# On Pricing Schemes in Data Center Network with Game Theoretic Approach


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***Abstract*—This paper aims at systematically analyzing the pricing schemes in data center network. The interaction between a monopolistic operator and customers in the network is modeled as Stackelberg game. In this model, both homogeneous- and heterogeneous-customer scenarios are analyzed. In homogeneous customer case, a special scenario is that only a single customer exists in the network. In this scenario, we observe that the Stackelberg equilibrium will lead to a Pareto-inefficient outcome. To address this problem, a two-part pricing scheme is proposed to derive a Pareto efficient outcome and benefit both the operator and customers. When there are an infinite number of homogeneous customers in the network, our analysis shows that customers' selfish action may incur zero utility to them and operator can achieve all the utility by announcing an appropriate price. As to the heterogeneous customer case, we not only analyse how the operator should price the network resources, but also introduce Paris Metro Pricing (PMP) scheme to further increase operator's profit. Since the operator's profit is not a concave function of the resource price, these studies are conducted by simulation.**



***Index Terms*—Data Center Network, Stackelberg game, Paris Metro Pricing**


## I. Introduction

In recent years, there are a growing number of researchers working on the network economics. The center issue of network economics is a pricing problem. Every customer should afford the corresponding charge for requesting resource, such as the bandwidth, application and data. Given this price, each customer selfishly determines how many resources it requests from the network. The operator can not only recover its cost and earn profit by announcing an appropriate price [1, 2], but also reduce the congestion of the network and control the traffic density [3, 4]. In this paper, we will focus on how a data center network operator can maximize its profit (We also say the profit of operator as its utility without discrimination) by announcing an appropriate price.

A data center is a facility used to host computer systems and associated components, such as telecommunications and storage systems. In data center network, each customer requests resource from the data center for its special purpose [5]. To the best of our knowledge, all the existing works on pricing problem focus on the traditional transit networks, where operator makes his "best effort" to deliver customer's data [1, 4, 6]. Therefore, researchers usually assume that the customer's utility is a continuous, increasing and concave function of the quality of service (QoS) [1, 7]. But there is still another form of utility function widely existing in data center networks. In this case, the customer's utility should be zero if the QoS is weaker than a customer's requirement, and it has a step at the QoS threshold. When the QoS is stronger than a customer's requirement, the utility has only a slight increase with the QoS improvement. Since the utility function is not convex/concave any more, the traditional analysis on transit network is not applicable. Our work exactly focuses on this case where the customer's utility function is of step-form.

Two charging methods can be used by the operator. One is the fixed pricing method that the operator charges each customer a deterministic fee $c$, which is independent of the quantity of resources the customer requesting. More realistically, the operator adopts a demand relative pricing method [1], *i.e.* the fee depends on the resource quantity customer request from the network. The later pricing method is assumed in this paper.

In terms of the number of operators in the data center network, there are three major conditions which should be studied. We say that all the customers who have the same QoS requirement in a monopolistic market are homogeneous, or they are heterogeneous customers. In this case, the operator may need a different pricing scheme from that in the market with homogenous customers. Further more, Paris Metro pricing (PMP) scheme has potential to further increase operator's utility. All the above scenarios in monopolistic market are studied in this paper.

There are many approaches in designing the pricing scheme, such as the cost-based approach, the optimization-based approach and edge pricing [8]. Besides these approaches, the approaches based on game theory are most commonly used. In the game theoretic model, There are three choices to model the interaction between operator and customer: two person game, leader-follower games (Stackelberg game [1]) and cooperative game (bargaining). The leader-follower game is a good method to model the interaction between operator and customers. But this method usually cannot obtain a Pareto-efficient solution. To solve this problem, researchers ask cooperative game for a

[1]the description of the Stackelberg Game model is given in [9]

help. Nash bargaining is one of the cooperative game methods to obtain a Pareto-efficient solution. The player can set the threat value [10] of the Nash bargaining as the outcome of the leader-follower game. But the solution of Nash bargaining is usually not equilibrium and each player has a incentive to deviate from this solution. One more serious drawback of the cooperative game is that it is difficult to design a protocol to apply bargaining in a realistic network.

In this paper, we also formulate the interaction between operator and customers as a leader-follower game. But different from previous work, we propose a new pricing scheme to get a Pareto-efficient solution and further increase the operator's utility. It is much more convenient to be used in the realistic network than Nash bargaining.

The main contributions of our work can be summarized as follows:

- We systematically analyze how a data center network operator should design pricing schemes in a monopolistic market to maximize its profit.
- When all the customers in the network are homogeneous, though the traditional analysis on transit network can be transplanted to our model with few modifications, the outcome may not be Pareto-efficient at Stackelberg equilibrium. We design a two-part pricing scheme to solve this problem. Our scheme is much more convenient than Nash bargaining method [1] to be used in the realistic network.
- When customers in the network are heterogeneous, by approximating customer's utility function as a standard step function, we not only study how the operator should price network resources to maximize its profit, but also analyze how the PMP scheme can be used to further increase operator's profit. As far as we know, there is no existing work on how to assign resources to each subnetwork when PMP scheme is used.

The rest of the paper is organized as follows. In Section II and Section III, homogeneous-customer cases are analyzed. Section II assumes that there is only a single customer in the network while Section III assumes there are an infinite number of customers. Section IV and Section V study the cases in which customers have different QoS requirement. Section IV shows how a operator should announce a price for resource to maximize its profit, and Section V introduces PMP schemes to help operator in further increasing its utility. After that, we discuss some related works in Section VI and conclude in Section VII.

## II. Single Customer

In this section, we study the case that there is only a single customer in the market who tries to maximize its utility by requesting resource from data center. We assume that the customer earns $f(d)$ from $d$ units of demands, where $f(d)$ is an increasing and concave function of $d$. It means that the more resources requested from the network, the more utility the customer will get, but the marginal utility will decrease due to the QoS deterioration (Since all the demands are of the same QoS requirement, we only study the case that the QoS requirement is met. Otherwise, the customer will get no utility and then it should decrease its demand quantity.). Let $g(d)$ denote the energy consumption of the operator if it gives $d$ units of resource to the customer and $v$ denotes the cost which the operator should pay for each unit of energy. Usually, $g(d)$ is a convex function [11]. Now, the customer's utility is:

$$U(d,p) = f(d) - pd \tag{1}$$

if the operator charges a fee $p$ for each unit of demands. And the operator's utility is:

$$V(d,p) = pd - vg(d) \tag{2}$$

For a given $p$, the customer may choose $d$ such that

$$\frac{\partial U(d,p)}{\partial d} = f'(d) - p = 0$$

to maximize its utility. That is to say

$$vd = f'^{-1}(p) = h(p) \tag{3}$$

where $h(\cdot)$ is the inverse function of $f'(\cdot)$. (Since $f(\cdot)$ is a concave function, $f'(\cdot)$ is a decreasing function and its inverse function must exist.) Combining (2) and (3), we have

$$V(d(p),p) = ph(p) - vg(h(p))$$

Therefore, the operator can maximize its utility by solving

$$\frac{dV(d(p),p)}{dp} = h(p) + ph'(p) - vg'(h(p(h))h'(p) = 0 \tag{4}$$

From (3) and (4), the Stackelberg equilibrium can be solved. But there remain some questions: 1) Does Stackelberg equilibrium always exist? 2) Is the outcome at Stackelberg equilibrium Pareto-efficient? 3) If 2) is not the case, how to get a Pareto-efficient solution? To answer these questions, we provide two numeric examples:

**Example1**: In this example, we set $f(d) = \ln d$, which is a commonly used utility function [7] and $g(d) = d^{\alpha}$, since when the load on an equipment is $x$, the energy curve of this equipment is often modeled by a polynomial function $g(x) = \mu x^{\alpha}$, where $\mu$ and $\alpha$ are device specific parameters [11]. Using (3), we know the customer will request $d(p) = 1/p$ units resource from the network for any given $p$. In this case

$$\frac{dV(d(p),p)}{dp} = \frac{1}{p} + p(-\frac{1}{p^2}) + vap^{-a-1} = vap^{-a-1} > 0$$

It means that the higher price the operator announces for each unit of resource, the larger utility it may get. Hence, there is no Stackelberg equilibrium.

**Example2**: The only difference between this example and Example 1 is that for $f(d) = M - Me^{-d}$ in this example, $M$ is a constant. Similarly, from (3), it is known that the customer will request $d(p) = -\ln(p/M)$ units of resource for a given $p$ and then we can use (4) to derive that the operator will choose a price $p$ such that

$$\ln\frac{p}{M} + 1 = v\alpha\frac{1}{p}(-\ln\frac{p}{M})^{\alpha-1}$$

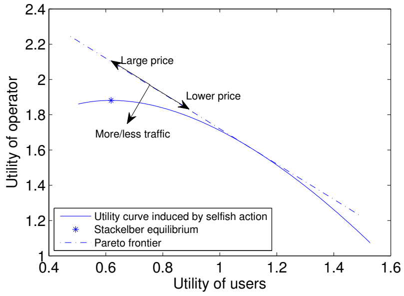

Fig. 1. How the operator determines its price in Example2

to maximize its utility. To get a numeric solution, we set $M$=10, $v$=5 and $\alpha$=2. Then, the operator will charge a fee 6.6961 for each unit of resource and the customer will request 0.4011 unit resource from network. Correspondingly, the outcome of this game is (0.6184,1.8814).

For each price $p$ the operator announces, the customer can determine its demand quantity through (3) and it results in a curve in the utility space $(U, V)$. Fig. 1 shows such curve associated with Example 2. A rational operator may choose the price which will maximize its own utility. This price is exactly corresponding to the Stackelberg equilibrium.

It is worth noting that the total utility of the system $U(d,p) + V(d,p) = f(d) - vg(d)$ is only determined by $d$. Fig. 1 also shows the Pareto frontier to Example 2, on which total utility of the system is 2.7023. On the Pareto frontier, if the customer sends more/less traffic into the network, it will result in a lower-left moving to the line. From the numeric example we can see that to get a Pareto efficient outcome and benefit both the customer and the operator, the customer should request more resource from the network, meanwhile, the operator should reduce the price of each unit of resource.

**Theorem 1**: Assume the demand quantity sent by the customer at Stackelberg equilibrium and Pareto efficient solution are $d^{(s)}$ and $d^{(o)}$, respectively, then $d^{(s)} \leq d^{(o)}$. If we refer $p^{(s)}$ and $p^{(o)}$ as the corresponding price which can force the customer send $d^{(s)}$ and $d^{(o)}$ units demand into the network, there must be $p^{(s)} \geq p^{(o)}$.

**Proof**: We first prove the later part of this theorem by contradiction. Assume $p^{(s)} < p^{(o)}$, from (3) we know $d^{(s)} = h(p^{(s)})$ and $d^{(o)} = h(p^{(o)})$. Since $f(d)$ is a concave function, $f'(d)$ must be a decreasing function and so is $h(p)$, which is the inverse function of $f'(d)$. Therefore, $d^{(s)} > d^{(o)}$.

$$
\begin{aligned}
&U(d^{(o)}, p^{(o)}) - U(d^{(s)}, p^{(s)}) \\
&= f(d^{(o)}) - f(d^{(s)}) - [f'(d^{(o)})d^{(o)} - f'(d^{(s)})d^{(s)}] \\
&= f'(\xi)[d^{(o)} - d^{(s)}] - [f'(d^{(o)})d^{(o)} - f'(d^{(s)})d^{(s)}] \\
&(\xi \in (d^{(o)}, d^{(s)})) \\
&< f'(\xi)[d^{(o)} - d^{(s)}] - f'(d^{(s)})[d^{(o)} - d^{(s)}] \\
&= [f'(\xi) - f'(d^{(s)})][d^{(o)} - d^{(s)}] \\
&< 0
\end{aligned}
$$

The first equation is derived from (3), the second equation can be obtained by using Lagrange's mean value theorem and the inequations in the fourth line and the last line are due to the fact that $f'(d)$ is a decreasing function and $f'(d^{(o)}) > f'(\xi) > f'(d^{(s)})$.

$d^{(o)}$ is the demand quantity inducing a Pareto efficient outcome, so that

$$U(d^{(o)}, p^{(o)}) + V(d^{(o)}, p^{(o)}) \geq U(d^{(s)}, p^{(s)}) + V(d^{(s)}, p^{(s)})$$

Therefore, we have $V(p^{(o)}, d^{(o)}) \geq V(p^{(s)}, d^{(s)})$, which means that if the operator set $p^{(o)}$ to be the price for each unit of resource, it may obtain more utility than that at Stackelberg equilibrium. Contradiction occurs here and $p^{(s)} \geq p^{(o)}$ must be held. Since $h(p)$ is a decreasing function of $p$, $d^{(s)} = h(p^{(s)})h(p^{(o)}) = d^{(o)}$. ■

Considering that the demand quantity sent by the customer is only determined by the resource margin cost, so the operator can require a fixed price for the customer's resource request to guarantee its utility and reduce the margin cost of resource to induce a Pareto efficient outcome, say the price for $d$ units of resource is:

$$B(d,p) = P + pd \tag{5}$$

Now, the utility function to the customer and the operator will be

$$U_a(d,p) = f(d) - B(d,p) \tag{6}$$

and

$$V_a(d,p) = B(d,p) - vg(d) \tag{7}$$

respectively.

If only the customer can get utility by requesting some demand from network, *i.e.* for given $P$ and $p$, there exists $d$, such that $U(d,p) > 0$, this pricing scheme will work.

**Theorem 2**: Assume

$$T(d) = U(d,p) + V(d,p)$$

If the operator set

$$\begin{cases} p = p^{(o)} \\ P = V(d^{(s)}, p^{(s)}) - V(d^{(o)}, p^{(o)}) + \frac{1}{2}(T(d^{(o)}) - T(d^{(s)})) \end{cases}$$

the outcome will be Pareto efficient and both the operator and the customer will get more utility than that at Stackelberg equilibrium.

**Proof**: For the cost given in Theorem 2,

$$U_a(d,p) = f(d) - (P + p^{(o)}d)$$

The customer will choose the demand quantity such that

$$\frac{\partial U_a(d,p)}{\partial d} = f'(d) - p^{(o)} = 0$$

*i.e.* $d = d^{(o)}$. In another word, the outcome will be social optimal and Pareto efficient.

The customers utility will be

$$
\begin{aligned}
U_a(d^{(o)}, p^{(o)}) &= f(d^{(o)}) - (P + p^{(o)}d^{(o)}) \\
&= U(d^{(o)}, p^{(o)}) - [V(d^{(s)}, p^{(s)}) - \\
&\quad V(d^{(o)}, p^{(o)}) + \tfrac{1}{2}(T(d^{(o)}) - T(d^{(s)}))] \\
&= \tfrac{1}{2}(T(d^{(o)}) + T(d^{(s)})) - V(d^{(s)}, p^{(s)}) \\
&> T(d^{(s)}) - V(d^{(s)}, p^{(s)}) = U(d^{(s)}, p^{(s)})
\end{aligned}
$$

The inequation is due to the fact that $T(d^{(o)}) > T(d^{(s)})$.

$$\begin{aligned} V_a(d^{(o)}, p^{(o)}) &= (P + p^{(o)} d^{(o)}) - vg(d^{(o)}) \\ &= P + V(d^{(o)}, p^{(o)}) \\ &= V(d^{(s)}, p^{(s)}) + \tfrac{1}{2}(T(d^{(o)}) - T(d^{(s)})) \\ &> V(d^{(s)}, p^{(s)}) \end{aligned}$$

Hence, both the operator and the customer will get more utility than that at Stackelberg equilibrium. ■

## III. Infinite Homogeneous Customers

In this section, we assume that there are infinite homogeneous customers in the network to decide whether to request resource from the network and a single individual has only an infinitesimal impact on the total demands in the network. Now, the operator's utility can also be presented as (2), but the margin utility for customers from one unit of resource is

$$U(d, p) = f(d)/d - p \tag{8}$$

If $U(d, p) > 0$, more rational customers will request resources from the network. Therefore, $U(d, p) = 0$ must be held at the Stackelberg equilibrium. Unfortunately, for a given $p$, $d$ is difficult to be solved explicitly. Accordingly, we cannot substitute $d$ in (2) with a function of $p$ and maximize operator's utility analytically. However, the Stackelberg equilibrium can also be solved by numeric computation.

**Lemma 1**: When the operator announces a higher price, there will be less demand sent into the network.

**Lemma 2**: The demand quantity $d$ is a concave function of price of each unit resource, if and only if $f''(d)d' > 2$. [2]

**Theorem 3**: If $f''(d)d' > 2$, $V(d(p), p)$ is a concave function of $p$.

**Proof**: Since $f(d)$ is a concave function while $g(d)$ is a convex function, $V(d, p) = f(d) - vg(d)$ is a concave function of $d$. From Lemma 1 and Lemma 2, $d$ is a decreasing and concave function of $p$. Therefore, $V(d(p), p)$ is a concave function of $p$ [13]. ■

Considering that

$$\begin{aligned} f''(d)d' &= \frac{f''(d)d^2}{f'(d)d - f(d)} \\ &= \frac{f''(d)d}{f'(d) - f'(d^*)} = \frac{f''(d)d}{f''(d^{**})(d - d^*)} \end{aligned}$$

where $d^{**} \in (d^*, d)$. Usually, the customers in a market have decreasing absolute risk aversion, therefore $f^{'''}(d) > 0$ [14]. It means $f''(d) > f''(d^{**})$ and $d^* > d/2$, so that $f''(d)d' > 2$ is usually held in realistic network.

Algorithm 1 is proposed to search such Stackelberg equilibrium if $V(d(p), p)$ is a concave function of $p$. There are two phases in this algorithm. The aim of Phase 1 is to set $p_3$ larger than $p_{max}$. In Phase 2, the size of feasible range is reduced by trisection method. With the range reduction, $p_i$ (for $i$=1, 2, 3, 4) will converge to the point where operator's utility is maximized.

When $V(d(p), p)$ is not a concave function of $p$, we can only leverage fixed-step-size searching method. In this method, we try different prices for operator and calculate the customers' response by solving $U(d, p) = 0$. The price at Stackelberg equilibrium $p(s)$ will be the one maximizing operator's utility and the corresponding demand quantity is the solution of $U(d, p^{(s)}) = 0$.

[2]For brevity, the proof of Lemma 1 and Lemma 2 is given in [12]

**Algorithm 1:** Algorithm to derive Stackelberg equilibrium

**Input**: The utility function users can earn from $d$ unit resource $f(d)$, energy consumption function $g(d)$, the cost of unit of energy $v$ and $a$ small number $\varepsilon$.
**Output**: The Stackelberg equilibrium ($p^*$, $d^*$)
**Initialize**: $p_1 = p_{min}$, $p_2 = p_{min} + 1/3(p_{max} - p_{min})$, $p_3 = p_{max} - 1/3(p_{max} - p_{min})$, $p_4 = p_{max}$. Given $p_i$, compute $d_i$ by solving equation (8) for $i$=1, 2, 3, 4
/* Phase 1*/
**while** $(V(p_3, d_3) < V(p_4, d_4))$ **do**
    $p_3 = 1/2(p_4 - p_3)$ and update $d_3$ by solving (8);
**end**
/*Phase 2*/
**while** $(p_4 - p_1 > \varepsilon)$ **do**
    **if** $V(p_3, d_3) > V(p_2, d_2)$ **then**
        $p_1 = p_2$, $p_2 = p_3$, $p_3 = 1/2(p_3 + p_4)$;
        $d_1 = d_2$, $d_2 = d_3$ and update $d_3$ by solving (8);
    **else**
        $p_4 = p_3$, $p_3 = p_2$, $p_2 = 1/2(p_1 + p_2)$;
        $d_4 = d_3$, $d_3 = d_2$ and update $d_2$ by solving (8);
    **end**
**end**
$p^* = 1/2(p_1 + p_4)$ and get $d^*$ by solving (8);
return ($p^*$, $d^*$);

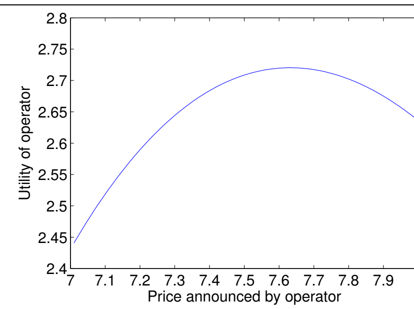

Fig. 2. Price of resource vs. operators utility in Example 3

**Example 3**: In this example, we set all the parameters as the same in Example 2. At first, we try different $p$ for the operator and make customers respond each price according to $U(d, p) = 0$. This interaction will lead to a curve in operator's utility space as shown in Fig. 2. It can be seen that when the price is relatively low, the operator announces a larger price may improve its utility. Using Algorithm 1, it can be found that the Stackelberg equilibrium is (7.63, 0.5671) and the operator obtains 2.7203 units utility at Stackelberg equilibrium which is exactly the maximal system total utility in Example 2.

As a result, the operator increases the price it announces and customer requests less resource from the network. Both of these actions hurt other's utility and lead to a Pareto-inefficient equilibrium.

## IV. Heterogeneous Customers without PMP

Usually, every customer in realistic network has its special QoS requirement and correspondingly obtains different marginal utility. In this section we will study such a heterogeneous-customer scenario. Assuming that each customer has a QoS requirement $s$, the total demand quantity in the network is $d$ and the network resource capacity is $C$. To continue our analysis, we give following assumptions: each customer has infinitesimal demands with QoS requirement $s$, $s$ has a distribution density function $q(s)$, $u(s)$ is a decreasing function of $s$, and the total demand quantity (even some are not sent into the network) in the network is $D$.

Given the price $p$, only the customers whose marginal utility is larger than $p$ (QoS requirement is stronger than $u^{-1}(p)$) would send demand into the network. Assume that the strongest QoS requirement can be satisfied is $s$, then all the customers whose QoS requirement are in $[s, u^{-1}(p)]$ will send demand into the network, hence

$$D \int_{s}^{u^{-1}(p)} q(x)dx = sC \tag{9}$$

Whenever the strongest/weakest QoS requirement can be satisfied in the network is fixed, (9) can be used to find the weakest/strongest QoS requirement in the network, so that we refer (9) as the QoS requirement constraint. The customers' total utility is

$$U(s,p) = D \int_{s}^{u^{-1}(p)} u(x)q(x)dx - pD \int_{s}^{u^{-1}(p)} q(x)dx \tag{10}$$

and the operator's utility is

$$V(s,p) = p \int_{s}^{u^{-1}(p)} Dq(x)dx - vg(\int_{s}^{u^{-1}(p)} Dq(x)dx) \tag{11}$$

If we can solve (9) explicitly, say $s = h(p)$, (11) can be modified to be

$$V(h(p),p) = p \int_{h(p)}^{u^{-1}(p)} Dq(x)dx - vg(\int_{h(p)}^{u^{-1}(p)} Dq(x)dx)$$

From $\frac{\partial V(s,p)}{\partial p} = 0$,

$$\begin{aligned} &pD[I'(p)q(I(p)) - h'(p)q(h(p))] + D \textstyle\int_{h(p)}^{I(p)} q(x)dx = \\ &vDg'(D \textstyle\int_{h(p)}^{I(p)} q(x)dx)[I'(p)q(I(p)) - q(h(p))h'(p)] \end{aligned} \tag{12}$$

is obtained, where $I(p) = u^{-1}(p)$. The Stackelberg equilibrium can be obtained by getting $p$ from (12) and then using $s = h(p)$ to calculate $s$. Unfortunately, (9) cannot be solved explicitly in most cases and we can only ask numeric computation for help solving Stackelberg equilibrium.

In this case, we cannot guarantee $V(s(p), p)$ is a concave function of $p$, so that Algorithm 1 dose not suit this case. To get the price which should be announced by the operator, we can only use the cruel fixed-step-size searching algorithm to find the price which can maximize the operator's utility. The algorithm is shown in Algorithm 2.

**Example 4**: In this example, we set $u(s) = e^{-s}$, $q(s) = e^{-s}$, $g(d) = d^2$, $D = 2$, $C = 1$ and $v = 1$. For a given $p$, (9) is used to get $2e^{-s} - 2p = s$ at first, in which $s$ cannot be solved explicitly. Algorithm 2 can also be used to calculate the utility of the operator and customers for different price. We can observe in Fig. 3 that there is only a single peak value at the operator's utility and the operator can increase its utility by announcing a larger price if $p \in (0, 0.56)$, otherwise larger price will result in a lower utility to the operator. Therefore, the operator may announce a price 0.56 to get its maximal utility 0.1556. In this case, the customers whose QoS requirement are in [0.3250, 0.5712] will send demand into the network and they will get 0.0264 unit utility in total.

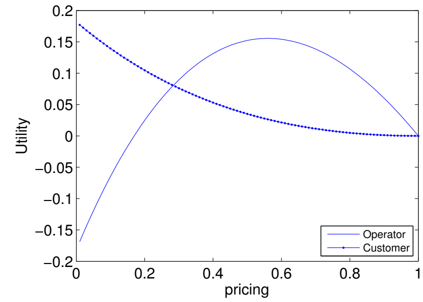

Fig. 3. Impact of the operator's price on players' utilities in Example 4

Just as seen in Section II, the maximal total utility of the system is only determined by the quantity of demand sent into the network. Fig. 4 shows how the total utility of the system is impacted by which demands are sent into the network. It can be observed that if the strongest QoS requirement metric of the demands sent into the network is 0.40, the total utility of the system will be maximized. In this case, the customers whose QoS requirement are in [0.40, 0.7543] will send demand into the network and the maximal total utility of the system is 0.1881 which is larger than that at the Stackelberg equilibrium 0.182.

## V. Heterogeneous Customers with PMP

Considering the scenario in Section IV, all the customers who send demand into the network share the same QoS but obtain different utility and the demand quantity in the network is limited by the strongest QoS requirement. Intuitively, we can partition all the resources into two parts to serve the demands. There remain two questions in this issue: 1) If the network resources have been partitioned into two parts, how to determine the resource price for each subnetwork? 2) If the network resources have not been assigned to each subnetwork, how should operator partition the network resources?

### A. *How to determine the price of each subnetwork*

In this subsection, we assume that operator's total resource have been partitioned into two equivalent parts (Subnetwork

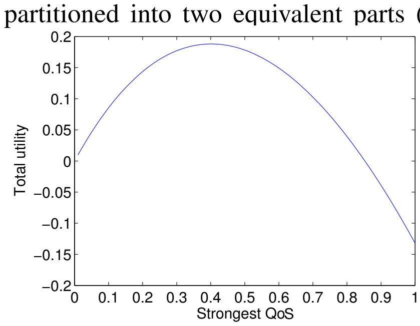

Fig. 4. Strongest QoS vs. total utility of the system in Example 4

**Algorithm 2:** Algorithm to determine the price customer should announce

**Input**: customers' marginal utility function $u(s)$, the density distribution of QoS requirement $q(s)$, the energy consumption function $g(d)$, network resource capacity $C$, total available demand quantity $D$, the cost of one unit of energy $v$ and the accuracy requirement $\varepsilon$.
**Output**: The price maximizing the operator's utility $p^*$
**initialize**: $p^* = p = p_{min}$, $s^*$ is derived by solving (9) for given $p^*$, $V^* = V(s^*, p^*)$ which is calculated by (11);
**while** *($p < p_{max}$)* **do**
  $s$ is derived by solving (9) for a given $p$;
  $V = V(s, p)$ which is calculated by (11);
  **if** *($V > V^*$)* **then**
    $p^* = p$;
    $V^* = V$;
  **end**
  $p = p + \varepsilon$;
**end**
return $p^*$;

1 and Subnetwork 2) whose resource capacity are $C_1$ and $C_2$ respectively, such that $C_1 = C_2$. The operator should determine the price it announces for each subnetwork $p_1$ and $p_2$. In this paper, we assume $p_1 < p_2$. (In realistic project, we can assume $p_1 > p_2$ and repeat the same process in this paper and employ the better solution.) Let $s_1$ and $s_2$ denote the strongest QoS requirements existing in each subnetwork, then $s_1 > s_2$. Now, the total utility of customers can be expressed as

$$\begin{aligned}&U(s_1, s_2, p_1, p_2)\\&= D\int_{s_1}^{u^{-1}(p_1)} u(x)g(x)dx + D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} u(x)g(x)dx\\&-p_1 D\int_{s_1}^{u^{-1}(p_1)} q(x)dx - p_2 D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} q(x)dx\end{aligned}$$

and $s_1$ and $s_2$ are determined by $D\int_{s_1}^{u^{-1}(p_1)} q(x)dx = s_1 C_1$ and $D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} q(x)dx = s_2 C_2$. The operator's utility is

$$\begin{aligned}&V(s_1, s_2, p_1, p_2)\\&= p_1 D\int_{s_1}^{u^{-1}(p_1)} q(x)dx + p_2 D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} q(x)dx\\&-vg(D\int_{s_1}^{u^{-1}(p_1)} q(x)dx + D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} q(x)dx)\end{aligned}$$

To study how operator should announce the resource price in each subnetwork, we first give following proposition.
**Proposition 1**: If the price announced in Subnetwork 1 is $p_1$, the price announced in Subnetwork 2 $p_2$ should be in $[u(s_1), p_{max}]$, where s1 is the strongest QoS requirement in Subnetwork 1 which can be calculated by solving , and pmax is determined by the strongest QoS requirement may exist in the network. [3]

From Proposition 1, we can re-formulate the utility of

[3]The proof of Proposition 1 is given in [12]

**Algorithm 3:** Algorithm to determine the price for each subnetwork

**Input**: Customers' marginal utility function $u(s)$, density distribution of QoS requirement $q(s)$, energy consumption function $g(d)$, resource capacity for each subnetworks $C_1$ and $C_2$, total available demand quantity $D$, the cost of unit of energy $v$ and the accuracy requirement $\varepsilon$.
**Output**: The price maximizing operator's utility $p_1^*$ and $p_2^*$
**initialize**: $p_1 = p_1^* = p_2^* = 0$, $V^* = 0$;
**while** *(1)* **do**
  $s_1$=the solution of (13);
  $p_2 = u(s_1)$;
  **if** *($p_2 > p_{max}$)* **then**
    return $p_1^*$, $p_2^*$;
  **end**
  **while** *($p_2 < p_{max}$)* **do**
    Calculate $V$ by (13);
    **if** *($V > V^*$)* **then**
      $V^* = V$; $p_1^* = p_1$; $p_2^* = p_2$;
    **end**
    $p_2 = p_2 + \varepsilon$;
  **end**
  $p_1 = p_1 + \varepsilon$;
**end**

operator as

$$\begin{aligned}&U(s_1, s_2, p_1, p_2)\\&= D\int_{s_1}^{u^{-1}(p_1)} u(x)g(x)dx + D\int_{s_2}^{u^{-1}(p_2)} u(x)g(x)dx\\&-p_1 D\int_{s_1}^{u^{-1}(p_1)} q(x)dx - p_2 D\int_{s_2}^{u^{-1}(p_2)} q(x)dx\end{aligned} \tag{13}$$

where $s_1$ and $s_2$ are determined by

$$D\int_{s_1}^{u^{-1}(p_1)} q(x)dx = s_1 C_1 \tag{14}$$

$$D\int_{s_2}^{u^{-1}(p_2)} q(x)dx = s_2 C_2 \tag{15}$$

and re-formulate the customers' total utility as

$$\begin{aligned}&V(s_1, s_2, p_1, p_2)\\&= p_1 D\int_{s_1}^{u^{-1}(p_1)} q(x)dx + p_2 D\int_{s_2}^{u^{-1}(p_2)} q(x)dx\\&-vg(D\int_{s_1}^{u^{-1}(p_1)} q(x)dx + D\int_{s_2}^{u^{-1}(p_2)} q(x)dx)\end{aligned} \tag{16}$$

When the network is divided into two parts, one more variable should be determined and the question is even more complex than it in Section IV. This Algorithm 3 is also based on fixed-step-size searching.
**Example 5**: In this example, we partition the network in Example 4 into two subnetworks with the same resource capacity 0.5 and 0.5. Using Algorithm 3, we derive that the optimal pricing scheme for operator is to set the resource price in Subnetwork 1 to be 0.52 and announce a price 0.6339 in Subnetwork 2. In this case, the utility for operator and customers are 0.1851 and 0.0199 respectively. Comparing with Example 4, the operator can obtain larger utility than that

without PMP scheme. More importantly, with PMP scheme, the system total utility 0.1851+0.0199=0.2050 is even larger than the maximal system total utility without PMP scheme, which is only 0.1882. The reason is that, without PMP, many customers share better QoS than they required and the operator does not charge these customers enough fee, so that there are a lot of utility in the system wasted.

For better understanding of this example, we further study how some of the variables change with the price announced in Subnetwork 1. Fig. 5 shows low price may attract more customers to request resource from Subnetwork 1 and cost operator more energy but the operator cannot get enough money from customers in Subnetwork 1 to cover the energy consumption cost. In Fig. 6, because if the price in Subnetwork 1 is relatively low, the increasing of its price will move out many demands from Subnetwork 1 and the operator should announce a lower price in Subnetwork 2 to attract these demands into Subnetwork 2 and get more utility from them. When the price in Subnetwork 2 is relatively large, price in Subnetwork 2 will increase with the increasing of the price in Subnetwork 1 due to the constraint $p_1 < p_2$. From Fig. 7, it is because the operator can charge more fee for per unit of resource in Subnetwork 1 while there will be more demands in Subnetwork 2 due to the reduction of its resource price. But at some price (0.46 in our example), operator will not get more profit from customers because there are fewer and fewer demands sent into the network with the increasing of resource price.

### *B. How to assign resource to each subnetworks*

In previous subsection, we assume that the operator's resources have already been assigned to each subnetworks. However, how to partition the total resource into two parts is still an important issue. We study how the operator partition its resource can maximize its utility by simulation. The algorithm is shown in Fig. 8 and we also give an example under this condition.

**Example 6**: In this example, all the parameters are set as them in Example 4. Fig. 8 shows how the operator's and customers' utility change with the resource capacity of Subnetwork 1. It can be observed that the operator should divide the network by $C_1 = 0.45$ and $C_2 = 0.55$. Fig. 9 depicts that price of resource in each subnetwork to be 0.52 and 0.6256, respectively when the operator get maximal utility. In this way, the operator can

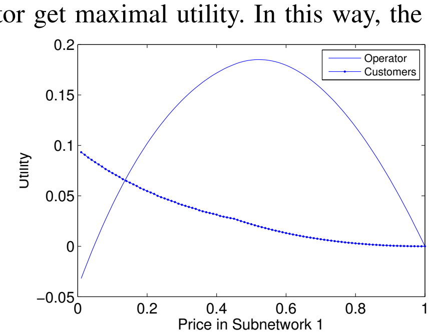


Fig. 5. Price announced in Subnetwork 1 vs. two players' utility in Example 5. The larger price will lead to the reduction of customers' utility, just as the same in Example 4.

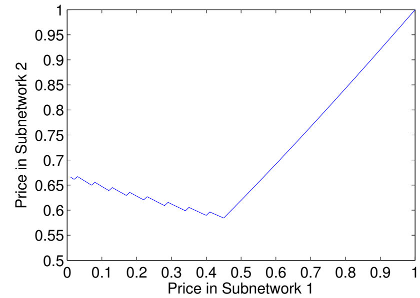


Fig. 6. Price announced in Subnetwork 1 vs. Price announced in Subnetwork 2 in Example 5. The price announced in Subnetwork 2 should first decrease with the increasing price in Subnetwork 1 when the price in Subnetwork 1 is relatively low.

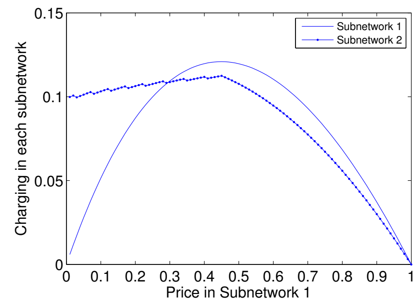


Fig. 7. Price announce in Subnetwork 1 vs. the charging in each subnetwork. When the price in Subnetwork 1 increases from 0, operator can charge more fee from both subnetworks.

obtain utility 0.1854 and customers' total utility is 0.0195. It shows that the operator get larger utility at the cost of loss of system total utility. Accordingly, it is difficult to get the optimal resource Example 6: In this example, all the parameters are set as them in Example 4. Fig. 8 shows how the operator's and customers' utility change with the resource capacity of Subnetwork 1. It can be observed that the operator should divide the network by $C_1 = 0.45$ and $C_2 = 0.55$. Another observation is that the operator and customers' pricing scheme are even not a convex/concave function of Subnetwork 1's resource capacity, so is the customers' utility. Therefore, it is difficult to get the optimal resource allocation scheme analytically.

## VI. Related Work

With the rapid development of communication networks, there has been a surging interest in network pricing research over the past decade. But most of these works are focusing on congestion control, *i.e.* reduce the congestion of the network and control the traffic density [3, 4, 15, 16]. In these works, some of them determine the pricing schemes by considering the priority property of service [4], while some others using bargaining methods [3]. Though all these works are not focusing on how to maximize operator's profits, the modeling methods can be employed in our issue on how to maximize operator's utility.

The pricing scheme is also used in wireless communication systems. It is applied in spectrum sharing [17], utility optimization [18], power control [19] and QoS guarantee [20]. The analysis in [20] is very similar to our work. It determines an appropriate pricing scheme for voice over WLAN through a micro-economic framework that considers the trade-off be-

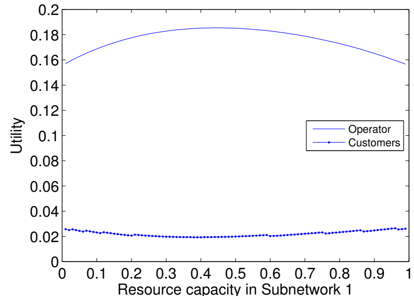


Fig. 8. Resource capacity in Subnetwork 1 vs. utility of operator and customers

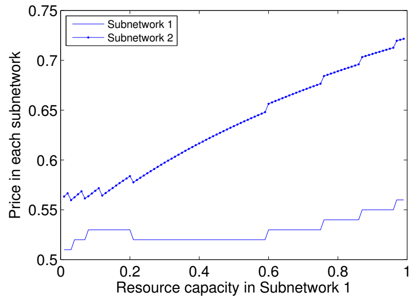


Fig. 9. Resource capacity in Subnetwork 1 vs. resource price in each subnetwork

tween perceived QoS and paid price in the users' request. However, [20] focus on the network QoS guarantee, while our work is on maximizing operator's profit.

There are also some works on operator's profit [1, 2]. We can treat [1] as a parallel story of our work. In parallel to [1], we focus on how to maximize operator's profit in data center networks, where customers' utility is a step function of network's QoS.

## VII. Conclusion and Future Work

In this paper, we study the network pricing schemes in data center network by modeling the pricing problem as a Stackelberg game. Both the cases that the customers in the network are homogeneous and heterogeneous are analyzed.

As for the case that customers in the network are heterogeneous, we study how the operator announces price by numeric computation and find that the Stackelberg equilibrium cannot maximize the total utility of the system. When PMP scheme is introduced into the network, we study not only how the operator should announce resource price in each subnetwork, but also how to assign its resource to each subnetwork.

This paper only focuses on the monopolistic market scenarios and the duopoly market cases are still unsolved. Our future work is to systematically analyze the pricing scheme in data center network in a duopoly market.

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