AN OPTIMIZATION MODEL FOR EMERGENCY VEHICLE LOCATION WITH CONSIDERATION OF INTEGRATION DISPATCHING

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Abstract. This paper aims to develop location opitimization and dispatching optimization techniques for emergency vehicles. In this research, Firstly, we seek to propose a optimization model to find the optimal location of emergency vehicles. Then, we generate the some scenarios to consider the uncertainty conditions and do numerical experiment. The numerical experiment results show that the proposed model achieves a good performance and worked well.

Keywords. emergency vechicle location and dispatching, optimization, uncertainty, integration dispatching.

1. INTRODUCTION

Utilizing the avaliable facilities and human resoures, fire service organizations shall protect the lives, physical being and property of public from fire and take precautions against disasters such as storms, floods fires and earthquakes, while mitigating the damage of these disaters. Fire service organizations are also responsible for the appropriate transport of persons who have sustained injuries due to a disaster (FDMA, 2015). When the Large-scale disaster or accident, serious accident and military attack happend, emergency vehicles belonging to the fire departments will rush to the emergency demand as soon as possible for rescue. However, In recent years, the number of emergency services for emergency call is increasing. In particular, the emergency vehicles tend to lengthen the average response time to the site because of the lack of fire departments and emergency vehicles in the suburbs. On the other hand, if we can locate more emergency vehicles in more fire departments, emergency vehicles can shorten the traveling time to the site, but at the same time location cost must be considered. Therefore, according to the Japan Fire and Disaster Management Agency Heisei 29th edition fire fighting white paper (FDMA, 2018) pointed out that consider large-scale distaers such as the Great East Japan Earthquake, higher future disater risks, and decrease in the population of Japan, we must enhance the structure of fire departments by expanding their jurisdictions. Accordingly, it is necessary to balance these two aspects and develop a more effective emergency vehicle location planning.

In the emergency vehicle location problem, the main objective to minimize the time it takes to respond to the sites (the traveling time between emergency call receipt and emergency vehicles arrival to the site) (Saeed et al., 2018). Because the site for emergency services is covered by the emergency vehicles located at fixed points, therefore, the location of emergency vehicles is important in service quality level. Moreover, as the uncertainty commonly exists in the real world. The unpredictability of the time and the location of emergency incidents are also the main issue in the emergency vehicle location (Xiao-Xia et al., 2013) (Lei et al., 2015) . Therefore, we should consider the uncertainty for the emergency vehicle location problem as follows:

- The uncertainty of call-in time and the site location
- The uncertainty of the emergency vehicle traveling time
- The uncertainty of service time at site

Based on the above, in this research, we developed an optimization model to find the optimal location of emergency vehicles with consideration of integration dispatching and consiered the uncertainty conditons by numerical experiment.

In Section 2, the process of emergency response system and the time and time periods in emergency vehicle dispathcing are described in detail. Next, the mathematical model will be proposed in Section 3. Then, in order to verify the performance of the proposed model, we generated some numerical instances are presented and solved in Section 4. Finally, we draw a conclusion in Section 5.

2. THE PROCESS OF EMREGENCY RESPONSE SYSTEM

The process of emergency response system usually covers a sequence of activities are shown as follows:

1. The emergency call comes to the system when the incident detection happened.
2. After call screening the dispatcher evaluates the system status and determines the appropriate emergency vehicle (EV) to dipatch.
3. Upon EV arrives at site and starts doing service.
4. After completing service at site, EV may be moves to the next site if there has another emergency call or returns back fire departments to await another call.

In this research, we assumed that once the service is completed at site, the EV must return back fire departments as shown in Fig. 1.

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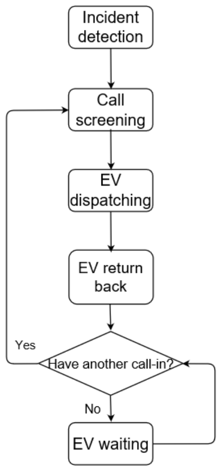

Fig. 1 The process of emergency response system

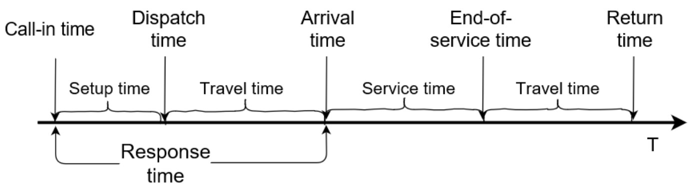

Fig. 2 The time and time periods in an EV dispathcing

In addition, we will introduce the time and time periods about an EV when it is dispatched to a site as showed in Fig. 2.

- The time period between call-in time and dispatch time is called setup time.
- The time period between dispatch time and arrival time is called travel time.
- The time period between call-in time and arrival time is called response time.
- The time period between arrival time and end-of-time is called service time.
- The time period between end-of-service time and return time is called travel time.

3. MATHEMATICAL FORMULATION

In this section, we formulate the optimization model to find the optimal location of emergency vehicles .

3.1. Notation

We use the following notations to describe our proposed model.

- Sets
  - I : the set of fire departments indexed by $i \in \{1, 2, \ldots, \alpha\}$ (α: the number of emergency stations),
  - J : the set of sites indexed by $j \in \{1, 2, \ldots, \beta\}$ (β: the number of demand points),
  - K : the set of emergency vehicles indexed by $k \in \{1, 2, \ldots, \gamma\}$ (γ: the number of emergency vehicles),
  - N : the set of the numbers of dispatching indexed by $n \in \{1, 2, \ldots, n_{\max}\}$ ($n_{\max}$: the maximum number of dispatches of each emergency vehicle),
  - A : the set of scenarios indexed by $a \in \{1, 2, \ldots, \delta\}$ (δ: the number of scenarios).
- Parameters
  - $u_{a,j}$: the call-in time from a site j under a scenario a,
  - $t_{a,i,j}$: the traveling time between an fire department i and a site j under a scenario a,
  - $s_{a,j}$: the service time of an emergency vehicle k at a site j under a scenario a,
  - M_1/M_2 : sufficiently large constant number,
  - e : the setup time,
  - P_a : the occurrence probability of a scenario a.
- Decision variables
  - $$x_{a,j,n,k} = \begin{cases} 1, & \text{an emergency vehicle } k \text{ is dispatched to a site } j \text{ with the } n\text{-th dispatch under a scenario } a, \\ 0, & \text{otherwise,} \end{cases}$$
  - $$y_{i,k} = \begin{cases} 1, & \text{an emergency vehicle } k \text{ is assigned to an fire department } i, \\ 0, & \text{otherwise,} \end{cases}$$
  - $h_{a,n,k}$: the dispatch time of an emergency vehicle k with the n-th dispatch under a scenario a,
  - $l_{a,n,k}$: the traveling time of an emergency vehicle k with the n-th dispatch under a scenario a,
  - $v_{a,n,k}$: the arrival time of an emergency vehicle k with the n-th dispatch under a scenario a,
  - $z_{a,n,k}$: the end-of-service time of an emergency vehicle k with the n-th dispatch under a scenario a,
  - $w_{a,n,k}$: the return time of an emergency vehicle k in with the n-th dispatch under a scenario a,
  - $p_{a,j,n,k}$: the response time of an emergency vehicle k for a site j with the n-th dispatch under a scenario a,
  - $m_{a,j,n,k}$: the penalty time of the response time of an emergency vehicle k for a site j with the n-th dispatch under a scenario a exceeds 30 minutes.

3.2. Constraints

In this research, we built the following constraints:

Affiliation and dispatching rule of vehicles

$$\sum_{i \in I} y_{i,k} = 1 \quad (k \in K) \tag{1}$$

$$\sum_{n \in N, k \in K} x_{a,j,n,k} = 1 \quad (a \in A, j \in J) \tag{2}$$

$$\sum_{j \in J} x_{a,j,n,k} \leq 1 \quad (a \in A, n \in N, k \in K) \tag{3}$$

$$\sum_{n\in N} x_{a,j,n,k} \le 1 \quad (a \in A, j \in J, k \in K) \tag{4}$$

The constraint (1) ensures that the emergency vehicles are assigned to the fire department. The Constraint (2) ensures that an emergency vehicle must be dispatched to a site. The constraint (3) ensures that each emergency vehicle can only be dispatched at most one time in each dispatching. The constraint (4) ensures each emergency vehicle can only be dispatched at most one time in every dispatching for each site.

Dispatching emergency vehicles

$$c_{a,n,k} \ge \sum_{j\in J} x_{a,j,n,k} u_{a,j} + e \quad (a \in A, n \in N, k \in K) \tag{5}$$

$$l_{a,n,k} \ge t_{a,i,j} - (1 - y_{i,k})M_1 - (1 - x_{a,j,n,k})M_1 \quad (a \in A, i \in I, j \in J, n \in N, k \in K) \tag{6}$$

$$v_{a,n,k} \ge h_{a,n,k} + l_{a,n,k} \quad (a \in A, n \in N, k \in K) \tag{7}$$

$$z_{a,n,k} = v_{a,n,k} + \sum_{a\in A, j\in J} x_{a,j,n,k} s_{a,j} \quad (a \in A, n \in N, k \in K) \tag{8}$$

$$w_{a,n,k} = z_{a,n,k} + l_{a,n,k} \quad (a \in A, n \in N, k \in K) \tag{9}$$

$$w_{a,n-1,k} + e \le h_{a,n,k} \quad (a \in A, n \in N, k \in K : n \ge 2) \tag{10}$$

The constraint (5) ensures that the setup time will be need before dispatching. The constraint (6) is $l_{a,n,k} \ge t_{a,i,j}$ and $l_{a,n,k}$ shows the traveling time between the fire department and the site. The constraints (7) and (8) determines the arrival time and the end-of-service time of an emergency vehicle at a site, respectively. The constraint (9) shows the return time of an emergency vehicle. The constraint (10) shows the relationship between the return time and the next dispatching time of an emergency vehicle.

Posteriority of time variables

$$v_{a,n,k} \ge h_{a,n,k} \quad (a \in A, n \in N, k \in K) \tag{11}$$

$$z_{a,n,k} \ge v_{a,n,k} \quad (a \in A, n \in N, k \in K) \tag{12}$$

$$w_{a,n,k} \ge z_{a,n,k} \quad (a \in A, n \in N, k \in K) \tag{13}$$

$$p_{a,j,n,k} \ge v_{a,n,k} - u_{a,j} - (1 - x_{a,j,n,k})M_2 \quad (a \in A, j \in J, n \in N, k \in K) \tag{14}$$

$$m_{a,j,n,k} \ge v_{a,n,k} - u_{a,j} - 30 - (1 - x_{a,j,n,k})M_2 \quad (a \in A, j \in J, n \in N, k \in K) \tag{15}$$

The constraints (11),(12) and (13) show the order of time variables as shown in Fig. 2. The constraint (14) computes the response time of each emergency vehicle.The constraint (14) computes the penalty of response time for each emergency vehicle exceeds 30 minutes.

Nonnegativity of variables

$$l_{a,n,k} \ge 0 \quad (a \in A, n \in N, k \in K) \tag{16}$$

$$p_{a,j,n,k} \ge 0 \quad (a \in A, j \in J, n \in N, k \in K) \tag{17}$$

$$m_{a,j,n,k} \ge 0 \quad (a \in A, j \in J, n \in N, k \in K) \tag{18}$$

3.3. Objective function

$$f_1 = \sum_{a\in A} \left(P_a \cdot \sum_{j\in J, n\in N, k\in K} p_{a,j,n,k} \right)$$

$$f_2 = \sum_{a\in A} \left(P_a \cdot \sum_{j\in J, n\in N, k\in K} m_{a,j,n,k} \right)$$

3.4. Mathematical model

The mathematical models are presented as follows:

$$(\star) \left| \begin{array}{ll} \text{minimize} & f_1 \\ \text{subject to} & (1) \sim (14), (16), (17). \end{array} \right.$$

$$(\clubsuit) \left| \begin{array}{ll} \text{minimize} & f_2 \\ \text{subject to} & (1) \sim (13), (15), (16), (18). \end{array} \right.$$

$(\star)$ is used to find the optimal location of emergency vehicles which minimize the expectation value of the total response time for every emergency vehicle. $(\clubsuit)$ is used to find the optimal location of emergency vehicles which minimize the expectation value of the total penalty time when the response time exceeds 30 minutes for each emergency vehicle.

4. NUMERICAL EXPERIMENT

In this section, we will solve some numerical instances which are generated by random in order to demonstrate the performance and validate the proposed model.

4.1. Parameter generation

First, we will generate a $L \times W$ (Unit:kilometres) rectangular region and divide the region into 12 parts (A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4). Next, the candidate location of fire departments ($Rhombus$, $Square$, $Triangle$, $Circle$, $Pentagon$) will be fixed on it as shown in Fig. 3.

The assumption of each parameter is shown as follows:

- The number of sites follows a possion distribution and the mean value $\lambda 1$ and the probability of sites occurrence according to the proportion of population density in each region as showed in Table. 1. According to the probability of sites occurrence of each parts, we can generate the location of each sites and follows uniform distribustion.
- The Call-in time $u_{a,j}$ follows an exponential distribution and the mean value is $\frac{1}{\lambda 1}$ minute during one day.
- We set the distances $d_{i,j}$ between fire departments and

sites are measured in the Euclidean sense and the emergency vehicle average speed is V kilometer one hour. Besides, we will consider the uncertainty of traveling time. Hence, we define a parameter $r_{i,j}$ follows a uniform distribution function defined by $U\left[\underline{l}, \overline{u}\right]$ that means the time besides $d_{i,j} \div V$. Accordingly, we defined $t_{a,i,j}$ $=\{d_{i,j} \div V + r_{i,j}\}$ under each scenario a.

- The service time $s_{a,j}$ follows an exponential distribution and the mean value is $\frac{1}{\lambda 2}$ minute.

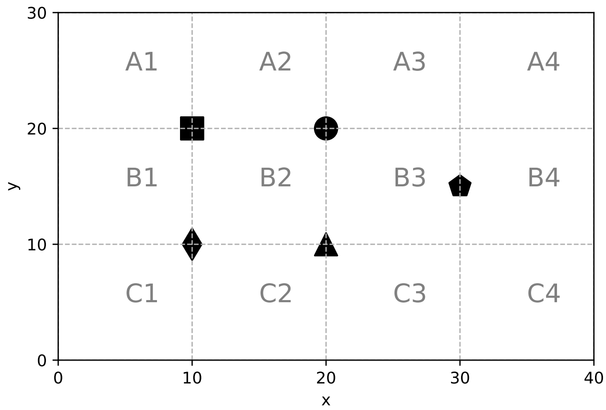

Fig. 3 The candidate location of fire departments

Table 1 The probability of sites occurrence in each region

	1	2	3	4
A	0.10	0.10	0.02	0.20
B	0.03	0.07	0.08	0.02
C	0.11	0.20	0.03	0.04

4.2. Experimental outline and results

In this subsection, we use the 4.1 and set the parameter values in Table. 2 to generate data for our numerical experiments and compare the performance for 6 different cases.

We use the mathematical model (★) and (♣) to slove instances by using the Gurobi Optimizer version 7.0.1 and the computation enviroment as shown in Table. 3.

Table 2 Parameter values in parameter generation

Param	L	W	V	$\underline{l}$	$\overline{u}$	$\lambda 2$
Value	40	30	30	0	15	20

Table 3 Computation enviroment

OS	Microsoft Windows 10 Home
CPU	Intel(R) Core(TM) i7-6600U CPU
Memory	16.0GB
Solver	Gurobi Optimizer version 7.0.1
Computaion time	3600sec

Table 4 The description and results of the instance

	$\lvert I\rvert$	$\lvert K\rvert$	$\lambda 1$	$n_{\max}$	$\lvert A\rvert$	Model	Obj	Gap
Case 1	5	3	10	8	30	(★)	947.74	100%
Case 2	5	3	10	8	30	(♣)	666.04	100%
Case 3	5	4	10	5	30	(★)	614.56	100%
Case 4	5	4	10	5	30	(♣)	304.13	100%
Case 5	5	5	10	4	30	(★)	426.81	100%
Case 6	5	5	10	4	30	(♣)	147.70	100%

when the candidate fire departments $|I|$=5, the mean value of sites $\lambda 1$=10, scenarios $|A|$=30, we change the number of emergency vehicles $|K|$ and the maximum number of dispatching of each emergency vehicle $n_{\max}$ to do the numerical experiment. The computation results are showed in Table. 4 and the optimal lcoation results are showed in Table. 5 for each case.

Firstly, we will analyze the Case 1, Case 3 and Case 5 solved by model (★).

- In the Case 1, the optimal lcoation of Case 1 is $\{Triangle, Circle, Circle\}$, the Objective value is 947.74 and the emergency vehicles belonging to fire stations are dispatched to the site as shown in Fig. 4
- In the Case 3, the optimal lcoation of Case 3 is $\{Rhombus, Circle, Circle, Circle\}$, the Objective value is 614.56 and the emergency vehicles belonging to fire stations are dispatched to the site as shown in Fig. 6.
- In the Case 5, the optimal lcoation of Case 5 is $\{Rhombus, Square, Triangle, Circle, Circle\}$, the Objective value is 426.81 and the emergency vehicles belonging to fire stations are dispatched to the site as shown in Fig. 8.

We can see that the Objective value (expectation value of response time) decreases as the number of emergency vehicles increases. Besides, the optimal lcoation of Case 1 is $\{Triangle, Circle, Circle\}$, the optimal lcoation of Case 3 is $\{Rhombus, Circle, Circle, Circle\}$. From this, Case 1 is not a partial set of Case 3, so the solution of Case 1 can not be used.

Secondly, we will analyze the Case 2, Case 4 and Case 6 solved by model (♣).

- In the Case 2, the optimal lcoation of Case 2 is $\{Triangle, Circle, Circle\}$, the Objective value is 666.04 and the emergency vehicles belonging to fire stations are dispatched to the site as shown in Fig. 5
- In the Case 4, the optimal lcoation of Case 1

is $\{Rhombus, Triangle, Circle, Circle\}$, the Objective value is 304.13 and the emergency vehicles belonging to fire stations are dispatched to the site as shown in Fig. 7.

- In the Case 6, the optimal lcoation of Case 1 is $\{Rhombus, Rhombus, Triangle, Circle, Circle\}$, the Objective value is 147.70 and the emergency vehicles belonging to fire stations are dispatched to the site as shown in Fig. 9.

Thirdly, we will compare Case 1 and Case 2, Case 3 and Case 4, Case 5 and Case 6 respectively. The optimal location of Case 1 is same as the optimal location of Case 2. As the number of vehicles increases, the optimal location of Case 3 and Case 5 are different with the optimal location of Case 4 and Case 6.

Finally, we analyze all cases. We can know that the *circle* fire department have the most emergency vehicles and the *pentagon* fire department have no emergency vehicle. Besides, when there is a bias of site occurrence probability in all regions, we should locate multiple emergency in one fire department. For example, the results show that locate two emergency vehicles at least in the *circle* fire departments. Because the population density is relatively high in A2 and A4 region.

Table 5 The optimal location results of each case

	Rhombus	*Square*	*Triangle*	*Circle*	*Pentagon*
Case 1	0	0	1	2	0
Case 2	0	0	1	2	0
Case 3	1	0	0	3	0
Case 4	1	0	1	2	0
Case 5	1	1	1	2	0
Case 6	2	0	1	2	0

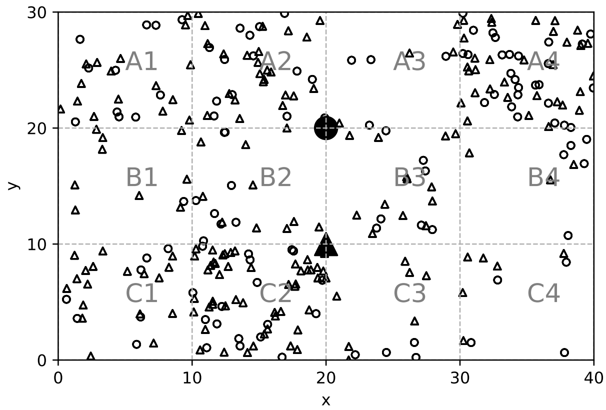

Fig. 4 The dispatching results of Case 1

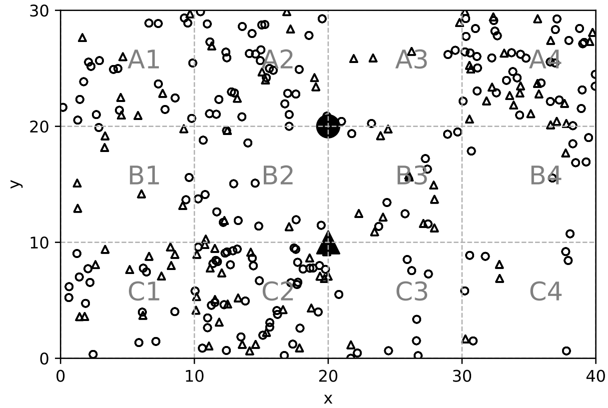

Fig. 5 The dispatching results of Case 2

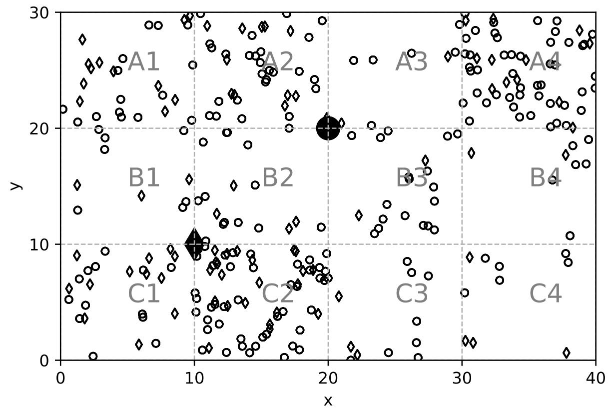

Fig. 6 The dispatching results of Case 3

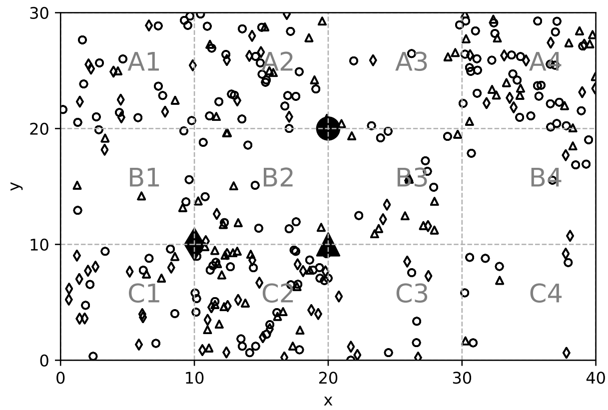

Fig. 7 The dispatching results of Case 4

4.3. Responding to large-scale problem cases

In order to know the limits of the mathematical model ($\star$) in the realistic time, we conducted an experiment to expand the scale of the problem. We set the computation time is 43200sec, if the solver can't find the feasible solution within 43200sec, we called it no feasiable solution.

The computation results are showed in Table. 6.

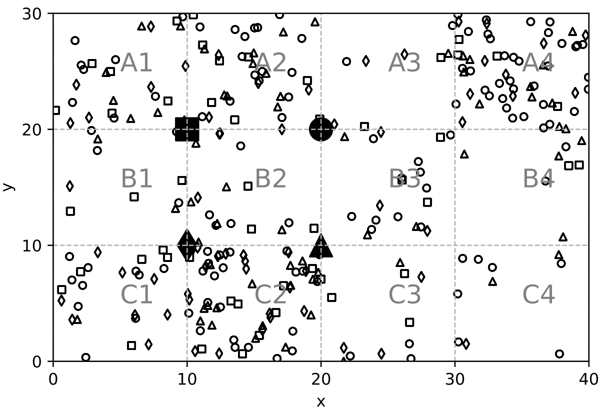

Fig. 8 The dispatching results of Case 5

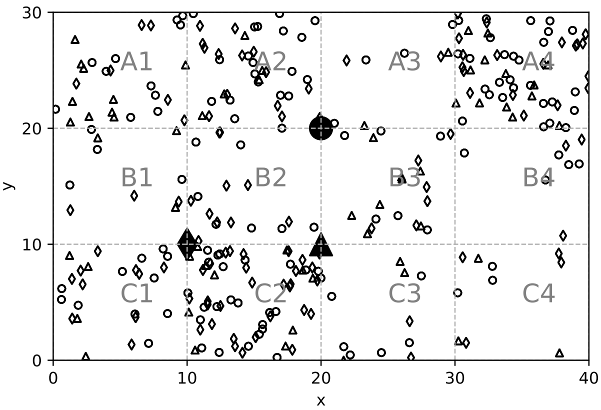

Fig. 9 The dispatching results of Case 6

- In the Case 7, the solver can find the feasible solution at 4691sec.
- In the Case 8
- In the Case 9

From the computation results, we can know that if the scale of problem as Case 7, our model ($\star$) can solve it. However, if the scale of problem as Case 8, Case 9, our model ($\star$) can't solve them smoothly, therefore , we should modify the optimization mdoel an design an algorithm for solving the large-scale problems effciently in the future.

Table 6 Large-scale problems computation results

	$\|I\|$	$\|K\|$	$\lambda 1$	$n_{\max}$	$\|A\|$	Fesaible solution	Gap
Case 7	20	20	40	4	30	Yes	100%
Case 8	30	30	60	4	30	None	100%
Case 9	40	40	80	4	30	None	100%

5. CONCLUSIONS AND FUTURE WORK

This paper focused on emergency vehicle location and dispatching problem with consideration of dispatching borad area. The proposed model aims to minimize the response time in order to arrive the emergenyc demands as fast as possible. For the purpose of model evalution, a set of test instances was generated to solve using the Gurobi Optimizer version 7.0.1 in 3 cases: Case 7, Case 8, Case 9. The computaiton results show the efficacy of the proposed model. However, if the problem scale over the Case 8 and Case 9, it is hard to solve the problems, therefore, we should design an algorithm to solve the large-scale problems in the future. Although many aspects of the real-world problem were comsidered in this mathematical model, still some issues need to be studied in the future.

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