OPTIMIZATION MODEL FOR EMERGENCY VEHICLE ALLOCATION WITH INTEGRATION OF DISPATCH

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Abstract. This paper aims to develop an optimization model for allocating emergency vehicles to fire departments. Our proposed model considers the integration of dispatch of emergency vehicles in a comparatively large area. We generated some random scenarios to consider uncertainty in the real world and conducted numerical experiments. However, the obtained solutions showed that usefulness of our model is very limited. So, we solved other problems in which the allocation of EVs was fixed. The results showed that our model can find the optimal or good dispatch under the fixed allocation.

Keywords. Allocation and dispatch of emergency vehicles, integration of dispatch, optimization, uncertainty.

1. INTRODUCTION

Fire and Disaster Management Organization Act defines the tasks of the fire service organization as follows (FDMA, 2015):

> Utilizing the available facilities and human resources, fire service organizations shall protect the lives, physical being, and property of public from fire and take precautions against disasters such as storms, floods fires, and earthquakes, while mitigating the damage of these disasters. Fire service organizations are also responsible for the appropriate transport of persons who have sustained injuries due to a disaster.

When serious accidents or disasters happen, emergency vehicles belonging to the fire departments will rush to the emergency site as soon as possible for rescue. However, in recent years, the number of emergency calls is increasing. In particular, the average traveling time of emergency vehicles to the emergency site tends to become longer because of the lack of fire departments and emergency vehicles in suburbs. If we allocate more emergency vehicles in more fire departments, we can shorten the traveling time to the emergency sites. However, at the same time the allocation cost must be considered. Therefore, the white paper of the Japan Fire and Disaster Management Agency (FDMA, 2018) pointed out that we must merge the responsibility areas of fire departments and integrate the operation for dispatching emergency vehicles. This is an urgent task because we have to prepare for disaster risks and the decrease of the population of Japan in the near future.

In the allocation problem of emergency vehicles, the main objective is to minimize the response time, that is, the elapsed time between the receiving time of the emergency call and the arrival time of emergency vehicles at the emergency site (Saeed et al., 2018). Because the emergency site would be covered by the emergency vehicles allocated at fire departments, therefore, the allocation of emergency vehicles is important for the level of service quality. Moreover, the unpredictability of the time and the location of emergency incidents are also the main issues in the emergency vehicle location (Inakawa et al., 2004) (Lei et al., 2015) (Xiao-Xia et al., 2013). We should consider the uncertainty for solving the emergency vehicle allocation problem:

- The uncertainty of the call-in time and the location of the emergency site
- The uncertainty of the traveling time of emergency vehicles
- The uncertainty of the service time at the emergency site

Based on the above, in this research, we develop an optimization model to find the optimal allocation of emergency vehicles with integration of dispatching of emergency vehicles. In our proposed model, we can deal with some uncertainties which are explained above.

In Section 2, the process of emergency response system is described in detail. Next, the mathematical optimization model will be proposed in Section 3. After that, in order to demonstrate the performance and validate the proposed model, we generated some numerical instances are presented and solved them by the proposed model in Section 4. Finally, we draw a conclusion in Section 5.

2. PROCESS OF EMERGENCY RESPONSE SYSTEM

In this section, we introduce the process of the emergency response system. The process of the emergency response system usually covers the sequence of the following activities.

1. The system receives the emergency call when the incident happens.
2. After call screening, dispatchers evaluate the system status and determine the appropriate emergency vehicle (EV) to dispatch.

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3. The dispatched EV arrives at the emergency site and starts service.
4. After completing service, the EV returns to the fire departments or goes to another emergency site.

In this research, we assume that dispatched EVs must return back to the fire departments when the service is completed (Fig. 1). Also, we use some notations of the times and the time periods for explaining this produce as shown in Fig. 2.

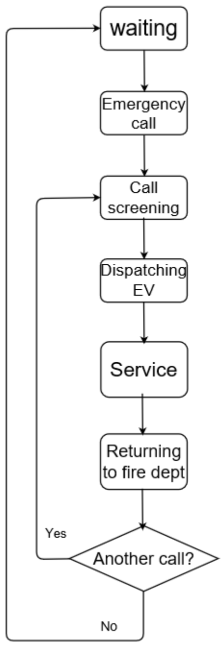

Fig. 1 The process of emergency response system

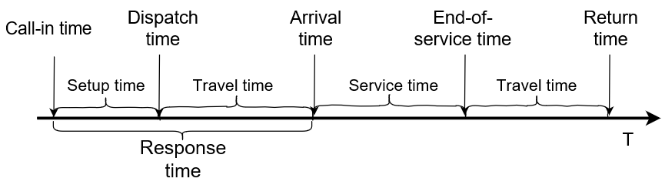

Fig. 2 Times and time periods in EV dispathcing

3. MATHEMATICAL FORMULATION

In this section, we formulate the optimization model to find the optimal allocation of emergency vehicles.

3.1. Notation

We use the following notations to describe our proposed model.

- Indices and Sets
  - $s \in S$: the set of scenarios,
  - $d \in D$: the set of fire departments,
  - $p_s \in P_s$: the set of emergency sites under s,
  - $v \in V$: the set of emergency vehicles,
  - $n \in N$: the set of the numbers of dispatching. Note that each emergency vehicle can be dispatched at most $|N|$ times for each scenario.
- Parameters
  - t^{call}_{s,p_s} : the call-in time from p_s under s,
  - $t^{\text{travel}}_{s,d,p_s}$: the traveling duration between d and p_s under s,
  - $t^{\text{service}}_{s,p_s}$: the service duration at p_s under s,
  - t^{setup} : the setup duration for dispatching,
  - t^{allow} : the threshold of allowable duration,
  - $\mathbb{P}_s$: the occurrence probability of s,
  - M_1/M_2 : sufficiently large constant numbers (so-called "big-M").
- Decision variables
  - $$x_{s,p_s,n,v} = \begin{cases} 1, & v \text{ is dispatched to } p_s \text{ with the } n\text{-th dispatch under } s, \\ 0, & \text{otherwise,} \end{cases}$$
  - $$y_{d,v} = \begin{cases} 1, & v \text{ is assigned to } d, \\ 0, & \text{otherwise,} \end{cases}$$
  - $z^{\text{dispatch}}_{s,n,v}$: the dispatch time of v with the n-th dispatch under s,
  - $z^{\text{travel}}_{s,n,v}$: the traveling duration of v with the n-th dispatch under s,
  - $z^{\text{arrival}}_{s,n,v}$: the arrival time of v with the n-th dispatch under s,
  - $z^{\text{delay}}_{s,p_s,n,v}$: the delay time of v for p_s with the n-th dispatch under s,
  - $z^{\text{end}}_{s,n,v}$: the end-of-service time of v with the n-th dispatch under s,
  - $z^{\text{return}}_{s,n,v}$: the return time of v with the n-th dispatch under s.

3.2. Constraints

In this research, we built the following constraints.

Constraints on allocation and dispatching rule of vehicles

$$\sum_{d \in D} y_{d,v} = 1 \quad (v \in V), \tag{1}$$

$$\sum_{n \in N, v \in V} x_{s,p_s,n,v} = 1 \quad (s \in S, p_s \in P_s), \tag{2}$$

$$\sum_{p_s \in P_s} x_{s,p_s,n,v} \leq 1 \quad (s \in S, n \in N, v \in V). \tag{3}$$

The constraint (1) ensures that the emergency vehicles are assigned to one fire department. The constraint (2) ensures that an emergency vehicle must be dispatched to an emergency site. The constraint (3) ensures that each emergency vehicle can only be dispatched to at most one emergency site in each dispatching.

Constraints on variables of time

$$z^{\text{dispatch}}_{s,n,v} \geq \sum_{p_s \in P_s} x_{s,p_s,n,v} t^{\text{call}}_{s,p_s} + t^{\text{setup}} \quad (s \in S, n \in N, v \in V), \tag{4}$$

$$z^{\text{travel}}_{s,n,v} \geq t^{\text{travel}}_{s,d,p_s} - (1 - y_{d,v})M_1 - (1 - x_{s,p_s,n,v})M_1 \quad (s \in S, d \in D, p_s \in P_s, n \in N, v \in V), \tag{5}$$

$$z^{\text{arrival}}_{s,n,v} \geq z^{\text{dispatch}}_{s,n,v} + z^{\text{travel}}_{s,n,v} \quad (s \in S, n \in N, v \in V), \tag{6}$$

$$z^{\text{end}}_{s,n,v} = z^{\text{arrival}}_{s,n,v} + \sum_{p_s \in P_s} x_{s,p_s,n,v} t^{\text{service}}_{s,p_s} \quad (s \in S, n \in N, v \in V), \tag{7}$$

$$z^{\text{return}}_{s,n,v} = z^{\text{end}}_{s,n,v} + z^{\text{travel}}_{s,n,v} \quad (s \in S, n \in N, v \in V), \tag{8}$$

$$z^{\text{return}}_{s,n-1,v} + t^{\text{setup}} \leq z^{\text{dispatch}}_{s,n,v} \quad (s \in S, n \in N, v \in V : n \geq 2), \tag{9}$$

$$z^{\text{delay}}_{s,p_s,n,v} \geq z^{\text{arrival}}_{s,n,v} - t^{\text{call}}_{s,p_s} - t^{\text{allow}} - (1 - x_{s,p_s,n,v})M_2 \quad (s \in S, p_s \in P_s, n \in N, v \in V). \tag{10}$$

The constraint (4) ensures that the setup duration is necessary for dispatching. The constraint (5) becomes $z^{\text{travel}}_{s,n,v} \geq t^{\text{travel}}_{s,d,p_s}$ when $x_{s,p_s,n,v} = 1$ and $y_{d,v} = 1$ hold, and $z^{\text{travel}}_{s,n,v}$ shows the upper bound of traveling duration between the fire department and the emergency site. The constraints (6) and (7) determine the arrival time and the end-of-service time of an emergency vehicle at an emergency site, respectively. The constraint (8) computes the return time of an emergency vehicle. The constraint (9) defines the relationship between the return time and the next dispatching time of an emergency vehicle. The constraint (10) computes the delay time. Now we consider the case that $x_{s,p_s,n,v}$ equals 1. If the response time is smaller than or equal to t^{allow}, then $z^{\text{delay}}_{s,p_s,n,v}$ equals 0. Otherwise, $z^{\text{delay}}_{s,p_s,n,v}$ equals the delay time, which shows the exceedance of t^{allow}.

Nonnegativity of variables

$$z^{\text{travel}}_{s,n,v} \geq 0 \quad (s \in S, n \in N, v \in V), \tag{11}$$

$$z^{\text{delay}}_{s,p_s,n,v} \geq 0 \quad (s \in S, p_s \in P_s, n \in N, v \in V). \tag{12}$$

The constraints (11) and (12) ensure the nonnegativity of $z^{\text{travel}}_{s,n,v}$ and $z^{\text{delay}}_{s,p_s,n,v}$.

3.3. Objective function

$$f = \sum_{s \in S} \left(\mathbb{P}_s \cdot \sum_{p_s \in P_s, n \in N, v \in V} z^{\text{delay}}_{s,p_s,n,v} \right)$$

f shows the expected value of the aggregation of all delay times.

3.4. Mathematical model

Our proposed model is presented as follows:

$$(\heartsuit) \left| \begin{array}{ll} \text{minimize} & f \\ \text{subject to} & (1) \sim (12). \end{array} \right.$$

$(\heartsuit)$ is used to find the optimal allocation of emergency vehicles by minimizing the total delay time for each emergency vehicle.

4. NUMERICAL EXPERIMENTS

In this section, we solve some numerical instances which are generated randomly. Through these numerical experiments, we confirm the validness and performance of the proposed model.

4.1. Generation of instances

First, we supposed a 40 (km) × 30 (km) rectangular region and divided it into 12 parts (A1–C4). Also, we set fire departments in the region. In our experiments, we try to find the optimal solution for (i) the case of five fire departments (Fig. 3) and (ii) the case of 20–30 fire departments (Fig. 4).

Next, we set the values of the parameters t^{call}_{s,p_s}, $t^{\text{travel}}_{s,d,p_s}$, $t^{\text{service}}_{s,p_s}$ and t^{setup}. The assumptions of each parameter are shown as follows:

- The number of emergency sites in each scenario p_s follows the Poisson distribution whose mean value equals λ_1.
- The number of emergency sites in each part is proportional to the probability of occurrence of emergency sites as shown in Table 1.
- The locations of emergency sites are distributed uniformly in each part.
- The interval of t^{call}_{s,p_s} follows the exponential distribution whose mean value equals $1/\lambda_1$ (day).
- The distance δ_{s,d,p_s} (km) between the fire department d and the emergency site p_s in each scenario s is measured in the Euclidean norm.
- The traveling time of an emergency vehicle from a fire department d to an emergency site p_s in a scenario s is defined as
$$t^{\text{travel}}_{s,d,p_s} = (\delta_{s,d,p_s} \ / \ q) \times 60 + r_{s,d,p_s} \text{ (min.)},$$
where q (km/h) is the average speed of emergency vehicles and r_{s,d,p_s} (min.) follows a uniform distribution on the interval $[l, u]$ (min.). Here, the term r_{s,d,p_s} is included to express a potential delay.
- The service time $t^{\text{service}}_{s,p_s}$ follows the exponential distribution whose mean value equals $1/\lambda_2$ (day).
- The occurrence probability $\mathbb{P}_s$ equals $1/|S|$.
- The setup time t^{setup} equals three minutes.

In both the cases (i) and (ii), we set the values of some parameters in common as shown in Table 2 to generate scenarios for our numerical experiments. In the following, we explain the detail of instance settings of both cases.

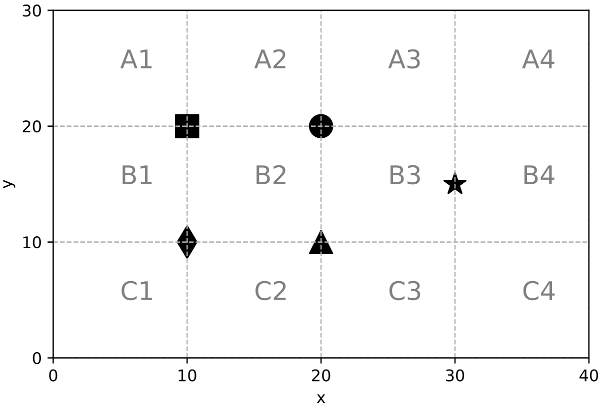

Fig. 3 (i) the location of five fire departments

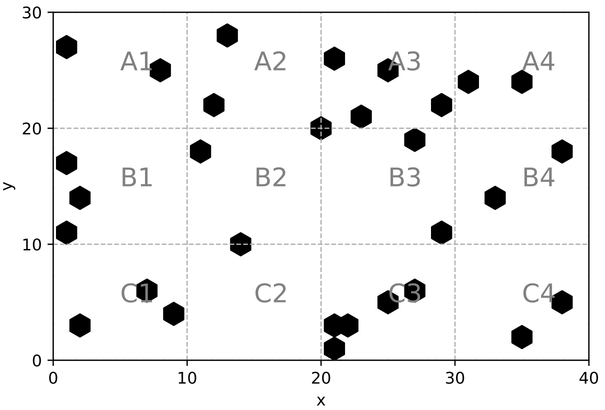

Fig. 4 (ii) the location of thirty fire departments

Table 1 Probability of occurrence of emergency sites

	1	2	3	4
A	0.10	0.10	0.02	0.20
B	0.03	0.07	0.08	0.02
C	0.11	0.20	0.03	0.04

Table 2 Parameter values

Parameter	$\lvert S\rvert$	q	l	u	λ_2
Value	30	30	0	15	20

- Instance settings of (i)

  We set the λ_1=10, and generated t^{call}_{s,p_s}, $t^{\text{travel}}_{s,d,p_s}$, $t^{\text{service}}_{s,p_s}$ and t^{setup}. Moreover, we solved (♡) with six different settings (Table 3). Note that these six cases use the same scenarios to examine the effect of some parameters.

Table 3 Parameter settings for small-scale experiments

	$\lvert V\rvert$	$\lvert N\rvert$	t^{allow}
Case (i)-1	3	8	0
Case (i)-2	3	8	20
Case (i)-3	4	5	0
Case (i)-4	4	5	20
Case (i)-5	5	4	0
Case (i)-6	5	4	20

- Instance settings of (ii)

  We generated three cases as shown in Table 4 for large-scale experiments.

  First, we set the value of $|D|$ and λ_1 as shown in Table 4 and generated scenarios. In these cases, the fire departments are distributed randomly in the region. After that, we solved (♡) with the settings in Table 4.

Table 4 Parameter settings for large-scale experiments

	$\lvert D\rvert$	λ_1	$\lvert V\rvert$	$\lvert N\rvert$	t^{allow}
Case (ii)-1	20	40	20	4	0
Case (ii)-2	25	50	25	5	0
Case (ii)-3	30	60	30	6	0

4.2. Experimental results

In this subsection, we show the results of the small-scale and large-scale experiments. In this research, we used the computational environment as shown in Table 5 to conduct the numerical experiments.

Table 5 Computational environment

OS	Microsoft Windows 10 Pro
CPU	Intel(R) Core(TM) i7-6950X CPU @ 3.00GHz
Memory	64.0 GB
Solver	Gurobi Optimizer version 7.0.1

4.2.1. Results of (i): small-scale experiments

In this case, we set the limit of computation time as 1800 seconds to solve each case. The objective value and the location of EVs obtained by solving (♡) is shown in Table 6.

Table 6 Location of EVs and objective value

	♦	■	▲	●	★	Obj.val.	Gap(%)
Case (i)-1	1	0	0	1	1	323.64	91.3
Case (i)-2	0	0	1	2	0	178.66	100
Case (i)-3	1	1	1	0	1	264.95	89.3
Case (i)-4	1	0	1	2	0	116.80	100
Case (i)-5	0	2	2	0	1	268.90	89.5
Case (i)-6	1	1	0	2	1	78.58	100

Gap(%): (upper bound - lower bound) / upper bound × 100

4.2.2. Results of (ii): large-scale experiments

In order to examine the ability of our proposed model (♡) for large-scale problems, we tried to solve three problems of the case (ii). We set the computation time as 43200 seconds for this case. The computation results are shown in Table 7.

Table 7 Computation results for the case (ii)

	Feasiblity	(∗)	Gap(%)
Case (ii)-1	Yes	4691sec	100
Case (ii)-2	Yes	20772sec	100
Case (ii)-3	None	None	100

(∗): the duration for finding the first feasible solution

4.3. Discussion

In this section, we discuss the experimental results above.

Actually, all the obtained solutions for Case (i) are incumbent: Table 6 shows the best solution at the end of the computation. Moreover, according to Table 6, we can see that the gaps are very large. Therefore, the usefulness of our model is very limited. So, we conducted another experiment: we used the location of EVs obtained by the experiments in Case (i) to fix the binary variable $y_{d,v}$, and try to solve each case again. In this additional experiment, the limit of computation time is 3600 seconds.

Table 8 shows the obtained objective value and Table 9 shows the numbers of emergency sites which are counted by the response time. In Case (i)-3 ∼ Case (i)-6, the solver can find the optimal solution in hundreds of seconds. Also, in Case (i)-1 and Case (i)-2, the gap is comparatively small.

As a result, if the allocation of EVs is fixed, then we can find the optimal dispatch in many cases. Therefore, we may construct another approach to solve (♡). The framework is as follows: we set an allocation of EVs by metaheuristics and find the optimal dispatch under the allocation by solving (♡). Also, we try to find the optimal allocation by local search.

Fig. 5 ∼ Fig. 10 show the result of dispatch in Case (i)-1 ∼ Case (i)-6. The figures (♦, ■, ▲, •, ⋆) show the places of emergency cites and the bases of EVs which are assigned to the cites in the optimal solutions.

Table 8 Objective value

	Obj.val.	Gap(%)	Time(sec.)
Case (i)-1	305.38	1.92	3600
Case (i)-2	166.14	9.07	3600
Case (i)-3	258.16	0	281
Case (i)-4	106.43	0	115
Case (i)-5	265.70	0	192
Case (i)-6	73.55	0	37

Table 9 Number of emergency sites

	≤ 20 (min.)	≤ 30	≤ 40	≤ 50	> 50
Case (i)-1	55	138	221	262	21
Case (i)-2	37	111	180	243	40
Case (i)-3	71	179	253	277	6
Case (i)-4	57	150	217	272	11
Case (i)-5	56	171	252	277	6
Case (i)-6	75	198	268	280	3

Next, we analyze the Case (ii)-1 ∼ Case (ii)-3. According to the computation results (Table 7), our model (♡) can find the feasible solution for Case (ii)-1 and Case (ii)-2 by the solver. However, the gap between the upper bound and the lower bound is too large. Moreover, our model (♡) cannot find the feasible solution for Case (ii)-3 by the solver. We plan to design a hybrid algorithm which is explained above in the near future.

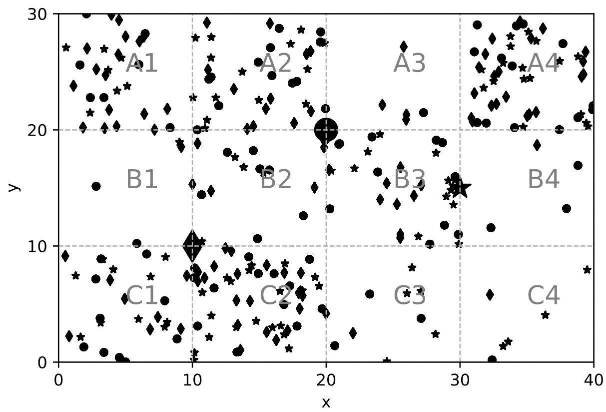

Fig. 5 Dispatching results of Case (i)-1

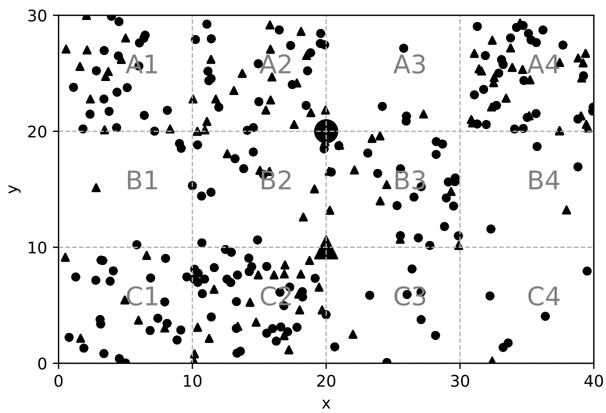

Fig. 6 Dispatching results of Case (i)-2

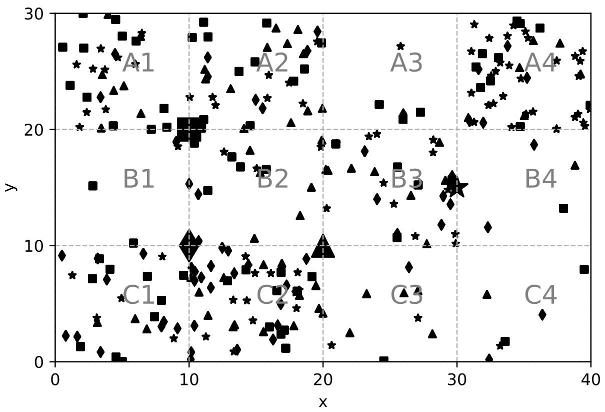

Fig. 7 Dispatching results of Case (i)-3

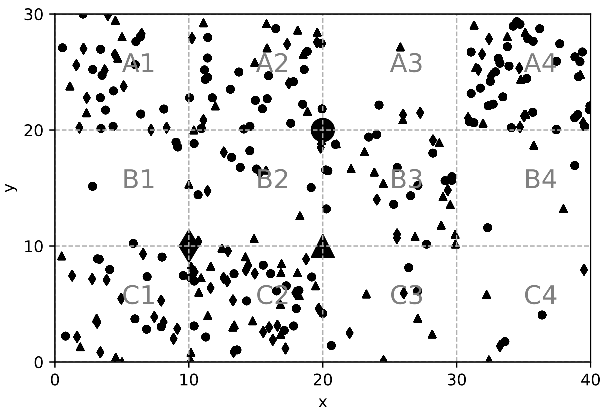

Fig. 8 Dispatching results of Case (i)-4

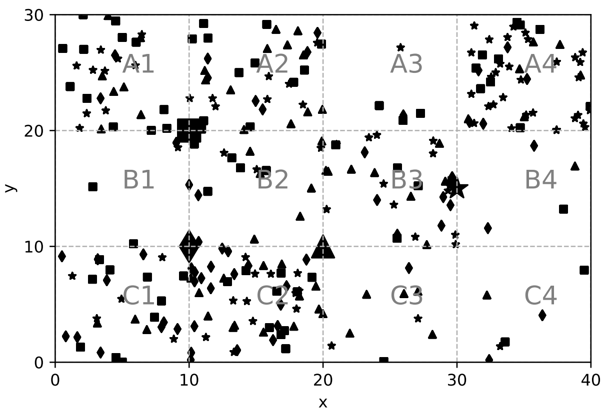

Fig. 9 Dispatching results of Case (i)-5

5. CONCLUSIONS AND FUTURE WORKS

This paper focused on the emergency vehicle allocation problem with integration of dispatch. The proposed model aims to minimize the delay time in order to arrive at the emergency site as fast as possible.

In order to examine the performance of the proposed model, some test instances were generated randomly and solved them. But the solver cannot ensure the accuracy of the obtained solutions: the gaps are comparatively large. However, if we fixed the allocation of EVs, then the solver can find the optimal or good dispatch. We would like to construct a new method for solving this problem by using this property.

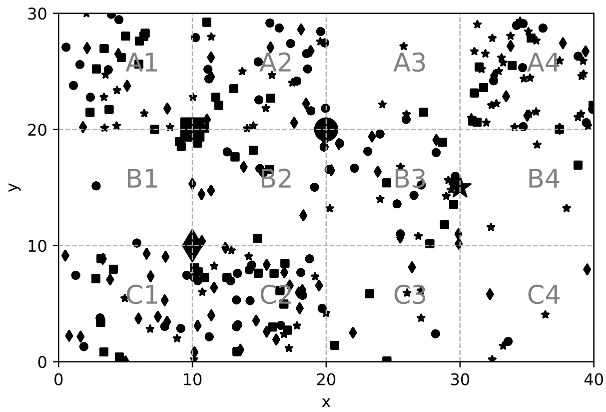

Fig. 10 Dispatching results of Case (i)-6

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