Quantitative Methods

CFA一级培训项目

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Topic Weightings in CFA Level I

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Framework

Quantitative Methods

- **Time Value Calculation**
  - R6 The Time Value of Money
  - R7 Discounted Cash Flow Applications
- **Probability & Statistics**
  - R8 Statistical Concepts and Market Returns
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- **Inferential statistics**
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Reading 6

Time Value of Money

Framework

1. Required interest rate on a security 的组成
2. EAR
3. Annuities的计算：FV, PV, required payment

Time Value of Money

- **Required rate of return is**
  - affected by the supply and demand of funds in the market;
  - the minimum rate of return an investor must receive to accept the investment.
  - usually for particular investment.
- **Discount rate is**
  - the interest rate we use to discount payments to be made in the future.
  - usually used interchangeably with the interest rate.
- **Opportunity cost is**
  - also understood as a form of interest rate. It is the value that investors forgo by choosing a particular course of action.

Time Value of Money

- **Decompose required rate of return:**
  - Nominal risk-free rate = real risk-free rate + expected inflation rate
  - Required interest rate on a security = nominal risk-free rate + default risk premium + liquidity risk premium + maturity risk premium

Example

- The sum of the real risk-free interest rate and the expected inflation premium is best described as:
  - A. the nominal risk-free interest rate
  - B. the default risk premium
  - C. the liquidity premium
- **Correct Answer: A**

Time Value of Money

- **EAR calculation:**

$$\text{EAR}=(1+\text{periodic rate})^{m}-1 \quad \longleftrightarrow \quad 1+EAR=\left(1+\frac{r}{m}\right)^{m}$$

  - If semi-annually compounding, then m=2
  - if quarterly compounding, then m=4
  - If continuous compounding, then EAR = $e^{\text{annual int}}$-1
- **Tips：**
  - Calculate——calculate EAR，or calculate the frequency of compounding
  - Feature
    - ✓ The more frequency of compounding，the larger the EAR
    - ✓ The largest EAR exists if it is continuously compounding

Example

- A money manager has $1,000,000 to invest for one year. She has identified two alternative one-year certificates of deposit (CD) shown below:

	Compounding frequency	Annual interest rate
CD1	Quarterly	4.00%
CD2	Continuously	4.95%

- Which CD has the highest effective annual rate (EAR) and how much interest will it earn?

	Highest EAR	Interest earned
A.	CD1	$41,902
B.	CD1	$40,604
C.	CD2	$50,700

Time Value of Money

- **What's annuities?**
  - is a finite set of level sequential cash flows.
    - ✓ equal intervals
    - ✓ equal amount of cash flows
    - ✓ same direction
- **Elements of annuity：**
  - N = number of periods
  - I/Y = interest rate per period
  - PV = present value
  - PMT = amount of each periodic payment
  - FV= future value

Time Value of Money

About ordinary annuities:

- Tom makes a saving plan with $1000 annual deposit at the end each year. Suppose the deposit rate is 5 percent annually, what's the future value at the end of year five?

- **Correct Answer:**
  - Enter relevant data for calculate.
    - ✓ N=5, I/Y=5, PMT=-1,000, PV=0, CPT→FV=5,525.63

Time Value of Money

- **About an annuity due**
  - The first cash flow occurs immediately(at t=0)
    - ✓ Example：rental fees, tuition fees, living expenses, etc.
  - **Calculation:**
    - ✓ **Measure 1:** use calculator, put the calculator in the **BGN** mode and input relevant data.
    - ✓ **Measure 2:** treat as an ordinary annuity and simply multiple the resulting PV by (1+I/Y)
      - ◆PV and FV calculation applies, while PMT not.
    - ✓ **Measure 3:** treat as an ordinary annuity and simply plus the resulting PV by PMT

Time Value of Money

- To see how a lump sum can generate an annuity, assume that we loan $3,170 from the bank today at 10 percent interest. Construct an **amortization** table to show the annuity payments over the next four years.

- **Correct Answer:**
  The amount of the annuity payments : N=4; I/Y=10; PV=-$3,170; FV=0; CPT: PMT=$1,000

How an Initial Present Value Funds an Annuity					
Time Period	Amount Available at beginning	Ending Amount before withdrawal		withdrawal	Amount Available after withdrawal
1	3,170	3,170*1.1	3,487	1,000	2,487
2	2,487	2,487*1.1	2,735.7	1,000	1,735.7
3	1,735.7	1,735.7*1.1	1,909.27	1,000	909.27
4	909.27	909.27*1.1	1,000	1,000	0

Example

- You are planning to purchase a $1,200,000 house by making a down payment of $200,000 and borrowing the remainder with a 30-year fixed-rate mortgage with monthly payments. Each payment will be made at the end of the month. Current mortgage annual interest rates are quoted at 10 percent with monthly compounding. Calculate the amount of the outstanding balance after the second payment?

- **Correct Answer:**
  - First, calculate the payment of each mouth
    - ✓ PV=-$1,000,000; I/Y=0.8333; N=360; FV=0; CPT: PMT=$8,776
  - Second, calculate the principal and interest component of the first and second payment, there are **two methods** to calculate.

Example

- **Methods 1:**
  - First Payment : $8,776
    - ✓Interest component = ($1,000,000)*(0.1/12) = $8,333
    - ✓Principal component = $8,746-$8,333= $442
    - ✓Ending balance = $1,000,000-$413= $999,587
  - Second Payment : $8,776
    - ✓Interest component = $999,587*(0.1/12) = $8,330
    - ✓Principal component = $8,776-$8,330=$446
    - ✓Ending balance = $1,000,000-$442-$446= $999,112
- **Methods 2:**
  - Using a calculator

1. Input PV=-1,000,000; I/Y=0.83; N=360; FV=0; CPT: PMT=$8776
2. Using 2ND+AMORT, INPUT P1=1,P2=1, calculate ending balance of first year
3. Input P1=2,P2=2, calculate BAL, PRN, INT

Ordinary annuities and annuity due

- A couple is preparing their newborn baby's university tuition, estimated at $30,000 per year for four years, with the first payment starting at the beginning of Year 18. To balance the account, they need to make 18 equal payments into an investment account, the first to be made one year from today. What will the annually payments be at a 8% rate of return?

- **Correct Answers:**
  - Compute PV at the beginning of Year 18 and Set your calculator to the **BGN mode**,
    - ✓ N=4; I/Y=8; FV=0; PMT=-30,000; CPT: PV18=107,312.91
  - At the beginning of 18, PV18 =FV(for ordinary annuity), Set your calculator to the **END mode**,
    - ✓ N=18; I/Y=8; PV=0; FV=-107,312.91; CPT: PMT=2,865.48

Time Value of Money

➢ **About perpetuity**

- A perpetuity is a set of level never-ending sequential cash flows, with the first cash flow occurring one period from now.

$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + \cdots \qquad (1)$$

$$(1+r)PV = A + \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots \qquad (2)$$

$$(2)-(1) \qquad r \times PV = A \Rightarrow PV = \frac{A}{r}$$

Reading 7

Discounted Cash Flow Applications

Framework

1. NPV & IRR
2. Calculate HPY, EAY, and transform
3. Money-weighted return & Time-weighted return

Discounted Cash Flow Applications

- **NPV (Net Present Value)**
  - When NPV>0, accept the project.(in most situation)

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_N}{(1+r)^N} = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

- **IRR (Internal Rate of Return)**
  - When NPV= 0, the discount rate.

$$NPV = 0 = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_N}{(1+IRR)^N} = \sum_{t=0}^{N} \frac{CF_t}{(1+IRR)^t}$$

  - IRR method assumes the project's cash flows will be reinvested at the IRR.
  - Multiple solutions or no solution problem of the IRR calculation (# sign changes)

Discounted Cash Flow Applications

- **Project Decision Rule**
  - **Single project Case**
    - ✓ NPV method: Accept it if NPV>0
    - ✓ IRR method: Accept it if IRR>r (required rate of return)
  - **Two Projects Case**
    - ✓ **Independent Projects**
      - ◆ Similar to Single projects case
    - ✓ **Mutually Exclusive Projects**
      - ◆ NPV method: Choose the one with higher NPV
      - ◆ IRR method: Choose the one with higher IRR
      - ◆ When the IRR and NPV rules conflict with each other, NPV method dominates.

Example

- Which of the following statements least accurately describes the IRR and NPV methods?
  - A. The discount rate that gives an investment an NPV of zero is the investment's IRR.
  - B. If the NPV and IRR methods give conflicting decisions for mutually exclusive projects, the IRR decision should be used to select the project.
  - C. The NPV method assumes that a project's cash flows will be reinvested at the cost of capital, while the IRR method assumes they will be reinvested at the IRR.
- **Correct Answer: B**

Example

- Calabash Crab House is considering an investment in two kitchen-upgrade projects with the following cash flows:

	Project A	Project B
Initial Year	-$10,000	-$9,000
Year 1	2,000	200
Year 2	5,000	-2,000
Year 3	8,000	11,000
Year 4	8,000	15,000

Assuming Calabash has a 12.5 percent cost of capital, which of the following investment decisions has the least justification? Accept:

A. Project B because the net present value (NPV) is higher than that of Project A.
B. Project A because the IRR is higher than the cost of capital.
C. Project A because the internal rate of return (IRR) is higher than that of Project B.

Holding Period Return

- **Define:** the holding period return is the return that an investor earns over a specified holding period.
- **Formula:** $HPR=\frac{P_1-P_0+CF_1}{P_0}$ **or,** $HPR=\frac{FV-PV}{PV}$

Example： Jane Peebles purchased a T-bill that matures in 200 days for $975. The face value of the bill is $1,000. What's the holding period return of the bond?

$$HPR=\frac{FV-PV}{PV}=\frac{1000-975}{975}=2.564\%$$

Money Market rate

- The r_{MM} (also called CD equivalent yield)is the annualized yield that is based on price and a 360-day year and dose not account for the effects of compounding – it assumes simple interest.
- **Formula:** $r_{MM}=HPY\times\frac{360}{t}$

Example： Jane Peebles purchased a T-bill that matures in 200 days for $975. The face value of the bill is $1,000. What's the money market rate of the bond?

$$r_{MM}=HPY\times\frac{360}{t}=2.56\%\times\frac{360}{200}=4.615\%$$

Effective annual yield(rate)

- The **EAY** is the annualized HPY on the basis of a 365-day year and incorporates the effects of compounding.
- **Formula:** $EAY=(1+HPY)^{365/t}-1$

Example: Jane Peebles purchased a T-bill that matures in 200 days for $975. The face value of the bill is $1,000. What's the effective annual rate of the bond?

$$EAY=(1+HPY)^{365/t}-1=(1+2.564\%)^{365/200}-1=4.729\%$$

Bond Equivalent Yield

- The **BEY** is the semi-annualized HPY on the basis of a 365-day year and incorporates the effects of compounding.
- **Formula:** $(1+BEY/2)^2=1+EAY=(1+HPY)^{365/t}$

Example: Jane Peebles purchased a T-bill that matures in 200 days for $975. The face value of the bill is $1,000. What's the bond equivalent yield of the bond?

$$(1+BEY/2)^2=(1+HPY)^{365/t}=(1000/975)^{365/200}$$

$$BEY=2\times[\sqrt{(1000/975)^{365/200}}-1]=4.674\%$$

Bank Discount Yield

- The **bank discount yield** (often called simply the discount yield) takes the dollar discount from par as a fraction of the face value of the T-bill ,and based a 360-day year and without the effects of compounding.
- **Formula:** $r_{BD}=\frac{(FV-PV)}{FV}\times\frac{360}{t}$

Example: Jane Peebles purchased a T-bill that matures in 200 days for $975. The face value of the bill is $1,000. What's the bank discount yield of the bond?

$$r_{BD}=\frac{(FV-PV)}{FV}\times\frac{360}{t}=\frac{(1000-975)}{1000}\times\frac{360}{200}=4.500\%$$

Example

- An investor buys a T-bill at 98,000 with 50 days to maturity. The par value of this T-bill is 100,000. The money market yield is closest to:
  - A. 14.6%.
  - B. 14.7%.
  - C. 14.9%.

- **Correct Answer: B.**

  Money market yield = [(par value – purchase price)/purchase price] * (360/50) = [(100,000 – 98,000)/98,000] * (360/50) = 14.69%

Discounted Cash Flow Applications

- **Time-weighted Rate of Return**
  - Time-weighted rate of return measures the compound rate of growth.
  - Calculation
    - ✓ Firstly, calculate the HPR on the portfolio for each subperiod;
    - ✓ then, compound the HPR to obtain an annual rate of return for the year.
    - ✓ Lastly, take the geometric mean of the annual returns over the measurement period.

Example

- An investor bought a fund at the beginning of 2015(t=0) when the fund had a market value of $10,000 . At the beginning of 2016 (t=1), he bought another share for $11,000. And he sold both shares at the beginning of 2017 (t=2), and the market value of the fund was $15,000 each. In addition, at the end of each year in the holding period, the stock offered a dividend of $200 each.
  - What is the fund's time-weighted rate of return?

- **Correct Answer:**

$$HPR_1 = \frac{11,000 - 10,000 + 200}{10,000} = 12\%$$

$$HPR_2 = \frac{15,000 - 11,000 + 200}{11,000} = 38.18\%$$

$$TMRR = \sqrt{(1+12\%) \times (1+38.18\%)} - 1 = 24.40\%$$

Discounted Cash Flow Applications

- **Money-weighted Rate of Return**
  - the IRR based on the cash flows related to the investment
  - Calculation
    - ✓ Firstly, determine the timing of each cash flow;
    - ✓ then, using the calculation to compute IRR, or using geometric mean.

Example

- An investor bought a fund at the beginning of 2015(t=0) when the fund had a market value of $10,000 . At the beginning of 2016 (t = 1), he bought another share for $11,000. And he sold both shares at the beginning of 2017 (t=2), and the market value of the fund was $15,000 each. In addition, at the end of each year in the holding period, the stock offered a dividend of $200 each.
  - What is the fund's money-weighted rate of return?

- **Correct Answer:**
  - CF_0 = -10,000,
  - CF_1 = -11,000+200 = -10,800,
  - CF_2 =15,000*2+200*2 = 30,400;
  - CPT MWRR = IRR = 28.53%

Discounted Cash Flow Applications

- **The relationship between TWRR and MWRR**
  - Both TWRR and MWRR are **annual rates**.
  - Time-weighted return **is not influenced by cash flow**, but money-weighted return will be affected by cash flow.

Example

- Which of the following statements most likely describes a characteristic of the time-weighted rate of return?
  - A. It is affected by the timing of cash flows.
  - B. It measures the compound rate of growth of $1 over a stated measurement period.
  - C. It is defined as the internal rate of return on an investment portfolio, taking into account all inflows and outflows.

- **Correct Answer: B.**
  - The money-weighted rate of return is the IRR of an investment's net cash flows. Time-weighted return is not influenced by cash flow, but money-weighted return will be affected by cash flow.

Example

- Would a client making additions or withdrawals of funds most likely affect their portfolio's:

	Time-weighted return?	Money-weighted return?
A.	No	No
B.	No	Yes
C.	Yes	No

- **Correct Answer: B.**
  The time-weighted return is not affected by cash withdrawals or addition to the portfolio, the money-weighted return measure would be affected by client additions or withdrawals, if a client adds funds at a favorable time the money-weighted return will be elevated.

Reading 8

Statistical Concepts and Market Returns

Framework

1. Types of measurement scales
2. Measures of central tendency
3. Quantile
4. Measures of dispersion
5. Chebyshev's inequality
6. CV & Sharp ratio
7. Skewness & Kurtosis

Statistical Concepts and Market Return

- **Descriptive statistics**
  - Quantitatively describe or summarize the important features of large data sets.
- **Inferential statistics**
  - Makes estimations about a large set of data (a population with smaller group of data.

Statistical Concepts and Market Return

- **Types of measurement scales:**
  - **Nominal scales**
    - ✓ Distinguishing **two different things**, no order, only has mode
    - ✓ Example: assigning the number 1 to male, the number 2 to female.
  - **Ordinal scales** (>, <)
    - ✓ **Making things in order**, but the difference are not meaningful
    - ✓ Example: ranking mutual funds based on their five-year cumulative returns, we might assign the number top1 to 10 for the funds performance.
  - **Interval scales** (>, <, +, -)
    - ✓ **Subtract is meaningful**
    - ✓ Example: temperature
  - **Ratio scales** (>, <, +, -, *, /)
    - ✓ With original point
    - ✓ Example: as is money, if we have twice as much money, then we have twice the purchasing power.

Example

- An analyst gathered the price-earnings ratios (P/E) for the firms in the S&P 500 and then ranked the firms from highest to lowest P/E. She then assigned the number 1 to the group with the lowest P/E ratios, the number 2 to the group with the second lowest P/E ratios, and so on. The measurement scale used by the analyst is *best* described as:

  A. Ratio.

  B. Ordinal.

  C. Interval.

- **Correct Answer: B**

Statistical Concepts and Market Return

- **Population**
  - A population is defined as all members of a specified group.
- **Sample**
  - A sample is a subset of a population.
- A **parameter** is used to describe the features of a population.
- A **sample statistic** is used to describes the features of a sample.

Statistical Concepts and Market Return

- **Relative frequency**
  - The relative frequency of observations in an interval is the number of observations(the absolute frequency) in the interval divided by the total number of observations.
- **Frequency Distribution**
  - A frequency distribution is a tabular display of data summarized into a relatively small number of intervals. Frequency distributions permit analyst to evaluate how data are distributed.
- **Cumulative frequency/Cumulative Relative Frequency**
  - The cumulative relative frequency cumulates (adds up) the relative frequencies as we move from the first interval to the last.

Statistical Concepts and Market Return

- **Frequency distribution**

Interval Relative	Absolute Frequency	Relative Frequency	Cumulative Absolute Frequency	Cumulative Frequency
-10 - -5	3	0.97%	3	0.97%
-5 – 0	35	11.29%	38	12.26%
0 – 5	176	56.77%	214	69.03%
5 – 10	74	23.87%	288	92.90%
10 - 15	22	7.10%	310	100%
Total	**310**	**100%**		

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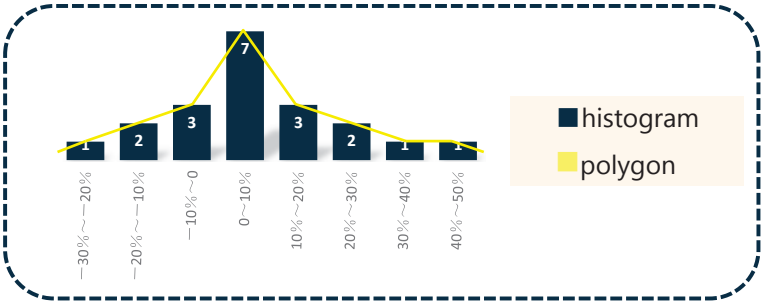
Statistical Concepts and Market Return

- **Histogram and Polygon**
  - A **histogram** is a bar chart of data that have been grouped into a frequency distribution.
  - A frequency **polygon** is a graph of frequency distributions obtained by drawing straight lines joining successive points representing the class frequencies.



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Example

- An analyst gathered the following information about the annual return on the S&P 500 over the period 1927 to 2016.

Interval	Return interval(%)	Frequency
I	-30.0 to -10.0	10
II	-10.0 to 10.0	34
III	10.0 to 30.0	30
IV	30.0 to 50.0	16

The relative frequency and the cumulative relative frequency, respectively, for interval III are closest to:

	Relative frequency	Cumulative relative frequency
A.	20%	82%
B.	33%	36%
C.	33%	82%

- **Correct Answer: C.**

Statistical Concepts and Market Return

The Arithmetic Mean: $\overline{X} = \frac{\sum_{i=1}^{N} X_i}{n}$

The Weighted Mean: $\overline{X_W} = \sum_{i=1}^{n} w_i X_i = (w_1 X_1 + w_2 X_2 + \cdots + w_n X_n)$

The Geometric Mean: $G = \sqrt[N]{X_1 X_2 X_3 \ldots X_N} = (\prod_{i=1}^{N} X_i)^{1/N}$

The Harmonic Mean: $\overline{X_H} = \frac{n}{\sum_{i=1}^{n} (1/X_i)}$

Harmonic Mean<= Geometric Mean<=Arithmetic Mean

Example

- Which is the most accurate?

	Harmonic mean	Arithmetic mean	Geometric mean
A.	13	15	18
B.	15	15	18
C.	13	18	15

- **Correct Answer: C**

Example

- An analyst obtains the following annual rates of return for a mutual fund:

Year	Return (%)
2008	14
2009	-10
2010	-2

The fund's holding period return over the three-year period is closest to:

A. 0.18%
B. 0.55%
C. 0.67%

- **Correct Answer: B**

- The fund's annual holding period return is closest to:

A. 0.18%
B. 0.55%
C. 0.67%

- **Correct Answer: A**

Statistical Concepts and Market Return

- The initial costs of the stock is $200. At the end of the first year, the stock is trading at $400. At the end of the second year, the stock price falls back to the price of $200. No dividends are paid during the two-year periods. Calculate the arithmetic and geometric mean of annual returns.

- **Correct Answer:**

  HPR_1= 400/200-1 = 100%

  HPR_2= 200/400-1 = -50%

  Arithmetic mean = (100%-50%)/2 =25%

  Geometric mean = $(2.0 \times 0.5)^{1/2} - 1 = 0\%$

  The geometric mean return reflects that the ending value of the investment in Year2 equals the starting value in Year1.

  The compound rate of return on the investment is 0%. The arithmetic mean return reflects the average of the one-year returns.

Statistical Concepts and Market Return

- **The use of arithmetic mean and geometric mean when determining investment returns**
  - The arithmetic mean is the statistically best estimator of the **next year's returns** given only the three years of return outcomes.
  - The geometric mean is especially important in reporting compound growth rates for time series data, which is good reflection of past performance.

Statistical Concepts and Market Return

- **Quantiles**
  - Quartile /Quintile/Deciles/Percentile
    - ✓ The third quintile: 60%, or there are three-fifths of the observations fall below that value.
  - Calculation formula: L_y = (n+1)y/100,
    - ✓ Where Ly is the quantile position expressed in percentage.

Example:

Observers: 5 8 11 12 14 16 16 18 19 21 23

Calculate the third quartile of the data set

N=11 , L_y=(11+1)*75%=9, i.e. the 9th number is 75%

The third quartiles = 19

Statistical Concepts and Market Return

- **Absolute dispersion:** is the amount of variability present without comparison to any reference point or benchmark.

Range = maximum value – minimum value

$$MAD = \frac{\sum_{i=1}^{N}|X_i - \bar{X}|}{n}$$

$$\text{For population: } \sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

$$\text{For sample: } s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

$$\text{Semivariance} = \frac{\sum_{\text{for all } X_i \le \bar{X}}(X_i - \bar{X})^2}{n-1}$$

$$\text{Target Semivariance} = \frac{\sum_{\text{for all } X_i \le B}(X_i - B)^2}{n-1}$$

Example

- As a mutual fund analyst, you are examining, as of early 2017, the most recent five years of total returns for two US large-cap value equity mutual fund.

Year	Portfolio return(%)
2012	-39.44
2013	31.64
2014	12.53
2015	-4.35
2016	12.82

The portfolio's mean absolute deviation and variance of annual returns, respectively, for the five-year period are closest to:

	Mean absolute deviation	Variance
A.	19.63%	0.05724
B.	2.64%	0.0968
C.	19.63%	0.0968

- **Correct Answer: A**

Statistical Concepts and Market Return

- For any set of observations (samples or population), the proportion of the values that **lie within k standard deviations** of the mean is at least $1 - 1/k^2$, where k is any constant **greater than 1.**

$$P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2}$$

- This relationship applies regardless of the shape of the distribution.

Example

- Assume a sample of beer prices is negatively skewed. Approximately what percentage of the distribution lies within plus or minus 2.40 standard deviations of the mean?
  - A. 82.6%
  - B. 58.3%
  - C. 17.36%

- **Correct Answer: A**

Example

- The arithmetic mean monthly return and standard deviation of monthly returns on the S&P 500 were 0.97 percent and 5.65 percent, respectively, during the 1926-2002 period, totaling 924 monthly observations. Using this information, address the following:

1. Calculate the endpoints of the interval that must contain at least 75 percent of monthly returns according to Chebyshev's inequality.
2. What is the minimum number of observations that must lie in the interval computed in Part 1, according to Chebyshev's inequality?

- **Correct Answer 1:**
  $1-1/k^2=75\% \rightarrow k=2$
  $0.97\% \pm 2(5.65\%)=0.97\% \pm 11.30\%$
  Lower endpoint of the interval: 0.97%-11.30%=-10.33%
  Upper endpoint of the interval: 0.97%+11.30%=12.27%
- **Correct Answer 2:**
  For a sample size of 924, at least 0.75(924)=693 observations must lie in the interval from -10.33% to 12.27% that we computed in Part 1.

Statistical Concepts and Market Return

- **Coefficient of variation measures the amount of dispersion in a distribution relative to the distribution's mean. (relative dispersion)**

$$CV=\frac{s_x}{\bar{X}}\times 100\%$$

- **The sharp ratio measures excess return per unit of risk.**

$$\text{Sharp ratio}=\frac{R_p-R_f}{\sigma_p}$$

Statistical Concepts and Market Return

- An analyst gathered the following information about a portfolio's performance over the past ten years:

Mean annual return	12.8%
Mean excess return	7.4%
Standard deviation of annual returns	15.7%

- The coefficient of variation and Sharpe measure, respectively, for the portfolio are *closest* to:

	Coefficient of variation	Sharpe measure
A	0.82	0.39
B	0.82	0.47
C	1.23	0.47

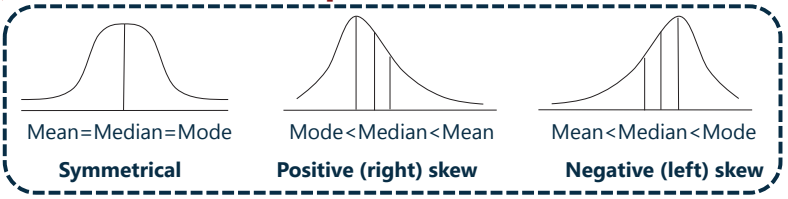
- **Correct Answer: C**

Example

- The scale-free measure of relative dispersion that is useful in making direct comparisons among different asset classes is the:
  - A. Range.
  - B. Variation.
  - C. Coefficient of variation.

- **Correct Answer: C**

Statistical Concepts and Market Return



- **Positive skewed:** Mode<median<mean, having a right fat tail
  - A return distribution with positive skew has frequent small losses and a few extreme gains
- **Negative skewed:** Mode>media>mean, having a left fat tail
  - A return distribution with negative skew has frequent small gains and a few extreme losses.
- Investors should be attracted by a *positive skew* because the mean return falls above the median.
- Sample skewness: $S_K = [\frac{n}{(n-1)(n-2)}] \frac{\sum_{i=1}^{n}(X_i - \overline{X})^3}{s^3} \approx (\frac{1}{n}) \frac{\sum_{i=1}^{n}(X_i - \overline{X})^3}{s^3}$

Example

- A distribution with mode 2.6, median 2.2, mean 2, the distribution can be described as:
  - A. long tail in the left and positively skewed.
  - B. long tail in the right and negatively skewed.
  - C. long tail in the left and negatively skewed.
- **Correct Solution: C**

Example

- As analyst gathered the following information about the return distribution of four investment. Based only on the information above, a well-diversified investor would most likely prefer Portfolio:

Portfolio	Skewness	Sharp Ratio
1	Positive	0.6
2	Positive	0.8
3	Negative	0.6

  - A. 1
  - B. 2
  - C. 3
- **Correct Answer: B**

Statistical Concepts and Market Return

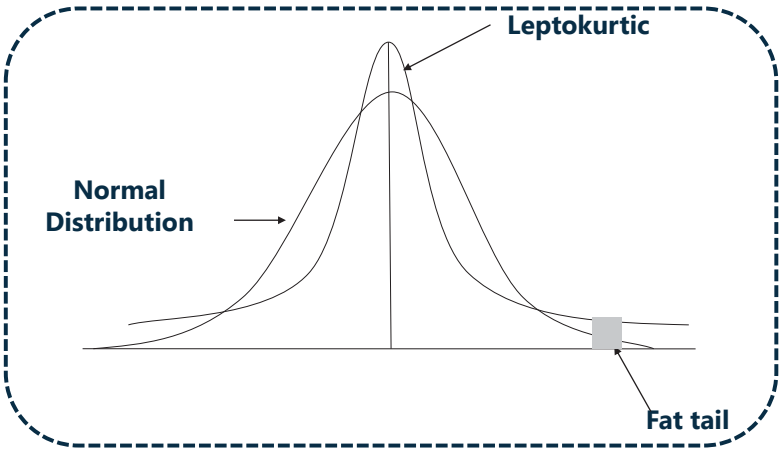
- **Leptokurtic vs. platykurtic**
  - It deals with whether or not a distribution is more or less "peaked" than a normal distribution
- **Excess kurtosis = sample kurtosis – 3**

	leptokurtic	Normal distribution	platykurtic
Sample kurtosis	>3	=3	<3
Excess kurtosis	>0	=0	<0

- **Sample Kurtosis:**

$$K_E = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \frac{\sum_{i=1}^{n}(X_i - \overline{X})^4}{s^4} \approx \frac{1}{n} \frac{\sum_{i=1}^{n}(X_i - \overline{X})^4}{s^4}$$

Statistical Concepts and Market Return



- **A leptokurtic** return distribution has more extremely large deviations from the mean than a normal distribution if they share **same variance**.

Example

- An analyst gathered the following information about the return distribution for two portfolios during the same time period:

Portfolio	Skewness	Kurtosis
A	-1.3	2.2
B	0.5	3.5

The analyst stated that the distribution for Portfolio A is less peaked than a normal distribution and that the distribution for Portfolio B has a long tail on the left side of the distribution. Is the analyst's statement correct with respect to:

	Portfolio A	Portfolio B
A.	No	No
B.	No	Yes
C.	Yes	No

- **Correct Solution: C**

Reading 9

Probability Concepts

Framework

1. Probability concepts
2. Two defining properties of probability
3. Empirical, subjective, and priori probabilities
4. Odds for or against
5. Multiplication rule and addition rule
6. Dependent and independent events
7. Covariance & correlation
8. Expected value, variance, and standard deviation of a random variable and of returns on a portfolio
9. Bayes' formula

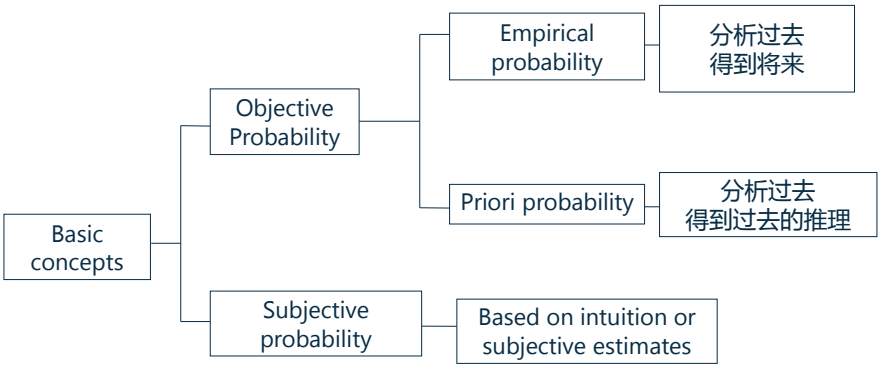
Probability Concepts

- **Basic Concepts**
  - **Random variable** a quantity whose value is uncertain.
  - Outcomes are the possible values of a random variable.
  - Event
    - ✓ Mutually exclusive events—can not happen at the same time.
    - ✓ Exhaustive events—include all possible outcomes.
- **Two Defining Properties of Probability**
  - $0 \leq P(E) \leq 1$
  - $P(E_1) + P(E_2) + \ldots\ldots + P(E_n) = 1$

Probability Concepts

- **Empirical probability 经验概率**
  - eg. Historically, the Dow Jones Industrial Average has closed higher than the previous close two out of every three trading days. Therefore, the probability of the Dow going up tomorrow is two-thirds, or 66.7%.
- **Priori probability 先验概率**
  - eg. Yesterday, 24 of the 30 DJIA stocks increased in value. Thus, if 1 of 30 stocks is selected at random, there is an 80%(24/30) probability that its value increased yesterday
- **Subjective probability 主观概率**
  - will close higher tomorrow is 90%.

Probability Concepts



Example

- An analyst adjusts the historical probability of default for high-yield bonds to reflect her perceptions of changes in the quality of high-yield bonds. The analyst is best characterized as obtaining a(n):
  A. A priori probability.
  B. Objective probability.
  C. Subjective probability.

- **Correct Answer: C**

Probability Concepts

- **Odds for an event**
  - P(E)/(1-P(E))
- **Odds against an event**
  - (1-P(E))/P(E)

- Last year, the average salary increase for Poultry Research Assistants was 2.5 percent. Of the 10,000 Poultry Research Assistants, 2,000 received raises in excess of this amount. The odds that a Poultry Research Assistant received a salary increase in excess of 2.5 percent are:
  A. 1 to 4.
  B. 2 to 10.
  C. 20%.

- **Correct Answer: A**

Probability Concepts

- **Unconditional Probability (marginal probability): P(A)**
  - What's the probability of event A?
- **Conditional probability : P(A|B)**
  - What's the probability of event A, given that B has occurred?

Probability Concepts

- **Probability that two events will happen at the same time: Joint probability : P(AB)**
  - **Multiplication rule:**
    - ✓ $P(AB)=P(A|B)\times P(B)= P(B|A)\times P(A)$
  - If A and B are mutually exclusive events, then:

    $P(AB)=P(A|B)=P(B|A)=0$
- **The probability that A or B occurs, or both occur:**
  - **Addition rule:**
    - ✓ P(A or B)=P(A)+P(B)-P(AB)
  - If A and B are mutually exclusive events, then:

    P(A or B)=P(A)+P(B)

Example

- The probability that two or more events will happen concurrently is best characterized as:
  - A. Joint probability.
  - B. Multiple probabilities.
  - C. Concurrent probability.
- **Correct Answer: A**

Probability Concepts

- **Independence**
  - The occurrence of A doesn't affect the occurrence of B
    - ✓ P(A|B)=P(A) or P(B|A)=P(B)
    - ✓ P(AB)=P(A)×P(B)
    - ✓ P(A or B)=P(A)+P(B)- P(A)×P(B)
  - Independence and Mutually Exclusive are quite different
    - ✓ **If exclusive, must not independence**;
      - ◆Cause exclusive means if A happen, B can not happen, A affects B.

Example

- A fundamental analyst studying 100 potential companies for inclusion in her stock portfolio uses the following three screening criteria:

Screening Criterion	Number of Companies meeting screen
Market-to-Book Ratio >4	20
Current Ratio >2	40
Return on Equity >10%	25

Assuming that the screening criteria are independent, the probability that a given company will meet all three screening criteria is closest to:

A. 2.0%.
B. 8.5%.
C. 20.0%

- **Correct answer: A**

Example

- P (A) =0.5, P (B) =0.5, odd for concurrent A and B is 3/5, the relationship between A and B?

A. dependent
B. Independent
C. Mutually exclusive

- **Correct Answer: A**
  - Odds for P(AB)= P(AB)/(1- P(AB))=3/5, solve for P(AB)=3/8≠P(A)×P(B)

Probability Concepts

- **Total probability formula**
  - For unconditional probability of event A,

$$P(A) = P(A|W_1)P(W_1) + P(A|W_2)P(W_2) + ... + P(A|W_N)P(W_N)$$

  - ✓ Where the set of events $\{W_1, W_2, ...W_N\}$ is mutually exclusive and exhaustive.
- **Expected value and variance:**

$$E(X) = \sum P_i \times X_i$$

$$\sigma^2 = \sum_{i=1}^{N} P_i (X_i - E(X))^2$$

Example

- An analyst gathered the following information: the probability of economy prosperity is 75%, the probability of economy recession is 25%. For a company, when the economy is prosperity, there is 10% of probability that its EPS is $2.0 and 90% of probability that the EPS is $4.0. However, when the economy is recession, there is 25% of probability that the EPS is $2.0 and 75% of probability that the EPS is $4.0. What is the variance of this company's EPS, when the economy is recession?
  - A. 3.55
  - B. 1.25
  - C. 0.75

- **Correct Answer: C**

  When the economy recession:

  E(EPS) = 25% * 2 + 75% *4 = 3.5

  Var(EPS) = 25% * $(2-3.5)^2$ + 75% * $(4-3.5)^2$ = 0.75

Probability Concepts

- **Covariance:**
  - Covariance is a measure of the co-movement between random variables

$$COV(X,Y) = E[(X-E(X))(Y-E(Y))]$$

  - The covariance of a random variable with itself is its own variance

$$COV(X,X) = E[(X-E(X))(X-E(X))] = \sigma^2(X)$$

  - Covariance ranges from negative infinity to positive infinity
- **Correlation:**
  - Correlation measures the co-movement (linear association) between two random variables

$$\rho_{XY} = \frac{COV(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

  - Correlation is a number between −1 and +1
  - Understand the difference between correlation and independence
    - ✓ If ρ=0, there is **no linear relationship** between two variables

Probability Concepts

- The joint probability of returns, for securities A and B, are as follows:

Joint Probability Function of Security A and Security B Returns (Entries are joint probabilities)		
	Return on security B=30%	Return on security B=20%
Return on security A=25%	0.60	0
Return on security A=20%	0	0.40

The covariance of the returns between securities A and B is closest to:

A. $3(\%)^2$.
B. $12\ (\%)^2$.
C. $24\ (\%)^2$.

- **Correct Answer: B**

Example

- The covariance of returns for two stocks:

A. must have a value between -1.0 and +1.0
B. must have a value equal to the weighted average of the standard deviations of the returns of the two stocks
C. will be positive if the actual returns on both stocks are consistently below their expected returns at the same time

- **Correct Answer: C**

Example

- The covariance of return between Portfolio A and Portfolio B is 0.0051. The correlation of returns between the two portfolios is 0.40, and the standard deviation of the return of Portfolio B is 24%. The standard deviation of the return for Portfolio A is:

A. 5.31%
B. 0.28%
C. 8.37%

- **Correct Answer: A**

- The correlation coefficient that indicates the weakest linear relationship between variables is:

A. -0.75
B. -0.22
C. 0.35

- **Correct Answer: B**

Probability Concepts

- **Expected return, variance and standard deviation of a portfolio**

$$E(r_p) = \sum_{i=1}^{n} w_i E(R_i)$$

$$\sigma^2{}_p = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \operatorname{cov}(R_i, R_j)$$

Example

- An individual wants to invest $100,000 and is considering the following stocks:

stock	Expected Return	Standard Deviation of Returns
A	12%	15%
B	16%	24%

The expected correlation of returns for the two stocks is +0.5. If the investor invests $40,000 in Stock A and $60,000 in Stock B, the expected standard deviation of returns on the portfolio will be:

A. equal to 20.4%
B. less than 20.4%
C. greater than 20.4% because the correlation coefficient is greater than zero

- **Correct Answer: B**

Example

- An fund manager has a portfolio of two mutual funds, A and B, 75 percent invested in A, as shown in the following table.

Covariance Matrix		
Fund	A	B
A	625	120
B	120	196

The correlation between A and B, and the portfolio standard deviation of return is closest to:

	Correlation between A and B	Portfolio standard deviation of return
A.	0.18	40.80%
B.	0.34	20.22%
C.	0.12	18.00%

- **Correct Answer: B**

Probability Concepts

- **Bayes' Formula**
  - P(AB)=P(A|B)×P(B) =P(B|A)×P(A)

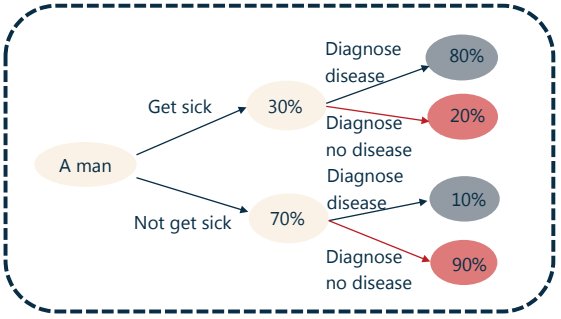
$$P(A \mid B) = \frac{P(B \mid A)}{P(B)} * P(A)$$

  - Where P(B) can be solved using total probability formula:
    - ✓ $P(B)=P(B|W_1)\times P(W_1)+P(B|W_2)\times P(W_2)+\ldots+ P(B|W_n)\times P(W_n)$
    - ✓ W_i is a set of **mutually exclusive and exhaustive** events

Probability Concepts

- The probability that a man has got sick is 30%, and if it does, a medical machine will have 80% chance to diagnose the disease. The probability that a man is healthy is 70%, and if it does, a medical machine will have 10% chance to diagnose the disease, and 90% chance not to diagnose disease. What's the probability that the man has actually got sick when the machine diagnoses disease ?

$$P(A|B) = \frac{30\% \times 80\%}{30\% \times 80\% + 70\% \times 10\%} = 77.42\%$$



Probability Concepts

- **Multiplication rule:** $n_1 \times n_2 \times \ldots\ldots \times n_k$
- **Factorial:** $n!$
- **Labeling (or Multinomial):** $\frac{n!}{n_1! \times n_2! \times \ldots \times n_k!}$
- **Combination:** ${}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)! \times r!}$
- **Permutation:** ${}_nP_r = \frac{n!}{(n-r)!}$

Probability Concepts

➢ **Labeling**

There is a portfolio consisting of nine bonds. You want to assign two of the bonds as "high risk", two of the bonds as "average risk", and five bonds as "low risk". How many ways can these nine bonds be labeled?

Probability Concepts

➢ **Correct Answer:**

- We can label any of the 9 bonds high risk, then any of 8 remaining bonds; then we can label any of 7 remaining bonds average risk, then any of 6 remaining bonds, and so forth. There can be 9! Possible sequences.
- However, order of assignment within a category does not matter.
  - ✓ For example, whether a bond occupies the first or second slot of the two bonds labeled high risk, the bond has the same label. There are 2! Ways to assign a given group of two funds to the two high risk slots. So, in this case, in total there are (2!)(2!)(4!) equivalent sequences.
  - ✓ To eliminate such redundancies from 9! total, we divide 9! by (2!)(2!)(4!). The calculation is shown below:

$$\frac{9!}{2!\times 2!\times 5!}=\frac{362,880}{2\times 2\times 120}=756$$

Reading 10

Common Probability Distributions

Framework

1. Properties of discrete distribution and continuous distribution
2. Uniform random variable and a binomial random variable
3. The key properties of the normal distribution
4. Standardize a random variable
5. Confidence interval for a normally distributed random variable
6. Lognormal distribution
7. Safety-first ratio
8. Monte Carlo simulation

Common Probability Distributions

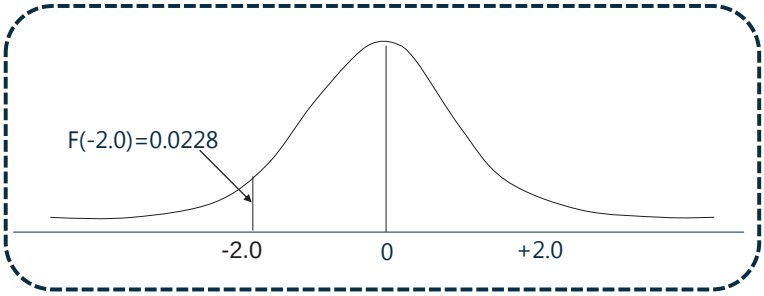
- **Probability Distribution**
  - Specifies the probabilities of the possible outcomes of a random variable.
- **Discrete and continuous random variables**
  - Discrete random variables take on at most a **countable** number of possible outcomes **but do not necessarily to be limited**.
  - Continuous random variables: cannot describe the possible outcomes of a continuous random variable Z with a list z_1, z_2, . . . because the outcome $(z_1 + z_2)/2$, not in the list, would always be possible.
    - ✓ P (x)=0 even though x can happen.
    - ✓ P (x_1<X<x_2)

Common Probability Distributions

- **Probability function: p(x)=P(X=x)**
  - For discrete random variables
  - $0 \le p(x) \le 1$
  - $\Sigma p(x)=1$
- **Probability density function (p.d.f) : f(x)**
  - For continuous random variable commonly
- **Cumulative probability function (c.p.f) : F(x)**
  - F(x)=P(X<=x)

Common Probability Distributions

- **Probability density function**



Example

- Which of the following statements about probability distributions is FALSE?
  - A. For a probability distribution for the number of days the air pollution is above a specified level, p(x) = 0 when x cannot occur, or p(x) > 0 when it can.
  - B. For a probability distribution for the specific level of air pollution on a given day, p(x) = 0 even if x can occur.
  - C. A cumulative distribution function gives the probability that a random variable takes a value equal to or greater than a given number.

- **Correct Answer: C**

  A cumulative distribution function gives the probability that a random variable takes a value equal to or *less* than a given number: $P(X \leq x)$, or F(X).

Common Probability Distributions

- **Discrete uniform distribution**
  - Discrete uniform distribution would be a known, finite number of outcomes equally likely to happen. Every one of n outcomes has equal probability 1/n.
  - For example, rolling a dice will have 6 possible outcomes as X={1,2,3,4,5,6}
    - ✓ In that case, the probability for each outcome is 0.167 [i.e. p(1)=p(2)=p(3)=p(4)=p(5)=p(6)=0.167].

Common Probability Distributions

- **Binomial distribution**
  - Bernoulli random variable

    P(Y=1)=p P(Y=0)=1-p
  - Binomial random variable，the probability of x successes in n trails

$$p(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

  - Expectations and variances

	Expectation	Variance
Bernoulli random variable (Y)	p	p(1-p)
Binomial random variable (X)	np	np(1-p)

Common Probability Distributions

- **Continuous Uniform Distribution**
  - All intervals of the same length on the Continuous Uniform Distribution's support are equally probable.
    - ✓ The support is defined by the two parameters, a and b, which are its minimum and maximum values
- **Properties of Continuous uniform distribution**
  - For all $a \le x_1 < x_2 \le b$:

$$P(x_1 \le X \le x_2) = (x_2 - x_1)/(b-a)$$

  - P (X<a or X>b) = 0

Example

- Which of the following statements about probability distributions is TRUE?
  - A. A continuous uniform distribution has a lower limit but no upper limit.
  - B. A cumulative distribution function defines the probability that a random variable is greater than a given value.
  - C. A binomial distribution counts the number of successes that occur in a fixed number of independent trials that have mutually exclusive (i.e. yes or no) outcomes.
- **Correct Answer: C**

- A random variable with a finite number of equally likely outcomes is best described by a:
  - A. Binomial distribution.
  - B. Bernoulli distribution.
  - C. Discrete uniform distribution.
- **Correct Answer: C**

Example

- A study indicated that 70% of the price of all stocks from Shanghai Stock Exchange will increase. From the binominal probability distribution table, the probability that exactly 6 stocks' price will increase in a random selection of ten stocks is:
  A. 0.136
  B. 0.200
  C. 0.319
- **Correct Answer: B**

- Assume that 40% of candidates who sit for the CFA examination pass it the first time. Of a random sample of 15 candidates who are sitting for the exam for the first time, what is the expected number of candidates that will pass?
  A. 0.375
  B. 4.000
  C. 6.000
- **Correct Answer: C**

Example

- An analyst has recently determined that only 60 percent of all U.S. pension funds have holdings in hedge funds. In evaluating this probability, a random sample of 50 U.S. pension funds is taken. The number of U.S. pension funds in the sample of 50 that have hedge funds in their portfolio would most accurately be described as:
  A. A binomial random variable.
  B. A Bernoulli random variable.
  C. A continuous random variable.

- **Correct Answer: A**

Example

- An energy analyst forecasts that the price per barrel of crude oil five years from now will range between USD$75 and USD$105. Assuming a continuous uniform distribution, the probability that the price will be less than USD$80 five years from now is closest to:
  A. 5.6%.
  B. 16.7%.
  C. 44.4%.

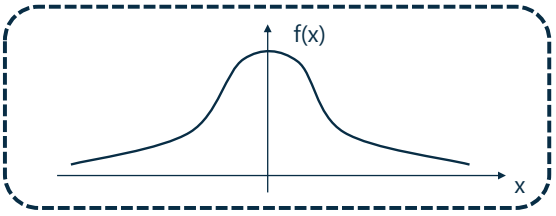
- **Correct Answer: B**

Common Probability Distributions

- **Tracking error** is the total return on a portfolio minus the total return on the benchmark index.
  - It can be use to measure the portfolio's performance.

Common Probability Distributions

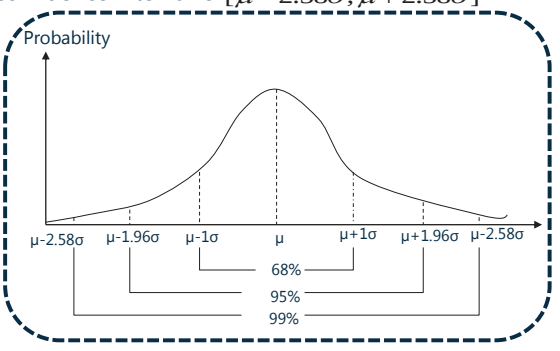
- **The shape of the density function**



- **Properties:**
  - $X \sim N(\mu, \sigma^2)$
  - Symmetrical distribution: skewness=0; kurtosis=3; excess kurtosis=0;
  - A linear combination of random variables these are in normally distribution is also normally distributed.
  - As the values of x gets farther from the mean, the probability density get smaller and smaller but are always positive.

Common Probability Distributions

- **The confidence intervals**
  - 68% confidence interval is $[\mu-\sigma, \mu+\sigma]$
  - 90% confidence interval is $[\mu-1.65\sigma, \mu+1.65\sigma]$
  - 95% confidence interval is $[\mu-1.96\sigma, \mu+1.96\sigma]$
  - 99% confidence interval is $[\mu-2.58\sigma, \mu+2.58\sigma]$



Common Probability Distributions

- The standard deviation of a stock annual returns is 21% and the average return is 9.6% per year. If returns are normally distributed, what is the 95% confidence interval for the stock return next year?

- **Correct Answer:**

  We can infer that μ and σ are 9.6% and 21%. The 95% confidence interval for the return, R, is:

  9.6%± 1.96*21%= -31.56% to 50.76%

  So we can conclude that:

  P(-31.56% < R < 50.76%) = 0.95 or 95%

  We can say that the annual return is expected to be within this interval 95% of the time, or 95 out of 100 years.

Example

- An analyst determined that approximately 99 percent of the observations of daily sales for a company were within the interval from $230,000 to $480,000 and that daily sales for the company were normally distributed. The mean daily sales and standard deviation of daily sales, respectively, for the company were closest to:

	Mean daily sales	Standard deviation of daily sales
A.	$351,450	$48,450
B.	$351,450	$83,333
C.	$355,000	$48,450

- **Correct Answer: C**

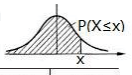
Common Probability Distributions

- **Standard normal distribution**
  - N(0,1) or Z
  - Standardization: if X~N(μ , σ^2), then $Z = \frac{X-\mu}{\sigma}$ ~ N(0,1)
  - Z-table
- **F(-z)=1-F(z)**
- **P(Z>z) = 1 –F(z)**

Common Probability Distributions

Cumulative Probabilities for a Standard Normal Distribution

$P(X \le x) = N(x)$ for $x \ge 0$



x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7582
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767

Example

- Assume the portfolio mean return is 12 percent and the standard deviation of return estimate is 22 percent per year. Assuming that a normal distribution describes returns. What is the probability that portfolio return will exceed 20 percent ?

- **Correct Answer:**

  For X=20%, Z=(20%-12%)/22%=0.3636.

  P(Z>x)=1-P($Z \le x$)=1-N(x)

  Rounding 0.3636 to 0.36, according to the table, N(0.36)=0.6406.

  Thus, P(X>20%)=1-0.6406=0.3594.

  The probability that portfolio return will exceed 20 percent is about 36 percent.

Common Probability Distributions

- **Shortfall risk: R_L= threshold level return, minimum return required**
  - Minimize ($R_p < R_L$)
- **Roy's safety-first criterion**
  - $[E(R_P) - R_L]/\sigma_P$
- **Maximize S-F-Ratio**
  - Maximize $SFR=\frac{E(R_P)-R_L}{\sigma_P}$ <=> Minimize $P(R_p < R_L)$

Example

- A portfolio manager gathered the following information about four possible asset allocations:

Allocation	Expected annual return	Standard deviation of return
A	10%	6%
B	25%	14%
C	18%	17%

The manager's client has stated that her minimum acceptable return is 8%. Based on Roy's safety-first criterion, the *most* appropriate allocation is:

A. Allocation A.
B. Allocation B.
C. Allocation C.

- **Correct Answer: B**

Example

- You are researching asset allocations for a client with an $1,000,000 portfolio. Although her investment objective is long-term growth, at the end of a year she may want to liquidate $40,000 of the portfolio to fund educational expenses. If that need arises, she would like to be able to take out he $40,000 without invading the initial capital of $1000,000. The following table shows three alternative allocations.

	A	B	C
Expected annual return	26	13	15
Standard deviation of return	28	9	21

Address these questions (assume normality for Parts 2 and 3):

1. Given the client's desire not to invade the $1,000,000 principal, what is the shortfall level, R_L? Use this shortfall level to answer Part 2.
2. According to the safety-first criterion, which of the tree allocations is the best?
3. What is the probability that the return on the safety-first optimal portfolio will be less than the shortfall level? (F(1.00)=0.8413)

Example

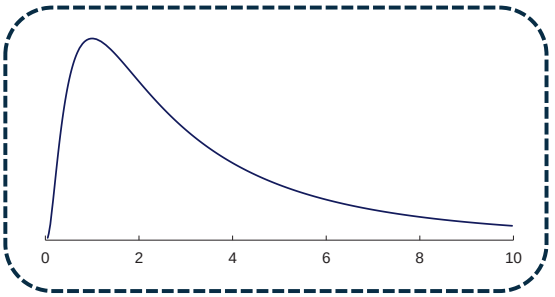
- **Correct Answer:**

1. R_L=40,000/1,000,000=4.00%
2. A: SFR_A=(26-4.00)/28=0.79; B: SFR_B=(13-4.00)/9=1.00; C: SFR_C=(15-4.00)/21=0.52; B is best.
3. $P(R_B<4.00)=P[(R_B-13)/9 < (4.00-13)/9] = F(-1.00) = 1-F(1.00)=1-0.8413=0.1587$

The safety-first optimal portfolio has a roughly 16% chance of not meeting a 4.00% return threshold.

Common Probability Distributions

- **Definition: If lnX is normal, then X is lognormal, which is used to describe the price of asset**
- **Features**
  - Right skewed
  - The values of random variables that follow lognormal distribution are always be positive, so it is useful for modeling asset prices



Common Probability Distributions

- **Discrete:** $EAY = (1+\frac{R}{m})^m - 1$
- **Continuous:** $EAR = \lim_{m\to\infty}(1+\frac{R}{m})^m - 1 = e^{R_c} - 1$
- $\frac{S_1}{S_0} = 1+HPR = e^{R_c}$ (持有一年)
- $1+HPR_T = e^{R_c \times T}$ (持有T年)

Example

- Compared to a normal distribution, a lognormal distribution is *least likely* to be:
  A. Skewed to the left.
  B. Skewed to the right.
  C. Useful in describing the distribution of stock prices.
- **Correct Answer: A**

- An analyst stated that lognormal distribution are suitable for describing asset returns and that normal distributions are suitable for describing distributions of asset prices. Is the analyst's statement correct with respect to:

	Lognormal distribution	Normal distribution
A.	No	No
B.	No	Yes
C.	Yes	No

- **Correct Answer: A**

Common Probability Distributions

- **Monte Carlo simulation vs Historical simulation**
  - Monte Carlo simulation is to generate a large number of random samples from specified probability distribution(s) to represent the operation of risk in the system. It is used in planning, in financial risk management, and in valuing complex securities;
    - ✓ Limitations:
      - ◆The operating of Monte Carlo simulation is very complex and we must assume a parameter distribution in advance.
      - ◆Monte Carlo simulation provides only statistical estimates, not exact results.
  - Historical simulation is to repeat sampling from a historical data series. Historical simulation is grounded in actual data but can reflect only risks represented in the sample historical data.
    - ✓ Limitations: Compared with Monte Carlo simulation, historical simulation does not lend itself to "what if " analyses.

Example

- Monte Carlo simulation is best described as:
  - A. An approach to back testing data
  - B. A restrictive form of scenario analysis
  - C. Providing a distribution of possible solutions to complex functions
- **Correct Answer: C**

Reading 11

Sampling and Estimation

Framework

1. Simple random and stratified random sampling, time-series and cross-sectional data
2. Central limit theorem
3. Standard error of the sample mean
4. The desirable properties of an estimator
5. Student's t-distribution
6. Criteria for selecting the appropriate test statistic
7. Calculate confidence interval
8. Five kinds of biases

Sampling and Estimation

- **Sampling and estimation**
  - Simple random sampling
  - Stratified random sampling: the population is divided into subpopulations based on some distinguishing characteristics. Stratum and cells=M*N
- **Sampling error: sampling error of the mean= sample mean- population mean**
- **The sample statistic itself is a random variable and has a probability distribution.**

Sampling and Estimation

- **Time-series data**
  - are a collection of observations at equally spaced intervals of time.
- **Cross-sectional data**
  - are a collection of observations at a single point in time.

Example

- Greg Goldman, research analyst in the fixed-income area of an investment bank, needs to determine the average duration of a sample of twenty 15-year fixed-coupon investment grade bonds. Goldman first categorizes the bonds by risk class and then randomly selects bonds from each class. After combining the bonds selected (bond ratings and other information taken as of March 31st of the current year), he calculates a sample mean duration of 10.5 years.

  Assuming that the actual population mean is 9.7 years, which of the following statements about Goldman's sampling process and sample is FALSE?

  A. Goldman used stratified random sampling.
  B. The sampling error of the means equals 0.8 years.
  C. Goldman is using time-series data.

- **Correct Answer: C**

Sampling and Estimation

- **Central Limit Theory**
  - For sufficiently large sample sizes n**(n ≥30)**, for any underlying distribution for a random variable with **known population mean and variance**, the sampling distribution
    - ✓ will be approximately normal,
    - ✓ has mean equals to the population mean μ
    - ✓ has variance equal to the population variance of the variable divided by sample size, which equals σ^2/n.

Standard error

- **Standard error of the sample mean**
  - Known population variance $\sigma_{\bar{x}} = \sigma / \sqrt{n}$
  - Unknown population variance $s_{\bar{x}} = s / \sqrt{n}$

Example

- An analyst gathered the following information:

Sample mean	12%
Sample size	50
Sample variance	30(%)^2

The standard error of the sample mean is *closest* to:

A. 0.47%.

B. 0.64%.

C. 0.77%.

- **Correct Answer: C**

Sampling and Estimation

- **The desirable properties of an estimator：**
  - **Unbiasedness:** the expected value of the estimator equals the population parameter.
  - **Efficiency:** the unbiased estimator has the smallest variance.
  - **Consistency:** the probability of accurate estimates increases as sample size increases.
    - ✓ the standard deviation of the parameter estimate decreases as the sample size increases
    - ✓ If the sample size raises, the standard error of the sample mean falls.

Example

- Shawn Choate is thinking about his graduate thesis. Still in the preliminary stage, he wants to choose a variable of study that has the most desirable statistical properties. The statistic he is presently considering has the following characteristics:
  - The expected value of the sample mean is equal to the population mean.
  - The variance of the sampling distribution is smaller than that for other estimators of the parameter.
  - As the sample size increases, the standard error of the sample mean rises and the sampling distribution is centered more closely on the mean.

Select the *best* choice. Choate's estimator is:

A. Unbiased, efficient, and consistent.

B. Efficient and consistent.

C. Unbiased and efficient.

- **Correct Answer: C**

Sampling and Estimation

- **Point estimate:** the statistic, computed from sample information, which is used to estimate the population parameter
- **Confidence interval estimate:** a range of values constructed from sample data so the parameter occurs within that range at a specified probability.

  α—the level of significance
- **Interval Estimation（also see Chapter: Hypothesis Testing ）**
  - Level of significance （alpha）
  - Degree of Confidence （1—alpha）
  - Confidence Interval = [Point Estimate +/- (reliability factor) * Standard error]

Example

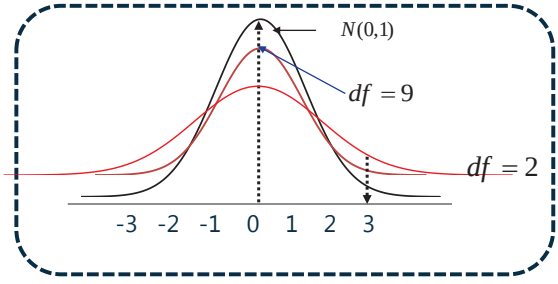
- The width of a confidence interval *most likely* will be smaller if the sample variance and number of observations, respectively, are:

	Sample variance	Number of observations
A.	Smaller	Smaller
B.	Smaller	Larger
C.	Larger	Smaller

- **Correct Answer: B**

Sampling and Estimation

- **Student's t-distribution**
  - Symmetrical
  - Degrees of freedom (df): n-1
  - Less peaked than a normal distribution ("fatter tails")
  - Student's t-distribution converges to the standard normal distribution as degrees of freedom goes to infinity.



Example

- An analyst stated that as degrees of freedom increase, a t-distribution will become more peaked and the tails of the t-distribution will become less fat. Is the analyst's statement correct with respect to the t-distribution:

	Become more peaked?	Tails becoming less fat?
A.	No	No
B.	No	Yes
C.	Yes	Yes

- **Correct Answer: C**

Sampling and Estimation

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \qquad \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

Sampling from:	Normal distribution with known variance	Normal distribution with unknown variance	Nonnormal distribution with known variance	Nonnormal distribution with unknown variance
Statistic for small sample size(n<30)	z- Statistic	t- Statistic	not available	not available
Statistic for large sample size(n>=30)	z- Statistic	t- Statistic/z	z- Statistic	t- Statistic/z

Example

- If the population is normally distributed and we don't know the variance, which test statistic should we select to construct confidence intervals for the population mean?
  - A. The t-statistic at α/2 with n-1 degrees of freedom
  - B. The t-statistic at α with n-1 degrees of freedom
  - C. The z-statistic at α with n-1 degrees of freedom
- **Correct Answer: A**

- When constructing a confidence interval for the population mean of nonnormal distribution when the population variance is unknown and the sample size large (n>30), an analyst may acceptably use:
  - A. Either a z-statistic or a t-statistic
  - B. Only a z-statistic or α with n degrees of freedom
  - C. Only a t-statistic or α/2 with n degrees of freedom
- **Correct Answer : A**

Sampling and Estimation

- **Data-mining bias**
  - Data-mining bias comes from finding models by repeatedly searching through databases for patterns.
- **Sample selection bias**
  - When data availability leads to certain assets being excluded from the analysis, we call the resulting problem sample selection bias.
- **Survivorship bias**
  - Survivorship bias occurs if companies are excluded from the analysis because they have gone out of business or because of reasons related to poor performance.
- **Look-ahead bias**
  - Look-ahead bias exists if the model uses data not available to market participants at the time the market participants act in the model.
- **Time-period bias**
  - time-period bias is present if the time period used makes the results time-period specific or if the time period used includes a point of structural change.

Example

- Sunil Hameed is a reporter with the weekly periodical The Fun Finance Times. Today, he is scheduled to interview a researcher who claims to have developed a successful technical trading strategy based on trading on the CEO's birthday (sample was taken from the Fortune 500). After the interview, Hameed summarizes his notes (partial transcript as follows). The researcher:
  - Used the same database of data for all his tests and has not tested the trading rule on out-of-sample data.
  - Excluded stocks for which he could not determine the CEO's birthday.
- Select the choice that *best* completes the following: Hameed concludes that the research is flawed because the data and process are biased by:
  - A. Data mining and sample selection bias.
  - B. Data mining and look-ahead bias.
  - C. Time-period bias and survivorship bias.
- **Correct Answer: A**

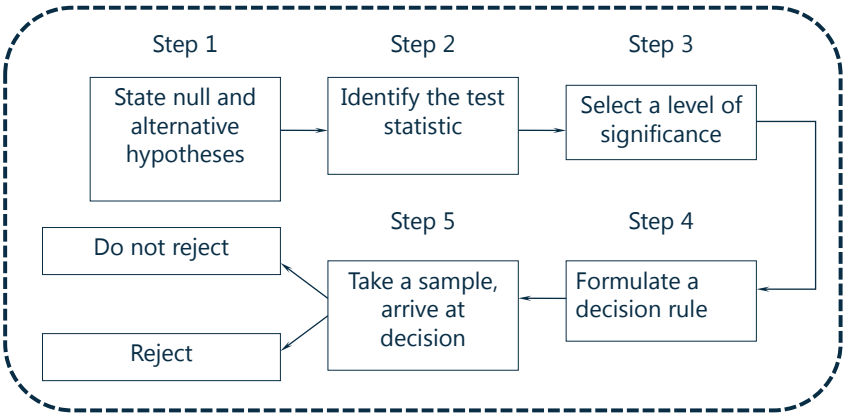
Reading 12

Hypothesis Testing

Framework

1. The steps of hypothesis testing
2. The null hypothesis and alternative hypothesis, one-tailed and two-tailed test
3. Test statistics
4. Type I and type II errors
5. Decision rule
6. The Chi-square test and F-test
7. Parameter tests and non-parameter tests

Hypothesis Testing



Hypothesis Testing

➢ **Step 1: Define Hypothesis**

- A hypothesis is a statement about one or more population parameters.
  - ✓ For population not sample
- Null hypothesis and Alternative hypothesis (we want to assess)
  - ✓ Null hypothesis is the fact we suspect and want to reject

$$H_0 : \mu = \mu_0 \qquad H_a : \mu \neq \mu_0$$

- One-tailed and Two-tailed tests of Hypothesis

Two-tailed	$H_0 : \mu = \mu_0 \qquad H_a : \mu \neq \mu_0$
One-tailed	$H_0 : \mu \leq \mu_0 \qquad H_a : \mu > \mu_0$ $or, H_0 : \mu \geq \mu_0 \qquad H_a : \mu < \mu_0$

Example

- Terry believes that the price of his houses is greater than $250,000. the appropriate alternative hypothesis is:
  - A. H_a: μ > $250,000
  - B. H_a: μ < $250,000
  - C. H_a: μ ≥ $250,000
- **Correct Answer: A**

- Mary is an analyst. She want to determine whether the mean time spent on research is different from two hours per day. The appropriate null hypothesis for the test is:
  - A. H_0: μ = 2 hours, one-tailed test.
  - B. H_0: μ ≥ 2 hours, two-tailed test.
  - C. H_0: μ = 2 hours, two-tailed test.
- **Correct Answer: C**

Hypothesis Testing

- **Step 2: Choose and Calculate Test statistic**

$$Test\ Statistic = \frac{Sample\ statistics - Hypothesized\ value}{stanard\ error\ of\ the\ sample\ statistic}$$

- Test Statistic follows Normal, T, Chi Square or F distributions
- Test Statistic has formula. Calculate it with the sample data. We should emphasize Test Statistic is calculated by ourselves not from the table.
- This is the general formula but only for Z and T distribution.

Examples：

$$Test\ Statistic = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \qquad Test\ Statistic = \frac{\overline{X} - \mu_0}{s / \sqrt{n}}$$

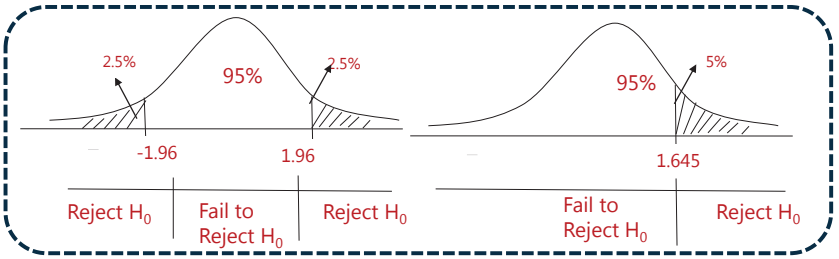
Hypothesis Testing

- **Step 3: Find Critical value**
  - Found in the Z, T, Chi Square or F distribution tables not calculated by us
  - Under given one tailed or two tailed assumption, critical value is determined solely by the significance level.

Hypothesis Testing

➢ **Step 4: Form Decision rule**

- Confidence interval (two tailed hypothesis only)
- Critical Value Method
  - ✓ Find Reject region with critical value
  - ✓ Reject H_0 if | test statistic | > critical value
  - ✓ Fail to reject H_0 if | test statistic | < critical value



Hypothesis Testing

➢ **Step 5: Draw a conclusion:**

- cannot say "accept the null hypothesis", only can say "cannot reject"
- ***** is significantly different from ******
- ***** is not significantly different from ******

Hypothesis Testing

➢ **Relation between Confidence Intervals and Hypothesis Tests**

- Confidence Interval = [sample statistic ± (critical value)(standard error)]
- Center of Interval = sample statistic
- Length of Interval = 2*(critical value)(standard error)

Example

- A financial analyst gathered data about the daily returns of a mutual fund over a recent 480-day period. The mean return is 0.2%, and the sample standard deviation of daily returns is 0.30%. The analyst considers that the mean daily return is unequal to zero. Construct a hypothesis test of the analyst's opinion.
- **Correct Answer:**
  - **Step 1:** We need to state null and alternative hypotheses. the analyst expects to reject the null hypothesis.

$$H_0 : \mu = 0 \ versus \ H_a : \mu \neq 0$$

  - **Step 2:** The standard error of the sample mean: the standard deviation of the sample is adjusted. IF the sample mean is $\overline{X}$ and the sample size is n , the standard error is calculated as: $S_{\overline{x}} = S / \sqrt{n}$ So the standard error of the sample mean for a sample size of 480 is $0.003 / \sqrt{480}$, and our test statistic is: $\frac{0.002}{0.003 / \sqrt{480}} = \frac{0.002}{0.000137} = 14.61$

Hypothesis Testing

- **Step 3:** Because the null hypothesis is an equality, this is a two-tailed test. The level of significance is 5% , so the critical z-values for the two-railed test are ± 1.96.
- **Step 4:** The decision rule can be stated as:
  Reject H_0 if test statistic < -1.96 or test statistic > +1.96
- **Step 5:** Since 14.61 > 1.96, we reject the null hypothesis, and we accept alternative hypotheses that the mean daily option return is unequal to zero. That is, the mean daily return of 0.002 is statistically different from zero given the sample's standard deviation and size.

Hypothesis Testing

- Perform a z-test using the mutual fund data from the previous example to test the opinion that option returns **are greater than zero**.
- **Correct Answer:**
  - **Step 1:** In this case, we use a one-tailed test and state null and alternative hypotheses:
    H_0: μ ≤ 0 versus H_a: μ > 0
  - **Step 2:** From the previous example, we know that the test statistic for the option return sample is 14.61.
  - **Step 3:** , this is a one-tailed test. The level of significance is 5% , so the critical z-values for the two-railed test are ± 1.645.
  - **Step 4:** The appropriate decision rule is:
    Reject H_0 if test statistic > 1.645
  - **Step 5:** Because 14.61 > 1.645, we reject the null hypothesis and conclude that mean returns are statistically greater than zero at a 5% level of significance.

Example

- An analyst conducts a two-tailed test to determine if earnings estimates are significantly different from reported earnings. The sample size was over 100. The computed Z-statistic is 1.25. At a 5 percent significance level, which of the following statements is TRUE?
  - A. Both the null and the alternative are significant.
  - B. You cannot determine what to do with the information given.
  - C. Fail to reject the null hypothesis and conclude that the earnings estimates are not significantly different from reported earnings.
- **Correct Answer: C**

Hypothesis Testing

- **P-value Method**
  - The **p-value** is the smallest level of significance based on the assumption that the null hypothesis will be rejected.
  - p-value < α: reject H_0; p-value > α: do not reject H_0.
  - P↓, easier to reject H_0

- The p-value for a two-tailed test of sample mean is 1.68%. Which of the following is true?
  - A. We can reject the null with 95% confidence
  - B. We can reject the null with 99% confidence
  - C. the largest probability of rejecting the null hypothesis is 1.68%
- **Correct Answer: A**

Hypothesis Testing

- **Type I error and Type II error**
  - **Type I error:** 拒真，reject the null hypothesis when it's actually true
    - ✓ Significance level (α): the probability of making a Type I error
    - ✓ Significance level =P(Type I error)=P(H_0×|H_0√)
  - **Type II error:** 取伪，fail to reject the null hypothesis when it's actually false
    - ✓ P(Type II error)=P(H_1×|H_1√)
    - ✓ **Power of a test:** the probability of correctly rejecting the null hypothesis when it is false
    - ✓ Power of a test = 1−P(Type II error)= P(H_1√|H_1√)

Hypothesis Testing

	H_0 is actually true	H_0 is actually false
Do not reject H_0	Correct	Type II error
Reject H_0	P (Type I error) =the significance level α	Correct Power of test =1- P (Type II error)

- With other conditions unchanged, either error probability arises at the cost of the other error probability decreasing.
- How to reduce both errors? **Increase the Sample Size.**

Example

- Kyra Mosby, M.D., has a patient who is complaining of severe abdominal pain. Based on an examination and the results from laboratory tests, Mosby states the following diagnosis hypothesis: Ho: Appendicitis, HA: Not Appendicitis. Dr. Mosby removes the patient's appendix and the patient still complains of pain. Subsequent tests show that the gall bladder was causing the problem. By taking out the patient's appendix, Dr. Mosby:
  A. Made a Type I error.
  B. Is correct.
  C. Made a Type II error.
- **Correct Answer: C**

Example

- If the sample size increases, the probability of get the Type I and Type II error will

	Type I	Type II
A.	increase	increase
B.	not change	not change
C.	decrease	decrease

- **Correct Answer: C**

Example

- All else equal, is specifying a larger significance level in a hypothesis test likely to increase the probability of a:

	Type I error?	Type II error?
A.	No	No
B.	No	Yes
C.	Yes	No

- **Correct Answer: C**

Example

- What is the definition of the power test? Power test is the probability to:
  - A. Reject the true null hypothesis while it is true
  - B. Reject the false null hypothesis while it is indeed false
  - C. Can not reject the true hypothesis
- **Correct Answer: B**

Hypothesis Testing

- **Test Population Mean**
  - One normal population with **known variance：Z distribution**
  - One normal population with **unknown variance：T distribution**

	Normal population, n<30	n>30
Known variance	z-test	z-test
Unknown variance	t-test	t-test or z-test

Summary of Hypothesis Testing

Test type	Assumptions	H_0	Test-statistic	Critical value
Mean hypothesis testing	Normally distributed population, known population variance	$\mu=0$	$Z=\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$	N(0,1)
	Normally distributed population, unknown population variance	$\mu=0$	$t=\frac{\bar{x}-\mu_0}{s/\sqrt{n}}$	t(n-1)
	Independent populations, unknown population variances assumed equal	$\mu_1-\mu_2=0$	$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{s_p^2/n_1+s_p^2/n_2}}$, where $s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$	$t(n_1+n_2-2)$
	Independent populations, unknown population variances not assumed equal	$\mu_1-\mu_2=0$	$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$	t*
	Samples not independent, paired comparisons test	$\mu_d=0$	$t=\bar{d}/s_{\bar{d}}$	t(n-1)

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Summary of Hypothesis Testing

Test type	Assumptions	H_0	Test-statistic	Critical value
Variance hypothesis testing	Normally distributed population	$\sigma^2=\sigma_0^2$	$\chi^2=\frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
	Two independent normally distributed populations	$\sigma_1^2=\sigma_2^2$	$F=s_1^2/s_2^2$	$F(n_1-1,n_2-1)$

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Hypothesis Testing

- An analyst collects the following data related to paired observations for Sample A and Sample B. Assume that both samples are drawn from normally distributed populations and that the population variances are not known.

Paired Observation	Sample A Value	Sample B Value
1	25	18
2	12	9
3	-5	-8
4	6	3
5	-8	1

- The *t*-statistic to test the hypothesis that the mean difference is equal to zero is *closest* to:
  A. 0.23
  B. 0.27
  C. 0.52
- **Correct Answer: C**

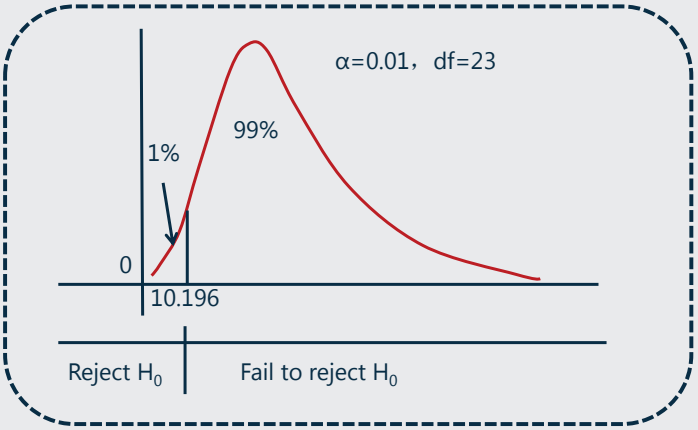
Hypothesis Testing

- **Example: Chi-square test for a single population variance**
  ABC Equity Fund has been in existence for 24 months. During this period, it has achieved a mean monthly return of 1.50 percent with a sample standard deviation of monthly returns of 3.70 percent. An analyst now wants to test a claim that the particular investment disciplines followed by ABC result in a standard deviation of monthly returns of less than 4 percent at the 0.01 level of significance.
- **Correct Answer:**
  - **Step 1**: We have a "less than" alternative hypothesis, where σ is the underlying standard deviation of return on ABC Equity Fund. Being careful to square standard deviation to obtain a test in terms of variance, the hypotheses are
    H_0: $\sigma^2 \geq 16.0$ versus H_a: $\sigma^2 < 16.0$.
  - **Step 2**: The test statistic is $\chi 2$ with 24 − 1 = 23 degrees of freedom.

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{23 \times 3.70^2}{4^2} = 19.68$$

Hypothesis Testing

- **Decision Rule for a One-Tailed Chi-Square Test of a Single Population Variance**



Hypothesis Testing

- **Step 3**: The lower 0.01 rejection point (α=0.01) is found on the line for df = 23, under the 0.99 column (99 percent probability in the right tail, to give 0.99 probability of getting a test statistic this large or larger). The rejection point is 10.196.
- **Step 4**: We will reject the null if we find that χ^2 is less than 10.196.
- **Step 5**: Because 19.68 (the calculated value of the test statistic) is not less than 10.196, we do not reject the null hypothesis. We cannot conclude that ABC's investment disciplines result in a standard deviation of monthly returns of less than 4 percent.

Hypothesis Testing

- **Example: F-test for equal variances**
- An analyst are investigating whether the population variance of returns on the KOSPI Index of the South Korean stock market changed subsequent to the global financial crisis that peaked in 2008. For this investigation, you are considering 2004 to 2006 as the pre-crisis period and 2010 to 2012 as the post-crisis period. You gather the data in Table for 156 weeks of returns during 2004 to 2006 and 156 weeks of returns during 2010 to 2012. You have specified a 0.01 level of significance.

KOSPI Index Returns and Variance before and after the Global Financial Crisis of the Late 2000s

	n	Mean Weekly Return (%)	Variance of Returns
Before crisis: 2004 to 2006	156	0.358	7.240
After crisis: 2010 to 2012	156	0.110	6.269

Hypothesis Testing

- **Step 1**: We have a "not equal to" alternative hypothesis:

  H_0: $\sigma^2_{Before} = \sigma^2_{after}$ *versus* H_a: $\sigma^2_{Before} \neq \sigma^2_{after}$

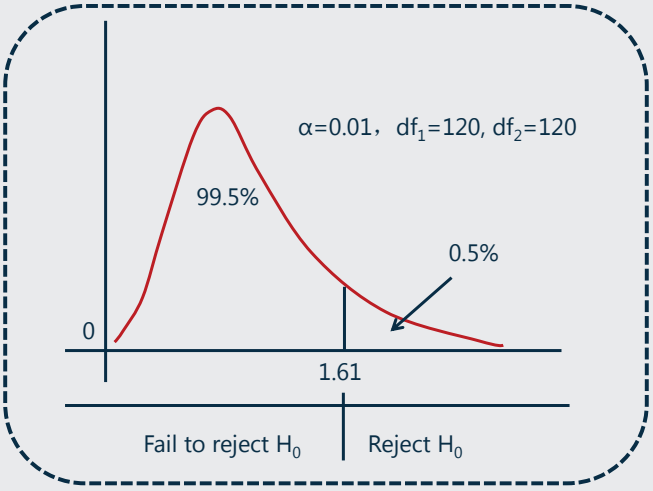
- **Step 2**: Select the appropriate test statistic. For tests of difference between variances, the appropriate test statistic is $F = s_1^2/s_2^2$ with 156 − 1 = 155 numerator and denominator degrees of freedom.

  $F = S_1^2/S_2^2 = 7.240/6.269 = 1.155$

- **Step 3**: This is a two-tailed test, we use F-tables for the 0.005 level (= 0.01/2) to give a 0.01 significance level (α=0.01). In the tables in the back of the volume, the closest value to 155 degrees of freedom is 120 degrees of freedom. At the 0.01 level, the rejection point is 1.61.

Hypothesis Testing

- **Decision rule for F-test**



Hypothesis Testing

- **Step 4**: We will reject the null if we find that χ^2 is more than 1.61.
- **Step 5**: Because 1.155 is less than the critical value 1.61, we cannot reject the null hypothesis that the population variance of returns is the same in the pre- and post-global financial crisis periods.

Example

- Which type of test is used to test if the square deviations of the two normal distribution population are equal?
  - A. T-test
  - B. χ^2-test
  - C. F-test
- **Correct Answer: C**
- Smith wants to know whether the mean returns of two stocks are the same. If the two normally distributed stock returns are dependent, the appropriate type of test and test statistic are:
  - A. Difference in means test, t-statistic
  - B. Paired comparisons test, t-statistic
  - C. Difference in means test, F-statistic
- **Correct Answer: B.**

Hypothesis Testing

- **Parametric tests**
  - based on specific distributional assumptions for the population
  - concerning a parameter of population.
  - For example, t-test.
- **Nonparametric tests**
  - a nonparametric test either **is not concerned with a parameter** or **makes minimal assumptions about the population** from which the sample comes.
  - Nonparametric tests are used:
    - ✓ when data do not meet distributional assumptions.
      - ◆Example: hypothesis test of the mean value for a variable, but the distribution of the variable is not normal and the sample size is small so that neither the t-test nor the z-test are appropriate.
    - ✓ when data are given in ranks.
    - ✓ when the hypothesis we are addressing does not concern a parameter.

Reading 13

Technical Analysis

Framework

1. The principles of technical analysis, its applications, and its underlying assumptions
2. Types of charts
3. The uses of trend
4. Common chart patterns
5. Common analysis indicators
6. The use of cycles

Technical Analysis

- **Principles:**
  - Prices are the result of the interaction of supply and demand in the real time.
  - The greater the volume of trades, the more impact that market participants will have on price.
  - Trades determine volume and price.
- **Assumptions:**
  - Market prices reflect both rational and irrational investor behavior.
    - ✓ Market trends and patterns reflect the irrational human behavior.
    - ✓ Efficient markets hypothesis dose not hold.
    - ✓ Market trends and patterns repeat themselves and are somewhat predictable.

Example

- Technical analysis relies most importantly on:
  A. price and volume data.
  B. accurate financial statements.
  C. fundamental analysis to confirm conclusions.

- **Correct Answer: A**

- Which of the following is not an assumption of technical analysis?
  A. Security markets are efficient.
  B. The security under analysis is freely traded.
  C. Market trends and patterns tend to repeat themselves.

- **Correct Answer: A**

Technical Analysis

- **The differences among technicians, fundamentalists and Efficient market followers.**
  - Fundamental analysis of a firm seeks to **determine the underlying long-term(intrinsic) value** of an asset by using the financial statements and other information.
  - While technical analysis uses more concrete data, primarily **price and volume data**, and seek to project the level at which a financial instrument will trade.
  - Fundamentalists believe that prices react quickly to changing stock values, while technicians believe that the reaction is slow.
  - Technicians look for changes in supply and demand, while fundamentalists look for changes in value.

Technical Analysis

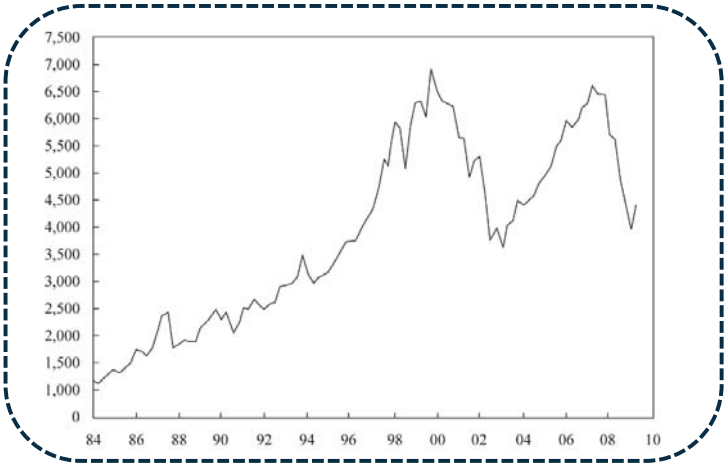
- **Advantages of technical analysis:**
  - Actual price and volume data is easy to access
  - Technical analysis is objective (although require subjective judgment), while much of the data used in fundamental analysis is subject to assumptions or restatements.
  - It can be applied to the prices of assets that do not produce future cash flows, such as commodities.
  - Fundamental analysis may have the risk of financial statement fraud, while technical analysis doesn't have.
- **Disadvantage:**
  - In markets that are subject to large outside manipulation, the application of technical analysis is limited.
  - Technical analysis is also limited in illiquid markets, where even modestly sized trades can have an inordinate impact on prices.

Technical Analysis

- Charts are the graphical display of **price and volume data**.
  - Horizontal axis: usually time interval (daily, weekly, monthly)
  - Vertical axis: Price level
- **Types of charts:**
  - Line charts
  - Bar charts
  - Candlestick charts
  - Point and figure charts

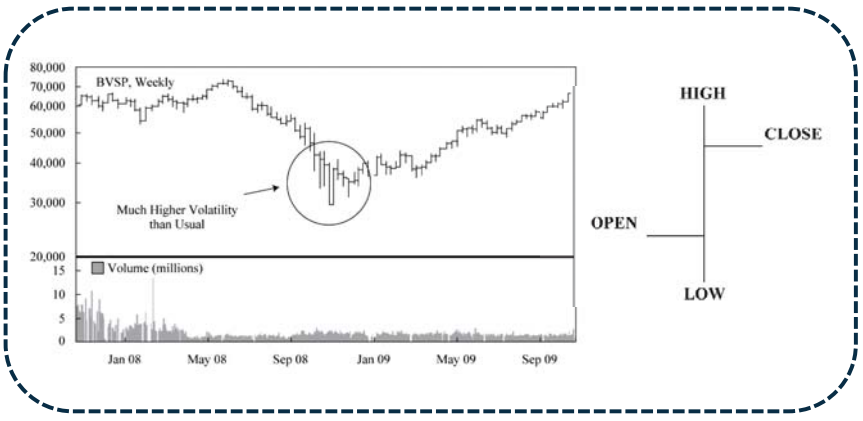
Technical Analysis

- **Line Charts** are a simple graphic display of price trends over time. Line charts are typically drawn with closing prices as the data points.



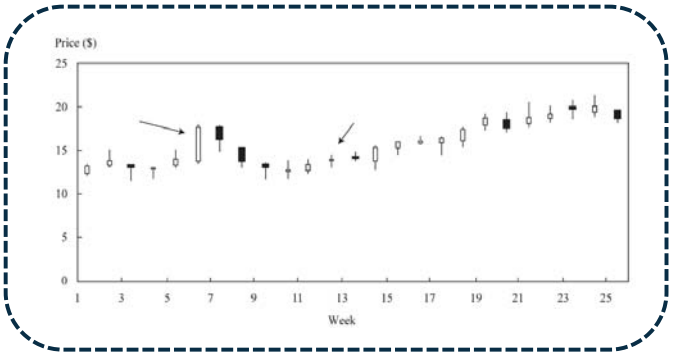
Technical Analysis

- **Bar charts** have four bits of data in each entry—the high and low price encountered during the time interval plus the opening and closing prices.



Technical Analysis

- **Candlestick charts** provides four prices per data point entry: the opening and closing prices and the high and low prices during the period.
  - Box is clear: closing price>opening price;
  - Box is filled: closing price<opening price



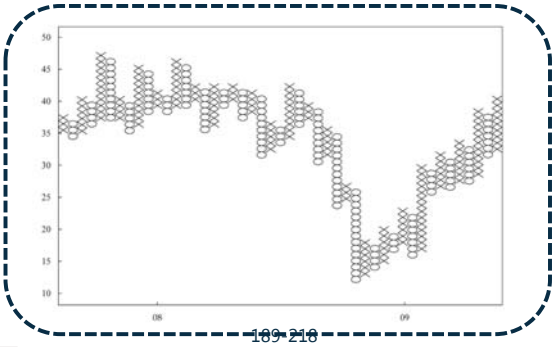
Example

- A candlestick chart is similar to a bar chart except that the candlestick chart:
  - A. represents upward movements in price with X's.
  - B. also graphically shows the range of the period's highs and lows.
  - C. has a body that is light or dark depending on whether the security closed higher or lower than its open.

- **Correct Answer: C**

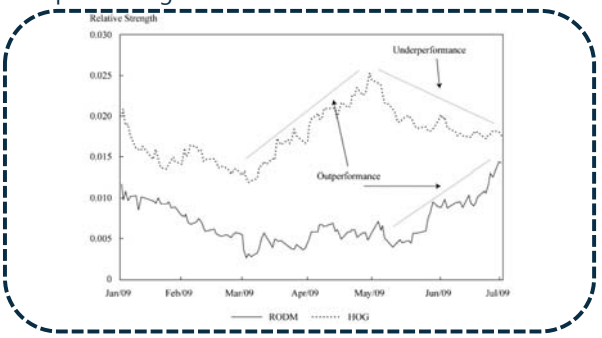
Technical Analysis

- **Point and figure charts** are helpful in identifying changes in the direction of price movements.
  - Starting form opening price;
  - X: increase of one box size, O: indicate a decrease.
  - Analyst will begin the next column when the price changes in the opposite direction by at least the reversal size (3 times the box size).



Technical Analysis

- **Relative strength analysis:** compare the performance of a particular asset, such as a common stock, with that of some benchmark.
  - **Positive relative strength**: an increasing trend indicates that the asset is outperforming the benchmark
  - **Negative relative strength**: an decreasing trend indicates that the asset is underperforming the benchmark



Technical Analysis

- **Trend:** is the most important aspect of technical analysis.
  - Uptrend: An uptrend for a security is when the price goes to higher highs and higher lows. (Demand>Supply)
  - Downtrend: is when a security makes lower lows and lower highs. (Demand<Supply)
- **Trend line:** can help to identify whether a trend is continuing or reversing.
  - Uptrend line: connecting the low of the price chart.
  - Downtrend line: connecting the highs of the price chart.
  - When price drops through and below the trend line by a significant amount, indicate that the uptrend is over and may signal a further deadline in the price.

Example：

- A downtrend line is constructed by drawing a line connecting:
  A. the lows of the price chart.
  B. the highs of the price chart.
  C. the highest high to the lowest low of the price chart.

- **Correct Answer: B**

Technical Analysis

- **Support level:** a low price range in which buying activity is sufficient to stop the decline in price.
- **Resistance level:** a price range in which selling is sufficient to stop the rise in price.
- **Change in polarity:**
  - once a support level is breached, it becomes a resistance level.
  - once a resistance levels breached, it becomes a support level.

Technical Analysis

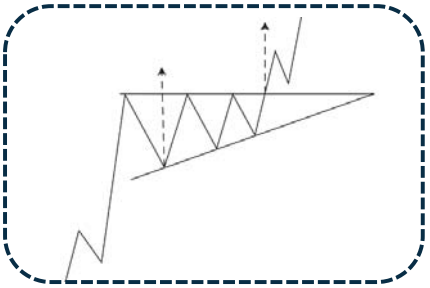
- **Common chart patterns.**
  - **Reversal patterns**
    - ✓ For uptrend: Head-and shoulders pattern, Double top and triple top
    - ✓ For downtrend: inverse head-and shoulders pattern, Double bottom, and triple bottom
  - **Continuation patterns**
    - ✓ Triangles
    - ✓ Rectangles

Technical Analysis

- **Head-and-shoulders pattern**
  - The size of the head-and-shoulders pattern: the difference in price between the head and the neckline.
  - **Price target** = Neckline – (Head – Neckline)
  - Inverse head and shoulders pattern: price target = neckline + (neckline – head)

Technical Analysis

- **Triangles:** A triangle pattern forms as the range between high and low prices narrows, visually forming a triangle.
- **Rectangles**: A rectangle pattern is a continuation pattern formed by two parallel trend lines, one formed by connecting the high prices during the pattern, and the other formed by the lows.
- **Flags and pennants:** the form over short periods of time-on a daily price chart, typically over a week.



Technical Analysis

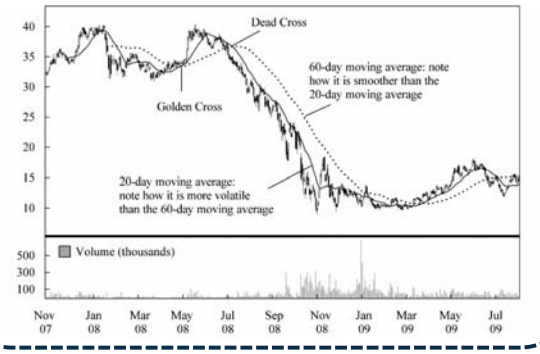
- **Technical Analysis Indicators**
  - **Price-based**
    - ✓ Moving average lines
    - ✓ Bollinger bands

Technical Analysis

- **Moving Average:** is the average of the closing price of a security over a specified number of periods.
  - Technicians commonly use a simple moving average, which weights each price equally in the calculation of the average price.
  - Some technicians prefer to use an exponential moving average (also called an exponentially smoothed moving average), which gives the greatest weight to recent prices while giving exponentially less weight to older prices.
  - **Trading strategies**
    - ✓ First, whether **price** is above or below its moving average is important.
      - ◆ A security that has been trending down in price will trade below its moving average, and a security that has been trending up will trade above its moving average.
    - ✓ Second, the **distance** between the moving-average line and price is also significant.
      - ◆ Once price begins to move back up toward its moving-average line, this line can serve as a resistance level.

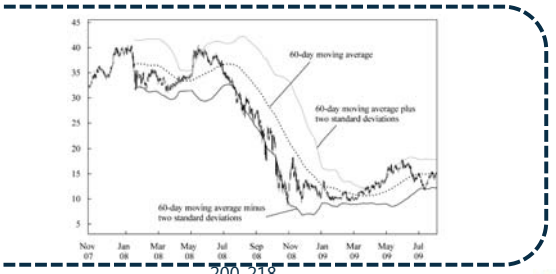
Technical Analysis

- When a short-term moving average crosses from underneath a longer-term average, this movement is considered **bullish** and is termed a **golden cross**.
- Conversely, when a short-term moving average crosses from above a longer-term moving average, this movement is considered **bearish** and is called a **dead cross**.



Technical Analysis

- **Bollinger bands**
  - Moving average +/- 2σ
  - **Trading strategies**
    - ✓ Investor sells when a security price reaches the upper band and buys when it reaches the lower band. (This strategy assumes that the security price will stay within the bands.)
    - ✓ The long-term investors might actually **buy** on a significant breakout above the upper boundary band.
    - ✓ The long-term investor would **sell** on a significant breakout below the lower band.



Technical Analysis

- **Technical Analysis Indicators**
  - **Momentum oscillators**
    - ✓ Rate of change oscillator
    - ✓ Relative Strength Index
    - ✓ Moving average convergence/divergence
    - ✓ Stochastic oscillator

Technical Analysis

➢ **Rate of Change Oscillator (ROC)**

$$M = (V - Vx) \times 100 \qquad M = \frac{V}{V_x} \times 100$$

- where
  - ✓ M = momentum oscillator value
  - ✓ V = last closing price
  - ✓ Vx = closing price x days ago, typically 10 days
- **Strategy**
  - ✓ If the ROC oscillator crosses into positive territory during an uptrend, it is a **buy signal.**
  - ✓ If it enters into negative territory during a downtrend, it is considered a **sell signal**.

Technical Analysis

➢ **Relative Strength Index (RSI)**

$$RSI = 100 - \frac{100}{1 + RS}$$

$$where\ RS = \frac{\sum(\text{Up changes for the period under consideration})}{\sum(\text{Down changes for the period under consideration})}$$

- An RSI is based on the ratio of total price increases to total price decreases over a selected number of periods.
- The index construction forces that RSI to lie within 0 to 100.
- **Strategy**
  - ✓ A value above 70 represents an **overbought** situation.
  - ✓ A Value below 30 suggests the asset is **oversold**.

Technical Analysis

➢ **Stochastic Oscillator**

$$\%K = 100\left(\frac{C - L14}{H14 - L14}\right)$$

where

C = latest closing price

$L14$ = lowest price in past 14 days

$H14$ = highest price in past 14 days

$\%D$ = average of the last three %K values calculated daily

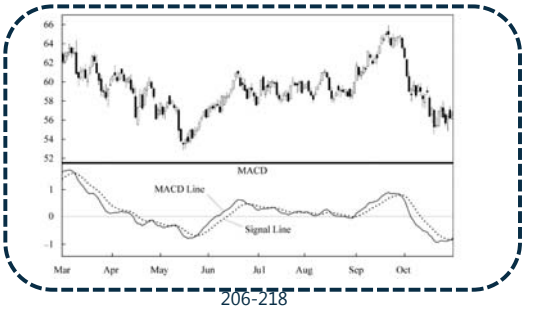
- The absolute level of the two lines should be considered in light of their normal range.
- **Strategy**
  - ✓ **Movements above** this range indicate to a technician an overbought security and are considered **bearish;**
  - ✓ **Movements below** this range indicate an oversold security and are considered **bullish.**

Technical Analysis

- **Stochastic Oscillator**
  - Crossovers of the two lines can also give trading signals the same way crossovers of two moving averages give signals.
    - ✓ When the %K moves from below the %D line to above it, this move is considered **a bullish short-term trading signal;**
    - ✓ Conversely, when %K moves from above the %D line to below it, this pattern is considered **bearish**.

Technical Analysis

- **Moving Average Convergence/Divergence (MACD)**
  - The MACD is the difference between a short-term and a long-term moving average of the security's price. The MACD is constructed by calculating two lines, **the MACD line and the signal line:**
  - **MACD line**: difference between two exponentially smoothed moving averages, generally 12 and 26 days.
  - **Signal line**: exponentially smoothed average of MACD line, generally 9 days.



Technical Analysis

- **Technical Analysis Indicators**
  - **Sentiment Indicators**
    - ✓ Put/call ratio
    - ✓ Volatility Index
    - ✓ Margin debt
    - ✓ Short interest ratio

Technical Analysis

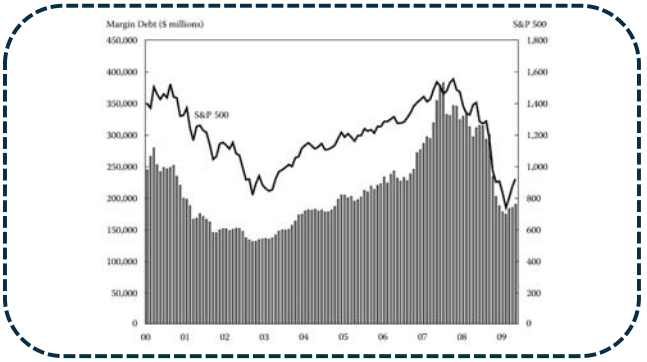
- **Put/call ratio**
  - The put /call ratio is the volume of put options traded divided by the volume of call option traded.
  - **Analysis:**
    - ✓ A **high** put/ call ratio usually indicates **bearish market**
      - ◆Investors who buy put options on a security are presumably bearish, and investors who buy call options are presumably bullish.
    - ✓ At **extreme highs** in the put/call ratio, market sentiment is said to be so extremely **negative** that an increase in price is likely.

Technical Analysis

- **Volatility index (VIX)**
  - The VIX is a measure of near-term market volatility calculated by the Chicago Board Options Exchange.
  - **Analysis:**
    - ✓ The VIX rises when market participants become fearful of an impending market decline. These participants then bid up the price of puts, and the result is an **increase** in the VIX level.
    - ✓ When other indicators suggest that the market is oversold and the VIX is at an extreme high, this combination is considered **bullish**.

Technical Analysis

- **Margin debt**
  - When stock margin debt is **increasing**, investors are aggressively buying and stock prices will **move higher** because of increased demand.
  - **Falling** prices may trigger margin calls and forced selling, thereby driving prices **even lower**.



Technical Analysis

- **Short interest ratio**
  - Investors sell shares short when they believe the share prices will decline. The number of shares of a particular security that are currently sold short is called **"short interest."**
  - **Short interest ratio = Short interest / Average daily trading volume**
  - **Analysis:**
    - ✓ If a large number of shares are sold short and the short interest ratio is high, the market should expect a falling price for the shares because of so much negative sentiment about them.
    - ✓ Therefore, the short interest ratio constitutes future demand for the shares.

Technical Analysis

- **Technical Analysis Indicators**
  - **Flow of funds**
    - ✓ Short-term trading index
    - ✓ Margin debt
    - ✓ Mutual fund cash position
    - ✓ New equity issuance

Technical Analysis

- **Arms index or Short-term trading index (TRIN)**
  - The TRIN is applied to a broad market to measure the relative extent to which money is moving into or out of rising and declining stocks.

$$\text{Arms Index}=\frac{\text{Number of advancing issues} \div \text{Number of declining issues}}{\text{Volume of advancing issues} \div \text{Volume of declining issues}}$$

  - **Analysis**
    - ✓ When the index value close to 1, the market is in balance.
    - ✓ Index values above 1 means that there is more volume in declining stocks, while an index value below 1 means that most trading activity is in rising stocks.

Technical Analysis

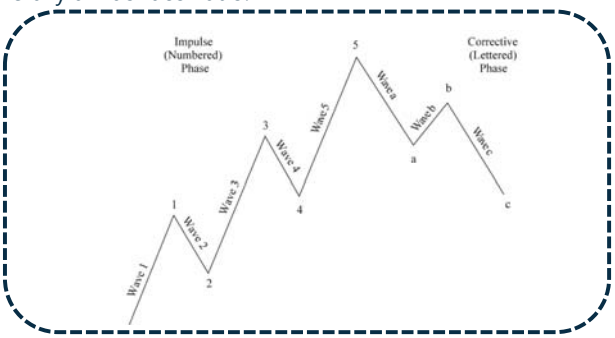
- **Mutual fund cash position**
  - The mutual fund cash position is another example of a **contrarian indicator.**
  - When mutual funds cash position is low, fund managers have already bought, and the effects of their purchases are already reflected in security prices.
- **New equity issuance**
  - The owners want to offer the shares when they can sell them at a premium price.
  - **Premium prices occur near market tops**.
  - The new equity issuance indicator suggests that as the number of initial public offerings increases, the upward price trend may be about to turn down.

Technical Analysis

- **Cycle theory: the study of cycles in the markets is part of broader cycle studies that exist in numerous fields of study.**
  - 4-year presidential cycles: related to election years in the USA
    - ✓ Decennial patterns: 10-year cycles
    - ✓ Kondratieff wave: 18-year cycles, 54-year cycles

Technical Analysis

- **Elliott wave theory:** the market moves in regular, repeated waves or cycles.
- **Waves:** how the market moved in a pattern of five waves moving up.
  - **up trend**: consist of 5 upward waves and 3 downward waves
  - **down trend**: consist of 5 downward waves and 3 upward waves
  - **Fibonacci ratios**: the ratio of the size of subsequent waves was generally a Fibonacci ratio.



Technical Analysis

- **Intermarket analysis:** is a field within technical analysis that combines analysis of major categories of securities-namely, equities, bonds, currencies, and commodities-to identify market trends and possible inflections in a trend.
- Intermarket analysis can also be used to **identify sectors** of the equity market to invest in-often in connection with technical observations of the business cycle at any time.

It's not the end but just beginning.

Life is short. If there was ever a moment to follow your passion and do something that matters to you, that moment is now.

生命苦短，如果你有一个机会跟随自己的激情去做你认为重要的事，那么这个机会就是现在。

Portfolio Management

CFA一级培训项目

讲师： 周琪

10/% Contribution Breeds Professionalism

师资介绍-周琪

工作职称：金程教育金融研究院CFA/FRM高级培训师
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Topic Weightings in CFA Level I

Session NO.	Content	Weightings
Study Session 1	Ethics & Professional Standards	15
Study Session 2-3	Quantitative Analysis	12
Study Session 4-5	Economics	10
Study Session 6-9	Financial Reporting and Analysis	20
Study Session 10-11	Corporate Finance	7
Study Session 12	**Portfolio Management and Wealth Planning**	**7**
Study Session 13-14	Equity Investment	10
Study Session 15-16	Fixed Income	10
Study Session 17	Derivatives	5
Study Session 18	Alternative Investments	4

Framework

Portfolio Management

- **Portfolio Management**
  - R40 Portfolio Management: An Overview
  - R41 Risk Management: An Introduction
  - R42 Portfolio Risk and Return: Part I
  - R43 Portfolio Risk and Return: Part II
  - R44 Basic of Portfolio Planning and Construction

Reading 40

Portfolio Management: An Overview

Portfolio Management: An Overview

- **Portfolio approach**
  - Definition:
    - ✓ **From the perspective of risk and returns**, evaluate individual securities in relation to their contribution to the investment characteristics of the whole portfolio.
  - **Diversification** provides an investor with a way to reduce the risk without necessarily decreasing their expected rate of return.
    - ✓ During times of severe market turmoil, correlations among assets tend to **increase**, which makes the **diversification less effective**.

Portfolio Management: An Overview

- **The types of investment management clients**
  - Individual investors
    - ✓ DC plan: the individual makes the investment decisions and bears the investment risk.
  - Institutional investors
    - ✓ DB plan: be funded by company contributions and have an obligation to provide specific benefits to retirees.
    - ✓ University endowments: dedicated to providing continuing financial support to a university and its students.
      - ◆E.g. Harvard University Endowment
    - ✓ Charitable foundations: established for funding grants that are consistent with the charitable foundation's objectives.
      - ◆E.g. Bill & Melinda Gates Foundation
    - ✓ Banks
    - ✓ Insurance companies
    - ✓ Investment companies. E.g. Mutual funds
    - ✓ Sovereign wealth funds (SWFs): investment funds owned by a government.

Portfolio Management: An Overview

- **Mutual funds and other forms of pooled investments**
  - Mutual funds: Open-end fund and Closed-end funds; Money market funds, Bond funds, Stock funds, Hybrid or balanced funds.
    - ✓ Index fund: track the performance of a particular index.
  - Exchange traded funds (ETFs)
  - Separately managed account
  - Hedge funds
  - Buyout funds
  - Venture capital funds

Portfolio Management: An Overview

- **ETF: investors buy the shares from other investors just as if they were buying or selling shares of stock.**
- **Comparison between Mutual funds and ETFs**
  - Expenses are lower for ETFs but, unlike mutual funds, investors do incur brokerage costs.
  - All purchases and redemptions in a mutual fund take place at the same price at the close of business. However, ETFs are constantly traded throughout the business day, and as such each purchase or sale takes place at the prevailing market price at that time.
  - For ETF, dividends are paid out to the shareholders, hence, there is a direct cash flow from the ETF. Index mutual funds usually reinvest the dividends that is not there with the index mutual fund.
  - The minimum required investment in an ETF is usually smaller. Investors can purchase as little as one share in an ETF, which is usually not the case with an index mutual fund.
  - ETFs are often cited as having tax advantages over index mutual funds.

Portfolio Management: An Overview

- **Comparison between Mutual funds and hedge funds**
  - Hedge fund strategies generally involve a significant amount of risk, driven in large measure by the liberal use of leverage and complexity. More recently, it has also involved the extensive use of derivatives.
  - A key difference between hedge funds and mutual funds is that the vast majority of hedge funds are exempt from many of the reporting requirements for the typical public investment company.

Portfolio Management: An Overview

- **Characteristics of different types of investors**

Investor	Time Horizon	Risk Tolerance	Liquidity Needs	Income Needs
Individuals	Varies by individual	Varies by individual	Varies by individual	Varies by individual
DB plan	Long	High	Quite low	High—mature funds Low—growing funds
Banks	Short	Quite low	High	Pay interest and operational expenses
Endowments and foundations	Very long	High	Quite low	Meet spending commitments
Insurance	Long—life Short—P&C	Quite low	High	Low
Mutual funds	Varies by fund	Varies by fund	High	Varies by fund

Portfolio Management Process

- **Planning step:**
  - Analyse the investor's needs: investment objectives and constraints
  - Develop an IPS: describes the investor's investment objectives and constraints; state an objective benchmark; reviewed and updated regularly.
- **Execution step:**
  - Asset allocation: top-down& bottom-up analysis;
  - Security analysis;
  - Portfolio construction.
- **Feedback step:**
  - Monitor and rebalance the portfolio;
  - Measure portfolio performance and report.

Reading 41

Risk Management: An Introduction

Risk management: An introduction

- **Risk**
  - Exposure to uncertainty
  - Many decision makers focus on return, which is not something that is easily controlled, as opposed to risk, or exposure to risk, which may actually be managed or controlled
- **Risk exposure**
  - The extent to which an entity's value may be affected through sensitivity to underlying risks.
- **Risk management**
  - Risk management is the process by which an organization or individual **defines** the level of risk to be taken, **measures** the level of risk being taken, and **adjusts** the latter toward the former; with the goal of **maximizing** the company's or portfolio's value or the individual's overall satisfaction, or utility.
  - It is comprises all the decisions and actions needed to best achieve organizational or personal objectives while **bearing a tolerable level of risk**.
  - **Not about minimizing risk.**

Risk management: An introduction

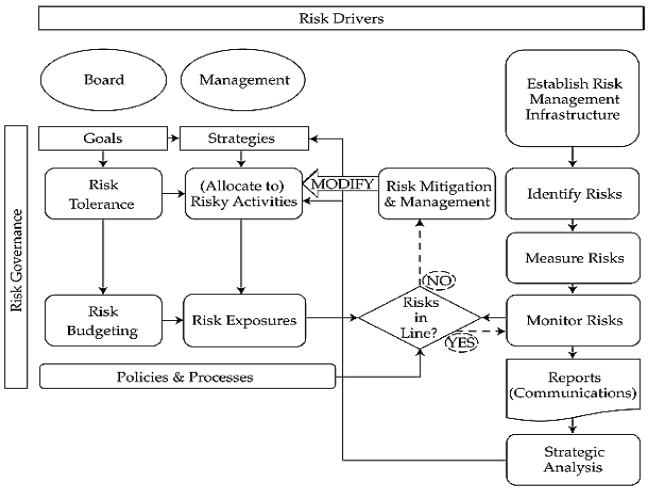
- **Risk management framework**
  - Risk governance
  - Risk identification and measurement
  - Risk infrastructure
  - Defined policies and processes
  - Risk monitoring, mitigation, and management
  - Communications
  - Strategic analysis or integration

Risk Management: An Introduction

- **Risk governance** is the top-level foundation for risk management, including risk oversight and setting risk tolerance for the organization.
- **Risk identification and measurement** is the quantitative and qualitative assessment of all potential sources of risk and the organization's risk exposures.
- **Risk infrastructure** comprises the resources and systems required to track and assess the organization's risk profile.
- **Risk policies and processes** are management's complement to risk governance at the operating level.
- **Risk mitigation and management** is the active monitoring and adjusting of risk exposures, integrating all the other factors of the risk management framework.
- **Communication** includes risk reporting and active feedback loops so that the risk process improves decision making.
- **Strategic risk analysis** and integration involves using these risk tools to rigorously sort out the factors that are and are not adding value as well as incorporating this analysis into the management decision process, with the intent of improving outcomes.

Risk Management: An Introduction

Exhibit 1. The Risk Management Framework in an Enterprise Context



Example:

- Which element of risk management makes up the quantitative part of the process?
  A. Communication
  B. Risk governance
  C. Risk identification and measurement

- **Correct Answer: C**

- Which of the following is the correct sequence of events for risk governance and management that focuses on the entire enterprise? Establishing:
  A. Risk tolerance, then risk budgeting, and then risk exposures.
  B. Risk exposures, then risk tolerance, and then risk budgeting.
  C. Risk budgeting, then risk exposures, and then risk tolerance.

- **Correct Answer: A**

Risk Management: An Introduction

- **Risk governance**
  - Risk governance is the foundation for risk management.
  - **Risk governance** refers to senior management's determination of the **risk tolerance** of the organization, the elements of its optimal **risk exposure strategy**, and the framework for oversight of the risk management function.
  - Employing a risk management committee, along with a chief risk officer (CRO), are hallmarks of a strong risk governance framework.
    - ✓ **Risk management committee** provides top decision makers with a forum for regularly considering risk management issues.

Risk Management: An Introduction

- **Risk tolerance**
  - At the governance level, the duty is generally not to select these activities—a job that usually falls to management—but to **establish the organization's risk appetite.**
    - ✓ Certain risks or levels of risks may be deemed acceptable, other risks deemed unacceptable, and in the middle are risks that may be pursued in a risk-limited fashion.
    - ✓ Said differently, risk tolerance identifies the extent to which the entity is willing to experience losses or opportunity costs and to fail in meeting its objectives
  - When analyzing risk tolerance, management should examine risks that may exist within the organization as well as those that may arise from outside. **("inside" view and "outside" view)**
  - The risk tolerance should be chosen and communicated **before** a crisis, and will serve as the high-level guidance for management in its strategic selection of risks.
  - If a company has **the ability to adapt quickly to adverse events** may allow for a higher risk tolerance.

Example

- Which of the following statements about risk tolerance is most accurate?
  - A. Risk tolerance us best discussed after a crisis, when awareness of risk is heightened.
  - B. The risk tolerance discussion is about the actions management will take to minimize losses.
  - C. The organization's risk tolerance describes the extent to which the organization is willing to experience losses.

- **Correct Answer: C**

  Risk tolerance identifies the extent to which the organization is willing to experience losses or opportunity costs and fail in meeting objectives. It is best discussed before a crisis and is primarily a risk governance or oversight issue at the board level, not a management or tactical one.

Risk Management: An Introduction

- **Risk budgeting** is any means of allocating investments or assets by their risk characteristics.
  - The process of risk budgeting forces the firm to consider **risk tradeoffs**.
  - The **goal** is to allocate the overall amount of acceptable risk to the mix of assets or investments that have the greatest expected returns over time. (**The return per unit of risk is the highest**.)

Risk Management: An Introduction

- Which of the following is not consistent with a risk-budgeting approach to portfolio management?
  - A. Limiting the beta of the portfolio to 0.75
  - B. Allocating investments by their amount of underlying risk sources or factors
  - C. Limiting the amount of money available to be spent on hedging strategies by each portfolio manager

- **Correct Answer: C**

  Risk budgeting does not require nor prohibit hedging, although hedging is available as an implementation tool to support risk budgeting and overall risk governance

Risk Management: An Introduction

- **Financial risks** refer to the risks that arise from events occurring in the financial markets. Examples are:
  - **Market risk**
    - ✓ Arises from movements in stock prices, interest rates, exchange rates, and commodity prices
  - **Credit risk**
    - ✓ The risk that a counterparty will not pay an amount owed
  - **Liquidity risk**
    - ✓ The risk that, as a result of degradation in market conditions or the lack of market participants, one will be unable to sell an asset without lowering the price to less than the fundamental value
    - ✓ Liquidity risk could also be called transaction cost risk and is most associated with **a widening bid-ask spread.**

Risk Management: An Introduction

- **Non-financial risks** consist of a variety of risks, including settlement risk, operational risk, legal risk, regulatory risk, accounting risk, tax risk, model risk, tail risk, and sovereign or political risk.
  - **Operational risk** is the risk that arises from within the operations of an organization and includes both human and system or process errors.
  - **Solvency risk** is the that an entity does not survive or succeed because it runs out of cash to meet its financial obligations.
- **Interaction between risks:**
  - Risks are not necessarily independent because many risks arise as a result of other risks; risk interactions can be extremely non-linear and harmful.

Risk Management: An Introduction

- Which of the following best describes an example of interactions among risks?
  - A. A stock in Russia declines at the same time as a stock in Japan declines
  - B. Political events cause a decline in economic conditions and an increase in credit spreads.
  - C. A market decline makes a derivative counterparty less creditworthy while causing it to owe more money on that derivative contract.
- **Correct Answer: C**

Risk Management: An Introduction

- **Risk metrics**
  - Standard deviation or volatility;
  - Asset-specific measures, such as beta or duration;
  - Derivative measures, such as delta, gamma, vega, and rho;
  - Tail measures such as VaR, CVaR and expected loss given default.
    - ✓ **Value at risk (VaR)** is a measure of the size of the tail of the distribution of profits on a portfolio or for an entity, which
      - ◆Three elements: an amount stated in units of currency, a time period, and a probability
      - ◆e.g. A VaR of $100 at 1% for one day means it is expected to lose a minimum of $100 in one day 1% of the time.
    - ✓ **Conditional VaR (CVaR)** is the weighted average of all loss outcomes in the statistical distribution that exceed the VaR loss.

Risk Management: An Introduction

- **Subjective and market-based estimates of risk**
  - Two methods of risk assessment that are used to supplement measures such as VaR and CVaR are **stress testing and scenario analysis.**
    - ✓ **Stress testing** examines the effects of a specific (usually extreme) change in a key variable such as an interest rate or exchange rate.
    - ✓ **Scenario analysis** refers to a similar what-if analysis of expected loss but incorporates changes in multiple inputs.

Risk Management: An Introduction

- **Methods of risk modification:**
  - **Risk prevention and avoidance**
    - ✓ Not engage in the activity with the uncertain outcome.
  - **Risk acceptance:** self-insurance and diversification
    - ✓ **Self-insurance** is obtained by setting aside sufficient capital to cover losses.
    - ✓ Another form of accepting risk, but doing so in the most efficient manner possible, is **diversification.**
  - **Risk transfer (insurance)**
    - ✓ Risk transfer is the process of passing on a risk to another party, often, but not always, in the form of an **insurance policy.**
  - **Risk shifting (derivatives)**
    - ✓ Whereas risk transfer refers to actions taken that pass the risk on to other parties, risk shifting refers to actions that change the distribution of risk outcomes. Risk shifting generally involves **derivatives** as the risk modification vehicle.
- The determinants of which method is best for modifying risk are the benefits weighed against the costs.

Example

- The best definition of value at risk is:
  - A. The expected loss if a counterparty defaults
  - B. The maximum loss an organization would expect to incur a holding period
  - C. The minimum loss expected over a holding period a certain percentage of the time.
- **Correct Answer: C**
- The choice of risk-modification method is based on:
  - A. Minimizing risk at the lowest cost
  - B. Maximizing return at the lowest cost
  - C. Weighing costs versus benefits in light of the entity's risk tolerance
- **Correct Answer: C**

Reading 42

Portfolio Risk and Return: Part I

Portfolio Risk and Return: Part I

- **HPR**
- **Average return**
  - Arithmetic mean return: has known statistical properties
  - Geometric mean return: accounts for the compounding of returns
  - Money-weighted rate of return: IRR
- **Other return measures**
  - Gross and net return: whether deduct M&A fees
  - Pretax nominal return: no adjustment has been made for taxes or inflation
  - After-tax nominal return: return after deducting the tax liability
  - Real return: nominal return adjusted for inflation. $(1+r_{real})=(1+r_{nominal})/(1+\pi)$
  - Leveraged return: the return on the investor's own money.
    - ✓ real estate

Portfolio Risk and Return: Part I

- **A higher return is not possible to attain in efficient markets and over long periods of time without accepting higher risk.**
- **Liquidity should be considered when invest, especially in emerging markets and for securities that trade infrequently.**

Portfolio Risk and Return: Part I

A single asset

- Expected return $E(R) = \sum_{i=1}^{n} P_i R_i = P_1 R_1 + P_2 R_2 + \cdots + P_n R_n$
- Variance of return $Var = \sigma^2 = \frac{1}{T}\sum_{i=1}^{T}(R_t - \mu)^2$
- Sample variance of return $s^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \bar{R})^2$
- Standard Deviation of Return

$$\sigma = \sqrt{\frac{1}{T}\sum_{i=1}^{T}(R_t - \mu)^2} \ (population) \qquad s = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_t - \bar{R})^2} \ (sample)$$

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Portfolio Risk and Return: Part I

Two asset portfolio

- Covariance of return

$$Cov_{1,2} = \frac{\sum_{t=1}^{T}(R_{t,1} - \mu_1)(R_{t,2} - \mu_2)}{T} \ (population)$$

$$Cov_{1,2} = \frac{\sum_{t=1}^{T}(R_{t,1} - \bar{R}_1)(R_{t,2} - \bar{R}_2)}{T-1} \ (sample)$$

- Correlation of return

$$\rho_{1,2} = \frac{Cov_{1,2}}{\sigma_1 \sigma_2} \qquad Cov_{1,2} = \rho_{1,2}\sigma_1\sigma_2$$

- Variance of return for the portfolio

$$\begin{aligned} Var_{portfolio} &= \sigma_p^2 \\ &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2} \\ &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2} \end{aligned}$$

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Portfolio Risk and Return: Part I

The portfolio standard deviation of many assets

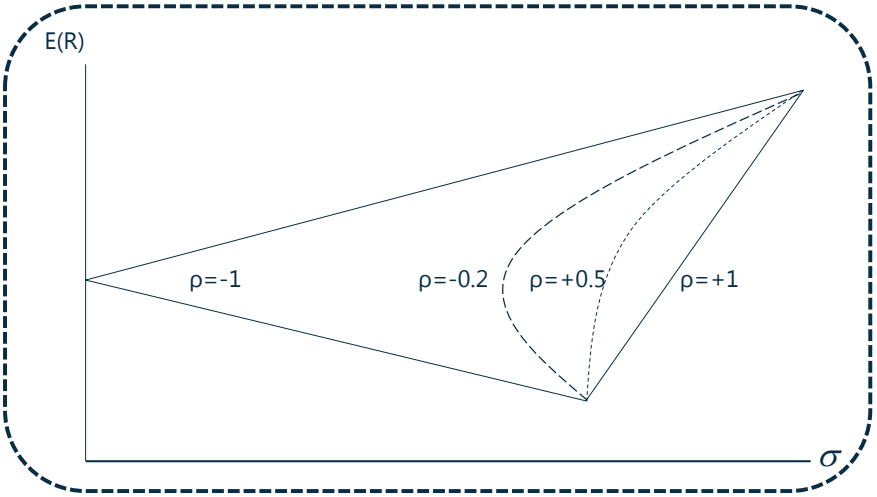
$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{\sum_{i=1,j=1}^{n} w_i w_j Cov_{i,j}} \text{ and } \sum_{i=1}^{n} w_i = 1$$

- The risk of a portfolio of risky assets depends on the asset weights and the standard deviations of the assets returns, and crucially on the correlation (covariance) of the asset returns.
- The lower the correlation between the returns of the stocks in the portfolio, all else equal, the greater the diversification benefits.
- Variance of N-asset portfolio **(same covariance**, **same weight**, **same volatility)**

$$\sigma_p^2 = \frac{\overline{\sigma^2}}{N} + \frac{N-1}{N}\overline{Cov}$$

Portfolio Risk and Return: Part I

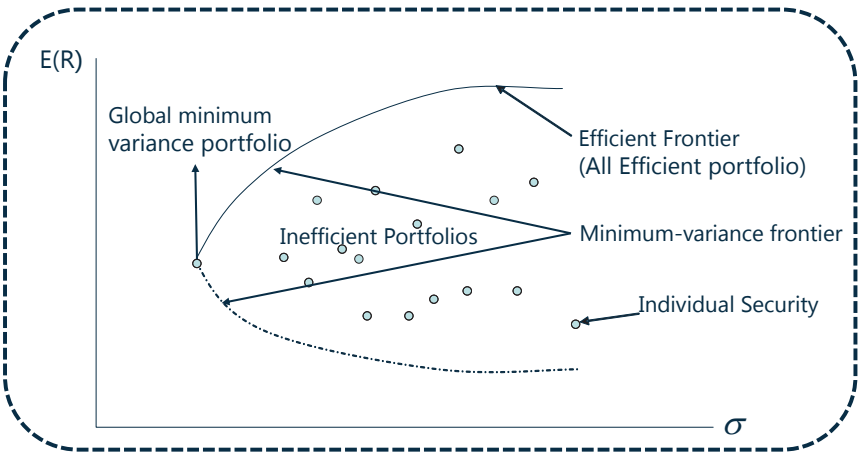
- **Risk and return for different values of correlation**



Portfolio Risk and Return: Part I

- **The Markowitz assumptions**
  - Returns distribution: Each investment can be measured by a probability distribution of expected returns over a given horizon
  - Utility maximization: Investor intends to maximize their expected utility over time horizon
  - Risk is variability: Risk is measured in terms of variance (standard deviation) of expected returns
  - Risk/return: Investors make their decision based on expected return and the risk
  - Risk aversion: Investors prefer less risk and given the same risks by given the same returns

Portfolio Risk and Return: Part I



Portfolio Risk and Return: Part I

- **Minimum variance frontier**
  - **Minimum-variance portfolio** is the portfolio available that has the lowest standard deviation with a given expected return.
  - **Minimum-variance frontier** is the entire collection of minimum-variance portfolios.
- **Global minimum-variance portfolio:** The portfolio with the **minimum variance** among all portfolios of risky assets, which is the **left-most** point on the minimum-variance frontier.
- **Efficient frontier**
  - The curve that **lies above and to the right** of the global minimum-variance portfolio is referred to as the **Markowitz efficient frontier.**
  - Those portfolios that have the greatest expected return with a given level of risk make up the efficient frontier.
  - All portfolios of risky assets that rational, risk-averse investors will choose.
  - **Efficient portfolio**: well-diversified or fully-diversified.

Example

- Which of the following statements is least accurate? The efficient frontier is the set of all attainable risky assets with the:
  - A. Highest expected return for a given level of risk.
  - B. Lowest amount of risk for a given level of return.
  - C. Highest expected return relative to the risk-free rate.
- **Correct Answer: C**

- The portfolio on the minimum-variance frontier with the lowest standard deviation is:
  - A. Unattainable.
  - B. The optimal risky portfolio.
  - C. The global minimum-variance portfolio
- **Correct Answer: C**

Portfolio Risk and Return: Part I

- **Risk seeking**
  - Prefer higher risk to lower risk for a given level of expected returns
  - Will accept less expected return because of the extra utility from the risk
  - The gamble has an uncertain outcome, but with the same expected value as the guaranteed outcome. Thus, an investor choosing the gamble means that the investor gets extra "utility" from the uncertainty associated with the gamble.
- **Risk neutral**
  - An investor is indifferent about the gamble or the guaranteed outcome
  - Risk neutrality investor cares only about return and not about risk, so higher return investments are more desirable even if they come with higher risk.
- **Risk averse**
  - Prefer lower to higher risk for a given level of expected returns
  - Will only accept a riskier investment if they are compensated in the form of greater expected return

Portfolio Risk and Return: Part I

- **Utility Theory**
  - Assumption:
    - ✓ Investors are risk averse.
    - ✓ They always prefer more to less (greater return to lesser return).
    - ✓ They are able to rank different portfolios in the order of their preference.

Portfolio Risk and Return: Part I

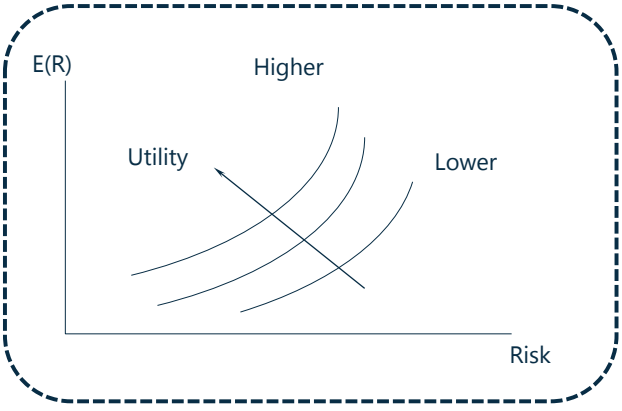
- **Utility Theory**
  - **Utility function**:

$$U = E(r) - \frac{1}{2} A\sigma^2$$

    - ✓ U: the utility of an investment
    - ✓ E(r): the expected return
    - ✓ σ^2: the variance of the investment
    - ✓ A: a measure of risk aversion, which is measured as the marginal reward that an investor requires to accept additional risk.
      - ◆A is higher for more risk-averse individuals.
      - ◆Risk-aversion: A>0
      - ◆Risk-neutral: A=0
      - ◆Risk-seeking: A<0

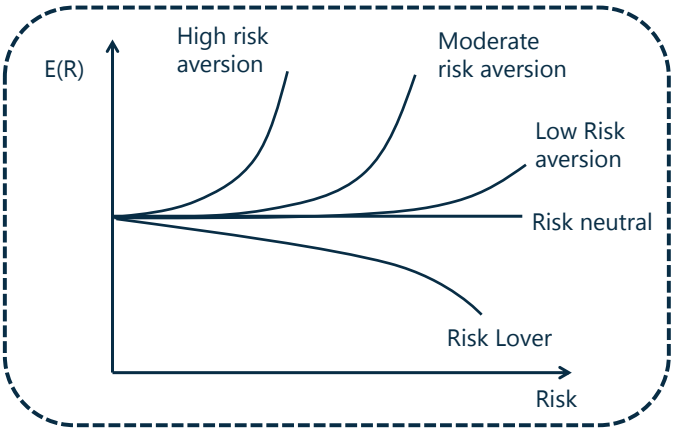
Portfolio Risk and Return: Part I

- **Indifference curve:** plots combinations of risk(standard deviation) and expected return among which an investor is indifferent.



Portfolio Risk and Return: Part I

- **Indifference Curve for various types of investors**

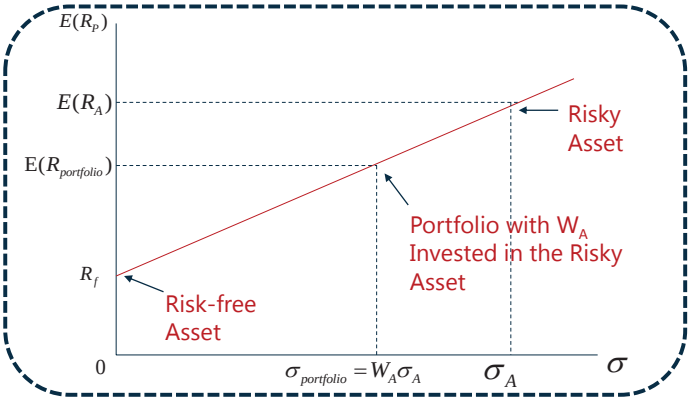


Portfolio Risk and Return: Part I

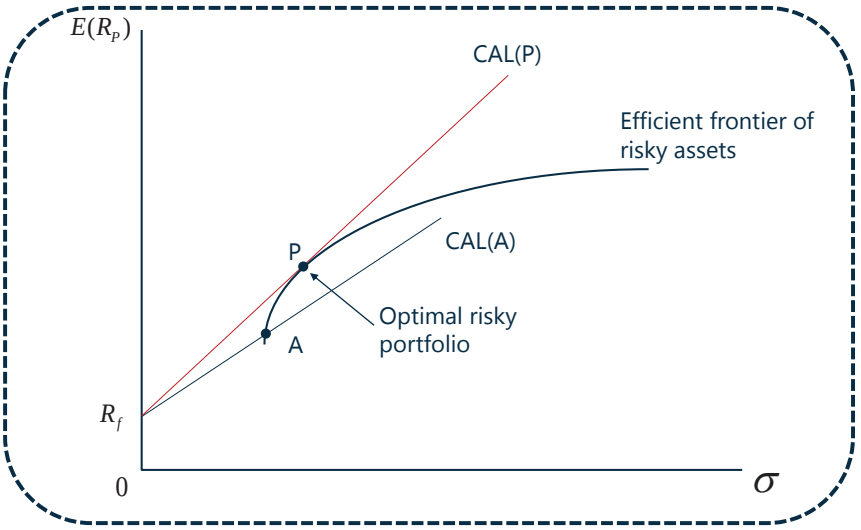
- If a risky asset is combined with a risk free asset, the relationship between the portfolio risk and return **is linear.**

$$E(R_P) = W_A E(R_A) + W_B E(R_B)$$

$$\sigma_P = \sqrt{W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_A W_B \rho_{AB}\sigma_A\sigma_B} = W_A\sigma_A$$



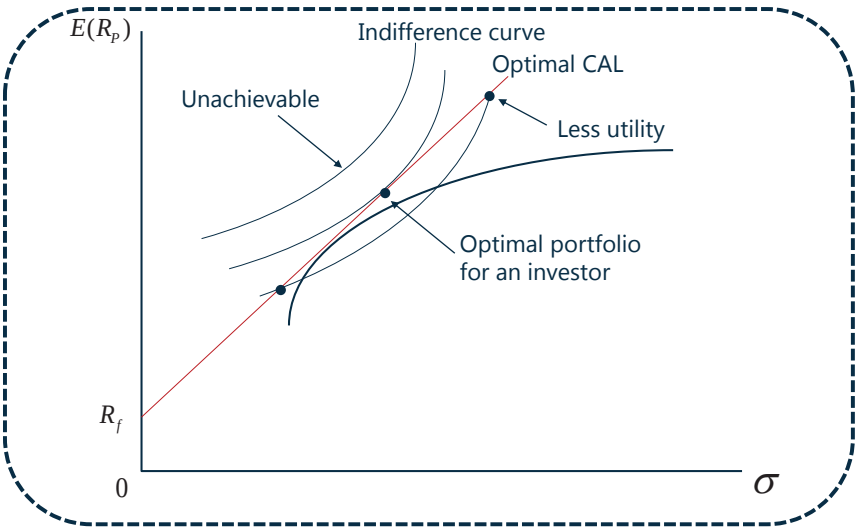
CAL



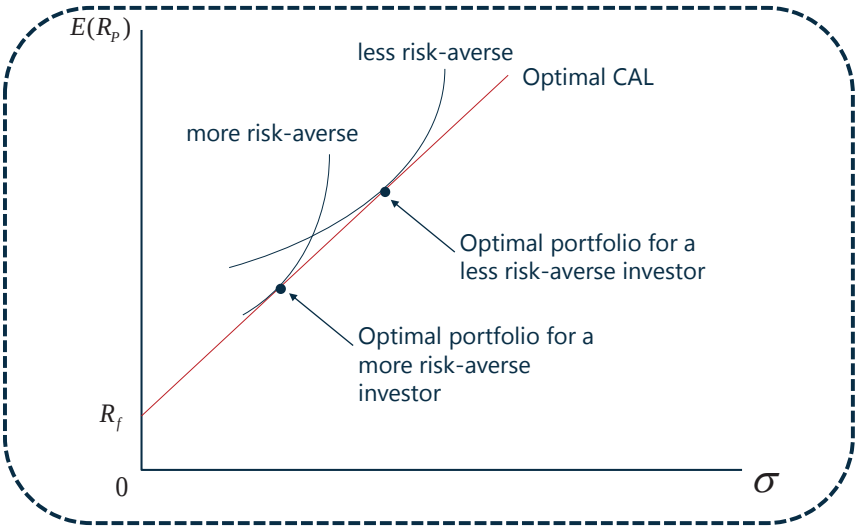
Portfolio Risk and Return: Part I

- **Optimal CAL**
  - The optimal capital allocation line connects the risk-free assets and **the optimal risky asset portfolio**.
  - The optimal risky portfolio is at the tangent of CAL and the efficient frontier of risky assets.
- **Two-fund separation theorem:**
  - All investors will hold a combination of two portfolios or funds: **a risk-free asset** and **an optimal portfolio of risky assets**.

Portfolio Selection



Portfolio Selection



Example

- The dominant capital allocation line is the combination of the risk-free asset and the:
  - A. Optimal risky portfolio
  - B. Levered portfolio of risky assets
  - C. Global minimum-variance portfolio
- **Correct Answer: A**

Example

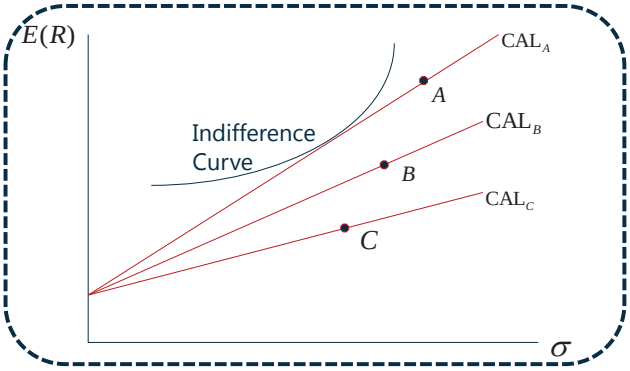
- With respect to the mean-variance theory, the optimal portfolio is determined by each individual investor's:
  - A. Risk-free rate
  - B. Borrowing rate
  - C. Risk preference
- **Correct Answer: C**

Reading 43

Portfolio Risk and Return: Part II

Portfolio Risk and Return: Part II

- Risky portfolios and their associated capital allocation lines for different investors



- If each investor has different expectations about the expected returns of, standard deviations of, or correlations between risky asset returns, each investor will have a different optimal risky asset portfolio and a different CAL

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Portfolio Risk and Return: Part II

- **Capital market line**
  - When investors share identical expectations about the mean returns, variance of returns, and correlations of risky assets, the CAL for all investors is the same and is known as the capital market line (CML):

$$E(R_P) = R_F + \frac{E(R_M) - R_F}{\sigma_M}\sigma_P$$

  - The Market Portfolio:
    - ✓ Is the tangent point where the CML touches the Markowitz efficient frontier.
    - ✓ Based on the assumption of **homogeneity of expectations.**
    - ✓ Consists of every risky assets.
    - ✓ The weights on each asset are equal to the percentage of the market value of the asset to the market value of the entire market portfolio.

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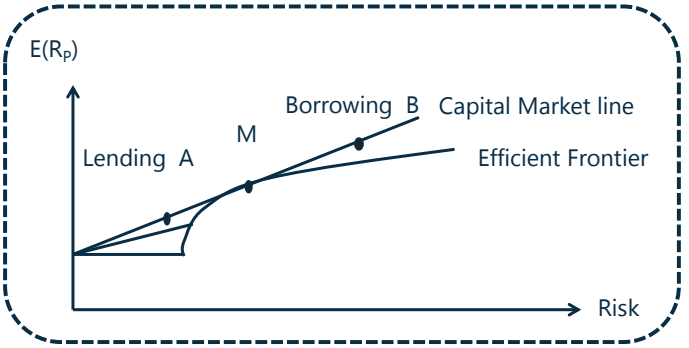
Portfolio Risk and Return: Part II

- **Explanation of the CML**
  - Difference between the CML and the CAL
    - ✓ homogeneity of expectations: there are many CALs, but only one CML.
  - Investment using CML follows a
    - ✓ passive investment strategy
      - ◆i.e., invest in an index of risky assets that serves as a proxy for the market portfolio and allocate a portion of their investable assets to a risk-free asset.
    - ✓ leverage strategy

Portfolio Risk and Return: Part II

- **Borrowing portfolio and lending portfolio**
  - If $\sigma_p > \sigma_m$, borrow money at risk free rate and invest the proceed in market portfolio.
  - If $\sigma_p < \sigma_m$, sell a portion of market portfolio and deposit the proceed in bank.



Example

- The capital market line, CML, is the graph of the risk and return of portfolio combinations consisting of the risk-free asset and:
  - A. Any risky portfolio
  - B. The market portfolio
  - C. The leveraged portfolio
- **Correct Answer: B**

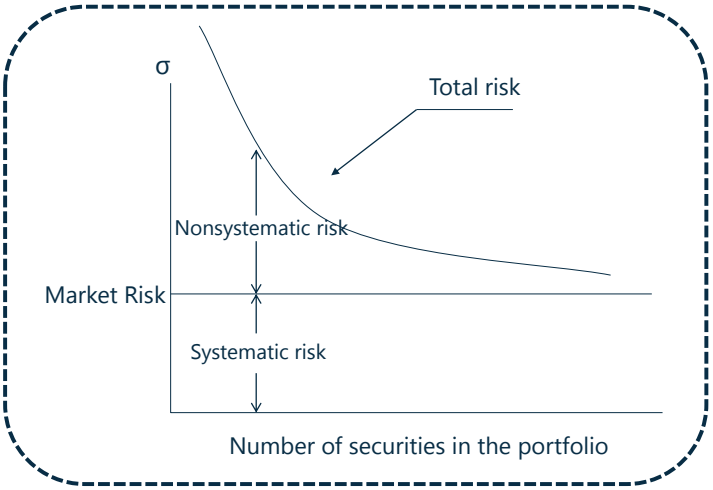
- A portfolio on the capital market line with returns greater than the returns on the market portfolio represents a(n):
  - A. Lending portfolio
  - B. Borrowing portfolio
  - C. Unachievable portfolio
- **Correct Answer: B**

Portfolio Risk and Return: Part II

- **Nonsystematic risk (or idiosyncratic, diversifiable, company-specific risk):**
  - Nonsystematic risk is local or limited to a particular asset or industry
  - The risk that disappears in the portfolio construction process
- **Systematic risk (or non-diversifiable, market risk):**
  - The risk that cannot be diversified away.
  - Total variance = systematic variance + nonsystematic variance, or
  - Total risk = systematic risk + nonsystematic risk
- **Since nonsystematic risk can be eliminated through diversification, only systematic risk is compensated.**

Portfolio Risk and Return: Part II

- **Risk vs. Number of portfolio Assets**



Portfolio Risk and Return: Part II

- **An important conclusion of capital market theory:**
  - A security's equilibrium return depends **only on systematic risk**, not its total risk which is measured by standard deviation.
- **An assumptions of the model :**
  - Diversification has no cost, because investors will not be compensated for bearing risk that can be eliminated for free.

Example

- Which of the following types of risk is most likely avoided by forming a diversified portfolio?
  - A. Total risk
  - B. Systematic risk
  - C. Nonsystematic risk
- **Correct Answer: C**

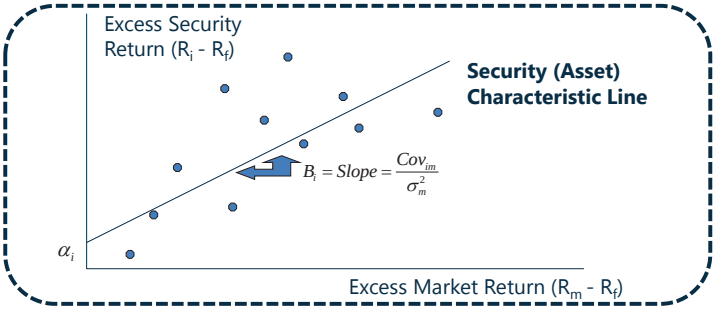
- Which of the following events is most likely an example of nonsystematic risk?
  - A. A decline in interest rates.
  - B. The resignation of chief executive officer.
  - C. An increase in the value of the U.S. dollar.
- **Correct Answer: B**

Portfolio Risk and Return: Part II

- **Beta:** a measure of how sensitive an asset's return is to the market as a whole. A standardized measure of systematic risk.

$$\beta_i = \frac{Cov_{i,mkt}}{\sigma^2_{mkt}} = (\frac{\sigma_i}{\sigma_{mkt}}) \times \rho_{i,mkt}$$

- **Estimation of Beta with Security Characteristic Line**(regression of excess security returns with excess market returns)



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Portfolio Risk and Return: Part II

- **Return generating models: multifactor models**

$$E(R_i) - R_f = \sum_{j=1}^{k} \beta_{i,j} \times E(Factor_j) = \beta_{i,1}[E(R_m) - R_f] + \sum_{j=2}^{k} \beta_{i,j} \times E(Factor_j)$$

  - Macroeconomic factors: GDP growth, interest rate, inflation rate, productivity, employment or consumer confidence
  - Fundamental factors: earnings, earnings growth, firm size, and research expenditures
  - Statistical factors: no obvious economic interpretations with asset returns
- **Market model**
  - The single factor model
  - The only factor is the expected excess return on the market portfolio (market index)

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f]$$

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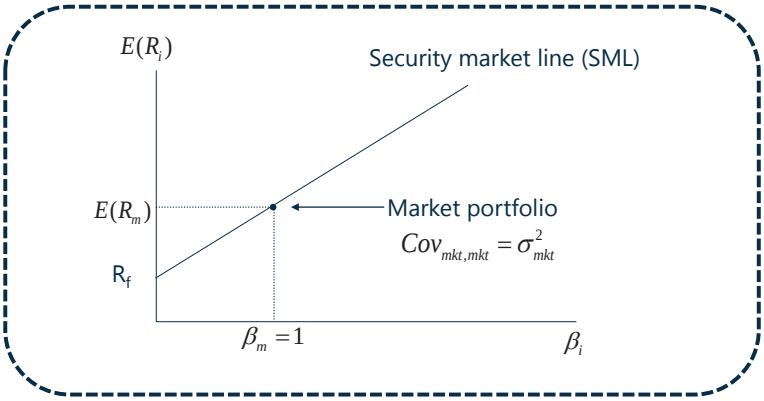
Portfolio Risk and Return: Part II

- **Assumptions of the CAPM**
  - Investors are **risk-averse**, **utility-maximizing**, rational individuals.
  - Markets are **frictionless**, including no transaction costs and no taxes.
  - Investors plan for the same **single holding period**.
  - Investors have **homogeneous expectations** or beliefs.
  - All investments are **infinitely divisible**.
  - Invstors are **price takers**.

Portfolio Risk and Return: Part II

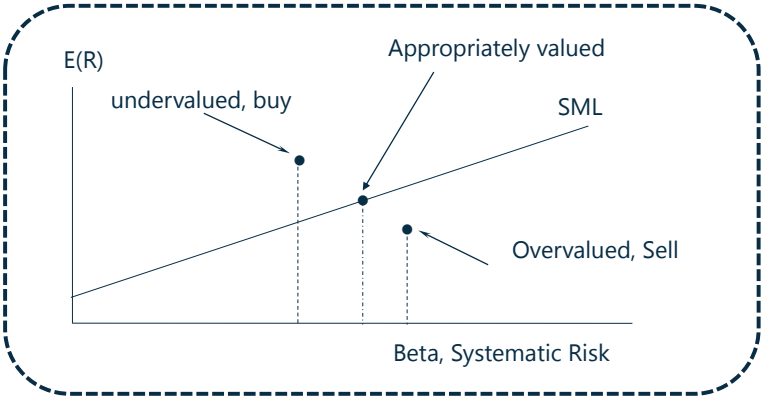
- **Security market line (SML):** Graphical representation of CAPM



- **The Equation of SML:** $E(R_i) = R_f + \beta_i[E(R_m) - R_f]$

Portfolio Risk and Return: Part II

- **How to judge if a stock is properly valued**



Portfolio Risk and Return: Part II

- **How to judge if a stock is properly valued**
  - Undervalued
    - ✓ Estimated return > Required return from the SML
    - ✓ Investors should buy.
  - Overestimated
    - ✓ Estimated return < Required return from the SML
    - ✓ Investors should sell.
  - Properly valued
    - ✓ Estimated return = Required return from the SML
    - ✓ Investors are indifferent between buying or selling

Portfolio Risk and Return: Part II

- **Differences between the SML and the CML**

	SML	CML
Measure of risk	Uses systematic risk (non-diversifiable risk)	Uses standard deviation (total risk)
Application	Tool used to determine the appropriate expected (benchmark) returns for securities	Tool used to determine the appropriate asset allocation (percentages allocated to the risk-free asset and to the market portfolio) for the investor
Definition	Graph of the capital asset pricing model	Graph of all the combinations of the risk-free asset and the market portfolio
Slope	Market risk premium	Market portfolio Sharpe ratio

Example

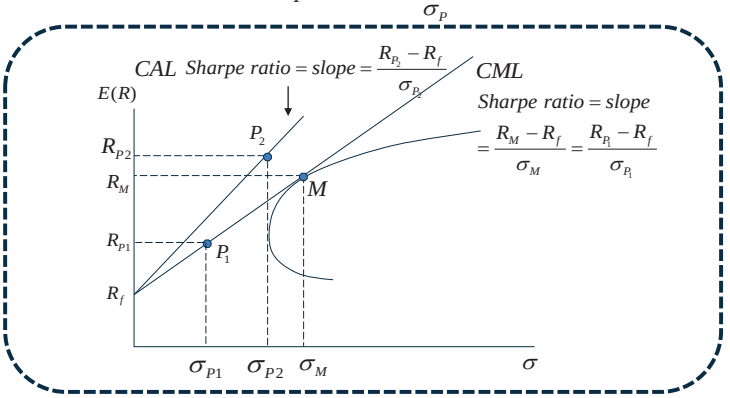
- With respect to the capital asset pricing model, the primary determinant of expected return of an individual asset is the:
  - A. Asset's beta.
  - B. Market risk premium
  - C. Asset's standard deviation
- **Correct Answer: A**

- Analysts who have estimated returns of an asset to be greater than the expected returns generated by the capital asset pricing model should consider the asset to be:
  - A. Overvalued
  - B. Undervalued
  - C. Properly valued
- **Correct Answer: B**

Portfolio Risk and Return: Part II

- **Evaluate relative portfolio performance (risk-adjusted returns)**

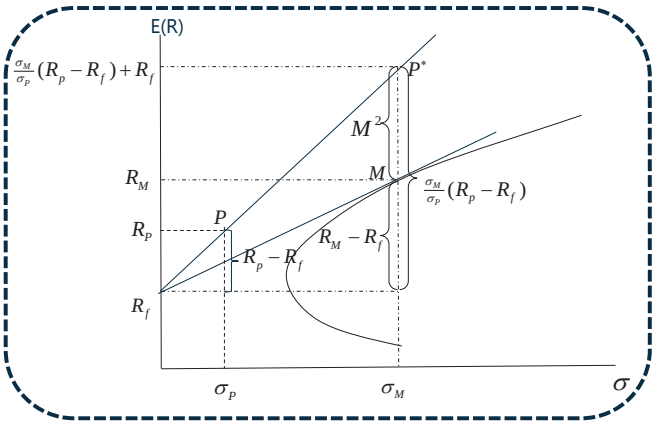
$$Sharpe\ ratio = \frac{R_P - R_f}{\sigma_P}$$



- **The Sharpe ratio for any portfolio along the CML is the same.**

Portfolio Risk and Return: Part II

- **The M-squared (M^2) measure gives identical rankings to those of Sharpe ratio, which can be applied to compare among different investments.** $M^2 = (R_P - R_f)\frac{\sigma_M}{\sigma_P} - (R_M - R_f)$



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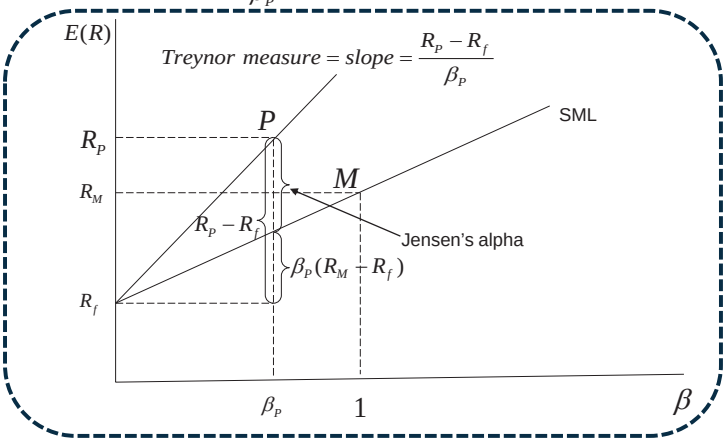
Portfolio Risk and Return: Part II

- **Treynor measure & Jensen's alpha (systematic risk)**

$$Treynor\ measure = \frac{R_P - R_f}{\beta_P} \qquad \alpha_P = (R_P - R_f) - \beta_P(R_M - R_f)$$



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Portfolio Risk and Return: Part II

- **Comparison of four measures**
  - Jensen's alpha and M-squared
    - ✓ We are not only able to determine the rank of a portfolio but also which, if any, of our portfolios **beat the market** on a risk-adjusted basis.
    - ✓ Comparable only when **sharing same beta** (Jensen's alpha) but no requirement to M-squared.
  - Sharpe ratio and Treynor measure
    - ✓ to rank portfolios, the Sharpe ratio or Treynor ratio of one portfolio must **be compared with the Sharpe ratio or Treynor ratio of another portfolio.**
  - Systematic risk and non-systematic risk
    - ✓ For non-diversified portfolio , Sharpe ratio and M-squared are appropriate.
    - ✓ For fully diversified portfolio , Jensen Alpha and Treynor are appropriate.

Example

- Which of the following performance measures is consistent with the CAPM?
  - A. M-squared.
  - B. Sharpe ratio.
  - C. Jensen's alpha.
- **Correct Answer: C**

- Which of the following performance measures does not require the measure to be compared to another value?
  - A. Sharpe ratio
  - B. Treynor ratio
  - C. Jensen's alpha
- **Correct Answer: C**

Reading 44

Basic of Portfolio Planning and Construction

Basic of Portfolio Planning and Construction

- **The need for a policy statement**
  - Understand and articulate realistic investor goals, needs and risk tolerance
  - Ensure that goals are realistic
  - Provide an objective measure of portfolio performance
- **Major components of IPS**
  - Introduction
  - Statement of purpose
  - Statement of duties and responsibilities
  - Procedures: the steps taken to keep the IPS updated in a timely manner and respond to various situations.
  - Investment objectives
  - Investment constraints
  - Investment guidelines
  - Evaluation and Review
  - Appendices: information on asset allocation

Basic of Portfolio Planning and Construction

- **Investment objectives: risk and return**
- **Risk objective**
  - The risk objective limits how high the investor can set the return objective
  - Risk measurement:
    - ✓ Absolute: variance or standard deviation
    - ✓ Relative: relate risk relative to one or more benchmarks perceived to represent appropriate risk standards (tracking risk),
    - ✓ Downside risk: VAR
  - Risk tolerance: willingness and ability

Situation		Risk tolerance
Willingness > Ability		Ability (Education)
Willingness < Ability	Return Objective = Willingness	Willingness (Reevaluation)
	Return Objective = Ability	Ability (Education)

Basic of Portfolio Planning and Construction

- **Return objectives: absolute or relative basis**
  - Return measurement:
    - ✓ Absolute basis:
      - ◆ percentage rate of return: total return(balance between capital gains and income), inflation-adjusted return(real)
    - ✓ Relative basis:
      - ◆ Relative to a benchmark return: Some institutions also set their return objective relative to a peer group or universe of managers
        - ☐ Limitation:
          - limited information about the investment strategies or the returns calculation methodology being used by peers,
          - the impossibility of all institutions being "above average."
        - ☐ Furthermore, a good benchmark should be investable
  - Stated return desire vs. Required return
  - Consistent with risk objective

Basic of Portfolio Planning and Construction

- **Investment constraints**
  - **Liquidity**—for cash spending needs (anticipated or unexpected)
  - **Time horizon**—the time between making an investment and needing the funds
  - **Tax concerns**—the tax treatments of various accounts, and the investor's marginal tax bracket
  - **Legal and regulatory factors**—restrictions on investments in retirement, personal, and trust accounts
  - **Unique circumstances**—investor preferences or other factors which has not been considered before
    - ✓ E.g. religions, ethical behavior

Basic of Portfolio Planning and Construction

- **Strategic asset allocation:**
  - the set of exposures to IPS-permissible asset classes that is expected to achieve the client's **long-term objectives** given the client's investment constrains.
  - Correlations within the class is **higher** than correlations between asset classes.

Asset Class	Target
Cash	0%
U.S. large-cap equity	12%
U.S. small-/mid-cap equity	6%
International (developed) equity	12%
Emerging market equity	6%
U.S. bonds	18%
Global bonds	8%
High –yield bonds	5%
Emerging market debt	3%
Inflation-protected bonds	3%
Real estate	5%
Hedge funds	5%
Private equity	2%
Commodities	0%
Tactical asset allocation and other	15%
TOTAL	100%

Basic of Portfolio Planning and Construction

- **Active portfolio management**
  - **Tactical asset allocation**: is the decision to **deliberately deviate** from the policy exposures to systematic risk factors with the intent to add value based on forecasts of the **near-term returns** of those asset classes.
    - ✓ The manager's ability to identify shot-term opportunities in specific asset classes;
    - ✓ The existence of such short-term opportunities.
  - **Security selection**: is an attempt to generate higher returns than the asset class benchmark by **selecting securities with a higher expected return**.
    - ✓ The manager's skill
    - ✓ The opportunities with in a particular asset class.

Example

- Which of the following factors is least likely to impact an individual's ability to take risk?
  - A. Time horizon
  - B. Personality type
  - C. Expected income
- **Correct Answer: B**

- Tactical asset allocation is best described as:
  - A. Attempts to exploit arbitrage possibilities among asset classes.
  - B. The decision to deliberately deviate from the policy portfolio.
  - C. Selecting asset classes with the desired exposures to sources of systematic risk in an investment portfolio
- **Correct Answer: B**

It's not the end but just beginning.

A true friend is the one who holds your hand and touches your heart.

一个真正的朋友会握着你的手，触动你的心。