AN OPTIMIZATION MODEL FOR EMERGENCY VEHICLE LOCATION WITH CONSIDERATION OF INTEGRATION DISPATCHING

You Zhao [1*] and Hiroshige Dan[†]

Abstract. This paper aims to develop location opitimization and dispatching optimization techniques for emergency vehicles. In this research, Firstly, we seek to propose a optimization model to find the optimal location of emergency vehicles. Then, we generate the some scenarios to consider the uncertainty conditions and do numerical experiment. The numerical experiment results show that the proposed model achieves a good performance and worked well.

Keywords. emergency vechicle location and dispatching, optimization, uncertainty, integration dispatching.

1. INTRODUCTION

Utilizing the avaliable facilities and human resoures, fire service organizations shall protect the lives, physical being and property of public from fire and take precautions against disasters such as storms, floods fires and earthquakes, while mitigating the damage of these disaters. Fire service organizations are also responsible for the appropriate transport of persons who have sustained injuries due to a disaster (FDMA, 2015). When the Large-scale disaster or accident, serious accident and military attack happend, emergency vehicles belonging to the fire departments will rush to the emergency demand as soon as possible for rescue. However, In recent years, the number of emergency services for emergency call is increasing. In particular, the emergency vehicles tend to lengthen the average response time to the site because of the lack of fire departments and emergency vehicles in the suburbs. On the other hand, if we can locate more emergency vehicles in more fire departments, emergency vehicles can shorten the traveling time to the emergency site, but at the same time location cost must be considered. Therefore, according to the Japan Fire and Disaster Management Agency Heisei 29th edition fire fighting white paper (FDMA, 2018) pointed out that consider Large-scale distaers such as the Great East Japan Earthquake, higher future disater risks, and decrease in the population of Japan, we must enhance the structure of fire departments by expanding their jurisdictions. Accordingly, it is necessary to balance these two aspects and develop a more effective emergency vehicle location planning.

In the emergency vehicle location problem, the main objective to minimize the time it takes to respond to the emergency sites (the traveling time between emergency call receipt and emergency vehicles arrival to the emergency site) (Saeed et al., 2018). Because the emergency site for emergency services is covered by the emergency vehicles located at fixed points, therefore, the location of emergency vehicles is important in service quality level. Moreover, as the uncertainty commonly exists in the real world. The unpredictability of the time and the location of emergency incidents are also the main issue in the emergency vehicle location (Xiao-Xia et al., 2013) (Lei et al., 2015) . Therefore, we should consider the uncertainty for the emergency vehicle location problem as follows:

- The uncertainty of call-in time and the location of emergency site
- The uncertainty of the emergency vehicle traveling time
- The uncertainty of service time at the emergency site

Based on the above, in this research, we developed an optimization model to find the optimal location of emergency vehicles with consideration of integration dispatching and consiered the uncertainty conditons by numerical experiment.

In Section 2, the process of emergency response system and the times and time periods in emergency vehicle dispathcing are described in detail. Next, the mathematical model will be proposed in Section 3. Then, in order to demonstrate the performance and validate the proposed model, we generated some numerical instances are presented and solved in Section 4. Finally, we draw a conclusion in Section 5.

2. PROCESS OF EMREGENCY RESPONSE SYSTEM

In this section, we introduce the process of emergency response system. The process of the emergency response system usually covers the sequence of the following activities.

1. The system receives the emergency call when the incident happenes.
2. After call screening, dispatchers evaluate the system status and determine the appropriate emergency vehicle (EV) to dispatch.
3. The dispatched EV arrives at emergency site and starts service.

1 Environmental and Applied System Engineering, Kansai University, 3-3-35, Yamate-cho, Suita, Osaka, 564-8680, Japan.
∗ E-mail address: k819381@kansai-u.ac.jp
† E-mail address: dan@kansai-u.ac.jp

4. After completing service, the EV returns to the fire departments or goes to another emergency site.

In this research, we assume that dispatched EVs must return back to the fire departments when the service is completed (Fig. 1).

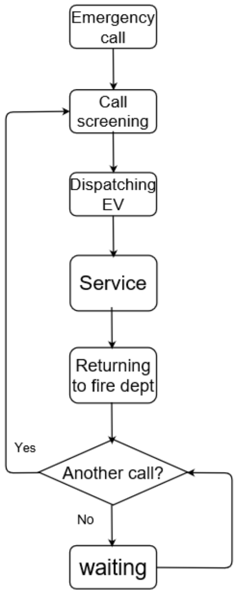

Fig. 1 The process of emergency response system

In this research, we have the times and the time periods which explain this produce as shown in Fig. 2.

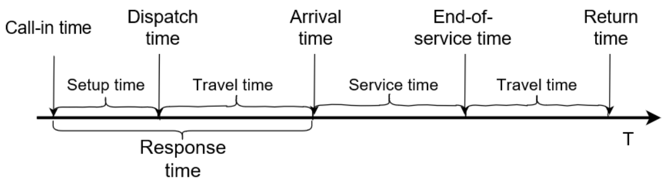

Fig. 2 The times and time periods in an EV dispathcing

3. MATHEMATICAL FORMULATION

In this section, we formulate the optimization model to find the optimal location of emergency vehicles.

3.1. Notation

We use the following notations to describe our proposed model.

- Sets
  - I : the set of fire departments indexed by $i \in \{1, 2, \ldots, \alpha\}$ (α: the number of fire departments),
  - J : the set of emergency sites indexed by $j \in \{1, 2, \ldots, \beta\}$ (β: the number of emergency sites),
  - K : the set of emergency vehicles indexed by $k \in \{1, 2, \ldots, \gamma\}$ (γ: the number of emergency vehicles),
  - N : the set of the numbers of dispatching indexed by $n \in \{1, 2, \ldots, n_{\max}\}$ ($n_{\max}$: the maximum number of dispatches of each emergency vehicle),
  - A : the set of scenarios indexed by $a \in \{1, 2, \ldots, \delta\}$ (δ: the number of scenarios).
- Parameters
  - $u_{a,j}$: the call-in time from j under a,
  - $t_{a,i,j}$: the traveling time between i and j under a,
  - $s_{a,j}$: the service time of k at j under a,
  - M_1/M_2 : sufficiently large constant numbers,
  - e : the setup time for dispatching,
  - b : the threshold amount,
  - P_a : the occurrence probability of a.
- Decision variables
  - $$x_{a,j,n,k} = \begin{cases} 1, & k \text{ is dispatched to a } j \text{ with the } n\text{-th dispatch under } a, \\ 0, & \text{otherwise,} \end{cases}$$
  - $$y_{i,k} = \begin{cases} 1, & k \text{ is assigned to } i, \\ 0, & \text{otherwise,} \end{cases}$$
  - $h_{a,n,k}$: the dispatch time of k with the n-th dispatch under a,
  - $l_{a,n,k}$: the traveling time of k with the n-th dispatch under a,
  - $v_{a,n,k}$: the arrival time of k with the n-th dispatch under a,
  - $z_{a,n,k}$: the end-of-service time of k with the n-th dispatch under a,
  - $w_{a,n,k}$: the return time of k with the n-th dispatch under a,
  - $p_{a,j,n,k}$: the response time of k for j with the n-th dispatch under a.

3.2. Constraints

In this research, we built the following constraints.

Affiliation and dispatching rule of vehicles

$$\sum_{i \in I} y_{i,k} = 1 \quad (k \in K), \tag{1}$$

$$\sum_{n \in N, k \in K} x_{a,j,n,k} = 1 \quad (a \in A, j \in J), \tag{2}$$

$$\sum_{j \in J} x_{a,j,n,k} \leq 1 \quad (a \in A, n \in N, k \in K). \tag{3}$$

The constraint (1) ensures that the emergency vehicles are assigned to one fire department. The constraint (2) ensures that an emergency vehicle must be dispatched to a emergency site. The constraint (3) ensures that each emergency vehicle can only be dispatched to at most one emergency site in each dispatching.

Dispatching emergency vehicles

$$h_{a,n,k} \geq \sum_{j \in J} x_{a,j,n,k} u_{a,j} + e \quad (a \in A, n \in N, k \in K), \quad (4)$$

$$l_{a,n,k} \geq t_{a,i,j} - (1 - y_{i,k})M_1 - (1 - x_{a,j,n,k})M_1$$
$$(a \in A, i \in I, j \in J, n \in N, k \in K), \quad (5)$$

$$v_{a,n,k} \geq h_{a,n,k} + l_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (6)$$

$$z_{a,n,k} = v_{a,n,k} + \sum_{a \in A, j \in J} x_{a,j,n,k} s_{a,j}$$
$$(a \in A, n \in N, k \in K), \quad (7)$$

$$w_{a,n,k} = z_{a,n,k} + l_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (8)$$

$$w_{a,n-1,k} + e \leq h_{a,n,k} \quad (a \in A, n \in N, k \in K : n \geq 2). \quad (9)$$

The constraint (4) ensures that the setup time is necessary for dispatching. The constraint (5) becomes $l_{a,n,k} \geq t_{a,i,j}$ when $x_{a,j,n,k} = 1$ and $y_{i,k} = 1$ holds, the $l_{a,n,k}$ shows the upper bound of traveling time between the fire department and the emergency site. The constraints (6) and (7) determine the arrival time and the end-of-service time of an emergency vehicle at a emergency site, respectively. The constraint (8) shows the return time of an emergency vehicle. The constraint (9) shows the relationship between the return time and the next dispatching time of an emergency vehicle.

Priority and posteriority of time variables

$$v_{a,n,k} \geq h_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (10)$$

$$z_{a,n,k} \geq v_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (11)$$

$$w_{a,n,k} \geq z_{a,n,k} \quad (a \in A, n \in N, k \in K), \quad (12)$$

$$p_{a,j,n,k} \geq v_{a,n,k} - u_{a,j} - b - (1 - x_{a,j,n,k})M_2$$
$$(a \in A, j \in J, n \in N, k \in K). \quad (13)$$

The constraints (10), (11) and (12) show the order of time variables (Fig. 2). The constraint (13) computes the response time of each emergency vehicle.

Nonnegativity of variables

$$l_{a,n,k} \geq 0 \quad (a \in A, n \in N, k \in K), \quad (14)$$

$$p_{a,j,n,k} \geq 0 \quad (a \in A, j \in J, n \in N, k \in K). \quad (15)$$

The constraints (14),(15) ensure the continuous variables nonnegative.

3.3. Objective function

$$f = \sum_{a \in A} \left(P_a \cdot \sum_{j \in J, n \in N, k \in K} p_{a,j,n,k} \right)$$

3.4. Mathematical model

The mathematical model is presented as follows:

$$(\star) \left| \begin{array}{ll} \text{minimize} & f \\ \text{subject to} & (1) \sim (15). \end{array} \right.$$

$(\star)$ is used to find the optimal location of emergency vehicles by minimizing the total response time for each emergency vehicle.

4. NUMERICAL EXPERIMENT

In this section, we solve some numerical instances which are generated by randomly. Through these numerical experiments, we confirm the performance and validness of the proposed model.

4.1. Parameter generation

First, we generated a $L \times W$ rectangular region and divide the region into 12 parts (A1 −− C4). Also, we set five fire departments in the region (Fig. 3). we chose all or some of them to make various cases and conducted numerical experiments.

Next, we set the value of parameters randomly, the assumptions of each parameter are shown as follows:

- The number of emergency sites in each scenario follows the Poisson distribution whose mean value equals λ_1.
- The number of emergency sites in each part is proportional to the probability of occurrence of emergency sites as shown in Table. 1.
- The location of emergency sites are distributed uniformly in each part.
- The interval of call-in time follows the exponential distribution whose mean value is $1/\lambda_1$.
- The distance $d_{a,i,j}$ between the fire department i and the emergency site j in each a is measured in the Euclidean norm.
- The average speed of emergency vehicles is q.
- The traveling time of emergency vehicles from a fire department i to an emergency site j in a scenario a is defined as
$$t_{a,i,j} = d_{a,i,j} \ / \ q + r_{a,i,j},$$
where $r_{a,i,j}$ follows a uniform distribution an the interval $[l, u]$. The term $r_{a,i,j}$ is included to express a potential delay.
- The service time $s_{a,j}$ follows the exponential distribution whose mean value is $1/\lambda_2$.
- The occurrence probability P_a equals $1/\delta$.

Table 1 Probability of occurrence of emergency sites

	1	2	3	4
A	0.10	0.10	0.02	0.20
B	0.03	0.07	0.08	0.02
C	0.11	0.20	0.03	0.04

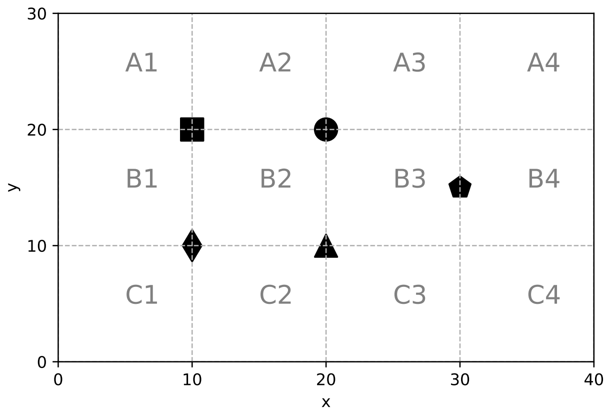

Fig. 3 The location of fire departments

4.2. Experimental outline and results

In this subsection, we use the 4.1 and set the parameter values in Table. 2 to generate data for our numerical experiments and compare the performance for 6 different cases.

We use the mathematical model ($\star$) to solve each case by using the Gurobi Optimizer version 7.0.1 and we set the limit of computation time is 3600 seconds. If the computation time is reached, we regard the best feasible solution as the optimal solution. The computation enviroment as shown in Table. 3.

Table 2 Parameter values in parameter generation

Param	L	W	V	l	u	λ_2	e
Value	40	30	30	0	15	20	3

Table 3 Computation enviroment

OS	Microsoft Windows 10 Home
CPU	Intel(R) Core(TM) i7-6600U CPU
Memory	16.0GB
Solver	Gurobi Optimizer version 7.0.1

Table 4 The description and results of the instance

	$\lvert I\rvert$	$\lvert K\rvert$	λ_1	$n_{\max}$	$\lvert A\rvert$	b	Obj	Gap
Case 1	5	3	10	8	30	0	947.74	100%
Case 2	5	3	10	8	30	30	666.04	100%
Case 3	5	4	10	5	30	0	614.56	100%
Case 4	5	4	10	5	30	30	304.13	100%
Case 5	5	5	10	4	30	0	426.81	100%
Case 6	5	5	10	4	30	30	147.70	100%

When the fire departments $|I|$=5, the mean value of emergency sites λ_1=10, scenarios $|A|$=30, we change the number of emergency vehicles $|K|$, the maximum number of dispatching of each emergency vehicle $n_{\max}$ and threshold amount b to observe the performance of the model by numerical experiment. Besides, the total number of emergency sites is 302. The computation results are shown in Table. 4 and the optimal location results are shown in Table. 5 for each case. The Table. 6 shows the number of dispatching for each fire departments in each case.

Firstly, we change the number of vehicles, the maximum number of dispatching of each emergency vehicle and set the b equals 0 in Case 1, Case 3 and Case 5. Besides, the emergency vehicles belonging to fire departments are dispatched to the emergency site as shown in Fig. 4, Fig. 6 and Fig. 8. we can see that the objective value (expectation value of response time) decrease as the number of emergency vehicles increase. It means that with 5 emergency vehicles at hand, we are able to arrive at the emergency site use little time. The Table. 7 shows that the number of the response time within various time periods. we can see that the Case 5 have a good performance in each time periods. Besides, the optimal location of Case 1 is $\{Triangle, Triangle, Circle\}$, the optimal location of Case 3 is $\{Rhombus, Circle, Circle, Circle\}$. From this, Case 1 is not a partial set of Case 3, so the solution of Case 1 can't be used.

Secondly, we change the number of vehicles, the maximum number of dispatching of each emergency vehicle and set the b equals 30 in Case 2, Case 4 and Case 6. Besides, the emergency vehicles belonging to fire departments are dispatched to the emergency site as shown in Fig. 5, Fig. 7 and Fig. 9.

we can see that the Objective value (expectation value of response time exceed 30 minutes) decrease as the number of emergency vehicles increase. It means that with 5 emergency vehicles at hand, the penalty time of the response time exceeds 30 minutes is smaller. The Table. 8 shows that the number of response time less or equal 30 minutes and greater the 30 minutes. we also can see that the Case 5 have a good performance whether less or equal 30 minutes or greater 30 minutes

Finally, we analyze the optimal location results in all cases (Table. 5 and Table. 6). We can know that the $circle$ fire department was used for each case because the population density is relatively high in A2 and A4 region. Then, the $Triangle$ fire department was used five times, because the population density is relatively high in C1 and C2 region. Next, the $Rhombus$ was used four times and the $Square$ was used one time. In addition, the $pentagon$ fire department wasn't used one time for each case. Based on the above, if we will locate the emergency vehicles in the fire departments, we may sort the importance of the fire departments from the optimal location results $\{Circle, Triangle, Square, Rhombus, Pentagon\}$.

Table 5 The optimal location results of each case

	Rhombus	*Square*	*Triangle*	*Circle*	*Pentagon*
Case 1	0	0	2	1	0
Case 2	0	0	1	2	0
Case 3	1	0	0	3	0
Case 4	1	0	1	2	0
Case 5	1	1	1	2	0
Case 6	2	0	1	2	0

Table 6 The number of dispatching in each fire departments for each case

	Rhombus	*Square*	*Triangle*	*Circle*	*Pentagon*
Case 1	0	0	191	111	0
Case 2	0	0	105	197	0
Case 3	90	0	0	212	0
Case 4	73	0	78	151	0
Case 5	62	50	64	126	0
Case 6	110	0	62	130	0

Table 7 The number of the response time within various time periods

	≤ 15	≤ 30	≤ 45	≤ 60	> 60
Case 1	4	58	138	183	119
Case 3	10	85	175	222	50
Case 5	24	122	219	259	43

Table 8 The number of the response time within 30 and greater than 30

	≤ 30	> 30
Case 2	51	251
Case 4	95	207
Case 6	114	188

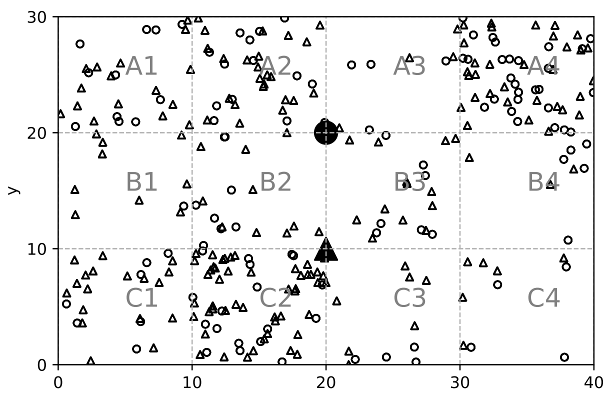

Fig. 4 The dispatching results of Case 1

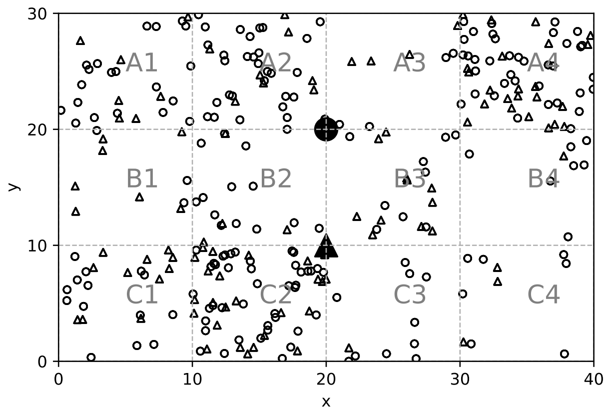

Fig. 5 The dispatching results of Case 2

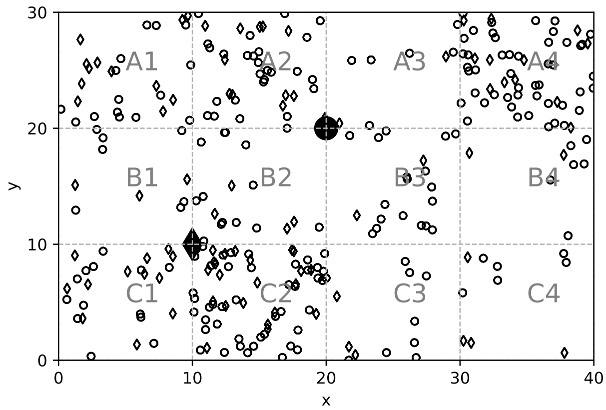

Fig. 6 The dispatching results of Case 3

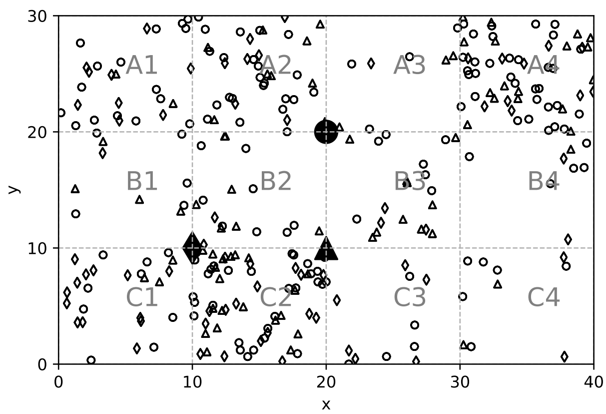

Fig. 7 The dispatching results of Case 4

4.3. Responding to large-scale problem cases

In order to know the limits of the mathematical model ($\star$) in the realistic time, we conducted an experiment to expand the scale of the problem (Case 7 ∼ Case 9). We set the computation time is 43200 seconds, if the solver can't find the feasible solution within 43200 seconds, we called it no feasible solution.

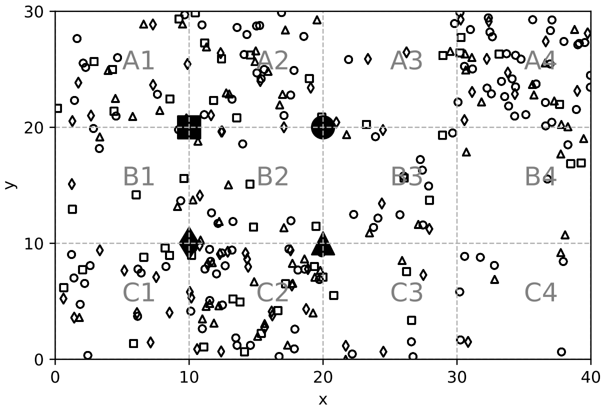

Fig. 8 The dispatching results of Case 5

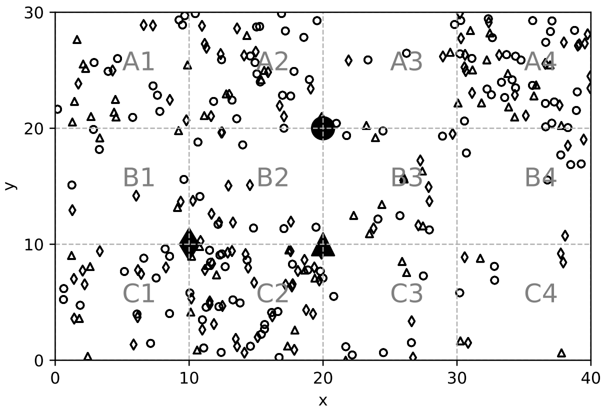

Fig. 9 The dispatching results of Case 6

The computation results are shown in Table. 9. from the computation results, we can know that if the scale of the problem as Case 7 and Case 8, our model ($\star$) can find the feasible solution by the solver. However, if the scale of the problem as Case 9, our model ($\star$) can't solve them smoothly by the solver. Besides, the Gap between the upper bound and the lower bound is too large. Therefore, we should modify the optimization model and design an algorithm for solving large-scale problems efficiently in the future.

Table 9 Large-scale problems computation results

	$\|I\|$	$\|K\|$	λ_1	$n_{\max}$	$\|A\|$	Feasiblity	Time	Gap
Case 7	20	20	40	4	30	Yes	4691sec	100%
Case 8	25	25	50	5	30	Yes	20772sec	100%
Case 9	30	30	60	6	30	None	None	100%

5. CONCLUSIONS AND FUTURE WORK

This paper focused on emergency vehicle location and dispatching problem with consideration of integration dispatching. The proposed model aims to minimize the response time in order to arrive at the emergency sites to rescue as fast as possible. In order to demonstrate the performance and validate the proposed model, a set of test instances was generated by random be solved by using the Gurobi Optimizer version 7.0.1. We know the model action by 6 cases (Case 1 ~ Case6) by changing some parameters. However, the call-in time and service time play an important role in the location of emergency vehicles, we should change them for each part and do the numerical experiment in the future. Next, we expanded the scale of problems (Case 7 ~ Case 9). The computation results show the efficacy of the proposed model. However, if the problem scale exceeds the Case 8 like Case 9, it is hard to be solved, therefore, we should we should modify the optimization model and design an algorithm to solve the large-scale problems in the future. Besides, we also need to use real data to validate the proposed model.

References

Fire and Disaster Management Agency, https://www.fdma.go.jp/neuter/about/pdf/en/2015/all.pdf, (2019/01/22 confirmed).

Fire and Disaster Management Agency, http://www.fdma.go.jp/html/hakusho/h29/h29/index.html, (2019/01/16 confirmed).

Xiao-Xia, Rong., Yi, Lu., Rui-Rui, Yin. and Jiang-Hua, Zhang (2013). A Robust Optimization Approach to Emergency Vehicle Scheduling. *Mathematical Problems in Engineering* **2013** https://doi.org/10.1155/2013/848312.

Lei, C., Lin, W.H. and Miao, L. (2015). A stochastic emergency vehicle redeployment model for an effective response to traffic incidents. *IEEE Transactions on Intelligent Transportation Systems* **16**(2): 898–909.

Saeed, Firooze1., Majid, Rafiee., Seyed, Mohammad, Zenouzzadeh. (2018). An Optimization Model for Emergency Vehicle Location and Relocation with Consideration of Unavailability Time. *SCIENTIA IRANICA* **25**(2): 3685–3699.