AN OPTIMIZATION MODEL FOR EMERGENCY VEHICLE LOCATION WITH CONSIDERATION OF DISPATCHING BOARD AREA

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Abstract. This paper aims to develop location opitimization and dispatching optimization techniques for emergency vehicles. In this research, Firstly, we seek to propose a optimization model for minimizing the total response time of vechicles to find the optimal emergency vehicles location solution. Then, we generate the some scenarios to consider the uncertainty conditions and do numerical experiment. The numerical experiment results show that the proposed model achieves a good performance and worked well.

Keywords. emergency vechicle location and dispatching, optimization, uncertainty, broad area.

1. INTRODUCTION

The emergency services ensure public safety and health by addressing different emergency incidents such as earthquake, typhoon, fire, traffic accident without taking a rest 24 hours, 365 days.

The emergency demand for emergency services is covered by the emergency vehicles (EVs) located at fixed points, therefore, the location of EVs is important in service quality level. The main challenge in emergency services is to minimize the time it takes to respond to the demands (the time between emergency call receipt and EVs arrival to the demand point) (Saeed et al., 2018). Moreover, as the uncertainty commonly exists in the real world. The unpredictability of the time and place of emergency demand is also the main issue in the EV location (Xiao-Xia et al., 2013) (Lei et al., 2015) . Therefore, we should consider the uncertainty when we seek to make a decision for the EV location as follows:

- The uncertainty of call-in time and the demand point location
- The uncertainty of the emergency vehicle travel time
- The uncertainty of service time at demand point

Besides, the depopulation of rual areas advanced as youths flowed into urban areas from the period of high economic growth to the stage of bubble economy in Japan. There are elderly people staying in the sparsely populated suburbs. In recent years, the EVs tend to lengthen the average response time to the demand point because of the lack of emergency stations and EVs in the depopulation of rual areas. On the other hand, if we can locate more EVs in more emergency stations, EVs can shorten the travel time to the demand point, but at the same time cost must also be considered. According to the Japan Fire and Disaster Management Agency Heisei 29th edition fire fighting white paper (FDMA, 2018) pointed out that it is necessary to promote cooperation between EVs in order to secure and enrich the necessary firefighting power even in areas where it is difficult to immediately promote cooperation in board area. Accordingly, it is necessary to balance these two aspects and develop a more effective EV location planning.

In this research, we developed an optimization model for finding the optimal EVs location solution with consideration of dispatching board area and consiered the uncertainty conditons by numerical experiment.

In Section 2, the process of emergency response system is described in detail. Then, a mathematical model will be proposed in Section 3. In order to verify the performance of the proposed model, a simple numerical example is presented and solved in Section 4. In order to apply it to real world, a case will be in Section **??**. Finally, we draw a conclusion in Section **??**.

2. THE PROCESS OF EMREGENCY RESPONSE SYSTEM

The process of emergency response system usually covers a sequence of activities are shown as follows:

1. The call (demand) comes to the system when the incident detection happened.
2. After call screening the dispatcher evaluates the system status and determines the appropriate EV to dipatch.
3. Upon EV arrives at demand point and starts doing service.
4. After completing service at demand point, EV may be moves to the next demand point if there has another emergency call or returns back emergency station to await another call.

In this research, we assumed that once the service is completed at demand point, the EV must return back emergency station as shown in Fig. 1.

In addition, we will introduce the time and time periods about an emergency vehicle when it is dispatched to a demand point as showed in Fig. 2.

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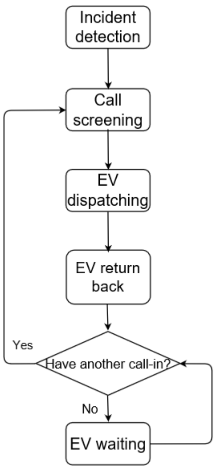

Fig. 1 The process of emergency response system

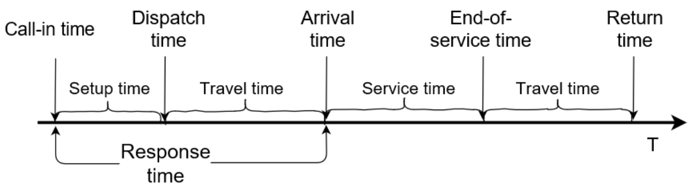

Fig. 2 The time and time periods in an emergency vehicle dispathcing

- The time period between call-in time and dispatch time is called setup time.
- The time period between dispatch time and arrival time is called travel time.
- The time period between call-in time and arrival time is called response time.
- The time period between arrival time and end-of-time is called service time.
- The time period between end-of-time and return time is called travel time.

3. MATHEMATICAL FORMULATION

In this section, we formulate the optimization model to find the optimal emergency vehicles location solution.

3.1. Notation

We use the following notations to describe our proposed model.

- Sets
  - I : the set of emergency stations indexed by $i \in \{1, 2, \ldots, \alpha\}$ (α: the number of emergency sations),
  - J : the set of demand points indexed by $j \in \{1, 2, \ldots, \beta\}$ (β: the number of demand points),
  - K : the set of emergency vehicles indexed by $k \in \{1, 2, \ldots, \gamma\}$ (γ: the number of emergency vehicles),
  - N : the set of the number of dispatches indexed by $n \in \{1, 2, \ldots, n_{\max}\}$ ($n_{\max}$: the maximum number of dispatches of each emergency vehicle),
  - A : the set of scenarios indexed by $a \in \{1, 2, \ldots, \delta\}$ (δ: the number of scenarios).
- Parameters
  - $u_{a,j}$: the call-in time of a demand point j under a scenario a.
  - $t_{a,i,j}$: the travel time between an emergency station i and a demand point j under a scenario a.
  - $s_{a,j}$: the service time of an emergency vehicle k at a demand point j under a scenario a.
  - M_1/M_2 : sufficiently large constant number.
  - e : the setup time.
  - P_a : the occurrence probability of a scenario a.
- Decision variables
  - $$x_{a,j,n,k} = \begin{cases} 1, & \text{an emergency vehicle } k \text{ is dispatched to a demand point } j \text{ with } n\text{-th dispatch under a scenario } a, \\ 0, & \text{otherwise,} \end{cases}$$
  - $$y_{i,k} = \begin{cases} 1, & \text{an emergency vehicle } k \text{ is assigned to an emergency satation } i, \\ 0, & \text{otherwise,} \end{cases}$$
  - $h_{a,n,k}$: the dispatch time of an emergency vehicle k with the n-th dispatch under a scenario a.
  - $l_{a,n,k}$: the travel time of an emergency vehicle k with the n-th dispatch under a scenario a.
  - $v_{a,n,k}$: the arrival time of an emergency vehicle k with the n-th dispatch under a scenario a.
  - $z_{a,n,k}$: the end-of-service time of an emergency vehicle k with the n-th dispatch under a scenario a.
  - $w_{a,n,k}$: the return time of an emergency vehicle k in with the n-th dispatch under a scenario a.
  - $p_{a,j,n,k}$: the response time of an emergency vehicle k for a demand point j with the n-th dispatch under a scenario a.

3.2. Constraints

The constraints about the affiliation and dispathcing number of vehicles

$$\sum_{i \in I} y_{i,k} = 1 \quad (k \in K) \tag{1}$$

$$\sum_{n \in N, k \in K} x_{a,j,n,k} = 1 \quad (a \in A, j \in J) \tag{2}$$

$$\sum_{j \in J} x_{a,j,n,k} \leq 1 \quad (a \in A, n \in N, k \in K) \tag{3}$$

$$\sum_{n \in N} x_{a,j,n,k} \leq 1 \quad (a \in A, j \in J, k \in K) \tag{4}$$

Constraint (1) ensures the emergency vehicles are assigned to the emergency station. Constraint (2) ensures the emergency vehicle is dispatched to a demand point. Constraint

(3) ensures each emergency vehicle can only be dispatched at most one time in each dispatching. Constraint (4) ensures each emergency vehicle can only be dispatched at most one time in every dispatching for each demand point.

The constraints about emergency vehicles dispatching and return

$$c_{a,n,k} \geq \sum_{j\in J} x_{a,j,n,k} u_{a,j} + e \quad (a \in A, n \in N, k \in K) \quad (5)$$

$$l_{a,n,k} \geq t_{a,i,j} - (1 - y_{i,k})M_1 - (1 - x_{a,j,n,k})M_1 \\ (a \in A, i \in I, j \in J, n \in N, k \in K) \quad (6)$$

$$v_{a,n,k} \geq h_{a,n,k} + l_{a,n,k} \quad (a \in A, n \in N, k \in K) \quad (7)$$

$$z_{a,n,k} = v_{a,n,k} + \sum_{a\in A, j\in J} x_{a,j,n,k} s_{a,j} \\ (a \in A, n \in N, k \in K) \quad (8)$$

$$w_{a,n,k} = z_{a,n,k} + l_{a,n,k} \quad (a \in A, n \in N, k \in K) \quad (9)$$

$$w_{a,n-1,k} + e \leq h_{a,n,k} \quad (a \in A, n \in N, k \in K : n \geq 2) \quad (10)$$

Constraint (5) ensures the setup time when each emergency vehicle will be dispatched. if an emergency vehicle k is dispatched to demand point j with the n-th dispatch under scenario a ($x_{a,j,n,k} = 1$) and this emergency vehicle k is assigned to an emergency station i ($y_{i,k} = 1$), Constraint (6) is $\bar{t}_{a,n,k} \geq t_{a,i,j}$ and $\bar{t}_{a,n,k}$ shows the travel time between the emergency station and the demand point. Constraint (7) shows the arrival time of an emergency vehicle at a demand point. Constraint (8) shows the end-of-service time of an emergency vehicle at a demand point. Constraint (9) shows the return time of an emergency vehicle. Constraint (10) shows the relationship between the return time of an emergency vehicle and the next dispatch time of this emergency vehicle.

The constraints about time variables

$$v_{a,n,k} \geq h_{a,n,k} \quad (a \in A, n \in N, k \in K) \quad (11)$$

$$z_{a,n,k} \geq v_{a,n,k} \quad (a \in A, n \in N, k \in K) \quad (12)$$

$$w_{a,n,k} \geq z_{a,n,k} \quad (a \in A, n \in N, k \in K) \quad (13)$$

$$p_{a,j,n,k} \geq v_{a,n,k} - u_{a,j} - (1 - x_{a,j,n,k})M_2 \\ (a \in A, j \in J, n \in N, k \in K) \quad (14)$$

Constraints (11),(12),(13) show the order of time variables. if an emergency vehicle k is dispatched to demand point j with the n-th dispatch under scenario a ($x_{a,j,n,k} = 1$), constraint (14) is $p_{a,j,n,k} \geq v_{a,n,k} - u_{a,j}$ and $p_{a,j,n,k}$ shows the response time of each emergency vehicle.

The constraints about nonnegative variables

$$l_{a,n,k} \geq 0 \quad (a \in A, n \in N, k \in K) \quad (15)$$

$$p_{a,j,n,k} \geq 0 \quad (a \in A, j \in J, n \in N, k \in K) \quad (16)$$

Constraints (15),(16) ensure the continuous variables nonnegative.

3.3. Objective function

$$f = \sum_{a\in A} \left(P_a \cdot \sum_{j\in J, n\in N, k\in K} p_{a,j,n,k} \right)$$

3.4. Mathematical model

The mathematical model is presented as follows:

$$(\star) \left| \begin{array}{ll} \text{minimize} & f \\ \text{subject to} & (1) \sim (16) \end{array} \right.$$

4. NUMERICAL EXPERIMENT

In this section, we will solve some numerical instances generated by random in order to demonstrate the performance and validate the proposed model.

4.1. Parameter generation

We will generate a $L \times W$ rectangular region and the candidate location of emergency stations will be fixed on it as shown in Fig. 3.

The assumption of each parameter is shown as follows:

- The number of emergency demands follow a possion distribution function defined by $P(\lambda 1)$ and the location of emergency demands follow a uniform distribution $U[L, W]$.
- The Call-in time $u_{a,j}$ follows a exponential distribution function defined by $P(\lambda 1)$ during one day.
- The distances $d_{i,j}$ between emergency stations and emergency demands are measured in the Euclidean sense and the emergency vehicle average speed is V. Besides, we will consider the uncertainty of travel time. Hence, we define a parameter $r_{i,j}$ follows a uniform distribution function defined by $U\left[\underline{l}, \overline{u}\right]$ that means the time besides $d_{i,j} \div V$. Accordingly, we defined $t_{a,i,j} = \{d_{i,j} \div V + r_{i,j}\}$
- The service time $s_{a,j}$ follows a exponential distribution function defined by $P(\lambda 2)$.

Table 1 Parameter values in parameter generation

Param	L	W	V	$\underline{l}$	$\overline{u}$	$\lambda 2$
Value	40	40	30	0	15	20

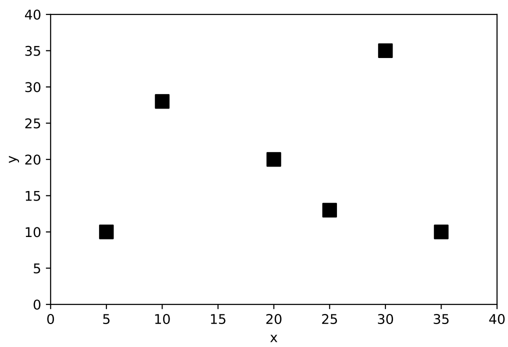

Fig. 3 The candidate location of emergency stations

4.2. Experimental outline

Table 2 Description of the instance

	I	K	$\lambda 1$	N	A
Case 1	10	50	20	8	30
Case 2	10	10	20	4	30
Case 3	10	15	20	3	30
Case 4	20	10	40	8	30
Case 5	20	20	40	4	30
Case 6	20	30	40	3	30

4.3. Computation results

We solve the mathematical model ($\star$) using the Gurobi Optimizer version 7.0.1 as the mixed-integer program solver and the computation enviroment as shown in Table. 3.

Table 3 Computation enviroment

OS	Microsoft Windows 10 Home
CPU	Intel(R) Core(TM) i7-6600U CPU
Memory	16.0GB
Solver	Gurobi Optimizer version 7.0.1

Table 4 Computation results

Case number	Objective value	Gap	Case 3
Case 1	40	40	12
Case 2	40	40	12
Case 3	40	40	12
Case 4	40	40	12
Case 5	40	40	12
Case 6	40	40	12

4.4. Responding to large-scale problem cases

In order to know the limits of the mathematical model ($\star$) in the realistic time, we conducted an experiment to expand the scale of the problem. We set the computation time is 43200sec, if the solver can't find the feasiable solution within 43200sec, we called it no feasiable solution.

$$gap = \frac{(upperbound - lowerbound) \times 100}{upperbound}$$

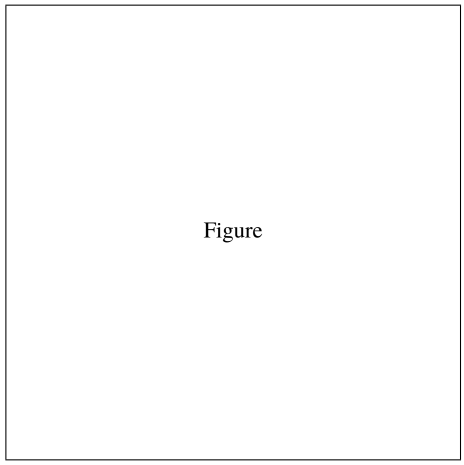

Fig. 4 Title of figure, left justified, subsequent text indented as in this example

Table 5 Large-scale problems computation results

I	K	$\lambda 1$	N	A	feasiable solution	Gap
20	20	40	4	30	None	100
20	30	60	4	30	None	100
20	40	80	4	30	None	100

5. SENSITIVITY ANALYSIS

6. CONCLUSIONS AND FUTURE WORK

Number the figures and tables and give them short informative captions. The style of captions should obey the descriptions in the captions of Fig. 4 and Table 6. Use "Figure" at the beginning of a sentence and "Fig." otherwise, e.g., "Figure 4 shows. . . "; ". . . is depicted in Fig. 4."

Place figures and tables at the top or bottom of a column wherever possible, as close as possible to the first references to them in the paper; never place all of them at the end of the paper. Do not use colored photographs or figures (they will be printed in black and white).

Equations are centered and numbered consecutively, from 1 upwards, e.g., equations (17) and (18) that are defined by

$$f(x) = 3x^2 + 2x \tag{17}$$

and

$$g(x) = 5x + 3. \tag{18}$$

Table 6 Title of table, left justified, subsequent text indented

1234	Normal	OK
3323	Normal	OK
342	Normal	OK
234	Normal	OK

7. REFERENCES

Citing publications When a publication is referred to in the text, enclose the authors' names and the year of publication within round brackets, e.g., (Kyoto and Shiga, 1994). For one author, use author's surname and the year (Osaka, 1996). For two authors, give both names and the year (Kyoto and Shiga, 1994). For three or more authors, use the first author, plus "et al.," and the year (**?**). When giving a list of reference, separate them using semicolons (Kobe et al., 1995b; Kyoto and Shiga, 1994; Osaka, 1995). (With LaTeX, this is realized just by typing, e.g., "`\cite{kobe95b,kyoto,osaka95}`," where kobe95b, kyoto, osaka95 are labels of papers.)

Put only the year in brackets when referring to the author(s) of referenced publication as noun (e.g., "This work was first developed by Kyoto and Shiga (1994), and later expanded by Kobe et al. (1995b), who demonstrated that. . . "). (With LaTeX, use for this purpose `\citeasnoun` instead of `\cite`, e.g., typing "`\citeasnoun{kobe95b}`" results in "Kobe et al. (1995b)", while "`\cite{kobe95b}`" results in "(Kobe et al., 1995b)," where kobe95b is the label of the paper.)

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8. MS WORD TEMPLATE FILE

We also offer a Word template file to authors for preparing a paper for the International Symposium on Scheduling. You can get the file from the symposium web site.

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