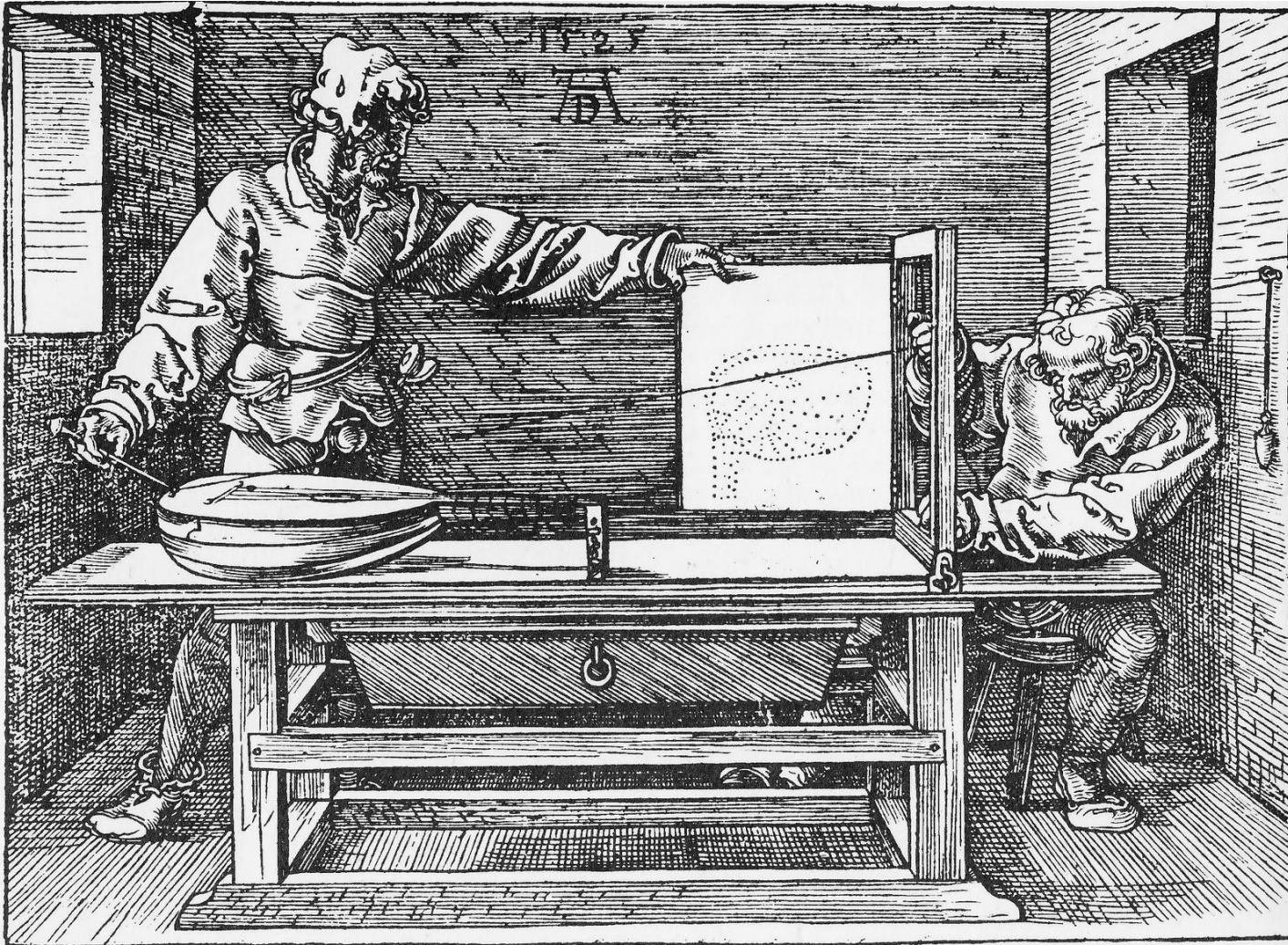


Fundamentals of Image Formation



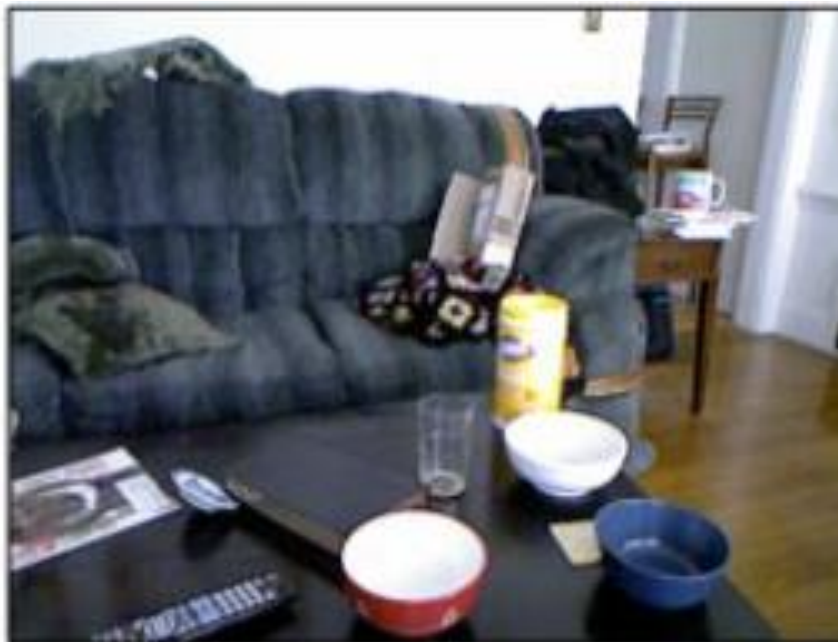
Mechanical creation of a perspective image,
Albrecht Dürer, 1525

Alexei Efros
CS280, Spring 2023

The Vision Story Begins...

“What does it mean, to see? The plain man's answer (and Aristotle's, too). would be, to know what is where by looking...”

“In other words, vision is the process of discovering from images what is present in the world, and where it is.”



Computer Vision: a split personality

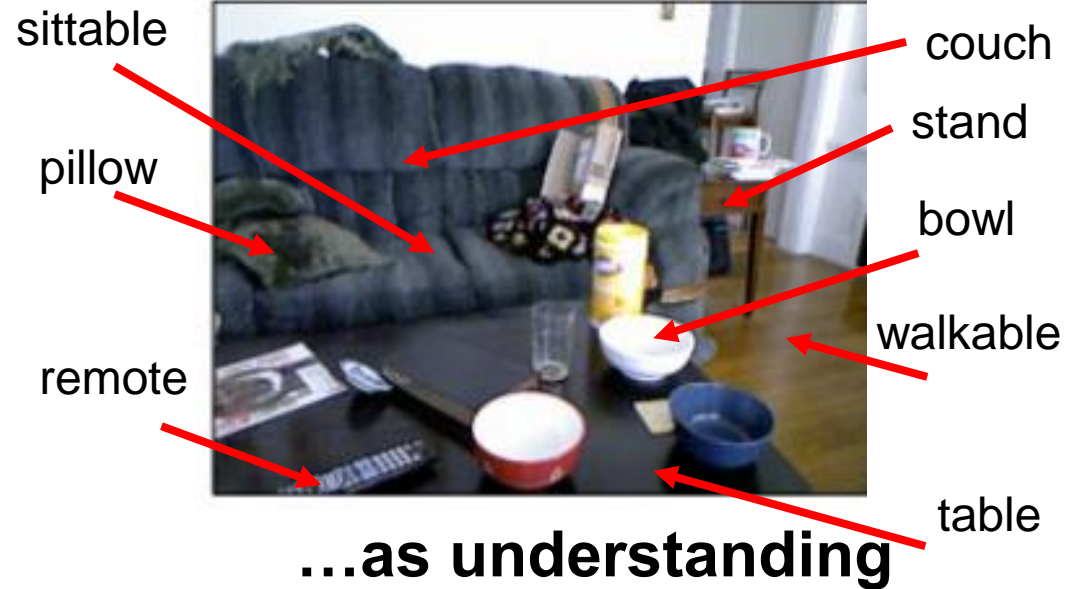


...as measurement

Goals: **Objective** (depth, distance, etc)

Represented by: meters, angles, 3D meshes, etc.

Related fields: mathematics, optics, physics, etc.



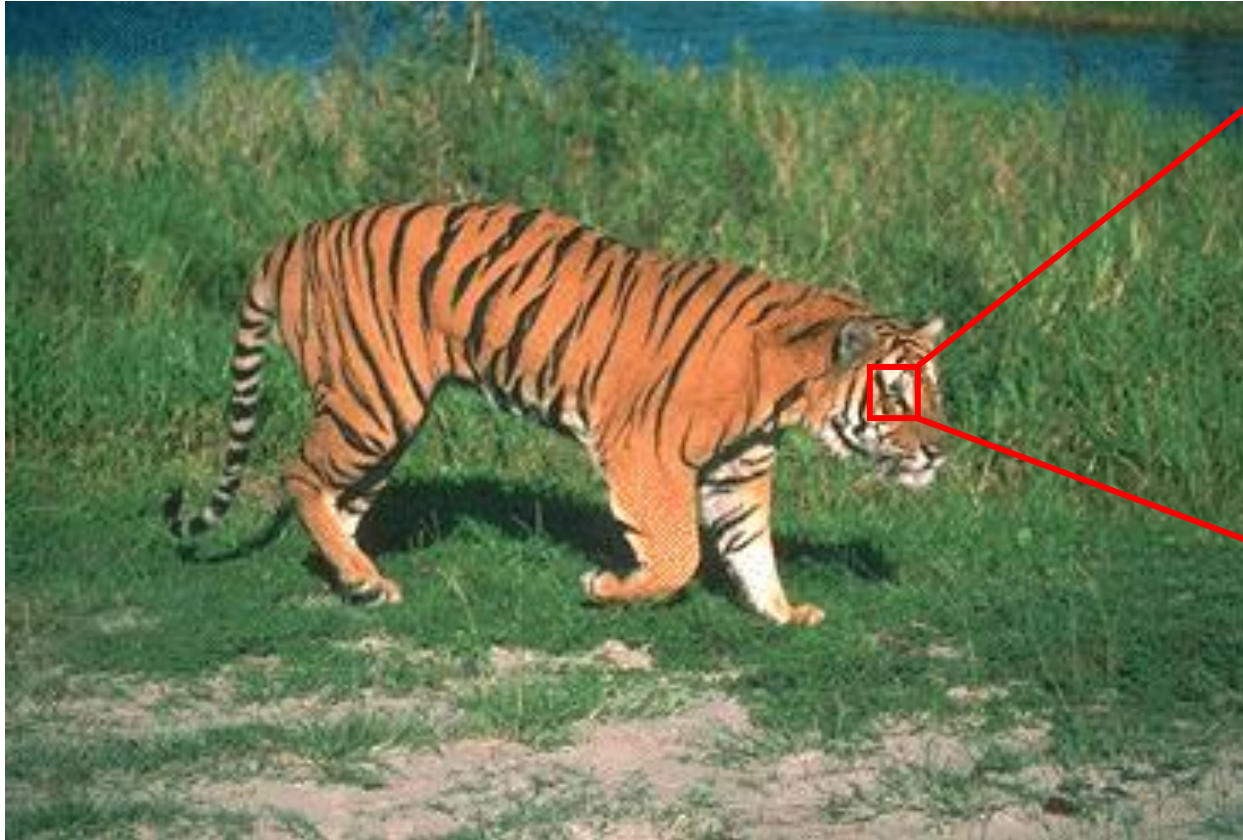
...as understanding

Goals: **Subjective** (objects, parts, affordances)

Represented by: words, human annotations, etc.

Related fields: statistics, learning, psychology, philosophy, etc.

Measurement vs. Understanding

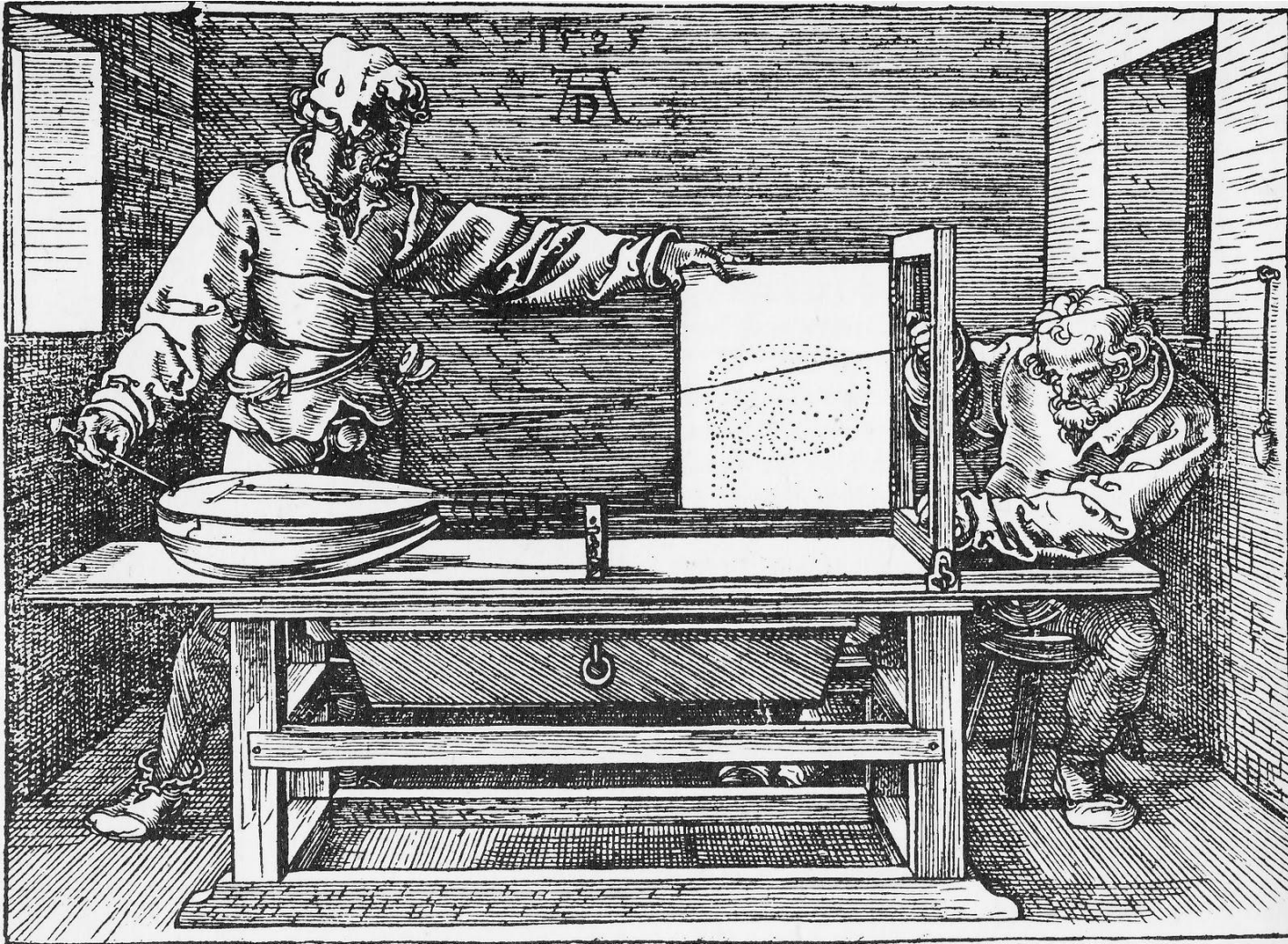


Measurement vs. Understanding



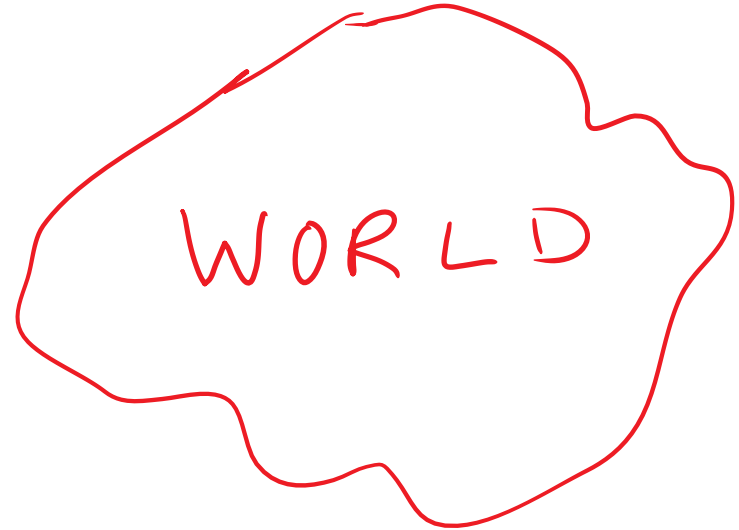
Pablo Picasso
The Guitar Player (1911)

Fundamentals of Image Formation



Mechanical creation of a perspective image,
Albrecht Dürer, 1525

A camera creates an image ...



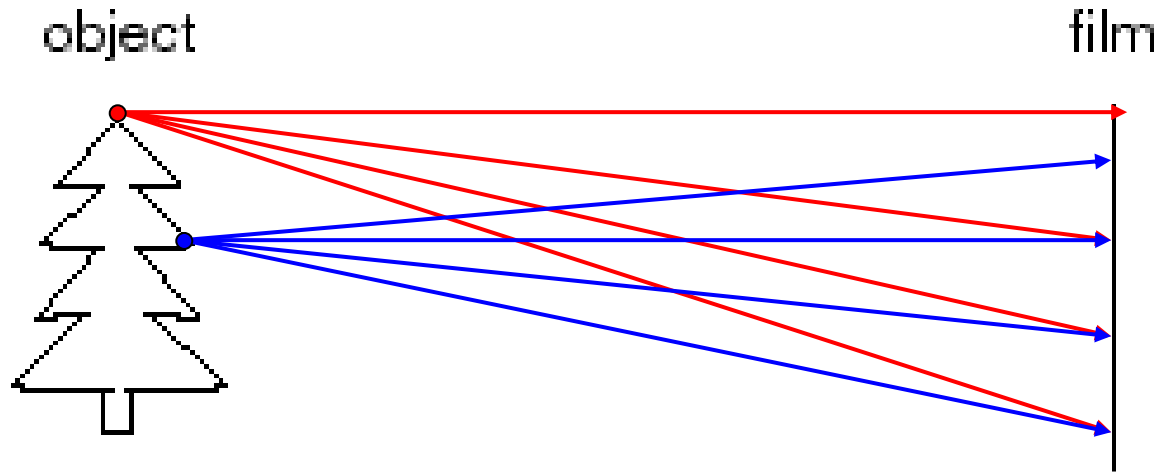
The image $I(x,y)$ measures how much light is captured at pixel (x,y)

We want to know:

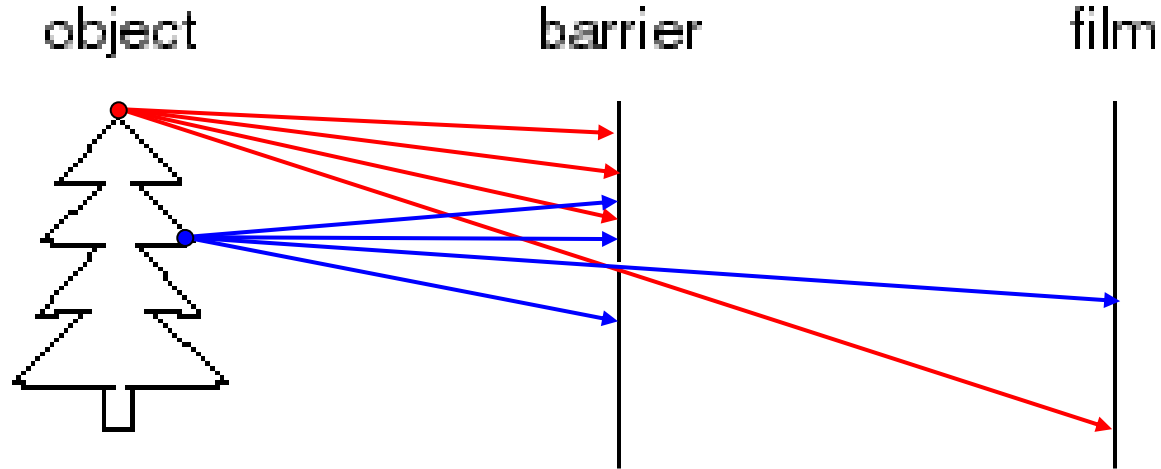
- **Where does a point (X,Y,Z) in the world get imaged?**
- What is the brightness at the resulting point (x,y) ?

Let's design a camera

Let's design a camera



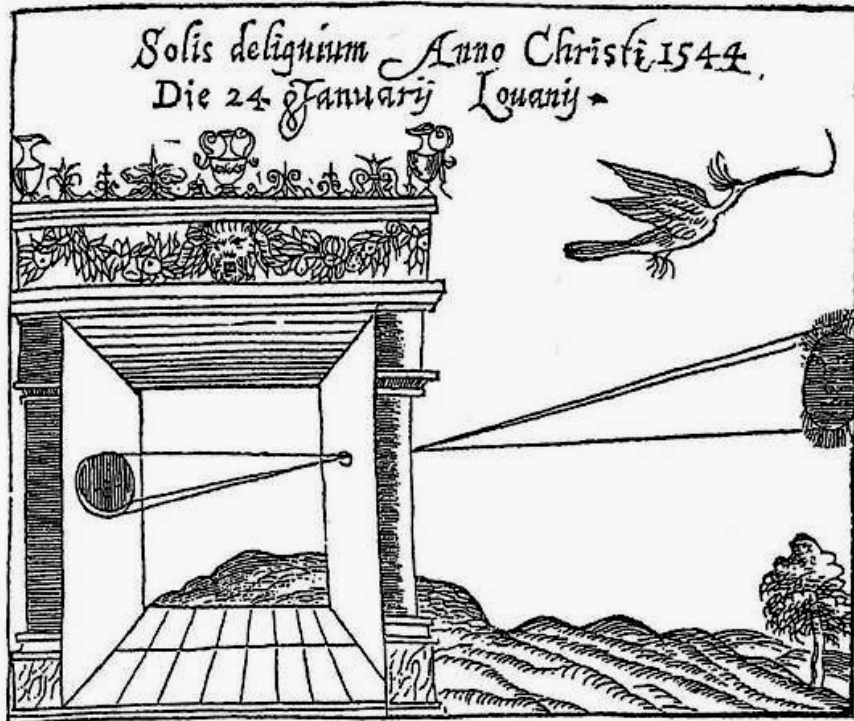
Pinhole camera



Add a barrier to block off most of the rays

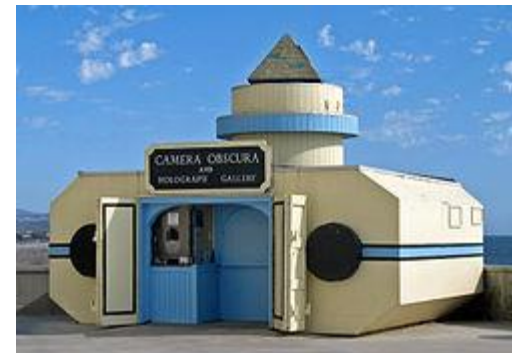
- The opening known as the **aperture**
- How does this transform the image?
- Is that a problem?

Camera Obscura: the pre-camera



Gemma Frisius, 1558

- First Idea: Mo-Ti, China (470-390 BC)
- First build: Al Hacen, Iraq/Egypt (965-1039 AD)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)



Camera Obscura near Cliff House

8-hour exposure (Abelardo Morell)



After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

After he's done inside, it gets harder. "I leave the room and I am constantly checking the weather, I'm hoping the maid reads my note not to come in, I'm worrying that the sun will hit the plastic masking and it will fall down, or that I didn't trigger the lens."



“Trashcam” Project



<http://petapixel.com/2012/04/18/german-garbage-men-turn-dumpsters-into-giant-pinhole-cameras/>

Accidental pinhole cameras

My hotel room,
contrast enhanced.



The view from my window



Accidental pinholes produce images that are
unnoticed or misinterpreted as shadows

Pinhole cameras everywhere



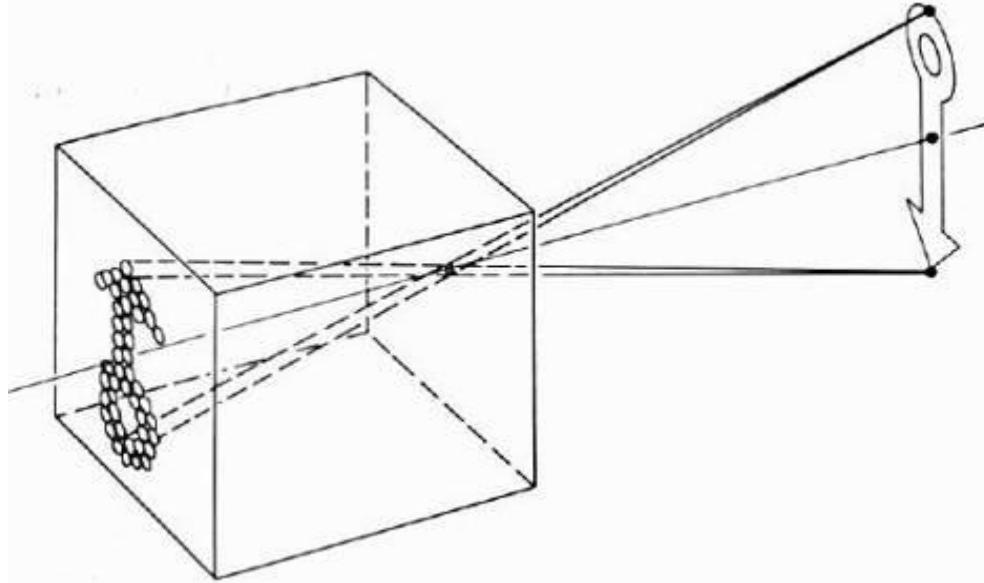
Tree shadows

photo

<http://www.fox.com>

© Trina Singley

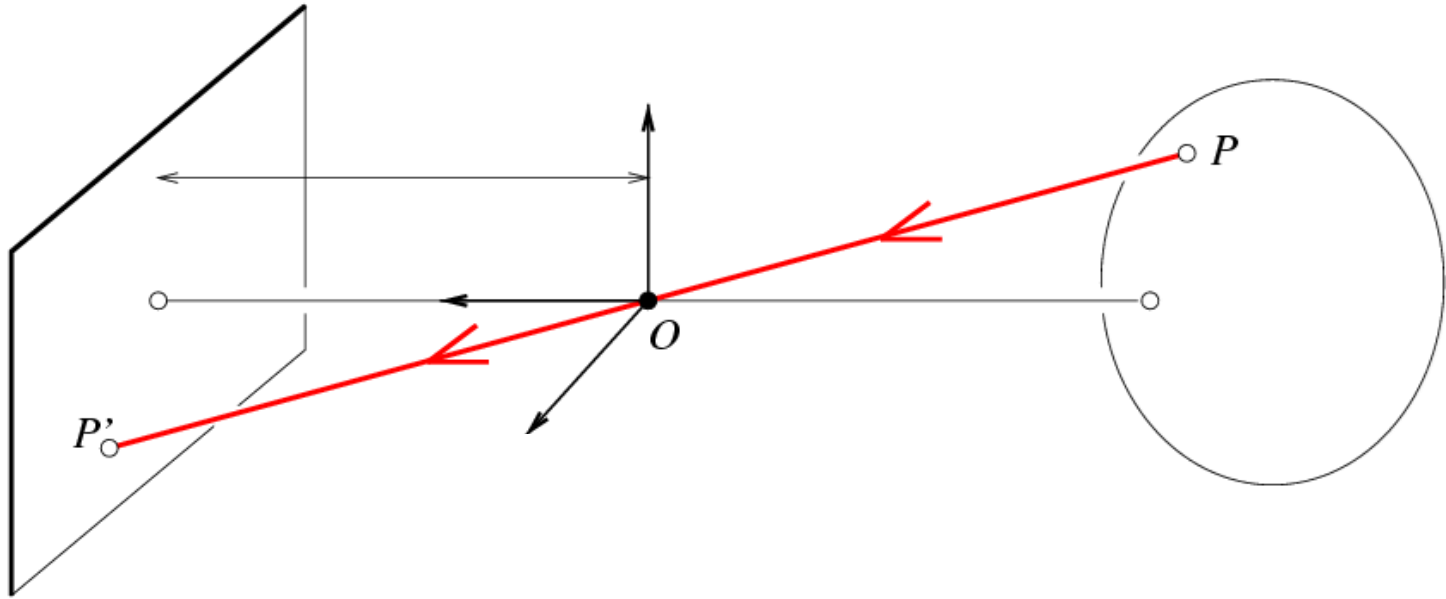
Back to pinhole camera model



Pinhole model:

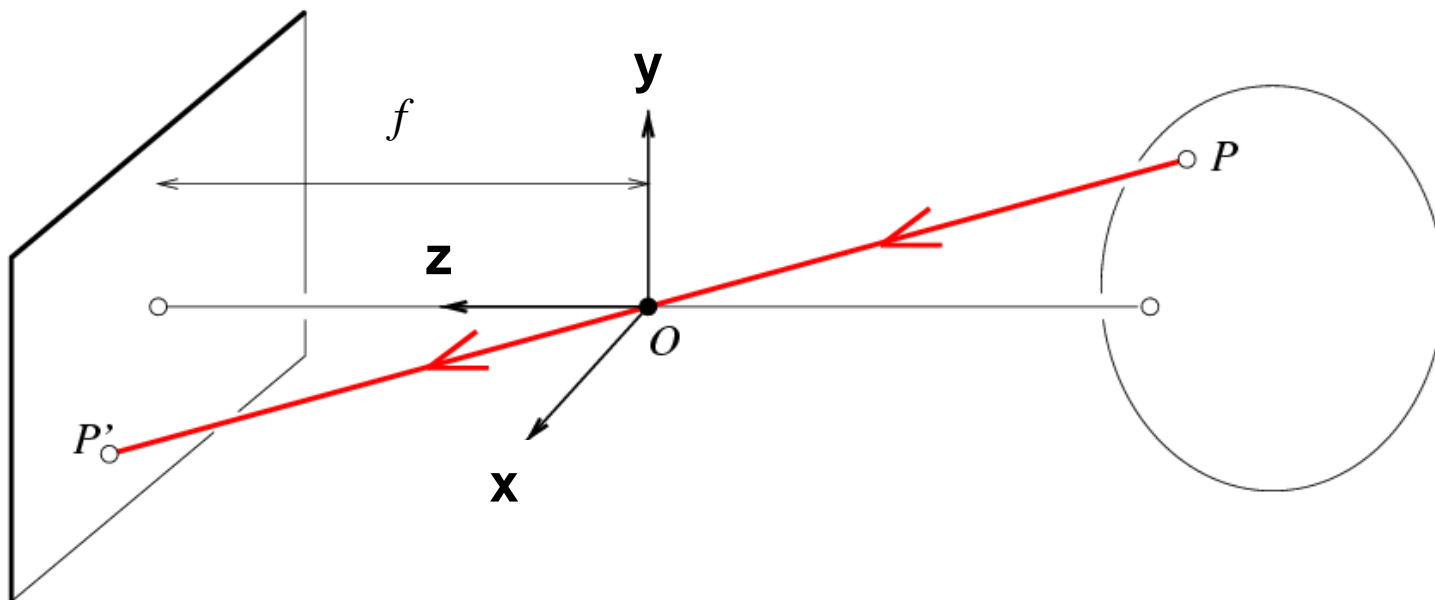
- Captures **pencil of rays** – all rays through a single point
- The point is called **Center of Projection (COP)**
- The image is formed on the **Image Plane**
- **Effective focal length f** is distance from COP to Image Plane

Modeling projection



- To compute the projection P' of a scene point P , form the **visual ray** connecting P to the camera center O and find where it intersects the image plane
 - All scene points that lie on this visual ray have the same projection in the image
 - Are there scene points for which this projection is undefined?

Modeling projection



The coordinate system

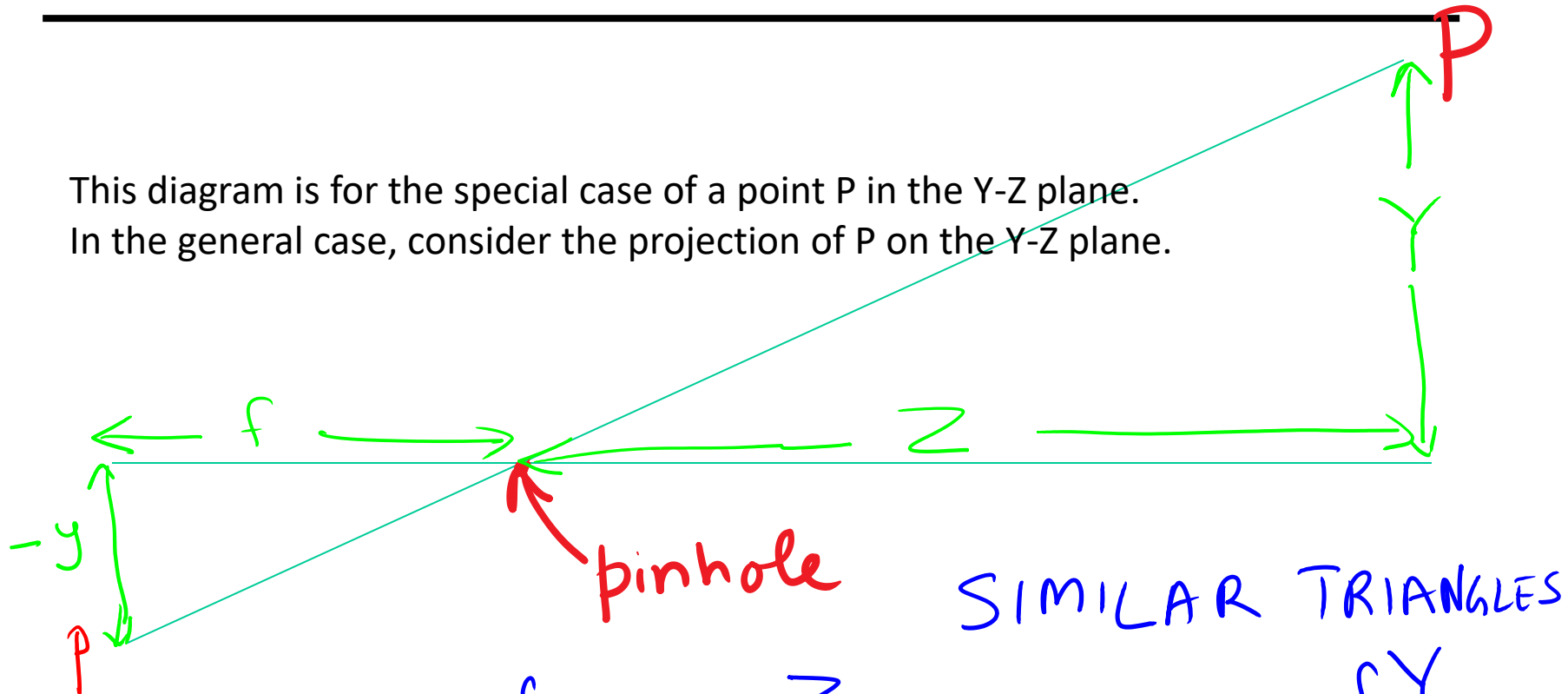
- The optical center (O) is at the origin
- The image plane is parallel to xy -plane (perpendicular to z axis)

Projection equations

$$x = f \frac{X}{Z}, y = f \frac{Y}{Z}$$

Let us prove this ...

This diagram is for the special case of a point P in the Y-Z plane.
In the general case, consider the projection of P on the Y-Z plane.

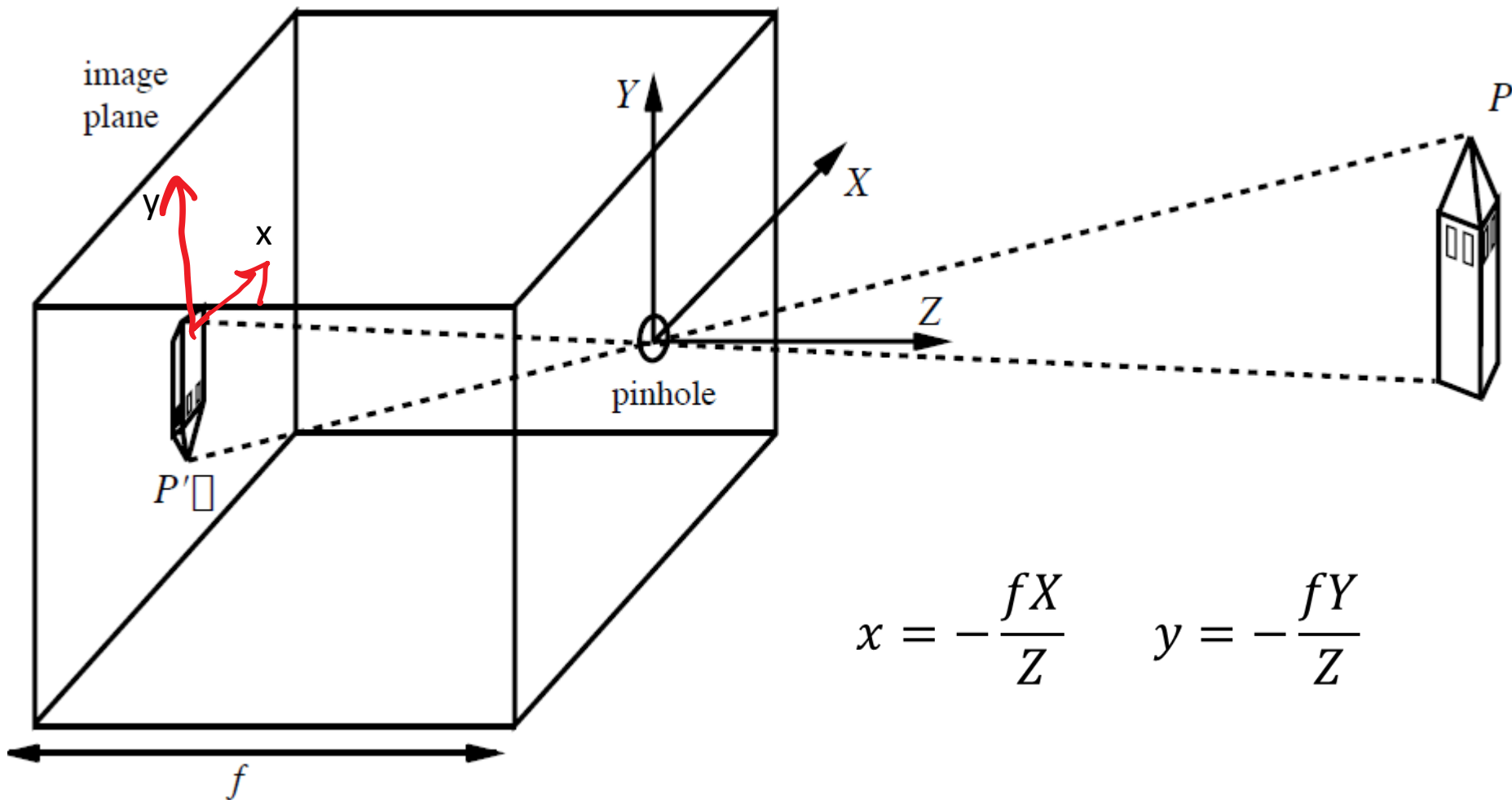


$$\frac{f}{-y} = \frac{Z}{Y} \Rightarrow y = -\frac{fY}{Z}$$

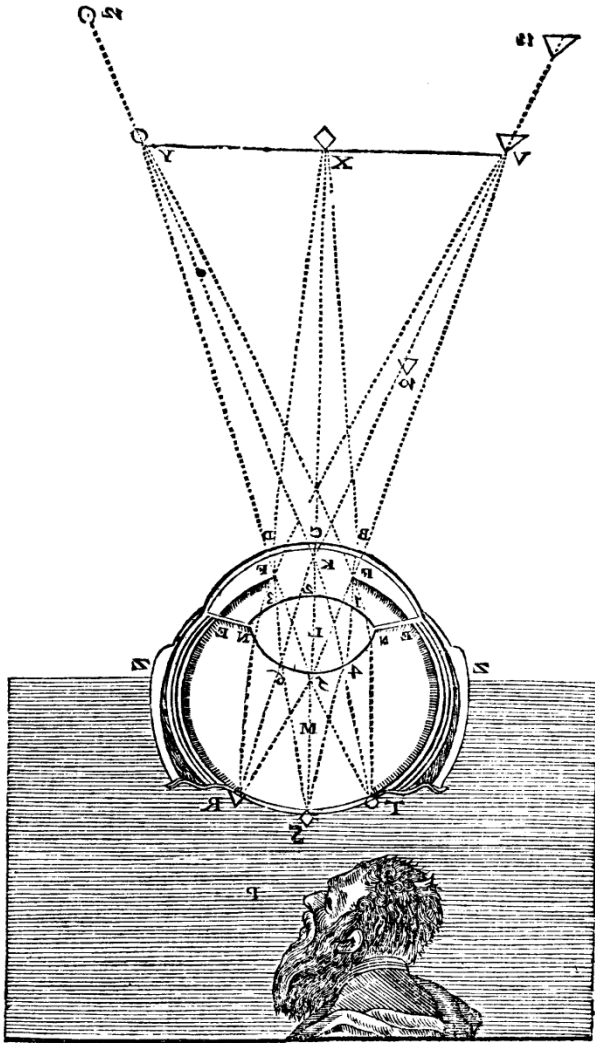
This is true even if the point P is not in the YZ plane.

By similar reasoning $x = -\frac{fX}{Z}$

The Pinhole Camera



The image is inverted



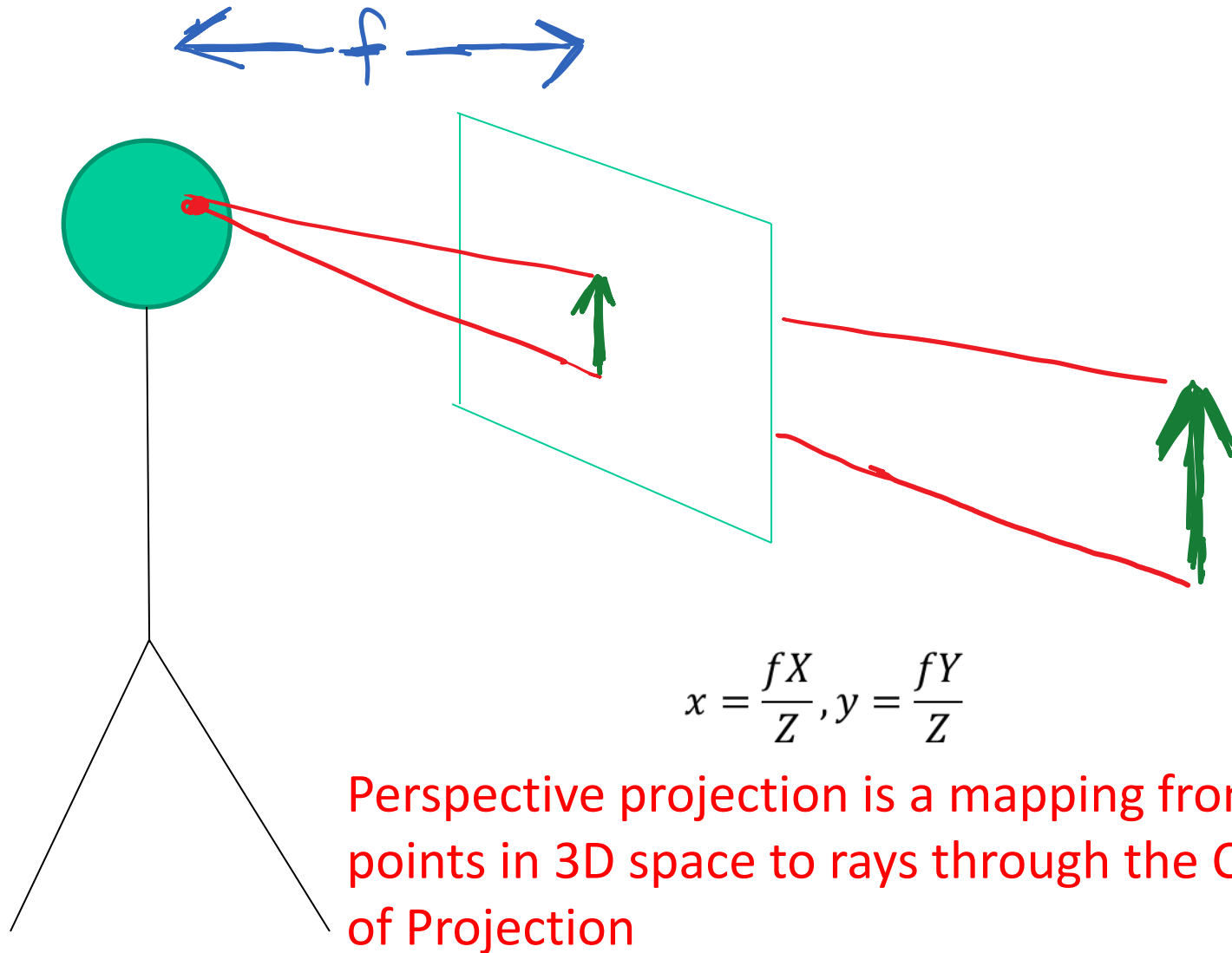
This was pointed out by Kepler in 1604

But this is no big deal. The brain can interpret it the right way. And for a camera, software can simply flip the image top-down and right-left. After this trick, we get

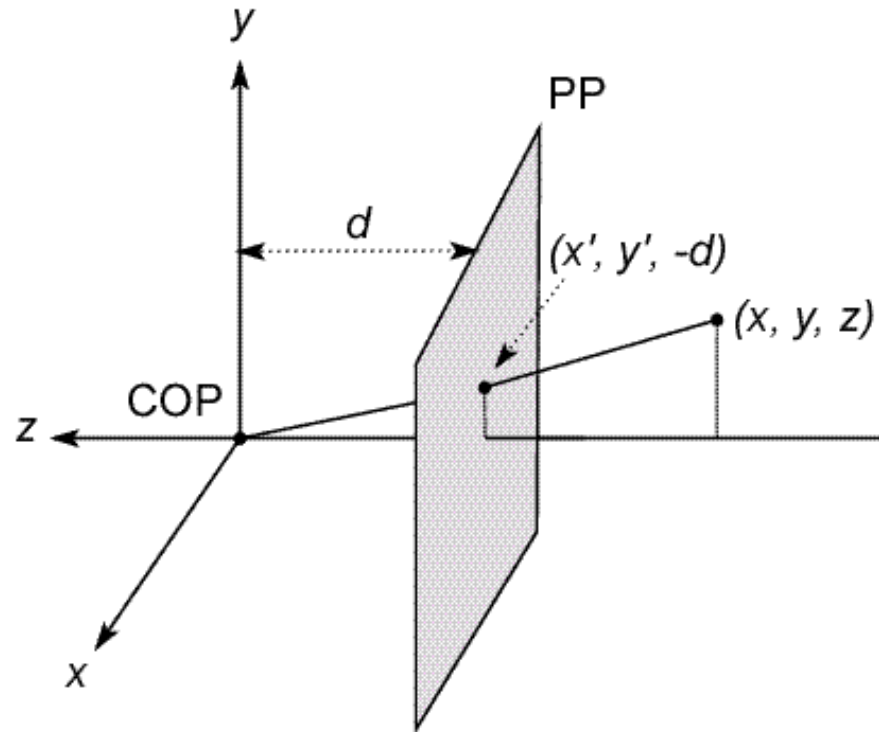
$$x = \frac{fX}{Z} \quad y = \frac{fY}{Z}$$

From Descartes(1637), La Dioptrique

~~A projection model that avoids inversion~~



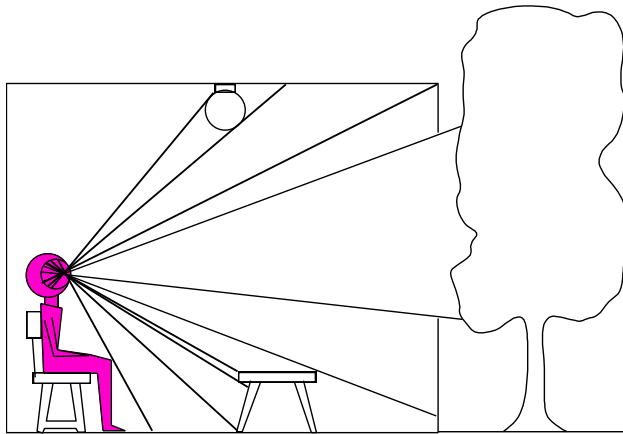
Simple trick to avoid inversion



—

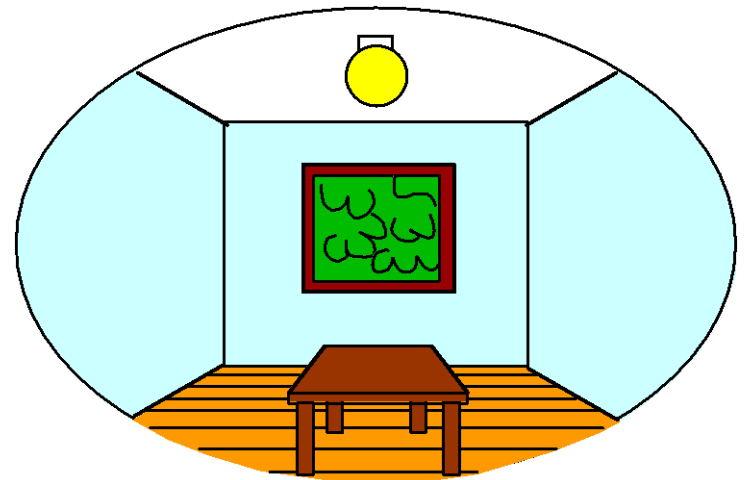
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image

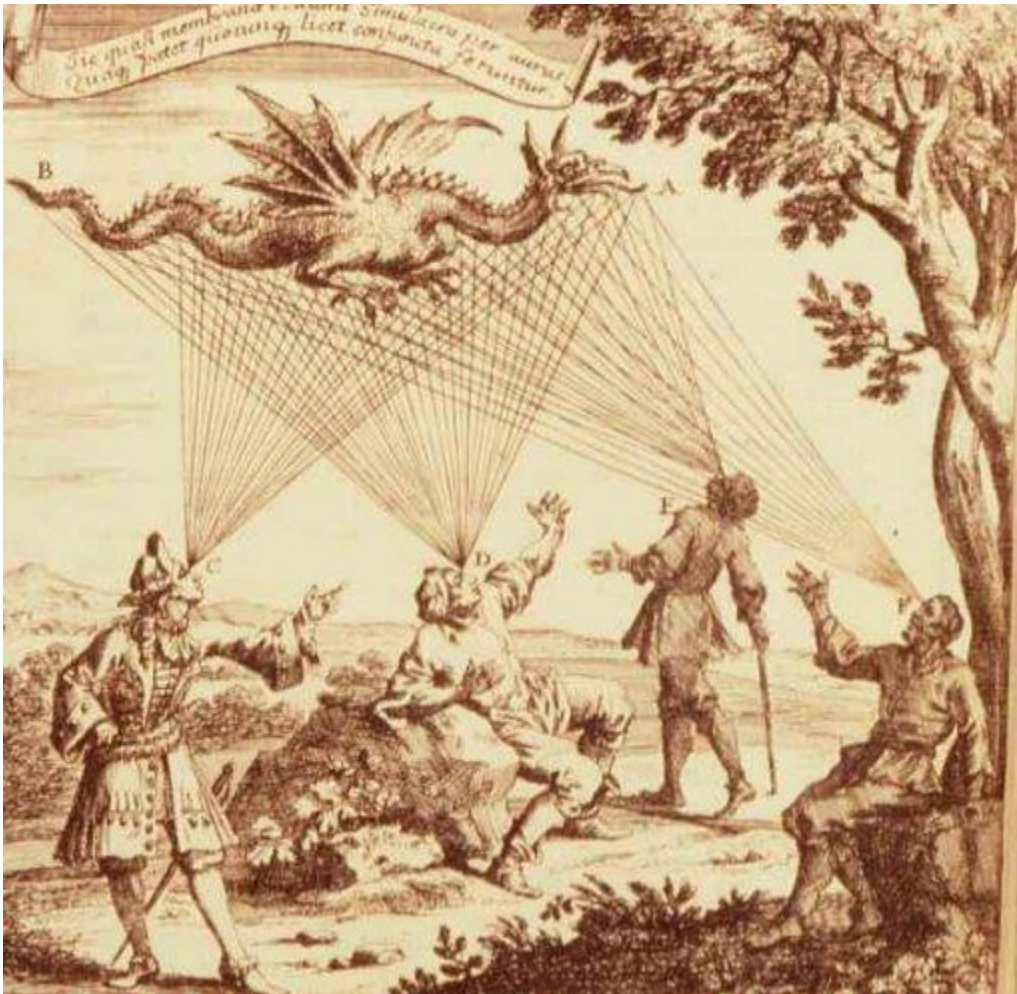


But there is a problem...

Emission Theory of Vision

“For every complex problem there is an answer that is clear, simple, and wrong.”

-- H. L. Mencken



Eyes send out “feeling rays” into the world

Supported by:

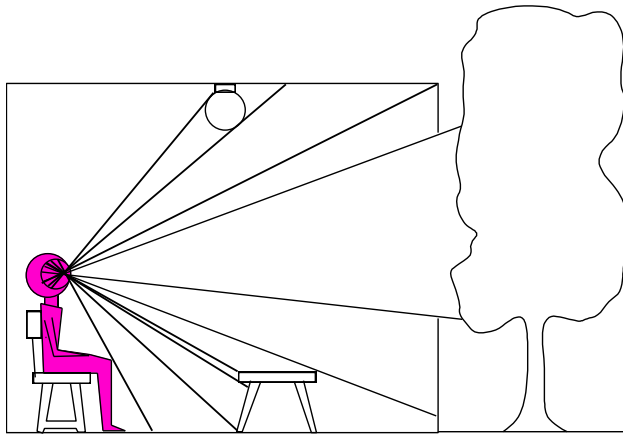
- Empedocles
- Plato
- Euclid (kinda)
- Ptolemy
- ...
- 50% of US college students*

[*http://www.ncbi.nlm.nih.gov/pubmed/12094435?dopt=Abstract](http://www.ncbi.nlm.nih.gov/pubmed/12094435?dopt=Abstract)



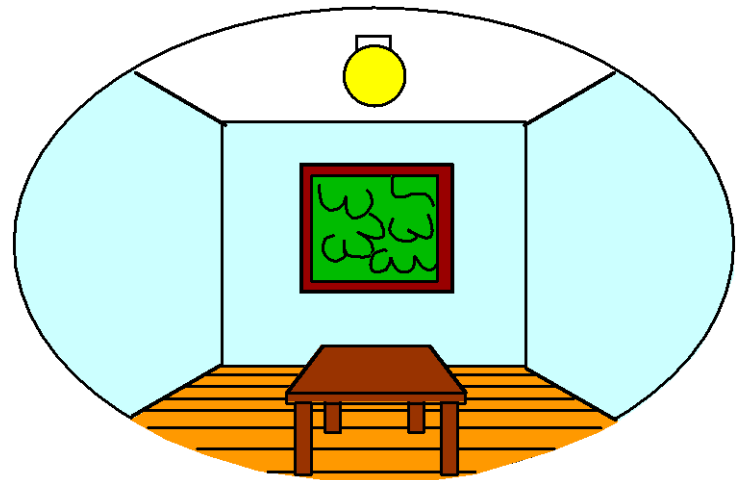
How we see the world

3D world



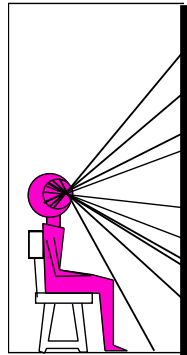
Point of observation

2D image



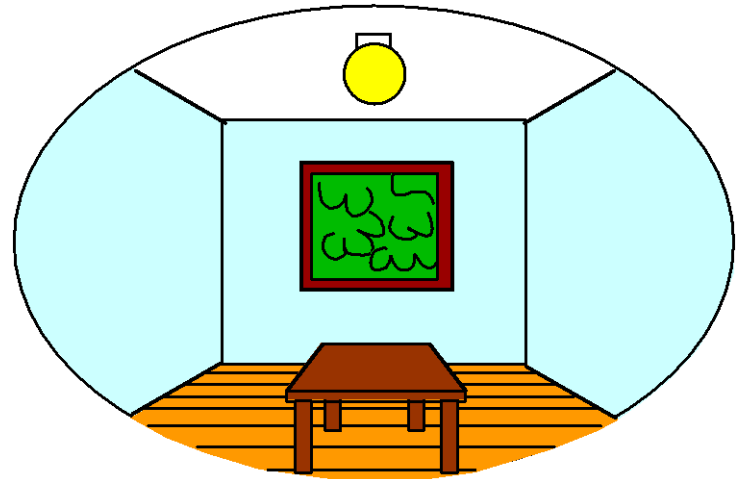
How we see the world

3D world



Painted
backdrop

2D image



Fooling the eye



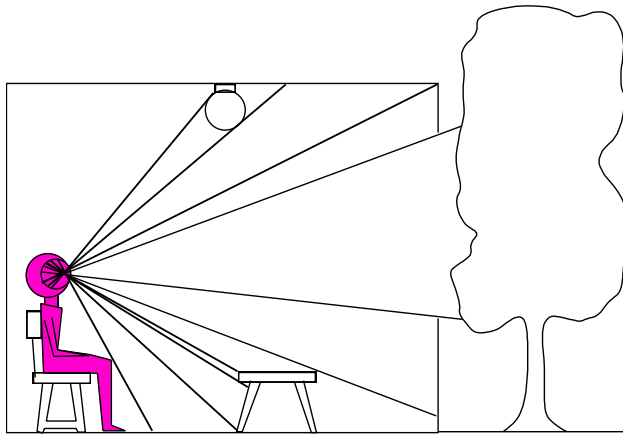
Fooling the eye



Making of 3D sidewalk art: <http://www.youtube.com/watch?v=3SNYtd0Ayt0>

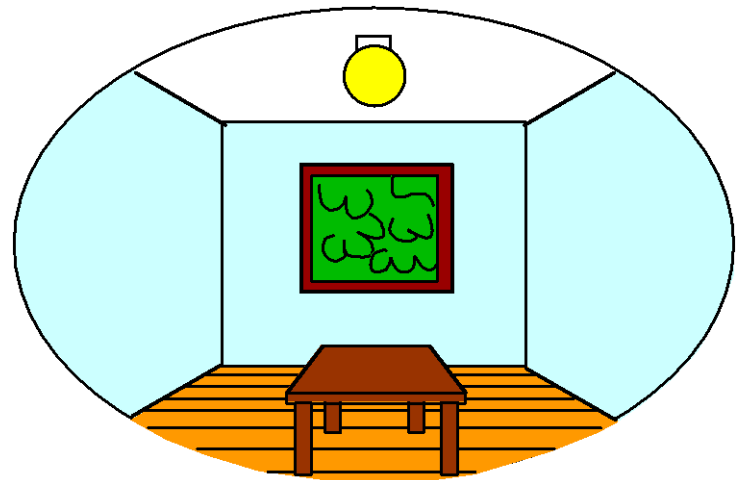
How we see the world

3D world



Point of observation

2D image

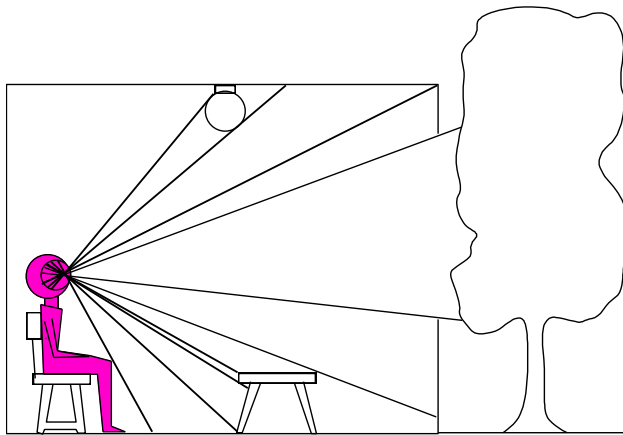


What is being lost?

- Distances (lengths)
- Angles

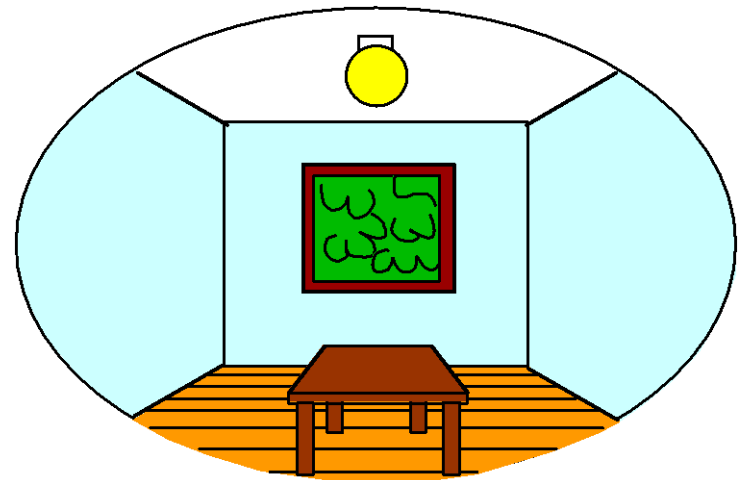
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

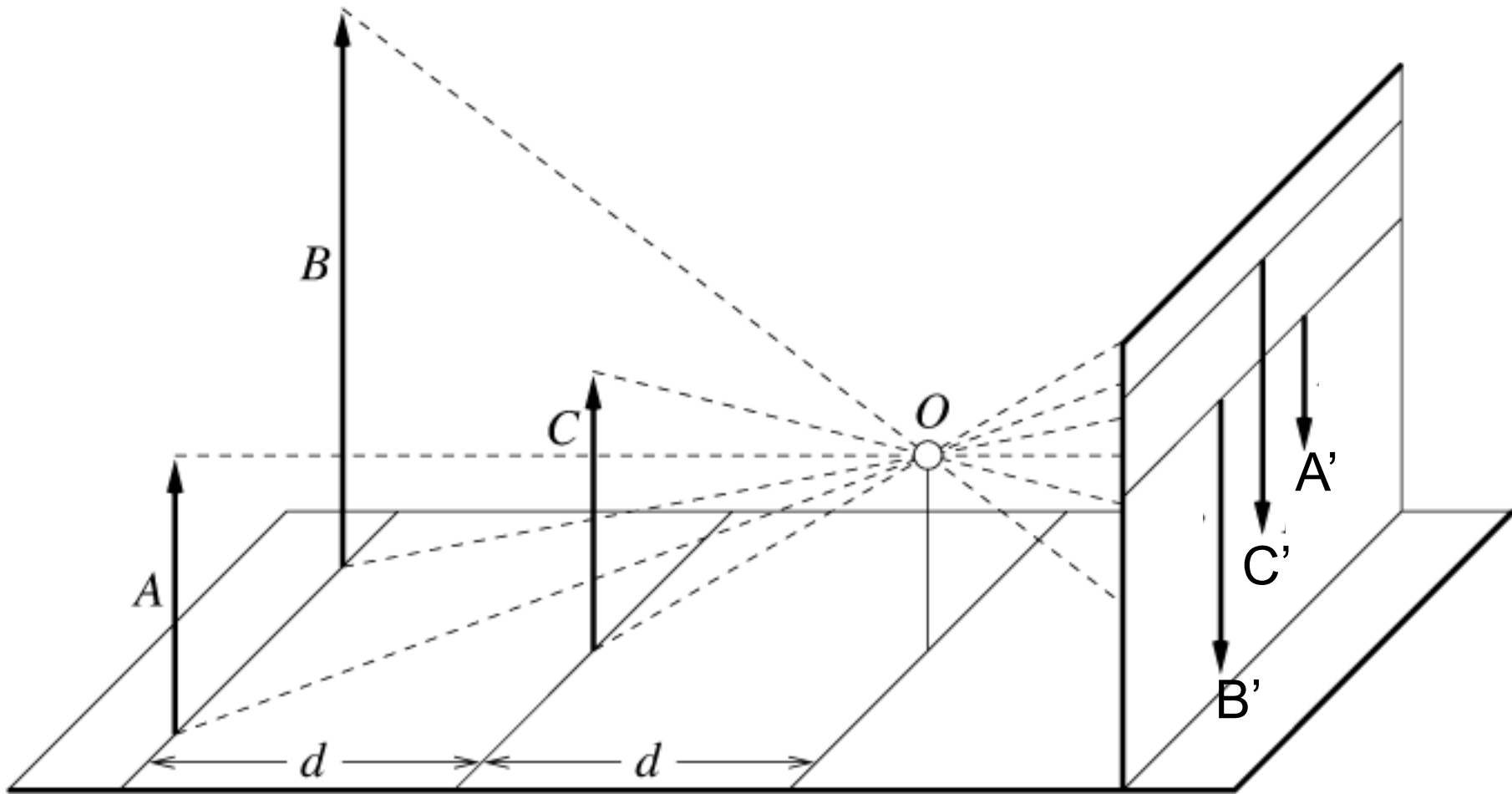
2D image



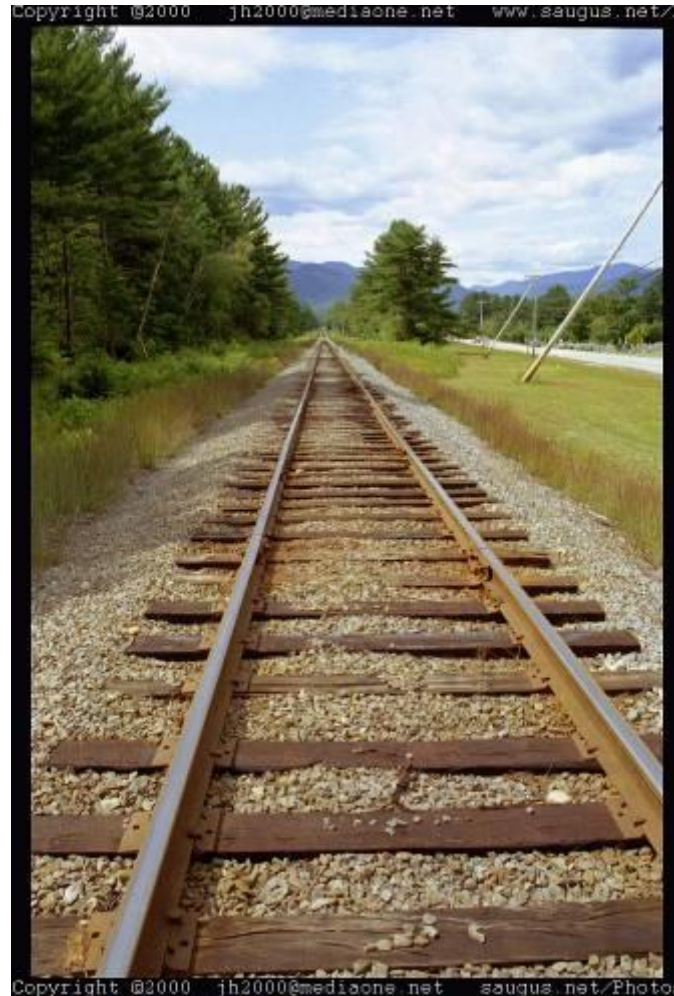
Why did evolution opt for such strange solution?

- Nice to have a passive, long-range sensor
- Can get 3D by moving around & experience

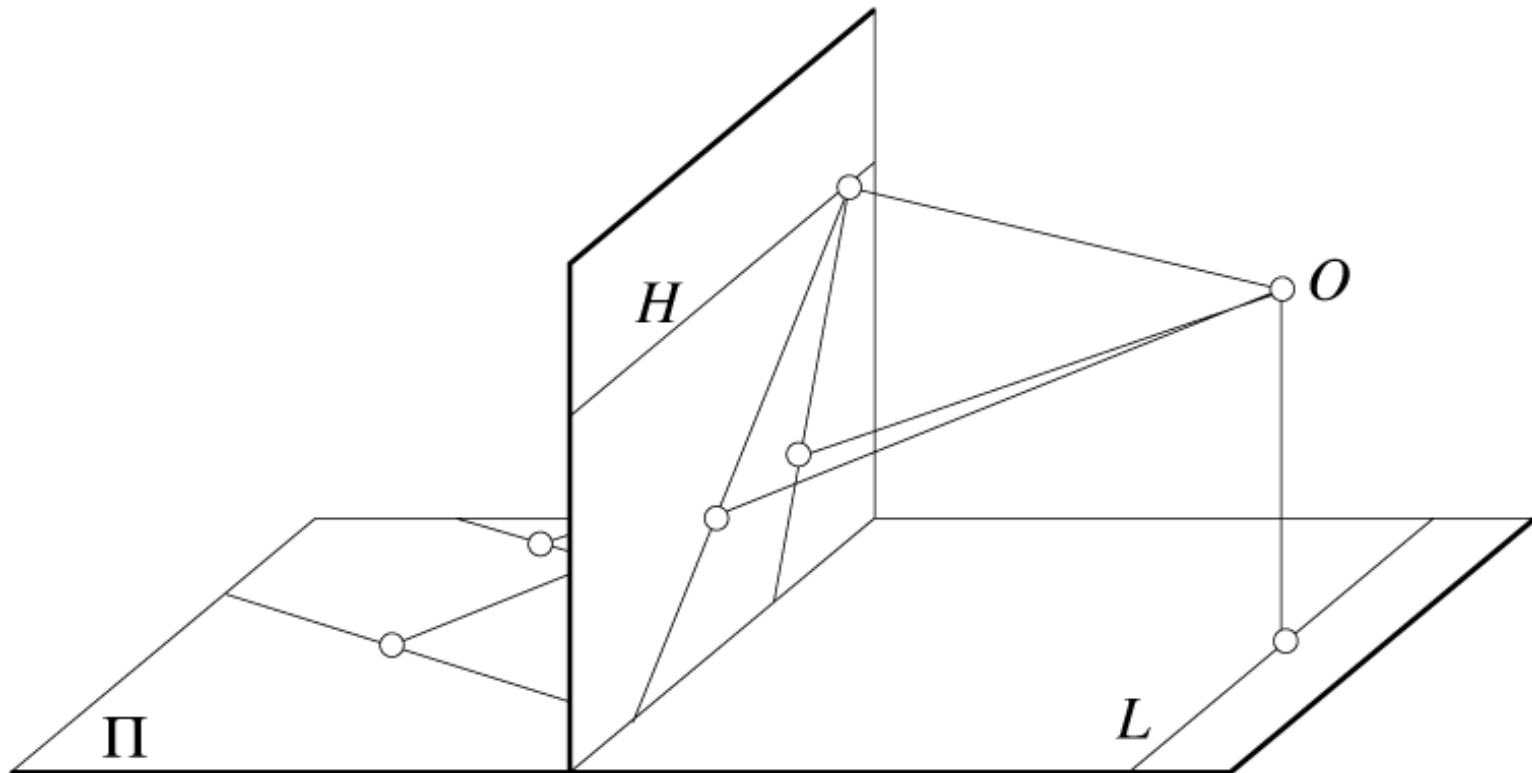
Funny things happen...



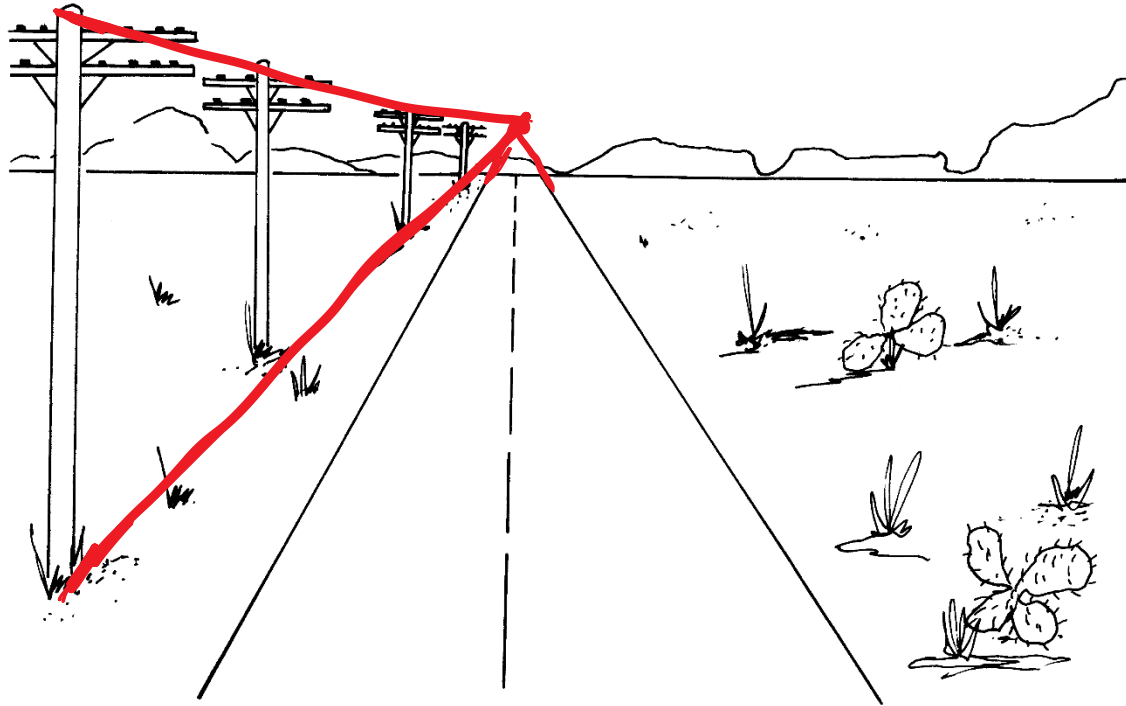
Silly Euclid...



Parallel lines aren't...



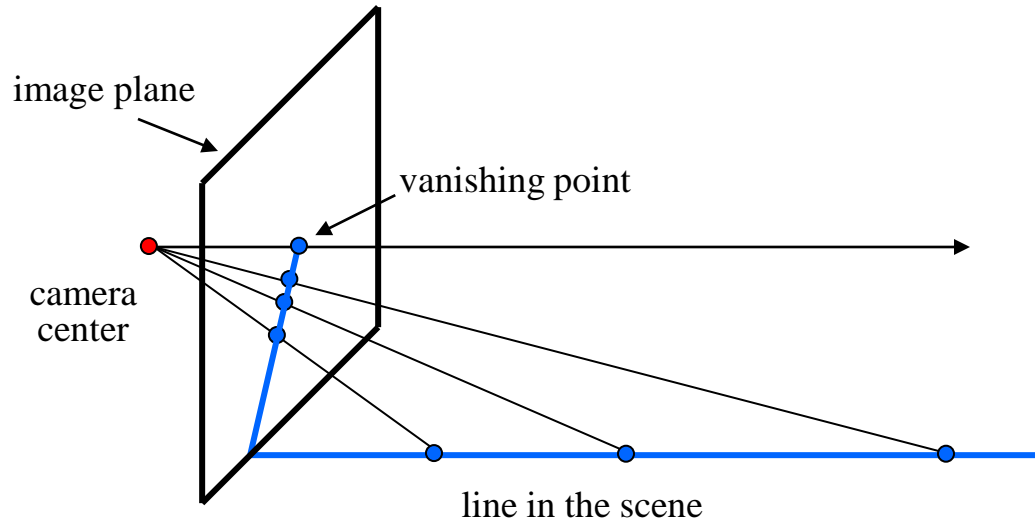
Parallel lines converge to a vanishing point



Proof

Let there be a point A and a direction vector D in three dimensional space.

Projection of a line



Vanishing point in vector notation

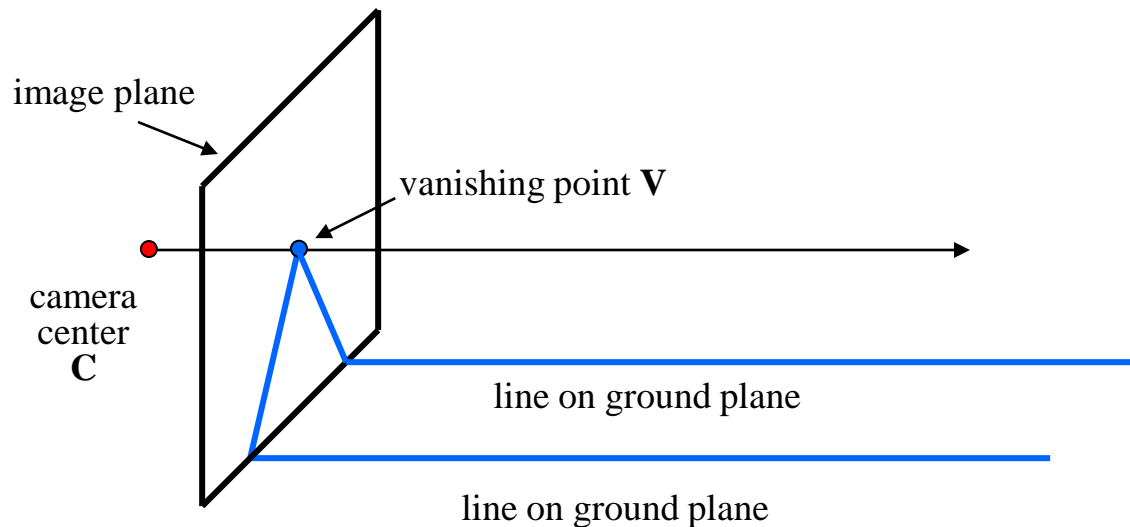
$$\mathbf{p} = f \frac{\mathbf{X}}{Z}$$

A line of points in 3D can be represented as $\mathbf{X} = \mathbf{A} + \lambda \mathbf{D}$, where \mathbf{A} is a fixed point, \mathbf{D} a unit vector parallel to the line, and λ a measure of distance along the line. As λ increases points are increasingly further away and in the limit:

$$\lim_{\lambda \rightarrow \infty} \mathbf{p} = f \frac{\mathbf{A} + \lambda \mathbf{D}}{A_Z + \lambda D_Z} = f \frac{\mathbf{D}}{D_Z}$$

i.e. the image of the line terminates in a *vanishing point* with coordinates $(f D_X / D_Z, f D_Y / D_Z)$, unless the line is parallel to the image plane ($D_Z = 0$). Note, the vanishing point is unaffected (invariant to) line position, \mathbf{A} , it only depends on line orientation, \mathbf{D} . Consequently, the family of lines parallel to \mathbf{D} have the same vanishing point.

Vanishing points



Properties

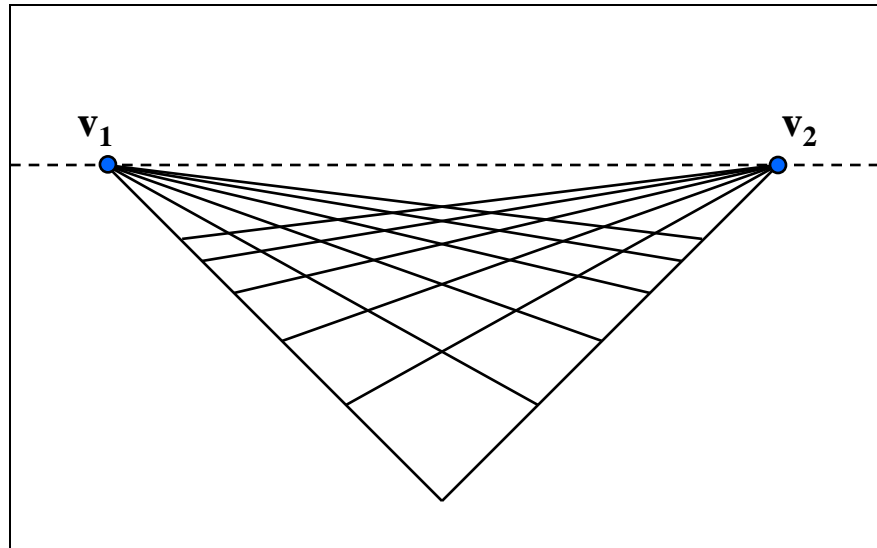
- Any two parallel lines have the same vanishing point v
- The ray from C through v is parallel to the lines
- An image may have more than one vanishing point

Each family of parallel lines has its own vanishing point



But this isn't true of the vertical lines. They stay parallel. Why?

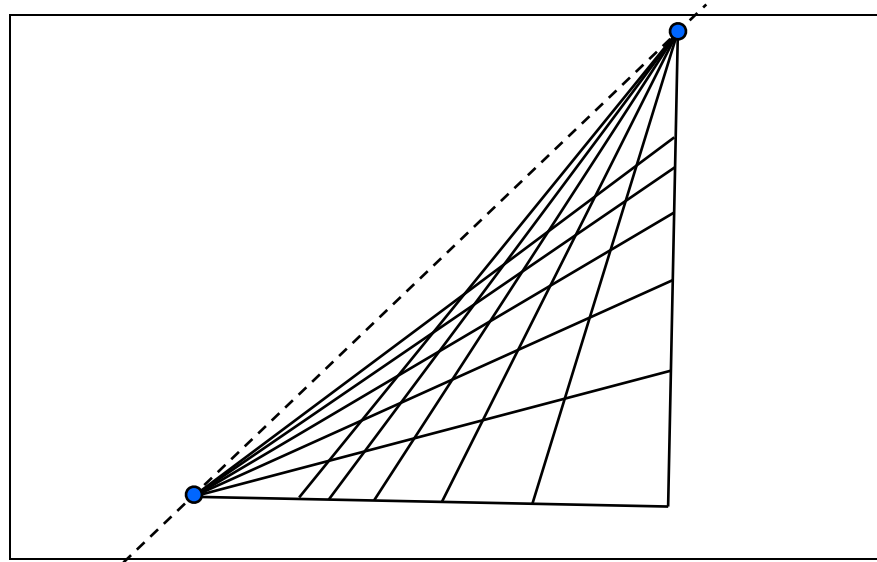
Vanishing lines



Multiple Vanishing Points

- Any set of parallel lines on the plane define a vanishing point
- The union of all of these vanishing points is the *vanishing line*
 - *horizon line* is a special case
- Note that different planes define different vanishing lines

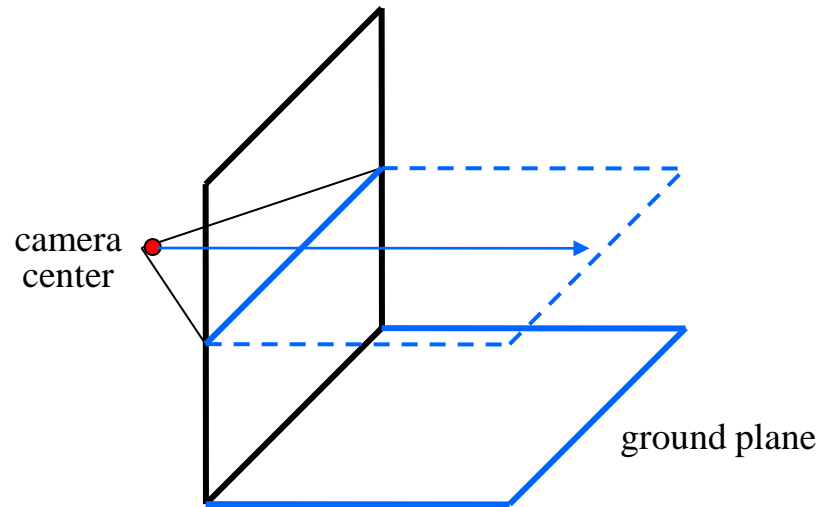
Vanishing lines



Multiple Vanishing Points

- Any set of parallel lines on the plane define a vanishing point
- The union of all of these vanishing points is the *vanishing line*
 - *horizon line* is a special case
- Note that different planes define different vanishing lines

Special case: ground plane & horizon

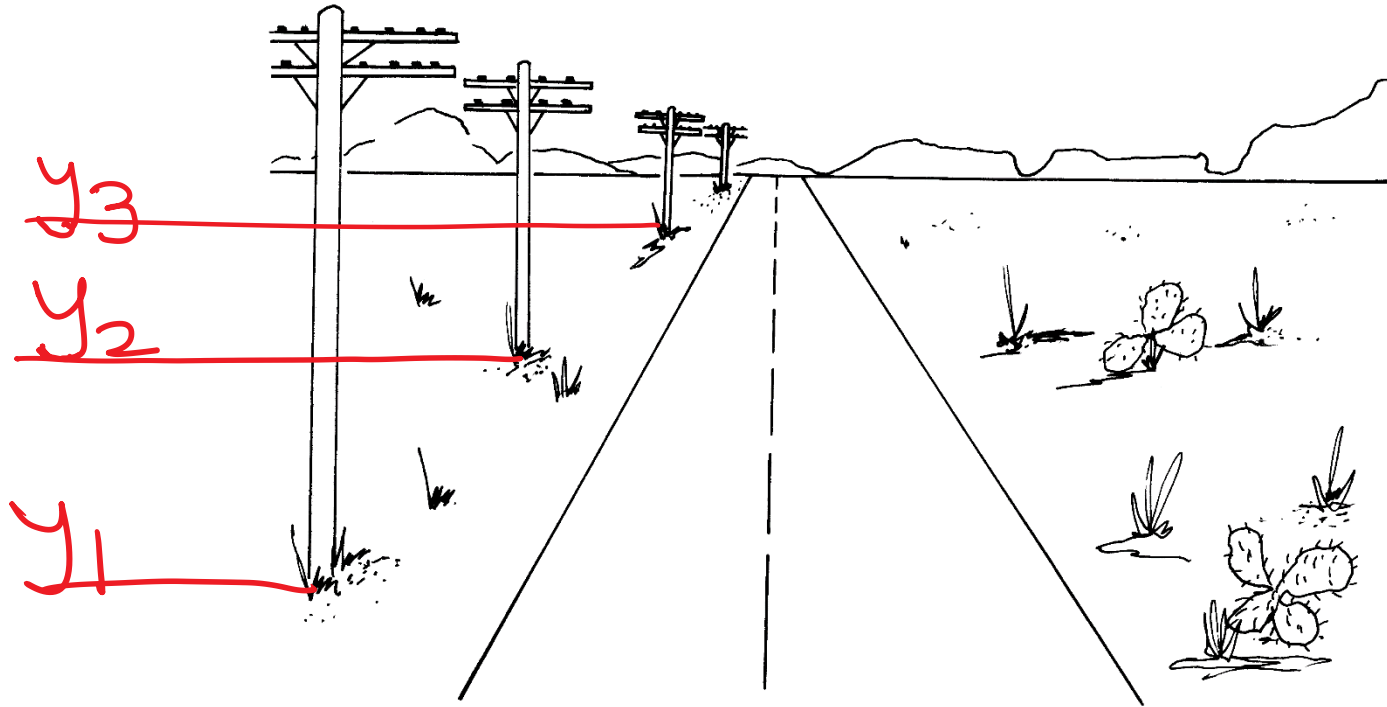


- Vanishing line of the ground plane
 - All points at the same height as the camera project to the horizon
 - Points higher than the camera project above the horizon
 - Provides way of comparing height of objects

The horizon



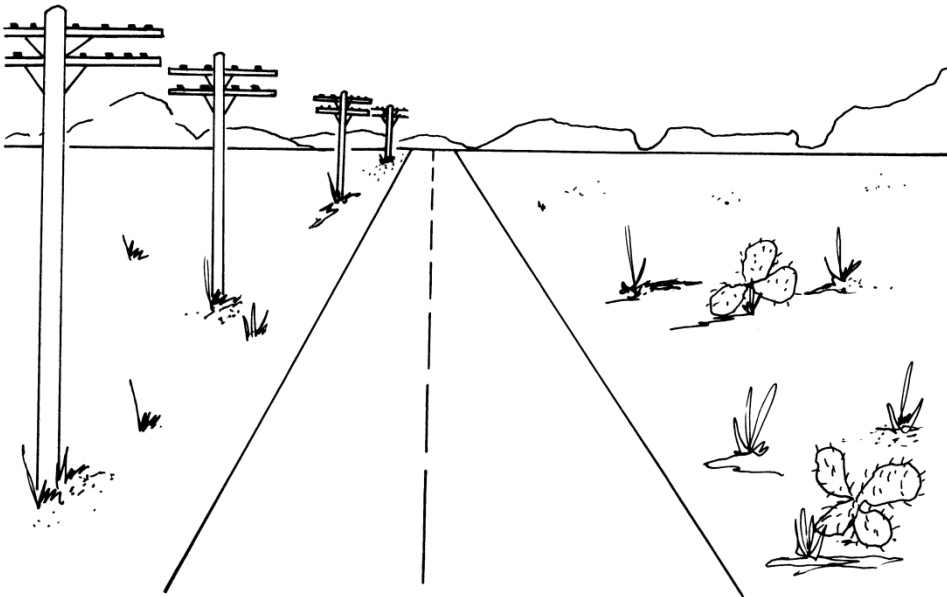
Nearer objects are lower in the image



Proof

The equation of the ground plane is $Y = -h$

A point on the ground plane will have y-coordinate $y = -fh/Z$



Exciting New Study!



Sailors Take Warning



A.V. CLUB



VIDEO

POLITICS

SPORTS

BUSINESS

SCIENCE/TECH

ENTERTAINMENT

LOCAL

Q search

RECENT NEWS

Ariel Castro Failed By System

New Skin Cream To Do Something

Assad Unable To Convince Putin
That He Used Chemical Weapons
On Syrians

Royal Baby Already Making New
Friends

College-Aged Female Finds
Unlikely Kindred Spirit In Audrey
Hepburn

Personal Trainer Has Desk

Bruce Springsteen On Fence About
Playing Assad's Birthday Gig

Study: People Far Away From You Not Actually Smaller

NEWS • Science & Technology • ISSUE 49•34 • Aug 22, 2013



Researchers say that, contrary to prior assertions, the subject above stands at equal height at left and at right, and does not grow smaller as he walks away from the camera.



10.2K



688

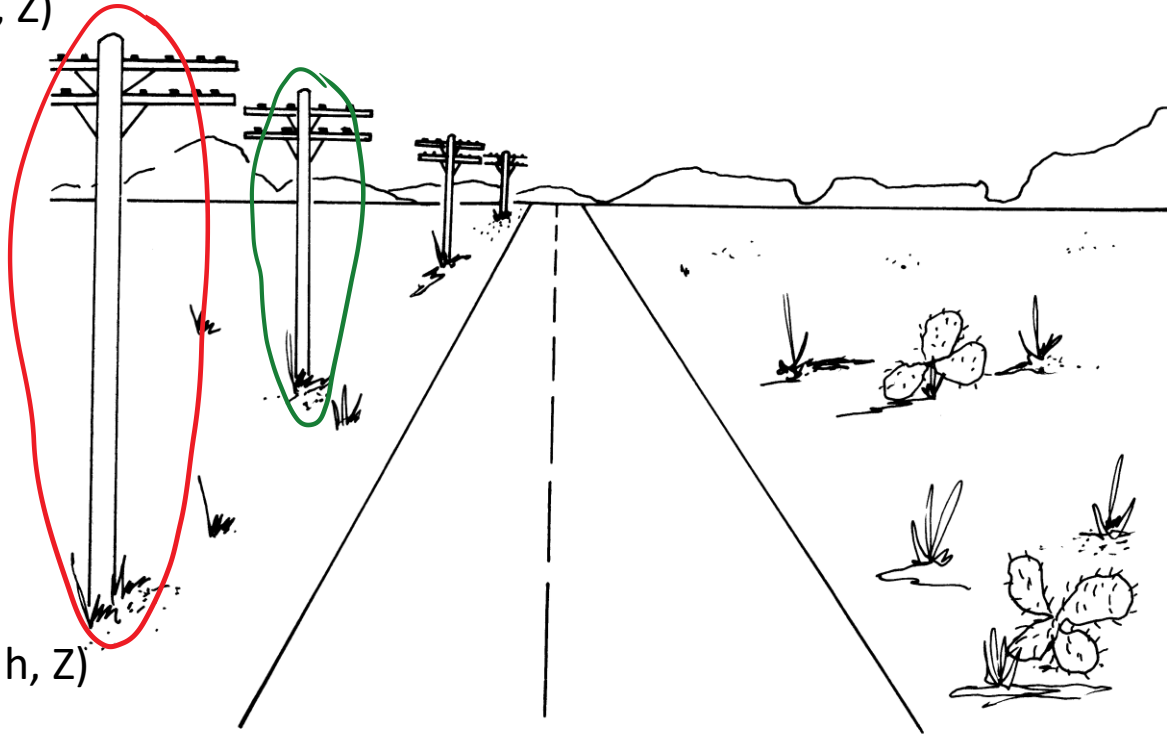


332

Nearer objects look bigger

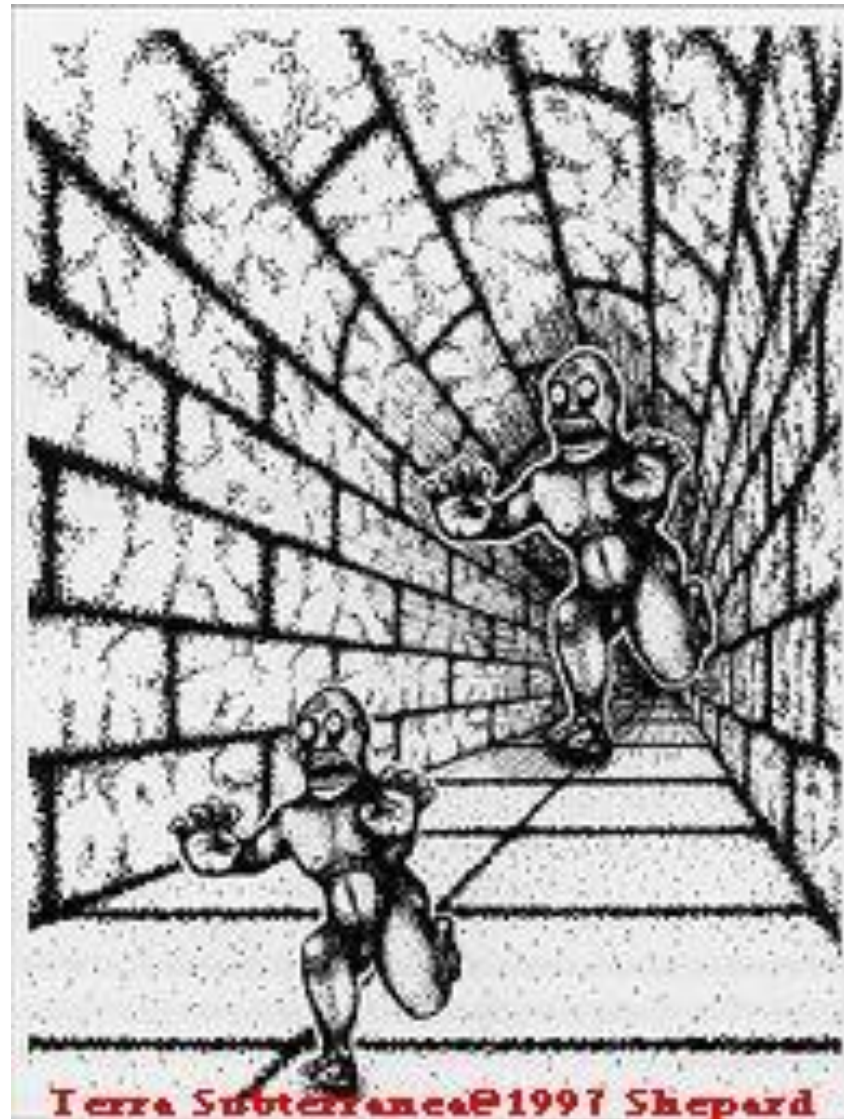
Top at $(X, L - h, Z)$

Bottom at $(X, -h, Z)$

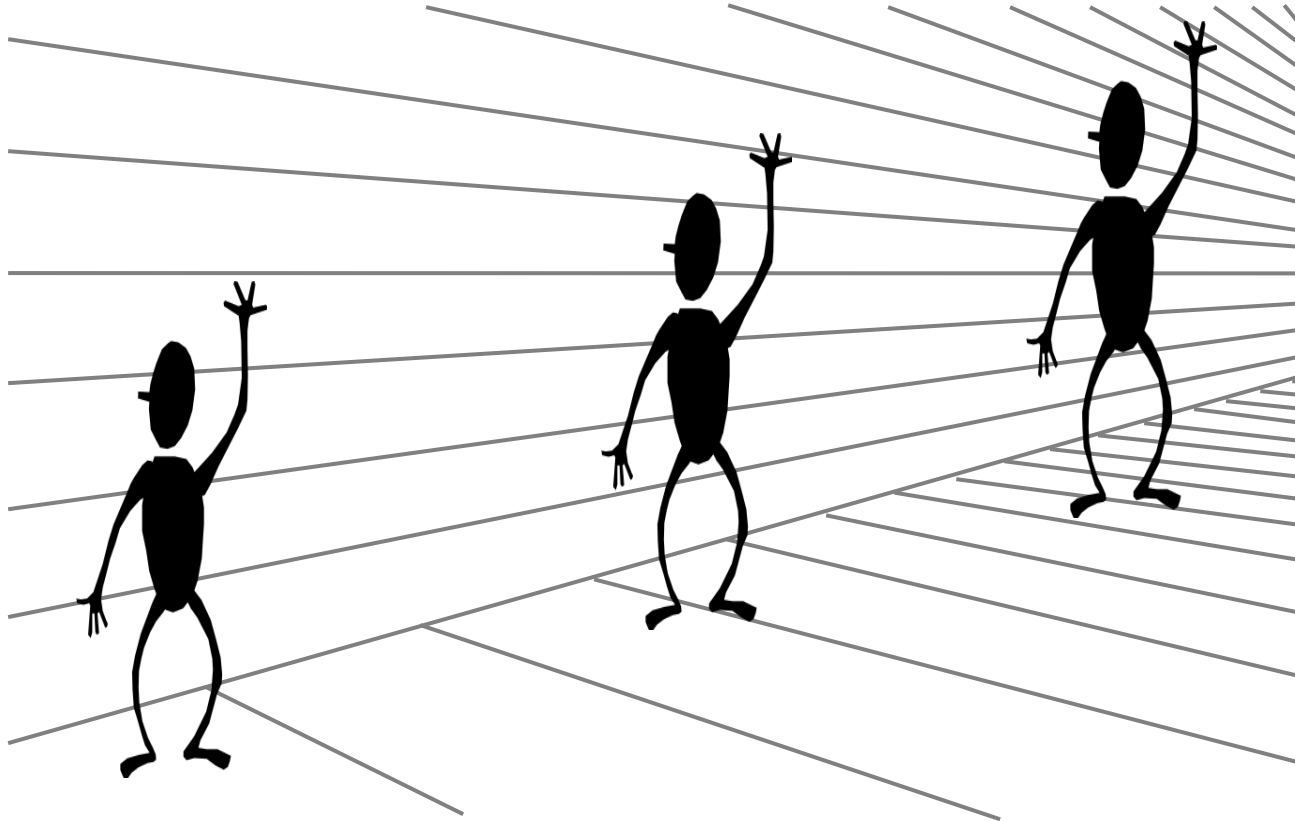


It is straightforward to calculate the projection of the top & bottom of the pole. The difference is the “apparent height”

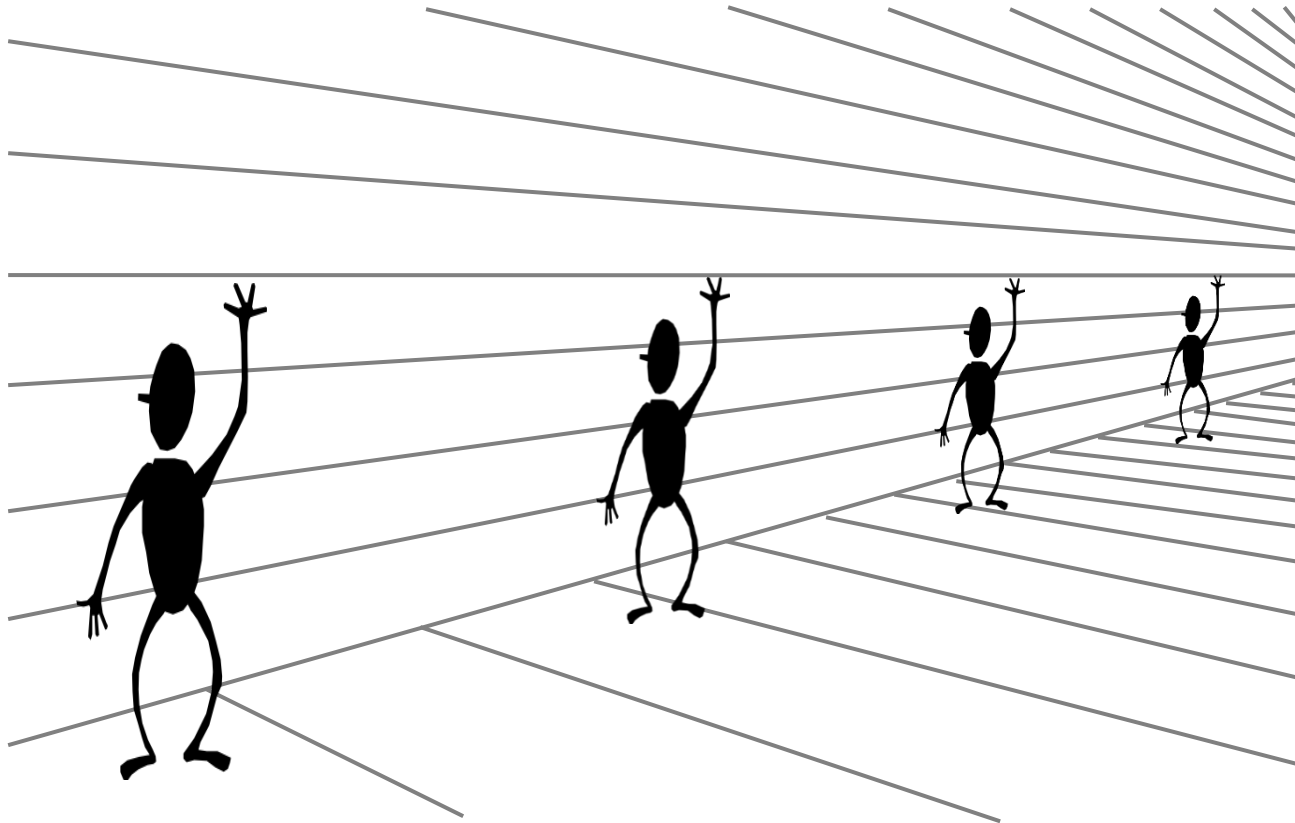
Perspective cues



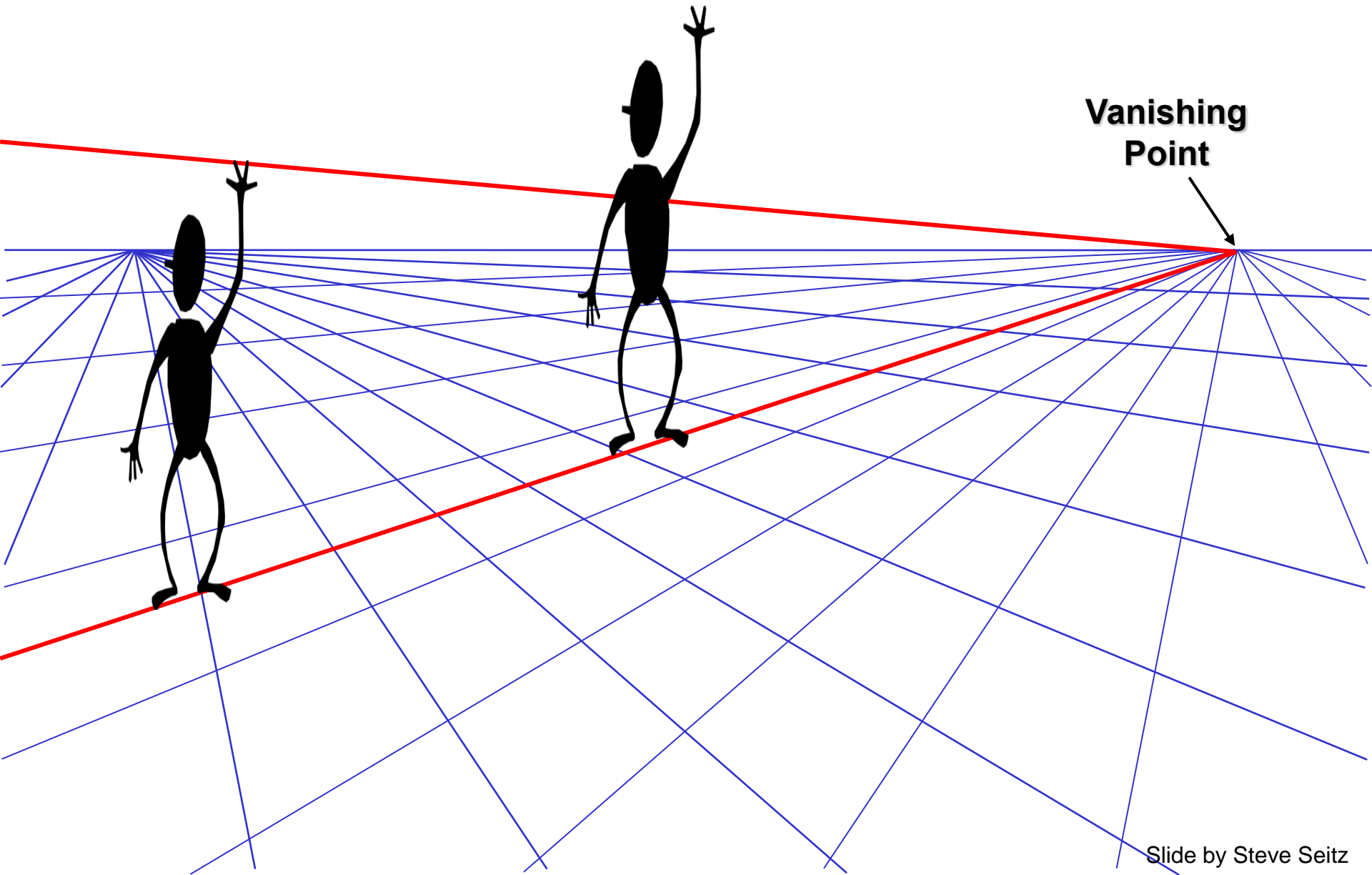
Vanishing points are perspective cues



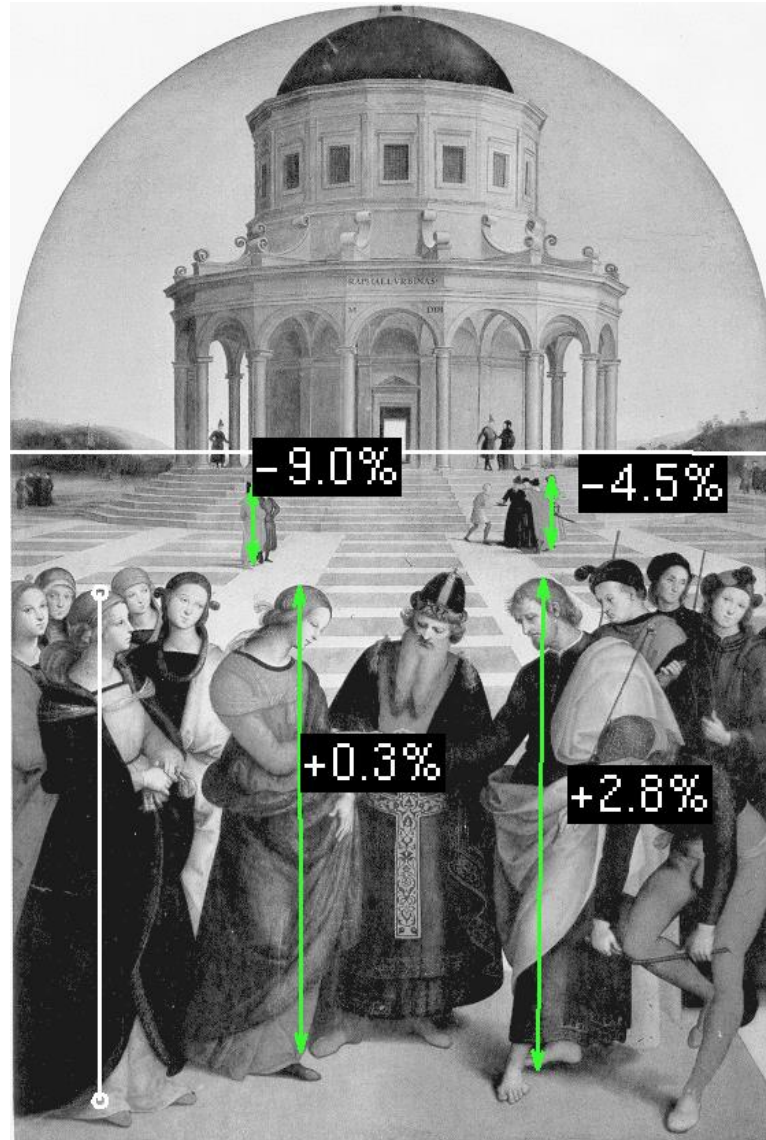
Vanishing points are perspective cues



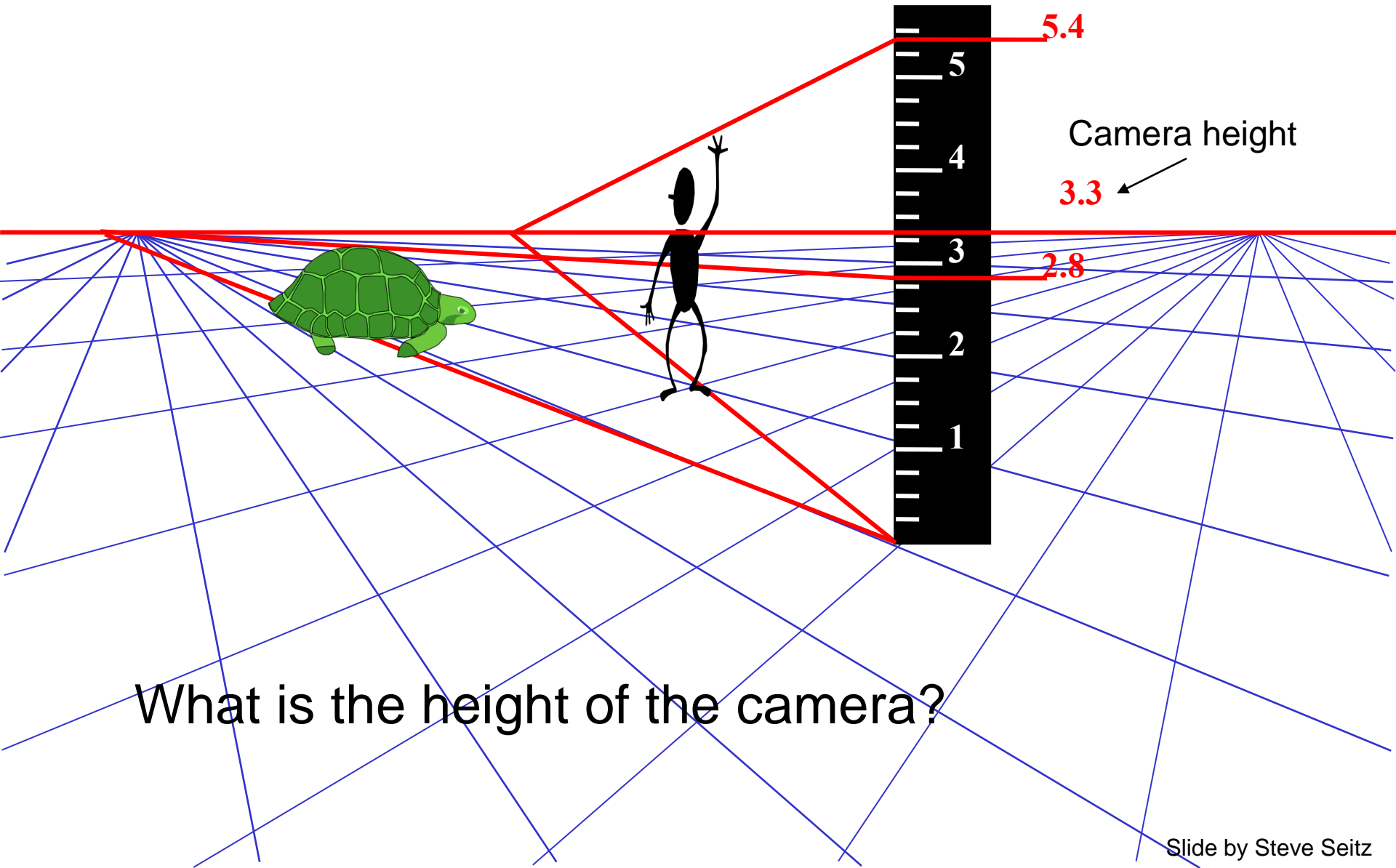
Comparing heights



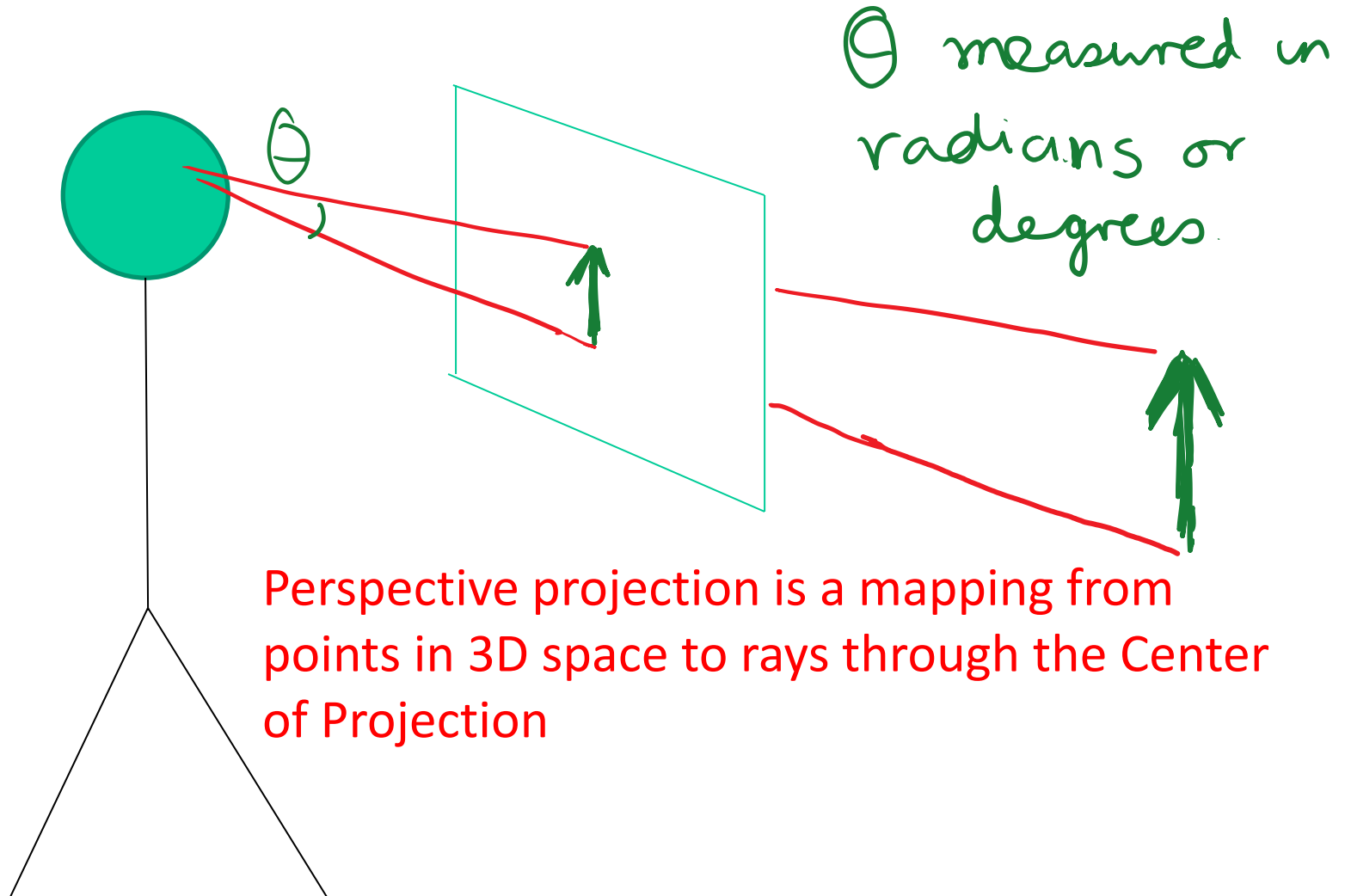
Too bad Rafael didn't know this...



Measuring height

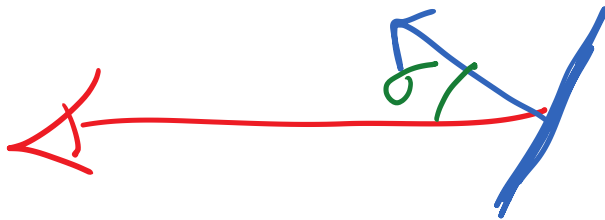


The natural measure of image size is visual angle



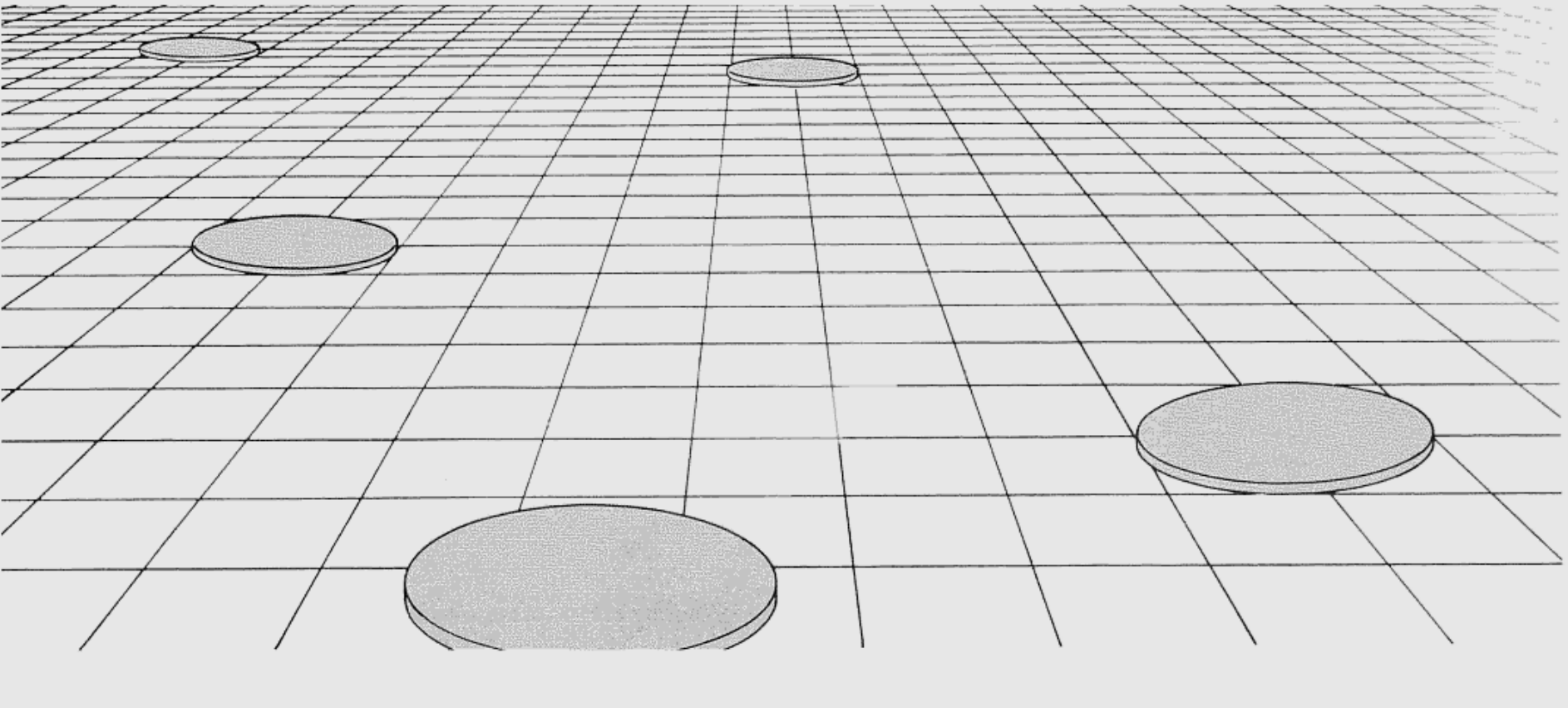
Two main effects of perspective projection

1. Distance – farther objects project to smaller sizes on the image plane. The scaling factor is $1/Z$
2. Foreshortening – objects that are slanted with respect to the line of sight project to smaller sizes on the image plane. The scaling factor is $\cos \sigma$

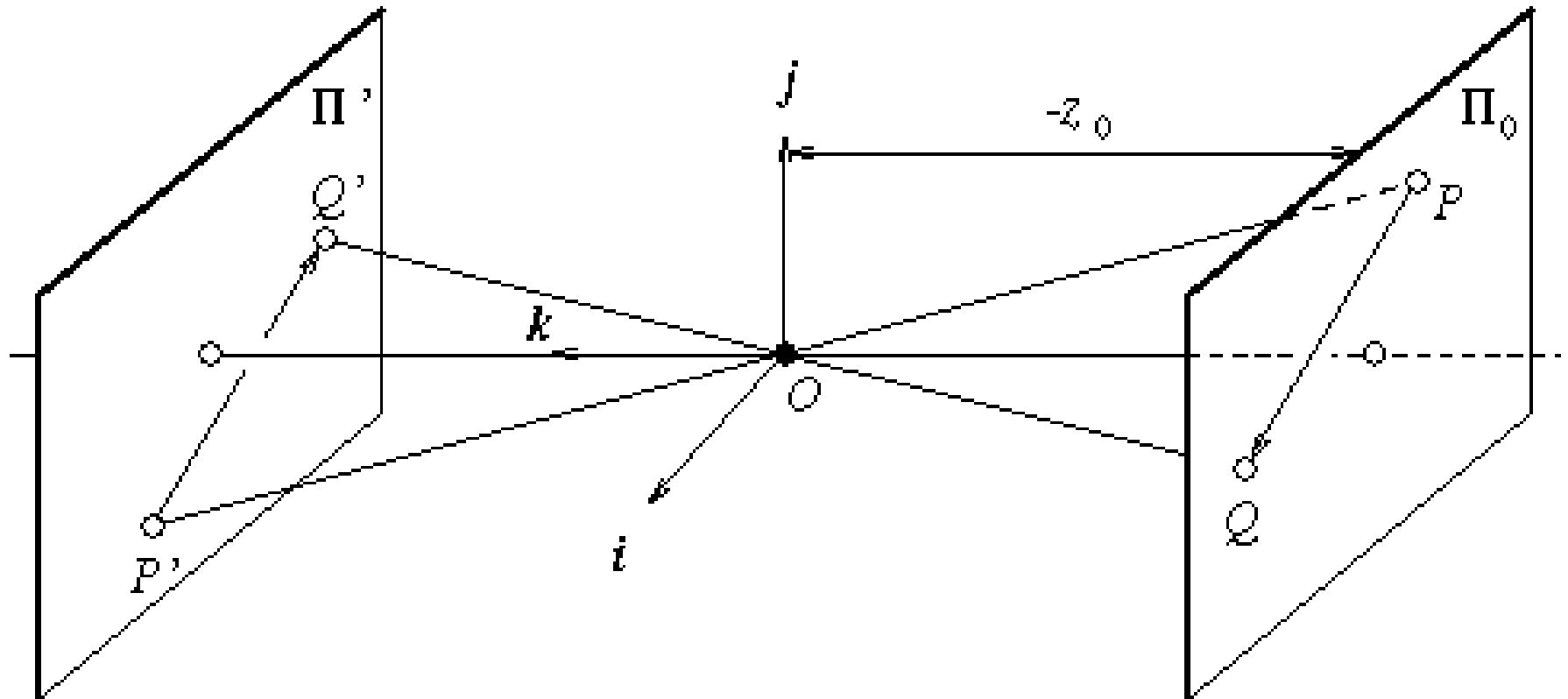


σ is the angle between the line of sight and the surface normal

The slabs that are far away not only look smaller, but also more foreshortened



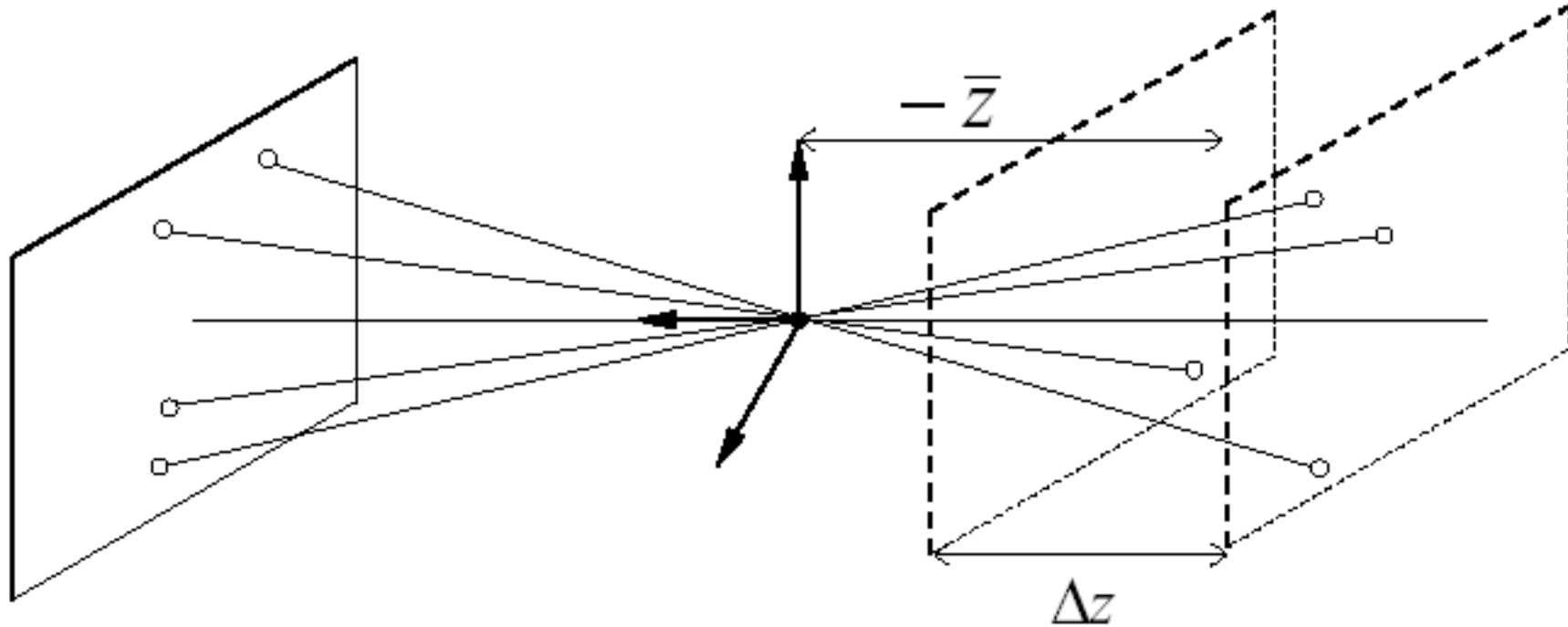
Special Case: Scaled Orthographic



$$\begin{aligned} x' &\approx -mx \\ y' &\approx -my \end{aligned} \quad m = -\frac{f'}{z_o}$$

If scene points are in a plane, projections are simply magnified by m

Special Case: Scaled Orthographic



$$\text{If } \Delta z \ll -\bar{z} : \begin{aligned} x' &\approx -mx \\ y' &\approx -my \end{aligned} \quad m = -\frac{f}{\bar{z}}$$

Justified if scene depth is small relative to average distance from camera