

# AN OPTIMIZATION MODEL FOR EMERGENCY VEHICLE LOCATION WITH CONSIDERATION OF INTEGRATION DISPATCHING

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**Abstract.** This paper aims to develop location optimization and dispatching optimization techniques for emergency vehicles. In this research, Firstly, we seek to propose a optimization model to find the optimal location of emergency vehicles. Then, we generate the some scenarios to consider the uncertainty conditions and do numerical experiment. The numerical experiment results show that the proposed model achieves a good performance and worked well.

**Keywords.** emergency vehicle location and dispatching, optimization, uncertainty, integration dispatching.

## 1. INTRODUCTION

Utilizing the available facilities and human resources, fire service organizations shall protect the lives, physical being, and property of public from fire and take precautions against disasters such as storms, floods fires, and earthquakes, while mitigating the damage of these disasters. Fire service organizations are also responsible for the appropriate transport of persons who have sustained injuries due to a disaster (FDMA, 2015). When the Large-scale disaster or accident, serious accident and the military attack happened, emergency vehicles belonging to the fire departments will rush to the emergency demand as soon as possible for rescue. However, in recent years, the number of emergency services for an emergency call is increasing. In particular, the emergency vehicles tend to lengthen the average response time to the emergency site because of the lack of fire departments and emergency vehicles in the suburbs. On the other hand, if we can locate more emergency vehicles in more fire departments, emergency vehicles can shorten the traveling time to the emergency site, but at the same time location cost must be considered. Therefore, according to the Japan Fire and Disaster Management Agency Heisei 29th edition fire fighting white paper (FDMA, 2018) pointed out that consider Large-scale disasters such as the Great East Japan Earthquake, higher future disaster risks, and the decrease in the population of Japan, we must enhance the structure of fire departments by expanding their jurisdictions. Accordingly, it is necessary to balance these two aspects and develop a more effective emergency vehicle location planning.

In the emergency vehicle location problem, the main objective to minimize the time it takes to respond to the emergency sites (the traveling time between emergency call receipt and emergency vehicles arrived at the emergency site) (Saeed et al., 2018). Because the emergency site for emergency services is covered by the emergency vehicles

located at fixed points, therefore, the location of emergency vehicles is important in service quality level. Moreover, as uncertainty commonly exists in the real world. The unpredictability of the time and the location of emergency incidents are also the main issues in the emergency vehicle location (Xiao-Xia et al., 2013) (Lei et al., 2015). Therefore, we should consider the uncertainty for the emergency vehicle location problem as follows:

- The uncertainty of call-in time and the location of the emergency site
- The uncertainty of the emergency vehicle traveling time
- The uncertainty of service time at the emergency site

Based on the above, in this research, we developed an optimization model to find the optimal location of emergency vehicles with consideration of integration dispatching and considered the uncertainty conditions by numerical experiment.

In Section 2, the process of emergency response system and the times and time periods in emergency vehicle dispatching are described in detail. Next, the mathematical model will be proposed in Section 3. Then, in order to demonstrate the performance and validate the proposed model, we generated some numerical instances are presented and solved in Section 4. Finally, we draw a conclusion in Section 5.

## 2. PROCESS OF EMERGENCY RESPONSE SYSTEM

In this section, we introduce the process of the emergency response system. The process of the emergency response system usually covers the sequence of the following activities.

1. The system receives the emergency call when the incident happened.
2. After call screening, dispatchers evaluate the system status and determine the appropriate emergency vehicle (EV) to dispatch.
3. The dispatched EV arrives at the emergency site and

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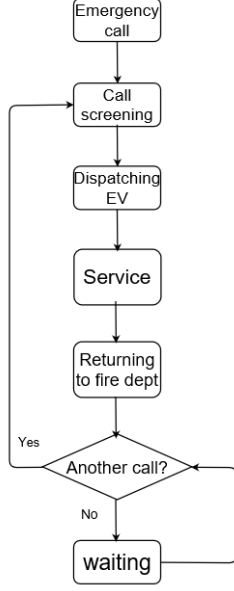
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starts service.

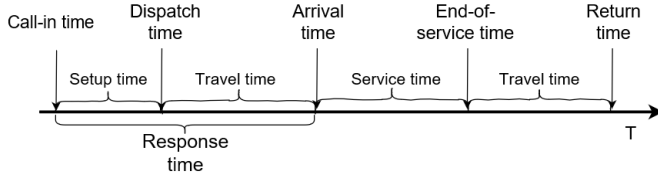
4. After completing service, the EV returns to the fire departments or goes to another emergency site.

In this research, we assume that dispatched EVs must return back to the fire departments when the service is completed (Fig. 1).



**Fig. 1** The process of emergency response system

In this research, we have the times and the time periods which explain this produce as shown in Fig. 2.



**Fig. 2** The times and time periods in an EV dispatching

### 3. MATHEMATICAL FORMULATION

In this section, we formulate the optimization model to find the optimal location of emergency vehicles.

#### 3.1. Notation

We use the following notations to describe our proposed model.

- Indices and Sets
  - $s \in S$  : the set of scenarios,
  - $d \in D$  : the set of fire departments,
  - $p_s \in P_s$  : the set of emergency sites under  $s$ ,

- $v \in V$  : the set of emergency vehicles,
- $n \in N$  : the set of the numbers of dispatching.

#### • Parameters

- $t_{s,p_s}^{\text{call}}$  : the call-in time from  $p_s$  under  $s$ ,
- $t_{s,d,p_s}^{\text{travel}}$  : the traveling time between  $d$  and  $p_s$  under  $s$ ,
- $t_{s,p_s}^{\text{service}}$  : the service time of  $v$  at  $p_s$  under  $s$ ,
- $t_{\text{setup}}$  : the setup time for dispatching,
- $t^{\text{thres}}$  : the threshold amount,
- $M_1/M_2$  : sufficiently large constant numbers,
- $\mathbb{P}_s$  : the occurrence probability of  $s$ .

#### • Decision variables

- $x_{s,p_s,n,v} = \begin{cases} 1, & v \text{ is dispatched to } p_s \text{ with the } n\text{-th} \\ & \text{dispatch under } s, \\ 0, & \text{otherwise,} \end{cases}$
- $y_{d,v} = \begin{cases} 1, & v \text{ is assigned to } d, \\ 0, & \text{otherwise,} \end{cases}$
- $z_{s,n,v}^{\text{dispatch}}$  : the dispatch time of  $v$  with the  $n$ -th dispatch under  $s$ ,
- $z_{s,n,v}^{\text{travel}}$  : the traveling time of  $v$  with the  $n$ -th dispatch under  $s$ ,
- $z_{s,n,v}^{\text{arrival}}$  : the arrival time of  $v$  with the  $n$ -th dispatch under  $s$ ,
- $z_{s,n,v}^{\text{end}}$  : the end-of-service time of  $v$  with the  $n$ -th dispatch under  $s$ ,
- $z_{s,n,v}^{\text{return}}$  : the return time of  $v$  with the  $n$ -th dispatch under  $s$ ,
- $z_{s,n,v}^{\text{response}}$  : the response time of  $v$  for  $p_s$  with the  $n$ -th dispatch under  $s$ .

### 3.2. Constraints

In this research, we built the following constraints.

#### Affiliation and dispatching rule of vehicles

$$\sum_{d \in D} y_{d,v} = 1 \quad (v \in V), \quad (1)$$

$$\sum_{n \in N, v \in V} x_{s,p_s,n,v} = 1 \quad (s \in S, p_s \in P_s), \quad (2)$$

$$\sum_{p_s \in P_s} x_{s,p_s,n,v} \leq 1 \quad (s \in S, n \in N, v \in V). \quad (3)$$

The constraint (1) ensures that the emergency vehicles are assigned to one fire department. The constraint (2) ensures that an emergency vehicle must be dispatched to an emergency site. The constraint (3) ensures that each emergency vehicle can only be dispatched to at most one emergency site in each dispatching.

#### Dispatching emergency vehicles

$$z_{s,n,v}^{\text{dispatch}} \geq \sum_{p_s \in P_s} x_{s,p_s,n,v} t_{s,p_s}^{\text{call}} + t_{\text{setup}} \quad (s \in S, n \in N, v \in V), \quad (4)$$

$$z_{s,n,v}^{\text{travel}} \geq t_{s,d,p_s}^{\text{travel}} - (1 - y_{d,v})M_1 - (1 - x_{s,p_s,n,v})M_1 \quad (s \in S, d \in D, p_s \in P_s, n \in N, v \in V), \quad (5)$$

$$z_{s,n,v}^{\text{arrival}} \geq z_{s,n,v}^{\text{dispatch}} + z_{s,n,v}^{\text{travel}} \quad (s \in S, n \in N, v \in V), \quad (6)$$

$$z_{s,n,v}^{\text{end}} = z_{s,n,v}^{\text{arrival}} + \sum_{s \in S, p_s \in P_s} x_{s,p_s,n,v} t_{s,p_s}^{\text{service}} \quad (s \in S, n \in N, v \in V), \quad (7)$$

$$z_{s,n,v}^{\text{return}} = z_{s,n,v}^{\text{end}} + z_{s,n,v}^{\text{travel}} \quad (s \in S, n \in N, v \in V), \quad (8)$$

$$z_{s,n-1,v}^{\text{return}} + t^{\text{setup}} \leq z_{s,n,v}^{\text{dispatch}} \quad (s \in S, n \in N, v \in V : n \geq 2). \quad (9)$$

The constraint (4) ensures that the setup time is necessary for dispatching. The constraint (5) becomes  $z_{s,n,v}^{\text{travel}} \geq t_{s,d,p_s}^{\text{travel}}$  when  $x_{s,p_s,n,v} = 1$  and  $y_{d,v} = 1$  holds, the  $z_{s,n,v}^{\text{travel}}$  shows the upper bound of traveling time between the fire department and the emergency site. The constraints (6) and (7) determine the arrival time and the end-of-service time of an emergency vehicle at a emergency site, respectively. The constraint (8) shows the return time of an emergency vehicle. The constraint (9) shows the relationship between the return time and the next dispatching time of an emergency vehicle.

#### Priority and posteriority of time variables

$$z_{s,n,v}^{\text{arrival}} \geq z_{s,n,v}^{\text{dispatch}} \quad (s \in S, n \in N, v \in V), \quad (10)$$

$$z_{s,n,v}^{\text{end}} \geq z_{s,n,v}^{\text{arrival}} \quad (s \in S, n \in N, v \in V), \quad (11)$$

$$z_{s,n,v}^{\text{return}} \geq z_{s,n,v}^{\text{end}} \quad (s \in S, n \in N, v \in V), \quad (12)$$

$$z_{s,p_s,n,v}^{\text{response}} \geq z_{s,n,v}^{\text{arrival}} - t_{s,p_s}^{\text{call}} - t^{\text{thres}} - (1 - x_{s,p_s,n,v})M_2 \quad (s \in S, p_s \in P_s, n \in N, v \in V). \quad (13)$$

The constraints (10), (11) and (12) show the order of time variables (Fig. 2). If  $t^{\text{thres}}$  greater than 0, the constraint (13) computes the penalty time of each emergency vehicle whose response time exceeds  $t^{\text{thres}}$ . If  $t^{\text{thres}}$  equals 0, the constraint (13) shows the penalty time equals the response time for each emergency vehicle.

#### Nonnegativity of variables

$$z_{s,n,v}^{\text{travel}} \geq 0 \quad (s \in S, n \in N, v \in V), \quad (14)$$

$$z_{s,p_s,n,v}^{\text{response}} \geq 0 \quad (s \in S, p_s \in P_s, n \in N, v \in V). \quad (15)$$

The constraints (14),(15) ensure the continuous variables nonnegative.

### 3.3. Objective function

$$f = \sum_{s \in S} \left( \mathbb{P}_s \cdot \sum_{p_s \in P_s, n \in N, v \in V} z_{s,p_s,n,v}^{\text{response}} \right)$$

### 3.4. Mathematical model

The mathematical model is presented as follows:

$$(\heartsuit) \begin{cases} \text{minimize} & f \\ \text{subject to} & (1) \sim (15). \end{cases}$$

( $\heartsuit$ ) is used to find the optimal location of emergency vehicles by minimizing the total penalty time for each emergency vehicle.

## 4. NUMERICAL EXPERIMENTS

In this section, we solve some numerical instances which are generated randomly. Through these numerical experiments, we confirm the validness and performance of the proposed model.

### 4.1. Parameter generation

First, we generated a 40 km  $\times$  30 km rectangular region and divided it into 12 parts (A1–C4). Also, we set fire departments in the region. In our experiments, we try to find the optimal solution for (i) the case of five fire departments (Fig. 3) and (ii) the case of 20–30 fire departments (Fig. 4).

Next, we set the value of parameters randomly. The assumptions of each parameter are shown as follows:

- The number of emergency sites in each scenario follows the Poisson distribution whose mean value equals  $\lambda_1$ .
- The number of emergency sites in each part is proportional to the probability of occurrence of emergency sites as shown in Table. 1.
- The location of emergency sites is distributed uniformly in each part.
- The interval of call-in times follows the exponential distribution whose mean value equals  $1/\lambda_1$  during one day.
- The distance  $d_{a,i,j}$  km between the fire department  $i$  and the emergency site  $j$  in each  $a$  is measured in the Euclidean norm.
- The traveling time of an emergency vehicle from a fire department  $i$  to an emergency site  $j$  in a scenario  $a$  is defined as

$$t_{a,i,j} = d_{a,i,j} / q + r_{a,i,j},$$

where  $q$  (unit: km/h) is the average speed of emergency vehicles and  $r_{a,i,j}$  (unit: minute) follows a uniform distribution on the interval  $[l, u]$  (unit: minute). Here, the term  $r_{a,i,j}$  is included to express a potential delay.

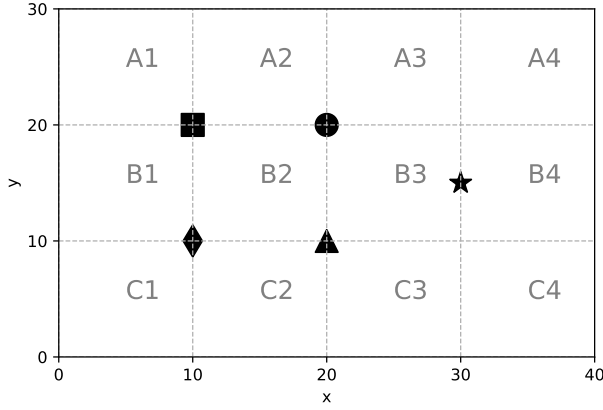
- The service time  $s_{a,j}$  follows the exponential distribution whose mean value equals  $1/\lambda_2$ .
- The occurrence probability  $P_a$  equals  $1/\delta$ .
- The setup time  $t^{\text{setup}}$  equals three minutes.

**Table 1** Probability of occurrence of emergency sites

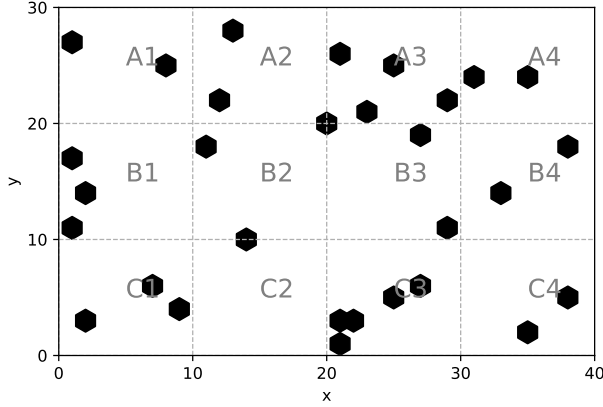
	1	2	3	4
A	0.10	0.10	0.02	0.20
B	0.03	0.07	0.08	0.02
C	0.11	0.20	0.03	0.04

### 4.2. Experimental outline and computational settings

In this subsection, we introduce the experimental outline and



**Fig. 3** (i) the location of five fire departments



**Fig. 4** (ii) the location of thirty fire departments

computational settings. In our experiments, we examine the performance of our model for the case (i) and (ii), which were explained above.

In both cases, we set the values of some parameters in common as shown in Table. 2 to generate data for our numerical experiments.

Table 2 Parameter values					
Parameter	$ S $	$q$	$l$	$u$	$\lambda_2$
Value	30	30	0	15	20

- Experiment cases of [Small]

We set the  $D=5$  and  $\lambda_1=10$  generated data and divided it into six cases by changing the  $|V|$ ,  $|N|$  and  $t^{\text{thres}}$  as shown in Table. 3 for small-scale experiments. Besides, the total number of emergency sites is 302.

**Table 3** Parameter settings for small-scale experiments

	$ V $	$ N $	$t^{\text{thres}}$
Case S-1	3	8	0
Case S-2	3	8	30
Case S-3	4	5	0
Case S-4	4	5	30
Case S-5	5	4	0
Case S-6	5	4	30

- Experiment cases of [Large]

We generated three cases as shown in Table. 4 for large-scale experiments.

**Table 4** Parameter settings for large-scale experiments

	$ D $	$ V $	$\lambda_1$	$t^{\text{thres}}$
Case L-1	20	20	40	4
Case L-2	25	25	50	5
Case L-3	30	30	60	6

In this research, we used the computational environment as shown in Table. 5 to conduct the numerical experiment.

**Table 5** Computational environment

OS	Microsoft Windows 10 Pro
CPU	Intel(R) Core(TM) i7-6950X CPU @ 3.00GHz
Memory	64.0 GB
Solver	Gurobi Optimizer version 7.0.1

### 4.3. Experimental results

In this subsection, we show the experiment results of small-scale experiments and large-scale experiments.

#### 4.3.1. Results of small-scale experiments

In this experiment, we use the model ( $\heartsuit$ ) to solve them and set the limit of computation time is 3600 seconds. If the solver can't find the optimal solution within the time limitation, we regard the best feasible solution as the optimal solution. The objective value of each case is shown in Table. 6. The optimal location results and the number of dispatching for each fire department are shown in Table. 7. The Table. 8 shows that the number of the response time within various time periods.

**Table 6** Objective value of each case

	Obj	Gap
Case S-1	947.74	100%
Case S-2	666.04	100%
Case S-3	614.56	100%
Case S-4	304.13	100%
Case S-5	426.81	100%
Case S-6	147.70	100%

**Table 7** Optimal location results and Number of dispatching for fire departments

	◆	■	▲	●	★	◆	■	▲	●	★
Case S-1	0	0	2	1	0	0	0	191	111	0
Case S-2	0	0	1	2	0	0	0	105	197	0
Case S-3	1	0	0	3	0	90	0	0	212	0
Case S-4	1	0	1	2	0	73	0	78	151	0
Case S-5	1	1	1	2	0	62	50	64	126	0
Case S-6	2	0	1	2	0	110	0	62	130	0

**Table 8** Number of the response time within various time periods

	$\leq 30$	$\leq 45$	$\leq 60$	$> 60$
Case S-1	58	138	183	119
Case S-2	51	137	181	121
Case S-3	85	175	222	80
Case S-4	95	198	230	72
Case S-5	122	219	259	43
Case S-6	114	263	267	35

#### 4.3.2. Results of large-scale experiments

In order to know the limit of the mathematical model (♥) in a realistic time, we set the computation time is 43200 seconds, if the solver can't find the feasible solution within 43200 seconds, we called it no feasible solution. The computation results are shown in Table. 9.

**Table 9** Large-scale problems computation results

	Feasibility	Time	Gap
Case L-1	Yes	4691sec	100%
Case L-2	Yes	20772sec	100%
Case L-3	None	None	100%

#### 4.4. Discussion

In this section, we discuss something based on the experimental results.

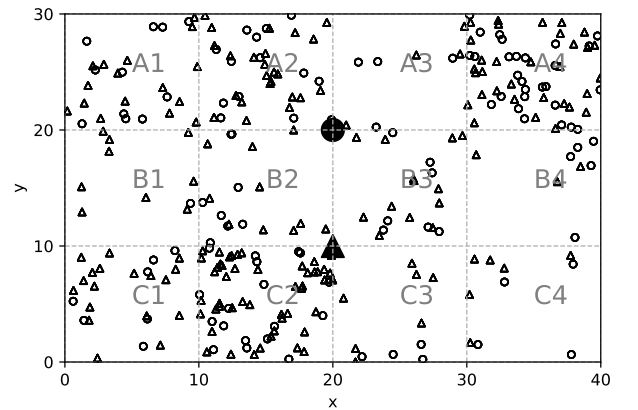
Firstly, in Case S-1, Case S-3 and Case S-5, we change the number of vehicles, the maximum number of dispatching of each emergency vehicle and set the  $t^{thres}$  equals 0. The emergency vehicle belonging to fire departments are dispatched to the emergency site as shown in Fig. 5, Fig. 7 and Fig. 9. In addition, we can see that the objective value (expectation value of response time) decreases as the number of emergency vehicles increases (Table. 6). It means that with five emergency vehicles at hand, we are able to arrive at the emergency site use little time. Moreover, we can see that the Case S-5 has a good performance in each time period (Table. 8). Besides, the optimal location of Case S-1 is  $\{\blacktriangle, \blacktriangle, \bullet\}$ , the optimal location of Case S-3 is  $\{\blacklozenge, \bullet, \bullet, \bullet\}$ . From this, Case S-1 is not a partial set of Case S-3, so the solution of Case S-1 can't be used.

Secondly, in Case S-2, Case S-4 and Case S-6, we change

the number of vehicles, the maximum number of dispatching of each emergency vehicle and set the  $t^{thres}$  equals 30. Besides, the emergency vehicle belonging to fire departments are dispatched to the emergency site as shown in Fig. 6, Fig. 8 and Fig. 10. In addition, we can see that the objective value (expectation value of the penalty time) decreases as the number of emergency vehicles increases (Table. 6). It means that with five emergency vehicles at hand, the penalty time of the response time exceeds 30 minutes is smaller. Moreover, we also can see that Case S-6 has a good performance in each time periods (Table. 8).

Thirdly, in Case S-1 ~ Case S-6, we analyze the optimal location results (Table. 7). We can know that the  $\bullet$  fire department was used for each case because the population density is relatively high in A2 and A4 region. Then, the  $\blacktriangle$  fire department was used five times because the population density is relatively high in C1 and C2 region. Next, the  $\blacklozenge$  fire department was used four times and the  $\blacksquare$  fire department was used one time. In addition, the  $\star$  fire department wasn't used one time for each case. Based on the above, if we will locate the emergency vehicles to the fire department, we may sort the importance of the fire department from the optimal location results  $\{\bullet, \blacktriangle, \blacklozenge, \blacksquare, \star\}$ .

Finally, we analyze the Case L-1 ~ Case L-3. From the computation results (Table. 9), we can know that if the scale of the problem as Case L-1 and Case L-2, our model (♥) can find the feasible solution by the solver. However, if the scale of the problem as Case L-3, our model (♥) can't solve them smoothly by the solver. Besides, the Gap between the upper bound and the lower bound is too large. Therefore, we should modify the optimization model and design an algorithm for solving large-scale problems efficiently in the future.



**Fig. 5** Dispatching results of Case S-1

## 5. CONCLUSIONS AND FUTURE WORK

This paper focused on the emergency vehicle location problem with consideration of integration dispatching. The pro-

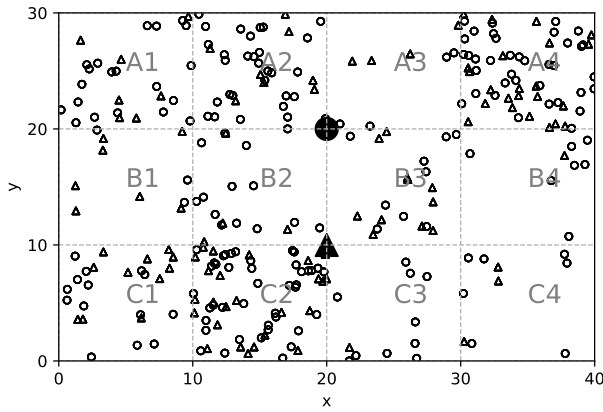


Fig. 6 Dispatching results of Case S-2

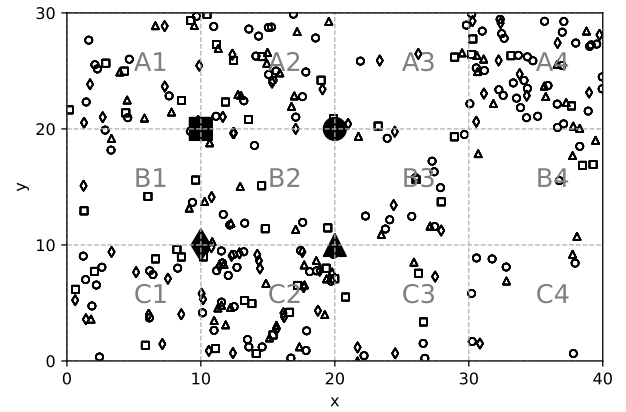


Fig. 9 Dispatching results of Case S-5

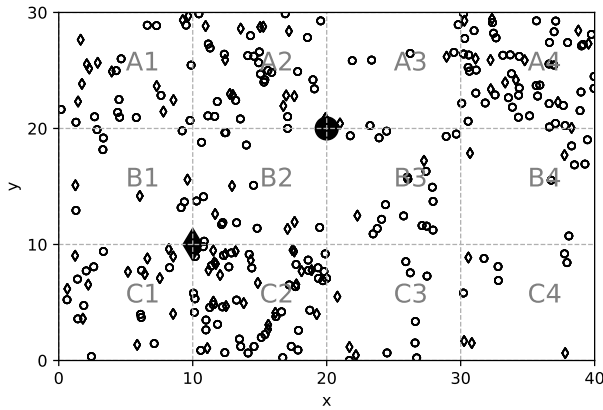


Fig. 7 Dispatching results of Case S-3

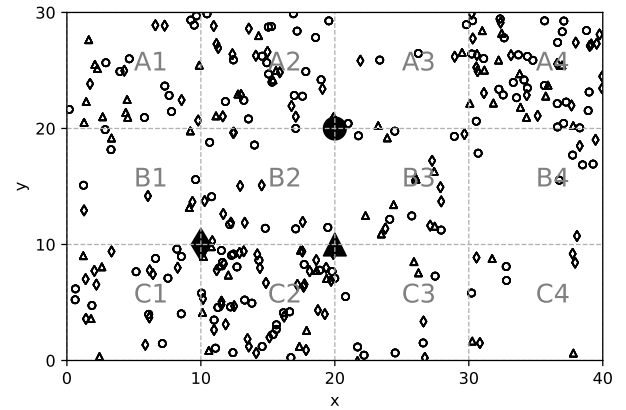


Fig. 10 Dispatching results of Case S-6

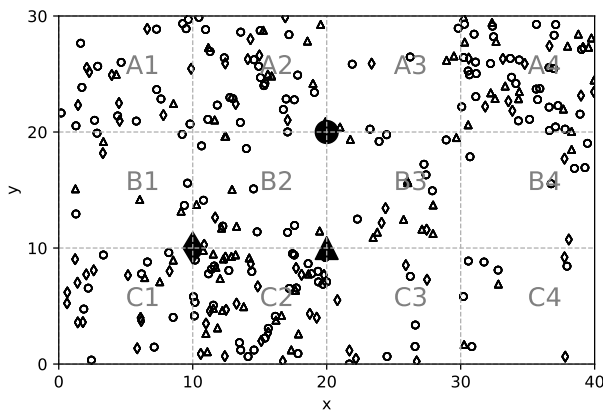


Fig. 8 Dispatching results of Case S-4

posed model aims to minimize the response time in order to arrive at the emergency site to rescue as fast as possible. In order to validate the proposed model and demonstrate the performance, a set of test instances was generated randomly be solved by using the Gurobi Optimizer version 7.0.1. We know the model action by small-scale cases (Case S-1 ~ Case S-6) by changing some parameters. However, the call-in time

and service time play an important role in the emergency vehicle location problem, we should change them for each part and conduct the numerical experiment in the future. Next, we expanded the scale of problems (Case L-1 ~ Case L-3). The computation results show the efficacy of the proposed model. However, if the problem scale exceeds the Case L-2 like Case L-3, it is hard to be solved. therefore, in the future, we should modify the optimization model and design an algorithm to solve the large-scale problem. Besides, we also need to use real data to validate the proposed model.

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