## **HW 11 solution**

**Zstar** 

2(1).

记
$$ar{u}(p,x)=L[u(y,x)]$$
, 得:  $\frac{d(par{u}-1)}{dx}=rac{1}{p}$ ,  $ar{u}|_{x=0}=ar{f}(p)\Longrightarrow ar{u}=rac{x}{p^2}+ar{f}(p)$ ,做反变换得:  $u=xy+y+1$ 

2(2).

做Laplace变换得:

$$\left\{egin{array}{l} p^2ar u-b=a^2rac{d^2ar u}{dx^2}\ ar u|_{x=0}=0,ar u|_{x=+\infty}$$
有界

找到通解:

$$ar{u}=Ce^{-rac{p}{a}x}+De^{rac{p}{a}x}+rac{b}{p^2}$$
,代入边界条件得: $ar{u}=-rac{b}{p^2}e^{-rac{p}{a}x}+rac{b}{p^2}$  $u=L^{-1}[ar{u}]=bt-b(t-rac{x}{a})H(t-rac{x}{a})$ 

2(3).

做Laplace变换得:

$$\left\{egin{array}{l} rac{d^2ar{u}}{dt^2}-rac{p}{a^2}ar{u}+rac{u_1}{a^2}=0\ rac{dar{u}}{dx}ig|_{x=0}=0, ar{u}ig|_{x=l}=u_0 \end{array}
ight.$$

可求得解:

$$ar{u}(p,x)=rac{u_0-u_1}{pchrac{\sqrt{p}}{a}l}chrac{\sqrt{p}}{a}x+rac{u_1}{p}$$

$$u = L^{-1}[ar{u}] = u_1 + (u_0 - u_1) L^{-1}[rac{u_0 - u_1}{pchrac{\sqrt{p}}{a}} chrac{\sqrt{p}}{a}x]$$

像函数比较复杂,考试不会涉及到,大家可以参考149页的做法。

$$chrac{\sqrt{p}}{a}l = \cos irac{\sqrt{p}}{a}l = 0. \Longrightarrow \sqrt{p_k} = \pm irac{(2k+1)a\pi}{2l}, p_k = -rac{(2k+1)^2a^2\pi^2}{4l^2}, (k=0,1,2,\dots)$$

$$\left. \left( pch rac{\sqrt{p}}{a} l 
ight)' 
ight|_{p=p_k} = -rac{\pi}{4} (2k+1) \sin rac{2k+1}{2} \pi = (-1)^{k+1} rac{\pi}{4} (2k+1)$$

还有:

$$\left.e^{pt}\operatorname{ch}rac{\sqrt{p}}{a}x
ight|_{p=p_{b}}=e^{-\left(rac{2k+1}{2l}\pi a
ight)^{2}}\cosrac{(2k+1)\pi}{2l}$$

最后:

$$u(t,x) = u_1 + rac{4}{\pi}(u_1 - u_0) \sum_{k=0}^{+\infty} rac{(-1)^k}{2k+1} e^{-\left(rac{2k+1}{2l}\pi a
ight)^2 t} \cosrac{(2k+1)\pi x}{2l}$$

2(4).

做Laplace变换得:

$$egin{cases} rac{d^2ar{u}}{dx^2} - rac{p^2}{a^2}ar{u} + rac{p}{a^2(p^2 + \omega^2)} = 0 \ ar{u}|_{x=0} = 0, \quad ar{u}|_{x=+\infty}$$
有界

可以得到解:

$$ar{u}(p,x) = -rac{1}{p\left(p^2+\omega^2
ight)}e^{-rac{p}{a}x} + rac{1}{p\left(p^2+\omega^2
ight)}$$

再做反变换:

$$egin{aligned} u(x,t) &= L^{-1}\left[-rac{1}{p\left(p^2+\omega^2
ight)}e^{-rac{p}{a}x}
ight] + L^{-1}\left[rac{1}{p\left(p^2+\omega^2
ight)}
ight] \ &= rac{2}{\omega^2}{
m sin}^2\,rac{\omega t}{2} - rac{2}{\omega^2}{
m sin}^2\,rac{\omega\left(t-rac{x}{a}
ight)}{2}H\left(t-rac{x}{a}
ight) \end{aligned}$$