 19, 23 (first eight primes greater than 2). Put these numbers to subsquares so that sums of numbers in all full columns and rows (there are obviously four of them in total) are the same. Note that each number must be used exactly once.

36) In chess, queens can attack horizontally, vertically, and diagonally. How can seven queens be placed on an $7x7$ chessboard so that no two of them attack each other?

37) If you break a stick at two points, what is the probability the three resulting sticks form a triangle?

38) In the following words, every letter represents a number: SEND+MORE=MONEY. Find the correspondence between letters and numbers.

39) Watch the video at $https://www.youtube.com/watch?v = 3RfYfMjZ5w0$ and write a program that tells this story. It is Ok if you do not go up to extraterrestrially large numbers.

40) Design and run a simulation study concerning sample size and statistical power. You can pick any parameter in any setting, and make any assumptions you want. Homework problems are designed to improve your algorithmic thinking skills. You need to be creative and convincing. You will face this problem at some point of your career sooner or later.