Notes of EFT LSS Multi-tracer

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December 15, 2021

Abstract

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1 Biased single-tracer in redshift space

Expand SPT kernals up to 3rd-order with counter-terms and stochastic terms:

$$\delta_{A,r} = \delta_{A,r}^{(1)} + \delta_{A,r}^{(2)} + \delta_{A,r}^{(3)} + \delta_{A,r}^{(3,\mathrm{ct})} + \delta_{A,r}^{(\varepsilon)} \tag{1}$$

where

$$\delta_{A,r}^{(n)}(\vec{k}) = \int d^3q_1 \ldots d^3q_n K_{A,r}^{(n)}(\vec{q}_1, \ldots, \vec{q}_n)_{\mathrm{sym}} \delta_D^3(\vec{k} - \vec{q}_1 \ldots - \vec{q}_n) \delta^{(1)}(\vec{q}_1) \ldots \delta^{(1)}(\vec{q}_n) \tag{2}$$

The kernal $K_{A,r}^{(1)}(\vec{q}_1), K_{A,r}^{(2)}(\vec{q}_1, \vec{q}_2), K_{A,r}^{(3)}(\vec{q}_1, \vec{q}_2, \vec{q}_3)$ have been expressed with real space kernals.

The counter-terms are as follows:

$$\delta_{A,r}^{(3,\mathrm{ct})} = (c_{\mathrm{ct}}^A + c_{r,1}^A \mu^2 + c_{r,2} \mu^4) k^2 \, \delta^{(1)} \tag{3}$$

where the RSD counter-term $c_{r,2}$ is independent of bia of the tracer A.

The stochasitic terms are as follows:

$$\langle \delta_{A,r} \delta_{A,r} \rangle = \frac{1}{\bar{n}_A} (c_{\varepsilon,1}^A + c_{\varepsilon,2}^A k^2 + c_{\varepsilon,3}^A f \mu^2 k^2) \tag{4}$$

Finally we get the single-tracer power spectrum P_{AA}:

$$\begin{aligned}
\langle \delta_{A,r}(\vec{k}) \delta_{A,r}(\vec{k}) \rangle &= \big\langle \delta_{A,r}^{(1)}(\vec{k}) \delta_{A,r}^{(1)}(\vec{k}) \big\rangle + \big\langle \delta_{A,r}^{(2)}(\vec{k}) \delta_{A,r}^{(2)}(\vec{k}) \big\rangle + 2 \big\langle \delta_{A,r}^{(1)}(\vec{k}) \delta_{A,r}^{(3)}(\vec{k}) \big\rangle \\
&\quad + 2 \big\langle \delta_{A,r}^{(1)}(\vec{k}) \delta_{A,r}^{(3,\mathrm{ct})}(\vec{k}) \big\rangle + \big\langle \delta_{A,r}^{(\varepsilon)}(\vec{k}) \delta_{A,r}^{(\varepsilon)}(\vec{k}) \big\rangle \\
&= \big(K_{A,r}^{(1)}\big)^2 P_{11}(k) + 2 \int d^3q \big(K_{A,r}^{(2)}(\vec{q}_1, \vec{k} - \vec{q})_{\mathrm{sym}}\big)^2 P_{11}(|\vec{k} - \vec{q}|) P_{11}(q) \\
&\quad + 6 \int d^3\vec{q} K_{A,r}^{(3)}(\vec{q}, -\vec{q}, \vec{k})_{\mathrm{sym}} K_{A,r}^{(1)} P_{11}(q) P_{11}(k) \\
&\quad + 2 K_{A,r}^{(1)} P_{11}(k) (c_{\mathrm{ct}}^A + c_{r,1}^A \mu^2 + c_{r,2} \mu^4) k^2 \\
&\quad + \frac{1}{\bar{n}_A} (c_{\varepsilon,1}^A + c_{\varepsilon,2}^A k^2 + c_{\varepsilon,3}^A f \mu^2 k^2)
\end{aligned} \tag{5}$$

2 Biased multi-tracer in redshift space

Assuming the stochasitic terms of the two tracers are uncorrelated with Gaussian initial conditions, we have:

$$\langle \delta_{A,r} \delta_{B,r} \rangle = \frac{1}{2} \bigg(\frac{1}{\bar{n}_A} + \frac{1}{\bar{n}_B} \bigg) \bigg(c_{\varepsilon,1}^{AB} + c_{\varepsilon,2}^{AB} \bigg(\frac{k}{k_M} \bigg)^2 + c_{\varepsilon,3}^{AB} f \mu^2 \bigg(\frac{k}{k_M} \bigg)^2 \bigg) \tag{6}$$

Therefore, we can write the cross power spectrum P_{AB} in this way:

$$\begin{aligned}
\langle\delta_{A,r}(\vec{k})\delta_{B,r}(\vec{k})\rangle &= \left\langle\delta^{(1)}_{A,r}(\vec{k})\delta^{(1)}_{A,r}(\vec{k})\right\rangle + \left\langle\delta^{(2)}_{A,r}(\vec{k})\delta^{(2)}_{A,r}(\vec{k})\right\rangle + \left\langle\delta^{(1)}_{A,r}(\vec{k})\delta^{(3)}_{B,r}(\vec{k})\right\rangle + \left\langle\delta^{(3)}_{A,r}(\vec{k})\delta^{(1)}_{B,r}(\vec{k})\right\rangle \\
&\quad + \left\langle\delta^{(1)}_{B,r}(\vec{k})\delta^{(3,\mathrm{ct})}_{A,r}(\vec{k})\right\rangle + \left\langle\delta^{(1)}_{A,r}(\vec{k})\delta^{(3,\mathrm{ct})}_{B,r}(\vec{k})\right\rangle + \left\langle\delta^{(\varepsilon)}_{A,r}(\vec{k})\delta^{(\varepsilon)}_{B,r}(\vec{k})\right\rangle \qquad (7)\\
&= K^{(1)}_{A,r}K^{(1)}_{B,r}P_{11}(k) + 2\int d^3\vec{q}K^{(2)}_{A,r}(\vec{q},\vec{k}-\vec{q})_{\mathrm{sym}}K^{(2)}_{B,r}(\vec{q},\vec{k}-\vec{q})_{\mathrm{sym}}P_{11}(|\vec{k}-\vec{q}|)P_{11}(q) \\
&\quad +3\int d^3\vec{q}K^{(3)}_{B,r}(\vec{q},-\vec{q},\vec{k})_{\mathrm{sym}}K^{(1)}_{A,r}P_{11}(q)P_{11}(k) \\
&\quad +3\int d^3\vec{q}K^{(3)}_{A,r}(\vec{q},-\vec{q},\vec{k})_{\mathrm{sym}}K^{(1)}_{B,r}P_{11}(q)P_{11}(k) \\
&\quad +K^{(1)}_{B,r}P_{11}(k)(c^A_{\mathrm{ct}}+c^A_{r,1}\mu^2+c_{r,2}\mu^4)k^2 \\
&\quad +K^{(1)}_{A,r}P_{11}(k)(c^B_{\mathrm{ct}}+c^B_{r,1}\mu^2+c_{r,2}\mu^4)k^2 \\
&\quad +\frac{1}{2}\left(\frac{1}{\bar{n}_A}+\frac{1}{\bar{n}_B}\right)\left(c^{AB}_{\varepsilon,1}+c^{AB}_{\varepsilon,2}\left(\frac{k}{k_M}\right)^2+c^{AB}_{\varepsilon,3}f\mu^2\left(\frac{k}{k_M}\right)^2\right)
\end{aligned}$$

For more clear expression, we organize all of the parameters needed:

$$\begin{aligned}
P_{AA} &: \{b^A_1, b^A_2, b^A_3, b^A_4, c^A_{\mathrm{ct}}, c^A_{r,1}, c_{r,2}, c^A_{\varepsilon,1}, c^A_{\varepsilon,2}, c^A_{\varepsilon,3}\} \qquad (8)\\
P_{BB} &: \{b^B_1, b^B_2, b^B_3, b^B_4, c^B_{\mathrm{ct}}, c^B_{r,1}, c_{r,2}, c^B_{\varepsilon,1}, c^B_{\varepsilon,2}, c^B_{\varepsilon,3}\} \\
P_{\mathrm{AB}} &: \{b^A_1, b^A_2, b^A_3, b^A_4, b^B_1, b^B_2, b^B_3, b^B_4, c^A_{\mathrm{ct}}, c^A_{r,1}, c^B_{\mathrm{ct}}, c^B_{r,1}, c_{r,2}, c^{AB}_{\varepsilon,1}, c^{AB}_{\varepsilon,2}, c^{AB}_{\varepsilon,3}\}
\end{aligned}$$