AN OPTIMIZATION MODEL FOR EMERGENCY VEHICLE LOCATION WITH CONSIDERATION OF INTEGRATION DISPATCHING

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Abstract. This paper aims to develop location opitimization and dispatching optimization techniques for emergency vehicles. In this research, Firstly, we seek to propose a optimization model to find the optimal location of emergency vehicles. Then, we generate the some scenarios to consider the uncertainty conditions and do numerical experiment. The numerical experiment results show that the proposed model achieves a good performance and worked well.

Keywords. emergency vechicle location and dispatching, optimization, uncertainty, integration dispatching.

1. INTRODUCTION

Utilizing the avaliable facilities and human resoures, fire service organizations shall protect the lives, physical being and property of public from fire and take precautions against disasters such as storms, floods fires and earthquakes, while mitigating the damage of these disaters. Fire service organizations are also responsible for the appropriate transport of persons who have sustained injuries due to a disaster (FDMA, 2015). When the Large-scale disaster or accident, serious accident and military attack happend, emergency vehicles belonging to the fire departments will rush to the emergency demand as soon as possible for rescue. However, In recent years, the number of emergency services for emergency call is increasing. In particular, the emergency vehicles tend to lengthen the average response time to the site because of the lack of fire departments and emergency vehicles in the suburbs. On the other hand, if we can locate more emergency vehicles in more fire departments, emergency vehicles can shorten the traveling time to the site, but at the same time location cost must be considered. Therefore, according to the Japan Fire and Disaster Management Agency Heisei 29th edition fire fighting white paper (FDMA, 2018) pointed out that consider large-scale distaers such as the Great East Japan Earthquake, higher future disater risks, and decrease in the population of Japan, we must enhance the structure of fire departments by expanding their jurisdictions. Accordingly, it is necessary to balance these two aspects and develop a more effective emergency vehicle location planning.

In the emergency vehicle location problem, the main objective to minimize the time it takes to respond to the sites (the traveling time between emergency call receipt and emergency vehicles arrival to the site) (Saeed et al., 2018). Because the site for emergency services is covered by the emergency vehicles located at fixed points, therefore, the location of emergency vehicles is important in service quality level. Moreover, as the uncertainty commonly exists in the real world. The unpredictability of the time and the location of emergency incidents are also the main issue in the emergency vehicle location (Xiao-Xia et al., 2013) (Lei et al., 2015) . Therefore, we should consider the uncertainty for the emergency vehicle location problem as follows:

- The uncertainty of call-in time and the site location
- The uncertainty of the emergency vehicle traveling time
- The uncertainty of service time at site

Based on the above, in this research, we developed an optimization model to find the optimal location of emergency vehicles with consideration of integration dispatching and consiered the uncertainty conditons by numerical experiment.

In Section 2, the process of emergency response system and the time and time periods in emergency vehicle dispathcing are described in detail. Next, the mathematical model will be proposed in Section 3. Then, in order to demonstrate the performance and validate the proposed model, we generated some numerical instances are presented and solved in Section 4. Finally, we draw a conclusion in Section 5.

2. THE PROCESS OF EMREGENCY RESPONSE SYSTEM

The process of emergency response system usually covers a sequence of activities are shown as follows:

1. The emergency call comes to the system when the incident detection happened.
2. After call screening the dispatcher evaluates the system status and determines the appropriate emergency vehicle (EV) to dipatch.
3. Upon EV arrives at site and starts doing service.
4. After completing service at site, EV may be moves to the next site if there has another emergency call or returns back fire departments to await another emergency call.

In this research, we assumed that once the service is completed at site, the EV must return back fire departments as

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shown in Fig. 1.

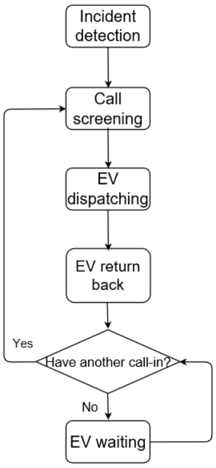

Fig. 1 The process of emergency response system

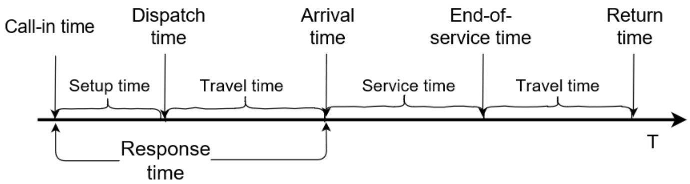

Fig. 2 The time and time periods in an EV dispathcing

In addition, we will introduce the time and time periods about an EV when it is dispatched to a site as shown in Fig. 2.

- The time period between call-in time and dispatch time is called setup time.
- The time period between dispatch time and arrival time is called travel time.
- The time period between call-in time and arrival time is called response time.
- The time period between arrival time and end-of-time is called service time.
- The time period between end-of-service time and return time is called travel time.

3. MATHEMATICAL FORMULATION

In this section, we formulate the optimization model to find the optimal location of emergency vehicles.

3.1. Notation

We use the following notations to describe our proposed model.

- Sets
  - I : the set of fire departments indexed by $i \in \{1, 2, \ldots, \alpha\}$ (α: the number of fire departments),
  - J : the set of sites indexed by $j \in \{1, 2, \ldots, \beta\}$ (β: the number of sites),
  - K : the set of emergency vehicles indexed by $k \in \{1, 2, \ldots, \gamma\}$ (γ: the number of emergency vehicles),
  - N : the set of the numbers of dispatching indexed by $n \in \{1, 2, \ldots, n_{\max}\}$ ($n_{\max}$: the maximum number of dispatches of each emergency vehicle),
  - A : the set of scenarios indexed by $a \in \{1, 2, \ldots, \delta\}$ (δ: the number of scenarios).
- Parameters
  - $u_{a,j}$: the call-in time from j under a,
  - $t_{a,i,j}$: the traveling time between i and j under a,
  - $s_{a,j}$: the service time of k at j under a,
  - M_1/M_2 : sufficiently large constant numbers,
  - e : the setup time for dispatching,
  - b : the threshold amount,
  - P_a : the occurrence probability of a.
- Decision variables
  - $$x_{a,j,n,k} = \begin{cases} 1, & k \text{ is dispatched to a } j \text{ with the } n\text{-th} \\ & \text{dispatch under } a, \\ 0, & \text{otherwise,} \end{cases}$$
  - $$y_{i,k} = \begin{cases} 1, & k \text{ is assigned to } i, \\ 0, & \text{otherwise,} \end{cases}$$
  - $h_{a,n,k}$: the dispatch time of k with the n-th dispatch under a,
  - $l_{a,n,k}$: the traveling time of k with the n-th dispatch under a,
  - $v_{a,n,k}$: the arrival time of k with the n-th dispatch under a,
  - $z_{a,n,k}$: the end-of-service time of k with the n-th dispatch under a,
  - $w_{a,n,k}$: the return time of k with the n-th dispatch under a,
  - $p_{a,j,n,k}$: the response time of k for j with the n-th dispatch under a.

3.2. Constraints

In this research, we built the following constraints.

Affiliation and dispatching rule of vehicles

$$\sum_{i \in I} y_{i,k} = 1 \quad (k \in K), \tag{1}$$

$$\sum_{n \in N, k \in K} x_{a,j,n,k} = 1 \quad (a \in A, j \in J), \tag{2}$$

$$\sum_{j \in J} x_{a,j,n,k} \le 1 \quad (a \in A, n \in N, k \in K). \tag{3}$$

The constraint (1) ensures that the emergency vehicles are assigned to one fire department. The constraint (2) ensures that an emergency vehicle must be dispatched to a site. The constraint (3) ensures that each emergency vehicle can only be dispatched to at most one site in each dispatching.

Dispatching emergency vehicles

$$h_{a,n,k} \geq \sum_{j \in J} x_{a,j,n,k} u_{a,j} + e \quad (a \in A, n \in N, k \in K), \quad (4)$$

$$l_{a,n,k} \geq t_{a,i,j} - (1 - y_{i,k}) M_1 - (1 - x_{a,j,n,k}) M_1$$
$$(a \in A, i \in I, j \in J, n \in N, k \in K), \quad (5)$$

$$v_{a,n,k} \geq h_{a,n,k} + l_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (6)$$

$$z_{a,n,k} = v_{a,n,k} + \sum_{a \in A, j \in J} x_{a,j,n,k} s_{a,j}$$
$$(a \in A, n \in N, k \in K), \quad (7)$$

$$w_{a,n,k} = z_{a,n,k} + l_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (8)$$

$$w_{a,n-1,k} + e \leq h_{a,n,k} \quad (a \in A, n \in N, k \in K : n \geq 2). \quad (9)$$

The constraint (4) ensures that the setup time is necessary for dispatching. The constraint (5) becomes $l_{a,n,k} \geq t_{a,i,j}$ when $x_{a,j,n,k} = 1$ and $y_{i,k} = 1$ holds, the $l_{a,n,k}$ shows the upper bound of traveling time between the fire department and the site. The constraints (6) and (7) determine the arrival time and the end-of-service time of an emergency vehicle at a site, respectively. The constraint (8) shows the return time of an emergency vehicle. The constraint (9) shows the relationship between the return time and the next dispatching time of an emergency vehicle.

Priority and posteriority of time variables

$$v_{a,n,k} \geq h_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (10)$$

$$z_{a,n,k} \geq v_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (11)$$

$$w_{a,n,k} \geq z_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (12)$$

$$p_{a,j,n,k} \geq v_{a,n,k} - u_{a,j} - b - (1 - x_{a,j,n,k}) M_2$$
$$(a \in A, j \in J, n \in N, k \in K). \quad (13)$$

The constraints (10), (11) and (12) show the order of time variables (Fig. 2). The constraint (13) computes the response time of each emergency vehicle.

Nonnegativity of variables

$$l_{a,n,k} \geq 0 \quad (a \in A, n \in N, k \in K), \quad (14)$$

$$p_{a,j,n,k} \geq 0 \quad (a \in A, j \in J, n \in N, k \in K). \quad (15)$$

3.3. Objective function

$$f = \sum_{a \in A} \left(P_a \cdot \sum_{j \in J, n \in N, k \in K} p_{a,j,n,k} \right)$$

3.4. Mathematical model

The mathematical model is presented as follows:

$$(\star) \left| \begin{array}{ll} \text{minimize} & f \\ \text{subject to} & (1) \sim (15). \end{array} \right.$$

$(\star)$ is used to find the optimal location of emergency vehicles by minimizing the total response time for each emergency vehicle.

4. NUMERICAL EXPERIMENT

In this section, we will some numerical instances which are generated by random in order to demonstrate the performance and validate the proposed model.

4.1. Parameter generation

First, we generated a $L \times W$ rectangular region and divide the region into 12 parts (A1 ~C4) and set the population density fo each part as shown in Table. 1. Next, we set the location of fire departments (Fig. 3).

The assumptions of each parameter are shown as follows:

- The number of sites of each scenario follows the Possion distribution whose mean value equals λ_1.
- The number of sites in each part is proportional to the population density of a part.
- The location of sites are distributed uniformly in each part.
- The call-in time $u_{a,j}$ follows the exponential distribution whose mean value is $\frac{1}{\lambda_1}$.
- The distance $d_{a,i,j}$ between the fire department i and the site j for each a is measured in the Euclidean norm.
- The average speed of emergency vehicles is q.
- The traveling time of emergency vehicles is defined as
$$t_{a,i,j} = d_{a,i,j} \ / \ q + r_{a,i,j}$$
where $r_{a,i,j}$ follows a uniform distribution function defined by $U\left[\underline{l}, \overline{u}\right]$ whose means that the rush time expect $d_{a,i,j}/q$.
- The service time $s_{a,j}$ follows the exponential distribution whose mean value is $\frac{1}{\lambda_2}$.
- The setup time for dispatching e equals 3 minute.
- The occurrence probability P_a equals $\frac{1}{A}$.

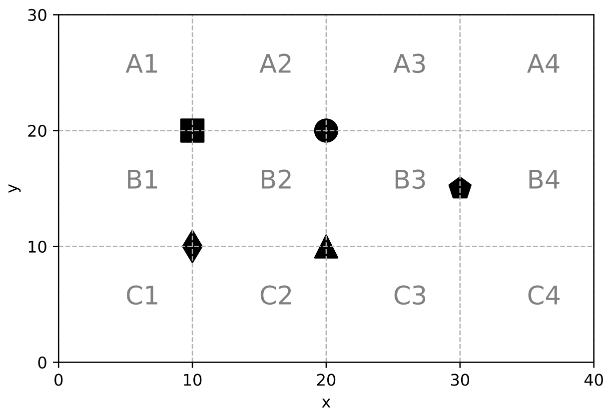

Fig. 3 The location of fire departments

Table 1 The probability of sites occurrence in each part

	1	2	3	4
A	0.10	0.10	0.02	0.20
B	0.03	0.07	0.08	0.02
C	0.11	0.20	0.03	0.04

4.2. Experimental outline and results

In this subsection, we use the 4.1 and set the parameter values in Table. 2 to generate data for our numerical experiments and compare the performance for 6 different cases.

We use the mathematical model ($\star$) to slove instances by using the Gurobi Optimizer version 7.0.1 and the computation enviroment as shown in Table. 3.

Table 2 Parameter values in parameter generation

Param	L	W	V	$\underline{l}$	$\overline{u}$	λ_2
Value	40	30	30	0	15	20

Table 3 Computation enviroment

OS	Microsoft Windows 10 Home
CPU	Intel(R) Core(TM) i7-6600U CPU
Memory	16.0GB
Solver	Gurobi Optimizer version 7.0.1
Computaion time	3600sec

Table 4 The description and results of the instance

	$\|I\|$	$\|K\|$	λ_1	$n_{\max}$	$\|A\|$	b	Obj	Gap
Case 1	5	3	10	8	30	0	947.74	100%
Case 2	5	3	10	8	30	30	666.04	100%
Case 3	5	4	10	5	30	0	614.56	100%
Case 4	5	4	10	5	30	30	304.13	100%
Case 5	5	5	10	4	30	0	426.81	100%
Case 6	5	5	10	4	30	30	147.70	100%

When the candidate fire departments $|I|$=5, the mean value of sites λ_1=10, scenarios $|A|$=30, we change the number of emergency vehicles $|K|$, the maximum number of dispatching of each emergency vehicle $n_{\max}$ and threshold amount b to do the numerical experiment. Besides, the total number of sites is 302. The computation results are shown in Table. 4 and the optimal lcoation results are shown in Table. 5 for each case. The Table. 6 shows the number of dispatching for each fire departments in each case.

Firstly, we will analyze the Case 1, Case 3 and Case 5, besides, the emergency vehicles belonging to fire departments are dispatched to the site as shown in Fig. 4, Fig. 6 and Fig. 8. we can see that the objective value (expectation value of response time) decrease as the number of emergency vehicles increases. In fact, the response time gradually decreases from the Table. 7, Besides, the optimal location of Case 1 is $\{Triangle, Triangle, Circle\}$, the optimal location of Case 3 is $\{Rhombus, Circle, Circle, Circle\}$. From this, Case 1 is not a partial set of Case 3, so the solution of Case 1 can't be used.

Secondly, we will analyze the Case 2, Case 4 and Case 6, besides, the emergency vehicles belonging to fire departments are dispatched to the site as shown in Fig. 5, Fig. 7 and Fig. 9. we can see that the Objective value decrease as the number of emergency vehicles increases. In fact, the response time gradually decreases from the Table. 8.

Thirdly, we will compare Case 1 and Case 2, Case 3 and Case 4, Case 5 and Case 6 respectively by the Table. 7 and Table. 8. If we compare the number of the response time less or equal 30 minutes for each case, we can know that the Case 1 is better than Case 2, and the Case 5 is better than Case 6. But the Case 4 is better than Case 3. We may can infer that when we have 3 vehicles the location of Case 1 may be the better choice, when we have 4 vehicles the lcoation of Case 4 may be the better choice and when we have 5 vehicles the location of Case 5 may be the better choice.

Finally, we analyze all cases. We can know that the *circle* fire department was used for each case because the population density is relatively high in A2 and A4 region. In addition, the *pentagon* fire department was't used one time for each case.

Table 5 The optimal location results of each case

	Rhombus	*Square*	*Triangle*	*Circle*	*Pentagon*
Case 1	0	0	2	1	0
Case 2	0	0	1	2	0
Case 3	1	0	0	3	0
Case 4	1	0	1	2	0
Case 5	1	1	1	2	0
Case 6	2	0	1	2	0

Table 6 The number of dispatching in each fire departments for each case

	Rhombus	*Square*	*Triangle*	*Circle*	*Pentagon*
Case 1	0	0	191	111	0
Case 2	0	0	105	197	0
Case 3	90	0	0	212	0
Case 4	73	0	78	151	0
Case 5	62	50	64	126	0
Case 6	110	0	62	130	0

Table 7 Simulaion advantageous effect

	≤ 15	≤ 30	≤ 45	≤ 60	> 60
Case 1	4	58	138	183	119
Case 3	10	85	175	222	50
Case 5	24	122	219	259	43

Table 8 Simulaion advantageous effect

	≤ 30	> 30
Case 2	51	251
Case 4	95	207
Case 6	114	188

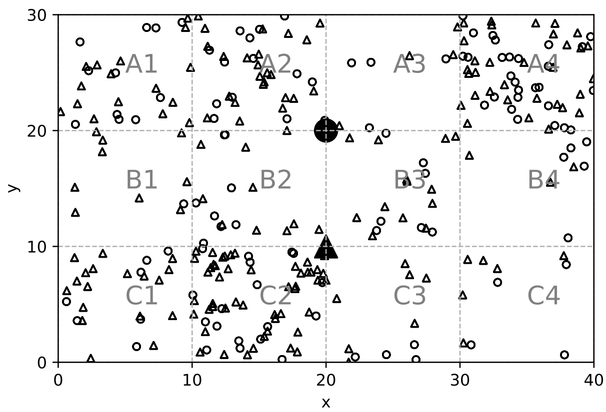

Fig. 4 The dispatching results of Case 1

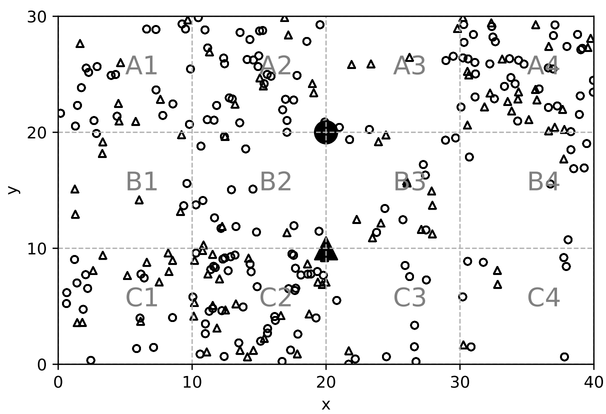

Fig. 5 The dispatching results of Case 2

4.3. System evaluation for 6 cases

In this subsection, we will use the DEA (Data Envelopment Analysis) CCR (Charnes Cooper Rhodes) model (♣) to evaluate each case for measuring the efficiency.

In model (♣), if the objective value equals 1 that means this case is efficiency. Besides, we set input item $i \in I$, output item $j \in J$, case $k \in K$, target case $\bar{k} \in \bar{K}$. In Table. 9 shown the data for (♣), among it, (♡) is the number of emergncy vehicles, (♠) is the number of fire departments, (◇) is the number of the response time is less or equal 30 minutes.

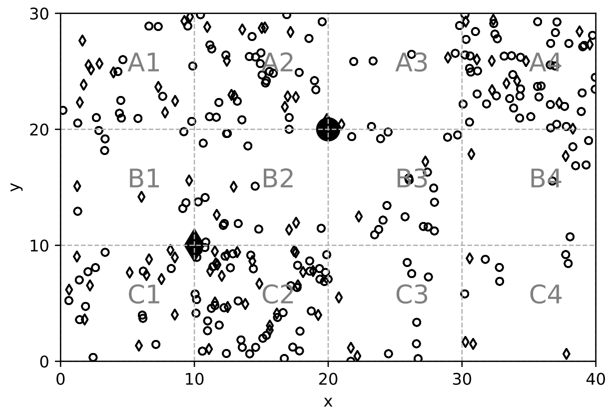

Fig. 6 The dispatching results of Case 3

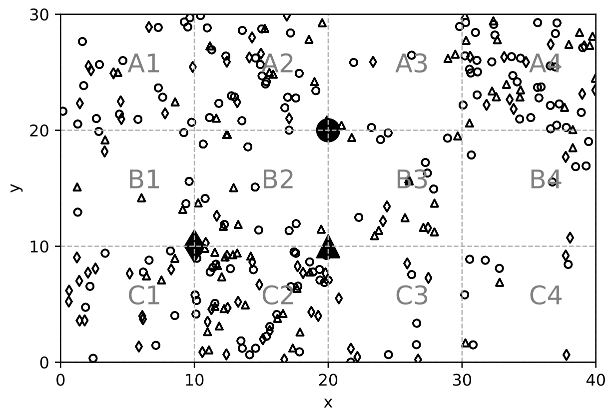

Fig. 7 The dispatching results of Case 4

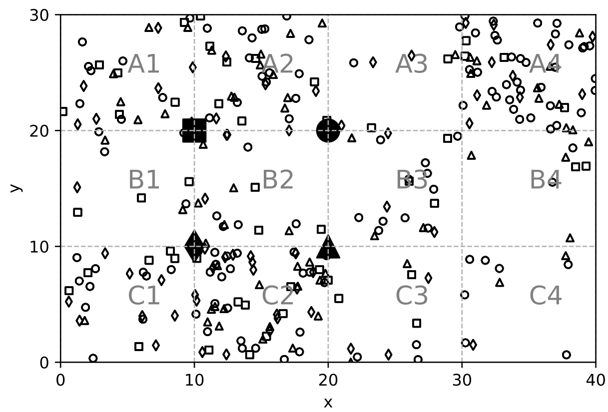

Fig. 8 The dispatching results of Case 5

$$
(\clubsuit)\quad
\begin{array}{|ll}
\text{maximize} & \sum_{\bar{k} \in \bar{K}, j \in J} outD_{\bar{k},j} outW_j \\
\text{subject to} & \sum_{i \in I} inD_{\bar{k},j} inW_i = 1 \quad (\bar{k} \in \bar{K}) \\
 & \sum_{j \in J} outD_{k,j} outW_j \leq \sum_{i \in I} inD_{k,i} inW_i \\
 & (k \in K) \\
 & inW_i \leq 0 \quad (i \in I) \\
 & outW_j \leq 0 \quad (j \in J)
\end{array}
$$

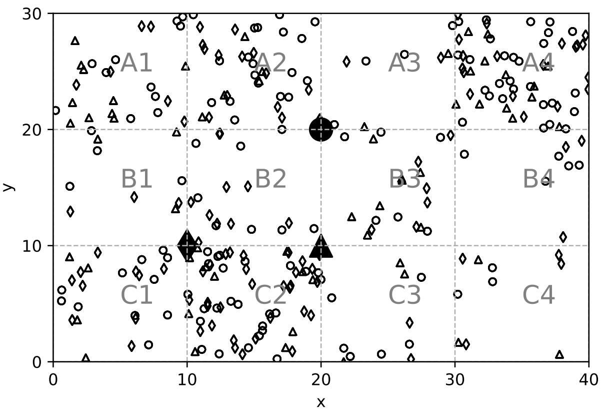

Fig. 9 The dispatching results of Case 6

where

- Variables
  - inW_i: the weight for i,
  - $outW_j$: the weight for j.
- Parameters
  - $inD_{k,i}$: the data of i for each k,
  - $outD_{k,j}$: the data of j for each k.

In DEA, an analysis object such as each case is called DMU (decision making unit). It is difficult to evaluate all DMU at one time, so we consider the problem of paying attention to the case $\bar{k} \in \bar{K} \subset K$ to maximize the efficiency by (♣). The computation results are shown in Table. 10, some of the results are represented by a rounded fourth number. We can know that Case number (3, 5, 6) which has the objective value of 1 is efficient, while the Case number (1, 2, 4) which is smaller than 1 is inefficient even if it selects any weight satifying the various constraints.

Table 9 Data for (♣)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
(♠)	2	2	2	3	4	3
(♡)	3	3	4	4	5	5
(◊)	58	51	85	95	122	114

Table 10 Results

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Obj	0.829	0.743	1	0.99	1	1
weight (♠)	0.114	0.114	0.182	0.083	0	0.07
weight (♡)	0.257	0.257	0.159	0.188	0.2	0.158
weight (◊)	0.014	0.014	0.118	0.01	0.008	0.008

4.4. Responding to large-scale problem cases

In order to know the limits of the mathematical model (⋆) in the realistic time, we conducted an experiment to expand the scale of the problem (Case 7, Case 8 and Case 9). We set the computation time is 43200sec, if the solver can't find the feasible solution within 43200sec, we called it no feasible solution.

The computation results are shown in Table. 11. from the computation results, we can know that if the scale of problem as Case 7 and Case 8, our model (⋆) can find the feasible solution by solver. However, if the scale of problem as Case 9, our model (⋆) can't solve them smoothly by solver, therefore , we should modify the optimization model an design an algorithm for solving the large-scale problems efficiently in the future.

Table 11 Large-scale problems computation results

	$\lvert I\rvert$	$\lvert K\rvert$	λ_1	$n_{\max}$	$\lvert A\rvert$	Feasiblity	Time	Gap
Case 7	20	20	40	4	30	Yes	4691sec	100%
Case 8	25	25	50	5	30	Yes	20772sec	100%
Case 9	30	30	60	6	30	None	None	100%

5. CONCLUSIONS AND FUTURE WORK

This paper focused on emergency vehicle location and dispatching problem with consideration of integration dispatching. The proposed model aims to minimize the response time in order to arrive the sites to rescue as fast as possible. In order to demonstrate the performance and validate the proposed model, a set of test instances was generated by random be solved by using the Gurobi Optimizer version 7.0.1. We know the model action by 6 cases (Case 1, Case 2, Case 3, Case 4, Case 5 and Case 6). However, the call-in time and service time are play an important role in the location of emergency vehicles, we should change them for each parts and do numerical experiment in the future. Besides, we do the evaluation for each case by model (♣), the reuluts shown that Case number (3, 5, 6) which has the objective value of 1 is efficient. Next, we expanded the scale of problems (Case 7, Case 8, Case 9). The computaiton results show the efficacy of the proposed model. However, if the problem scale exceeds the Case 8 like Case 9, it is hard to be solved, therefore, we should design an algorithm to solve the large-scale problems in the future. Besides, we also need to use the real data to validate the proposed model.

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