Quantum Fourier Transform

Group 8

Little Background...

- Use a Quantum Computer
- Qubit vs Bit (2^n vs 1)
- Discrete Fourier Transform

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j \omega_N^{jk} \text{ where } \omega_N^{jk} = e^{2\pi i \frac{jk}{N}}.$$

- Fast Fourier Transform

$$X_k = \sum_{m=0}^{N/2-1} x_{2m} \cdot e^{-i\, 2\pi\, k\, m\, /\, (N/2)} + e^{-i\, 2\pi\, k\, /\, N} \sum_{m=0}^{N/2-1} x_{2m+1} \cdot e^{-i\, 2\pi\, k\, m\, /\, (N/2)}$$

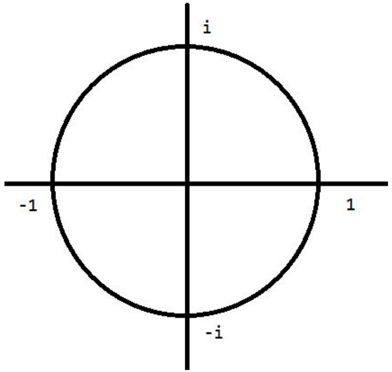
QFT

- Quantum solution of DFT
- $\sum_{i=0}^{N-1} x_i|i\rangle \rightarrow \sum_{i=0}^{N-1} y_i|i\rangle$ using DFT

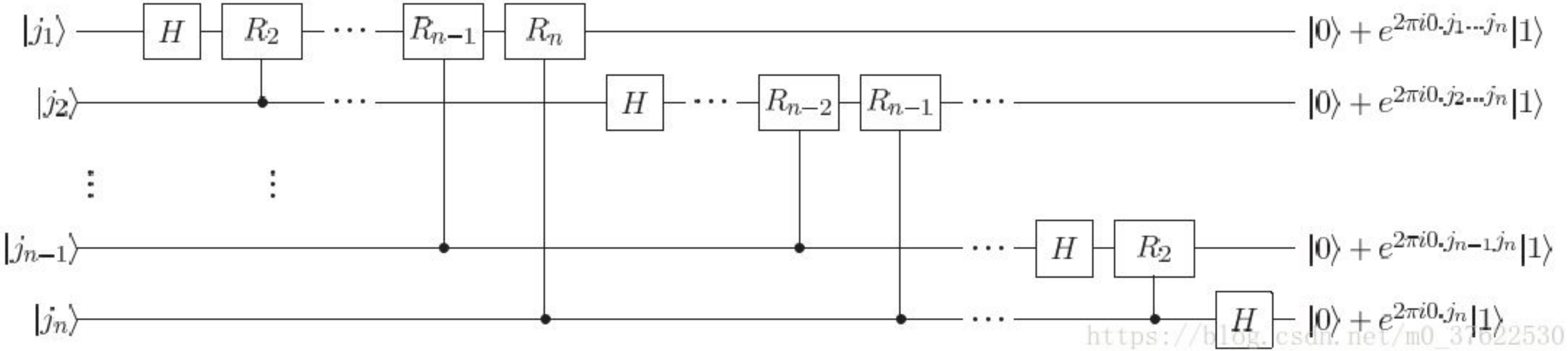
$$|\text{State in Computational Basis}\rangle \xrightarrow{\text{QFT}} |\text{State in Fourier Basis}\rangle$$

$$\text{QFT}|x\rangle = |\tilde{x}\rangle$$

- Computational Basis (Z axis)
- Fourier basis



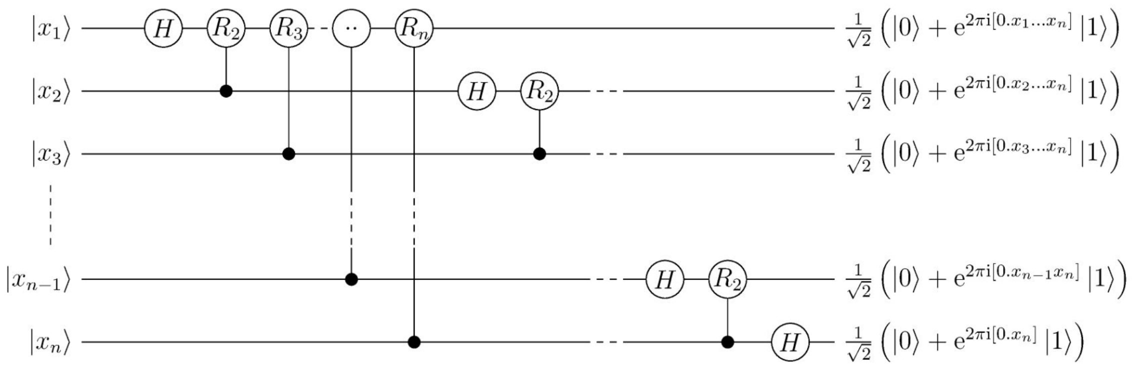
Quantum Circuit



H -> Hadarmard Operator

Ri -> $\begin{bmatrix} 1 & 0 \\ 0 & \exp\left(\frac{2\pi i}{2^k}\right) \end{bmatrix}$

Speed Comparison



n(n+1)/2 H and Ri gates!

Complexity

QFT O((log2N)^2)

FFT *O*(*Nlog*2*N*)

Verify the correctness of QFT algorithm

QFT = iFFT

- Applying QFT to a vector corresponds to applying inverse FFT to it

- Check wether the wavefunction of QFT is the same to that of the iFFT

- If same, we can verify the correctness of QFT algorithm.

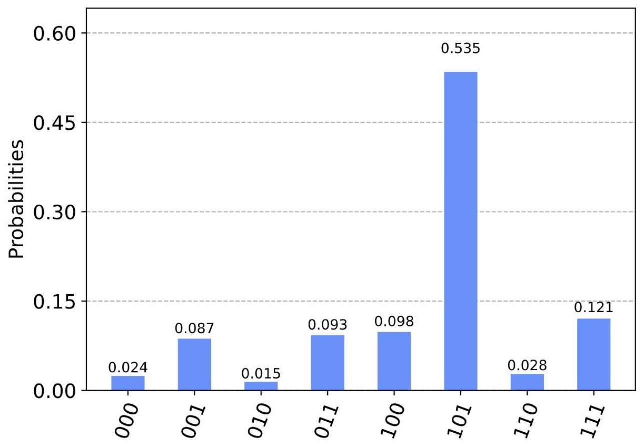
Experiments and Results

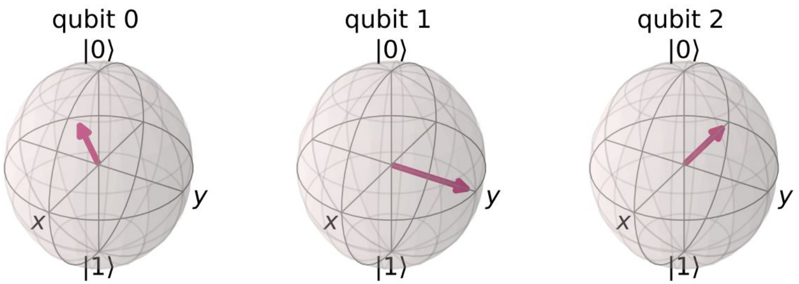
```
{'counts': {'0x0': 1},
 'statevector': array([ 3.53553391e-01-4.32978028e-17j,  2.50000000e-01+2.50000000e-01j,
         6.49467042e-17+3.53553391e-01j, -2.50000000e-01+2.50000000e-01j,
        -3.53553391e-01+4.32978028e-17j, -2.50000000e-01-2.50000000e-01j,
        -6.49467042e-17-3.53553391e-01j,  2.50000000e-01-2.50000000e-01j])}
```

```
array([ 0.35355339+0.j        ,  0.25      +0.25j      ,
        0.        +0.35355339j, -0.25      +0.25j      ,
       -0.35355339+0.j        , -0.25      -0.25j      ,
        0.        -0.35355339j,  0.25      -0.25j      ])
```

Experiments and Results

```
counts = job.result().get_counts()
plot_histogram(counts)
```





Conclusions

- Not meaningful to test real running time of QFT
- Our test results prove the correctness of algorithm

Reference

https://blog.csdn.net/m0_37622530/article/details/83215945

https://qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html