Level 9: Group A&B: Exact Pricing Mothods

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A.Exact Solutions of One-factor Plain Options

a). & **b).**

Below are the results , as we can see, Batch 1 to 4 all satisfy the put-call parity relationship. In addition, different methods give the same results.

```
Batch 1:
Use Option functions:  C= 2.13337, P= 5.84628
Use instances of EuropeanOption class :  C = 2.13337, P = 5.84628
Results from Put-Call parity : C = 2.13337, P = 5.84628
Parity Check result: 1
Batch 2:
Use Option functions:  C= 7.96557, P= 7.96557
Use instances of EuropeanOption class :  C = 7.96557, P = 7.96557
Results from Put-Call parity : C = 7.96557, P = 7.96557
Parity Check result: 1
Batch 3:
Use Option functions:  C= 0.204058, P= 4.07326
Use instances of EuropeanOption class :  C = 0.204058, P = 4.07326
Results from Put-Call parity : C = 0.204058, P = 4.07326
Parity Check result: 1
Batch 4:
Use Option functions:  C= 92.1757, P= 1.2475
Use instances of EuropeanOption class :  C = 92.1757, P = 1.2475
Results from Put-Call parity : C = 92.1757, P = 1.2475
Parity Check result: 1
```

c.) Computing option prices for a monotonically increasing range of underlying value of S [10,50]. Using the global *GenerateMeshArray*() function to create a vector of doubles separated by 1. The results shown below for each batch may be a little redundant, so I just exhibit the results for Batch 1.

```
Batch 1:
S: 10, C: 7.792e-36, P: 53.7129
S: 11, C: 1.88964e-32, P: 52.7129
S: 12, C: 1.64204e-29, P: 51.7129
S: 13, C: 6.19303e-27, P: 50.7129
S: 14, C: 1.17207e-24, P: 49.7129
S: 15, C: 1.24529e-22, P: 48.7129
S: 16, C: 8.11387e-21, P: 47.7129
S: 17, C: 3.47944e-19, P: 46.7129
S: 18, C: 1.03991e-17, P: 45.7129
S: 19, C: 2.27042e-16, P: 44.7129
S: 20, C: 3.76503e-15, P: 43.7129
S: 21, C: 4.89964e-14, P: 42.7129
S: 22, C: 5.1436e-13, P: 41.7129
S: 23, C: 4.45934e-12, P: 40.7129
S: 24, C: 3.25772e-11, P: 39.7129
S: 25, C: 2.0405e-10, P: 38.7129
```

```
S: 26, C: 1.11246e-09, P: 37.7129
S: 27, C: 5.34891e-09, P: 36.7129
S: 28, C: 2.29454e-08, P: 35.7129
S: 29, C: 8.8713e-08, P: 34.7129
S: 30, C: 3.11926e-07, P: 33.7129
S: 31, C: 1.00545e-06, P: 32.7129
S: 32, C: 2.99241e-06, P: 31.7129
S: 33, C: 8.27565e-06, P: 30.7129
S: 34, C: 2.13896e-05, P: 29.7129
S: 35, C: 5.19359e-05, P: 28.713
S: 36, C: 0.000119024, P: 27.713
S: 37, C: 0.000258548, P: 26.7132
S: 38, C: 0.000534403, P: 25.7134
S: 39, C: 0.00105474, P: 24.714
S: 40, C: 0.00199417, P: 23.7149
S: 41, C: 0.00362244, P: 22.7165
S: 42, C: 0.00633924, P: 21.7193
S: 43, C: 0.010714, P: 20.7236
S: 44, C: 0.0175282, P: 19.7304
S: 45, C: 0.0278174, P: 18.7407
S: 46, C: 0.0429083, P: 17.7558
S: 47, C: 0.0644467, P: 16.7774
S: 48, C: 0.0944125, P: 15.8073
S: 49, C: 0.135117, P: 14.848
S: 50, C: 0.189181, P: 13.9021
```

d).

(1). A mesh for expiry time T from 20 to 40, and compute the Prices for different expiry time. For brevity's sake, I only show the results for Batch 1 (All the other parameters are the same as Batch 1 except the expiry time).

```
Batch 1:
T: 20    C: 48.9643        P: 2.0876
T: 21    C: 49.8003        P: 1.91462
T: 22    C: 50.5726        P: 1.75547
T: 23    C: 51.286         P: 1.60916
T: 24    C: 51.9453        P: 1.47471
T: 25    C: 52.5544        P: 1.35124
T: 26    C: 53.1174        P: 1.23789
T: 27    C: 53.6377        P: 1.13388
T: 28    C: 54.1187        P: 1.03847
T: 29    C: 54.5632        P: 0.950971
T: 30    C: 54.9741        P: 0.870751
T: 31    C: 55.3539        P: 0.79722
T: 32    C: 55.705         P: 0.729836
T: 33    C: 56.0296        P: 0.668096
T: 34    C: 56.3297        P: 0.611536
T: 35    C: 56.6071        P: 0.559729
T: 36    C: 56.8635        P: 0.512282
T: 37    C: 57.1006        P: 0.468834
T: 38    C: 57.3198        P: 0.429051
T: 39    C: 57.5224        P: 0.392627
T: 40    C: 57.7097        P: 0.359283
```

(2). compute the prices of different volatility[0.1, 1]:

```
Batch 1:
Sigma: 0.1      C: 0.761015      P: 5.24309
Sigma: 0.2      C: 1.70096       P: 5.66924
Sigma: 0.3      C: 2.54494       P: 6.00351
Sigma: 0.4      C: 3.31764       P: 6.27057
Sigma: 0.5      C: 4.03758       P: 6.4889
Sigma: 0.6      C: 4.71666       P: 6.67035
Sigma: 0.7      C: 5.36283       P: 6.82288
Sigma: 0.8      C: 5.98171       P: 6.95204
Sigma: 0.9      C: 6.5774        P: 7.06191
Sigma: 1        C: 7.15299       P: 7.15556
```

(3). prices as a function of any the option pricing parameters, taking r:[0,0.5] and Sigma[0.1,1.0] as an example:

```
Batch 1:
Sigma: 0.1      r:0       C: 0.176593      P: 3.96451
Sigma: 0.2      r:0.05    C: 1.04929       P: 4.79016
Sigma: 0.3      r:0.1     C: 2.12273       P: 5.81712
Sigma: 0.4      r:0.15    C: 3.23325       P: 6.88176
Sigma: 0.5      r:0.2     C: 4.34026       P: 7.94344
Sigma: 0.6      r:0.25    C: 5.42937       P: 8.98779
Sigma: 0.7      r:0.3     C: 6.49428       P: 10.0085
Sigma: 0.8      r:0.35    C: 7.5318        P: 11.0024
Sigma: 0.9      r:0.4     C: 8.54019       P: 11.9676
Sigma: 1        r:0.45    C: 9.51842       P: 12.9033
```

Option Sensitivities, aka the Greeks

a). Results:

```
Delta and Gamma:
Delta of Call : 0.594629, Delta of Put: -0.356601
Gamma of Call : 0.0134936, Gamma of Put: 0.0134936
```

b) compute the delta price for a monotonically increasing range of underlying value S:

```
Computing Delta for range of underlying values:
S = 80  Delta of Call = 0.215772         Delta of Put = -0.735458
S = 81  Delta of Call = 0.230011         Delta of Put = -0.721218
S = 82  Delta of Call = 0.244562         Delta of Put = -0.706668
S = 83  Delta of Call = 0.259392         Delta of Put = -0.691837
S = 84  Delta of Call = 0.274471         Delta of Put = -0.676759
S = 85  Delta of Call = 0.289765         Delta of Put = -0.661465
S = 86  Delta of Call = 0.305241         Delta of Put = -0.645989
S = 87  Delta of Call = 0.320867         Delta of Put = -0.630363
S = 88  Delta of Call = 0.33661          Delta of Put = -0.61462
S = 89  Delta of Call = 0.352438         Delta of Put = -0.598792
S = 90  Delta of Call = 0.368319         Delta of Put = -0.58291
S = 91  Delta of Call = 0.384223         Delta of Put = -0.567007
S = 92  Delta of Call = 0.400118         Delta of Put = -0.551111
S = 93  Delta of Call = 0.415977         Delta of Put = -0.535252
S = 94  Delta of Call = 0.431772         Delta of Put = -0.519457
S = 95  Delta of Call = 0.447475         Delta of Put = -0.503754
S = 96  Delta of Call = 0.463062         Delta of Put = -0.488168
```

```
S = 97   Delta of Call = 0.478508        Delta of Put = -0.472721
S = 98   Delta of Call = 0.493791        Delta of Put = -0.457438
S = 99   Delta of Call = 0.50889         Delta of Put = -0.442339
S = 100  Delta of Call = 0.523785        Delta of Put = -0.427444
S = 101  Delta of Call = 0.538459        Delta of Put = -0.412771
S = 102  Delta of Call = 0.552894        Delta of Put = -0.398335
S = 103  Delta of Call = 0.567076        Delta of Put = -0.384153
S = 104  Delta of Call = 0.580992        Delta of Put = -0.370237
S = 105  Delta of Call = 0.594629        Delta of Put = -0.356601
S = 106  Delta of Call = 0.607976        Delta of Put = -0.343253
S = 107  Delta of Call = 0.621025        Delta of Put = -0.330205
S = 108  Delta of Call = 0.633767        Delta of Put = -0.317463
S = 109  Delta of Call = 0.646196        Delta of Put = -0.305034
S = 110  Delta of Call = 0.658306        Delta of Put = -0.292923
```

c). Compute delta with different option parameters:

```
Computing Delta for range of different Option Parameters:
S = 80   r=0.1   Delta of Call = 0.215772        Delta of Put = -0.735458
S = 81   r=0.11  Delta of Call = 0.228864        Delta of Put = -0.717621
S = 82   r=0.12  Delta of Call = 0.242128        Delta of Put = -0.699636
S = 83   r=0.13  Delta of Call = 0.25553         Delta of Put = -0.681537
S = 84   r=0.14  Delta of Call = 0.269036        Delta of Put = -0.663358
S = 85   r=0.15  Delta of Call = 0.28261         Delta of Put = -0.645133
S = 86   r=0.16  Delta of Call = 0.29622         Delta of Put = -0.626897
S = 87   r=0.17  Delta of Call = 0.309831        Delta of Put = -0.608682
S = 88   r=0.18  Delta of Call = 0.323411        Delta of Put = -0.59052
S = 89   r=0.19  Delta of Call = 0.33693         Delta of Put = -0.572443
S = 90   r=0.2   Delta of Call = 0.350356        Delta of Put = -0.554481
S = 91   r=0.21  Delta of Call = 0.363661        Delta of Put = -0.536664
S = 92   r=0.22  Delta of Call = 0.376817        Delta of Put = -0.519017
S = 93   r=0.23  Delta of Call = 0.389799        Delta of Put = -0.501567
S = 94   r=0.24  Delta of Call = 0.402581        Delta of Put = -0.484339
S = 95   r=0.25  Delta of Call = 0.415142        Delta of Put = -0.467355
S = 96   r=0.26  Delta of Call = 0.42746         Delta of Put = -0.450635
S = 97   r=0.27  Delta of Call = 0.439515        Delta of Put = -0.4342
S = 98   r=0.28  Delta of Call = 0.451291        Delta of Put = -0.418067
S = 99   r=0.29  Delta of Call = 0.462771        Delta of Put = -0.402252
S = 100  r=0.3   Delta of Call = 0.47394         Delta of Put = -0.386767
```

d).

- performing the same calculations as **a).** using the divided differences:

```
S = 105, h = 0.001, Delta of Call = 0.594629
S = 105, h = 0.001, Delta of Put = -0.356601
S = 105, h = 0.0535789, Delta of Call = 0.594629
S = 105, h = 0.0535789, Delta of Put = -0.356601
S = 105, h = 0.106158, Delta of Call = 0.594628
S = 105, h = 0.106158, Delta of Put = -0.356601
S = 105, h = 0.158737, Delta of Call = 0.594627
S = 105, h = 0.158737, Delta of Put = -0.356602
S = 105, h = 0.211316, Delta of Call = 0.594627
S = 105, h = 0.211316, Delta of Put = -0.356603
S = 105, h = 0.263895, Delta of Call = 0.594625
S = 105, h = 0.263895, Delta of Put = -0.356604
S = 105, h = 0.316474, Delta of Call = 0.594624
```

```
S = 105, h = 0.316474, Delta of Put = -0.356606
S = 105, h = 0.369053, Delta of Call = 0.594622
S = 105, h = 0.369053, Delta of Put = -0.356607
S = 105, h = 0.421632, Delta of Call = 0.59462
S = 105, h = 0.421632, Delta of Put = -0.356609
S = 105, h = 0.474211, Delta of Call = 0.594618
S = 105, h = 0.474211, Delta of Put = -0.356612
S = 105, h = 0.526789, Delta of Call = 0.594615
S = 105, h = 0.526789, Delta of Put = -0.356614
S = 105, h = 0.579368, Delta of Call = 0.594612
S = 105, h = 0.579368, Delta of Put = -0.356617
S = 105, h = 0.631947, Delta of Call = 0.594609
S = 105, h = 0.631947, Delta of Put = -0.35662
S = 105, h = 0.684526, Delta of Call = 0.594606
S = 105, h = 0.684526, Delta of Put = -0.356623
S = 105, h = 0.737105, Delta of Call = 0.594602
S = 105, h = 0.737105, Delta of Put = -0.356627
S = 105, h = 0.789684, Delta of Call = 0.594599
S = 105, h = 0.789684, Delta of Put = -0.356631
S = 105, h = 0.842263, Delta of Call = 0.594594
S = 105, h = 0.842263, Delta of Put = -0.356635
S = 105, h = 0.894842, Delta of Call = 0.59459
S = 105, h = 0.894842, Delta of Put = -0.356639
S = 105, h = 0.947421, Delta of Call = 0.594585
S = 105, h = 0.947421, Delta of Put = -0.356644
S = 105, h = 1, Delta of Call = 0.59458
S = 105, h = 1, Delta of Put = -0.356649
```

From the results above, the results with different "h" are very close to the exact value, and the larger "h" is corresponding to bigger errors.

- performing the same calculations as **b).** using the divided differences (Only part of the results) :

```
S = 40, h = 0.001, Delta of Call = 9.08606e-18, Delta of Put = -0.951229
S = 40, h = 0.0535789, Delta of Call = 9.77119e-18, Delta of Put = -0.951229
S = 40, h = 0.106158, Delta of Call = 1.05536e-17, Delta of Put = -0.951229
S = 40, h = 0.158737, Delta of Call = 1.14429e-17, Delta of Put = -0.951229
S = 40, h = 0.211316, Delta of Call = 1.24504e-17, Delta of Put = -0.951229
S = 40, h = 0.263895, Delta of Call = 1.35884e-17, Delta of Put = -0.951229
S = 40, h = 0.316474, Delta of Call = 1.48711e-17, Delta of Put = -0.951229
S = 40, h = 0.369053, Delta of Call = 1.63143e-17, Delta of Put = -0.951229
S = 40, h = 0.421632, Delta of Call = 1.79357e-17, Delta of Put = -0.951229
S = 40, h = 0.474211, Delta of Call = 1.97552e-17, Delta of Put = -0.951229
S = 40, h = 0.526789, Delta of Call = 1.12536e-16, Delta of Put = -0.951229
S = 40, h = 0.579368, Delta of Call = 1.12918e-16, Delta of Put = -0.951229
S = 40, h = 0.631947, Delta of Call = 1.14039e-16, Delta of Put = -0.951229
S = 40, h = 0.684526, Delta of Call = 1.1591e-16, Delta of Put = -0.951229
S = 40, h = 0.737105, Delta of Call = 1.18549e-16, Delta of Put = -0.951229
S = 40, h = 0.789684, Delta of Call = 1.21981e-16, Delta of Put = -0.951229
S = 40, h = 0.842263, Delta of Call = 1.26241e-16, Delta of Put = -0.951229
S = 40, h = 0.894842, Delta of Call = 1.31368e-16, Delta of Put = -0.951229
S = 40, h = 0.947421, Delta of Call = 1.37411e-16, Delta of Put = -0.951229
S = 40, h = 1, Delta of Call = 1.44431e-16, Delta of Put = -0.951229
S = 41, h = 0.001, Delta of Call = 9.77119e-18, Delta of Put = -0.951229
S = 41, h = 0.0535789, Delta of Call = 1.05536e-17, Delta of Put = -0.951229
S = 41, h = 0.106158, Delta of Call = 1.14429e-17, Delta of Put = -0.951229
```

```
S = 41, h = 0.158737, Delta of Call = 1.24504e-17, Delta of Put = -0.951229
S = 41, h = 0.211316, Delta of Call = 1.35884e-17, Delta of Put = -0.951229
S = 41, h = 0.263895, Delta of Call = 1.48711e-17, Delta of Put = -0.951229
S = 41, h = 0.316474, Delta of Call = 1.63143e-17, Delta of Put = -0.951229
S = 41, h = 0.369053, Delta of Call = 1.79357e-17, Delta of Put = -0.951229
S = 41, h = 0.421632, Delta of Call = 1.97552e-17, Delta of Put = -0.951229
S = 41, h = 0.474211, Delta of Call = 1.12536e-16, Delta of Put = -0.951229
S = 41, h = 0.526789, Delta of Call = 1.12918e-16, Delta of Put = -0.951229
S = 41, h = 0.579368, Delta of Call = 1.14039e-16, Delta of Put = -0.951229
S = 41, h = 0.631947, Delta of Call = 1.1591e-16, Delta of Put = -0.951229
S = 41, h = 0.684526, Delta of Call = 1.18549e-16, Delta of Put = -0.951229
S = 41, h = 0.737105, Delta of Call = 1.21981e-16, Delta of Put = -0.951229
S = 41, h = 0.789684, Delta of Call = 1.26241e-16, Delta of Put = -0.951229
S = 41, h = 0.842263, Delta of Call = 1.31368e-16, Delta of Put = -0.951229
S = 41, h = 0.894842, Delta of Call = 1.37411e-16, Delta of Put = -0.951229
S = 41, h = 0.947421, Delta of Call = 1.44431e-16, Delta of Put = -0.951229
S = 41, h = 1, Delta of Call = 1.52492e-16, Delta of Put = -0.951229
```