AN OPTIMIZATION MODEL FOR EMERGENCY VEHICLE ALLOCATION WITH CONSIDERATION OF INTEGRATION DISPATCHING

You Zhao [1*] and Hiroshige Dan[1†]

Abstract. This paper aims to develop location opitimization and dispatching optimization techniques for emergency vehicles. In this research, Firstly, we seek to propose a optimization model to find the optimal location of emergency vehicles. Then, we generate the some scenarios to consider the uncertainty conditions and do numerical experiment. The numerical experiment results show that the proposed model achieves a good performance and worked well.

Keywords. emergency vechicle allocation and dispatching, optimization, uncertainty, integration dispatching.

1. INTRODUCTION

Utilizing the available facilities and human resources, fire service organizations shall protect the lives, physical being, and property of public from fire and take precautions against disasters such as storms, floods fires, and earthquakes, while mitigating the damage of these disasters. Fire service organizations are also responsible for the appropriate transport of persons who have sustained injuries due to a disaster (FDMA, 2015). When the Large-scale disaster or accident, serious accident and the military attack happened, emergency vehicles belonging to the fire departments will rush to the emergency demand as soon as possible for rescue. However, in recent years, the number of emergency services for an emergency call is increasing. In particular, the emergency vehicles tend to lengthen the average response time to the emergency site because of the lack of fire departments and emergency vehicles in the suburbs. On the other hand, if we can locate more emergency vehicles in more fire departments, emergency vehicles can shorten the traveling time to the emergency site, but at the same time location cost must be considered. Therefore, according to the Japan Fire and Disaster Management Agency Heisei 29th edition fire fighting white paper (FDMA, 2018) pointed out that consider Large-scale disasters such as the Great East Japan Earthquake, higher future disaster risks, and the decrease in the population of Japan, we must enhance the structure of fire departments by expanding their jurisdictions. Accordingly, it is necessary to balance these two aspects and develop a more effective emergency vehicle location planning.

In the emergency vehicle allocation problem, the main objective to minimize the time it takes to respond to the emergency sites (the traveling time between emergency call receipt and emergency vehicles arrived at the emergency site) (Saeed et al., 2018). Because the emergency site for emergency services is covered by the emergency vehicles located at fixed points, therefore, the location of emergency vehicles is important in service quality level. Moreover, as uncertainty commonly exists in the real world. The unpredictability of the time and the location of emergency incidents are also the main issues in the emergency vehicle location (Xiao-Xia et al., 2013) (Lei et al., 2015). Therefore, we should consider the uncertainty for the emergency vehicle allocation problem as follows:

- The uncertainty of call-in time and the location of the emergency site
- The uncertainty of the emergency vehicle traveling time
- The uncertainty of service time at the emergency site

Based on the above, in this research, we developed an optimization model to find the optimal location of emergency vehicles with consideration of integration dispatching and considered the uncertainty conditions by numerical experiment.

In Section 2, the process of emergency response system and the times and time periods in emergency vehicle dispatching are described in detail. Next, the mathematical model will be proposed in Section 3. Then, in order to demonstrate the performance and validate the proposed model, we generated some numerical instances are presented and solved in Section 4. Finally, we draw a conclusion in Section 5.

2. PROCESS OF EMERGENCY RESPONSE SYSTEM

In this section, we introduce the process of the emergency response system. The process of the emergency response system usually covers the sequence of the following activities.

1. The system receives the emergency call when the incident happened.
2. After call screening, dispatchers evaluate the system status and determine the appropriate emergency vehicle (EV) to dispatch.
3. The dispatched EV arrives at the emergency site and

1 Environmental and Applied System Engineering, Kansai University, 3-3-35, Yamate-cho, Suita, Osaka, 564-8680, Japan.
∗ E-mail address: k819381@kansai-u.ac.jp
† E-mail address: dan@kansai-u.ac.jp

starts service.

4. After completing service, the EV returns to the fire departments or goes to another emergency site.

In this research, we assume that dispatched EVs must return back to the fire departments when the service is completed (Fig. 1).

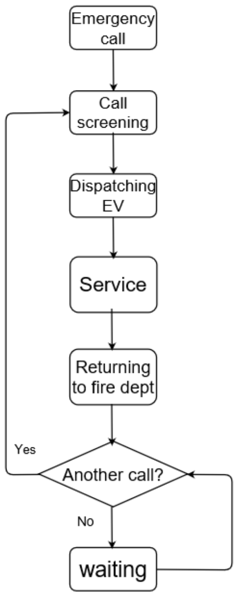

Fig. 1 The process of emergency response system

In this research, we have the times and the time periods which explain this produce as shown in Fig. 2.

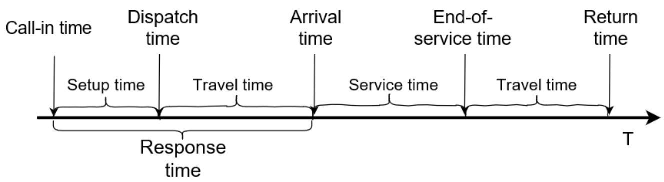

Fig. 2 The times and time periods in an EV dispathcing

3. MATHEMATICAL FORMULATION

In this section, we formulate the optimization model to find the optimal location of emergency vehicles.

3.1. Notation

We use the following notations to describe our proposed model.

- Indices and Sets
  - $s \in S$: the set of scenarios,
  - $d \in D$: the set of fire departments,
  - $p_s \in P_s$: the set of emergency sites under s,
  - $v \in V$: the set of emergency vehicles,
  - $n \in N$: the set of the numbers of dispatching. Note that each emergency vehicle can be dispatched at most $|N|$ times for each scenario.
- Parameters
  - t^{call}_{s,p_s} : the call-in time duration from p_s under s,
  - $t^{\text{travel}}_{s,d,p_s}$: the traveling duration between d and p_s under s,
  - $t^{\text{service}}_{s,p_s}$: the service duration at p_s under s,
  - t^{setup} : the setup duration for dispatching,
  - t^{allow} : the threshold of allowable duration,
  - $\mathbb{P}_s$: the occurrence probability of s,
  - M_1/M_2 : sufficiently large constant numbers.
- Decision variables
  - $$x_{s,p_s,n,v} = \begin{cases} 1, & v \text{ is dispatched to } p_s \text{ with the } n\text{-th dispatch under } s, \\ 0, & \text{otherwise,} \end{cases}$$
  - $$y_{d,v} = \begin{cases} 1, & v \text{ is assigned to } d, \\ 0, & \text{otherwise,} \end{cases}$$
  - $z^{\text{dispatch}}_{s,n,v}$: the dispatch time of v with the n-th dispatch under s,
  - $z^{\text{travel}}_{s,n,v}$: the traveling duration of v with the n-th dispatch under s,
  - $z^{\text{arrival}}_{s,n,v}$: the arrival time of v with the n-th dispatch under s,
  - $z^{\text{delay}}_{s,p_s,n,v}$: the delay time of v for p_s with the n-th dispatch under s,
  - $z^{\text{end}}_{s,n,v}$: the end-of-service time of v with the n-th dispatch under s,
  - $z^{\text{return}}_{s,n,v}$: the return time of v with the n-th dispatch under s.

3.2. Constraints

In this research, we built the following constraints.

Constraints on affiliation and dispatching rule of vehicles

$$\sum_{d \in D} y_{d,v} = 1 \quad (v \in V), \tag{1}$$

$$\sum_{n \in N, v \in V} x_{s,p_s,n,v} = 1 \quad (s \in S, p_s \in P_s), \tag{2}$$

$$\sum_{p_s \in P_s} x_{s,p_s,n,v} \leq 1 \quad (s \in S, n \in N, v \in V). \tag{3}$$

The constraint (1) ensures that the emergency vehicles are assigned to one fire department. The constraint (2) ensures that an emergency vehicle must be dispatched to an emergency site. The constraint (3) ensures that each emergency vehicle can only be dispatched to at most one emergency site in each dispatching.

Constraints on variables for time

$$z^{\text{dispatch}}_{s,n,v} \geq \sum_{p_s \in P_s} x_{s,p_s,n,v} t^{\text{call}}_{s,p_s} + t^{\text{setup}} \qquad (s \in S, n \in N, v \in V), \tag{4}$$

$$z^{\text{travel}}_{s,n,v} \geq t^{\text{travel}}_{s,d,p_s} - (1 - y_{d,v})M_1 - (1 - x_{s,p_s,n,v})M_1$$
$$(s \in S, d \in D, p_s \in P_s, n \in N, v \in V), \quad (5)$$

$$z^{\text{arrival}}_{s,n,v} \geq z^{\text{dispatch}}_{s,n,v} + z^{\text{travel}}_{s,n,v} \quad (s \in S, n \in N, v \in V), \quad (6)$$

$$z^{\text{end}}_{s,n,v} = z^{\text{arrival}}_{s,n,v} + \sum_{s \in S, p_s \in P_s} x_{s,p_s,n,v} t^{\text{service}}_{s,p_s}$$
$$(s \in S, n \in N, v \in V), \quad (7)$$

$$z^{\text{return}}_{s,n,v} = z^{\text{end}}_{s,n,v} + z^{\text{travel}}_{s,n,v} \quad (s \in S, n \in N, v \in V), \quad (8)$$

$$z^{\text{return}}_{s,n-1,v} + t^{\text{setup}} \leq z^{\text{dispatch}}_{s,n,v} \quad (9)$$
$$(s \in S, n \in N, v \in V : n \geq 2), \quad (10)$$

$$z^{\text{delay}}_{s,p_s,n,v} \geq z^{\text{arrival}}_{s,n,v} - t^{\text{call}}_{s,p_s} - t^{\text{allow}} - (1 - x_{s,p_s,n,v})M_2$$
$$(s \in S, p_s \in P_s, n \in N, v \in V). \quad (11)$$

The constraint (4) ensures that the setup time is necessary for dispatching. The constraint (5) becomes $z^{\text{travel}}_{s,n,v} \geq t^{\text{travel}}_{s,d,p_s}$ when $x_{s,p_s,n,v} = 1$ and $y_{d,v} = 1$ hold, and $z^{\text{travel}}_{s,n,v}$ shows the upper bound of traveling time between the fire department and the emergency site. The constraints (6) and (7) determine the arrival time and the end-of-service time of an emergency vehicle at a emergency site, respectively. The constraint (8) computes the return time of an emergency vehicle. The constraint (10) defines the relationship between the return time and the next dispatching time of an emergency vehicle. The constraint (11) computes the delay time. Now we consider the case that $x_{s,p_s,n,v}$ equals 1. If the response time is smaller than or equal to t^{allow}, then $z^{\text{delay}}_{s,p_s,n,v}$ equals 0. Otherwise, $z^{\text{delay}}_{s,p_s,n,v}$ equals the delay time, which shows the exceedance of t^{allow}.

Nonnegativity of variables

$$z^{\text{travel}}_{s,n,v} \geq 0 \quad (s \in S, n \in N, v \in V), \quad (12)$$

$$z^{\text{delay}}_{s,p_s,n,v} \geq 0 \quad (s \in S, p_s \in P_s, n \in N, v \in V). \quad (13)$$

The constraints (12) and (13) ensure the nonnegativity of $z^{\text{travel}}_{s,n,v}$ and $z^{\text{delay}}_{s,p_s,n,v}$.

3.3. Objective function

$$f = \sum_{s \in S} \left(\mathbb{P}_s \cdot \sum_{p_s \in P_s, n \in N, v \in V} z^{\text{delay}}_{s,p_s,n,v} \right)$$

f shows the expected value of the aggregation of all delay times.

3.4. Mathematical model

Our proposed model is presented as follows:

$$(\heartsuit) \left| \begin{array}{ll} \text{minimize} & f \\ \text{subject to} & (1) \sim (13). \end{array} \right.$$

($\heartsuit$) is used to find the optimal location of emergency vehicles by minimizing the total delay time for each emergency vehicle.

4. NUMERICAL EXPERIMENTS

In this section, we solve some numerical instances which are generated randomly. Through these numerical experiments, we confirm the validness and performance of the proposed model.

4.1. Generation of instances

First, we supposed a 40 (km) × 30 (km) rectangular region and divided it into 12 parts (A1–C4). Also, we set fire departments in the region. In our experiments, we try to find the optimal solution for (i) the case of five fire departments (Fig. 3) and (ii) the case of 20–30 fire departments (Fig. 4).

Next, we set the value of t^{call}_{s,p_s}, $t^{\text{travel}}_{s,d,p_s}$, $t^{\text{service}}_{s,p_s}$ and t^{setup}. The assumptions of each parameter are shown as follows:

- The number of emergency sites in each scenario p_s follows the Poisson distribution whose mean value equals λ_1.
- The number of emergency sites in each part is proportional to the probability of occurrence of emergency sites as shown in Table 1.
- The location of emergency sites is distributed uniformly in each part.
- The interval of t^{call}_{s,p_s} follows the exponential distribution whose mean value equals $1/\lambda_1$ (day).
- The distance δ_{s,d,p_s} (km) between the fire department d and the emergency site p_s in each scenario s is measured in the Euclidean norm.
- The traveling time of an emergency vehicle from a fire department d to an emergency site p_s in a scenario s is defined as
$$t^{\text{travel}}_{s,d,p_s} = (\delta_{s,d,p_s} \ / \ q) \times 60 + r_{s,d,p_s} \text{ (min.)},$$
where q (km/h) is the average speed of emergency vehicles and r_{s,d,p_s} (min.) follows a uniform distribution on the interval $[l, u]$ (min.). Here, the term r_{s,d,p_s} is included to express a potential delay.
- The service time $t^{\text{service}}_{s,p_s}$ follows the exponential distribution whose mean value equals $1/\lambda_2$ (day).
- The occurrence probability $\mathbb{P}_s$ equals $1/|S|$.
- The setup time t^{setup} equals three minutes.

Table 1 Probability of occurrence of emergency sites

	1	2	3	4
A	0.10	0.10	0.02	0.20
B	0.03	0.07	0.08	0.02
C	0.11	0.20	0.03	0.04

In both the cases (i) and (ii), we set the values of some parameters in common as shown in Table 2 to generate scenarios for our numerical experiments. In the following, we

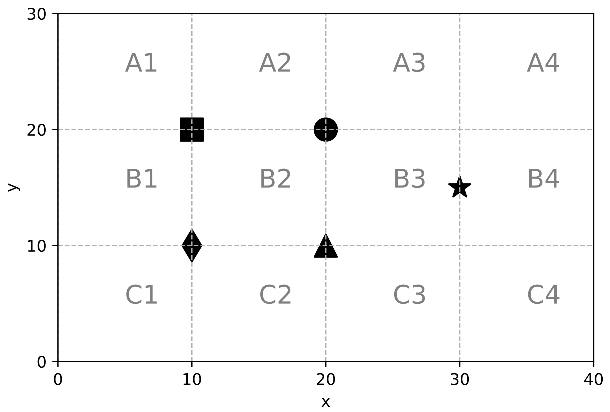

Fig. 3 (i) the location of five fire departments

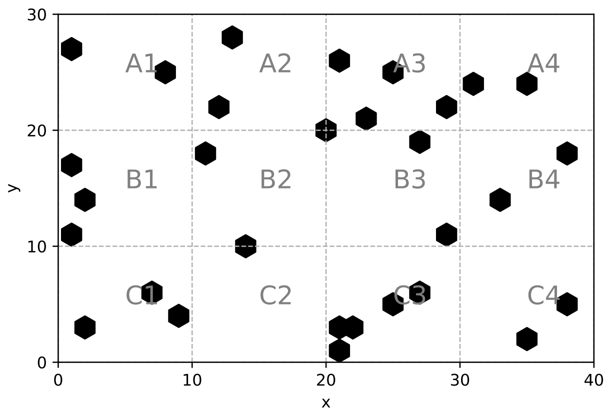

Fig. 4 (ii) the location of thirty fire departments

explain the detail of instance settings of both cases.

Table 2 Parameter values

Parameter	$\lvert S\rvert$	q	l	u	λ_2
Value	30	30	0	15	20

- Instance settings of (i)

  We set the λ_1=10, and generated t^{call}_{s,p_s}, $t^{\text{travel}}_{s,d,p_s}$, $t^{\text{service}}_{s,p_s}$ and t^{setup}. Moreover, we solve ($\heartsuit$) with six different settings (Table 3). Note that these six cases use the same scenarios to examine the effect of some parameters.

Table 3 Parameter settings for small-scale experiments

	$\lvert V\rvert$	$\lvert N\rvert$	t^{allow}
Case (i)-1	3	8	0
Case (i)-2	3	8	20
Case (i)-3	4	5	0
Case (i)-4	4	5	20
Case (i)-5	5	4	0
Case (i)-6	5	4	20

- Instance settings of (ii)

  We generated three cases as shown in Table 4 for large-scale experiments.

  First, we set the value of $|D|$ and λ_1 as shown in Table 4 and generated scenarios. In these cases, the fire departments are distributed randomly in the region. After that, we solved ($\heartsuit$) with the settings in Table 4.

Table 4 Parameter settings for large-scale experiments

	$\lvert D\rvert$	λ_1	$\lvert V\rvert$	$\lvert N\rvert$	t^{allow}
Case (ii)-1	20	40	20	4	0
Case (ii)-2	25	50	25	5	0
Case (ii)-3	30	60	30	6	0

4.2. Experimental results

In this subsection, we show the experiment results of small-scale and large-scale experiments. In this research, we used the computational environment as shown in Table 5 to conduct the numerical experiments.

Table 5 Computational environment

OS	Microsoft Windows 10 Pro
CPU	Intel(R) Core(TM) i7-6950X CPU @ 3.00GHz
Memory	64.0 GB
Solver	Gurobi Optimizer version 7.0.1

4.2.1. Results of (i): small-scale experiments

In this case, we set the limit of computation time as 1800 seconds. If the solver cannot find the optimal solution within the time limitation, we regard the best feasible solution as the optimal solution. The objective value of each case is shown in Table 6. Also, the number of allocated emergency vehicles at each fire department and the number of dispatching from each fire department are shown in Table 7. Moreover, Table 8 shows that the number of emergency sites which are counted by the delay time.

Table 6 Objective value

	Obj.val.	Gap
Case (i)-1	323.64	91.3%
Case (i)-2	178.66	100%
Case (i)-3	264.95	89.3%
Case (i)-4	116.80	100%
Case (i)-5	268.90	89.5%
Case (i)-6	78.58	100%

Table 7 Optimal location of EVs and Number of dispatching

	Location of EVs					Number of dispatching				
	♦	■	▲	●	⋆	♦	■	▲	●	⋆
Case (i)-1	1	0	0	1	1	98	0	0	95	90
Case (i)-2	0	0	1	2	0	0	0	102	181	0
Case (i)-3	1	1	1	0	1	58	74	68	0	83
Case (i)-4	1	0	1	2	0	70	0	72	141	0
Case (i)-5	0	2	2	0	1	0	105	104	0	74
Case (i)-6	1	1	0	2	1	67	56	0	89	71

Table 8 Number of emergency sites

	≤ 20 (min.)	≤ 30	≤ 40	≤ 50	> 50
Case (i)-1	52	134	209	250	33
Case (i)-2	37	103	171	231	52
Case (i)-3	66	177	248	272	11
Case (i)-4	50	140	210	266	17
Case (i)-5	57	169	245	274	9
Case (i)-6	70	181	267	279	4

4.2.2. Results of (ii): large-scale experiments

In order to examine the ability of our proposed model (♡) for large-scale problems, we tried to solve three problems of the case (ii). We set the computation time as 43200 seconds for this case, The computation results are shown in Table 9.

Table 9 Computation results for the case (ii)

	Feasiblity	Time	Gap
Case (ii)-1	Yes	4691sec	100%
Case (ii)-2	Yes	20772sec	100%
Case (ii)-3	None	None	100%

4.3. Discussion

In this section, we discuss the experimental results above.

First, let us pay attention to Case (i)-1, Case (i)-3 and Case (i)-5. We changed $|D|$, $|N|$ and set the t^{allow} equals 0. We can see that the best objective value is 264.95 in the Case (i)-3 (Table 6). It means that we are able to arrive at the emergency site in a relatively short time in this case. Simultaneously, we observed that the Case (i)-3 has a good performance because its number in each time period is dominant except > 50 (Table 8). Also, the solution of emergency vehicles are dispatched to each emergency site is illustrated in Fig. 5, Fig. 7 and Fig. 9 for observing intuitively. Furthermore, we can know that the optimal location of Case (i)-1 is {♦, ●, ⋆}, the optimal location of Case (i)-3 is {♦, ■, ▲, ⋆}. According to this, we discover that Case (i)-1 is not a partial set of Case (i)-3, so the solution of Case (i)-1 cannot be used (Table 7).

Second, let us pay attention to Case (i)-2, Case (i)-4 and Case (i)-6. We changed $|D|$, $|N|$ and set the t^{allow} equals 20. We can see that the objective value (expectation value of the aggregation of all delay times) decreases as the number of emergency vehicles increases (Table 6). It means that with five emergency vehicles at hand, the exceedance of twenty minutes is smaller. Simultaneously, we observed that Case (i)-6 has a good performance because its number in each time period is dominant (Table 8). Also, the solution of emergency vehicles are dispatched to each emergency site is illustrated in Fig. 6, Fig. 8 and Fig. 10 for observing intuitively. Furthermore, we can know that the optimal location of Case (i)-4 is {♦, ▲, ●, ●}, the optimal location of Case (i)-6 is {♦, ■, ●, ●, ⋆}. According to this, we discover that Case (i)-4 is not a partial set of Case (i)-6, so the solution of Case (i)-4 cannot be used (Table 7).

Third, let us pay attention to Case (i)-1 ∼ Case (i)-6, we analyze the optimal location of emergency vehicles (Table 7). We can see that the ⋆ fire department was used in common at Case (i)-1, Case (i)-3 and Case (i)-5, and the ● fire department was used in common at Case (i)-2, Case (i)-4 and Case (i)-6. The ⋆ and ● fire departments are relatively important in the situation $t^{\text{allow}} = 0$ and $t^{\text{allow}} = 20$. Then we focus on the number of emergency sites (Table 8). For example, if we compare the number of the response time is less than or equal to 20. We can observe that the Case (i)-6 has the best performance for all cases which equals 70. In contrast, the Case (i)-2 has a poor performance for all cases which equals 37. According to this, by increasing the number of emergency vehicles, we can arrive at the emergency sites as fast as possible. Moreover, we can see that if we allocate the emergency vehicles in more fire departments, we can get a high service quality. For example, we compare the Case (i)-1 and case (i)-2. Three fire departments are used in the Case (i)-1 and two fire departments used Case (i)-2. According to Table 8, Case (i)-1 has a good performance than Case (i)-2 in each time periods. Note that if compare the Case (i)-3 and Case (i)-4, the Case (i)-5 and Case (i)-6, relatively. We can get the same conclusion.

Finally, we analyze the Case (ii)-1 ∼ Case (ii)-3. According to the computation results (Table 9), we can know that if the scale of the problem as Case (ii)-1 and Case (ii)-2, our model (♡) can find the feasible solution by the solver. However, if the scale of the problem as Case (ii)-3, our model (♡) cannot solve it smoothly by the solver. Moreover, the Gap between the upper bound and the lower bound is too large. Therefore, we should modify the optimization model and design an algorithm for solving large-scale problems efficiently in the future.

5. CONCLUSIONS AND FUTURE WORK

This paper focused on the emergency vehicle allocation problem with consideration of integration dispatching. The proposed model aims to minimize the delay time in order to arrive at the emergency site to rescue as fast as possible. In order to validate the proposed model and demonstrate the performance, a set of test instances was generated randomly

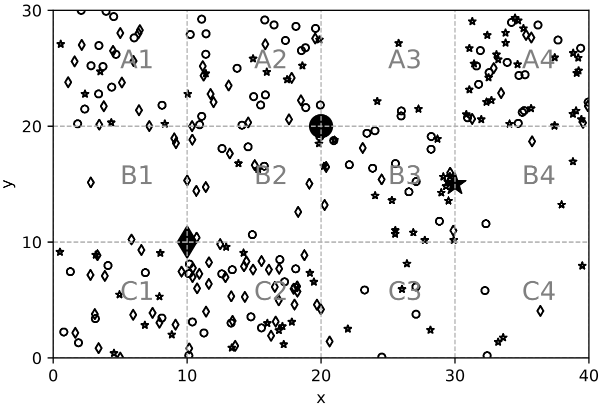

Fig. 5 Dispatching results of Case (i)-1

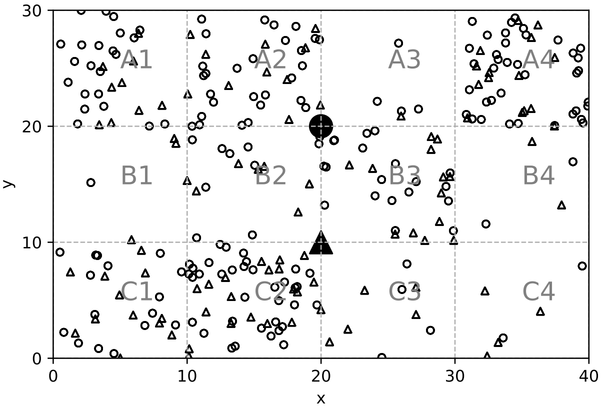

Fig. 6 Dispatching results of Case (i)-2

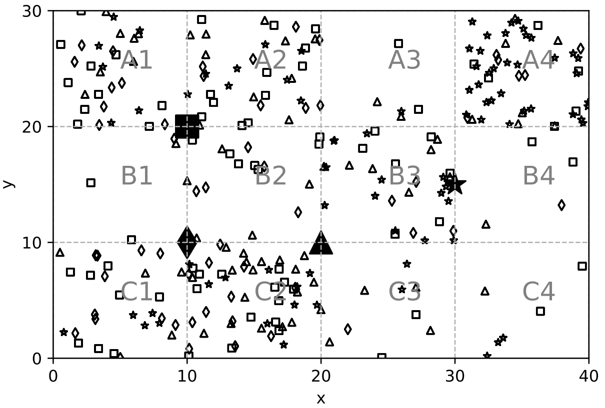

Fig. 7 Dispatching results of Case (i)-3

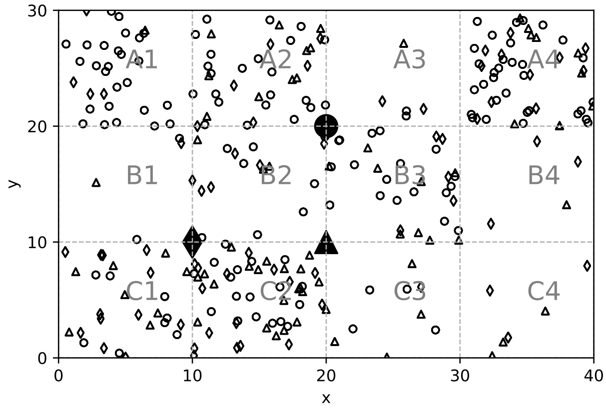

Fig. 8 Dispatching results of Case (i)-4

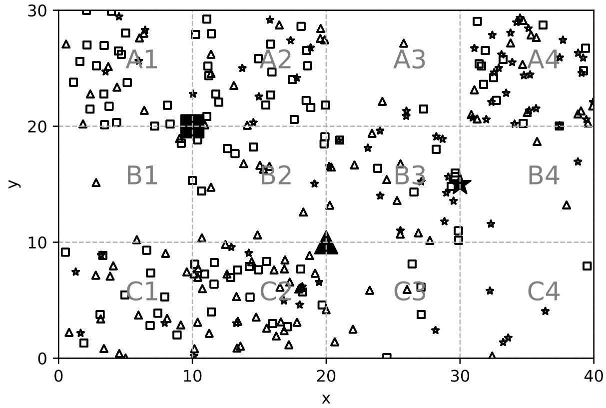

Fig. 9 Dispatching results of Case (i)-5

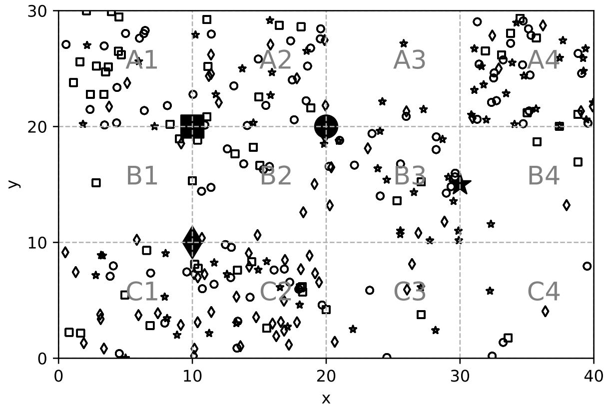

Fig. 10 Dispatching results of Case (i)-6

and solved by using the Gurobi Optimizer version 7.0.1. We know the model action by small-scale cases (Case (i)-1 ∼ Case (i)-6) by changing some parameters. However, the call-in time and service time play an important role in the emergency vehicle location problem, we should change them for each part and conduct the numerical experiments in the future. Next, we expanded the scale of problems (Case (ii)-1 ∼ Case (ii)-3). The computation results show the efficacy of the proposed model. However, if the problem scale exceeds the Case (ii)-2 like Case (ii)-3, it is hard to be solved. therefore, we should modify the optimization model and design an algorithm to solve the large-scale problem. Moreover, we also need to use real data to validate the proposed model in the future.

References

Fire and Disaster Management Agency, https://www.fdma.go.jp/neuter/about/pdf/en/2015/all.pdf, (2019/01/22 confirmed).

Fire and Disaster Management Agency, http://www.fdma.go.jp/html/hakusho/h29/h29/index.html, (2019/01/16 confirmed).

Xiao-Xia, Rong., Yi, Lu., Rui-Rui, Yin. and Jiang-Hua, Zhang (2013). A Robust Optimization Approach to Emergency Vehicle Scheduling. *Mathematical Problems in Engineering* **2013** https://doi.org/10.1155/2013/848312.

Lei, C., Lin, W.H. and Miao, L. (2015). A stochastic emergency vehicle redeployment model for an effective response to traffic incidents. *IEEE Transactions on Intelligent Transportation Systems* **16**(2): 898–909.

Saeed, Firooze1., Majid, Rafiee., Seyed, Mohammad, Zenouzzadeh. (2018). An Optimization Model for Emergency Vehicle Location and Relocation with Consideration of Unavailability Time. *SCIENTIA IRANICA* **25**(2): 3685–3699.