Functional Feynman-Kac Formula [Dupire (2009)]

Let x be a process given by the SDE

$$dx_s = \mu(X_s)ds + \sigma(X_s)dw_x.$$

Consider functionals $g : \Lambda_T \longrightarrow R$, $\lambda : \Lambda \longrightarrow R$ and $k : \Lambda \longrightarrow R$ and define the functional f

$$f(Y_t) = E\left[e^{-\int_t^T \lambda(X_u)du} g(X_T) + \int_t^T e^{-\int_t^s \lambda(X_u)du} k(X_s\right.$$

for any path $Y_t \in \Lambda, t \in [0, T]$ Thus, if $f \in C^{1,2}$ and k, λ, μ and σ are Λ-continuous, then f sat
Equation (PPDE):

$$\Delta_t f(Y_t) + \mu(Y_t)\Delta_x f(Y_t) + \frac{1}{2}\sigma^2(Y_t)\Delta_{xx} f(Y_t) - \lambda(Y_t)f(Y_t$$

with $f(Y_T) = g(Y_T)$, for any Y_t in the topological support of x.

Geometric Asian Option

Under Black--Scholes setting, where the spot value follows a Geometric Brownian Motion with cor

$$dx_t = (r - q)x_t dt + \sigma x_t dw_t.$$

Under this model, the price of a geometric Asian optin with maturity T and payoff $g : \Lambda_T \longrightarrow R$

$$\begin{cases} \Delta_t f(Y_t) + (r - q)y_t \Delta_x f(Y_t) + \frac{1}{2}\sigma^2 y_t^2 \Delta_{xx} f(Y_t) - \\ f(Y_T) = \left(\exp\left\{\frac{1}{T}\int_0^T \log y_t dt\right\} - K\right)^+. \end{cases}$$

Our Method

Approximate $f(X_t)$ using a combination of LSTM network and a feedforward NN, and use finite
paper.

```
"""
        Python  3.5
        Keras  2.2.2
        Tensorflow  1.10.0
"""
import  numpy  as  np
import  matplotlib.pyplot  as  plt
%matplotlib  inline
from  scipy.stats  import  norm
import  tensorflow  as  tf
import  time  as  ttt
import  os
import  pprint

import  keras.backend  as  K
from  keras.backend.tensorflow_backend  import  set_session
```

```
from keras.backend.tensorflow_backend import set_session
from keras.models import Sequential
from keras.layers import Dense, Activation,LSTM
from keras.optimizers import Adam
from keras.regularizers import L1L2
```

```
Using TensorFlow backend.
```

```
# """
#     GPU Setting
# """

# config = tf.ConfigProto()
# config.gpu_options.allow_growth = True
# config.log_device_placement = True
# sess = tf.InteractiveSession(config=config)
# set_session(sess)    # set TensorFlow session as the default session for Keras

"""
    TPU Setting
"""

if 'COLAB_TPU_ADDR' not in os.environ:
    print('ERROR: Not connected to a TPU runtime; please see the first cell in this r
else:
    tpu_address = 'grpc://' + os.environ['COLAB_TPU_ADDR']
    print ('TPU address is', tpu_address)

    sess = tf.InteractiveSession(tpu_address)
    devices = sess.list_devices()
    set_session(sess)    # set this TensorFlow session as the default session for Keras

    print('TPU devices:')
    pprint.pprint(devices)
```

```
TPU address is grpc://10.27.31.178:8470
TPU devices:
[_DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:CPU:0, CPU, -1, 147629099583308828
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:XLA_CPU:0, XLA_CPU, 17179869184, 4
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU:0, TPU, 17179869184, 100854866
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU:1, TPU, 17179869184, 160001474
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU:2, TPU, 17179869184, 114603824
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU:3, TPU, 17179869184, 184307966
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU:4, TPU, 17179869184, 901219672
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU:5, TPU, 17179869184, 113263592
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU:6, TPU, 17179869184, 168292315
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU:7, TPU, 17179869184, 164658353
 _DeviceAttributes(/job:tpu_worker/replica:0/task:0/device:TPU_SYSTEM:0, TPU_SYSTEM, 85899345
```

```
M = 128 # number of samples in a batch
T = 1 # terminal time
dt = 0.01 # detla_t = 0.01, 0.005, 0.002, 0.001
steps = int(T/dt) # number of time steps
```

```
r = 0.03
q = 0.01
sigma = 1
x_0 = 1.0 # X_0
Strike = 0.4 #Strike Price


def generate_t(T, steps, M = M):
        '''
        time discretization (M * (steps+1 +1))
        for computing the time derivative, we need an extra time step.
        '''
        t_temp = np.linspace(1e-6, T- 1e-6, steps +1, dtype = np.float32)
        return np.tile(np.concatenate((t_temp, [T + dt])), (M,1)) # extra after terminal


# for example
t = generate_t(T, steps)[0][0:-1]
print("the length of t is {}".format(len(t)+1))
```

```
the length of t is 102
```

```
def Create_paths(i, M):
        '''
        GBM paths (M * (steps+1)) with seed i
        number of steps could be 100, 200, 500, 1000.
        '''


        # generate time steps for each path
        np.random.seed(i)
        x = np.tile(x_0, (M, 1))
        path = np.tile(x_0, (M, 1))
        delta = T/1000

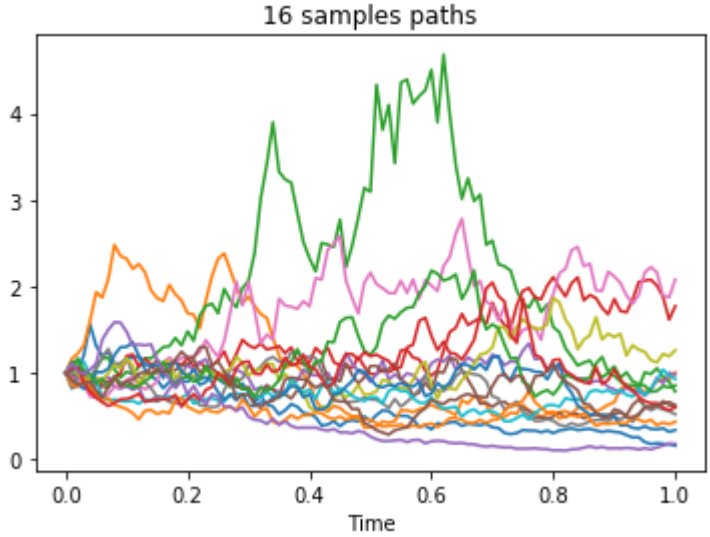
        dW = np.sqrt(delta)*np.random.normal(size=(M, 1000))
        for k in range(1000):
                x += (r-q) * x * delta + sigma * x * dW[:,k:k+1]
                path = np.concatenate((path, x), axis = 1)

        # select # of steps (100, 200, 500, 1000) from total 1000 timesteps
        selection = np.linspace(0,1000, steps+1, dtype = np.int)
        path = np.array(path, dtype=np.float32)
        path_selection = path[:,selection]

        return np.array(path_selection, dtype=np.float32)


for i in range(16):
        plt.plot(t, Create_paths(100000000, M)[i])
plt.title("16 samples paths")
plt.xlabel("Time")
```

```
Text(0.5, 0, 'Time')
```




```
# input time and path as placeholders
path = tf.placeholder(dtype=tf.float32, shape=[M,steps+1])
time = tf.placeholder(dtype = tf.float32, shape = [M, steps +1 + 1]) # extra after
```

True Solution and Terminal Condition

A closed-form solution is available in this case:

$$f(Y_t) = e^{-r(T-t)} \left(G_t^{t/T} y_t^{1-t/T} e^{\bar{\mu}+\bar{\sigma}^2/2} \Phi(d_1) - K\Phi(\right.$$

where Φ is the cumulative distribution function of the standard normal,

$$G_t = \exp\left\{ \frac{1}{t} \int_0^t \log y_u du \right\},$$

$$\bar{\mu} = \left(r - q - \frac{\sigma^2}{2} \right) \frac{1}{2T} (T-t)^2,$$

$$\bar{\sigma} = \frac{\sigma}{T} \sqrt{\frac{1}{3}(T-t)^3},$$

$$d_2 = \frac{(t/T)\log G_t + (1-t/T)\log y_t + \bar{\mu} - \log}{\bar{\sigma}}$$

$$d_1 = d_2 + \bar{\sigma}.$$

In the numerical examples below, we fix the strike price to $K = 0.4$.

```
def true_solution(apath):
    '''
    Givin a path, it outputs the true solution.
    '''
    G = 1
    mu_bar = (r-q-sigma**2/2)/2/T * (T-t[0])**2
    sigma_bar = sigma/T * np.sqrt(1/3*(T-t[0])**3)
    d_2 = 1/sigma_bar * (t[0]/T * np.log(G) + (1 - t[0]/T) * np.log(apath[0]) + r
    d_1 = d_2 + sigma_bar
    true_solution = [np.exp(-r*(T - t[0])) * ( G**(t[0]/T) * (apath[0] **(1-t[0]/T))
```

```
                                    - Strike * norm.cdf(d_2))]

    for k in range(1, steps+1):
        G = np.exp(1/t[k] * np.sum((np.log(apath[1:k+1]) + np.log(apath[:k]) )/2)*d
        mu_bar = (r-q-sigma**2/2)/2/T * (T-t[k])**2
        sigma_bar = sigma/T * np.sqrt(1/3*(T-t[k])**3)
        d_2 = 1/sigma_bar * (t[k]/T * np.log(G) + (1 - t[k]/T) * np.log(apath[l
        d_1 = d_2 + sigma_bar
        true_solution.append(np.exp(-r*(T - t[k])) * (G**(t[k]/T) * (apath[k] **(1-
                                    - Strike * norm.cdf(d_2)))

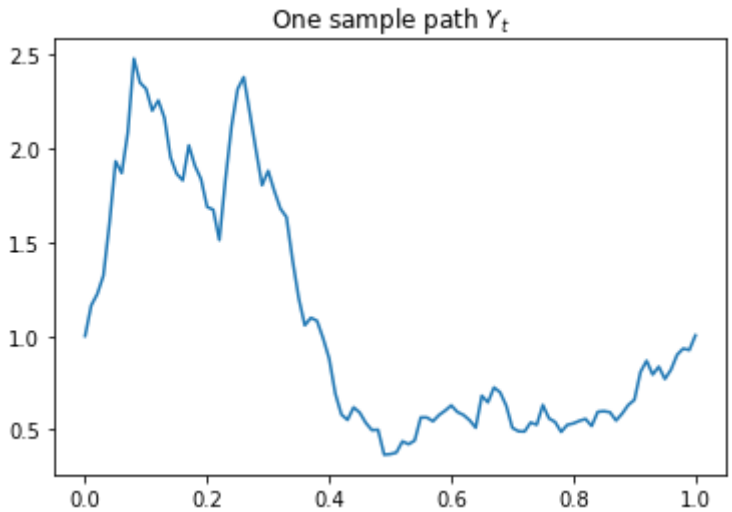
    return true_solution

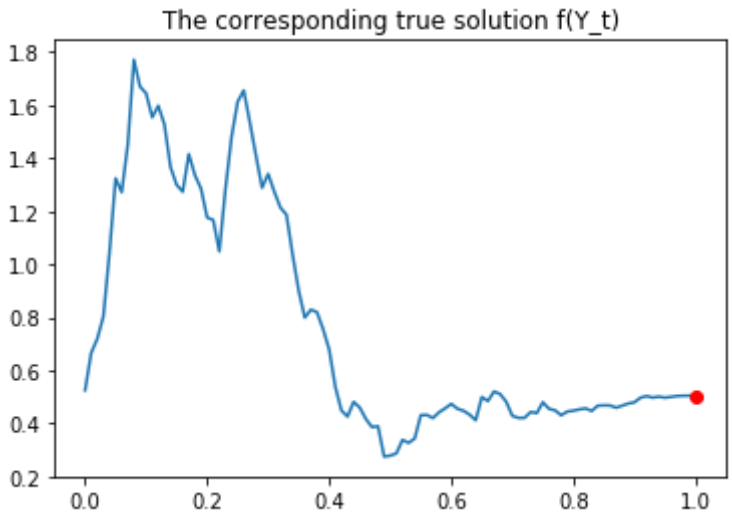

def terminal_condition(apath):
    '''
    Givin a path, it outputs the terminal condition g(Y_T).
    '''

    return np.maximum(np.exp(1/T*sum( (np.log(apath[1:]) + np.log(apath[:-1]) )/2)*dt) -


# for example given a GBM path
path1 = Create_paths(100000000, M)[1]
plt.plot(t, path1)
plt.title("One sample path $Y_t$")
plt.show()

plt.plot(t, true_solution(path1))
plt.plot(T, terminal_condition(path1), "ro")
plt.title("The corresponding true solution f(Y_t)")
plt.show()
```





Neural Network Setup

The path-dependent function $f(X_t)$ is been approximated by a combination of LSTM and a feedf

```
n_a = 128 # number of hidden neurons in the LSTM network
LSTM_cell = LSTM(n_a, return_state = True) # This is used to capture the long term

# This feedforward neural network is used to compute the derivatives.
# input dimension is (1+1+n_a) = (space, time, path (which is characterized by n_a h
NN = Sequential([
        Dense(128, input_shape=(1+1+n_a,)),
        Activation('tanh'),
        Dense(128),
        Activation('tanh'),
        Dense(128),
        Activation('tanh'),
        Dense(1)
])
```

Objective Function

$$J_{N,M}(\theta) = \frac{1}{M}\frac{1}{N}\sum_{j=1}^{M}\sum_{i=0}^{N}\left(\Delta_t u(Y_{t_i}^{(j)};\theta) + Lu(Y_{t_i}^{(j)};\theta)\right)^2 + \frac{1}{M}\sum_{j=1}^{M}($$

```
def loss_function(time,path, M):

        a = tf.Variable(tf.zeros([M, n_a]), trainable=False)
        c = tf.Variable(tf.zeros([M, n_a]), trainable=False)

##. Approximate f(X_{t_0})
        input_x = K.slice(path, [0,0], [M,1])
        input_t = K.slice(time, [0,0], [M,1])
        inputt_f = K.concatenate([input_x, input_t, a])
        f = NN(inputt_f)

##.      calculate space derivatives
        bump = 0.01 * input_x
        input_x_up = input_x + bump
        inputt_up = K.concatenate([input_x_up, input_t, a])
        f_up = NN(inputt_up)
        input_x_down = input_x - bump
        inputt_down = K.concatenate([input_x_down, input_t, a])
        f_down = NN(inputt_down)
        partial_x_f = (f_up - f)/bump
        partial_xx_f = (f_up - 2*f + f_down)/bump/bump

##. Next LSTM cell
        input_x_reshape = tf.reshape(input_x, (M,1,1))
        input_t_reshape = tf.reshape(input_t, (M,1,1))
        inputt = K.concatenate([input_x_reshape, input_t_reshape])
        a, _, c = LSTM_cell(inputt, initial_state = [a, c])

##.      calculate time derivative
        input_t_time = K.slice(time, [0, 1], [M, 1])
        inputt_time = K.concatenate([input_x, input_t_time, a])
        f_flat = NN(inputt_time)
        partial_t_f = (f_flat - f)/dt

        Loss = tf.reduce_sum(tf.square(partial_t_f + (r-q)*input_x * partial_x_f + 0.5 *

        solution = f
        time_derivative = partial_t_f
        space_derivative = partial_x_f
        space_2nd_derivative = partial_xx_f

        ##########################################################################
        for i in range(1, steps+1): # Iterate through every timestep

                ##. Approximate f(X_{t_i})
                input_x = K.slice(path, [0,i], [M,1])
                input_t = K.slice(time, [0,i], [M,1])
```

```
        inputt_f = K.concatenate([input_x, input_t, a])
        f = NN(inputt_f)

        ##. calculate space derivative
        bump = 0.01 * input_x
        input_x_up = input_x + bump
        inputt_up = K.concatenate([input_x_up, input_t, a])
        f_up = NN(inputt_up)
        input_x_down = input_x - bump
        inputt_down = K.concatenate([input_x_down, input_t, a])
        f_down = NN(inputt_down)
        partial_x_f = (f_up - f)/bump
        partial_xx_f = (f_up - 2*f + f_down)/bump/bump

        ##. Next LSTM cell
        input_x_reshape = tf.reshape(input_x, (M,1,1))
        input_t_reshape = tf.reshape(input_t, (M,1,1))
        inputt = K.concatenate([input_x_reshape, input_t_reshape])
        a, _, c = LSTM_cell(inputt, initial_state = [a, c])

        ##. calculate time derivative
        input_t_time = K.slice(time, [0, i+1], [M, 1])
        inputt_time = K.concatenate([input_x, input_t_time, a])
        f_flat = NN(inputt_time)
        partial_t_f = (f_flat - f)/dt

        Loss += tf.reduce_sum(tf.square(partial_t_f + (r-q) * input_x * partial_x_

        solution = K.concatenate([solution, f])
        time_derivative = K.concatenate([time_derivative, partial_t_f])
        space_derivative = K.concatenate([space_derivative, partial_x_f])
        space_2nd_derivative = K.concatenate([space_2nd_derivative, partial_xx_f])

    ##############################################################################
    ##. Terminal cost
    Loss += tf.reduce_sum( tf.square( f - tf.math.maximum(tf.exp(1/T*tf.reduce_sum((tf.l

    return Loss/M/steps, solution, time_derivative, space_derivative, space_2nd_derivative
```

SGD

```
Epoch = 15000+1


loss, solution, time_derivative, space_derivative, space_2nd_derivative = loss_function(time,


global_step = tf.Variable(0, trainable=False)
starter_learning_rate = 0.01
# exponential decay learning rate
learning_rate = tf.maximum(tf.train.exponential_decay(starter_learning_rate, global_step, 50,
# adam optimizer
optimizer = tf.train.AdamOptimizer(learning_rate = learning_rate)
optimizer = tf.contrib.estimator.clip_gradients_by_norm(optimizer, clip_norm=5.0)
```

```
train_op = optimizer.minimize(loss, global_step=global_step)
```

```
WARNING:tensorflow:
The TensorFlow contrib module will not be included in TensorFlow 2.0.
For more information, please see:
  * https://github.com/tensorflow/community/blob/master/rfcs/20180907-contrib-sunset.md
  * https://github.com/tensorflow/addons
  * https://github.com/tensorflow/io (for I/O related ops)
If you depend on functionality not listed there, please file an issue.

WARNING:tensorflow:From /usr/local/lib/python3.6/dist-packages/tensorflow_core/python/ops/mat
Instructions for updating:
Use tf.where in 2.0, which has the same broadcast rule as np.where
```

Training

```
init = tf.global_variables_initializer()
sess.run(init)


np.random.seed(8)
# GBM
path_test = Create_paths(100000000, M)
time_feed = generate_t(T, steps, M)
pred_dict = {path: path_test, time: time_feed}

train_loss_list = []
test_loss_list = []


start_time = ttt.time()

for it in range(Epoch):
        seed = it % 100
        path_feed = Create_paths(seed, M)
        feed_dict = {path: path_feed, time: time_feed}
        sess.run(train_op, feed_dict)

        if (it % 10 == 0): # every 10 epoch print and record
                elapsed = ttt.time() - start_time
                print("{}th Epoch:".format(it+1))
                loss_value = sess.run(loss, feed_dict)
                lr = sess.run(learning_rate)
                loss_test = sess.run(loss, pred_dict)
                print("traing loss is {}, test loss is {}, and learning rate is {}, e
                print(" ")

                train_loss_list.append(loss_value)
                test_loss_list.append(loss_test)
                start_time = ttt.time()

        if (it % 100 == 0): # every 100 epoch, show picture
```

```
solution_pred, time_derivative_pred, space_derivative_pred, space_2nd_derivative
num_epoches = len(train_loss_list)
x_axis_epoches = np.linspace(0, num_epoches*10, num_epoches)

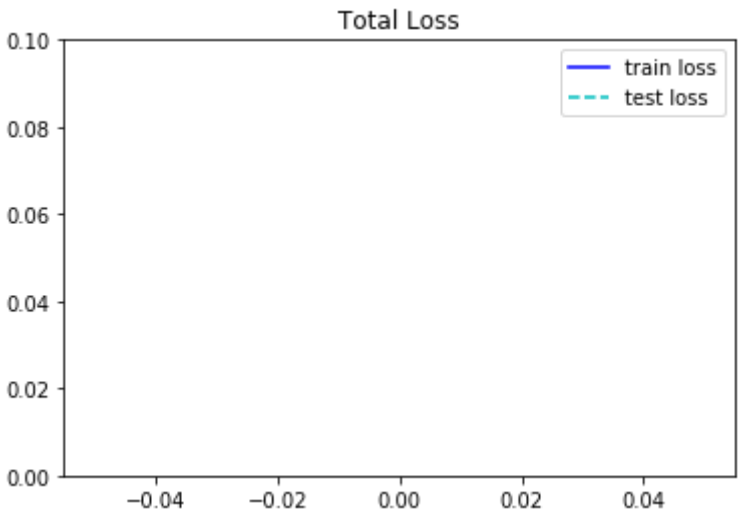
# plot train/test loss
plt.plot(x_axis_epoches, train_loss_list, 'b-', label = "train loss")
plt.plot(x_axis_epoches, test_loss_list, 'c--', label = "test loss")
plt.legend()
plt.ylim([0, 0.1])
plt.title("Total Loss")
plt.show()

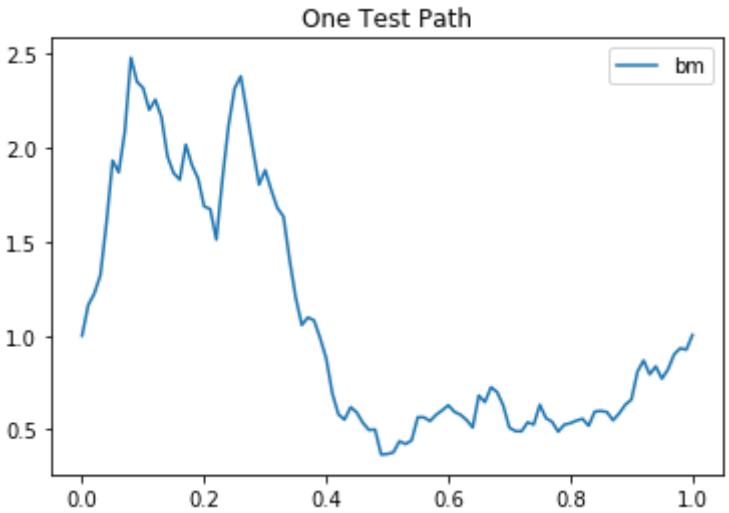
# plot one example path
plt.plot(t, path_test[1], label = "bm")
plt.legend()
plt.title("One Test Path")
plt.show()

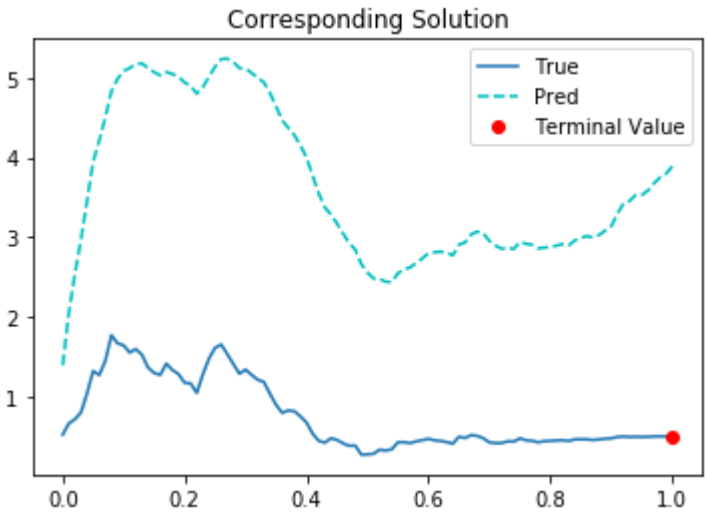
# plot the true solution vs predicted solution
plt.plot(t, true_solution(path_test[1]), label = "True")
plt.plot(t, solution_pred[1], 'c--', label = "Pred")
plt.plot(T, terminal_condition(path_test[1]), "ro", label = "Terminal Value")
plt.legend()
plt.title("Corresponding Solution")
plt.show()

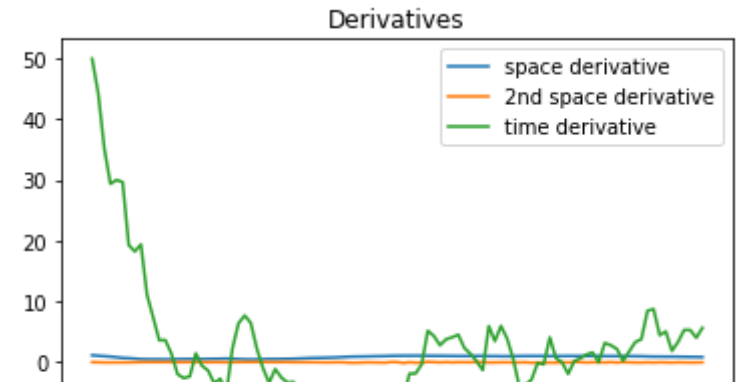
# plot the time and space derivatives.
plt.plot(np.linspace(0,T, steps+1), space_derivative_pred[1], label = "space
plt.plot(np.linspace(0,T, steps+1), space_2nd_derivative_pred[1], label = "2n
plt.plot(np.linspace(0,T, steps+1), time_derivative_pred[1], label = "time de
plt.legend()
plt.title("Derivatives")
plt.show()
```

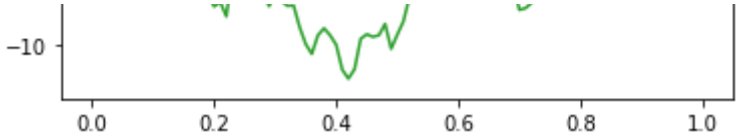
```
1th Epoch:
traing loss is 84.66207885742188, test loss is 84.60501861572266, and learning rate is 0.0099
```












```
11th Epoch:
traing loss is 0.46921080350875854, test loss is 0.44406890869140625, and learning rate is 0.

21th Epoch:
traing loss is 0.1912405788898468, test loss is 0.1602650284767151, and learning rate is 0.00

31th Epoch:
traing loss is 0.10561057925224304, test loss is 0.11739112436771393, and learning rate is 0.

41th Epoch:
traing loss is 0.09054793417453766, test loss is 0.10226331651210785, and learning rate is 0.

51th Epoch:
traing loss is 0.10457169264554977, test loss is 0.09029576182365417, and learning rate is 0.

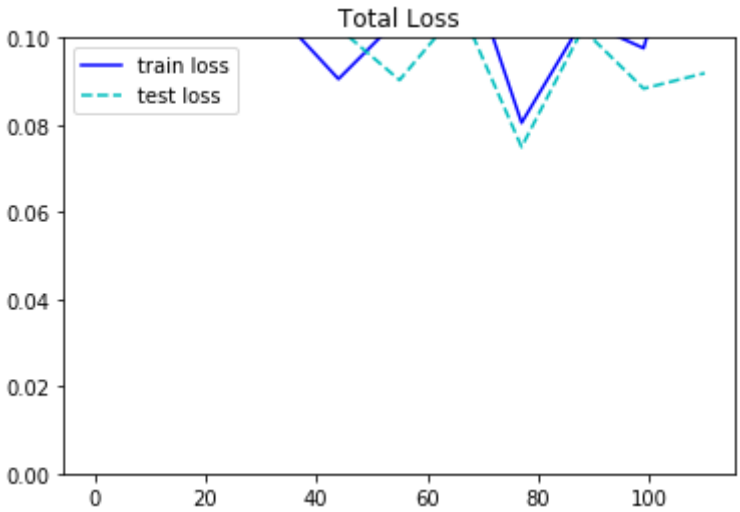
61th Epoch:
traing loss is 0.12227378785610199, test loss is 0.10690470784902573, and learning rate is 0.

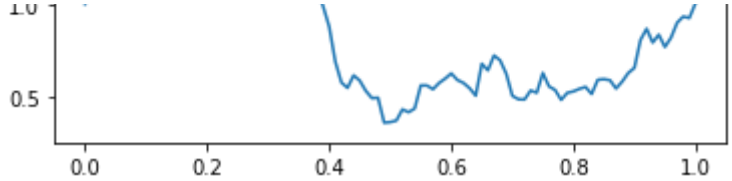
71th Epoch:
traing loss is 0.08049429953098297, test loss is 0.07493965327739716, and learning rate is 0.

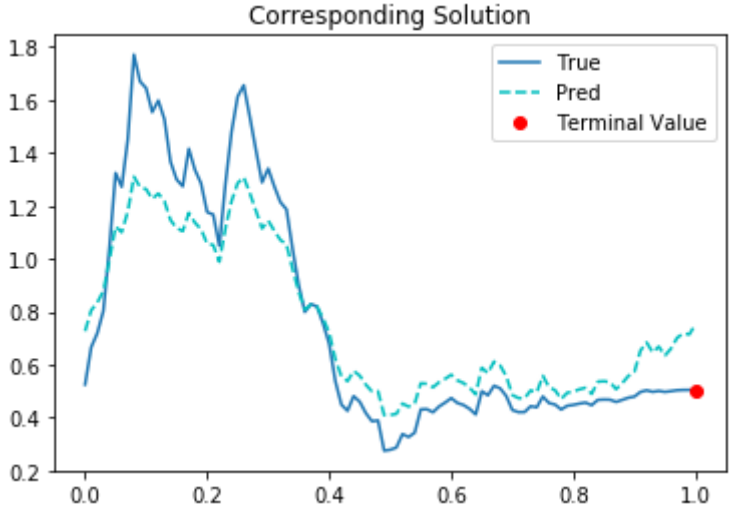
81th Epoch:
traing loss is 0.10393344610929489, test loss is 0.10202565789222717, and learning rate is 0.

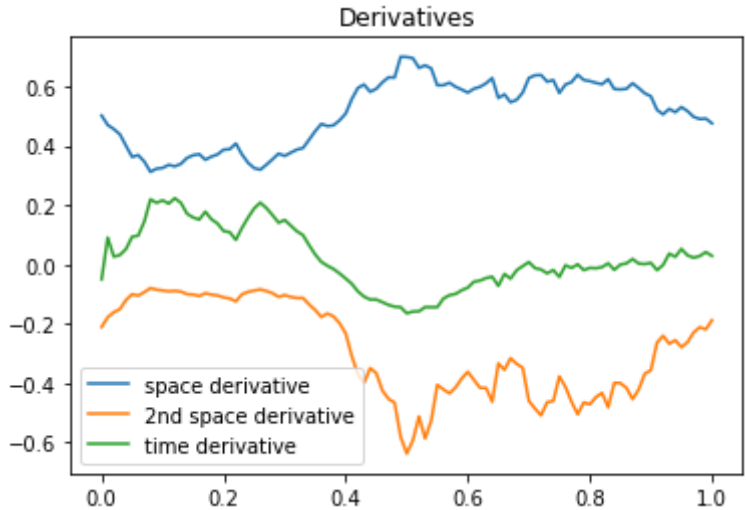
91th Epoch:
traing loss is 0.0976191982626915, test loss is 0.08834338933229446, and learning rate is 0.0

101th Epoch:
traing loss is 0.1474895477294922, test loss is 0.0918881744146347, and learning rate is 0.00
```







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111th Epoch:
traing loss is 0.11011618375778198, test loss is 0.10634177178144455, and learning rate is 0.

121th Epoch:
traing loss is 0.08608138561248779, test loss is 0.08070850372314453, and learning rate is 0.

131th Epoch:
traing loss is 0.045655667781829834, test loss is 0.06481418758630753, and learning rate is 0

141th Epoch:
traing loss is 0.10590299218893051, test loss is 0.11085785925388336, and learning rate is 0.

151th Epoch:
traing loss is 0.10541129857301712, test loss is 0.10473129153251648, and learning rate is 0.

161th Epoch:
traing loss is 0.0922241061925888, test loss is 0.08185625821352005, and learning rate is 0.0

171th Epoch:
traing loss is 0.0743260383605957, test loss is 0.07263746857643127, and learning rate is 0.0

181th Epoch:
traing loss is 0.09187650680541992, test loss is 0.07160701602697372, and learning rate is 0.
```

```
191th Epoch:
traing loss is 0.06870204210281372, test loss is 0.05879988521337509, and learning rate is 0.

201th Epoch:
traing loss is 0.14117147028446198, test loss is 0.07228219509124756, and learning rate is 0.
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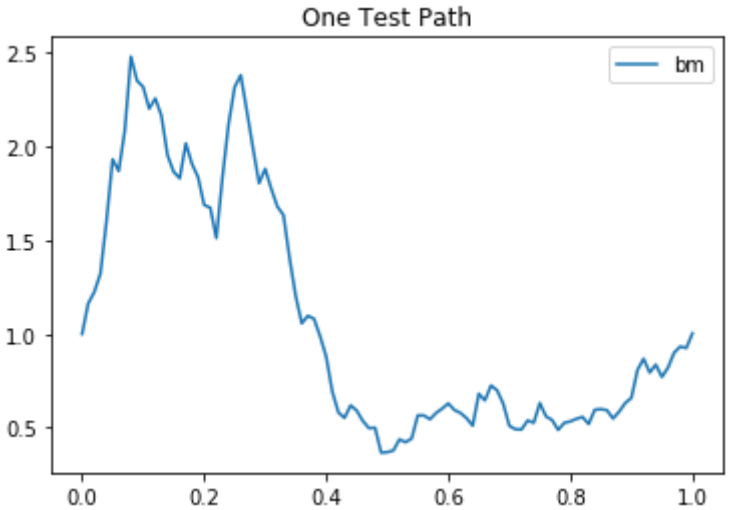

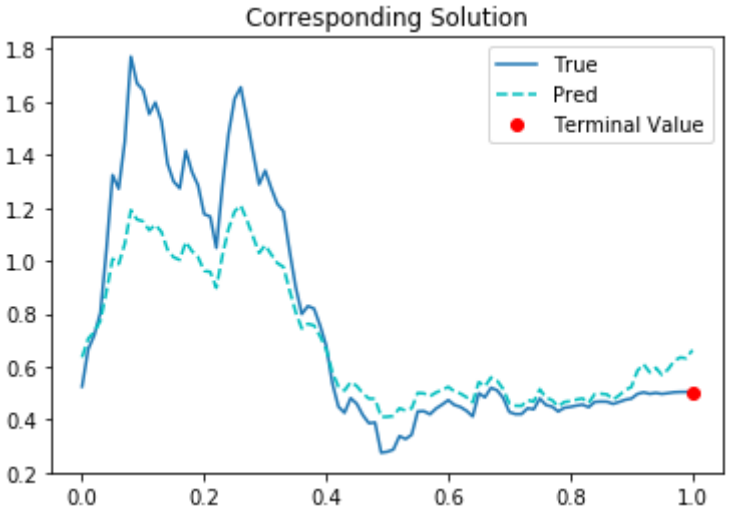

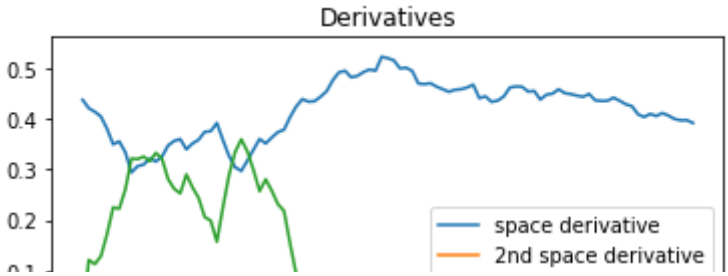

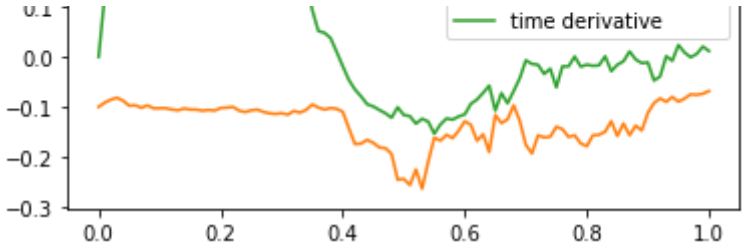


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211th Epoch:
traing loss is 0.07304307073354721, test loss is 0.06431138515472412, and learning rate is 0.

221th Epoch:
traing loss is 0.06033444404602051, test loss is 0.05917743593454361, and learning rate is 0.

231th Epoch:
traing loss is 0.053882043808698654, test loss is 0.058416947722435, and learning rate is 0.0

241th Epoch:
traing loss is 0.062196116894483566, test loss is 0.06191418692469597, and learning rate is 0

251th Epoch:
traing loss is 0.05439292639493942, test loss is 0.0361931212246418, and learning rate is 0.0

261th Epoch:
traing loss is 0.053385667502880096, test loss is 0.03455734625458717, and learning rate is 0

271th Epoch:
traing loss is 0.0326850563287735, test loss is 0.03248490393161774, and learning rate is 0.0

281th Epoch:
traing loss is 0.07046321034431458, test loss is 0.06004779040813446, and learning rate is 0.

291th Epoch:
traing loss is 0.03935333341360092, test loss is 0.03042793832719326, and learning rate is 0.

301th Epoch:
traing loss is 0.07548198103904724, test loss is 0.038338854908943176, and learning rate is 0
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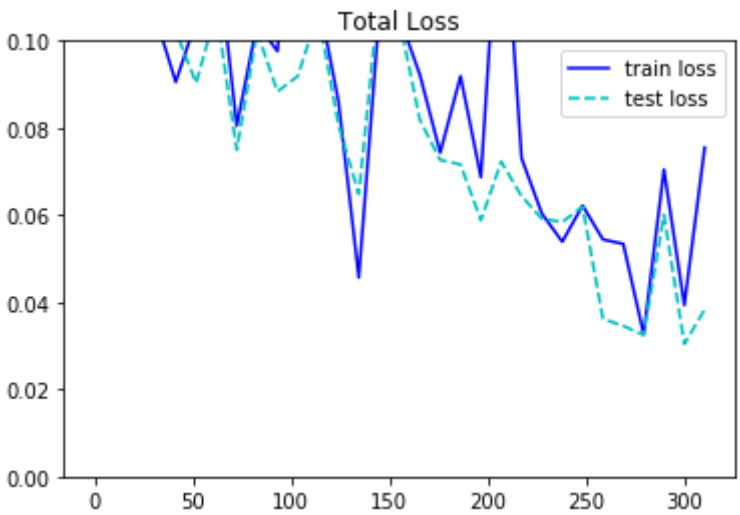

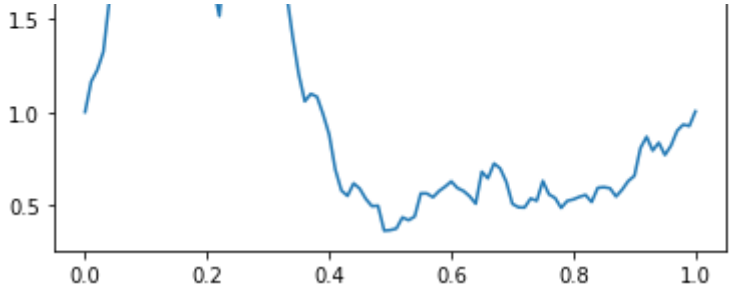

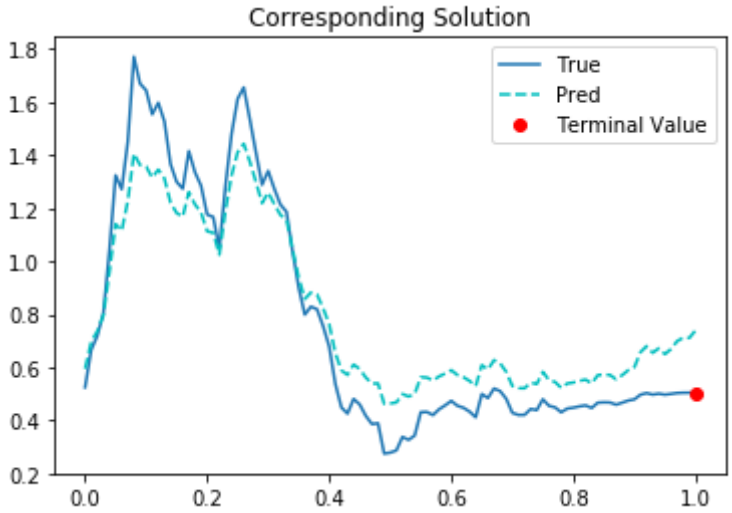

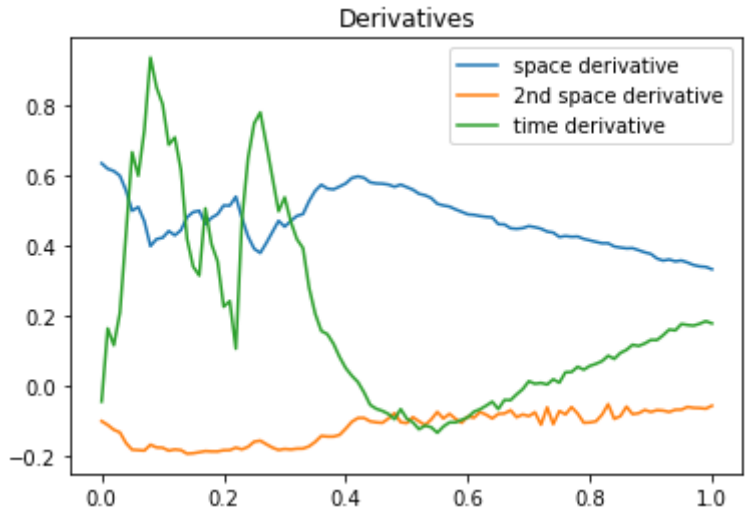


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311th Epoch:
traing loss is 0.03381170332431793, test loss is 0.024400055408477783, and learning rate is 0

321th Epoch:
traing loss is 0.024355951696634293, test loss is 0.017865022644400597, and learning rate is

331th Epoch:
traing loss is 0.01780388504266739, test loss is 0.02094743214547634, and learning rate is 0.

341th Epoch:
traing loss is 0.03160648047924042, test loss is 0.03129154443740845, and learning rate is 0.

351th Epoch:
traing loss is 0.044045839458703995, test loss is 0.03669213503599167, and learning rate is 0

361th Epoch:
traing loss is 0.028154335916042328, test loss is 0.018731575459241867, and learning rate is

371th Epoch:
traing loss is 0.014339098706841469, test loss is 0.013542554341256618, and learning rate is
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```
381th Epoch:
traing loss is 0.01407715305685997, test loss is 0.013759023509919643, and learning rate is 0

391th Epoch:
traing loss is 0.016568109393119812, test loss is 0.012520842254161835, and learning rate is

401th Epoch:
traing loss is 0.036128636449575424, test loss is 0.010484526865184307, and learning rate is
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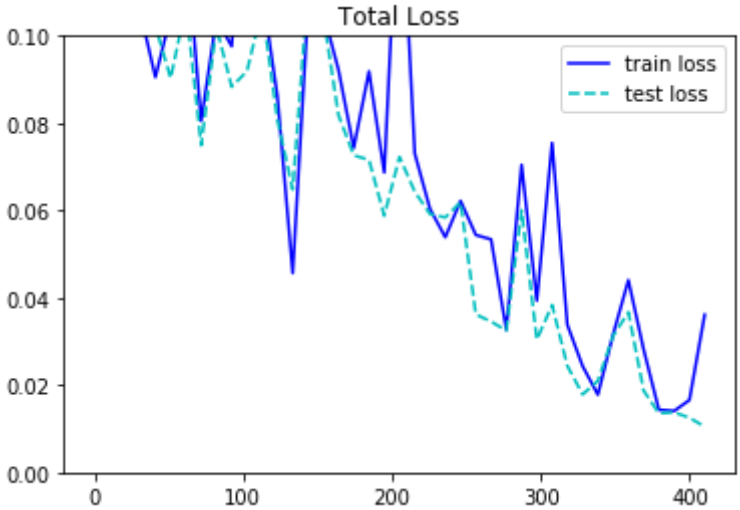

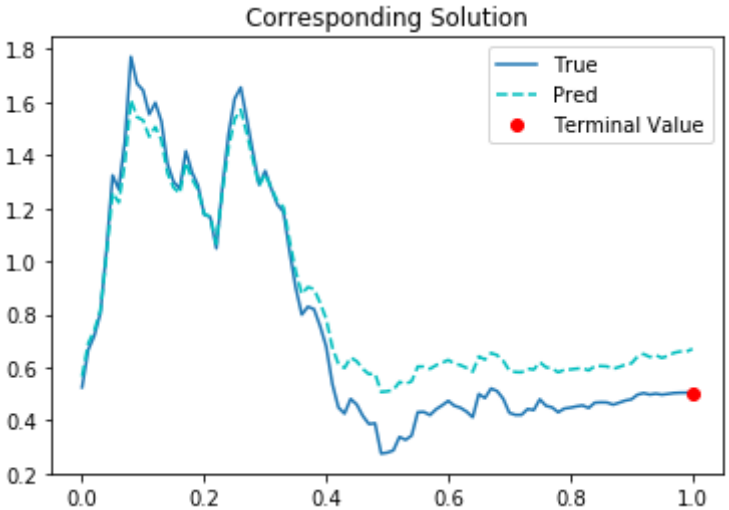

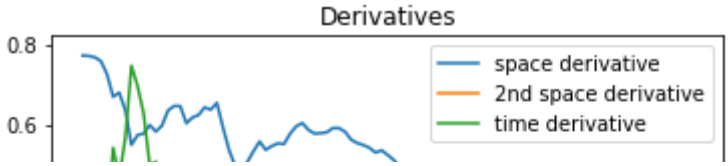

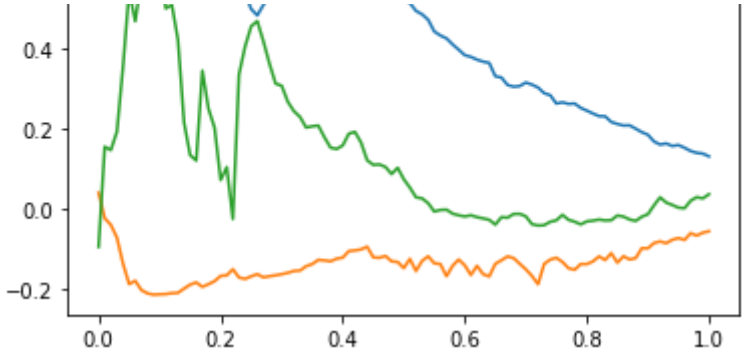


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411th Epoch:
traing loss is 0.009814845398068428, test loss is 0.00861534383147955, and learning rate is 0

421th Epoch:
traing loss is 0.027839673683047295, test loss is 0.027449658140540123, and learning rate is

431th Epoch:
traing loss is 0.012161940336227417, test loss is 0.01300858985632658, and learning rate is 0

441th Epoch:
traing loss is 0.02245485782623291, test loss is 0.02408873476088047, and learning rate is 0.

451th Epoch:
traing loss is 0.0128142349421978, test loss is 0.011395702138543129, and learning rate is 0.

461th Epoch:
traing loss is 0.016900133341550827, test loss is 0.013244356960058212, and learning rate is

471th Epoch:
traing loss is 0.022833576425909996, test loss is 0.024701740592718124, and learning rate is

481th Epoch:
traing loss is 0.015660902485251427, test loss is 0.014894217252731323, and learning rate is

491th Epoch:
traing loss is 0.010185208171606064, test loss is 0.008127767592668533, and learning rate is

501th Epoch:
traing loss is 0.019845595583319664, test loss is 0.005464973393827677, and learning rate is
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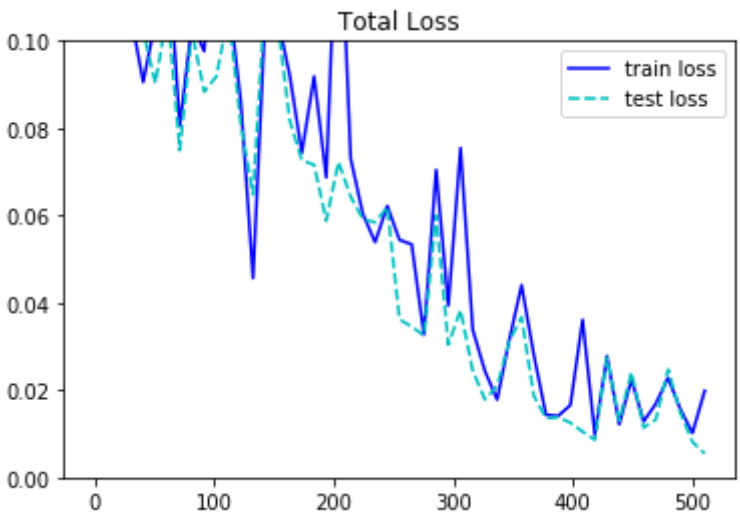

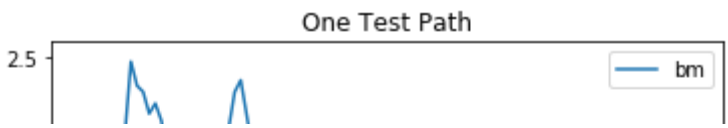

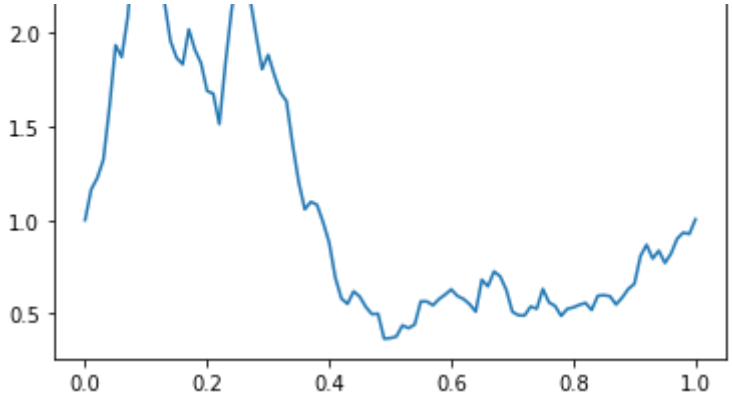

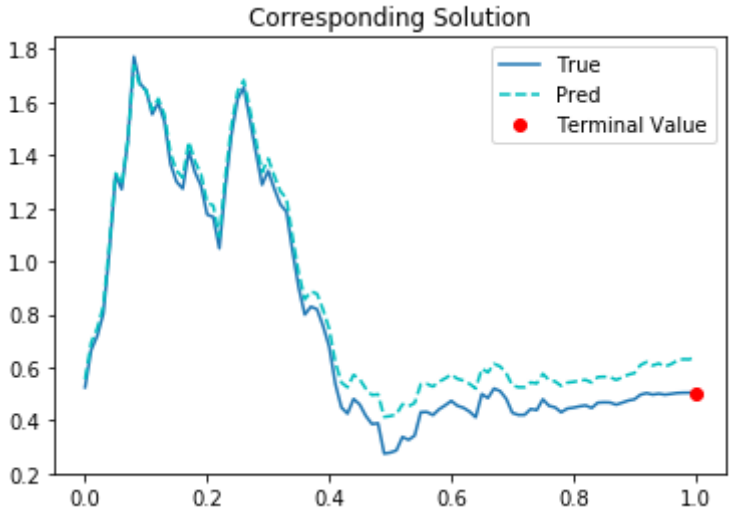

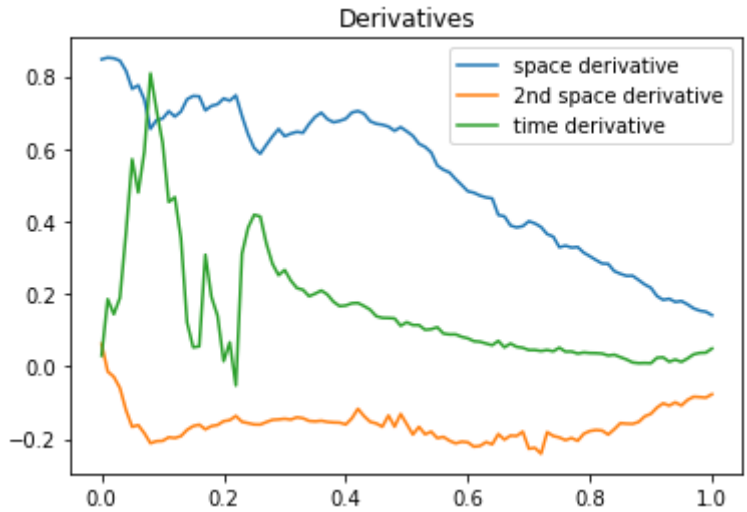


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511th Epoch:
traing loss is 0.009556375443935394, test loss is 0.010201910510659218, and learning rate is

521th Epoch:
traing loss is 0.0111152408644557, test loss is 0.010855428874492645, and learning rate is 0.

531th Epoch:
traing loss is 0.0033418769016861916, test loss is 0.0032609899062663317, and learning rate i

541th Epoch:
traing loss is 0.007779509760439396, test loss is 0.006574647966772318, and learning rate is

551th Epoch:
traing loss is 0.007701471913605928, test loss is 0.007486597169190645, and learning rate is

561th Epoch:
traing loss is 0.006815012078732252, test loss is 0.005247191525995731, and learning rate is
```

```
571th Epoch:
traing loss is 0.016452917829155922, test loss is 0.016505233943462372, and learning rate is

581th Epoch:
traing loss is 0.006133869290351868, test loss is 0.00455455994233489, and learning rate is 0

591th Epoch:
traing loss is 0.013834274373948574, test loss is 0.013979997485876083, and learning rate is

601th Epoch:
traing loss is 0.023361200466752052, test loss is 0.012532140128314495, and learning rate is
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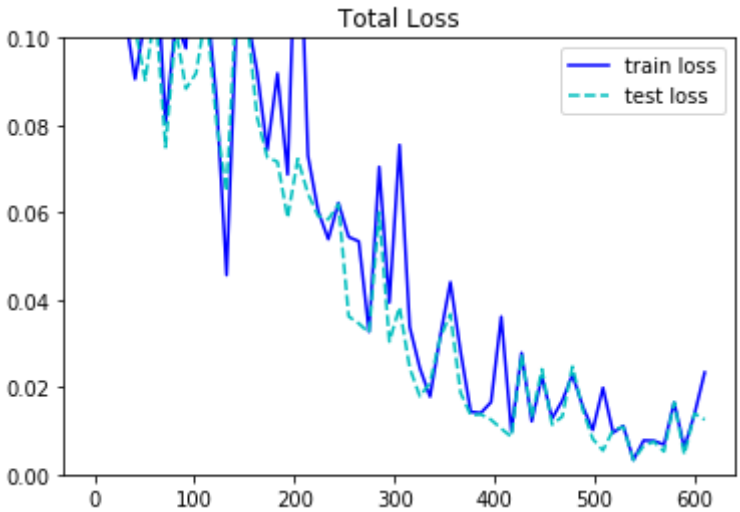

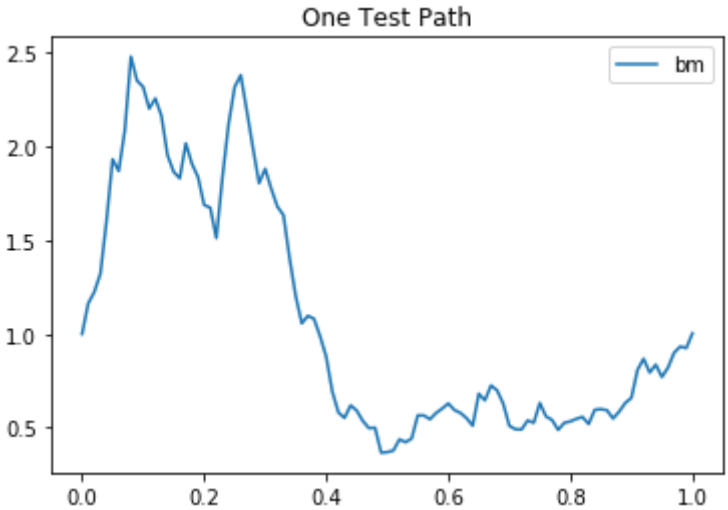

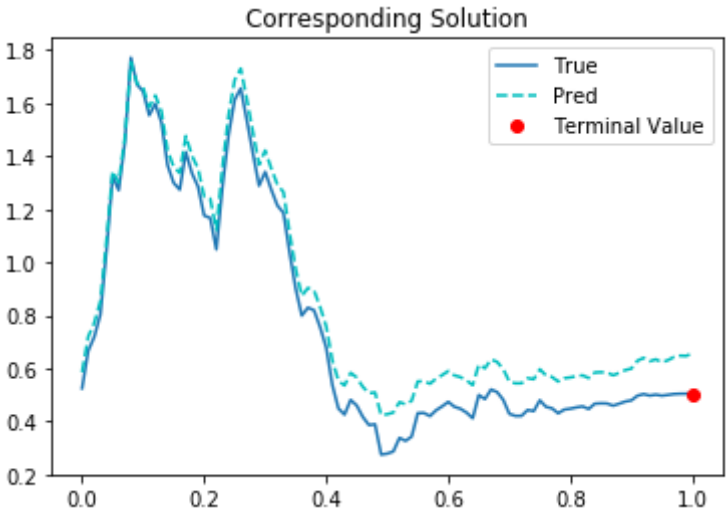

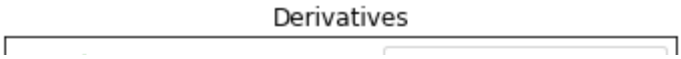

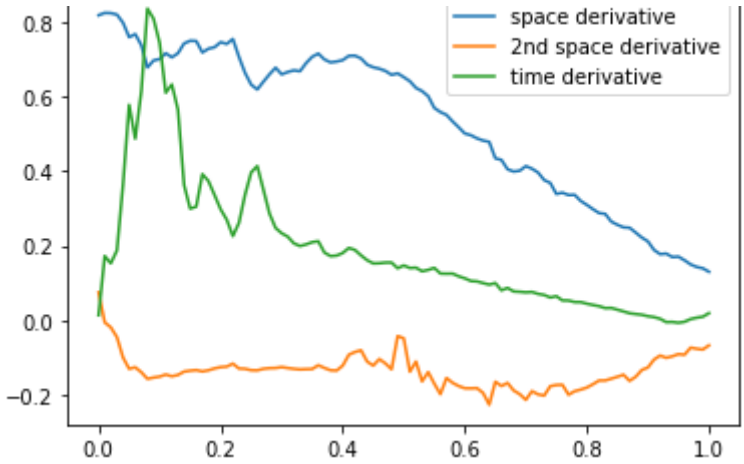


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611th Epoch:
traing loss is 0.010086915455758572, test loss is 0.008046713657677174, and learning rate is

621th Epoch:
traing loss is 0.013891584239900112, test loss is 0.011206112802028656, and learning rate is

631th Epoch:
traing loss is 0.010198874399065971, test loss is 0.009879709221422672, and learning rate is

641th Epoch:
traing loss is 0.00716280797496438, test loss is 0.0067865848541259766, and learning rate is

651th Epoch:
traing loss is 0.010781519114971161, test loss is 0.008787899278104305, and learning rate is

661th Epoch:
traing loss is 0.008867957629263401, test loss is 0.007374798879027367, and learning rate is

671th Epoch:
traing loss is 0.0036322360392659903, test loss is 0.003223649226129055, and learning rate is

681th Epoch:
traing loss is 0.008345592767000198, test loss is 0.005795541685074568, and learning rate is

691th Epoch:
traing loss is 0.0048421649262309074, test loss is 0.004736926406621933, and learning rate is

701th Epoch:
traing loss is 0.008880218490958214, test loss is 0.0030277231708168983, and learning rate is
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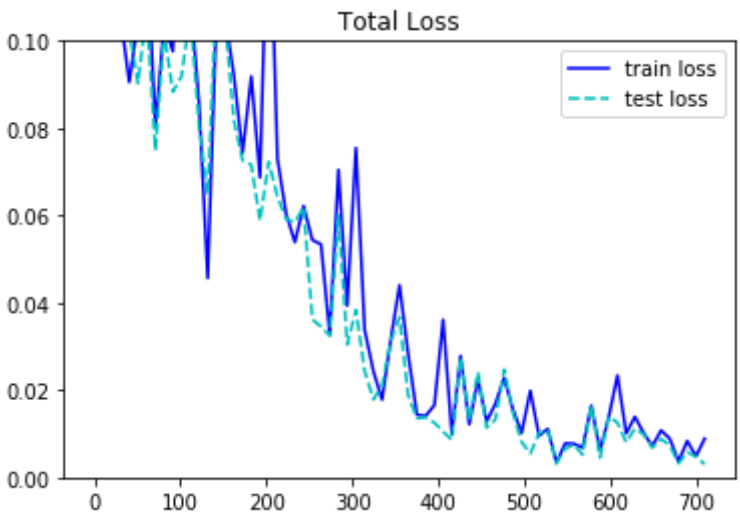

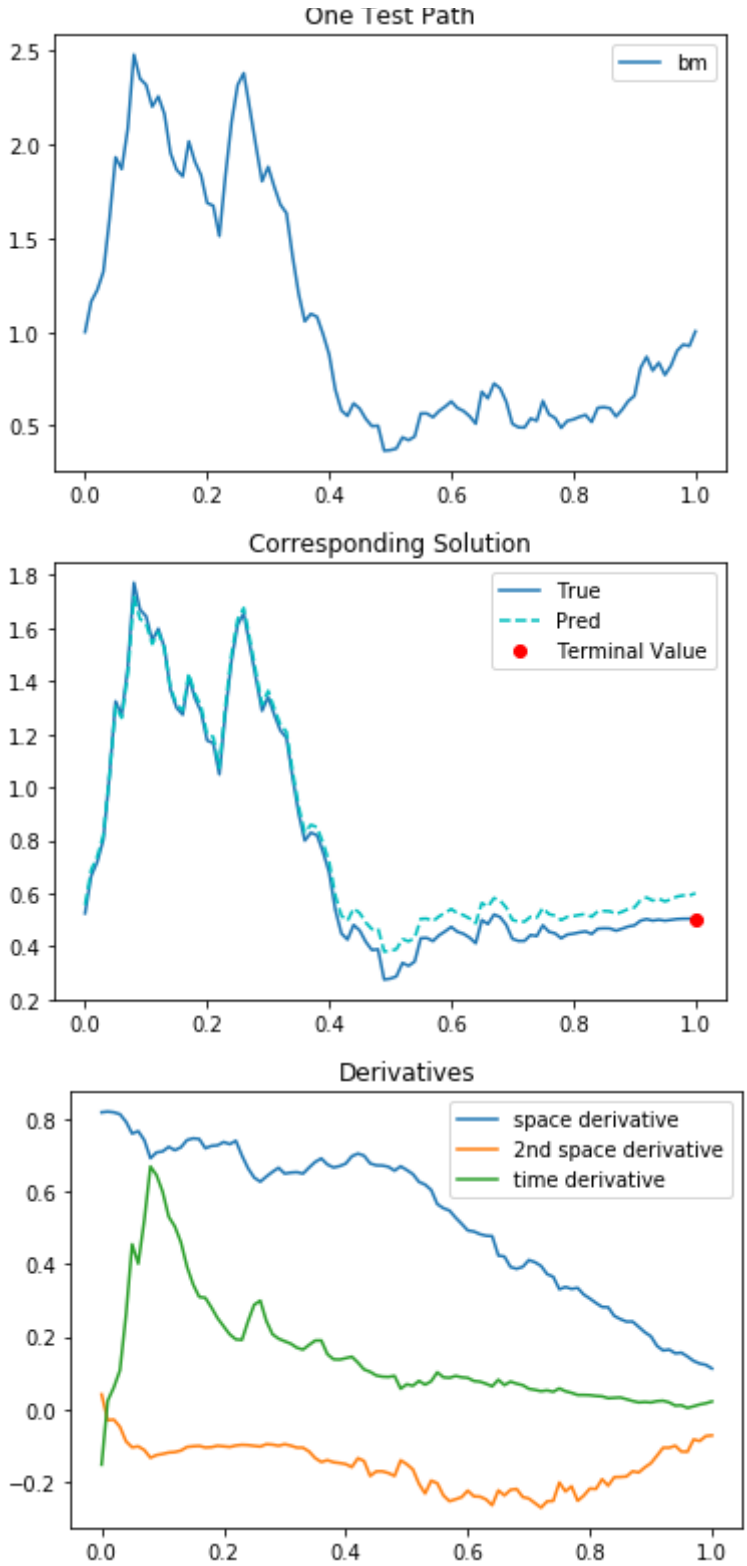


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711th Epoch:
traing loss is 0.005702255759388208, test loss is 0.0034732806961983442, and learning rate is

721th Epoch:
traing loss is 0.004584294278174639, test loss is 0.0044006588868796825, and learning rate is

731th Epoch:
traing loss is 0.005497845821082592, test loss is 0.005748781841248274, and learning rate is

741th Epoch:
traing loss is 0.010188786312937737, test loss is 0.010784272104501724, and learning rate is

751th Epoch:
traing loss is 0.005694190505892038, test loss is 0.005135614890605211, and learning rate is
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761th Epoch:
traing loss is 0.008562319912016392, test loss is 0.005643621552735567, and learning rate is

771th Epoch:
traing loss is 0.005484871100634336 5, test loss is 0.00594043405726552, and learning rate is

781th Epoch:
traing loss is 0.00393559830263257, test loss is 0.003138049738481641, and learning rate is 0

791th Epoch:
traing loss is 0.0023953223135322332, test loss is 0.0021725171245634556, and learning rate i

801th Epoch:
traing loss is 0.006260850932449102, test loss is 0.0024967698846012354, and learning rate is
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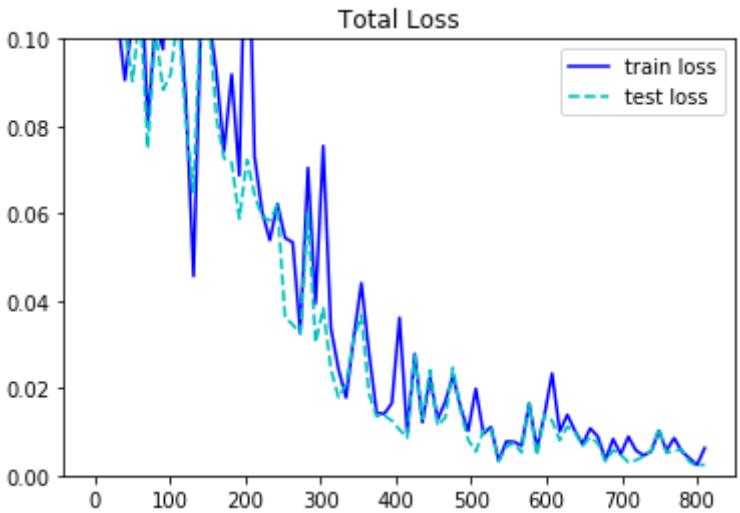

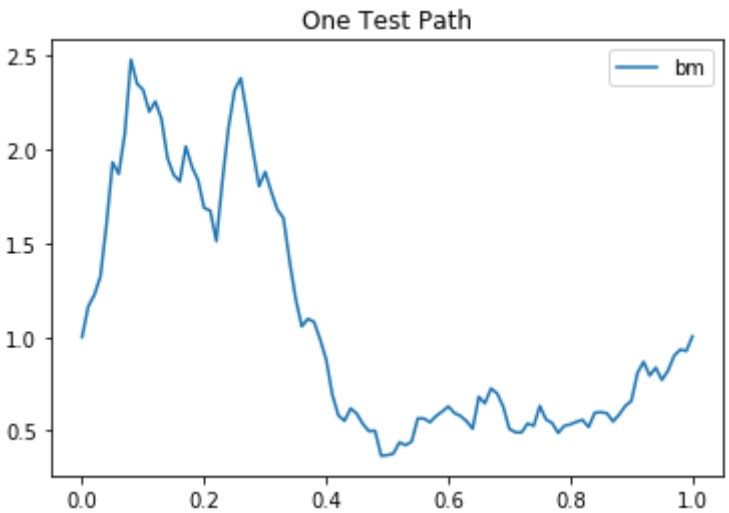

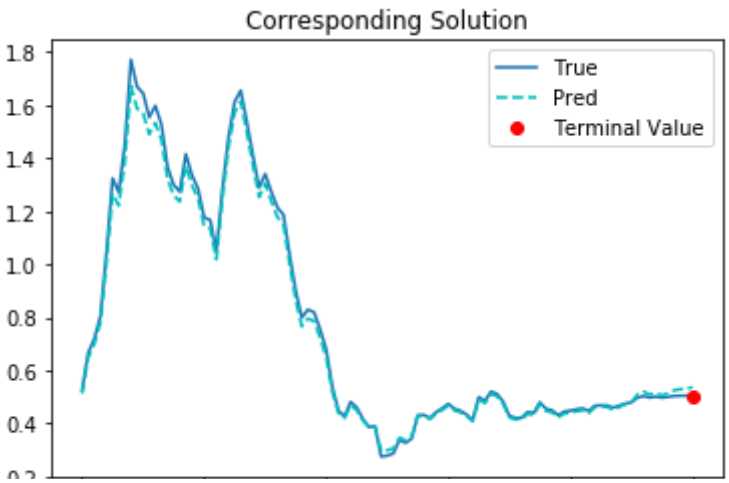

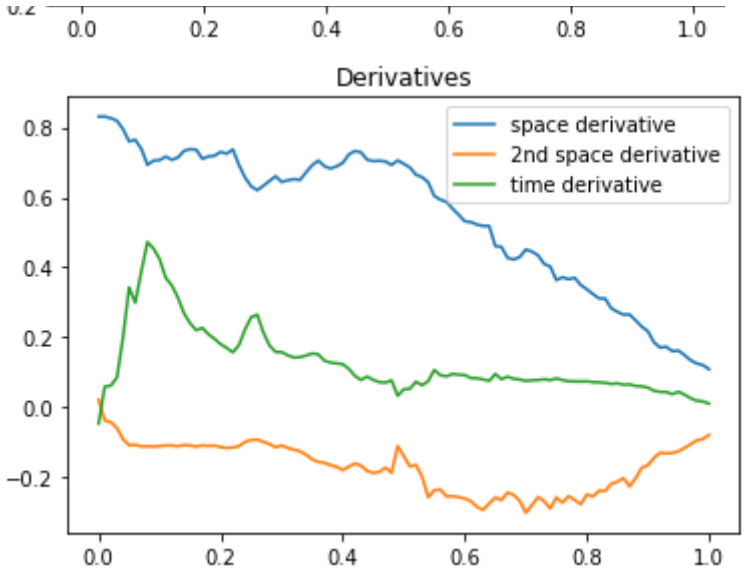


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811th Epoch:
traing loss is 0.007853060960769653, test loss is 0.0036208312958478928, and learning rate is

821th Epoch:
traing loss is 0.013470466248691082, test loss is 0.014293625950813293, and learning rate is

831th Epoch:
traing loss is 0.005343160592019558, test loss is 0.005256006494164467, and learning rate is

841th Epoch:
traing loss is 0.004318578634411097, test loss is 0.004201287869364023, and learning rate is

851th Epoch:
traing loss is 0.005707475822418928, test loss is 0.004755805712193251, and learning rate is

861th Epoch:
traing loss is 0.004168103449046612, test loss is 0.002360084094107151, and learning rate is

871th Epoch:
traing loss is 0.004394847899675369, test loss is 0.004774951376020908, and learning rate is

881th Epoch:
traing loss is 0.003002069890499115, test loss is 0.00232604774646461, and learning rate is 0

891th Epoch:
traing loss is 0.0030360212549567223, test loss is 0.0026121544651687145, and learning rate i

901th Epoch:
traing loss is 0.006984333973377943, test loss is 0.0019507232354953885, and learning rate is
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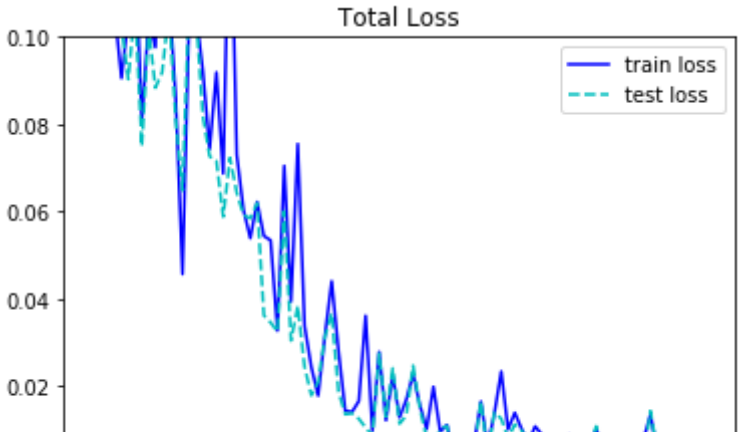
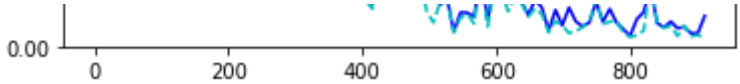
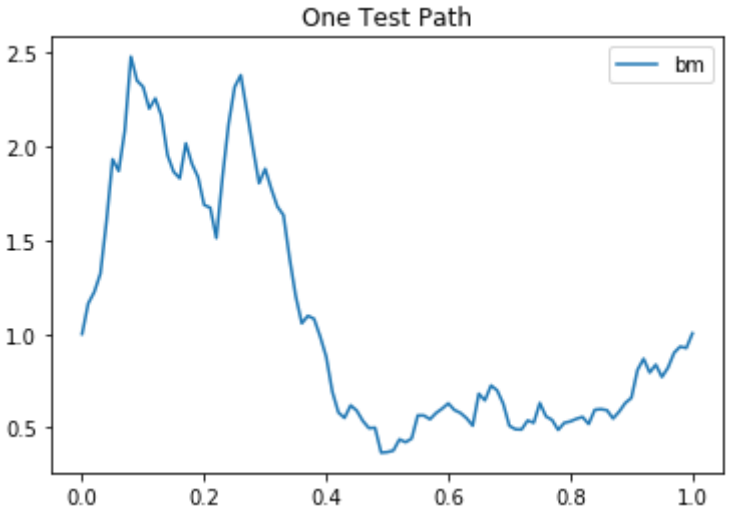
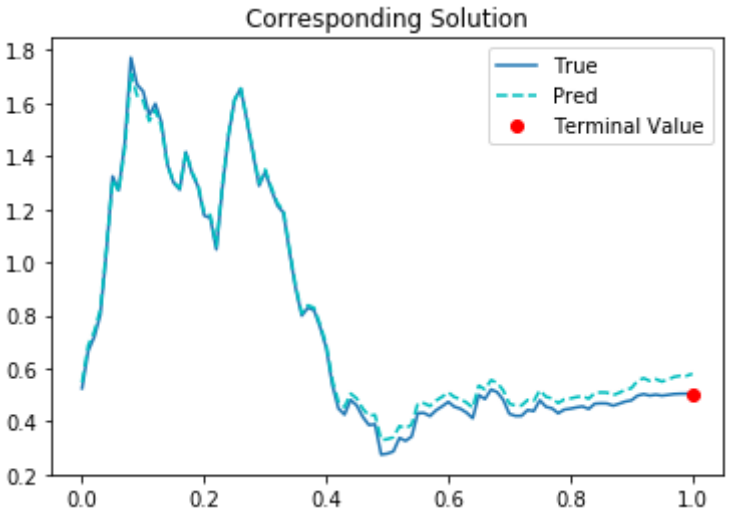
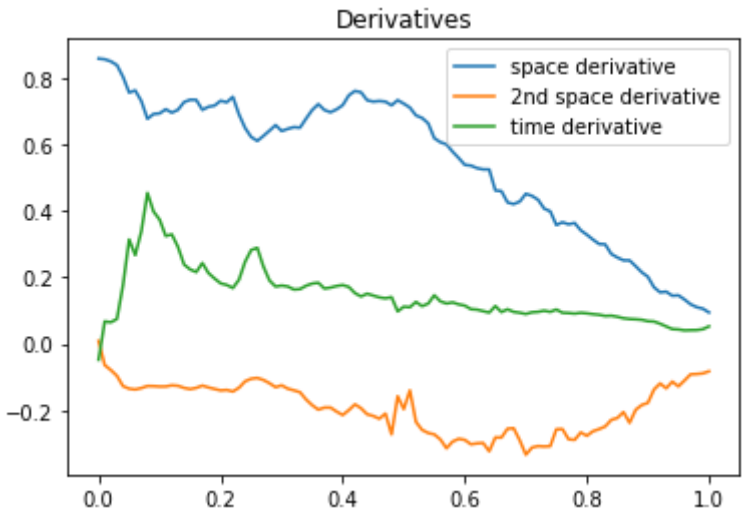











```
911th Epoch:
traing loss is 0.0022508916445076466, test loss is 0.0013465716037899256, and learning rate i

921th Epoch:
traing loss is 0.0025456841103732586, test loss is 0.002310937037691474, and learning rate is

931th Epoch:
traing loss is 0.0038812868297100067, test loss is 0.0033348239958286285, and learning rate i

941th Epoch:
traing loss is 0.005309956148266792, test loss is 0.006403265055269003, and learning rate is
```

```
951th Epoch:
traing loss is 0.005729391239583492, test loss is 0.004756008740514517, and learning rate is

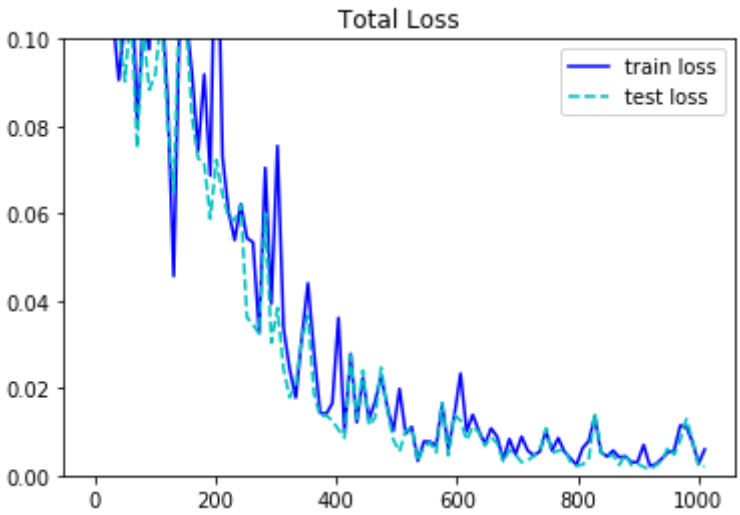
961th Epoch:
traing loss is 0.011448727920651436, test loss is 0.008102201856672764, and learning rate is

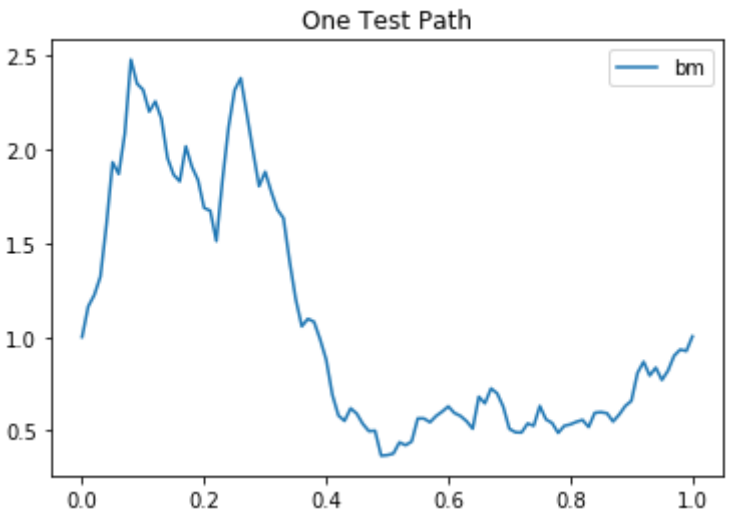
971th Epoch:
traing loss is 0.011082891374826431, test loss is 0.012896006926894188, and learning rate is

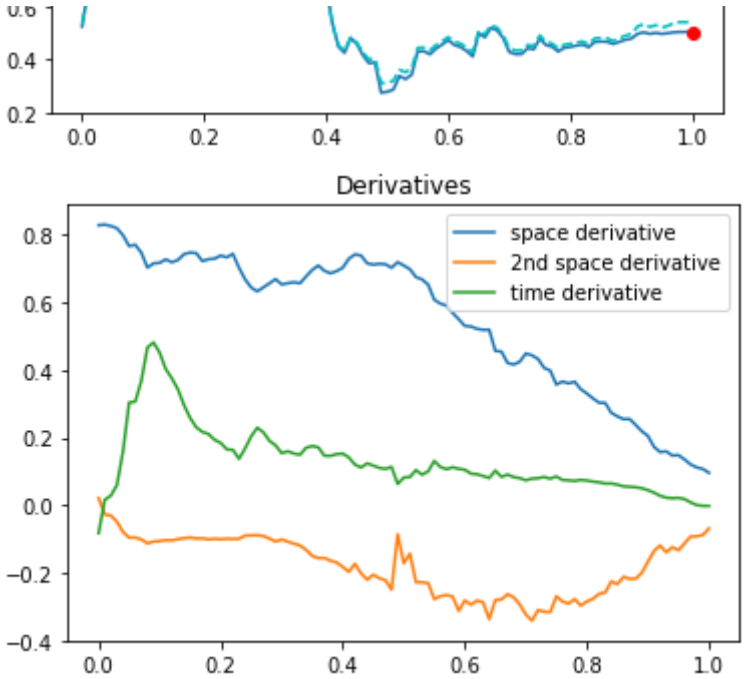
981th Epoch:
traing loss is 0.0076649426482617855, test loss is 0.008110462687909603, and learning rate is

991th Epoch:
traing loss is 0.002723106648772955, test loss is 0.002163039054721594, and learning rate is

1001th Epoch:
traing loss is 0.005938567221164703, test loss is 0.002061741892248392, and learning rate is
```





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1011th Epoch:
traing loss is 0.0030045374296605587, test loss is 0.0015749306185171008, and learning rate i

1021th Epoch:
traing loss is 0.00659715523943305, test loss is 0.006637251004576683, and learning rate is 0

1031th Epoch:
traing loss is 0.0032823814544826746, test loss is 0.003447955707088113, and learning rate is

1041th Epoch:
traing loss is 0.006192763336002827, test loss is 0.006988644599914551, and learning rate is

1051th Epoch:
traing loss is 0.004824972245842218, test loss is 0.00420595146715641, and learning rate is 0

1061th Epoch:
traing loss is 0.01221160776913166, test loss is 0.0066541521809995174, and learning rate is

1071th Epoch:
traing loss is 0.0037139039486646652, test loss is 0.003960666712373495, and learning rate is

1081th Epoch:
traing loss is 0.010925797745585442, test loss is 0.009340780787169933, and learning rate is

1091th Epoch:
traing loss is 0.007618919480592012, test loss is 0.006356942001730204, and learning rate is

1101th Epoch:
traing loss is 0.01456054300069809, test loss is 0.004327085334807634, and learning rate is 0
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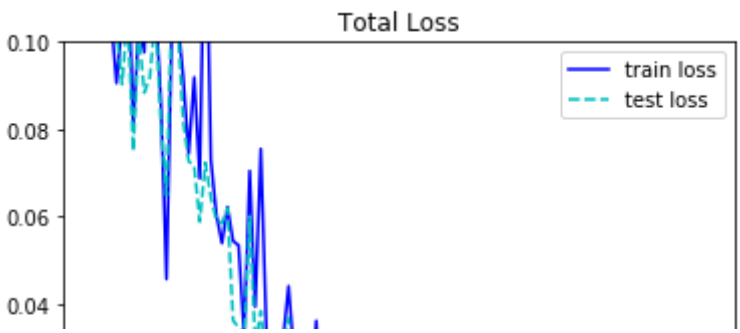

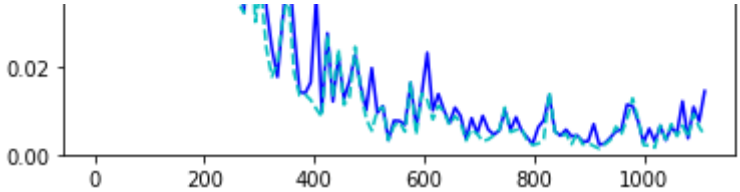

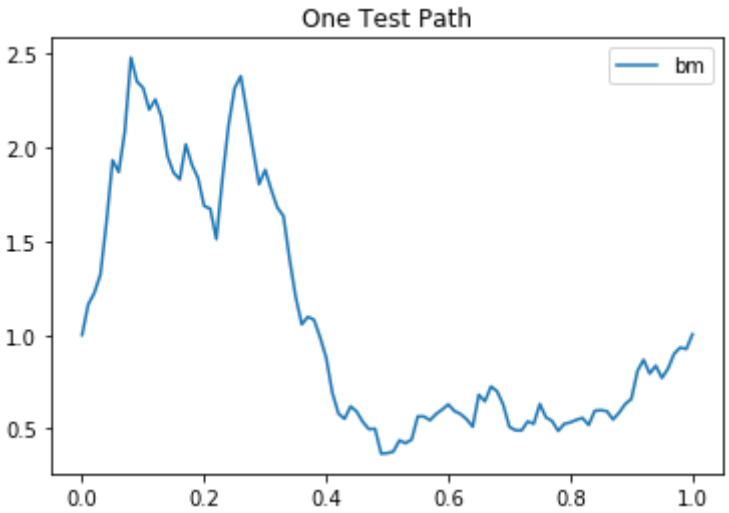

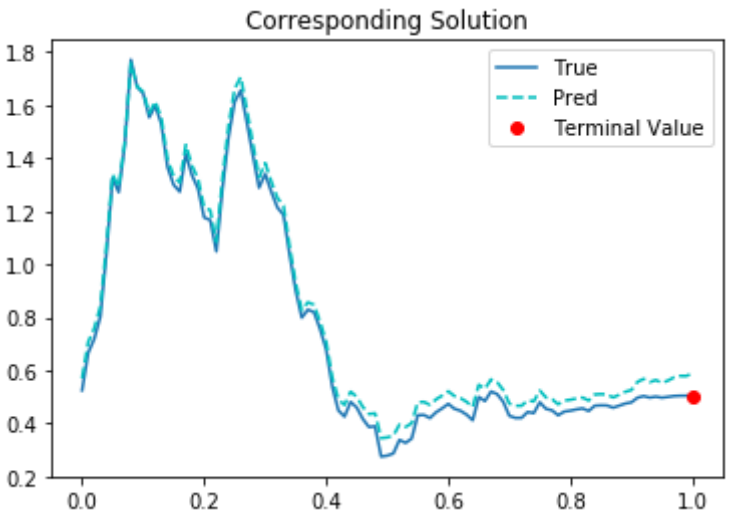

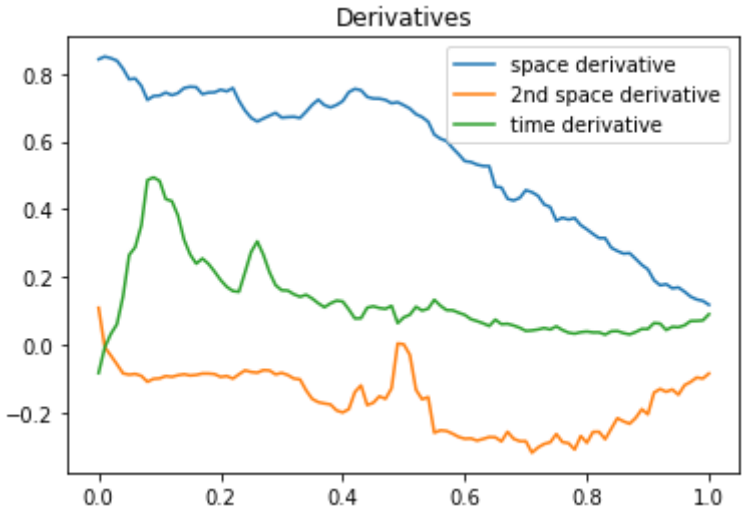


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1111th Epoch:
traing loss is 0.005518908146768808, test loss is 0.004063802771270275, and learning rate is

1121th Epoch:
traing loss is 0.006737154442816973, test loss is 0.005983466282486916, and learning rate is

1131th Epoch:
traing loss is 0.0028335859533399343, test loss is 0.003352662781253457, and learning rate is
```

```
1141th Epoch:
traing loss is 0.006802571937441826, test loss is 0.006551042664796114, and learning rate is

1151th Epoch:
traing loss is 0.004659063648432493, test loss is 0.004179652780294418, and learning rate is

1161th Epoch:
traing loss is 0.014214393682777882, test loss is 0.008201668970286846, and learning rate is

1171th Epoch:
traing loss is 0.008226285688579082, test loss is 0.00871012918651104, and learning rate is 0

1181th Epoch:
traing loss is 0.0028274308424443007, test loss is 0.0022378084249794483, and learning rate i

1191th Epoch:
traing loss is 0.003535543568432331, test loss is 0.0033342360984534025, and learning rate is

1201th Epoch:
traing loss is 0.005228555295616388, test loss is 0.001793279661796987, and learning rate is
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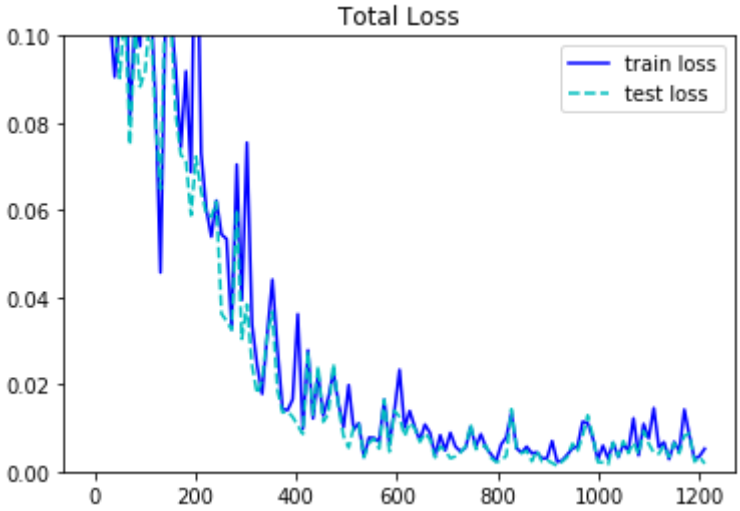

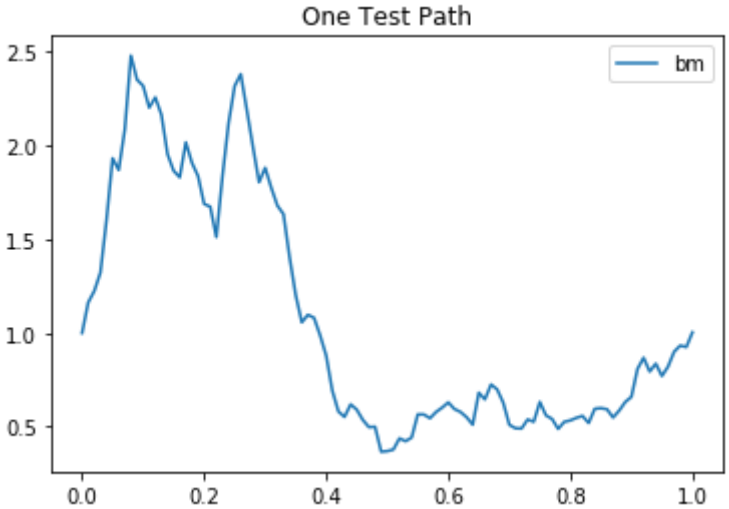

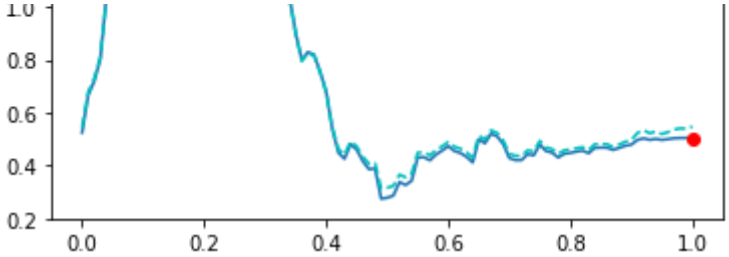

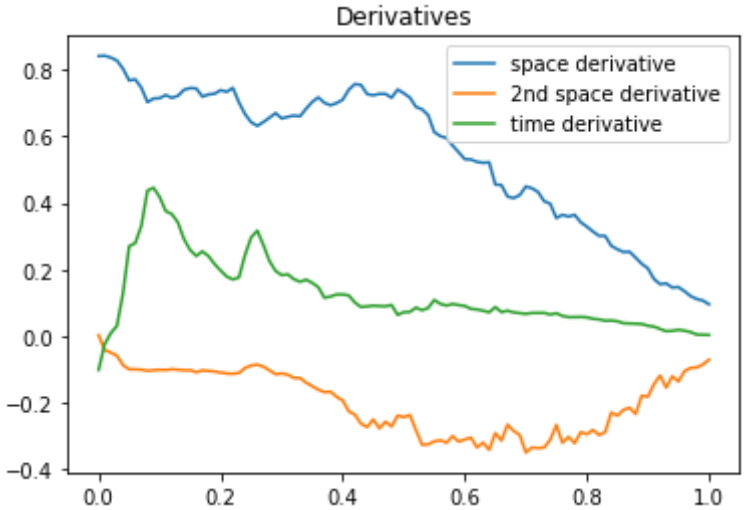


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1211th Epoch:
traing loss is 0.004081549122929573, test loss is 0.0033763402607291937, and learning rate is

1221th Epoch:
traing loss is 0.0034659483935683966, test loss is 0.002790390281006694, and learning rate is

1231th Epoch:
traing loss is 0.002845368580892682, test loss is 0.0027624606154859066, and learning rate is

1241th Epoch:
traing loss is 0.003126339754089713, test loss is 0.003343799849972129, and learning rate is

1251th Epoch:
traing loss is 0.0015088982181623578, test loss is 0.001441791420802474, and learning rate is

1261th Epoch:
traing loss is 0.010233468376100063, test loss is 0.007364999502897263, and learning rate is

1271th Epoch:
traing loss is 0.008379102684557438, test loss is 0.009179436601698399, and learning rate is

1281th Epoch:
traing loss is 0.006125265266746283, test loss is 0.004961266182363033, and learning rate is

1291th Epoch:
traing loss is 0.0033438163809478283, test loss is 0.0028948241379112005, and learning rate i

1301th Epoch:
traing loss is 0.0056725540198385715, test loss is 0.0030946091283112764, and learning rate i
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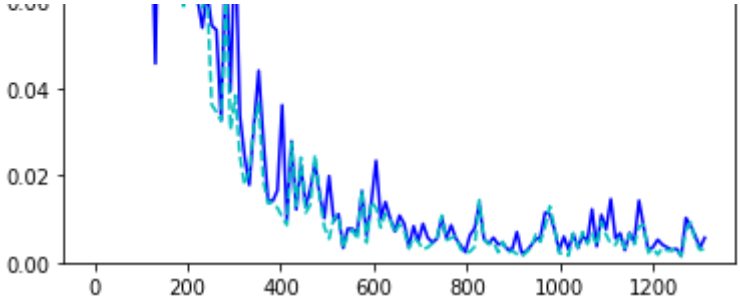

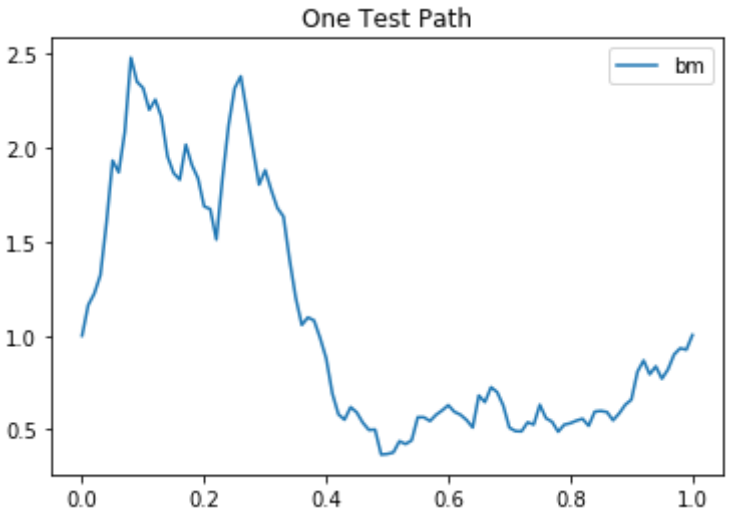

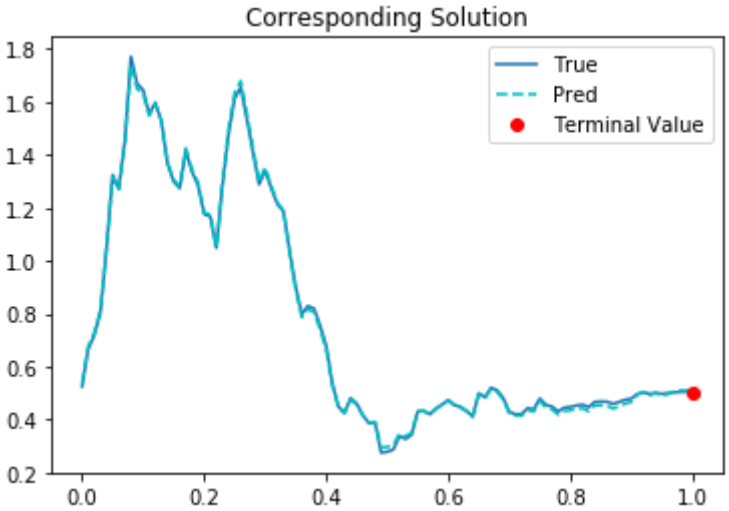

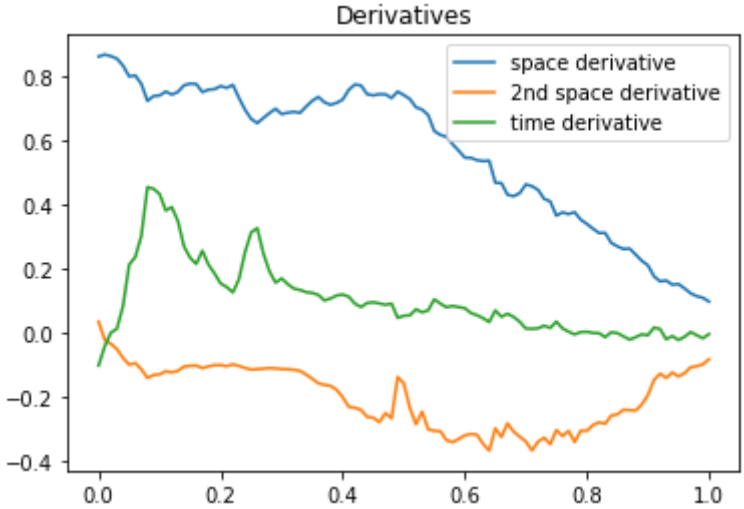

1311th Epoch:
traing loss is 0.00310142757371068, test loss is 0.0026394438464194536, and learning rate is

1321th Epoch:
traing loss is 0.001559991156682372, test loss is 0.001555071328766644, and learning rate is

```
1331th Epoch:
traing loss is 0.0012303938856348395, test loss is 0.0013504248345270753, and learning rate i

1341th Epoch:
traing loss is 0.002365649212151766, test loss is 0.0021028518676757812, and learning rate is

1351th Epoch:
traing loss is 0.004238607361912727, test loss is 0.003757848171517253, and learning rate is

1361th Epoch:
traing loss is 0.017329739406704903, test loss is 0.01249083224684, and learning rate is 0.00

1371th Epoch:
traing loss is 0.005093522369861603, test loss is 0.005270997993648052, and learning rate is

1381th Epoch:
traing loss is 0.006000186782330275, test loss is 0.004970268812030554, and learning rate is

1391th Epoch:
traing loss is 0.002323657274246216, test loss is 0.002110961591824889, and learning rate is

1401th Epoch:
traing loss is 0.0042419712990522385, test loss is 0.0020483890548348427, and learning rate i
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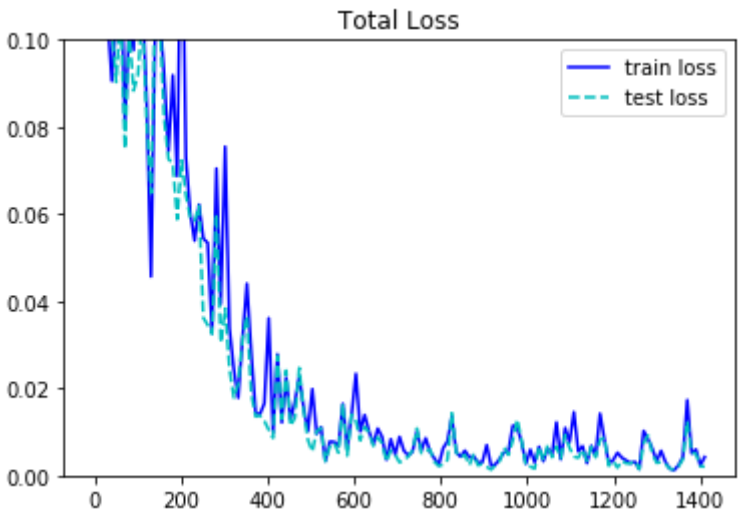

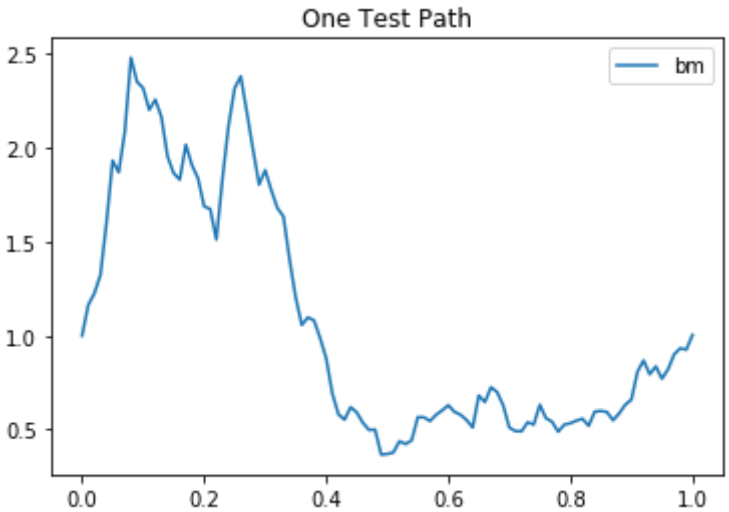

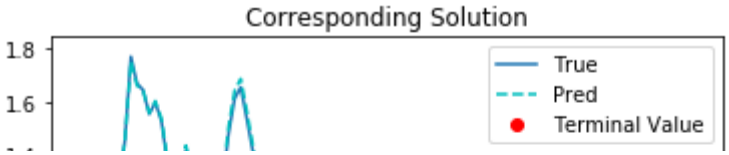

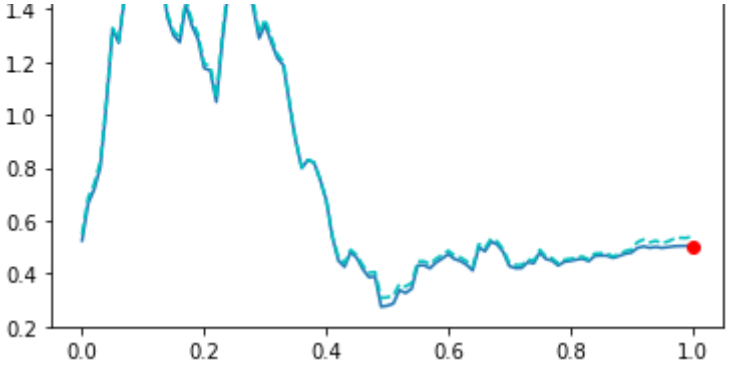

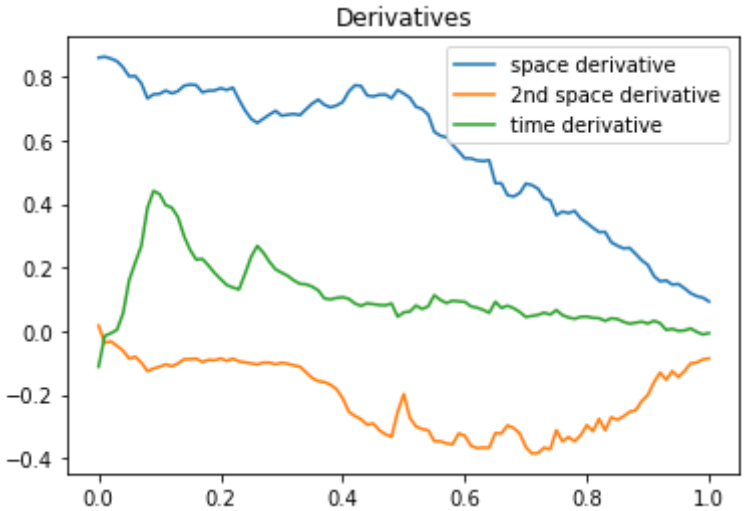


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1411th Epoch:
traing loss is 0.0017970765475183725, test loss is 0.0013710373314097524, and learning rate i

1421th Epoch:
traing loss is 0.002077588113024831, test loss is 0.0018168764654546976, and learning rate is

1431th Epoch:
traing loss is 0.0018368784803897142, test loss is 0.0019767391495406628, and learning rate i

1441th Epoch:
traing loss is 0.003364137839525938, test loss is 0.003671936923637986, and learning rate is

1451th Epoch:
traing loss is 0.002079453319311142, test loss is 0.00181692058686167, and learning rate is 0

1461th Epoch:
traing loss is 0.00887671671807766, test loss is 0.007223797962069511, and learning rate is 0

1471th Epoch:
traing loss is 0.006583343725651503, test loss is 0.006985141430050135, and learning rate is

1481th Epoch:
traing loss is 0.007229122798889875, test loss is 0.006355857476592064, and learning rate is

1491th Epoch:
traing loss is 0.003060583956539631, test loss is 0.002598644932731986, and learning rate is

1501th Epoch:
traing loss is 0.005809570197016001, test loss is 0.004027181304991245, and learning rate is
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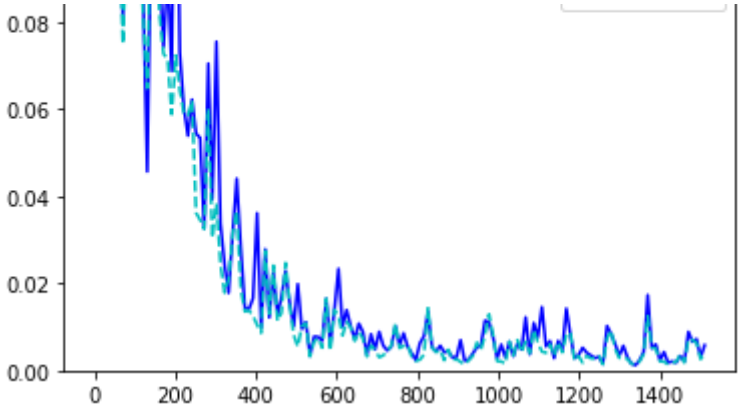

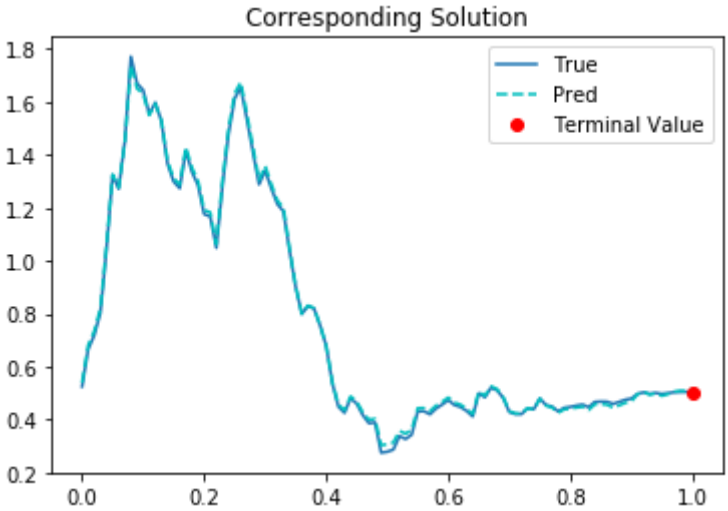

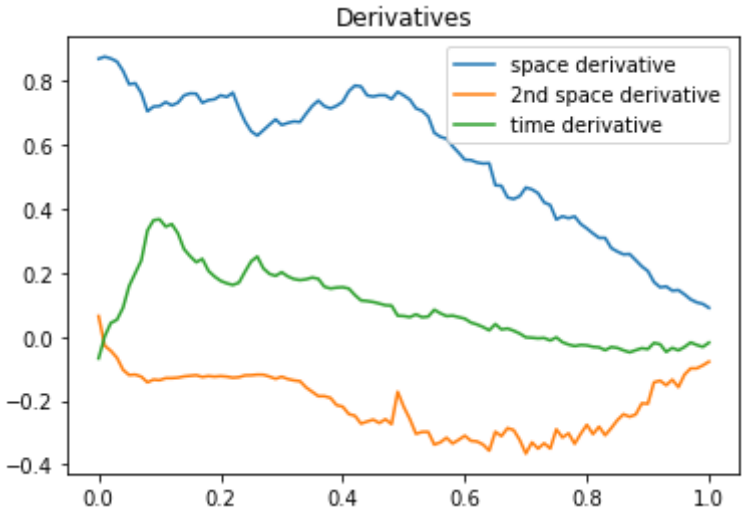

1511th Epoch:
traing loss is 0.002647962886840105, test loss is 0.002035844372585416, and learning rate is

```
1521th Epoch:
traing loss is 0.0033292346633970737, test loss is 0.003864315105602145, and learning rate is

1531th Epoch:
traing loss is 0.0013654815265908837, test loss is 0.0014749583788216114, and learning rate i

1541th Epoch:
traing loss is 0.002877846360206604, test loss is 0.003120971145108342, and learning rate is

1551th Epoch:
traing loss is 0.0016815649578347802, test loss is 0.0016534164315089583, and learning rate i

1561th Epoch:
traing loss is 0.007022638339549303, test loss is 0.0059537021443247795, and learning rate is

1571th Epoch:
traing loss is 0.003439475316554308, test loss is 0.0035660164430737495, and learning rate is

1581th Epoch:
traing loss is 0.0015057207783684134, test loss is 0.0013197805965319276, and learning rate i

1591th Epoch:
traing loss is 0.0009354367502965033, test loss is 0.0009518570732325315, and learning rate i

1601th Epoch:
traing loss is 0.0034310738556087017, test loss is 0.0019646072760224342, and learning rate i
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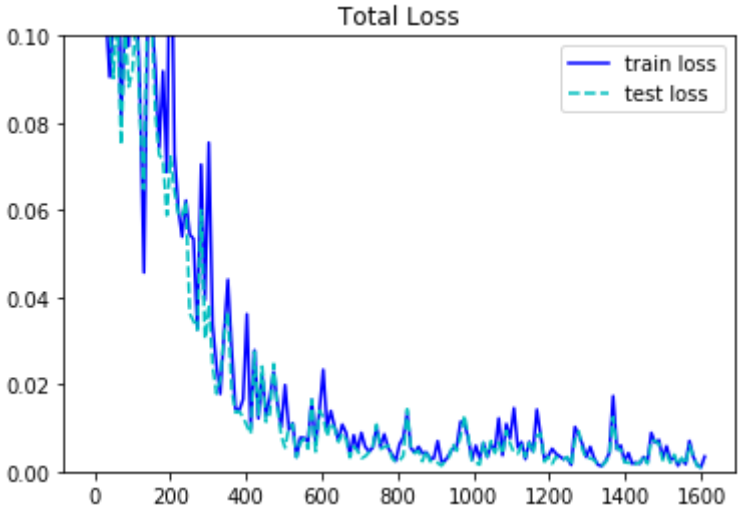
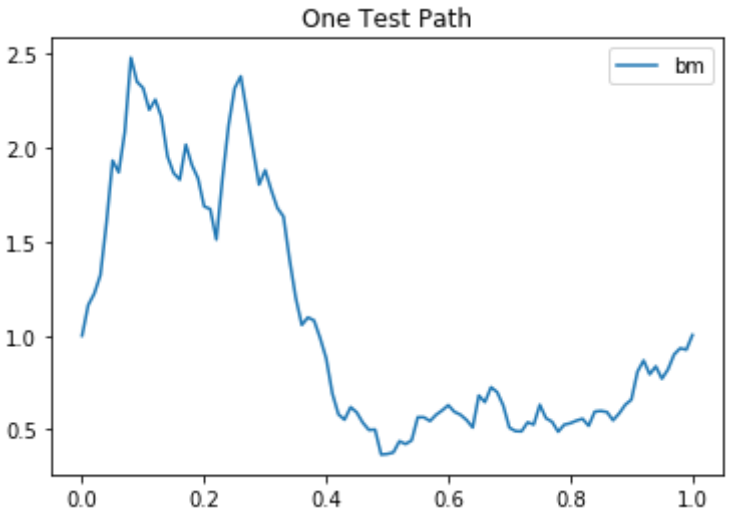
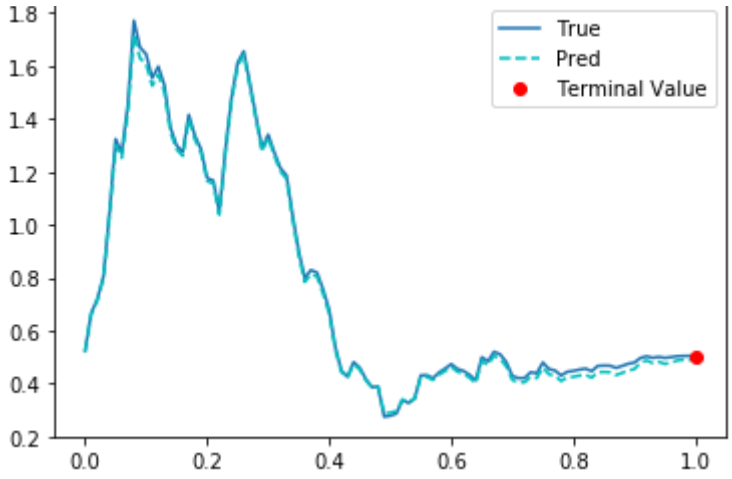
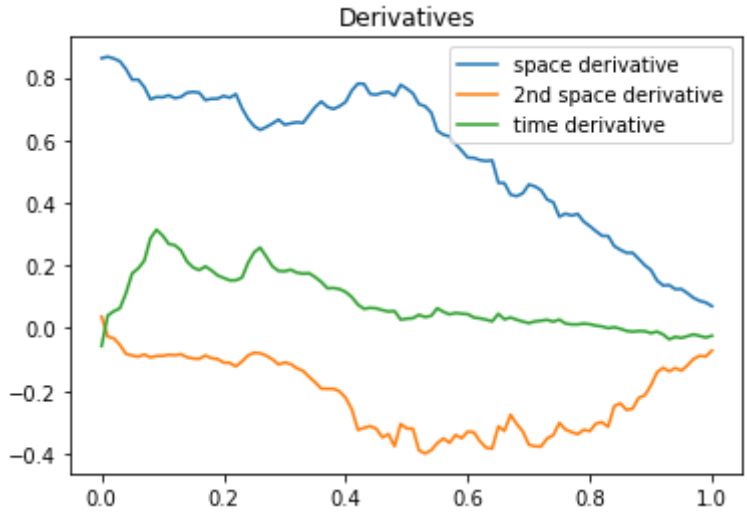




Corresponding Solution



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1611th Epoch:
traing loss is 0.0025752848014235497, test loss is 0.0017187756020575762, and learning rate i

1621th Epoch:
traing loss is 0.0013359073782339692, test loss is 0.0014957489911466837, and learning rate i

1631th Epoch:
traing loss is 0.0007906437385827303, test loss is 0.0009491575765423477, and learning rate i

1641th Epoch:
traing loss is 0.0023438327480107546, test loss is 0.0020236882846802473, and learning rate i

1651th Epoch:
traing loss is 0.0035744505003094673, test loss is 0.003415085608139634, and learning rate is

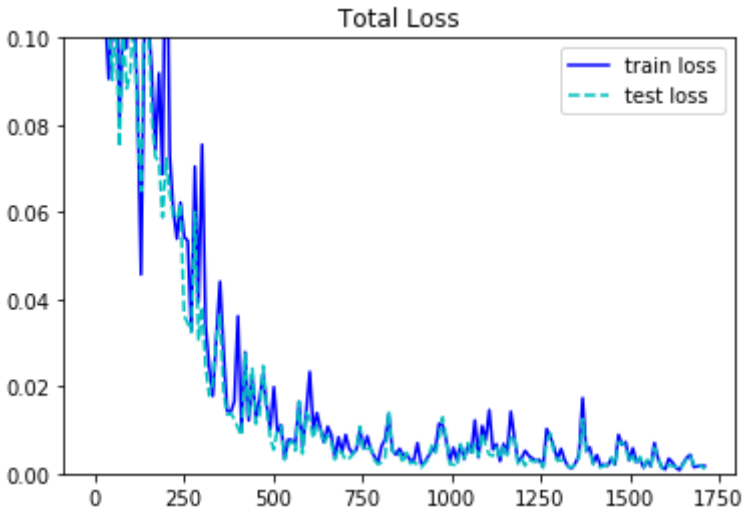
1661th Epoch:
traing loss is 0.004319101572036743, test loss is 0.004123153630644083, and learning rate is

1671th Epoch:
traing loss is 0.001472967560403049, test loss is 0.0016820463351905346, and learning rate is

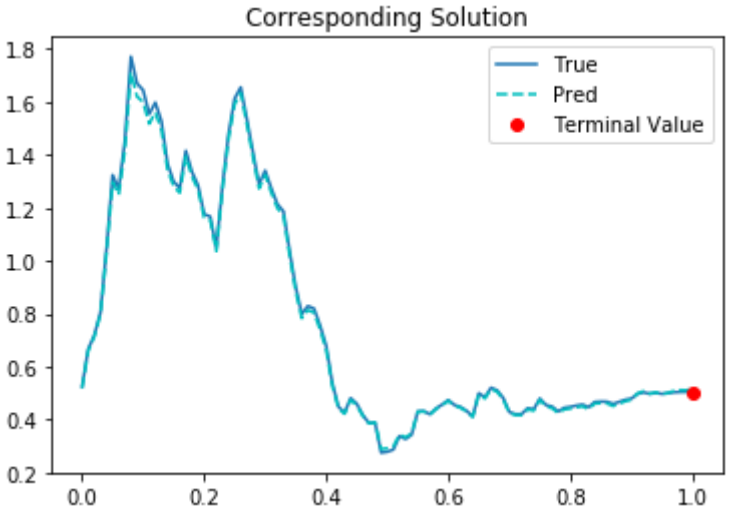
1681th Epoch:
traing loss is 0.0016307890182361007, test loss is 0.0015977869043126702, and learning rate i

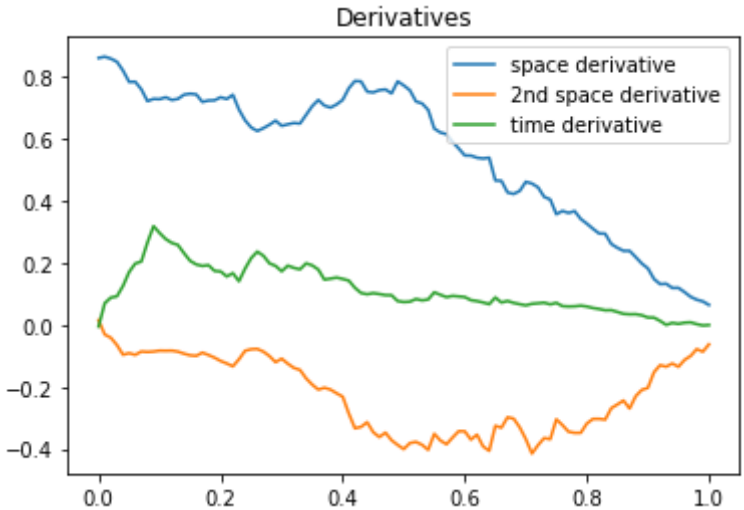
1691th Epoch:
traing loss is 0.0018659421475604177, test loss is 0.0018332403851673007, and learning rate i

1701th Epoch:
traing loss is 0.0017206004122272134, test loss is 0.0009413372026756406, and learning rate i
```


Total Loss
train loss
test loss
0.10
0.08
0.06
0.04
0.02
0.00
0
250
500
750
1000
1250
1500
1750

One Test Path
bm
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0


Corresponding Solution
True
Pred
Terminal Value
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


Derivatives
space derivative
2nd space derivative
time derivative
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0.0
0.2
0.4
0.6
0.8
1.0

```
1711th Epoch:
traing loss is 0.0018624758813530207, test loss is 0.001127137802541256, and learning rate is

1721th Epoch:
traing loss is 0.0011498953681439161, test loss is 0.0012092252727597952, and learning rate i

1731th Epoch:
traing loss is 0.001085592433810234, test loss is 0.0013112957822158933, and learning rate is

1741th Epoch:
traing loss is 0.003029429819434881, test loss is 0.002861452056095004, and learning rate is

1751th Epoch:
traing loss is 0.00143859488889575, test loss is 0.0014001545496284962, and learning rate is

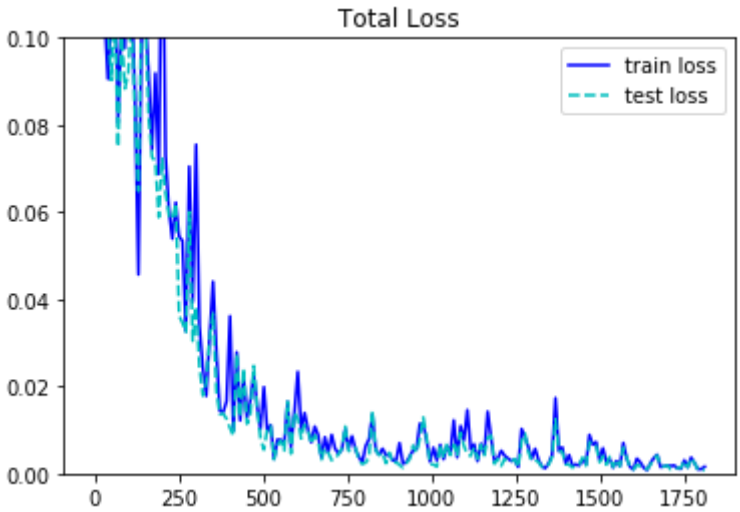
1761th Epoch:
traing loss is 0.0036614772398024797, test loss is 0.003486165078356862, and learning rate is

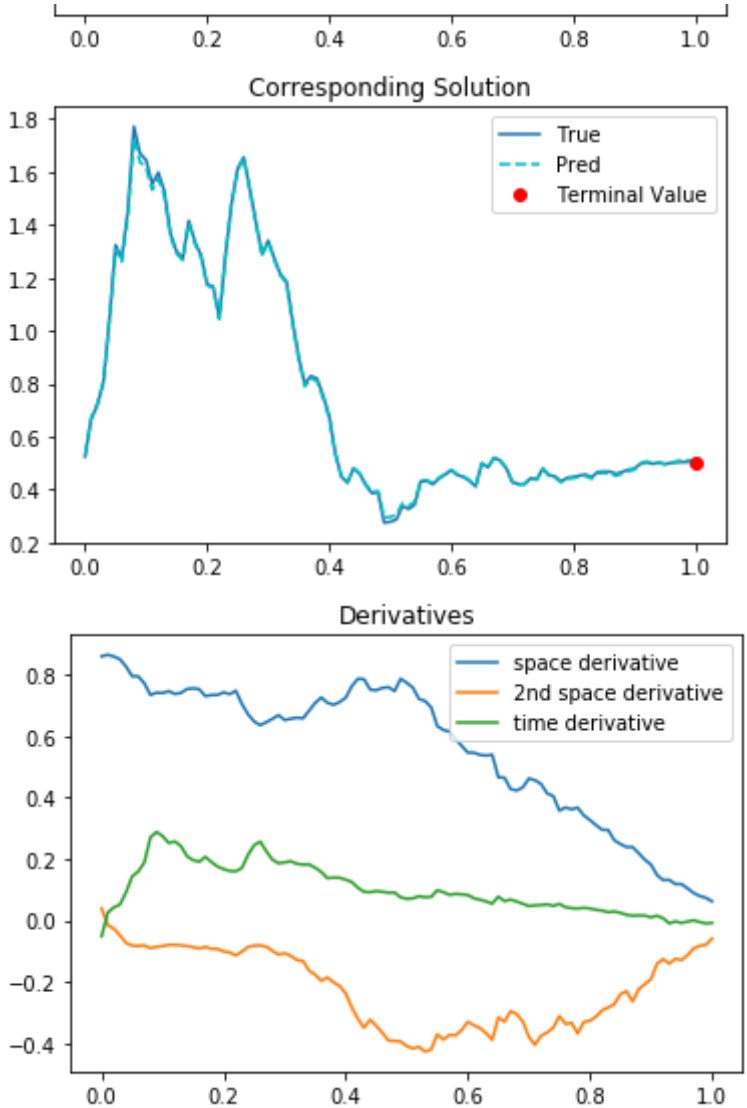
1771th Epoch:
traing loss is 0.002400720026344061, test loss is 0.0024272995069622993, and learning rate is

1781th Epoch:
traing loss is 0.0009895242983475327, test loss is 0.0008521608542650938, and learning rate i

1791th Epoch:
traing loss is 0.0010066338581964374, test loss is 0.0008857157663442194, and learning rate i

1801th Epoch:
traing loss is 0.0015561024192720652, test loss is 0.0007767925853841007, and learning rate i
```






```
1811th Epoch:
traing loss is 0.0010405922075733542, test loss is 0.0008166424231603742, and learning rate i

1821th Epoch:
traing loss is 0.0007294875103980303, test loss is 0.000692568311933428, and learning rate is

1831th Epoch:
traing loss is 0.0006502567557618022, test loss is 0.0007529787835665047, and learning rate i

1841th Epoch:
traing loss is 0.0021134763956069946, test loss is 0.0021364435087889433, and learning rate i

1851th Epoch:
traing loss is 0.003279841272160411, test loss is 0.0030248810071498156, and learning rate is

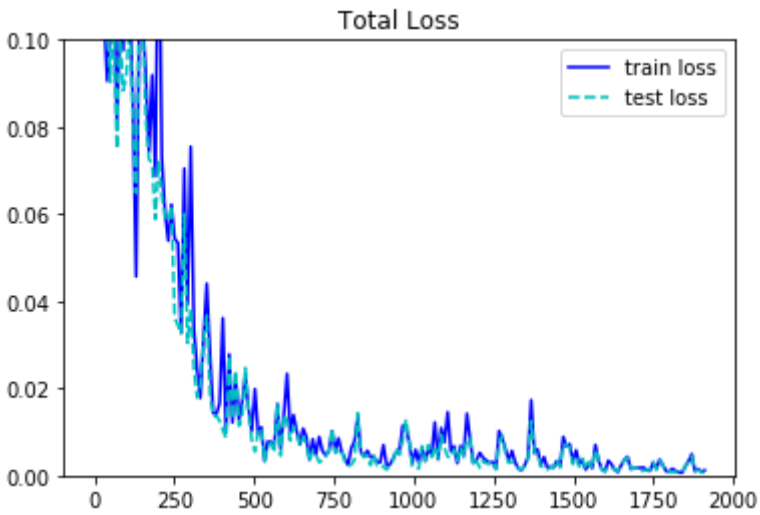
1861th Epoch:
traing loss is 0.004924208391457796, test loss is 0.004409229848533869, and learning rate is

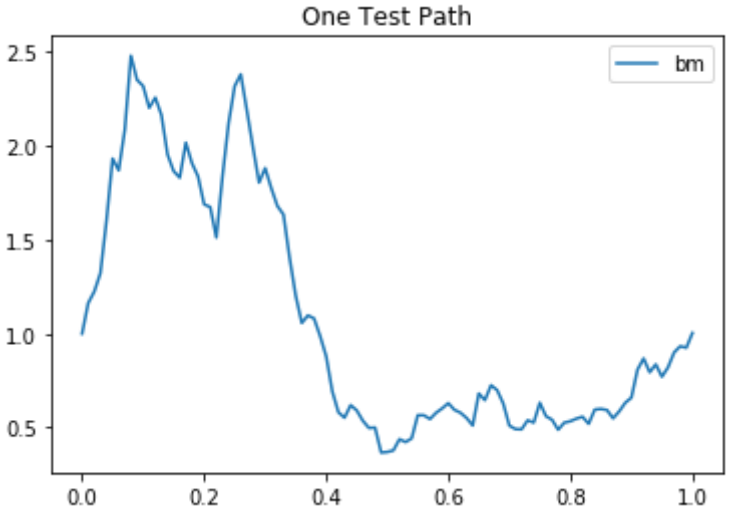
1871th Epoch:
traing loss is 0.0009240471990779042, test loss is 0.0010522797238081694, and learning rate i

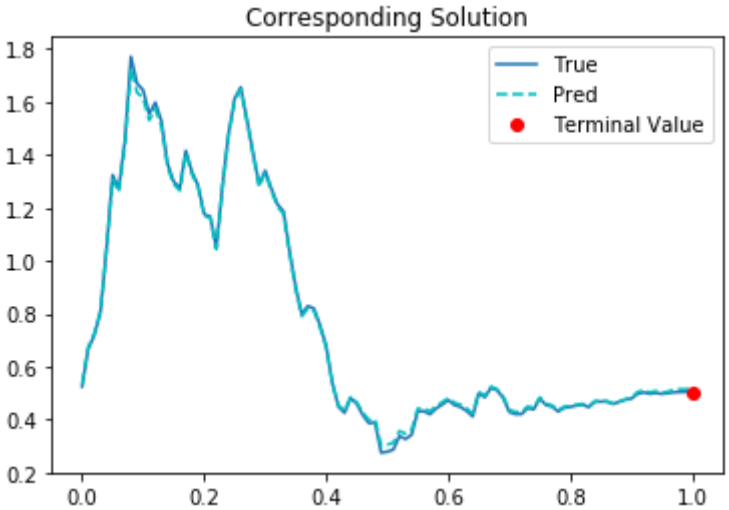
1881th Epoch:
traing loss is 0.0014824767131358385, test loss is 0.0012376883532851934, and learning rate i

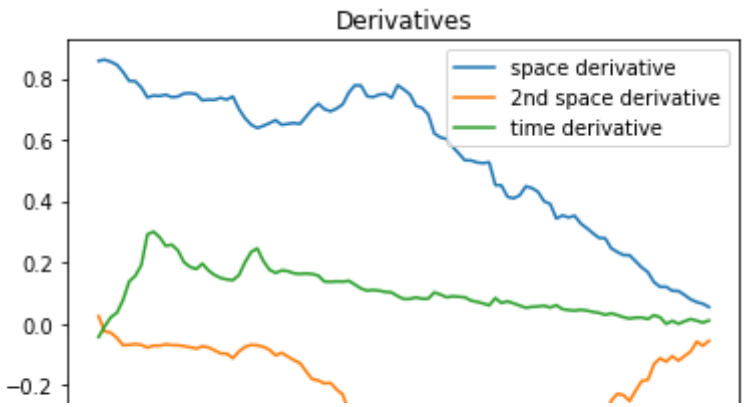
1891th Epoch:
traing loss is 0.0008755962480790913, test loss is 0.0007585376151837409, and learning rate i
```

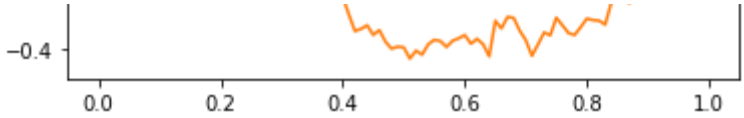
```
1901th Epoch:
traing loss is 0.0011917406227439642, test loss is 0.0006174746085889637, and learning rate i
```












```
1911th Epoch:
traing loss is 0.0010545895202085376, test loss is 0.0007332325330935419, and learning rate i

1921th Epoch:
traing loss is 0.0007374646374955773, test loss is 0.0007125681731849909, and learning rate i

1931th Epoch:
traing loss is 0.0008177891722880304, test loss is 0.0009971120161935687, and learning rate i

1941th Epoch:
traing loss is 0.004145507700741291, test loss is 0.003186540910974145, and learning rate is

1951th Epoch:
traing loss is 0.002334322314709425, test loss is 0.0020536372903734446, and learning rate is

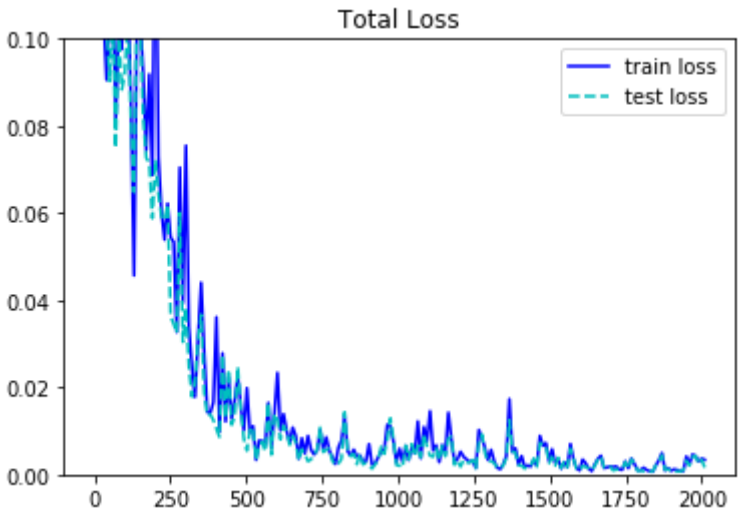
1961th Epoch:
traing loss is 0.0045862565748393536, test loss is 0.0040489425882697105, and learning rate i

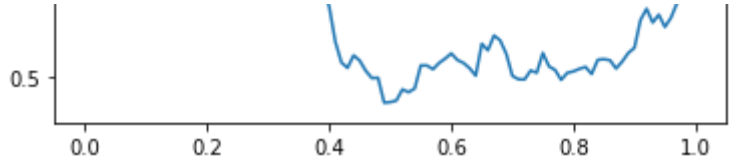
1971th Epoch:
traing loss is 0.004253133200109005, test loss is 0.004254811909049749, and learning rate is

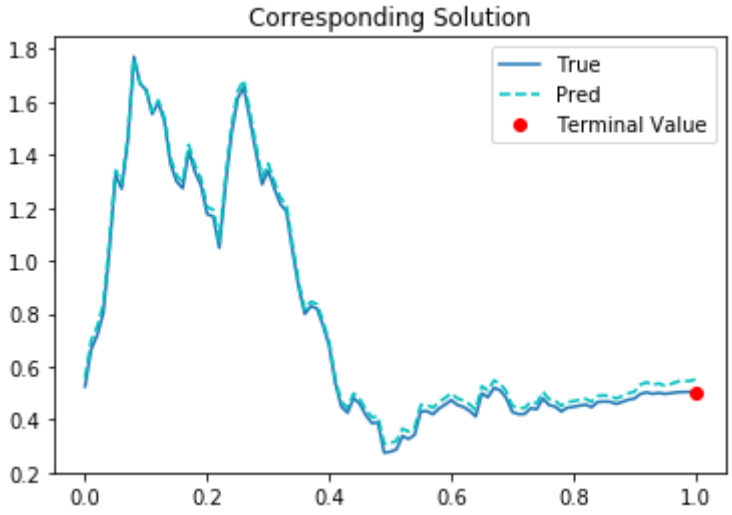
1981th Epoch:
traing loss is 0.0030024831648916006, test loss is 0.0024592923000454903, and learning rate i

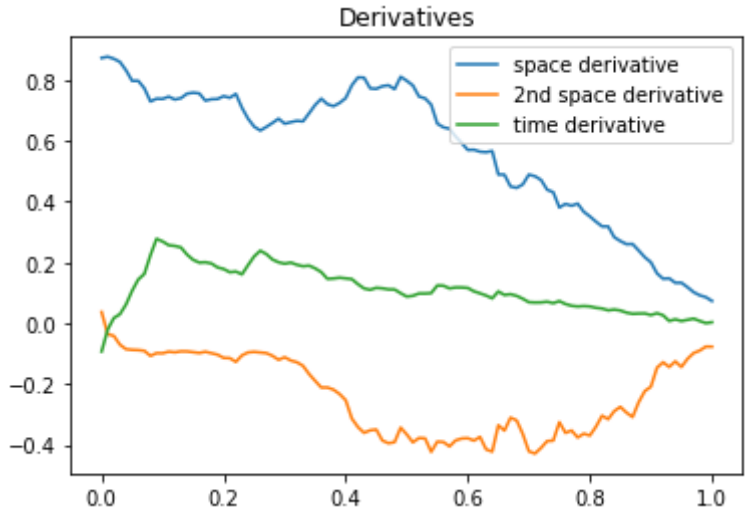
1991th Epoch:
traing loss is 0.003648066194728017, test loss is 0.0034067940432578325, and learning rate is

2001th Epoch:
traing loss is 0.003382725641131401, test loss is 0.0015903611201792955, and learning rate is
```










```
2011th Epoch:
traing loss is 0.0009663946693763137, test loss is 0.0007488819537684321, and learning rate i

2021th Epoch:
traing loss is 0.000894157390575856, test loss is 0.0008995358948595822, and learning rate is

2031th Epoch:
traing loss is 0.0007438243483193219, test loss is 0.0008699946920387447, and learning rate i

2041th Epoch:
traing loss is 0.0018643297953531146, test loss is 0.0021507504861801863, and learning rate i

2051th Epoch:
traing loss is 0.000891357020009309, test loss is 0.0009248705464415252, and learning rate is

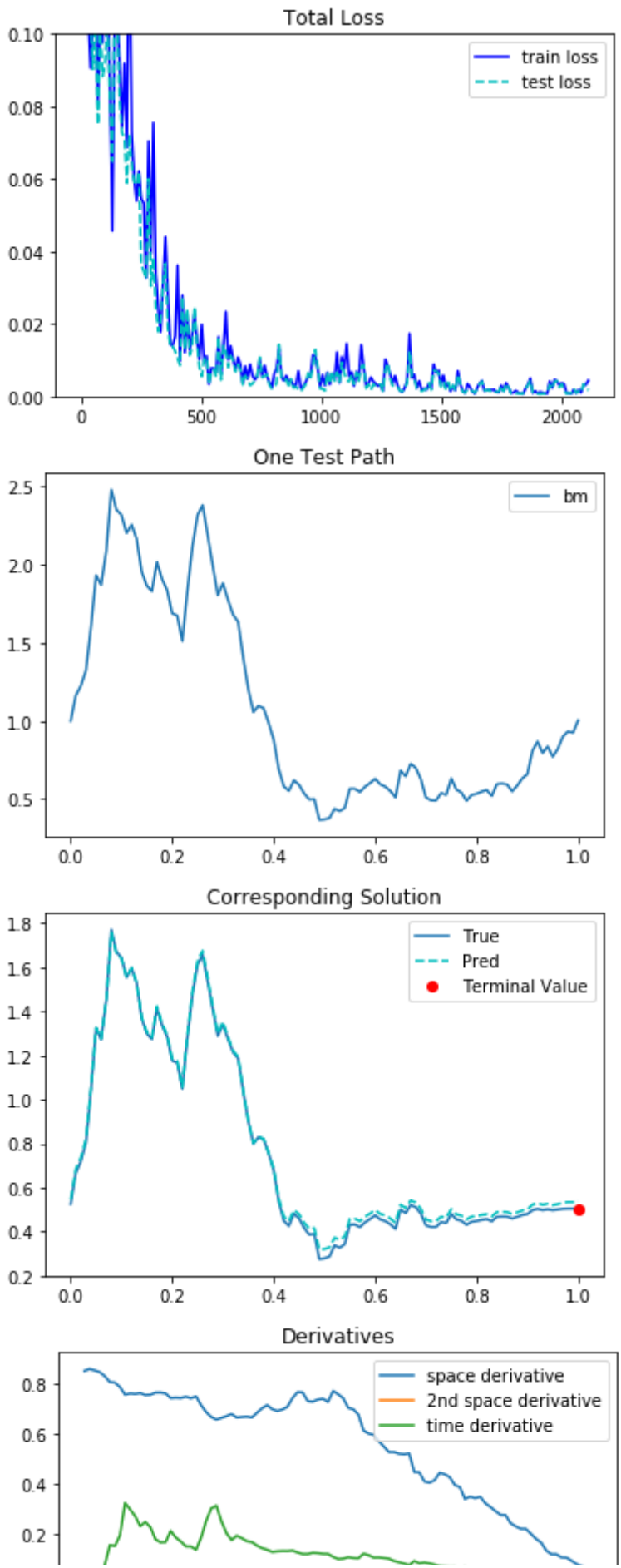
2061th Epoch:
traing loss is 0.0018012946238741279, test loss is 0.0014568337937816978, and learning rate i

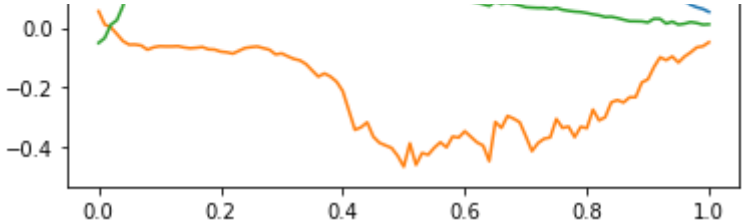
2071th Epoch:
traing loss is 0.0009999240282922983, test loss is 0.0010273553198203444, and learning rate i

2081th Epoch:
traing loss is 0.003172253957018256, test loss is 0.0034092573914676905, and learning rate is
```

```
2091th Epoch:
traing loss is 0.00319733377546072, test loss is 0.00266986689530313, and learning rate is 0.

2101th Epoch:
traing loss is 0.004391457419842482, test loss is 0.0016251815250143409, and learning rate is
```






```
2111th Epoch:
traing loss is 0.005269259680062532, test loss is 0.004603834822773933, and learning rate is

2121th Epoch:
traing loss is 0.00291791302151978, test loss is 0.0027207029052078724, and learning rate is

2131th Epoch:
traing loss is 0.002140561817213893, test loss is 0.002201796742156148, and learning rate is

2141th Epoch:
traing loss is 0.008610202930867672, test loss is 0.009951218962669373, and learning rate is

2151th Epoch:
traing loss is 0.00268154195509851, test loss is 0.002493801526725292, and learning rate is 0

2161th Epoch:
traing loss is 0.003101780312135815 6, test loss is 0.0027272275183349848, and learning rate i

2171th Epoch:
traing loss is 0.0017228687647730112, test loss is 0.0017660566372796893, and learning rate i

2181th Epoch:
traing loss is 0.00420153746381402, test loss is 0.0038586752489209175, and learning rate is

2191th Epoch:
traing loss is 0.0032515095081180334, test loss is 0.0023011553566902876, and learning rate i

2201th Epoch:
traing loss is 0.007121616043150425, test loss is 0.0029383401852101088, and learning rate is
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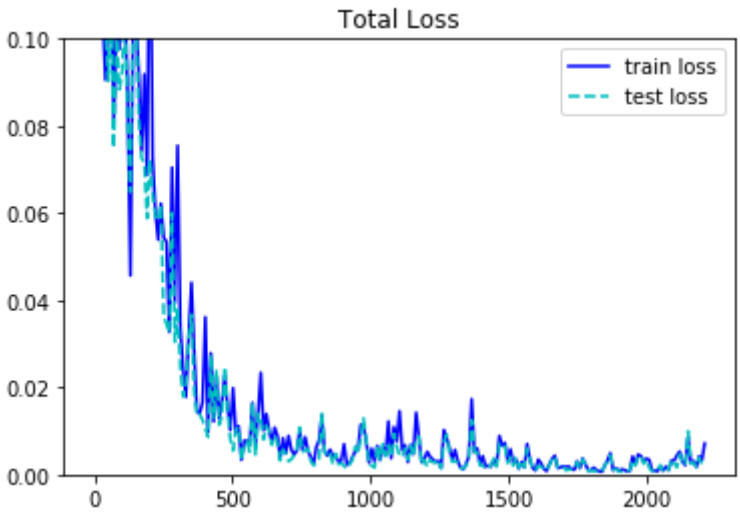
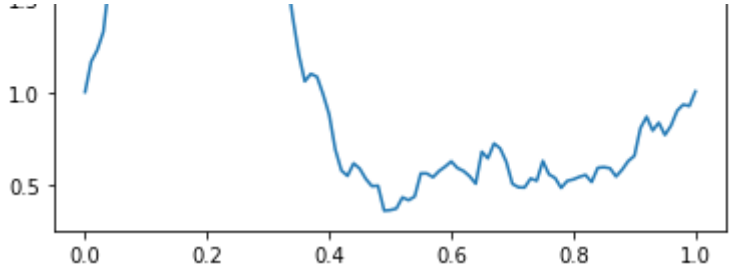
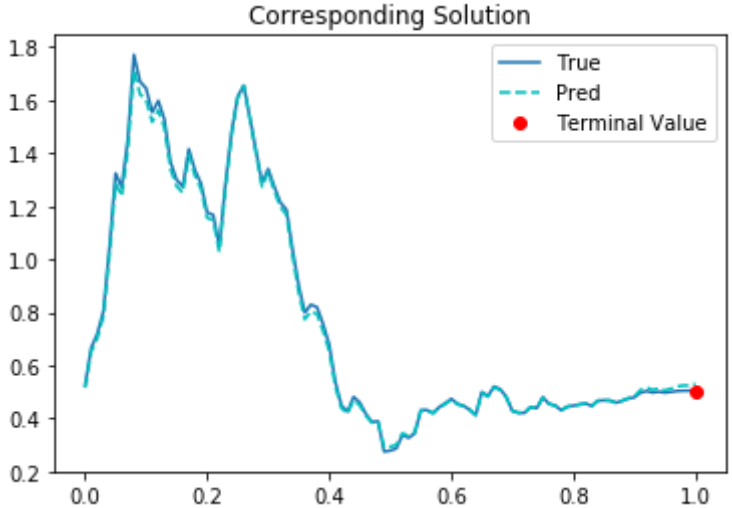
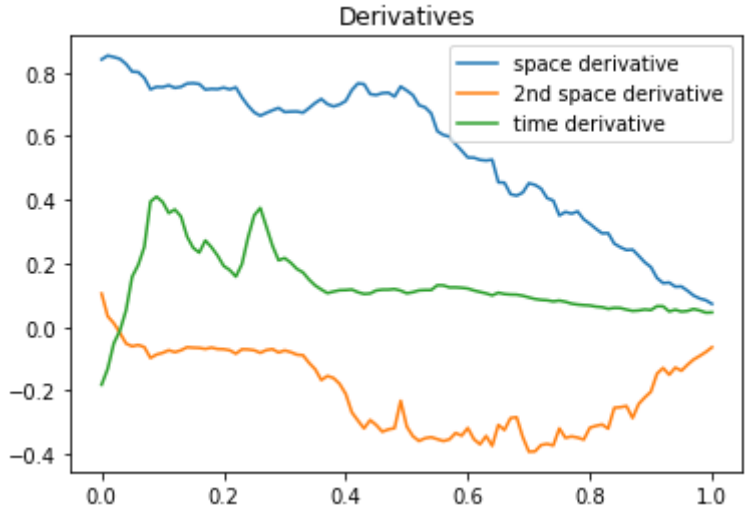









```
2211th Epoch:
traing loss is 0.0033701329957693815, test loss is 0.0023770281113684177, and learning rate i

2221th Epoch:
traing loss is 0.0069239456206560135, test loss is 0.006588997785001993, and learning rate is

2231th Epoch:
traing loss is 0.0016167080029845238, test loss is 0.001830193679779768, and learning rate is

2241th Epoch:
traing loss is 0.004784781951457262, test loss is 0.0046225604601204395, and learning rate is

2251th Epoch:
traing loss is 0.001752868527546525, test loss is 0.0018695252947509289, and learning rate is

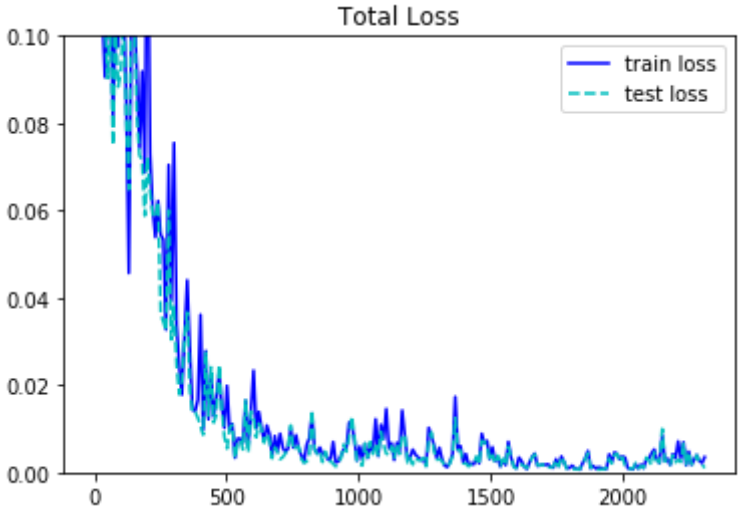
2261th Epoch:
traing loss is 0.002857040148228407, test loss is 0.001968485303223133, and learning rate is

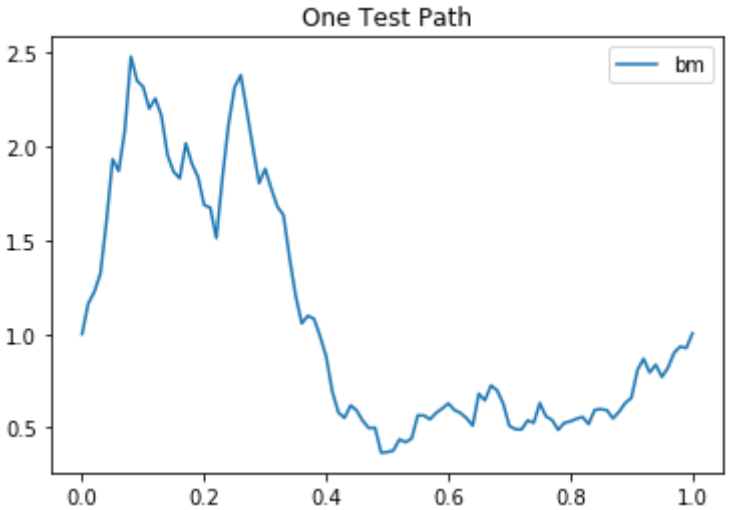
2271th Epoch:
traing loss is 0.003949351608753204, test loss is 0.0039497013203799725, and learning rate is
```

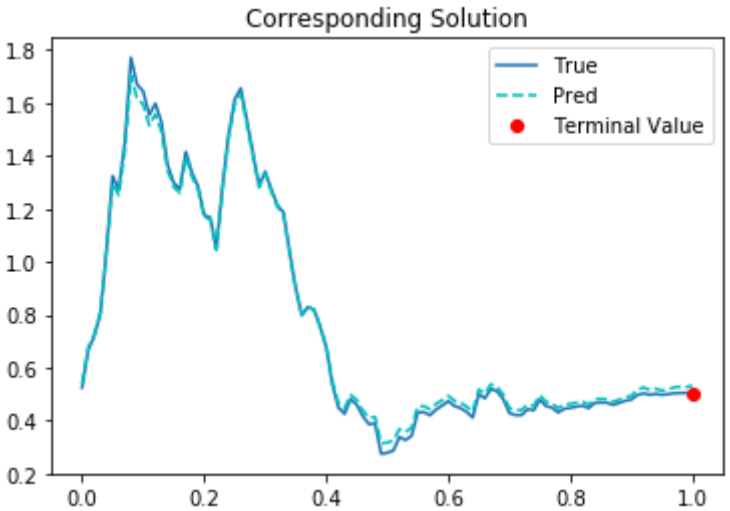
```
2281th Epoch:
traing loss is 0.0026723688933998346, test loss is 0.0024025917518883944, and learning rate i

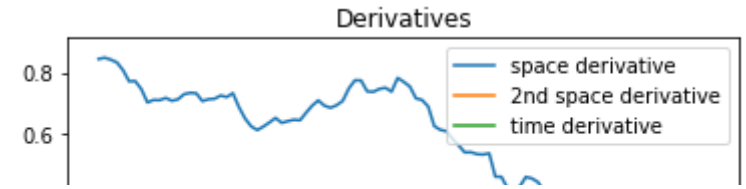
2291th Epoch:
traing loss is 0.0021544508635997772, test loss is 0.001844361424446106, and learning rate is

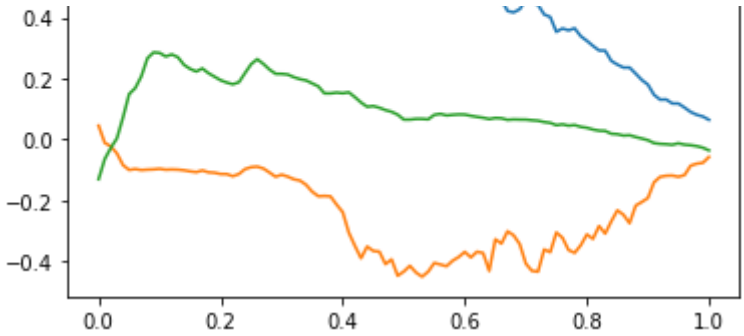
2301th Epoch:
traing loss is 0.003501318860799074, test loss is 0.0010101617081090808, and learning rate is
```












```
2311th Epoch:
traing loss is 0.0015990687534213066, test loss is 0.001114382641389966, and learning rate is

2321th Epoch:
traing loss is 0.0012253528693690896, test loss is 0.0012548657832667232, and learning rate i

2331th Epoch:
traing loss is 0.0008220101008191705, test loss is 0.0010267820907756686, and learning rate i

2341th Epoch:
traing loss is 0.0024826412554830313, test loss is 0.00234013213776052, and learning rate is

2351th Epoch:
traing loss is 0.0008159406133927405, test loss is 0.0007765424670651555, and learning rate i

2361th Epoch:
traing loss is 0.0017745805671438575, test loss is 0.001293409732170403, and learning rate is

2371th Epoch:
traing loss is 0.002566289622336626, test loss is 0.002610736060887575, and learning rate is

2381th Epoch:
traing loss is 0.0017009442672133446, test loss is 0.001599968527443707, and learning rate is

2391th Epoch:
traing loss is 0.0012535430723801255, test loss is 0.0010909668635576963, and learning rate i

2401th Epoch:
traing loss is 0.003083817195147276, test loss is 0.0007558417273685336, and learning rate is
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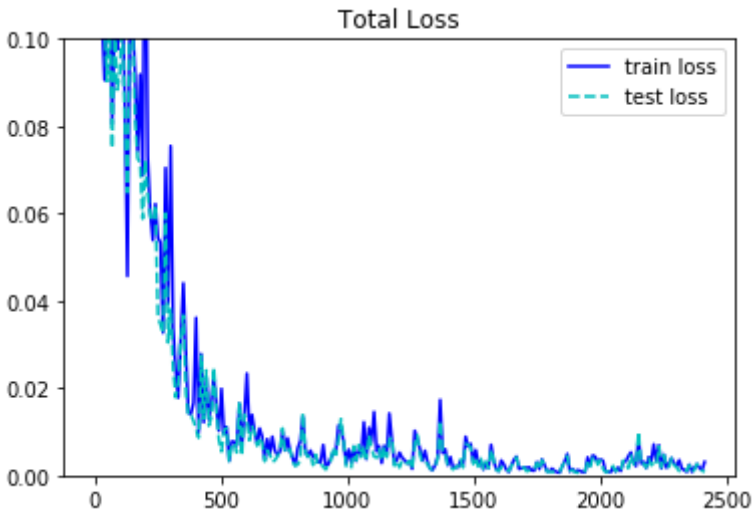

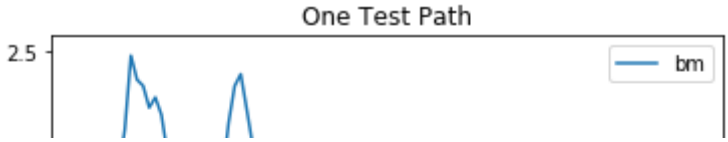

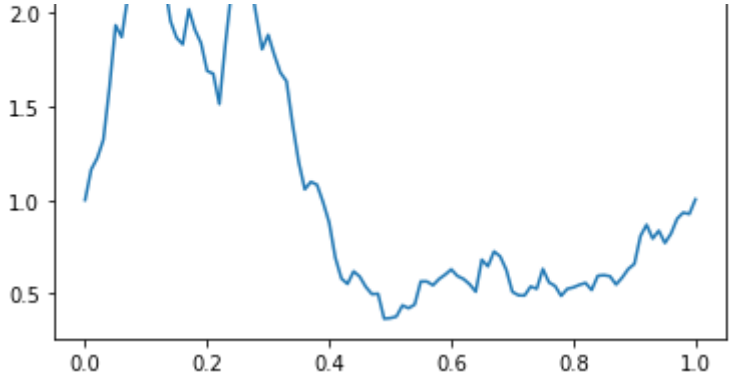

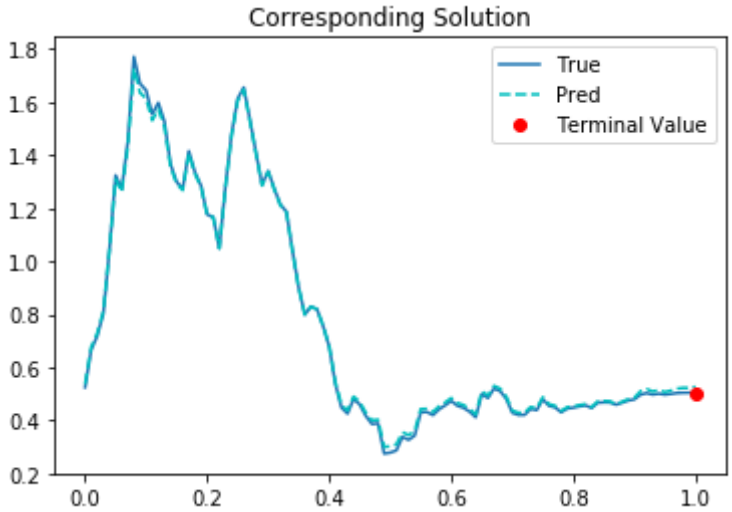

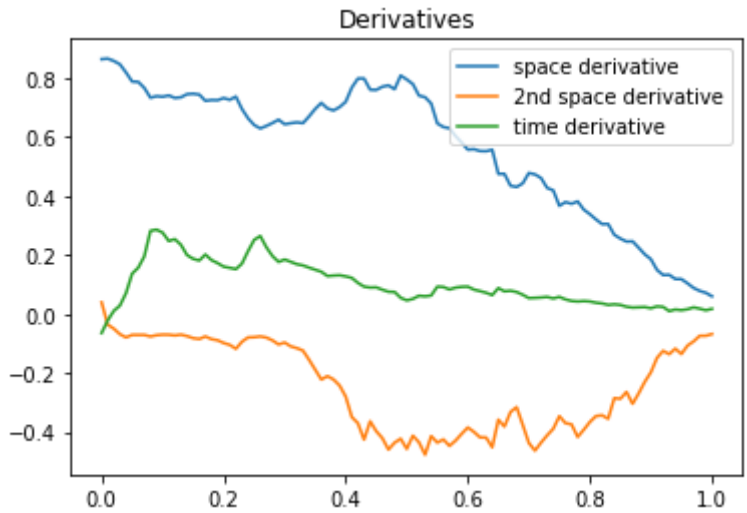


```
2411th Epoch:
traing loss is 0.0012437256518751383, test loss is 0.0007457656902261078, and learning rate i

2421th Epoch:
traing loss is 0.0007379456656053662, test loss is 0.0007331352680921555, and learning rate i

2431th Epoch:
traing loss is 0.000865480862557888, test loss is 0.0010057708714157343, and learning rate is

2441th Epoch:
traing loss is 0.0011292872950434685, test loss is 0.0010911681456491351, and learning rate i

2451th Epoch:
traing loss is 0.00047380916657857597, test loss is 0.000479580310638994, and learning rate i

2461th Epoch:
traing loss is 0.0016686758026480675, test loss is 0.0013533426681533456, and learning rate i
```

```
2471th Epoch:
traing loss is 0.0009028465719893575, test loss is 0.0011359500931575894, and learning rate i

2481th Epoch:
traing loss is 0.002279473701491952, test loss is 0.002231363905593753, and learning rate is

2491th Epoch:
traing loss is 0.0011879391968250275, test loss is 0.0009483795147389174, and learning rate i

2501th Epoch:
traing loss is 0.0022116638720035553, test loss is 0.0013975173933431506, and learning rate i
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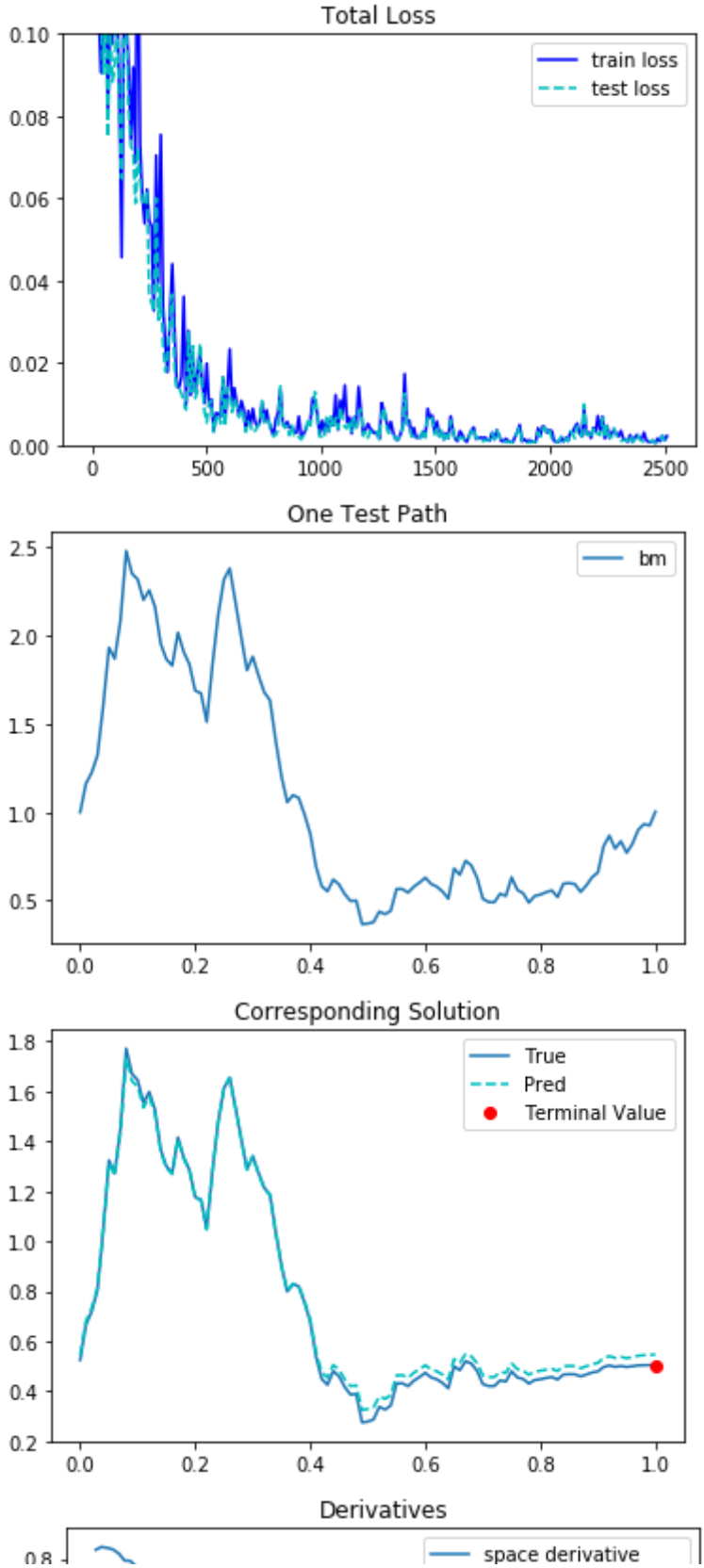

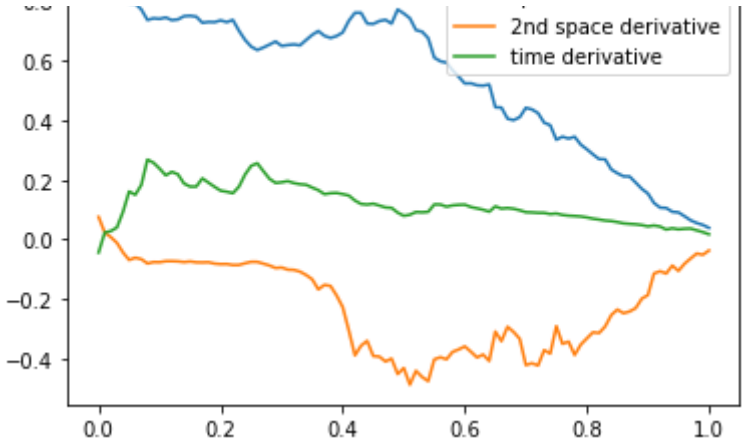


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2511th Epoch:
traing loss is 0.0011270764516666532, test loss is 0.000710076536051929, and learning rate is

2521th Epoch:
traing loss is 0.000667878775857389, test loss is 0.0006172156427055597, and learning rate is

2531th Epoch:
traing loss is 0.0007540945080108941, test loss is 0.0009504054323770106, and learning rate i

2541th Epoch:
traing loss is 0.0012935764389112592, test loss is 0.0011789306299760938, and learning rate i

2551th Epoch:
traing loss is 0.0011825382243841887, test loss is 0.0011462155962362885, and learning rate i

2561th Epoch:
traing loss is 0.0017920243553817272, test loss is 0.0015742474934086204, and learning rate i

2571th Epoch:
traing loss is 0.0007625621510669589, test loss is 0.0007782223401591182, and learning rate i

2581th Epoch:
traing loss is 0.0008793835295364261, test loss is 0.0008423959952779114, and learning rate i

2591th Epoch:
traing loss is 0.0004426891973707825, test loss is 0.0004362726176623255, and learning rate i

2601th Epoch:
traing loss is 0.0011634224792942405, test loss is 0.000455886940471828, and learning rate is
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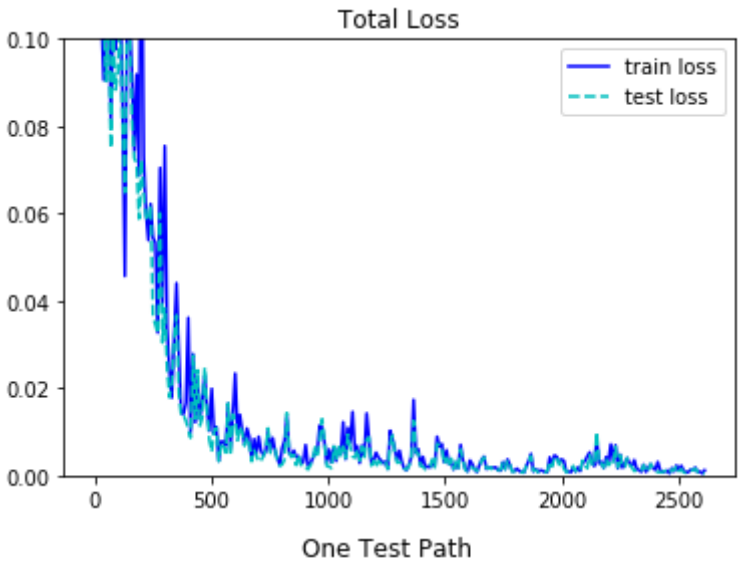

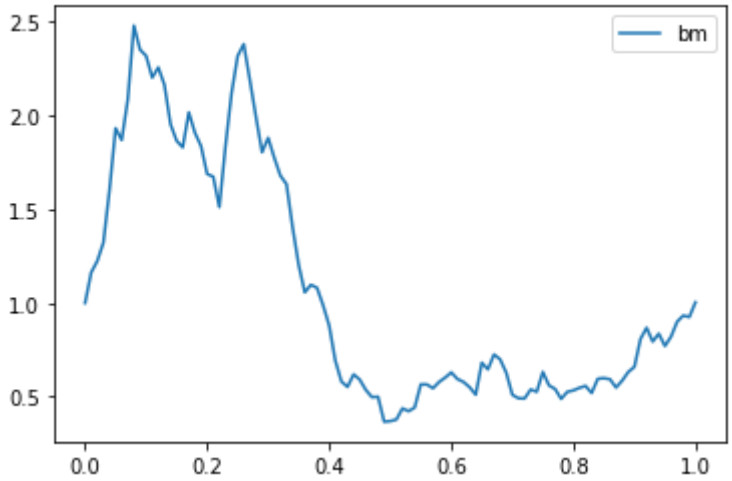

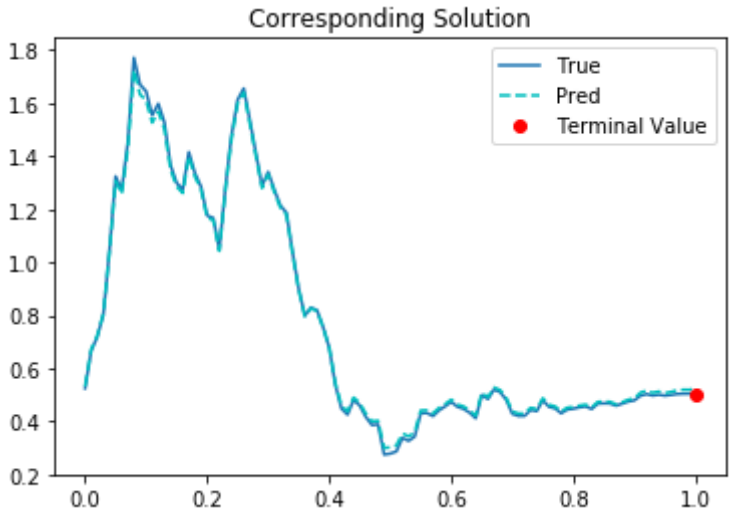

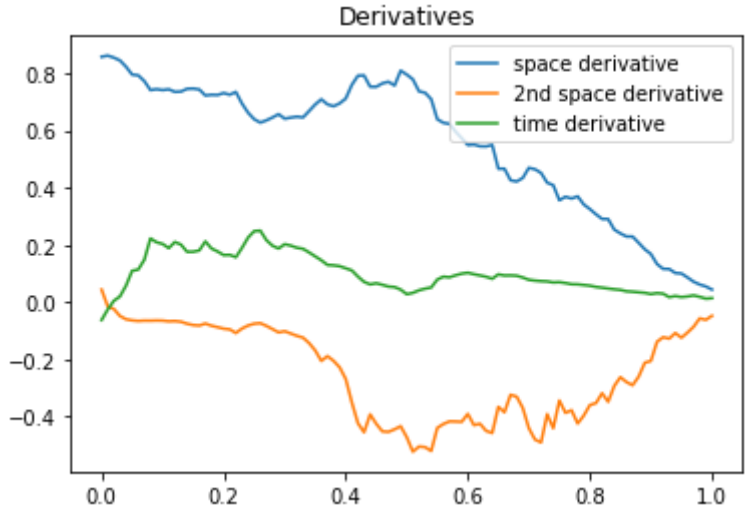


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2611th Epoch:
traing loss is 0.0011118323309347034, test loss is 0.00059083744417876, and learning rate is

2621th Epoch:
traing loss is 0.0015851148637011647, test loss is 0.001352406106889248, and learning rate is

2631th Epoch:
traing loss is 0.0005974721862003207, test loss is 0.0006747220177203417, and learning rate i

2641th Epoch:
traing loss is 0.0020903879776597023, test loss is 0.0019890584517270327, and learning rate i

2651th Epoch:
traing loss is 0.0018002813449129462, test loss is 0.0016636807704344392, and learning rate i
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```
2661th Epoch:
traing loss is 0.002192447194829583, test loss is 0.0017485981807112694, and learning rate is

2671th Epoch:
traing loss is 0.000925268221180886, test loss is 0.0008941657142713666, and learning rate is

2681th Epoch:
traing loss is 0.0010400102473795414, test loss is 0.0010172381298616529, and learning rate i

2691th Epoch:
traing loss is 0.0006038454594090581, test loss is 0.00041253300150856376, and learning rate

2701th Epoch:
traing loss is 0.0011183549650013447, test loss is 0.0005152071826159954, and learning rate i
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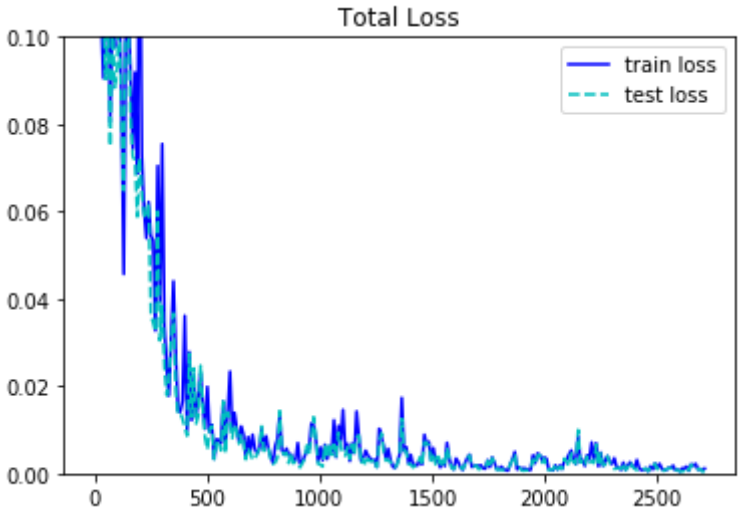

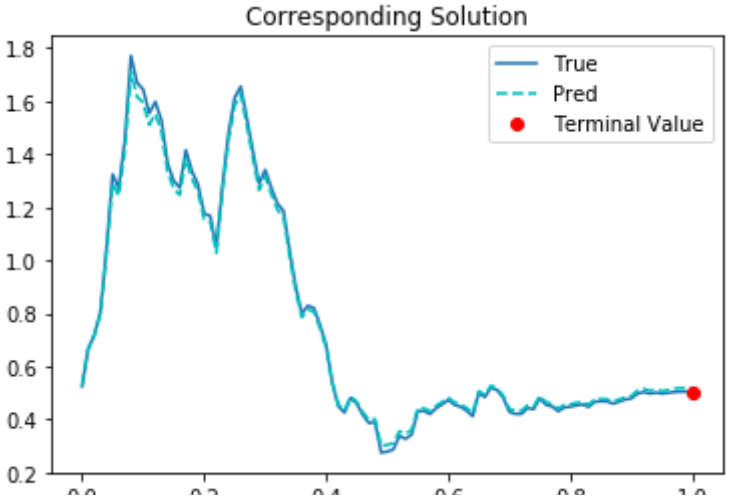

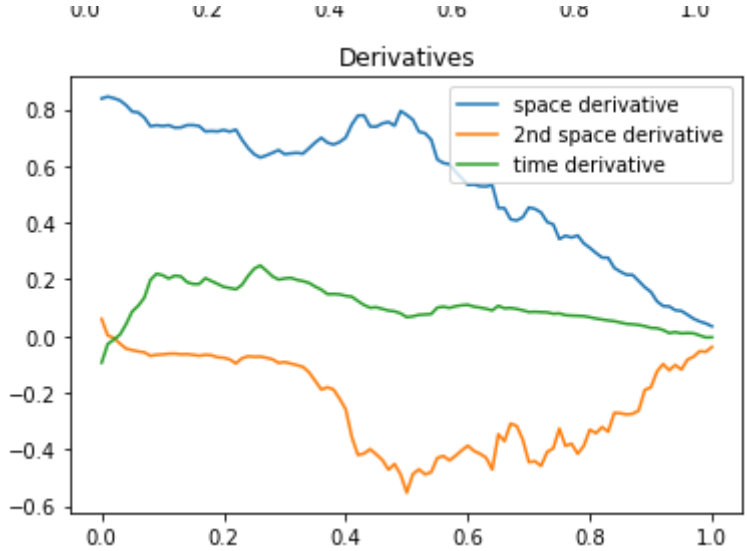


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2711th Epoch:
traing loss is 0.0008885642164386809, test loss is 0.000613383948802948, and learning rate is

2721th Epoch:
traing loss is 0.0011069192551076412, test loss is 0.0009298691875301301, and learning rate i

2731th Epoch:
traing loss is 0.0004508321871981025, test loss is 0.0005337087786756456, and learning rate i

2741th Epoch:
traing loss is 0.0014735221629962325, test loss is 0.0012907636119052768, and learning rate i

2751th Epoch:
traing loss is 0.0011415196349844337, test loss is 0.0010701861465349793, and learning rate i

2761th Epoch:
traing loss is 0.002726578852161765, test loss is 0.002174215391278267, and learning rate is

2771th Epoch:
traing loss is 0.0018149190582334995, test loss is 0.0018028857884928584, and learning rate i

2781th Epoch:
traing loss is 0.0009904955513775349, test loss is 0.0009821067797020078, and learning rate i

2791th Epoch:
traing loss is 0.00075491052120924, test loss is 0.0006328667514026165, and learning rate is

2801th Epoch:
traing loss is 0.001031261752359569, test loss is 0.0005542936851270497, and learning rate is
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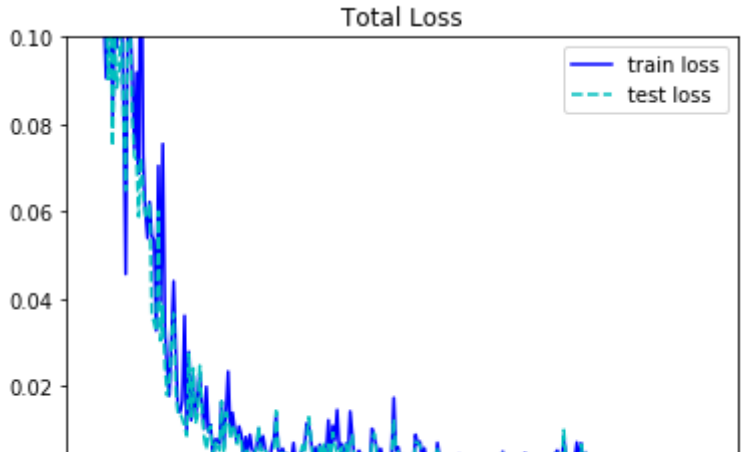

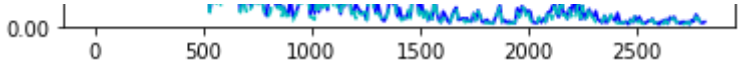

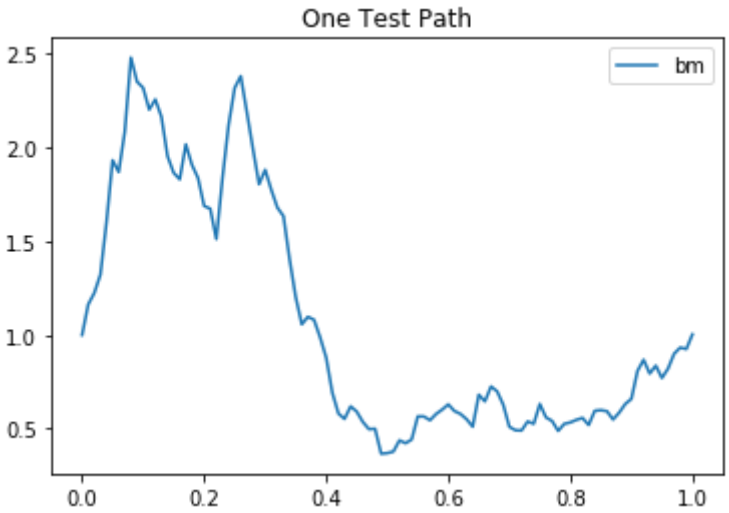

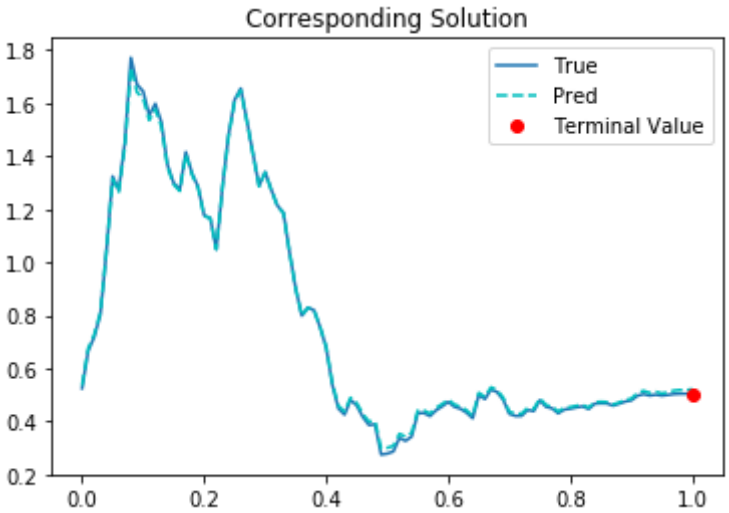

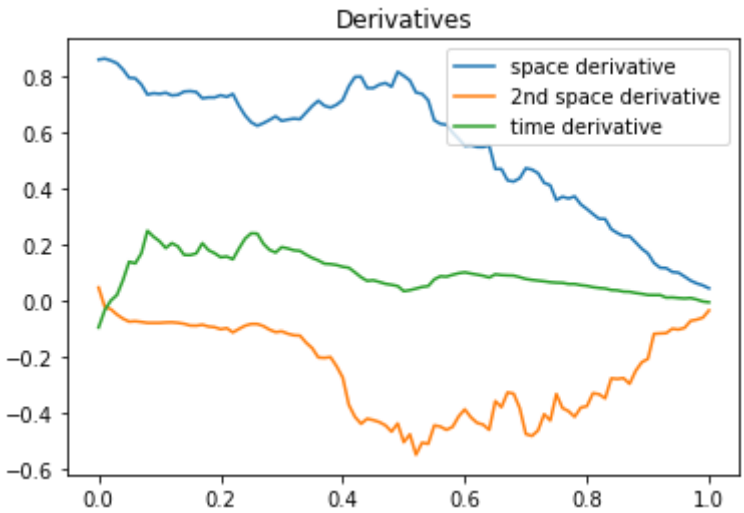


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2811th Epoch:
traing loss is 0.0010685102315619588, test loss is 0.0005207006470300257, and learning rate i

2821th Epoch:
traing loss is 0.0013167799916118383, test loss is 0.001113027916289866, and learning rate is

2831th Epoch:
traing loss is 0.00047039685887284577, test loss is 0.0005463141133077443, and learning rate

2841th Epoch:
traing loss is 0.004181291908025742, test loss is 0.0039436365477740765, and learning rate is
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2851th Epoch:
traing loss is 0.0019752010703086853, test loss is 0.001694757491350174, and learning rate is

2861th Epoch:
traing loss is 0.002287520095705986, test loss is 0.0018523865146562457, and learning rate is

2871th Epoch:
traing loss is 0.0011243766639381647, test loss is 0.0010599559172987938, and learning rate i

2881th Epoch:
traing loss is 0.0013837867882102728, test loss is 0.0009028262575156987, and learning rate i

2891th Epoch:
traing loss is 0.0007976740598678589, test loss is 0.0005844071274623275, and learning rate i

2901th Epoch:
traing loss is 0.001672555459663272, test loss is 0.0006805431912653148, and learning rate is
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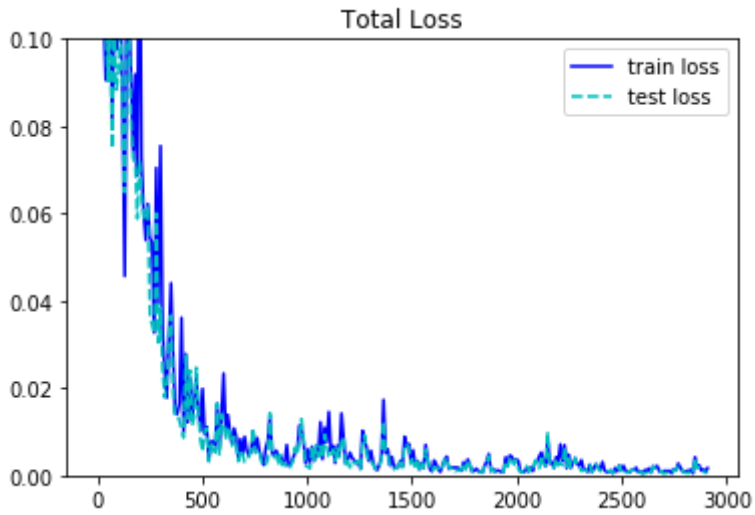

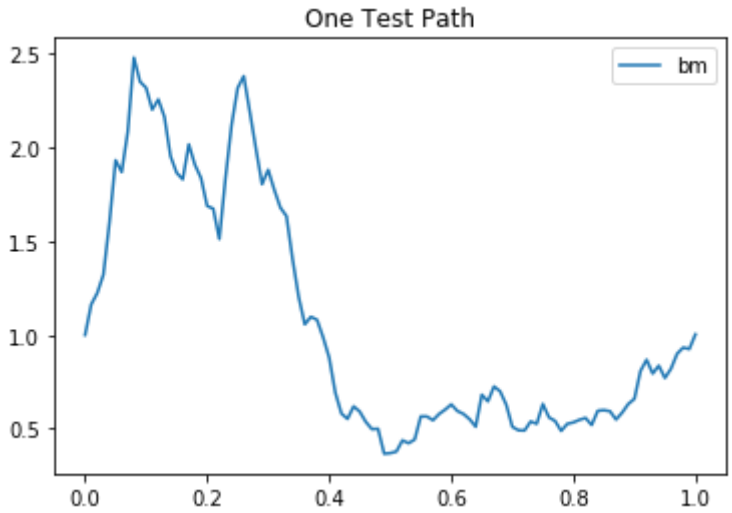

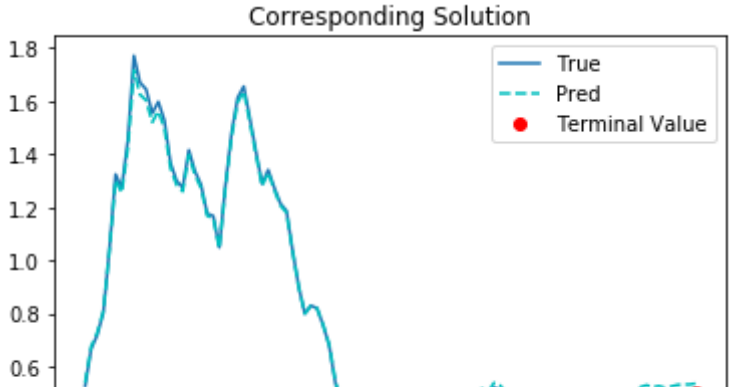

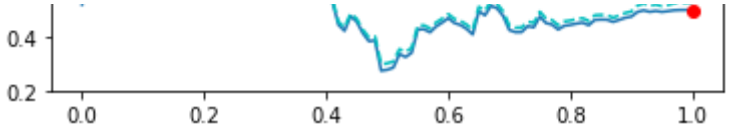

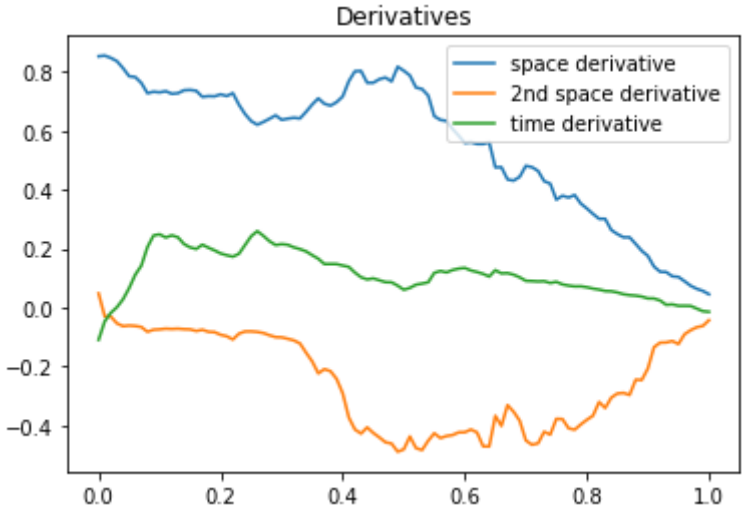


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2911th Epoch:
traing loss is 0.0007920082425698638, test loss is 0.0006588817923329771, and learning rate i

2921th Epoch:
traing loss is 0.0008351809810847044, test loss is 0.0007000303012318909, and learning rate i

2931th Epoch:
traing loss is 0.00041075667832046747, test loss is 0.00047121764509938657, and learning rate

2941th Epoch:
traing loss is 0.0011850550072267652, test loss is 0.0011905829887837172, and learning rate i

2951th Epoch:
traing loss is 0.0013120555086061358, test loss is 0.0013263931032270193, and learning rate i

2961th Epoch:
traing loss is 0.002662396989762783, test loss is 0.0019076609751209617, and learning rate is

2971th Epoch:
traing loss is 0.0018208202673122287, test loss is 0.0018235447350889444, and learning rate i

2981th Epoch:
traing loss is 0.0010776324197649956, test loss is 0.0009244100656360388, and learning rate i

2991th Epoch:
traing loss is 0.0008200444281101227, test loss is 0.0007518439088016748, and learning rate i

3001th Epoch:
traing loss is 0.0014112131902948022, test loss is 0.0006049774237908423, and learning rate i
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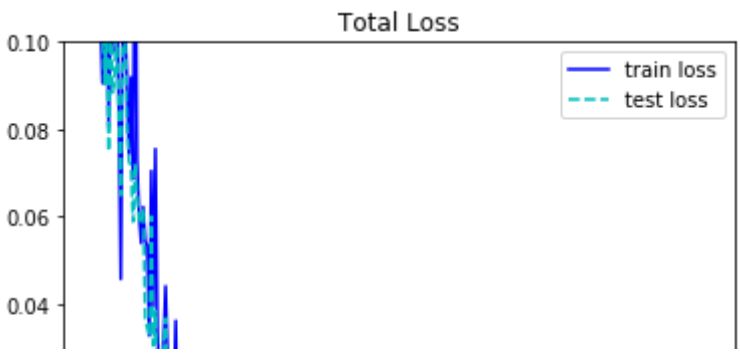

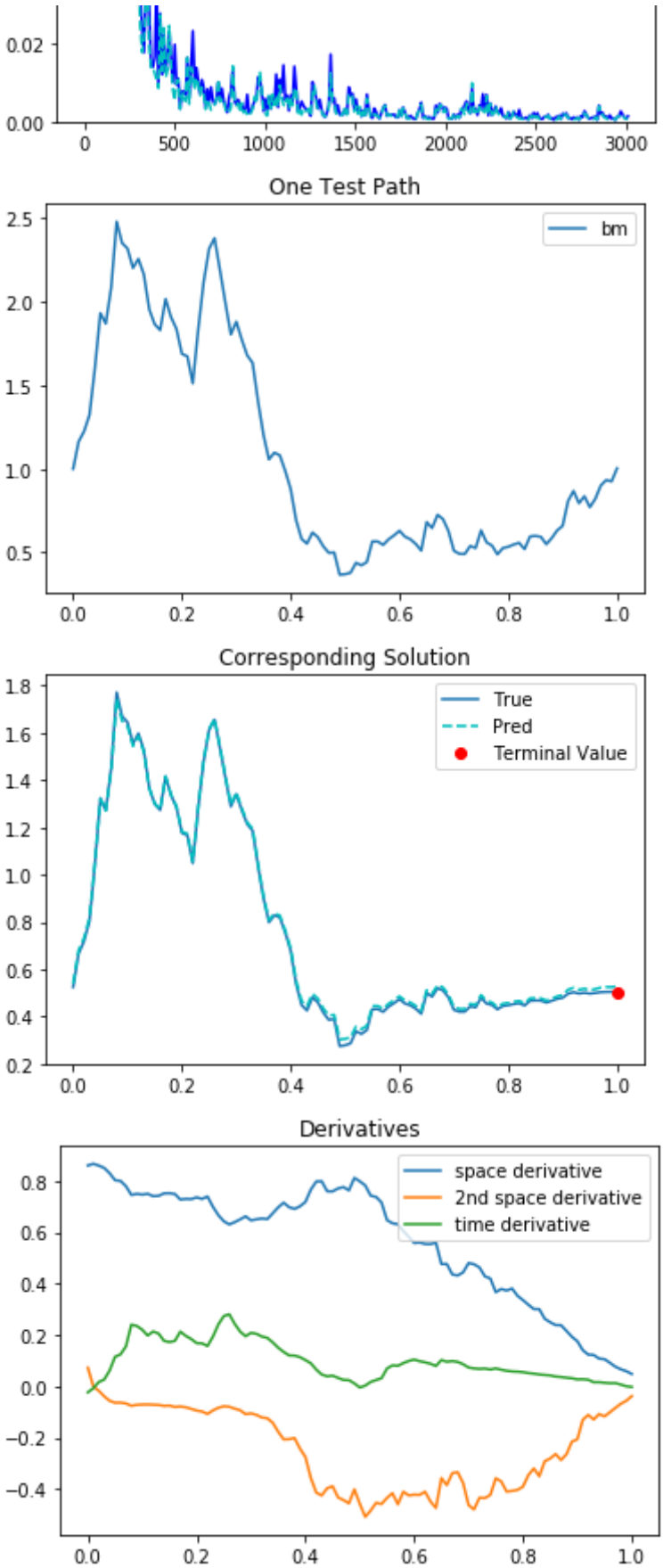


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3011th Epoch:
traing loss is 0.0011750453850254416, test loss is 0.0008498847018927336, and learning rate i

3021th Epoch:
traing loss is 0.0014667629729956388, test loss is 0.0013457202585414052, and learning rate i

3031th Epoch:
traing loss is 0.0005308391409926116, test loss is 0.00064394052606076, and learning rate is
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3041th Epoch:
traing loss is 0.002223557559773326, test loss is 0.0021602942142635584, and learning rate is

3051th Epoch:
traing loss is 0.0008277875604107976, test loss is 0.0007031139102764428, and learning rate i

3061th Epoch:
traing loss is 0.0013583899708464742, test loss is 0.0007745350594632328, and learning rate i

3071th Epoch:
traing loss is 0.0005876086070202291, test loss is 0.0006201434880495071, and learning rate i

3081th Epoch:
traing loss is 0.0007388200610876083, test loss is 0.0006793540669605136, and learning rate i

3091th Epoch:
traing loss is 0.0005690395482815802, test loss is 0.00047348253428936005, and learning rate

3101th Epoch:
traing loss is 0.0016608398873358965, test loss is 0.0015453619416803122, and learning rate i
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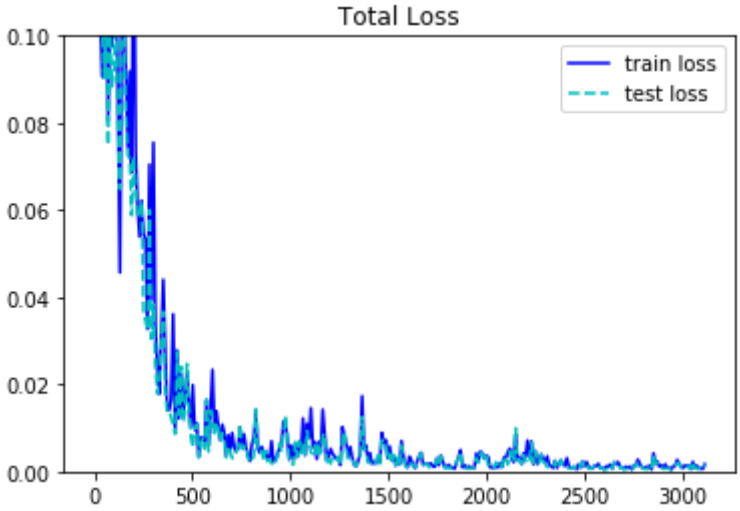

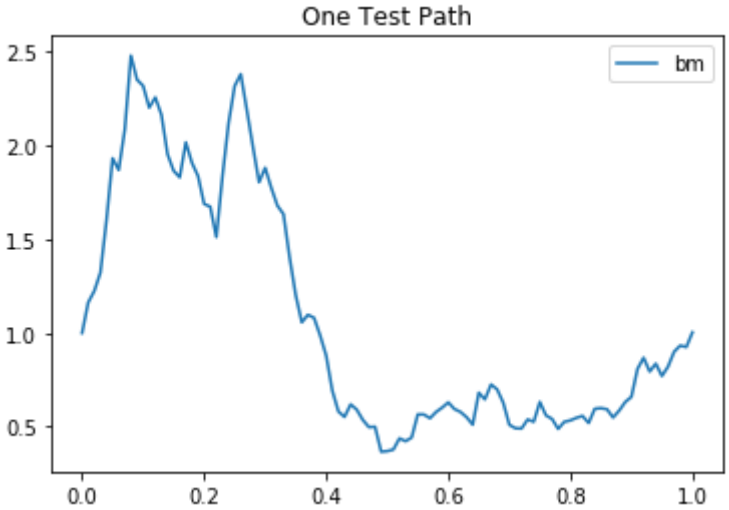

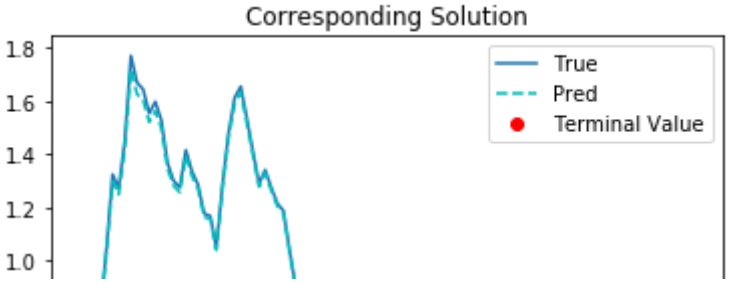

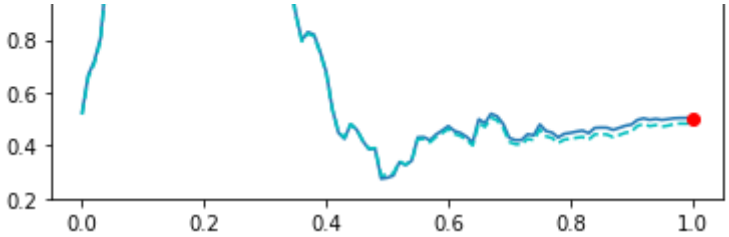

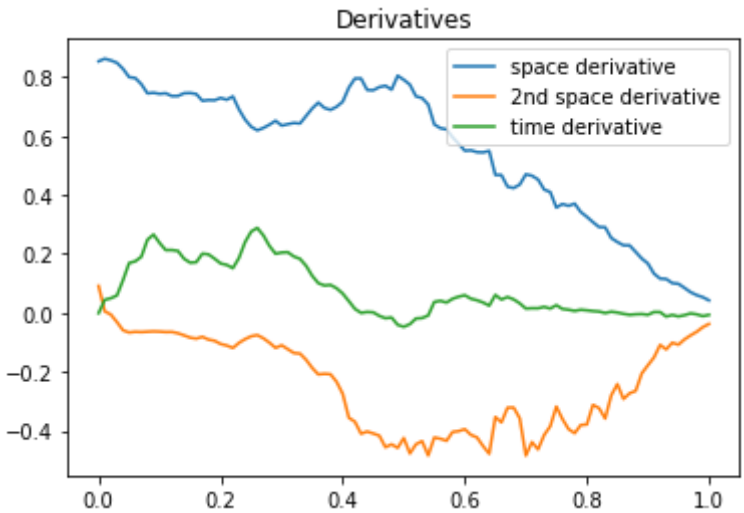


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3111th Epoch:
traing loss is 0.0009377406095154583, test loss is 0.0006573458085767925, and learning rate i

3121th Epoch:
traing loss is 0.0005964968004263937, test loss is 0.0005971499485895038, and learning rate i

3131th Epoch:
traing loss is 0.000480563729070127, test loss is 0.0005261572077870369, and learning rate is

3141th Epoch:
traing loss is 0.0006088617956265807, test loss is 0.0005675303982570767, and learning rate i

3151th Epoch:
traing loss is 0.0006350042531266809, test loss is 0.0006215222529135644, and learning rate i

3161th Epoch:
traing loss is 0.001198513200506568, test loss is 0.0010328295174986124, and learning rate is

3171th Epoch:
traing loss is 0.000716935028322041, test loss is 0.000789788318797946, and learning rate is

3181th Epoch:
traing loss is 0.0013834064593538642, test loss is 0.0013855010038241744, and learning rate i

3191th Epoch:
traing loss is 0.0013419828610494733, test loss is 0.001115354709327221, and learning rate is

3201th Epoch:
traing loss is 0.0007432065904140472, test loss is 0.0005061204428784549, and learning rate i
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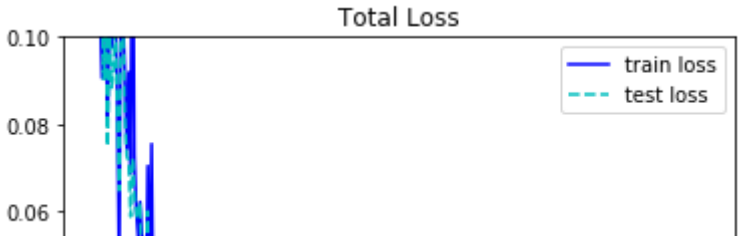

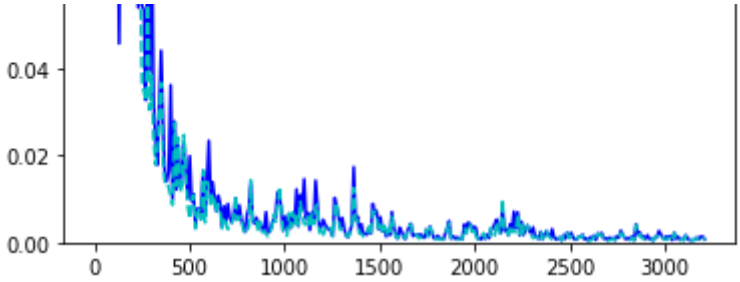

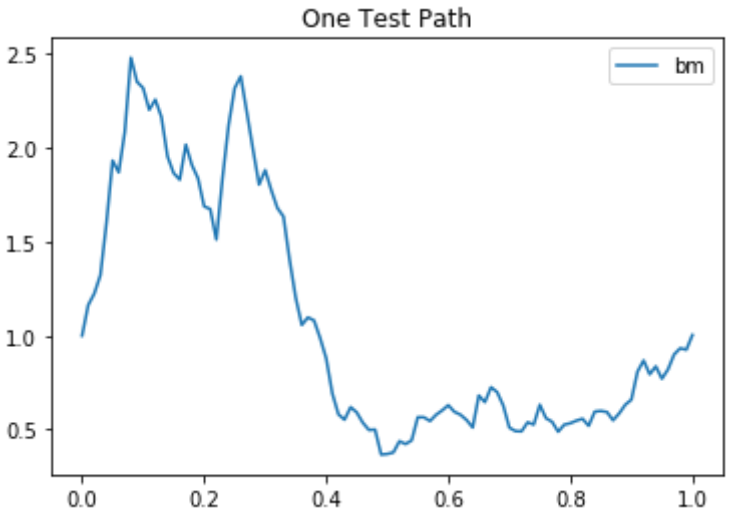

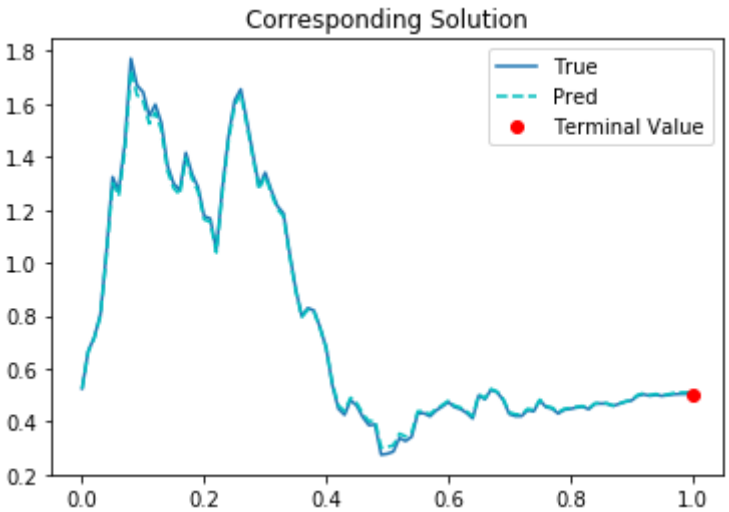

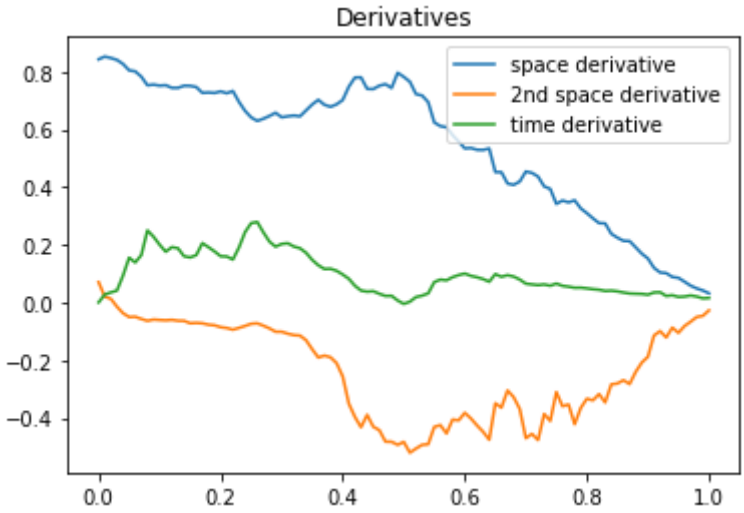

3211th Epoch:
traing loss is 0.0006566053489223123, test loss is 0.00042501359712332487, and learning rate

3221th Epoch:
traing loss is 0.0006818894762545824, test loss is 0.0006157933967188001, and learning rate i

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3231th Epoch:
traing loss is 0.0002655652933754027, test loss is 0.0003196742618456483, and learning rate i

3241th Epoch:
traing loss is 0.001152370939962566, test loss is 0.0012621600180864334, and learning rate is

3251th Epoch:
traing loss is 0.00046049317461438477, test loss is 0.00042130055953748524, and learning rate

3261th Epoch:
traing loss is 0.0006838847766630352, test loss is 0.0004018935433123261, and learning rate i

3271th Epoch:
traing loss is 0.0003626531397458166, test loss is 0.00038934193435125053, and learning rate

3281th Epoch:
traing loss is 0.0008449966553598642, test loss is 0.0008919320534914732, and learning rate i

3291th Epoch:
traing loss is 0.0008081879350356758, test loss is 0.0005488503957167268, and learning rate i

3301th Epoch:
traing loss is 0.000772442203015089, test loss is 0.000522380752954632, and learning rate is
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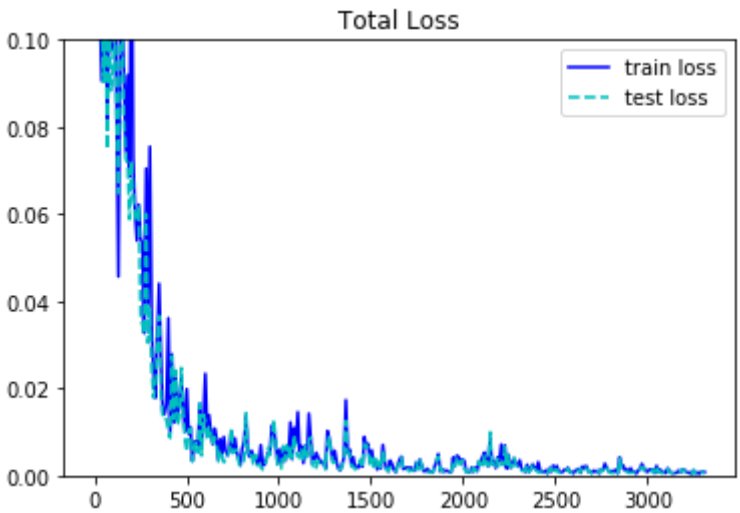
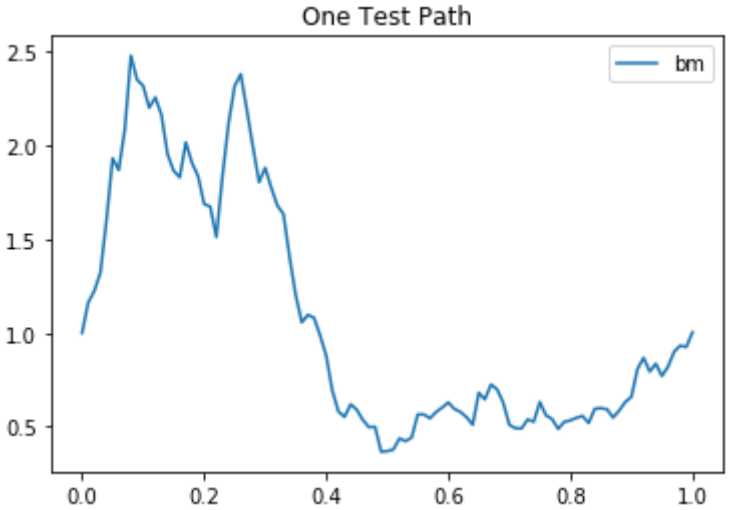
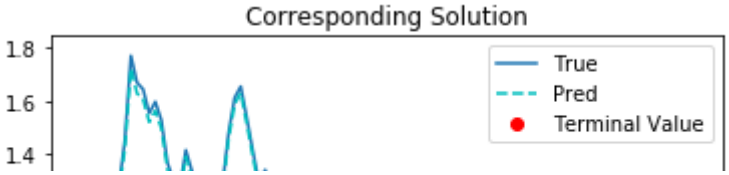
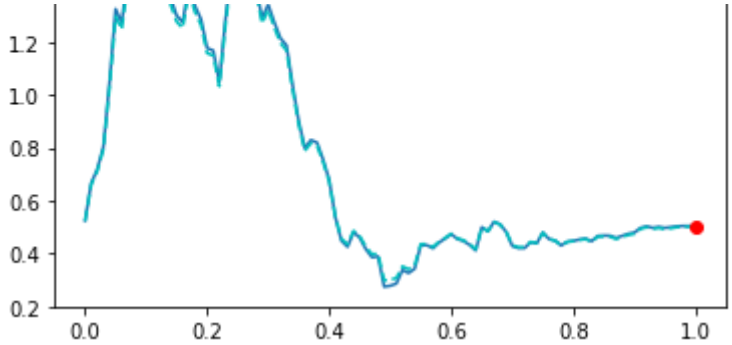
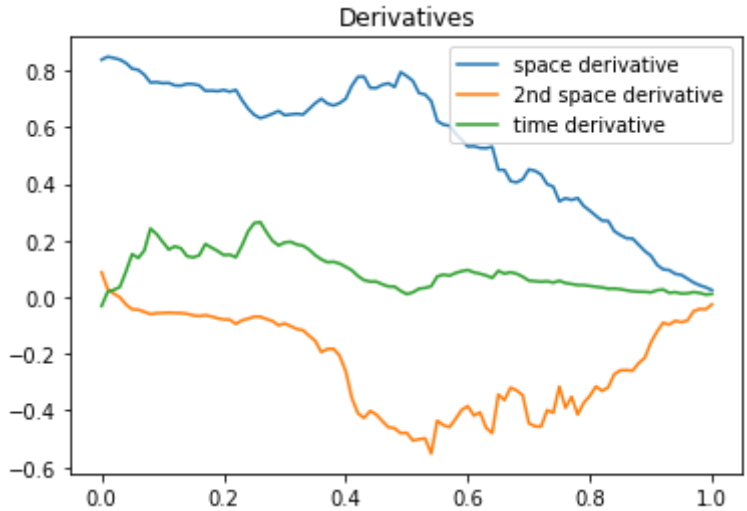











```
3311th Epoch:
traing loss is 0.0004960626247338951, test loss is 0.0003555406292434782, and learning rate i

3321th Epoch:
traing loss is 0.0009081336902454495, test loss is 0.0007594877970404923, and learning rate i

3331th Epoch:
traing loss is 0.0002563372254371643, test loss is 0.0003424003953114152, and learning rate i

3341th Epoch:
traing loss is 0.0007397404406219721, test loss is 0.0006573798600584269, and learning rate i

3351th Epoch:
traing loss is 0.0007365011842921376, test loss is 0.0007316356641240418, and learning rate i

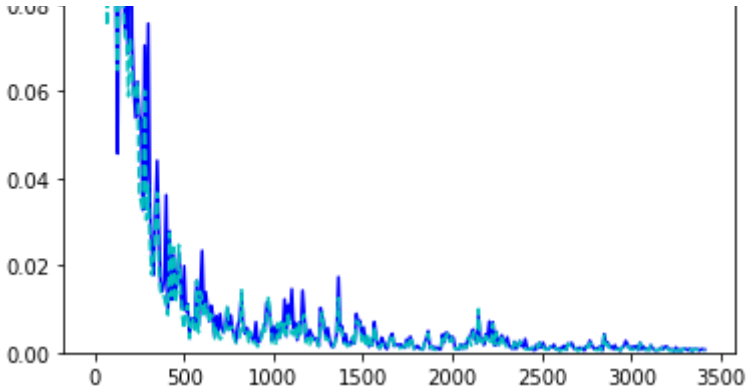
3361th Epoch:
traing loss is 0.0007173587218858302, test loss is 0.0006325691938400269, and learning rate i

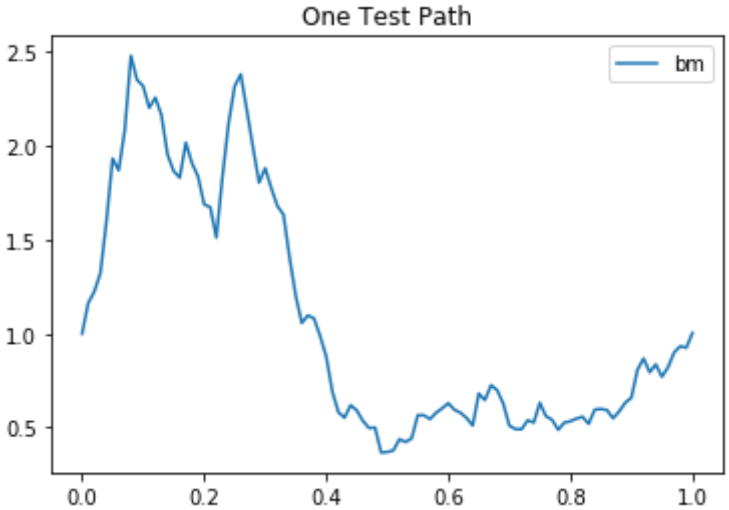
3371th Epoch:
traing loss is 0.0004390031681396067, test loss is 0.0004937007906846702, and learning rate i

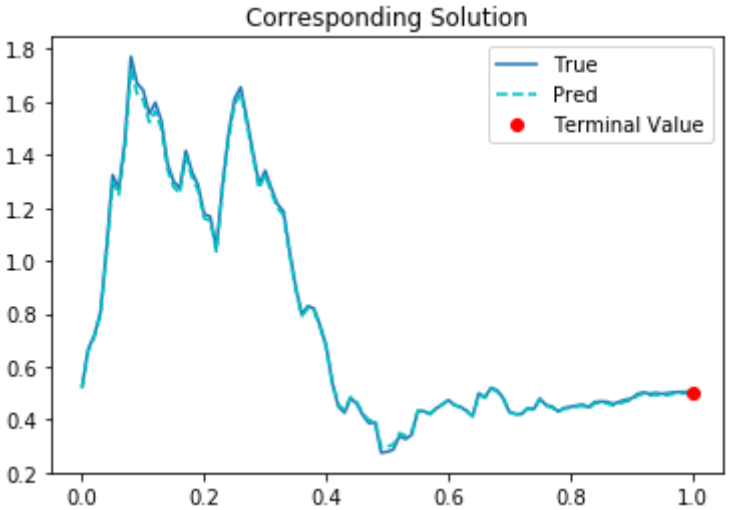
3381th Epoch:
traing loss is 0.0007615546928718686, test loss is 0.0007276982651092112, and learning rate i

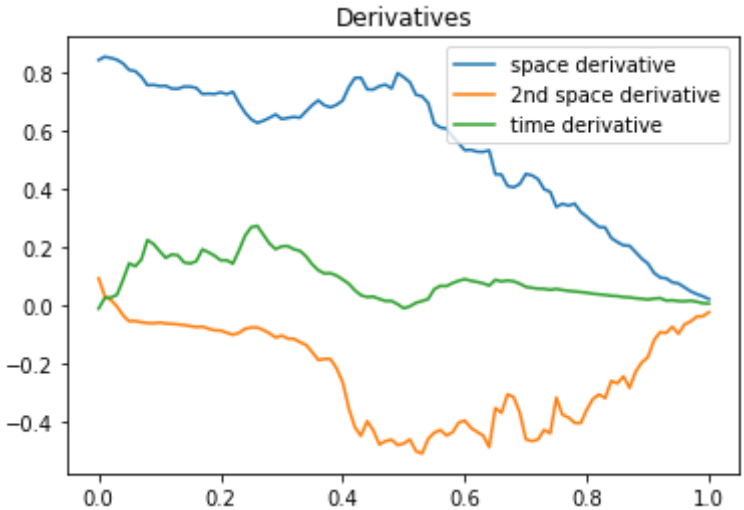
3391th Epoch:
traing loss is 0.0007093578460626304, test loss is 0.0004906676476821303, and learning rate i

3401th Epoch:
traing loss is 0.0006487938808277249, test loss is 0.0004860825720243156, and learning rate i
```









3411th Epoch:
traing loss is 0.0005301933269947767, test loss is 0.0003035813570022583, and learning rate i

```
3421th Epoch:
traing loss is 0.0005582452286034822, test loss is 0.00042088492773473263, and learning rate

3431th Epoch:
traing loss is 0.00019291738863103092, test loss is 0.000258799729635939, and learning rate i

3441th Epoch:
traing loss is 0.0010168537264689803, test loss is 0.0008510152692906559, and learning rate i

3451th Epoch:
traing loss is 0.0006354995421133935, test loss is 0.000652564107440412, and learning rate is

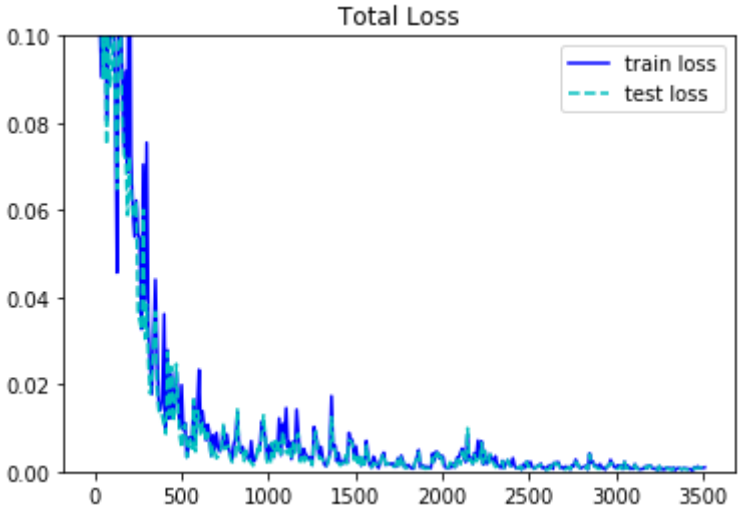
3461th Epoch:
traing loss is 0.001177661819383502, test loss is 0.0008337112958543003, and learning rate is

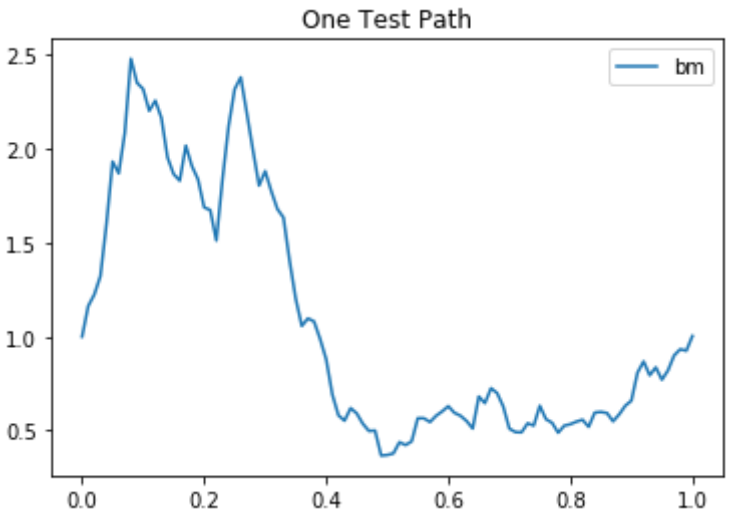
3471th Epoch:
traing loss is 0.0007393734995275736, test loss is 0.0007669194019399583, and learning rate i

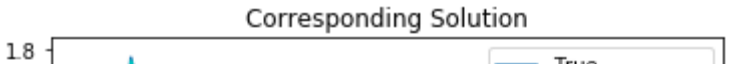
3481th Epoch:
traing loss is 0.0008289181860163808, test loss is 0.0009386512683704495, and learning rate i

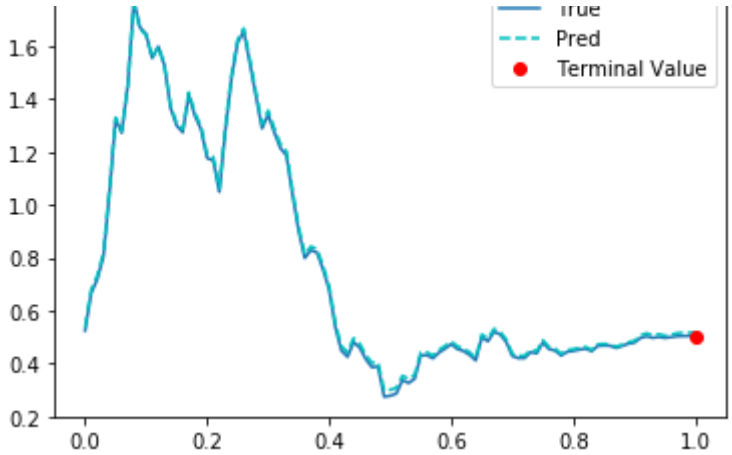
3491th Epoch:
traing loss is 0.0008328068070113659, test loss is 0.0007908038678579032, and learning rate i

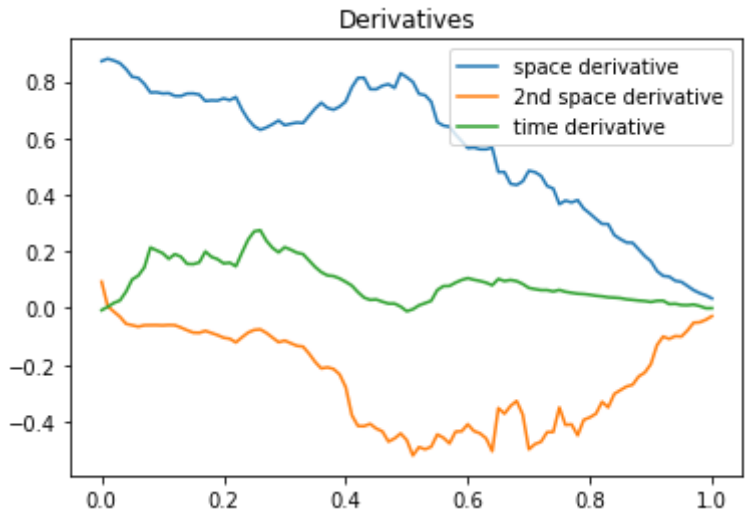
3501th Epoch:
traing loss is 0.0009513239492662251, test loss is 0.0006099868915043771, and learning rate i
```












```
3511th Epoch:
traing loss is 0.0007528469432145357, test loss is 0.0005294802249409258, and learning rate i

3521th Epoch:
traing loss is 0.0009736461215652525, test loss is 0.0008916815277189016, and learning rate i

3531th Epoch:
traing loss is 0.000259023392572999, test loss is 0.0003085738862864673, and learning rate is

3541th Epoch:
traing loss is 0.000442714081145823, test loss is 0.0003950884856749326, and learning rate is

3551th Epoch:
traing loss is 0.0002085809683194384, test loss is 0.00021692279551643878, and learning rate

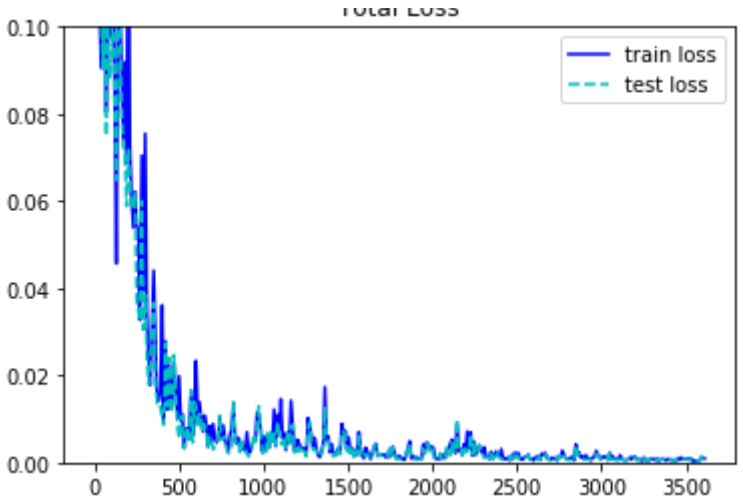
3561th Epoch:
traing loss is 0.00033964356407523155, test loss is 0.00025455388822592795, and learning rate

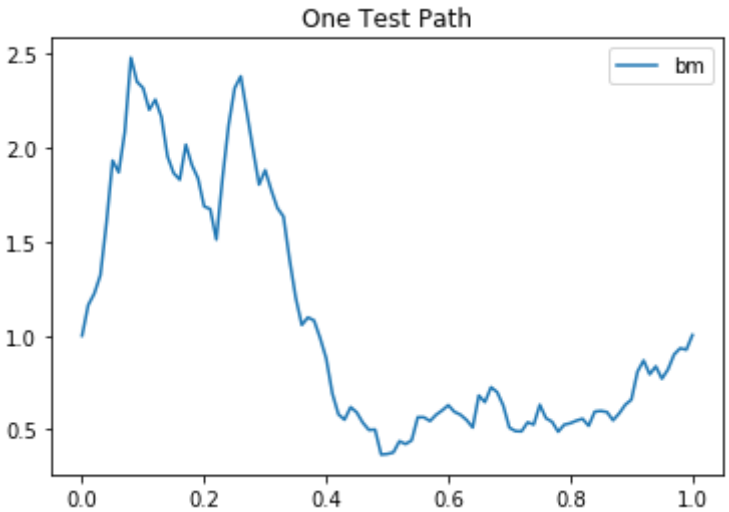
3571th Epoch:
traing loss is 0.0004434554139152169, test loss is 0.0005056100781075656, and learning rate i

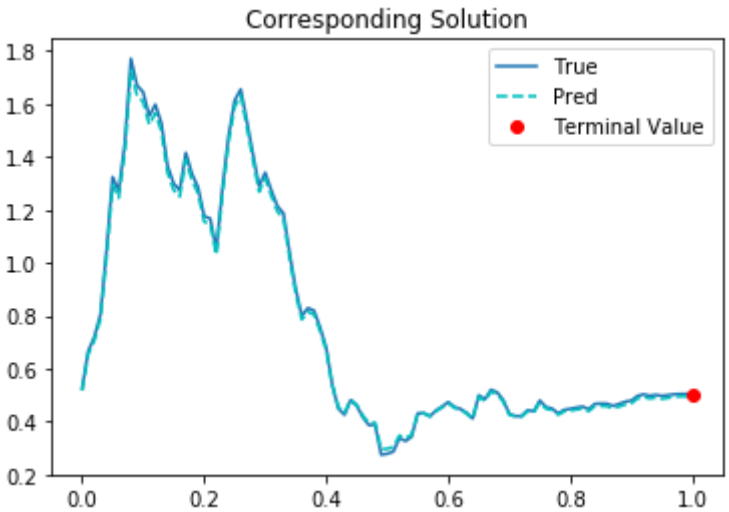
3581th Epoch:
traing loss is 0.0011971191270276904, test loss is 0.001259816694073379, and learning rate is

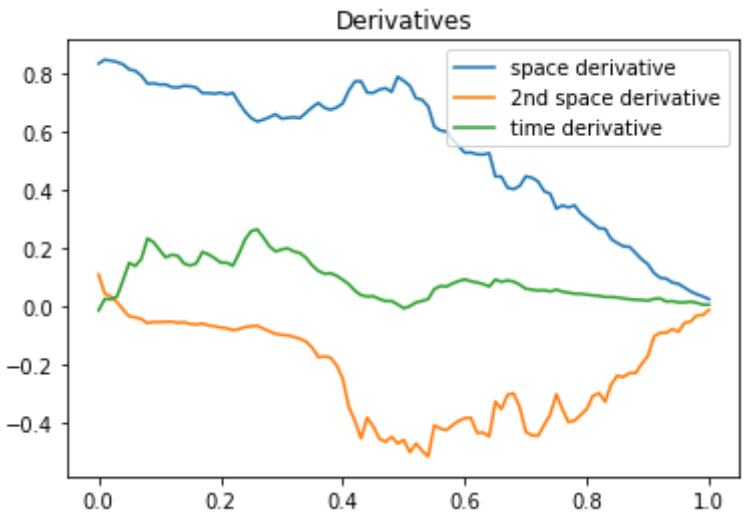
3591th Epoch:
traing loss is 0.0010856017470359802, test loss is 0.0007659620605409145, and learning rate i

3601th Epoch:
traing loss is 0.0008982415311038494, test loss is 0.0006779333925805986, and learning rate i
```


Total Loss
train loss
test loss
0.10
0.08
0.06
0.04
0.02
0.00
0
500
1000
1500
2000
2500
3000
3500


One Test Path
bm
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0


Corresponding Solution
True
Pred
Terminal Value
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


Derivatives
space derivative
2nd space derivative
time derivative
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0.0
0.2
0.4
0.6
0.8
1.0

```
3611th Epoch:
traing loss is 0.00045518833212554455, test loss is 0.00029579646070487797, and learning rate

3621th Epoch:
traing loss is 0.00037844976759515703, test loss is 0.0003133745049126446, and learning rate

3631th Epoch:
traing loss is 0.00019502815848682076, test loss is 0.00025590320001356304, and learning rate

3641th Epoch:
traing loss is 0.0005846984568051994, test loss is 0.00054056232329458, and learning rate is

3651th Epoch:
traing loss is 0.0005898186936974525, test loss is 0.0005895612412132323, and learning rate i

3661th Epoch:
traing loss is 0.0009572884300723672, test loss is 0.0007336764829233289, and learning rate i

3671th Epoch:
traing loss is 0.0005450873868539929, test loss is 0.0005875964416190982, and learning rate i

3681th Epoch:
traing loss is 0.0007127247517928481, test loss is 0.0006681725499220192, and learning rate i

3691th Epoch:
traing loss is 0.00051535275997594, test loss is 0.0004504545941017568, and learning rate is

3701th Epoch:
traing loss is 0.000949630921240896, test loss is 0.0005118367844261229, and learning rate is
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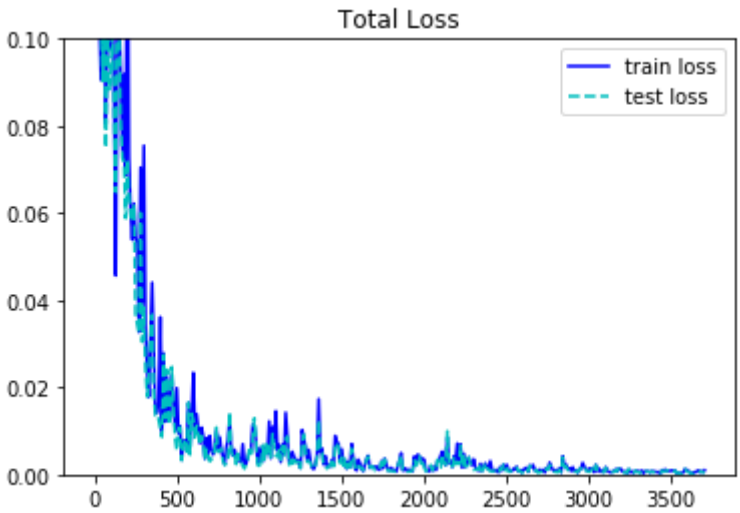
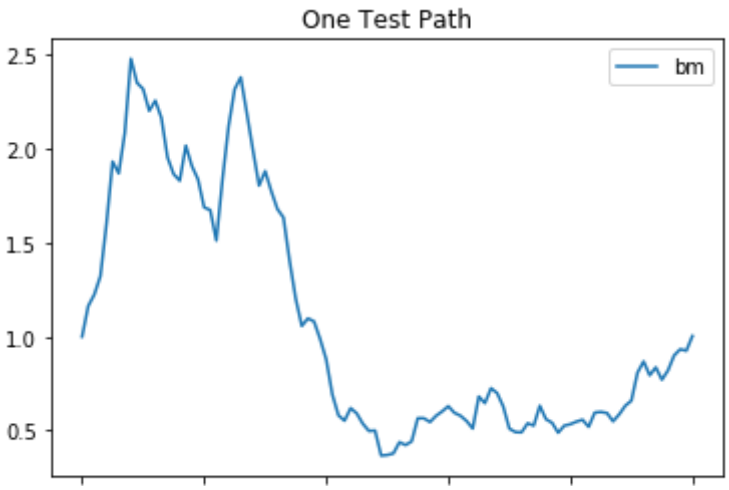
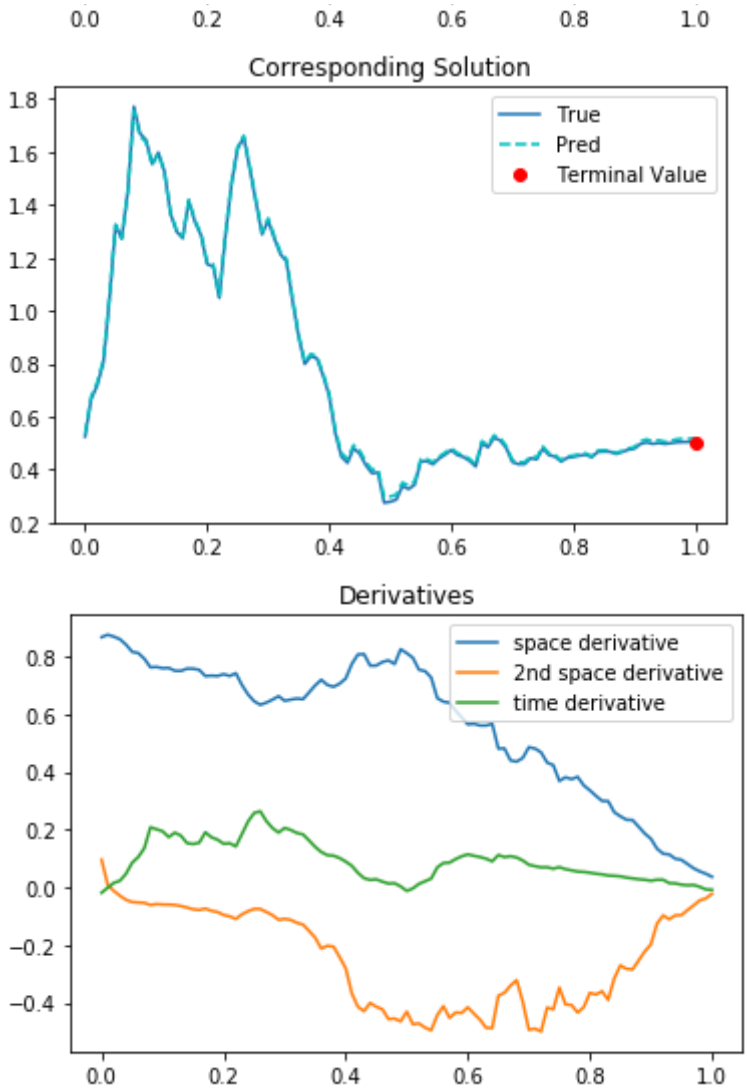







```
3711th Epoch:
traing loss is 0.0005522890714928508, test loss is 0.0003871543158311397, and learning rate i

3721th Epoch:
traing loss is 0.0007329157087951899, test loss is 0.0006866691983304918, and learning rate i

3731th Epoch:
traing loss is 0.000355030526407063, test loss is 0.0004263616865500808, and learning rate is

3741th Epoch:
traing loss is 0.0006748961750417948, test loss is 0.0005954528460279107, and learning rate i

3751th Epoch:
traing loss is 0.0003677853965200484, test loss is 0.00032583734719082713, and learning rate

3761th Epoch:
traing loss is 0.0006381577113643289, test loss is 0.00040585032547824085, and learning rate

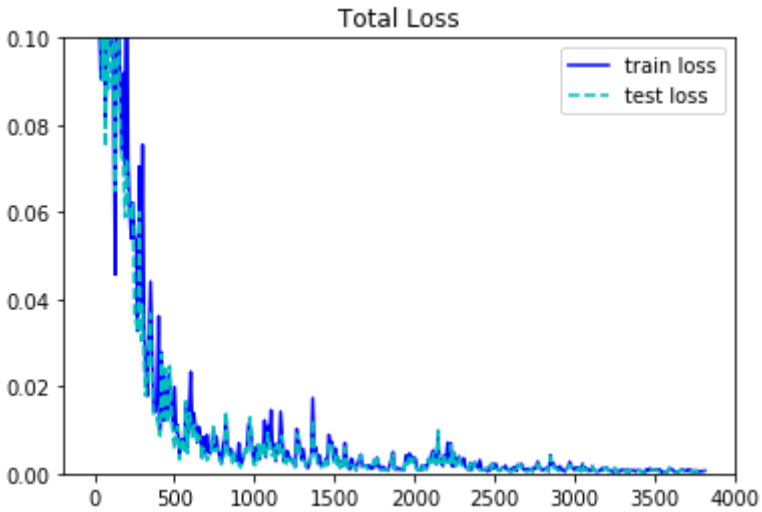
3771th Epoch:
traing loss is 0.0003762412816286087, test loss is 0.0004314950492698699, and learning rate i

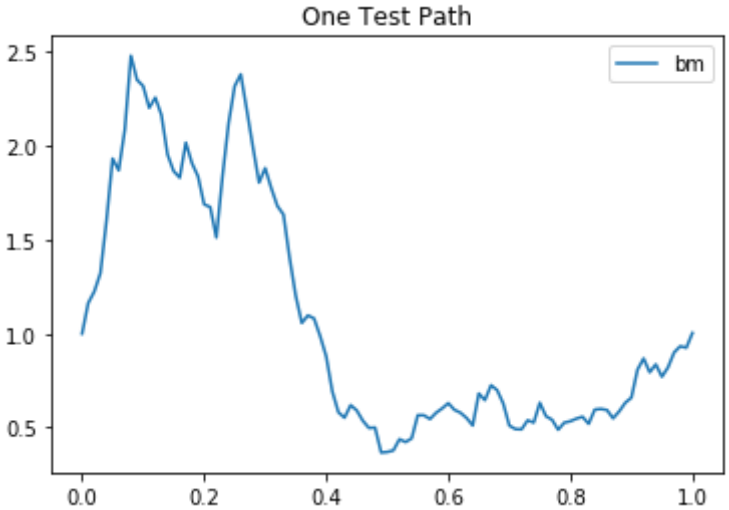
3781th Epoch:
traing loss is 0.0005516581004485488, test loss is 0.0005824593245051801, and learning rate i

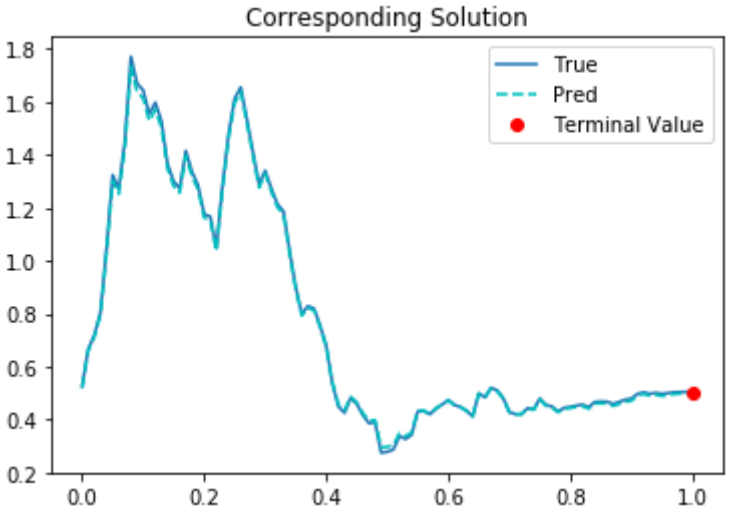
3791th Epoch:
traing loss is 0.00046248253784142435, test loss is 0.00032576938974671066, and learning rate

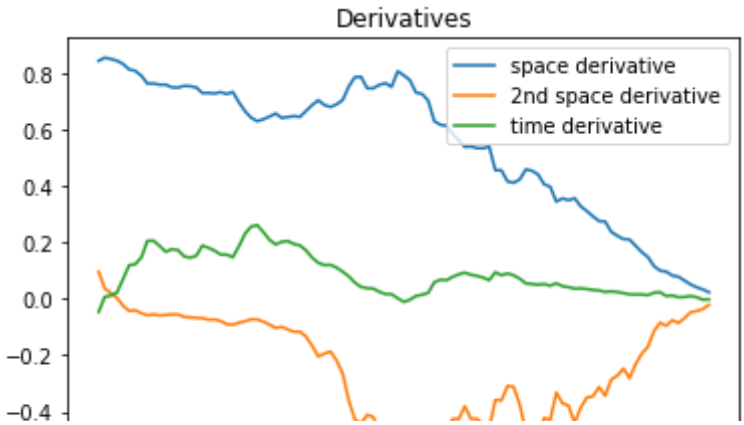
3801th Epoch:
```

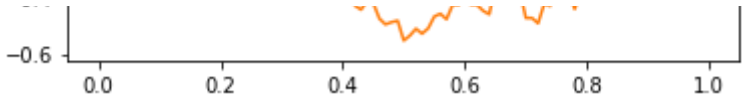
```
traing loss is 0.000652759219519794, test loss is 0.00048228114610537887, and learning rate i
```












```
3811th Epoch:
traing loss is 0.0003382669237907976, test loss is 0.00023808307014405727, and learning rate

3821th Epoch:
traing loss is 0.0006108818924985826, test loss is 0.0004788423248101026, and learning rate i

3831th Epoch:
traing loss is 0.00016363500617444515, test loss is 0.0002190284722018987, and learning rate

3841th Epoch:
traing loss is 0.0012006788747385144, test loss is 0.0008914864738471806, and learning rate i

3851th Epoch:
traing loss is 0.0005534301162697375, test loss is 0.0006461297743953764, and learning rate i

3861th Epoch:
traing loss is 0.0015990572283044457, test loss is 0.0007668753387406468, and learning rate i

3871th Epoch:
traing loss is 0.0008482140256091952, test loss is 0.0008573171216994524, and learning rate i

3881th Epoch:
traing loss is 0.0007466980605386198, test loss is 0.0006734564667567611, and learning rate i

3891th Epoch:
traing loss is 0.0005224251071922481, test loss is 0.00046602694783359766, and learning rate

3901th Epoch:
traing loss is 0.0006479076109826565, test loss is 0.0004038067418150604, and learning rate i
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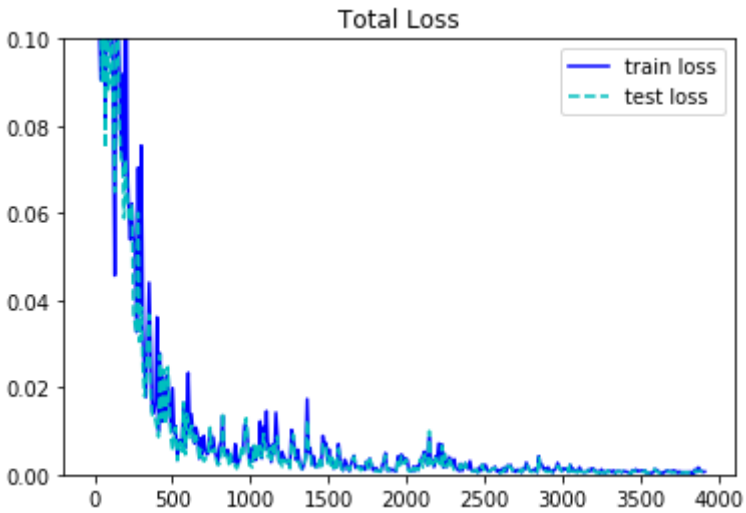

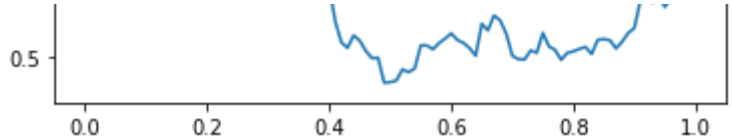

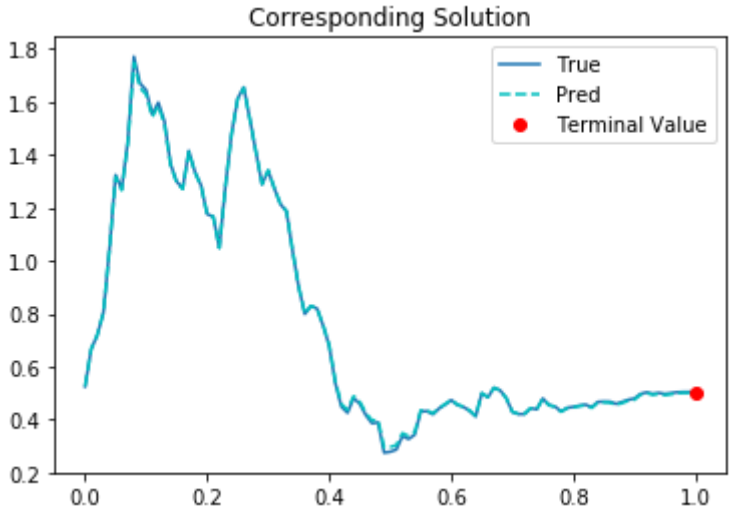

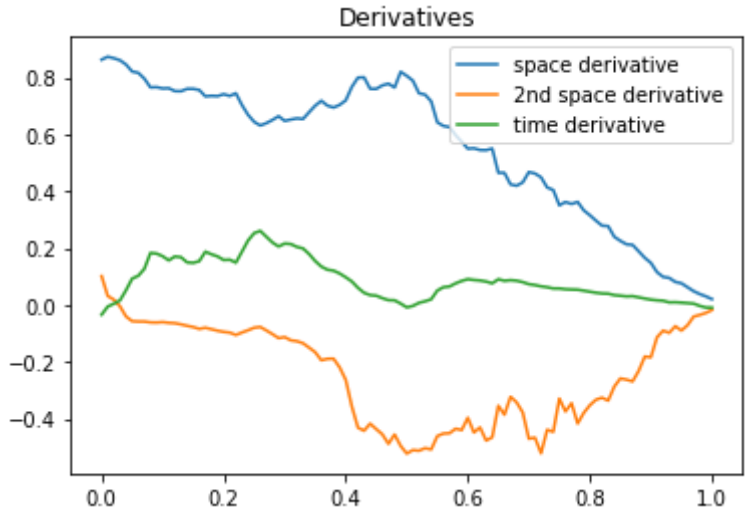


```
3911th Epoch:
traing loss is 0.0006667355191893876, test loss is 0.0003089202509727329, and learning rate i

3921th Epoch:
traing loss is 0.0005920691764913499, test loss is 0.0004412894486449659, and learning rate i

3931th Epoch:
traing loss is 0.00015018924023024738, test loss is 0.00019236281514167786, and learning rate

3941th Epoch:
traing loss is 0.0005590195651166141, test loss is 0.0003943643532693386, and learning rate i

3951th Epoch:
traing loss is 0.0008033945923671126, test loss is 0.0007301855948753655, and learning rate i

3961th Epoch:
traing loss is 0.0012847190955653787, test loss is 0.0010823761112987995, and learning rate i

3971th Epoch:
traing loss is 0.0009266083943657577, test loss is 0.0009064617333933711, and learning rate i

3981th Epoch:
traing loss is 0.00188232003711164, test loss is 0.0015554827405139804, and learning rate is

3991th Epoch:
```

```
traing loss is 0.0008220949675887823, test loss is 0.0007418685709126294, and learning rate i

4001th Epoch:
traing loss is 0.0019682832062244415, test loss is 0.0007157286745496094, and learning rate i
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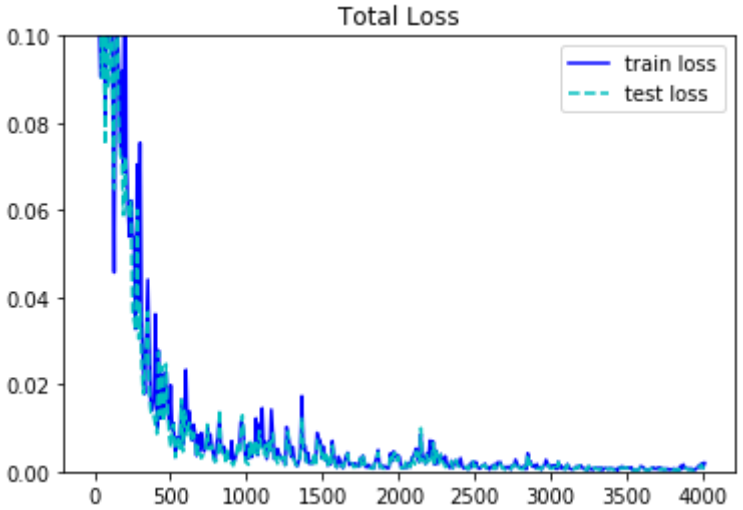

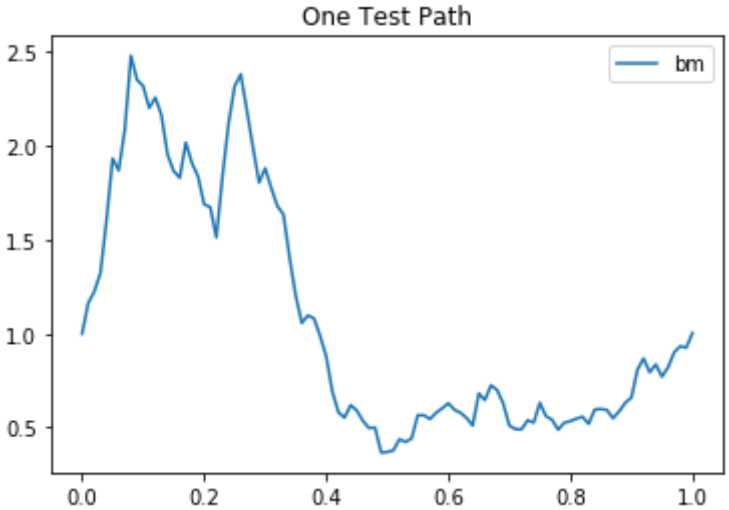

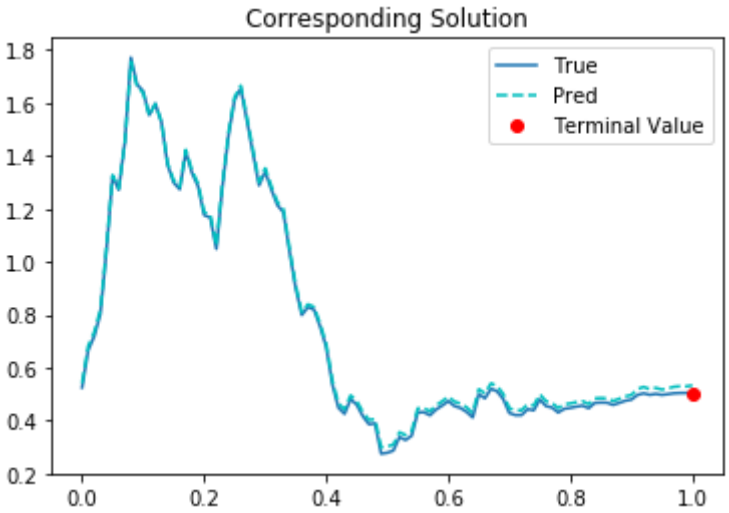

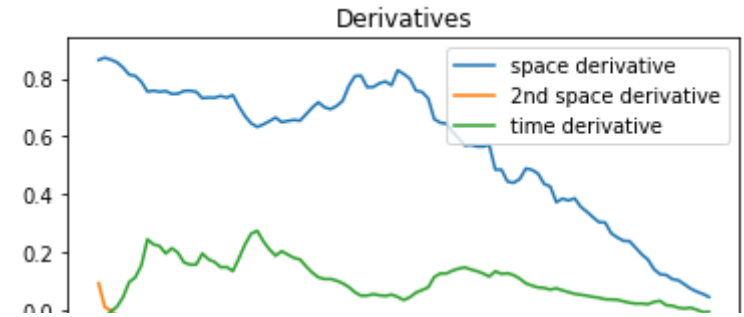

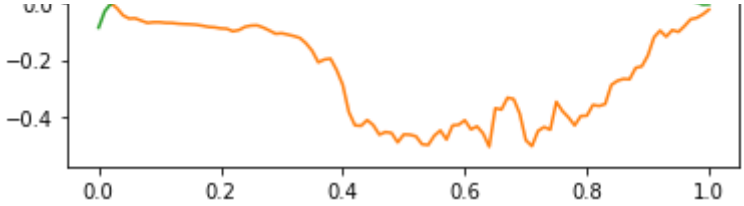


```
4011th Epoch:
traing loss is 0.0009122813935391605, test loss is 0.0006714969058521092, and learning rate i

4021th Epoch:
traing loss is 0.0007810076931491494, test loss is 0.0007006805972196162, and learning rate i

4031th Epoch:
traing loss is 0.0003452491364441812, test loss is 0.000386109109967947, and learning rate is

4041th Epoch:
traing loss is 0.001243304810486734, test loss is 0.0010545566910877824, and learning rate is

4051th Epoch:
traing loss is 0.0011252091499045491, test loss is 0.0011071892222389579, and learning rate i

4061th Epoch:
traing loss is 0.004432205576449633, test loss is 0.002398192649707198, and learning rate is

4071th Epoch:
traing loss is 0.0020159906707704067, test loss is 0.0020057104993611574, and learning rate i

4081th Epoch:
traing loss is 0.0014865344855934381, test loss is 0.001246370025910437, and learning rate is

4091th Epoch:
traing loss is 0.0012334708590060472, test loss is 0.0011250866809859872, and learning rate i

4101th Epoch:
traing loss is 0.0020123994909226894, test loss is 0.0006014956161379814, and learning rate i
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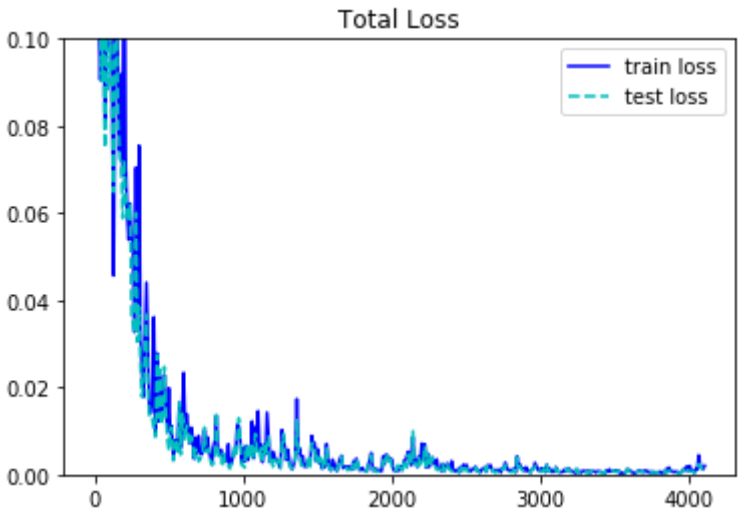

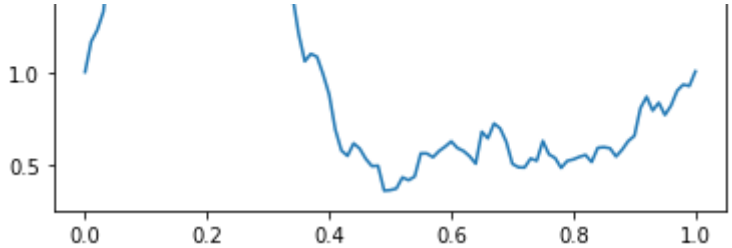

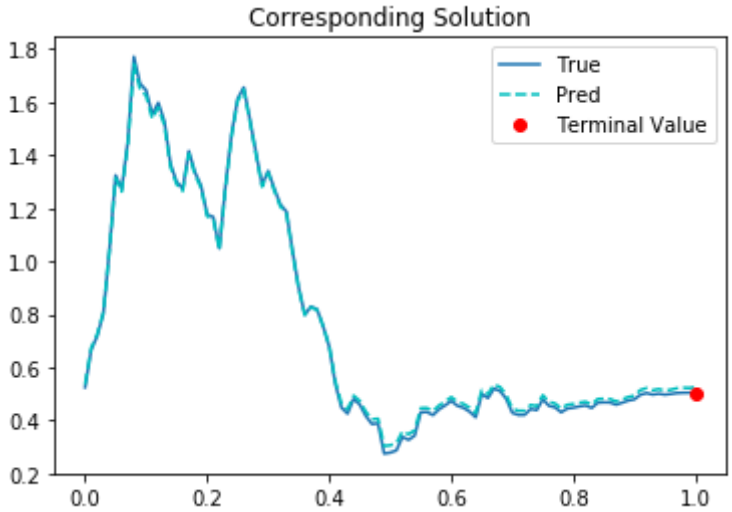

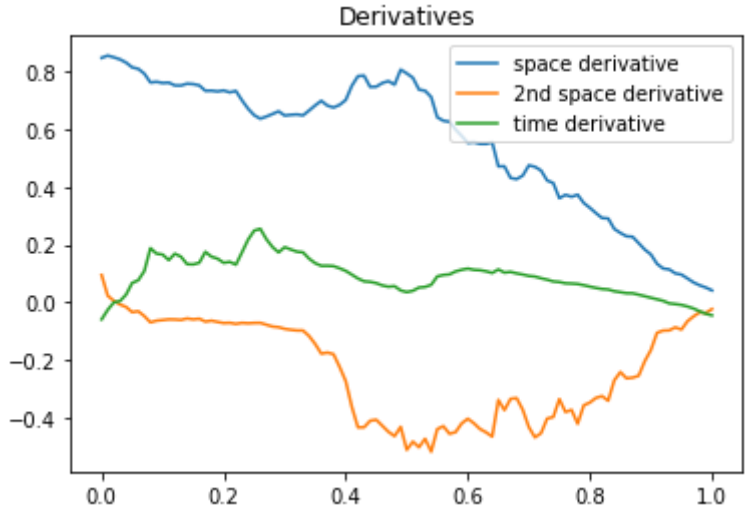


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4111th Epoch:
traing loss is 0.0007778556318953633, test loss is 0.00039277991163544357, and learning rate

4121th Epoch:
traing loss is 0.000526794174220413, test loss is 0.0005156373372301459, and learning rate is

4131th Epoch:
traing loss is 0.0005899132811464369, test loss is 0.0006854452076368034, and learning rate i

4141th Epoch:
traing loss is 0.0016956421313807368, test loss is 0.0016711631324142218, and learning rate i

4151th Epoch:
traing loss is 0.0012924486072733998, test loss is 0.0012209098786115646, and learning rate i

4161th Epoch:
traing loss is 0.0027250417042523623, test loss is 0.001183392945677042, and learning rate is

4171th Epoch:
traing loss is 0.0012774094939231873, test loss is 0.0012443289160728455, and learning rate i

4181th Epoch:
```

```
traing loss is 0.0016871081897988915, test loss is 0.0012132974807173014, and learning rate i

4191th Epoch:
traing loss is 0.001235556905157864, test loss is 0.0008827833808027208, and learning rate is

4201th Epoch:
traing loss is 0.002377270720899105, test loss is 0.0006594194564968348, and learning rate is
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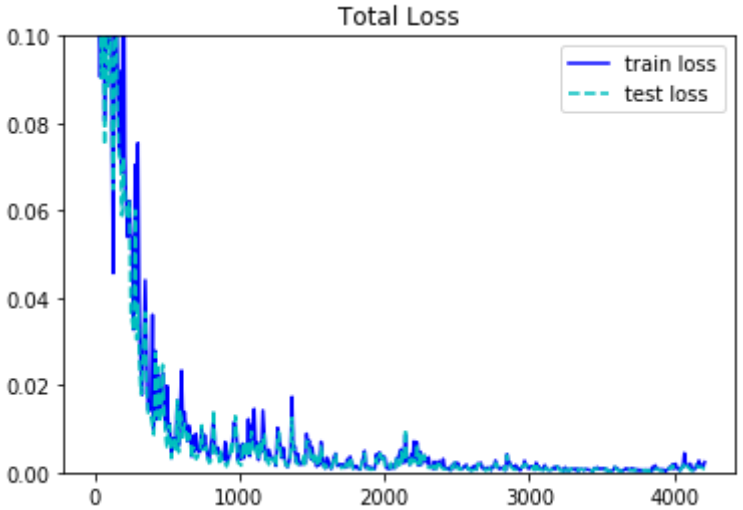

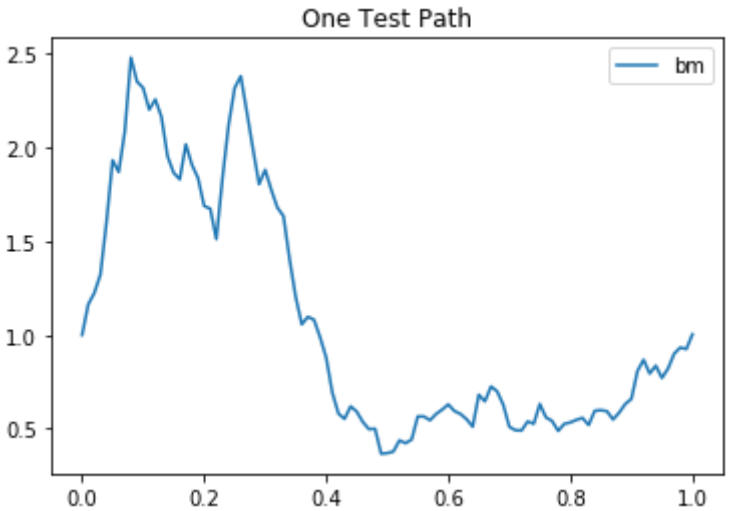

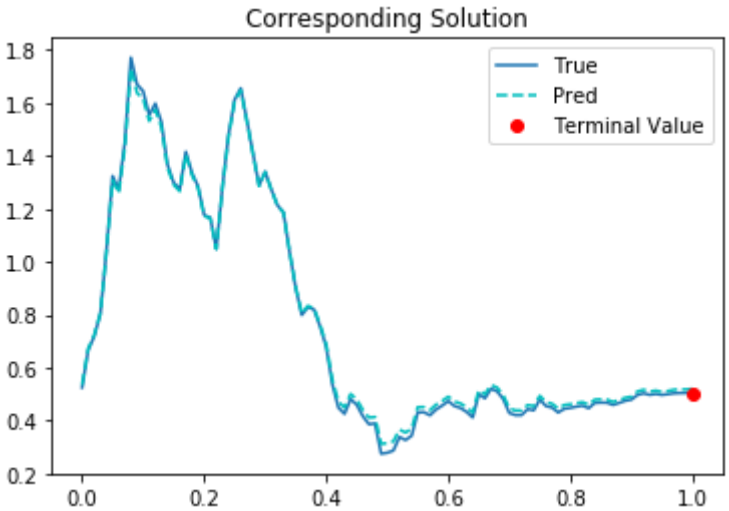

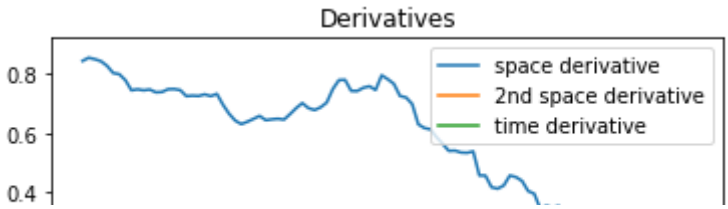

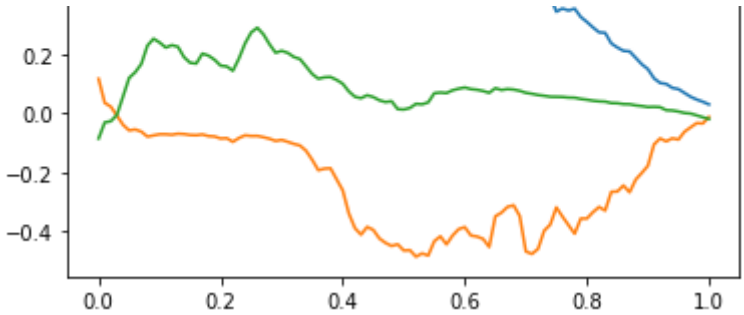


```
4211th Epoch:
traing loss is 0.0011635129339993, test loss is 0.0005777818732894957, and learning rate is 0

4221th Epoch:
traing loss is 0.0007840257603675127, test loss is 0.0006112807895988226, and learning rate i

4231th Epoch:
traing loss is 0.00031280089751817286, test loss is 0.000337546895025298, and learning rate i

4241th Epoch:
traing loss is 0.0018226495012640953, test loss is 0.0014546727761626244, and learning rate i

4251th Epoch:
traing loss is 0.0008321769419126213, test loss is 0.0007657023961655796, and learning rate i

4261th Epoch:
traing loss is 0.001372037106193602, test loss is 0.0007095349137671292, and learning rate is

4271th Epoch:
traing loss is 0.0007308421772904694, test loss is 0.0006796247325837612, and learning rate i

4281th Epoch:
traing loss is 0.0007861803169362247, test loss is 0.0005483580171130598, and learning rate i

4291th Epoch:
traing loss is 0.0006199807394295931, test loss is 0.0005597681156359613, and learning rate i

4301th Epoch:
traing loss is 0.0012665855465456843, test loss is 0.0004122894606553018, and learning rate i
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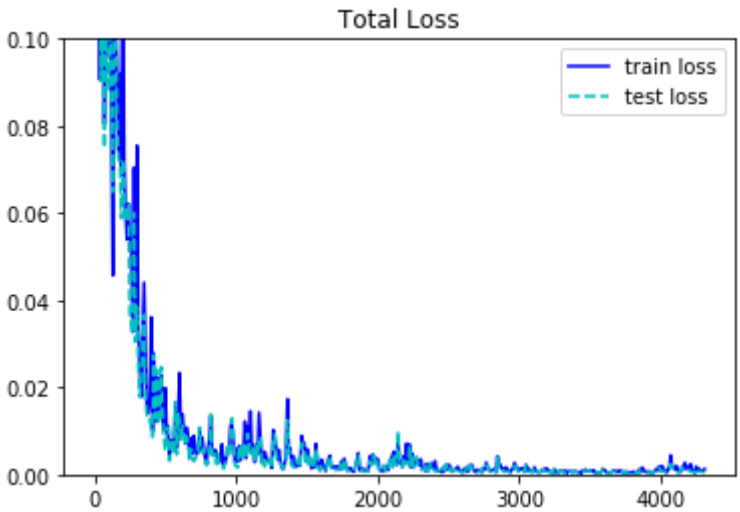

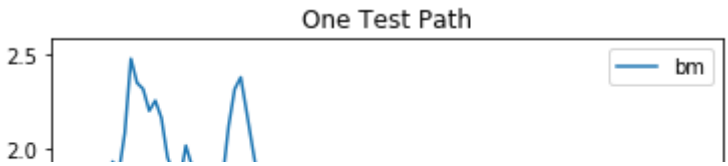

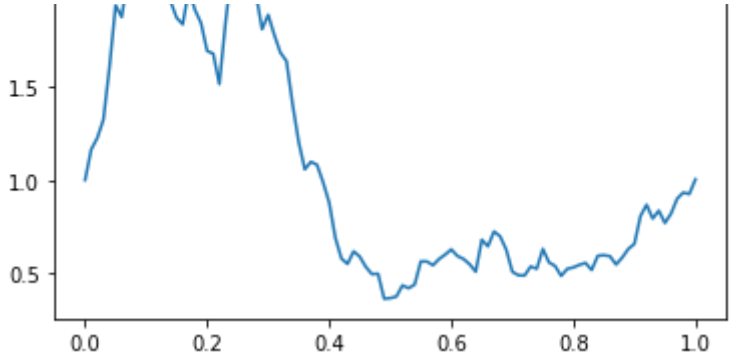

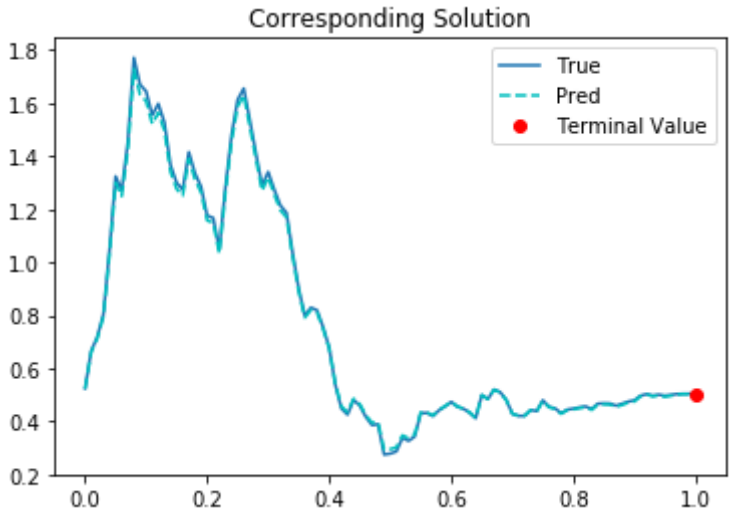

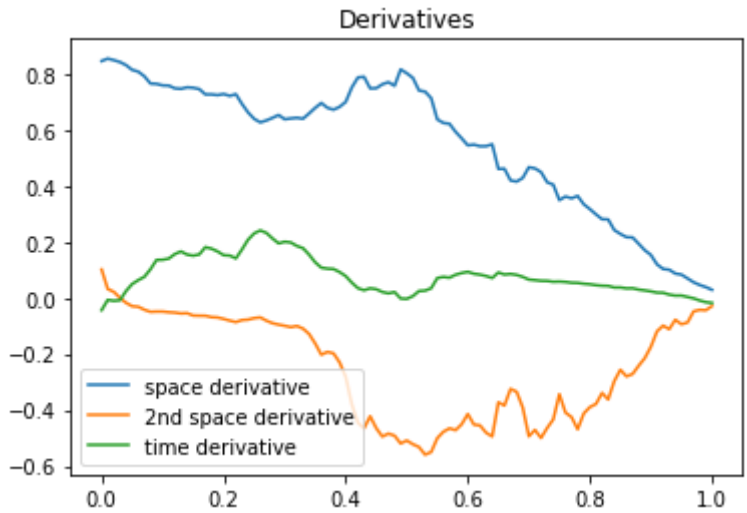


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4311th Epoch:
traing loss is 0.0006923043401911855, test loss is 0.000362322258297354, and learning rate is

4321th Epoch:
traing loss is 0.0004487113037612289, test loss is 0.00045954479719512165, and learning rate

4331th Epoch:
traing loss is 0.00037806242471560836, test loss is 0.0004420675104483962, and learning rate

4341th Epoch:
traing loss is 0.0012667658738791943, test loss is 0.0009882819140329957, and learning rate i

4351th Epoch:
traing loss is 0.0007287884363904595, test loss is 0.0006841709837317467, and learning rate i

4361th Epoch:
traing loss is 0.003487570444121957, test loss is 0.0020871651358902454, and learning rate is

4371th Epoch:
```

```
traing loss is 0.0008954863296821713, test loss is 0.0009716800414025784, and learning rate i

4381th Epoch:
traing loss is 0.0007539730286225677, test loss is 0.0005523950676433742, and learning rate i

4391th Epoch:
traing loss is 0.0005732069257646799, test loss is 0.0004307142226025462, and learning rate i

4401th Epoch:
traing loss is 0.0014117881655693054, test loss is 0.00042882448178716004, and learning rate
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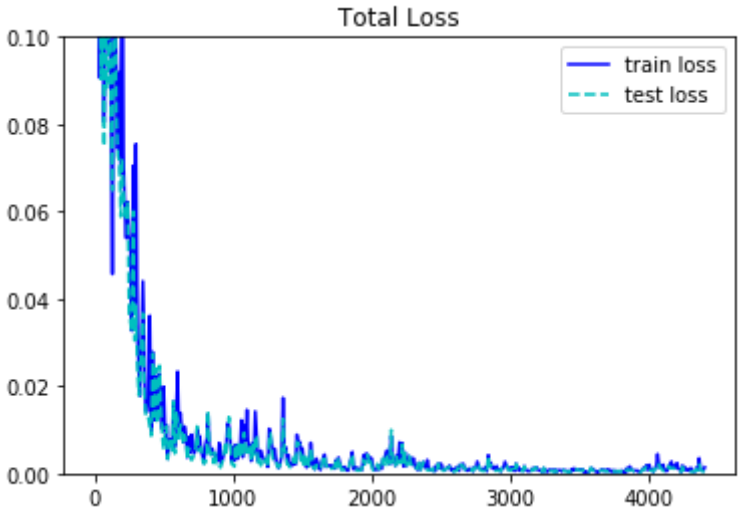

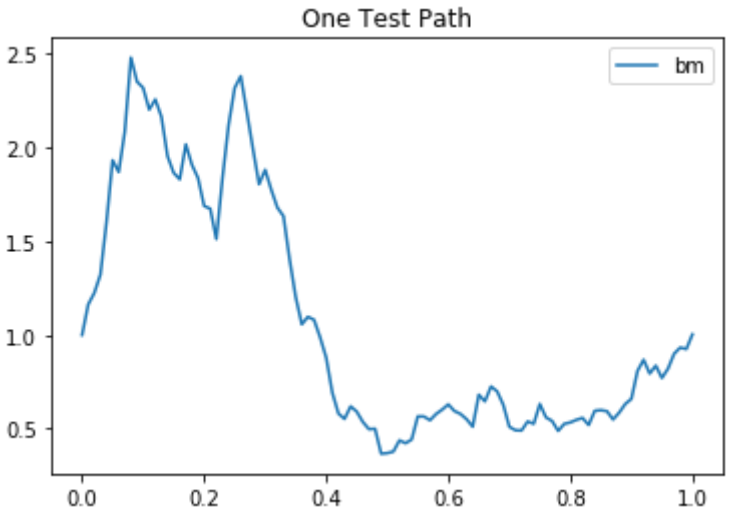

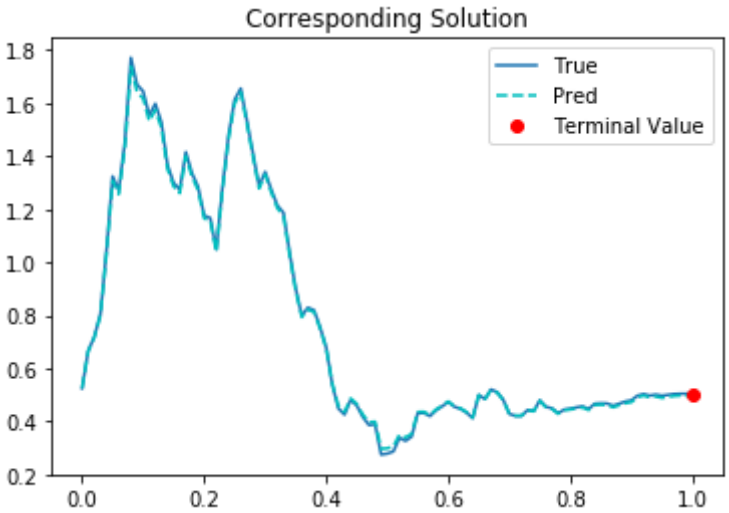

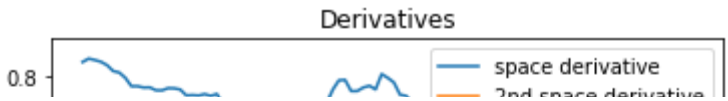

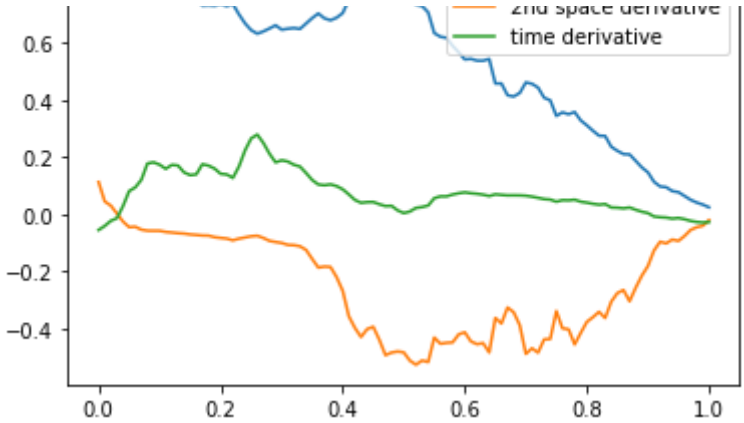


```
4411th Epoch:
traing loss is 0.000965892628300935, test loss is 0.0004300708242226392, and learning rate is

4421th Epoch:
traing loss is 0.0003977040760219097, test loss is 0.00036781150265596807, and learning rate

4431th Epoch:
traing loss is 0.0003035546687897295, test loss is 0.00034198976936750114, and learning rate

4441th Epoch:
traing loss is 0.0018430391792207956, test loss is 0.0013541592052206397, and learning rate i

4451th Epoch:
traing loss is 0.0014580913120880723, test loss is 0.001427294104360044, and learning rate is

4461th Epoch:
traing loss is 0.003800526726990938, test loss is 0.0022930805571377277, and learning rate is

4471th Epoch:
traing loss is 0.0028260033577680588, test loss is 0.0025860124733299017, and learning rate i

4481th Epoch:
traing loss is 0.0022664808202534914, test loss is 0.0015572508564218879, and learning rate i

4491th Epoch:
traing loss is 0.001497497665695846, test loss is 0.0010284979362040758, and learning rate is

4501th Epoch:
traing loss is 0.0025638388469815254, test loss is 0.0005662519251927733, and learning rate i
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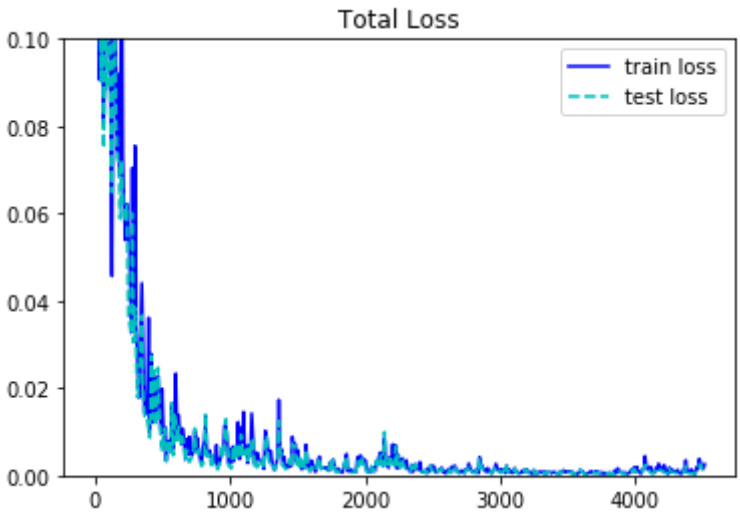
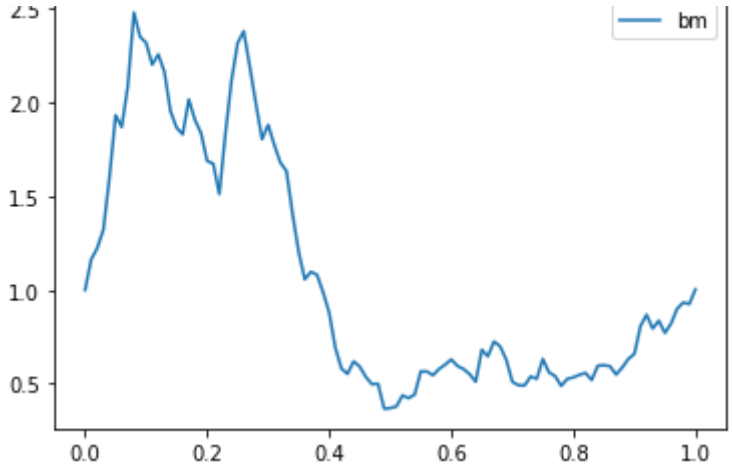
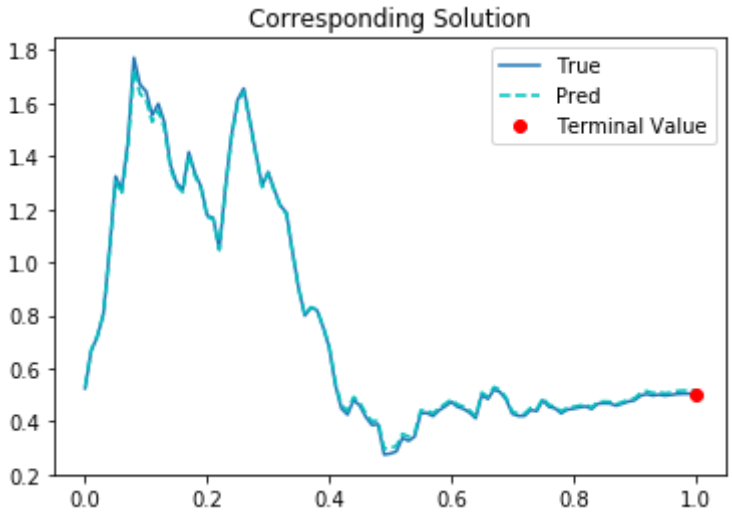
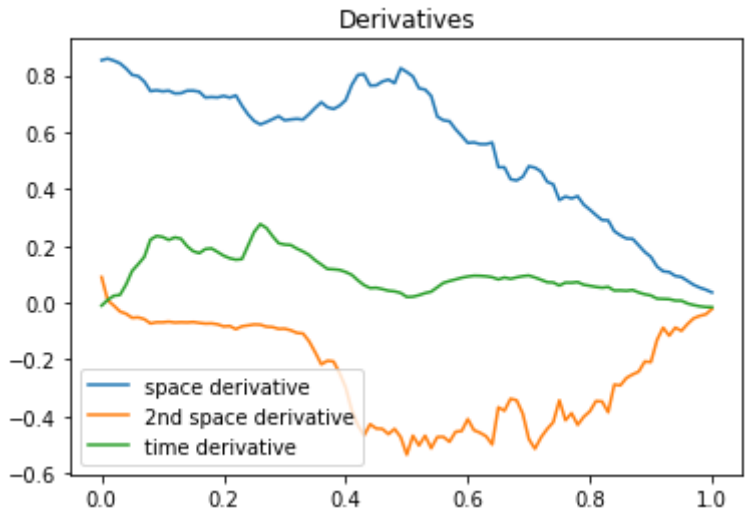


One Test Path








```
4511th Epoch:
traing loss is 0.0015769129386171699, test loss is 0.0012591303093358874, and learning rate i

4521th Epoch:
traing loss is 0.0013037014286965132, test loss is 0.000936998229008168, and learning rate is

4531th Epoch:
traing loss is 0.0007831223192624748, test loss is 0.0007992914179340005, and learning rate i

4541th Epoch:
traing loss is 0.005258979741483927, test loss is 0.004417409189045429, and learning rate is

4551th Epoch:
traing loss is 0.0027194800786674023, test loss is 0.002348807407543063, and learning rate is

4561th Epoch:
```

```
4561th Epoch:
traing loss is 0.006837455090135336, test loss is 0.003913396969437599, and learning rate is

4571th Epoch:
traing loss is 0.0033659394830465317, test loss is 0.0034791226498782635, and learning rate i

4581th Epoch:
traing loss is 0.0012730535818263888, test loss is 0.0008798303315415978, and learning rate i

4591th Epoch:
traing loss is 0.0008632256067357957, test loss is 0.0005482810083776712, and learning rate i

4601th Epoch:
traing loss is 0.0029185395687818527, test loss is 0.0006361319101415575, and learning rate i
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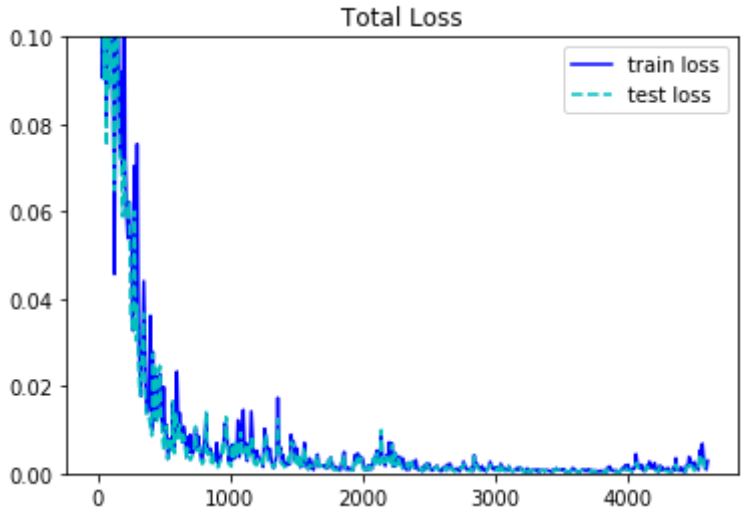

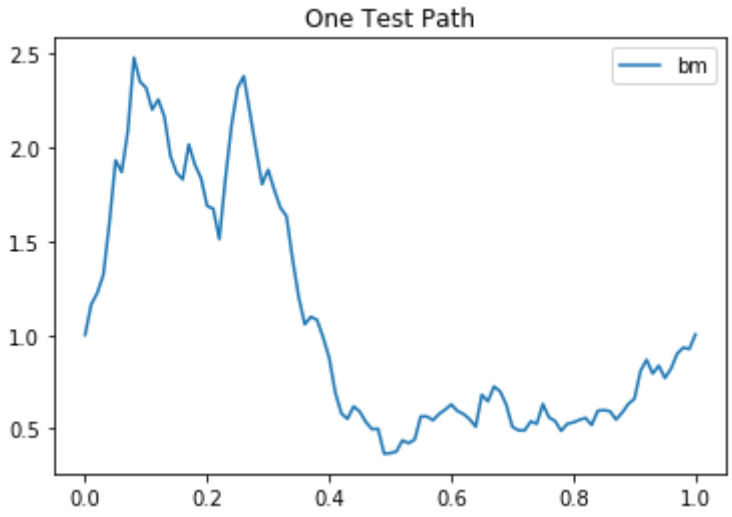

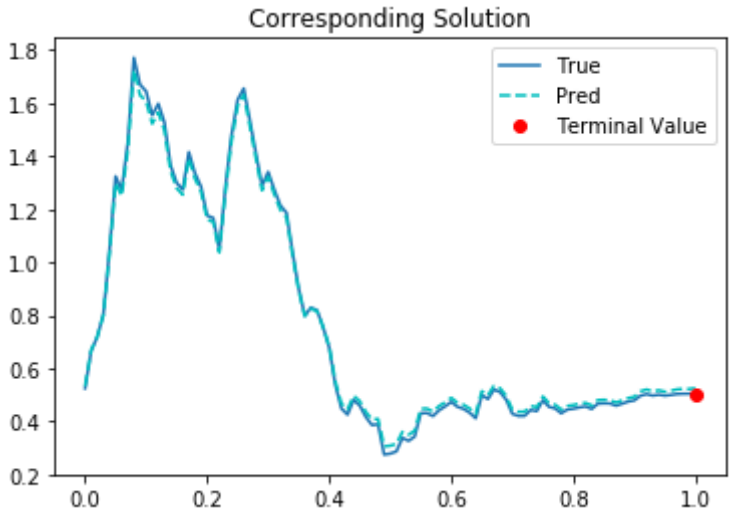

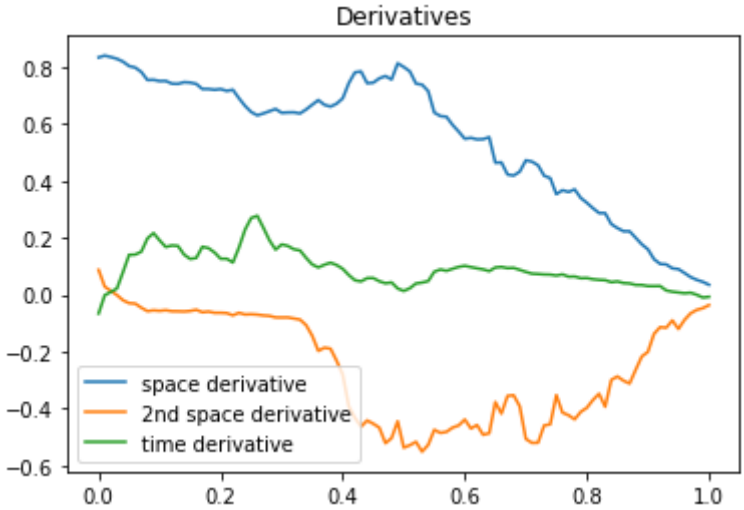


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4611th Epoch:
traing loss is 0.0010368642397224903, test loss is 0.0007546068518422544, and learning rate i

4621th Epoch:
traing loss is 0.0008061989792622626, test loss is 0.0009434486855752766, and learning rate i

4631th Epoch:
traing loss is 0.00035672966623678803, test loss is 0.00046672672033309937, and learning rate

4641th Epoch:
traing loss is 0.0019019597675651312, test loss is 0.0017871875315904617, and learning rate i

4651th Epoch:
traing loss is 0.0009025246254168451, test loss is 0.0008421990205533803, and learning rate i

4661th Epoch:
traing loss is 0.0009711672319099307, test loss is 0.0005882192053832114, and learning rate i

4671th Epoch:
traing loss is 0.0006233470048755407, test loss is 0.0006171864224597812, and learning rate i

4681th Epoch:
traing loss is 0.0007949834689497948, test loss is 0.0005505759618245065, and learning rate i

4691th Epoch:
traing loss is 0.0005573274102061987, test loss is 0.0004063713422510773, and learning rate i

4701th Epoch:
traing loss is 0.0008617384009994566, test loss is 0.0003341561241541058, and learning rate i
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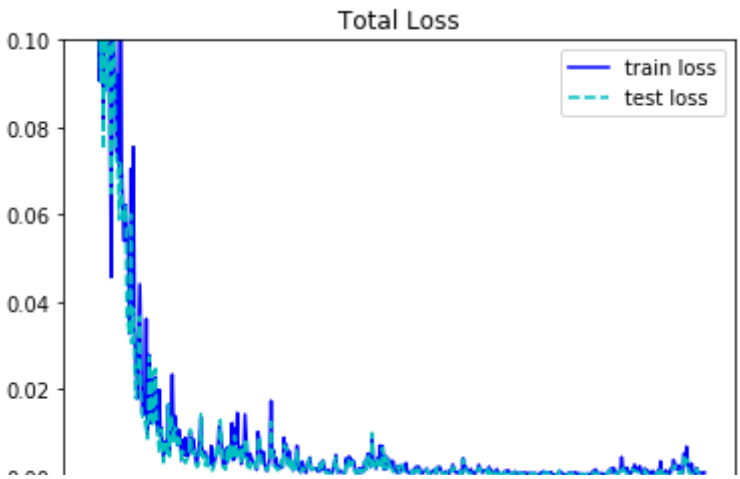

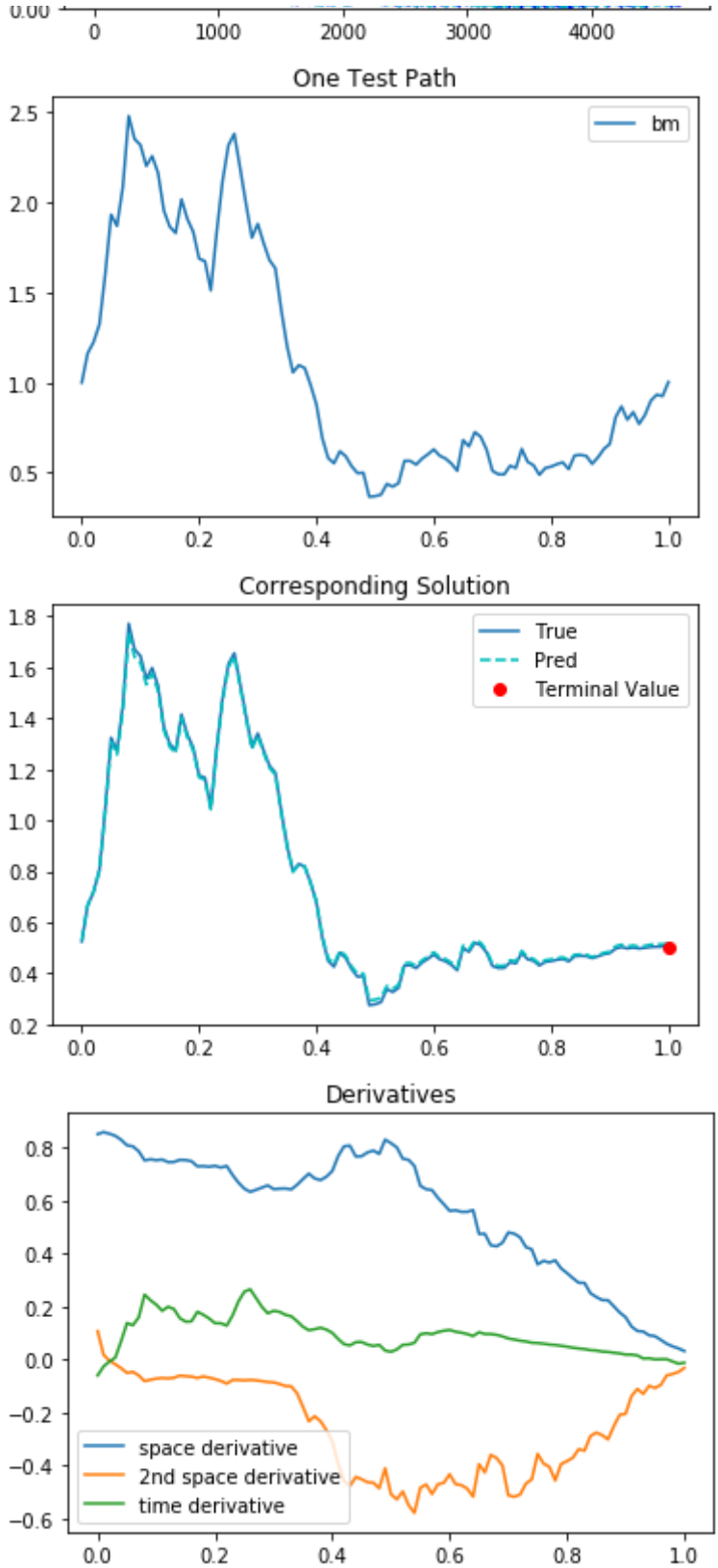


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4711th Epoch:
traing loss is 0.0005205384804867208, test loss is 0.0003718608058989048, and learning rate i

4721th Epoch:
traing loss is 0.00041491957381367683, test loss is 0.000393141177482903, and learning rate i

4731th Epoch:
traing loss is 0.0002572907542344183, test loss is 0.00032034292235039175, and learning rate

4741th Epoch:
traing loss is 0.0014452870236709714, test loss is 0.0013251975178718567, and learning rate i

4751th Epoch:
```

```
traing loss is 0.0005478266393765807, test loss is 0.0004516974731814116, and learning rate i

4761th Epoch:
traing loss is 0.000912786868866533, test loss is 0.0008441989193670452, and learning rate is

4771th Epoch:
traing loss is 0.00037849906948395073, test loss is 0.0004536615451797843, and learning rate

4781th Epoch:
traing loss is 0.0006039405707269907, test loss is 0.0005716665182262659, and learning rate i

4791th Epoch:
traing loss is 0.0005552630755119026, test loss is 0.0004112218739464879, and learning rate i

4801th Epoch:
traing loss is 0.000732794520445168, test loss is 0.0002544612798374146, and learning rate is
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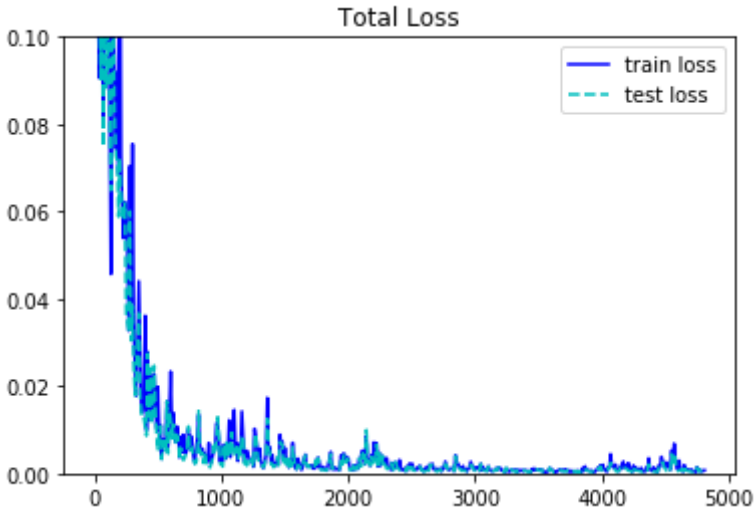

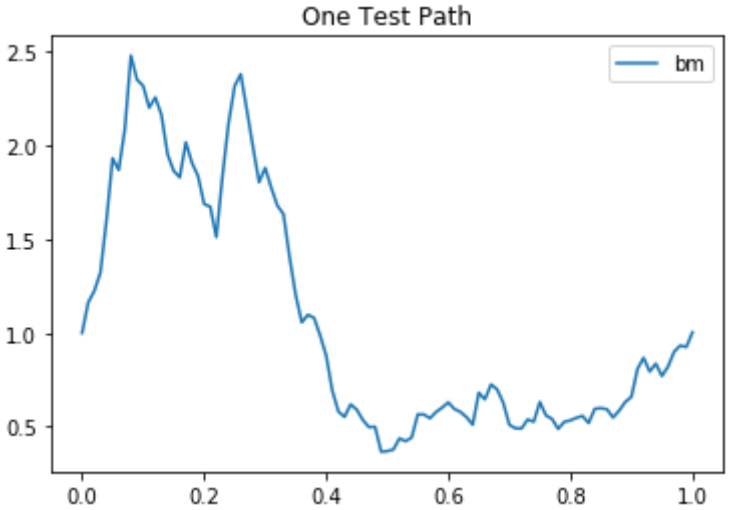

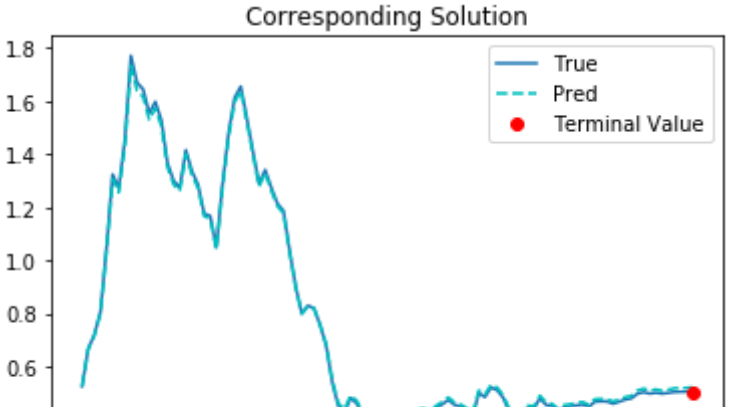

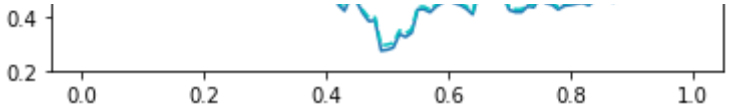

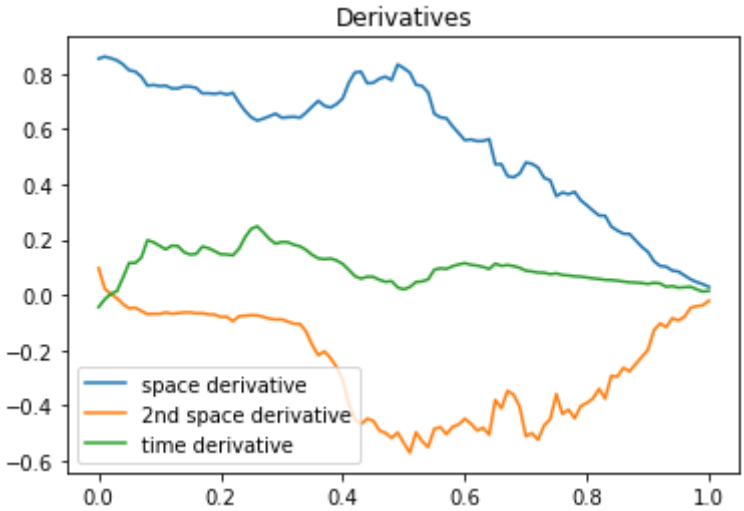


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4811th Epoch:
traing loss is 0.0006096595898270607, test loss is 0.0004847931850235909, and learning rate i

4821th Epoch:
traing loss is 0.0005517459358088672, test loss is 0.00046890764497220516, and learning rate

4831th Epoch:
traing loss is 0.00024801582912914455, test loss is 0.0003066842327825725, and learning rate

4841th Epoch:
traing loss is 0.000798584776930511, test loss is 0.0005539841367863119, and learning rate is

4851th Epoch:
traing loss is 0.00034402470919303596, test loss is 0.0003586716775316745, and learning rate

4861th Epoch:
traing loss is 0.0010854623978957534, test loss is 0.0009533509146422148, and learning rate i

4871th Epoch:
traing loss is 0.00037171831354498863, test loss is 0.0004251654609106481, and learning rate

4881th Epoch:
traing loss is 0.0005125253810547292, test loss is 0.0005390968290157616, and learning rate i

4891th Epoch:
traing loss is 0.0004795393324457109, test loss is 0.00035847691469825804, and learning rate

4901th Epoch:
traing loss is 0.000554934493266046, test loss is 0.00020975302322767675, and learning rate i
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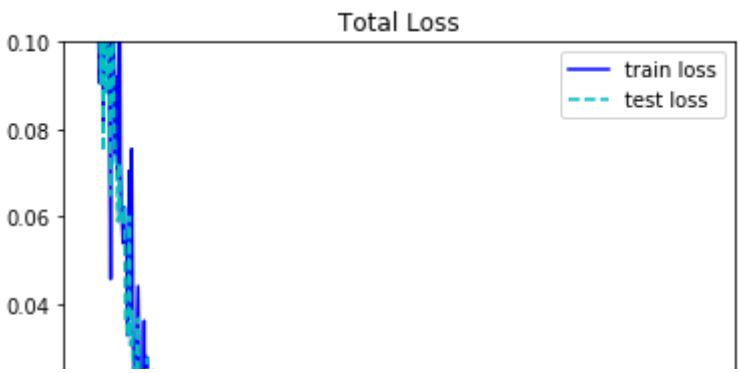

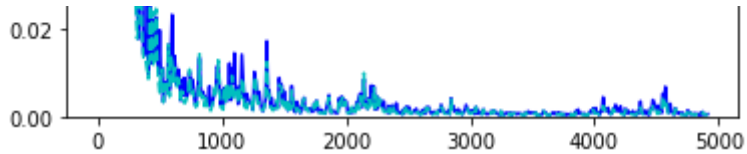

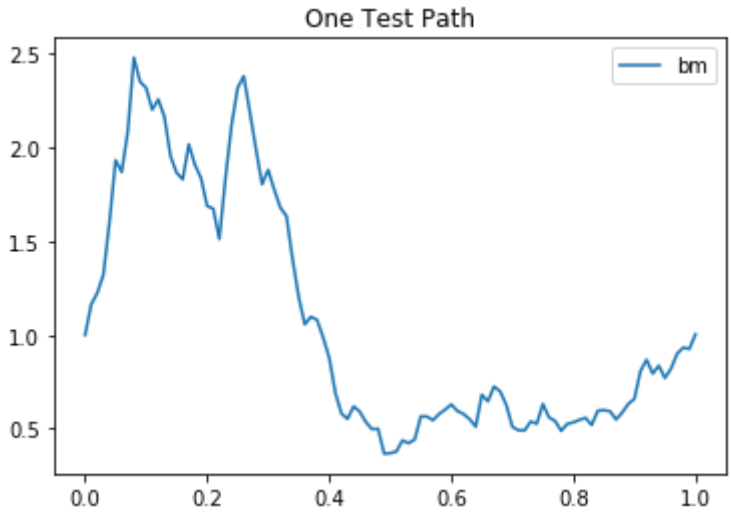

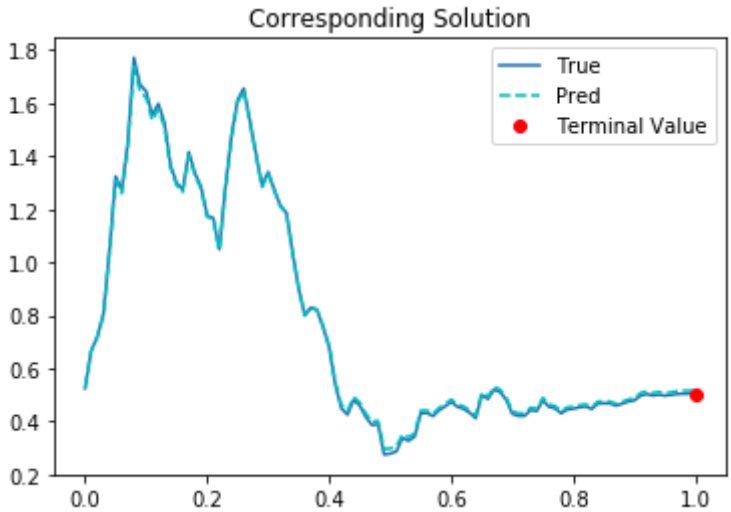

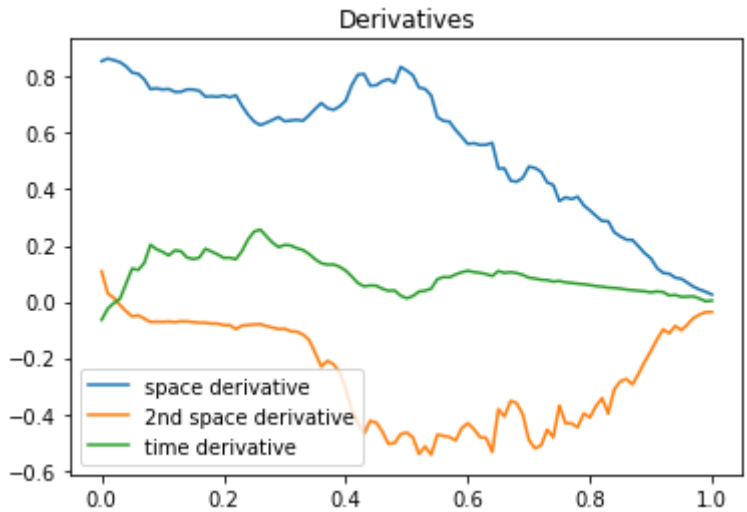


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4911th Epoch:
traing loss is 0.00032402510987594724, test loss is 0.00024436632520519197, and learning rate

4921th Epoch:
traing loss is 0.00033651344710960984, test loss is 0.000326530629536137, and learning rate i

4931th Epoch:
traing loss is 0.00017026416026055813, test loss is 0.00021529644436668605, and learning rate

4941th Epoch:
```

```
traing loss is 0.00048274215077981353, test loss is 0.000310434348648414, and learning rate i

4951th Epoch:
traing loss is 0.00021745743288192898, test loss is 0.00023078695812728256, and learning rate

4961th Epoch:
traing loss is 0.0005411526071839035, test loss is 0.00048778593190945685, and learning rate

4971th Epoch:
traing loss is 0.00033031575731001794, test loss is 0.00037150568095967174, and learning rate

4981th Epoch:
traing loss is 0.0009244122193194926, test loss is 0.0009606624371372163, and learning rate i

4991th Epoch:
traing loss is 0.0006153044523671269, test loss is 0.0004839822940994054, and learning rate i

5001th Epoch:
traing loss is 0.0005572827649302781, test loss is 0.00021191954147070646, and learning rate
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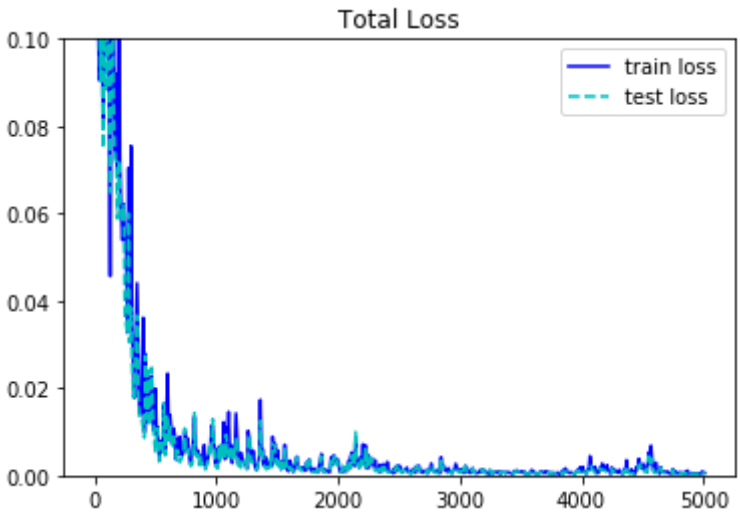

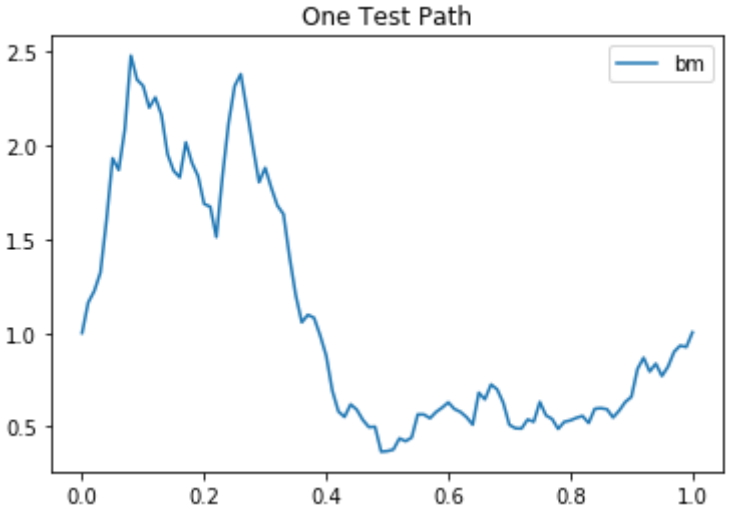

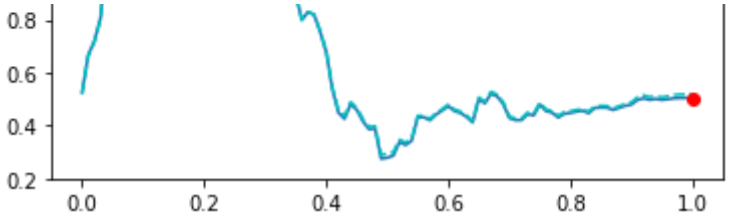

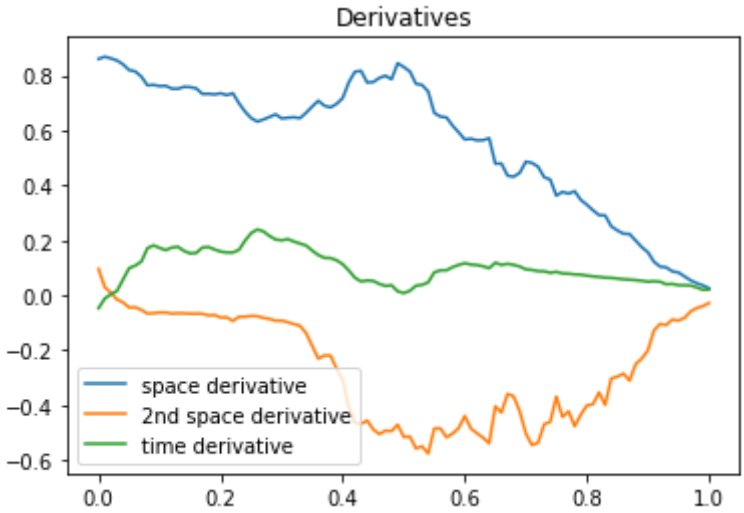


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5011th Epoch:
traing loss is 0.0007405707146972418, test loss is 0.0005507260211743414, and learning rate i

5021th Epoch:
traing loss is 0.0005315208109095693, test loss is 0.0004996951902285218, and learning rate i

5031th Epoch:
traing loss is 0.00030072458321228623, test loss is 0.00036098159034736454, and learning rate

5041th Epoch:
traing loss is 0.0009568360983394086, test loss is 0.0005589267238974571, and learning rate i

5051th Epoch:
traing loss is 0.00027312900056131184, test loss is 0.0003198245831299573, and learning rate

5061th Epoch:
traing loss is 0.0008725423831492662, test loss is 0.0004328286449890584, and learning rate i

5071th Epoch:
traing loss is 0.00036682470818050206, test loss is 0.0004174864443484694, and learning rate

5081th Epoch:
traing loss is 0.0005154324462637305, test loss is 0.0005351853906176984, and learning rate i

5091th Epoch:
traing loss is 0.0005296157323755324, test loss is 0.0004164789570495486, and learning rate i

5101th Epoch:
traing loss is 0.0005456713843159378, test loss is 0.00023631847579963505, and learning rate
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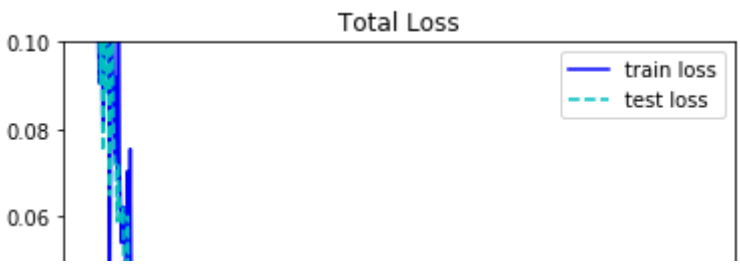

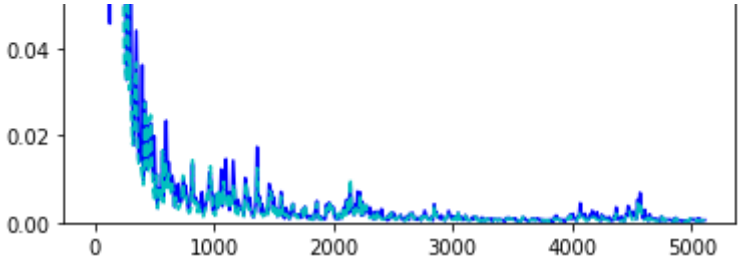

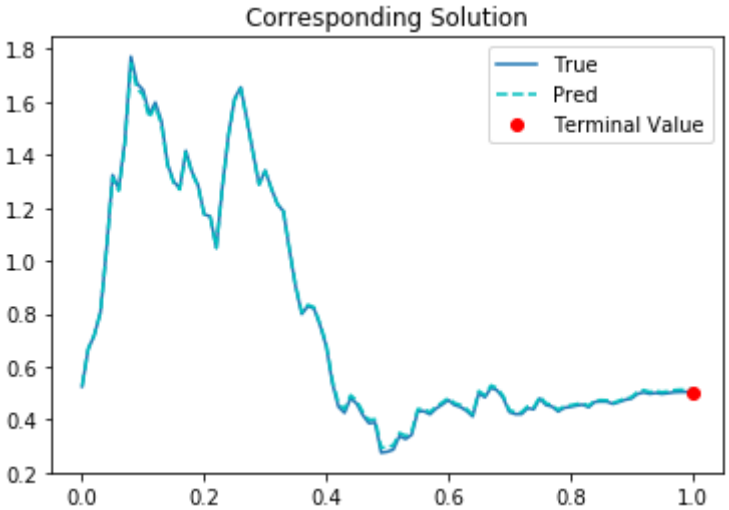

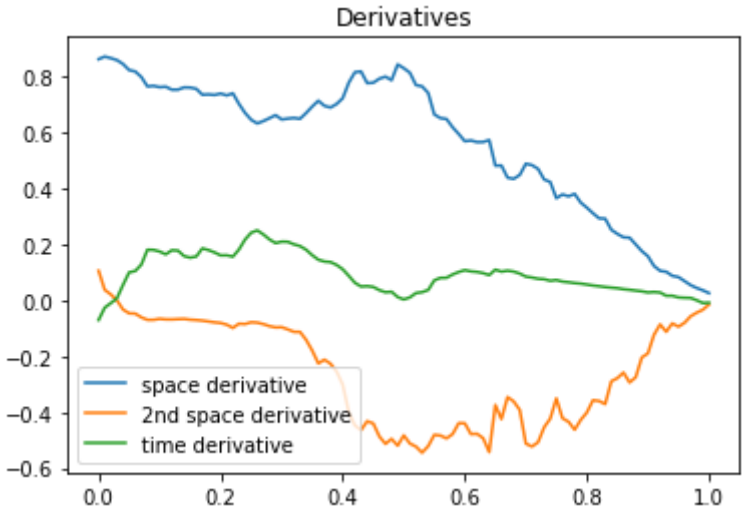


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5111th Epoch:
traing loss is 0.00030042766593396664, test loss is 0.00018180183542426676, and learning rate

5121th Epoch:
traing loss is 0.00026950048049911857, test loss is 0.0002763515221886337, and learning rate

5131th Epoch:
```

```
traing loss is 0.0001336813875241205, test loss is 0.00016495499585289508, and learning rate

5141th Epoch:
traing loss is 0.0006203782395459712, test loss is 0.00036046787863597274, and learning rate

5151th Epoch:
traing loss is 0.00021155639842618257, test loss is 0.00023224062169902027, and learning rate

5161th Epoch:
traing loss is 0.0007046133396215737, test loss is 0.00040407522465102375, and learning rate

5171th Epoch:
traing loss is 0.00048684008652344346, test loss is 0.0005182876484468579, and learning rate

5181th Epoch:
traing loss is 0.000965381390415132, test loss is 0.0010250381892547011, and learning rate is

5191th Epoch:
traing loss is 0.000775299733504653, test loss is 0.0006393977673724294, and learning rate is

5201th Epoch:
traing loss is 0.0005529025220312178, test loss is 0.00023240972950588912, and learning rate
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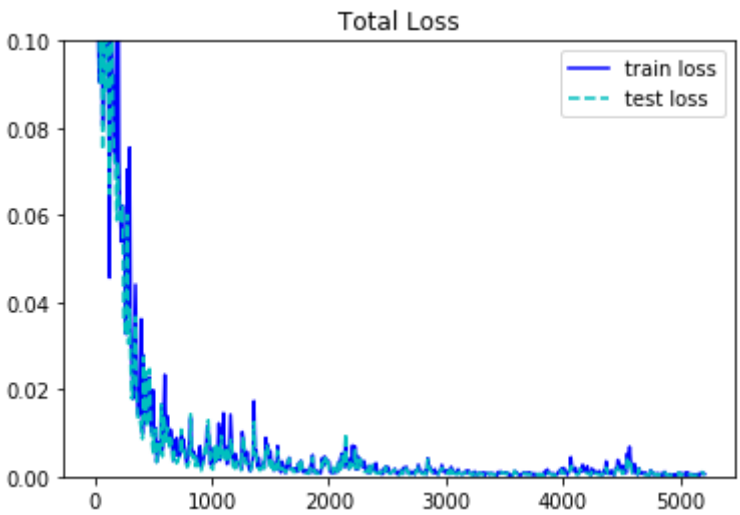

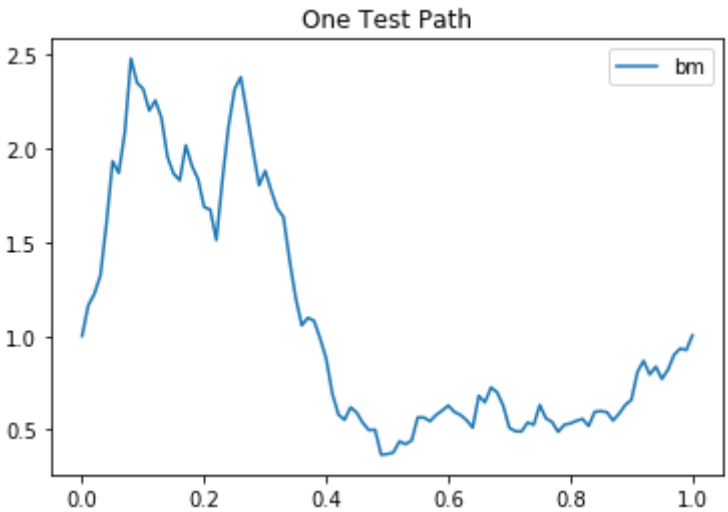

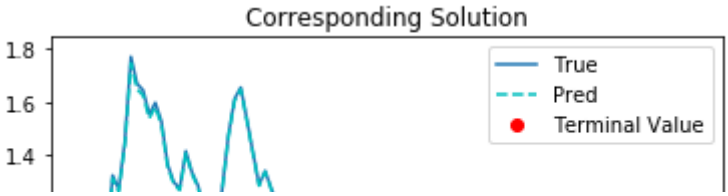

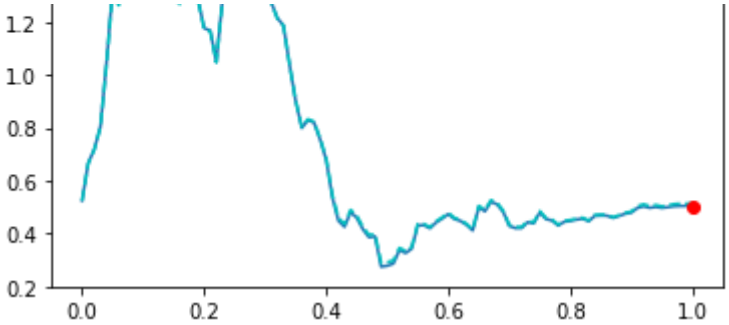

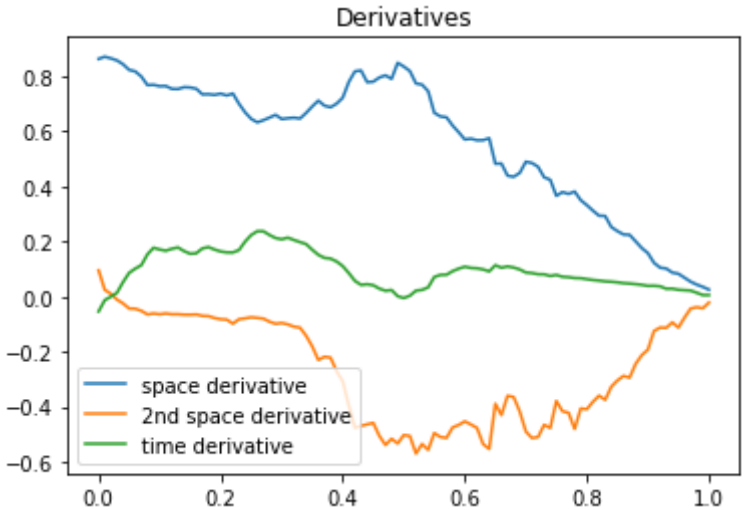


```
5211th Epoch:
traing loss is 0.0004864551010541618, test loss is 0.0003377644170541316, and learning rate i

5221th Epoch:
traing loss is 0.00040691375033929944, test loss is 0.0003853569505736232, and learning rate

5231th Epoch:
traing loss is 0.0002072824427159503, test loss is 0.0002449588500894606, and learning rate i

5241th Epoch:
traing loss is 0.0013220914406701922, test loss is 0.000743392389267683, and learning rate is

5251th Epoch:
traing loss is 0.00036298405029810965, test loss is 0.00041636251262389123, and learning rate

5261th Epoch:
traing loss is 0.0014869668520987034, test loss is 0.0006151380948722363, and learning rate i

5271th Epoch:
traing loss is 0.0009099746239371598, test loss is 0.0010381625033915043, and learning rate i

5281th Epoch:
traing loss is 0.0006124606006778777, test loss is 0.0005738530890084803, and learning rate i

5291th Epoch:
traing loss is 0.0008415293414145708, test loss is 0.0007152605103328824, and learning rate i

5301th Epoch:
traing loss is 0.0005148196360096335, test loss is 0.0002338581543881446, and learning rate i
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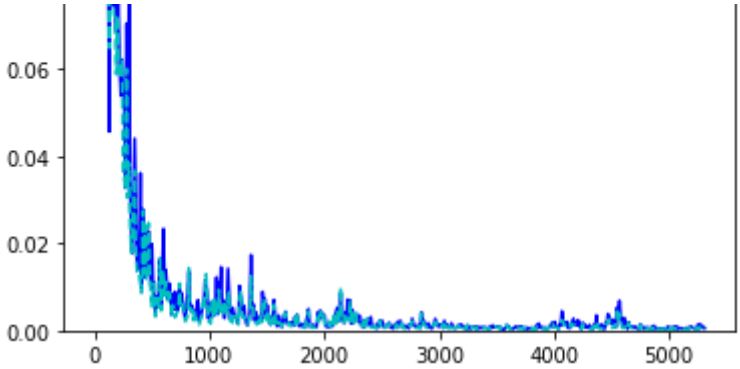

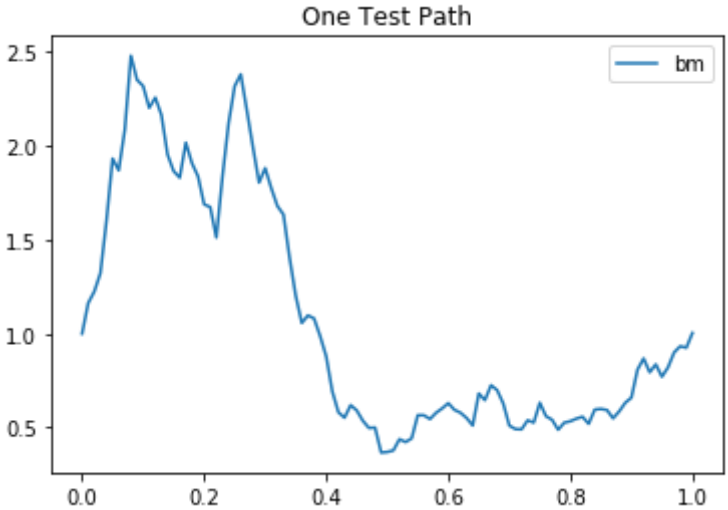

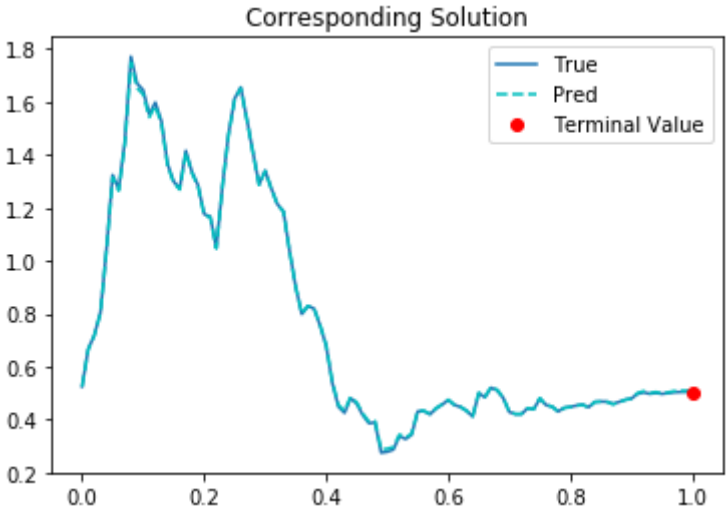

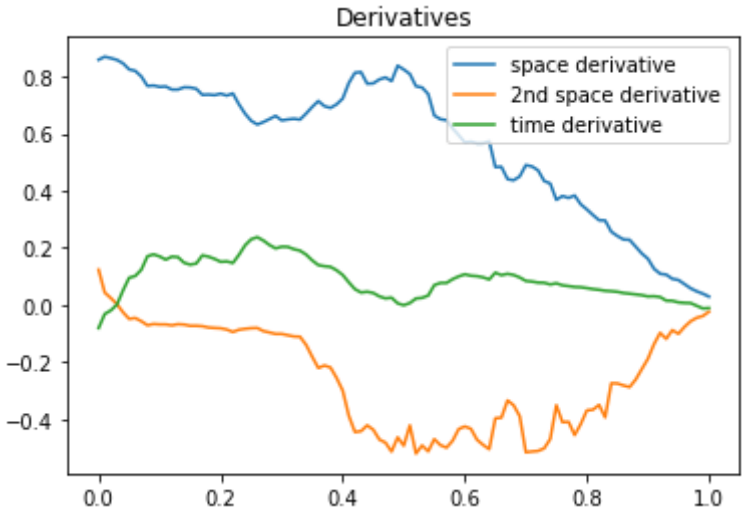

5311th Epoch:
traing loss is 0.0003834773087874055, test loss is 0.00018619160982780159, and learning rate

5321th Epoch:

```
traing loss is 0.0003931175160687417, test loss is 0.00041609167237766087, and learning rate

5331th Epoch:
traing loss is 0.0001627673045732081, test loss is 0.00018165635992772877, and learning rate

5341th Epoch:
traing loss is 0.0006373457727022469, test loss is 0.00040923431515693665, and learning rate

5351th Epoch:
traing loss is 0.0002183913456974551, test loss is 0.0002166780614061281, and learning rate i

5361th Epoch:
traing loss is 0.0007135614869184792, test loss is 0.00033408968010917306, and learning rate

5371th Epoch:
traing loss is 0.0004193662025500089, test loss is 0.00043789861956611276, and learning rate

5381th Epoch:
traing loss is 0.0004448284162208438, test loss is 0.0004182435222901404, and learning rate i

5391th Epoch:
traing loss is 0.0005241030594334006, test loss is 0.00042961465078406036, and learning rate

5401th Epoch:
traing loss is 0.0005451038596220315, test loss is 0.0002332318399567157, and learning rate i
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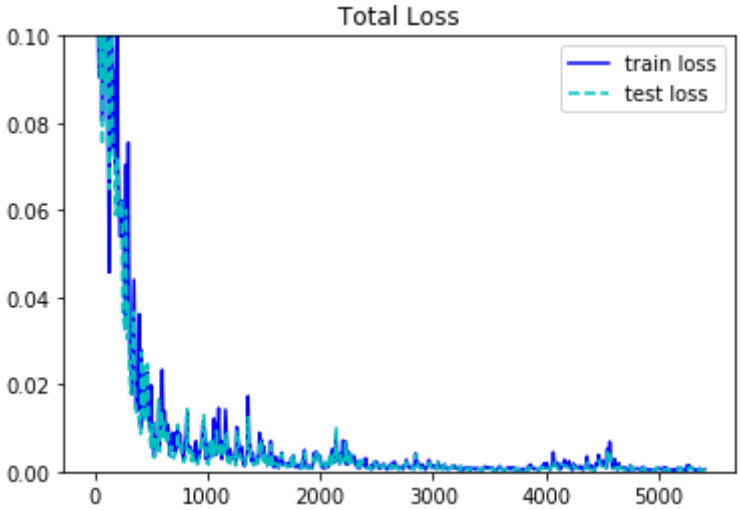

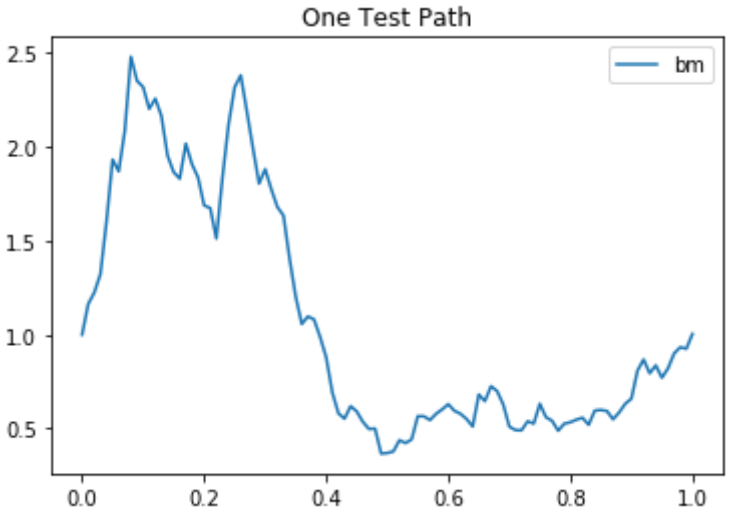

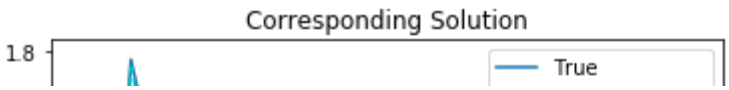

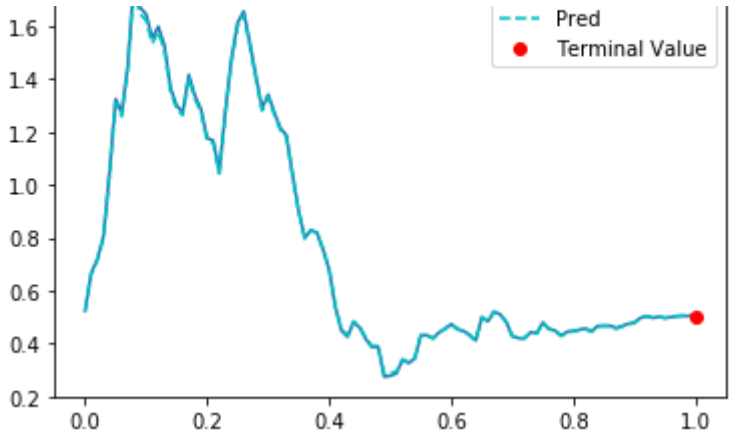

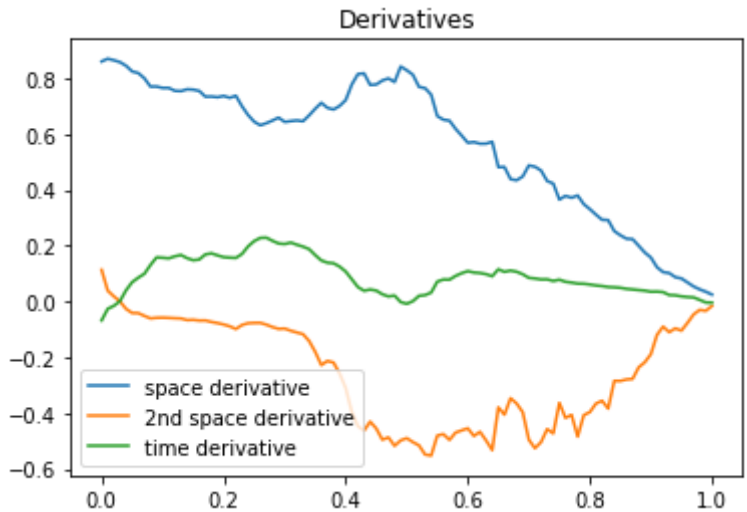


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5411th Epoch:
traing loss is 0.00033283952507190406, test loss is 0.00022984544921200722, and learning rate

5421th Epoch:
traing loss is 0.0005649105878546834, test loss is 0.0005834799958392978, and learning rate i

5431th Epoch:
traing loss is 0.00018910040671471506, test loss is 0.00019827138748951256, and learning rate

5441th Epoch:
traing loss is 0.0014725556829944253, test loss is 0.0009236892801709473, and learning rate i

5451th Epoch:
traing loss is 0.0003433480451349169, test loss is 0.0003590252308640629, and learning rate i

5461th Epoch:
traing loss is 0.0015004979213699698, test loss is 0.0009639835334382951, and learning rate i

5471th Epoch:
traing loss is 0.0007565375999547541, test loss is 0.00079047039616853, and learning rate is

5481th Epoch:
traing loss is 0.0004952827002853155, test loss is 0.000423221878008917, and learning rate is

5491th Epoch:
traing loss is 0.0005959153641015291, test loss is 0.0004158174560870975, and learning rate i

5501th Epoch:
traing loss is 0.0004884966183453798, test loss is 0.00022839636949356645, and learning rate
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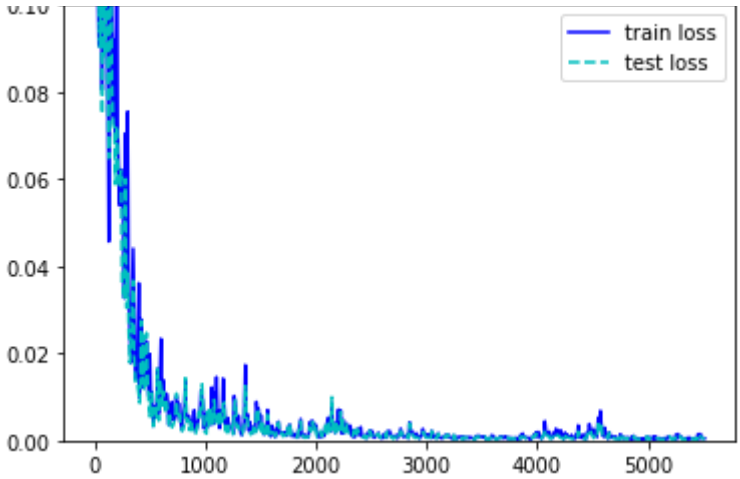

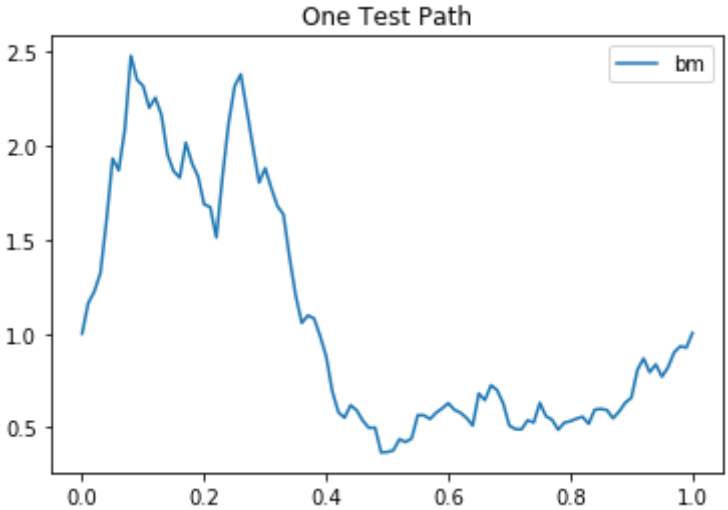

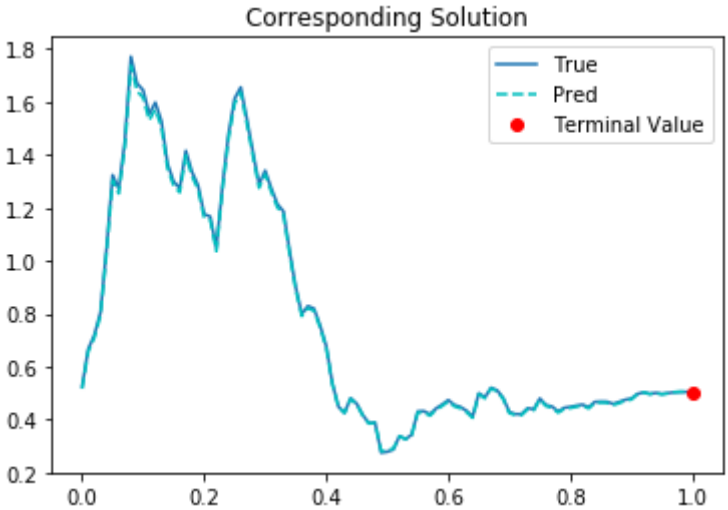

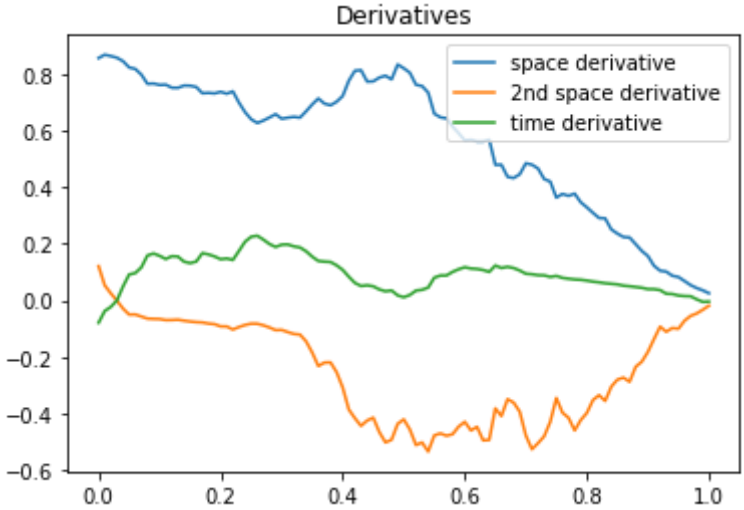

5511th Epoch:

```
traing loss is 0.000517610867973417, test loss is 0.0003584908845368773, and learning rate is

5521th Epoch:
traing loss is 0.0006914911791682243, test loss is 0.0006395973614417017, and learning rate i

5531th Epoch:
traing loss is 0.00031493447022512555, test loss is 0.0002975663519464433, and learning rate

5541th Epoch:
traing loss is 0.0006493813125416636, test loss is 0.000544697162695229, and learning rate is

5551th Epoch:
traing loss is 0.0002522732538636774, test loss is 0.00021757424110546708, and learning rate

5561th Epoch:
traing loss is 0.0006428724154829979, test loss is 0.0005170704098418355, and learning rate i

5571th Epoch:
traing loss is 0.00018923665629699826, test loss is 0.00018964189803227782, and learning rate

5581th Epoch:
traing loss is 0.00021571818797383457, test loss is 0.0001653970975894481, and learning rate

5591th Epoch:
traing loss is 0.00022160254593472928, test loss is 0.00014892032777424902, and learning rate

5601th Epoch:
traing loss is 0.00038981749094091356, test loss is 0.00021024781744927168, and learning rate
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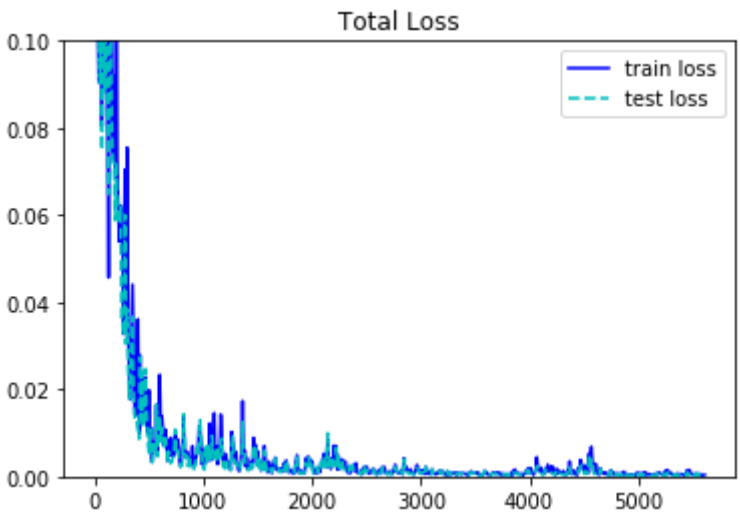

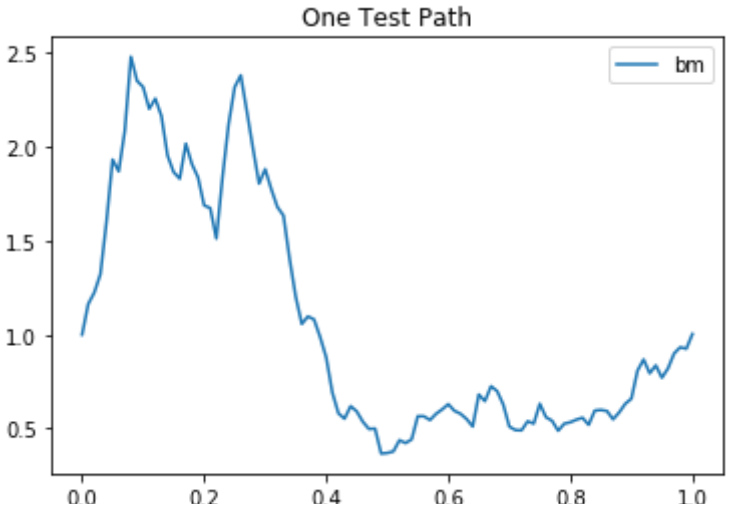

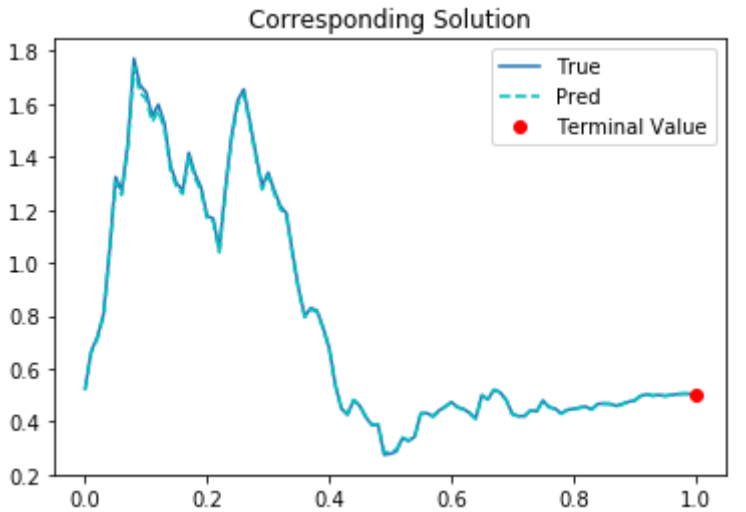

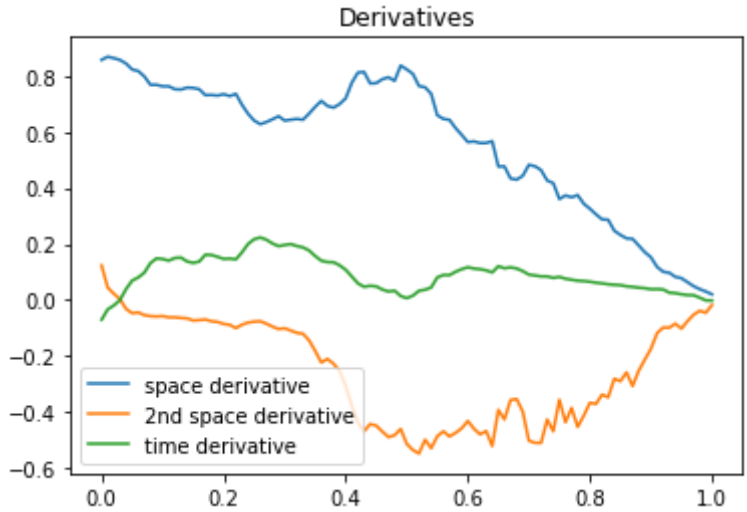


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5611th Epoch:
traing loss is 0.00038255559047684073, test loss is 0.00028151675360277295, and learning rate

5621th Epoch:
traing loss is 0.0006411399808712304, test loss is 0.0005947425379417837, and learning rate i

5631th Epoch:
traing loss is 0.00023872269957792014, test loss is 0.00022965043899603188, and learning rate

5641th Epoch:
traing loss is 0.0008095487719401717, test loss is 0.0006975465803407133, and learning rate i

5651th Epoch:
traing loss is 0.00032197684049606323, test loss is 0.00029756661388091743, and learning rate

5661th Epoch:
traing loss is 0.0009433653904125094, test loss is 0.0008102610008791089, and learning rate i

5671th Epoch:
traing loss is 0.0002514644293114543, test loss is 0.00025688984896987677, and learning rate

5681th Epoch:
traing loss is 0.00031141782528720796, test loss is 0.000261780311120674, and learning rate i

5691th Epoch:
traing loss is 0.00030120572773739696, test loss is 0.0002070278424071148, and learning rate

5701th Epoch:
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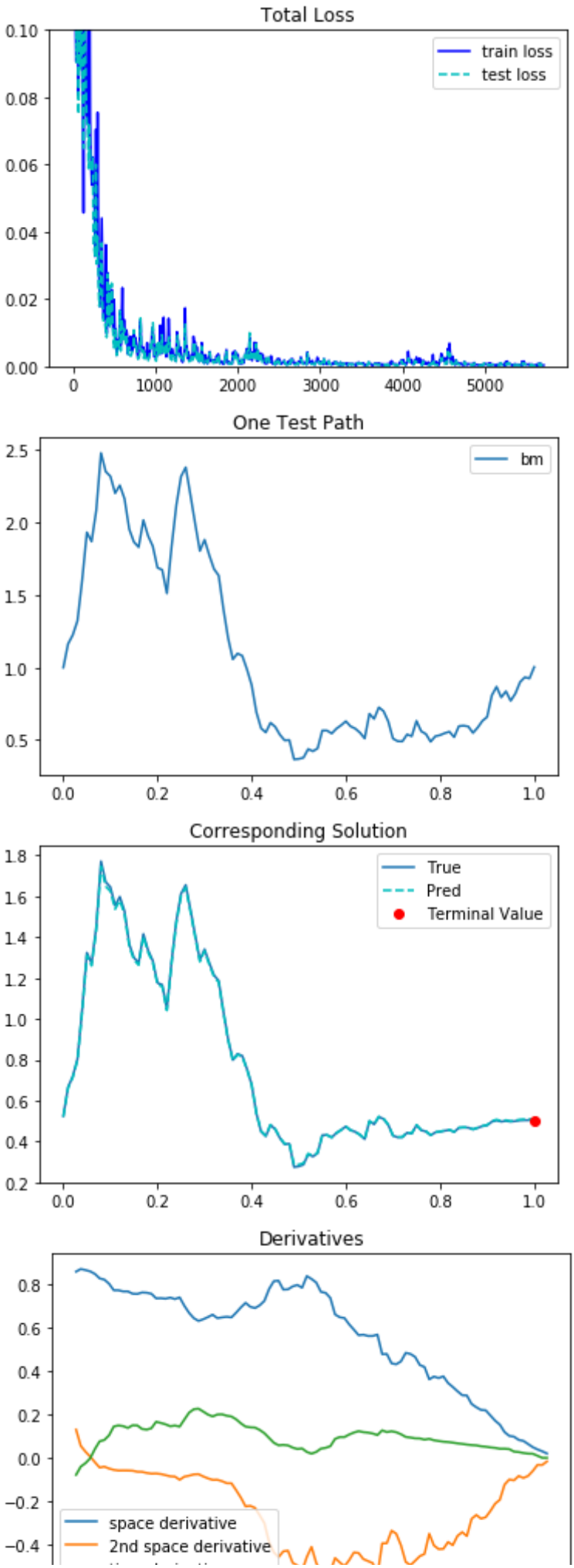
traing loss is 0.00035150215262547135, test loss is 0.00015366238949354738, and learning rate



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5711th Epoch:
traing loss is 0.0003682284732349217, test loss is 0.0002484110009390861, and learning rate i

5721th Epoch:
traing loss is 0.0006315652281045914, test loss is 0.0005993033992126584, and learning rate i

5731th Epoch:
traing loss is 0.0002587373892311007, test loss is 0.00026354240253567696, and learning rate

5741th Epoch:
traing loss is 0.0004747686325572431, test loss is 0.0004457013274077326, and learning rate i

5751th Epoch:
traing loss is 0.00023684091866016388, test loss is 0.0002211136306868866, and learning rate

5761th Epoch:
traing loss is 0.00033284342498518527, test loss is 0.00024029347696341574, and learning rate

5771th Epoch:
traing loss is 0.0001535213232273236, test loss is 0.00016066664829850197, and learning rate

5781th Epoch:
traing loss is 0.00016659233369864523, test loss is 0.00012039143621223047, and learning rate

5791th Epoch:
traing loss is 0.0002527124306652695, test loss is 0.0001712959201540798, and learning rate i

5801th Epoch:
traing loss is 0.00038962045800872147, test loss is 0.0001383024937240407, and learning rate
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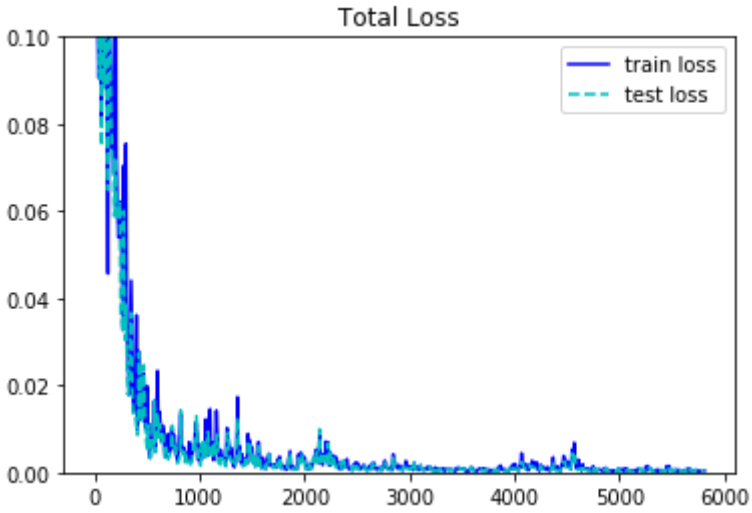

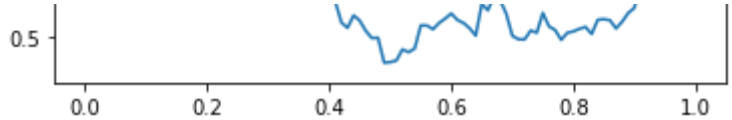

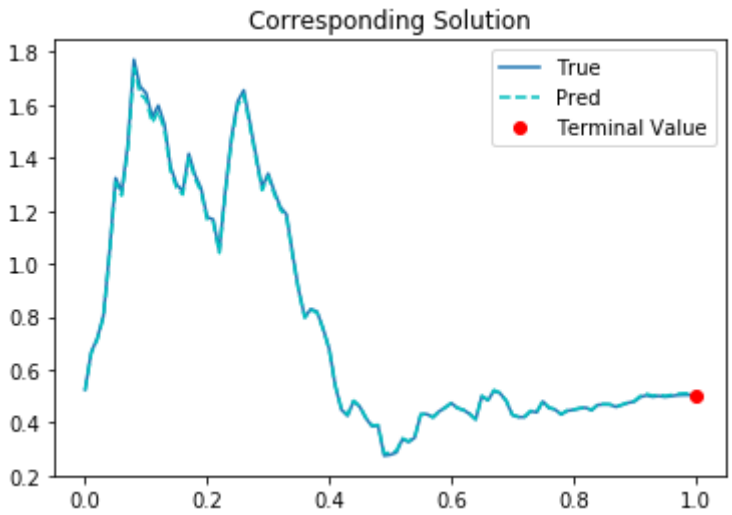

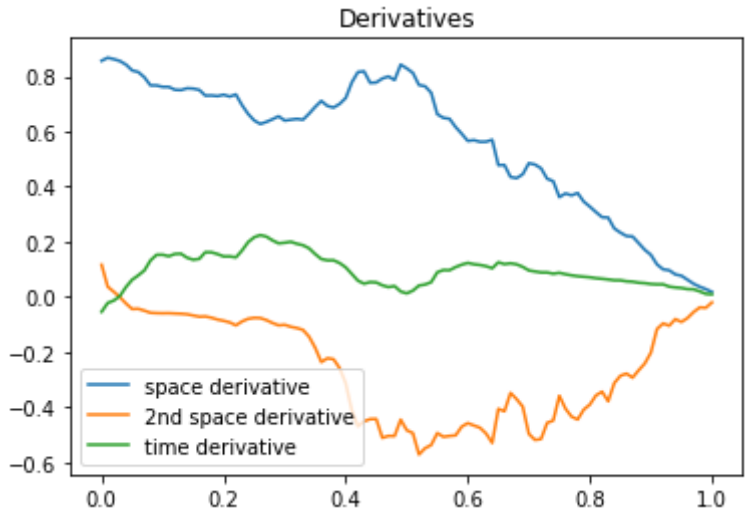


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5811th Epoch:
traing loss is 0.00046507350634783506, test loss is 0.00034653610782697797, and learning rate

5821th Epoch:
traing loss is 0.0008589032804593444, test loss is 0.0008118581026792526, and learning rate i

5831th Epoch:
traing loss is 0.0005059695104137063, test loss is 0.0005068830214440823, and learning rate i

5841th Epoch:
traing loss is 0.0003703966795001179, test loss is 0.0003084953932557255, and learning rate i

5851th Epoch:
traing loss is 0.00017672937246970832, test loss is 0.00015570639516226947, and learning rate

5861th Epoch:
traing loss is 0.0006935144774615765, test loss is 0.0005340795614756644, and learning rate i

5871th Epoch:
traing loss is 0.00021893135271966457, test loss is 0.0002392679889453575, and learning rate

5881th Epoch:
traing loss is 0.0003413400845602155, test loss is 0.00028549684793688357, and learning rate

5891th Epoch:
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```
traing loss is 0.0003742351254913956, test loss is 0.0002613975666463375, and learning rate i

5901th Epoch:
traing loss is 0.0005674117128364742, test loss is 0.0002072910574497655, and learning rate i
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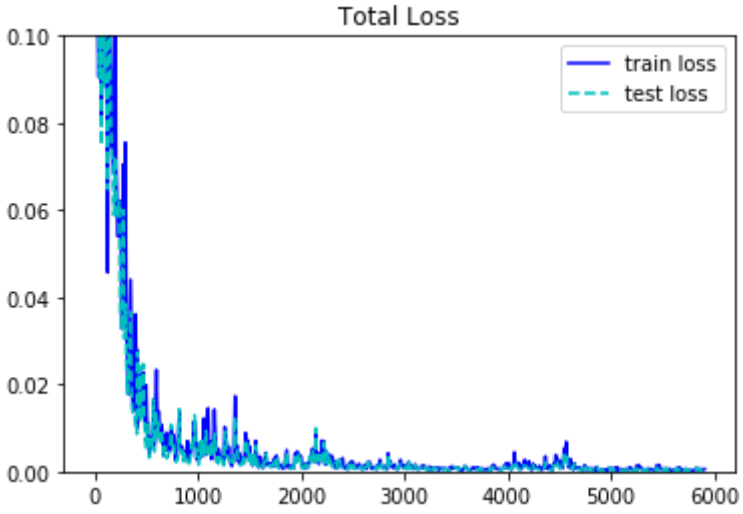

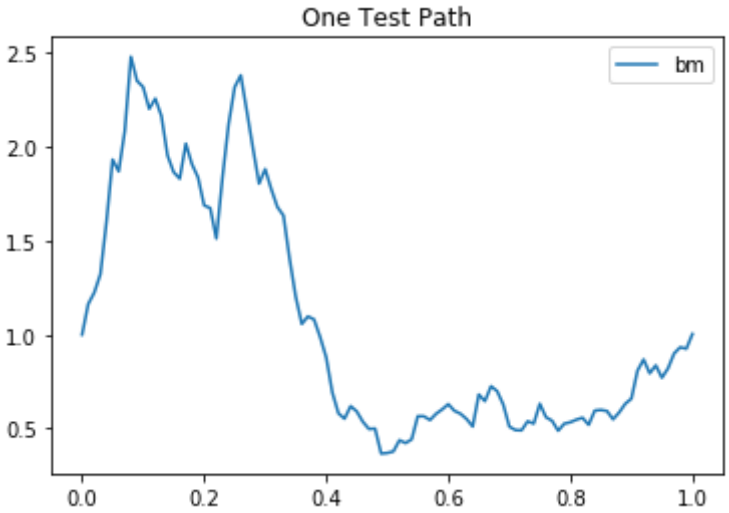

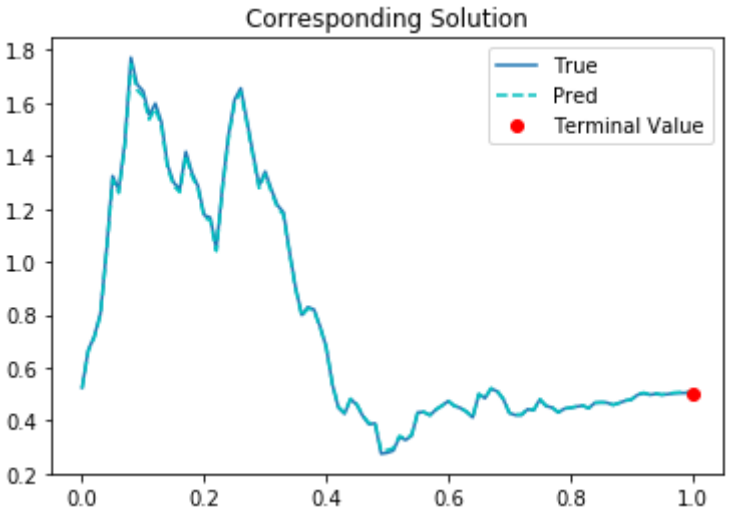

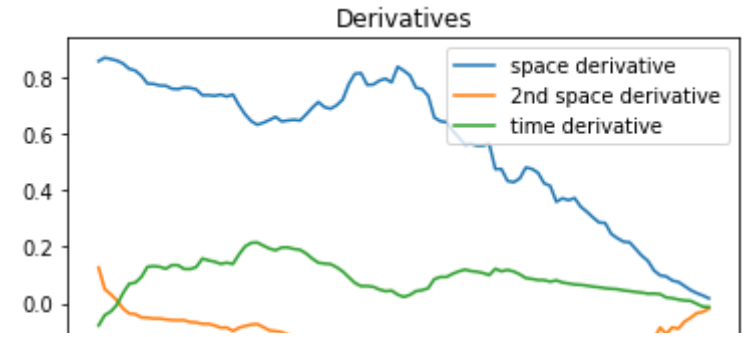

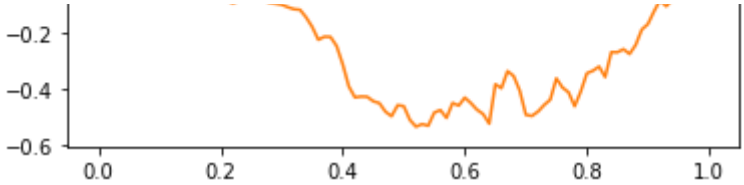


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5911th Epoch:
traing loss is 0.00048378994688391685, test loss is 0.0003082128241658211, and learning rate

5921th Epoch:
traing loss is 0.0006024794420227408, test loss is 0.0005776111502200365, and learning rate i

5931th Epoch:
traing loss is 0.0003397028485778719, test loss is 0.00038120688986964524, and learning rate

5941th Epoch:
traing loss is 0.0007616435759700835, test loss is 0.000623200205154717, and learning rate is

5951th Epoch:
traing loss is 0.00047292798990383744, test loss is 0.00044187650200910866, and learning rate

5961th Epoch:
traing loss is 0.0007013223948888481, test loss is 0.0003285295097157359, and learning rate i

5971th Epoch:
traing loss is 0.0003697572974488139, test loss is 0.00033235110458917916, and learning rate

5981th Epoch:
traing loss is 0.000405726459575817, test loss is 0.0002811583399306983, and learning rate is

5991th Epoch:
traing loss is 0.0002875878708437085, test loss is 0.0001765973138390109, and learning rate i

6001th Epoch:
traing loss is 0.0009297425276599824, test loss is 0.0001718760177027434, and learning rate i
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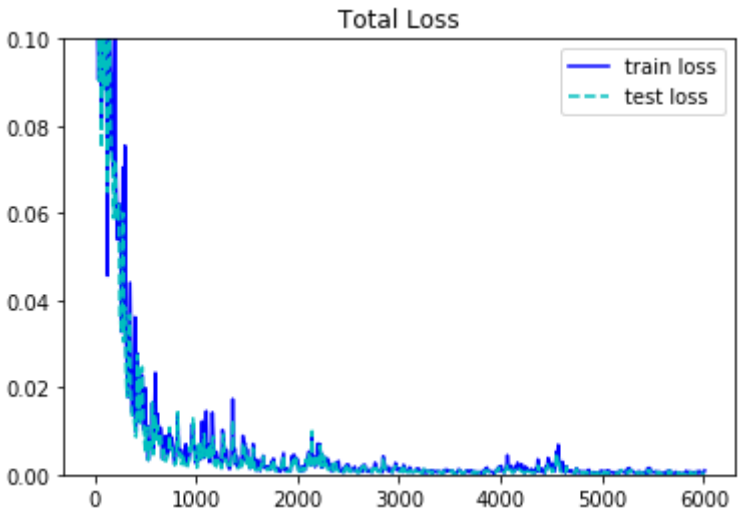

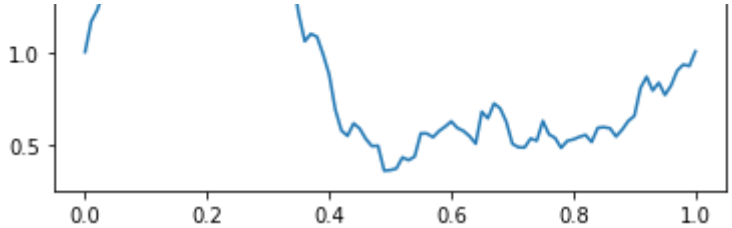

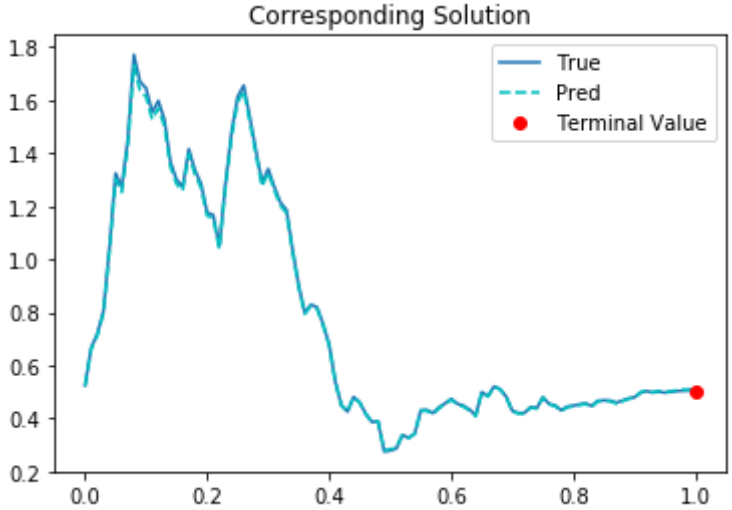

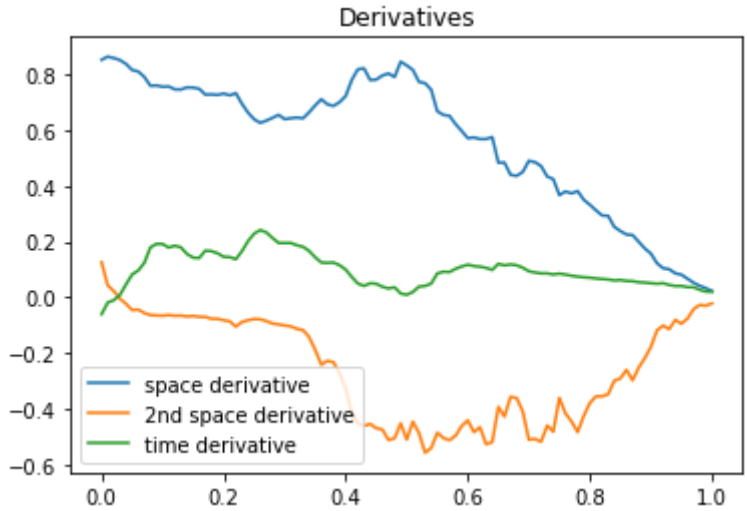


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6011th Epoch:
traing loss is 0.00038706263876520097, test loss is 0.00025280489353463054, and learning rate

6021th Epoch:
traing loss is 0.0005089431069791317, test loss is 0.0004946988774463534, and learning rate i

6031th Epoch:
traing loss is 0.00034286812297068536, test loss is 0.0003449733485467732, and learning rate

6041th Epoch:
traing loss is 0.0010575951309874654, test loss is 0.0005366362165659666, and learning rate i

6051th Epoch:
traing loss is 0.00046597293112426996, test loss is 0.0004119145742151886, and learning rate

6061th Epoch:
traing loss is 0.0019299400737509131, test loss is 0.0013599065132439137, and learning rate i

6071th Epoch:
traing loss is 0.000944577157497406, test loss is 0.0008651926764287055, and learning rate is

6081th Epoch:
```

```
traing loss is 0.00090132013428956 27, test loss is 0.00065339088905602 69, and learning rate i

6091th Epoch:
traing loss is 0.000368970213457942, test loss is 0.00020923701231367886, and learning rate i

6101th Epoch:
traing loss is 0.0015062046004459262, test loss is 0.00023376985336653888, and learning rate
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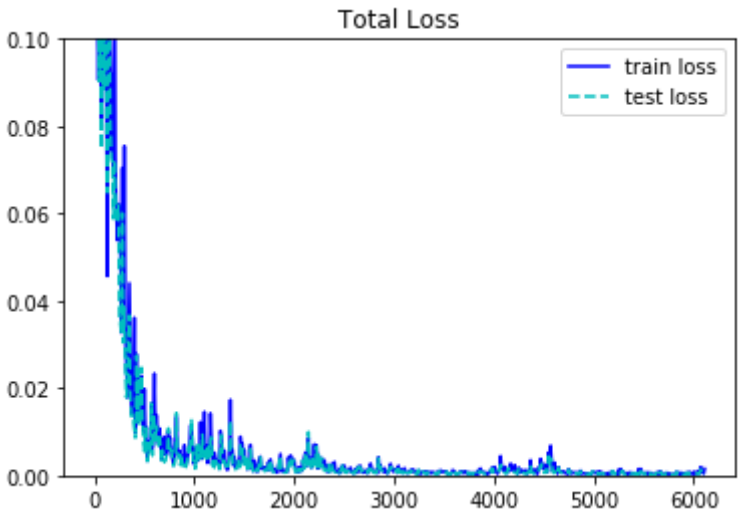

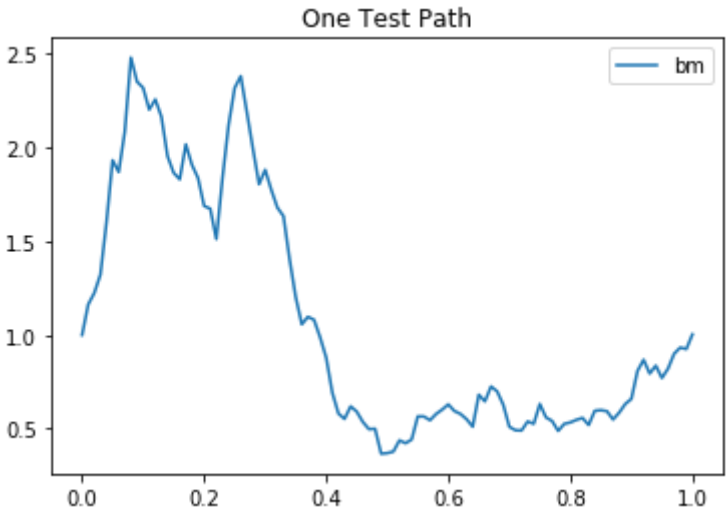

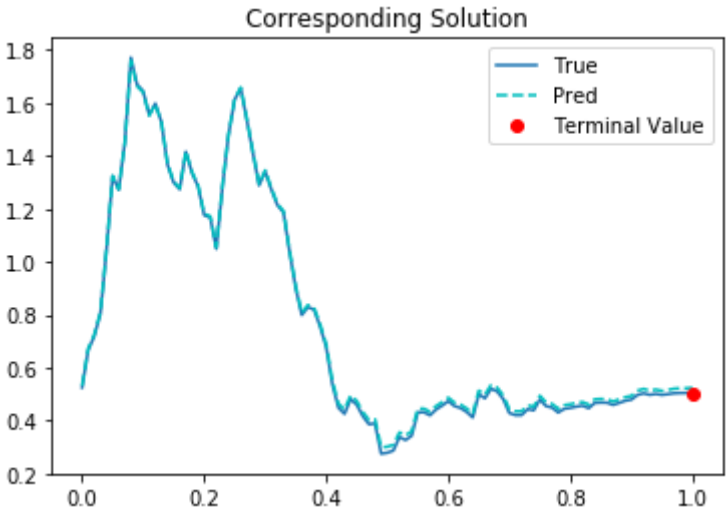

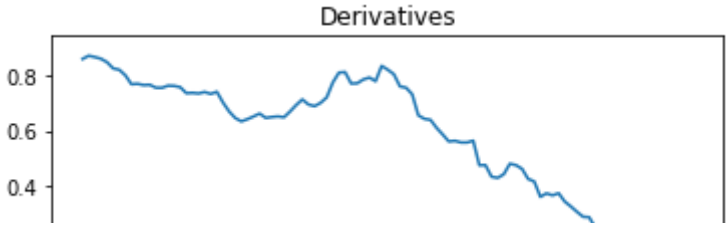

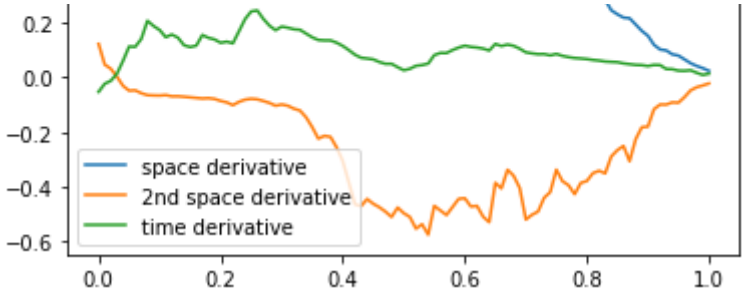


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6111th Epoch:
traing loss is 0.0007910068379715085, test loss is 0.0003426407929509878, and learning rate i

6121th Epoch:
traing loss is 0.0004032781289424747, test loss is 0.00037421914748847485, and learning rate

6131th Epoch:
traing loss is 0.00023660525039304048, test loss is 0.000301433727145195, and learning rate i

6141th Epoch:
traing loss is 0.0010486190440133214, test loss is 0.0009907570201903582, and learning rate i

6151th Epoch:
traing loss is 0.0006869806675240397, test loss is 0.0006624571979045868, and learning rate i

6161th Epoch:
traing loss is 0.001334688626229763, test loss is 0.0007590248133055866, and learning rate is

6171th Epoch:
traing loss is 0.0008563713636249304, test loss is 0.0007575718918815255, and learning rate i

6181th Epoch:
traing loss is 0.0015753470361232758, test loss is 0.0015911548398435116, and learning rate i

6191th Epoch:
traing loss is 0.00038499446236528456, test loss is 0.0002859003725461662, and learning rate

6201th Epoch:
traing loss is 0.0012092210818082094, test loss is 0.0002290632837684825, and learning rate i
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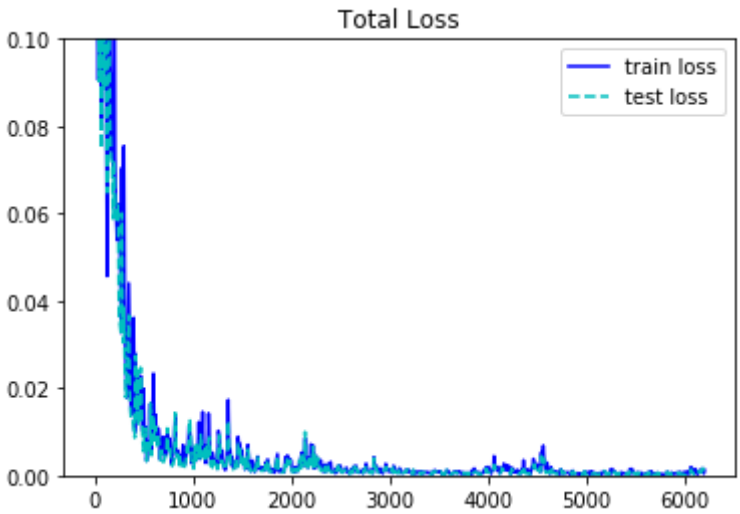

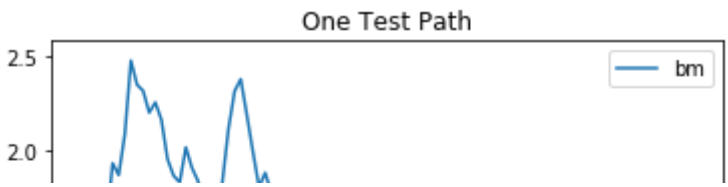

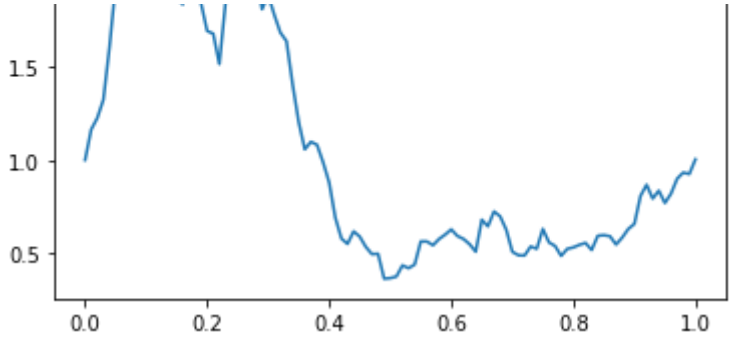

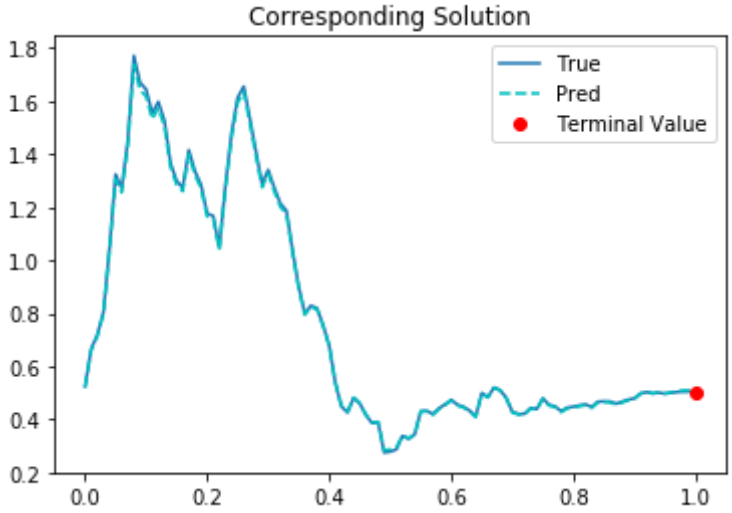

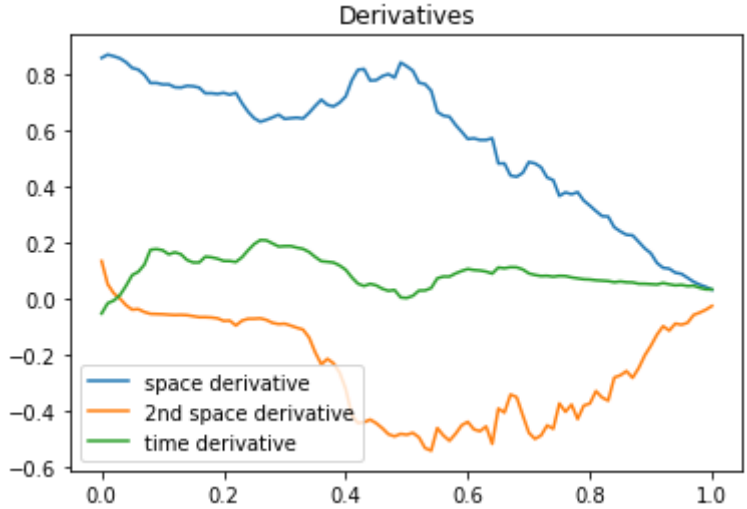


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6211th Epoch:
traing loss is 0.0007023828802630305, test loss is 0.0004106470732949674, and learning rate i

6221th Epoch:
traing loss is 0.001259895390830934, test loss is 0.0010721497237682343, and learning rate is

6231th Epoch:
traing loss is 0.0003281915851403028, test loss is 0.00037830157089047134, and learning rate

6241th Epoch:
traing loss is 0.0005439206724986434, test loss is 0.0004637623205780983, and learning rate i

6251th Epoch:
traing loss is 0.0003081726608797908, test loss is 0.00028140898211859167, and learning rate

6261th Epoch:
traing loss is 0.000871404365170747, test loss is 0.0004394267743919045, and learning rate is

6271th Epoch:
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```
traing loss is 0.00078253610990941 52, test loss is 0.00070999772287905 22, and learning rate i

6281th Epoch:
traing loss is 0.0003248208959121257, test loss is 0.00027896667597815394, and learning rate

6291th Epoch:
traing loss is 0.00027895031962543726, test loss is 0.00020399679488036782, and learning rate

6301th Epoch:
traing loss is 0.0007965471595525742, test loss is 0.00020149424381088465, and learning rate
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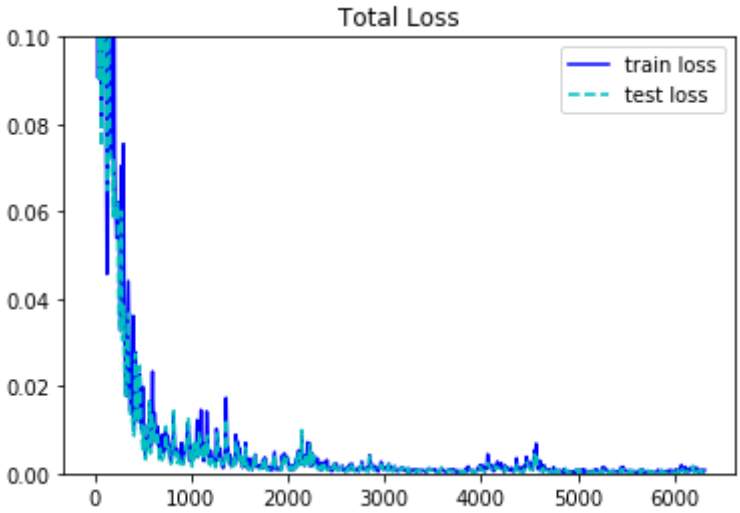

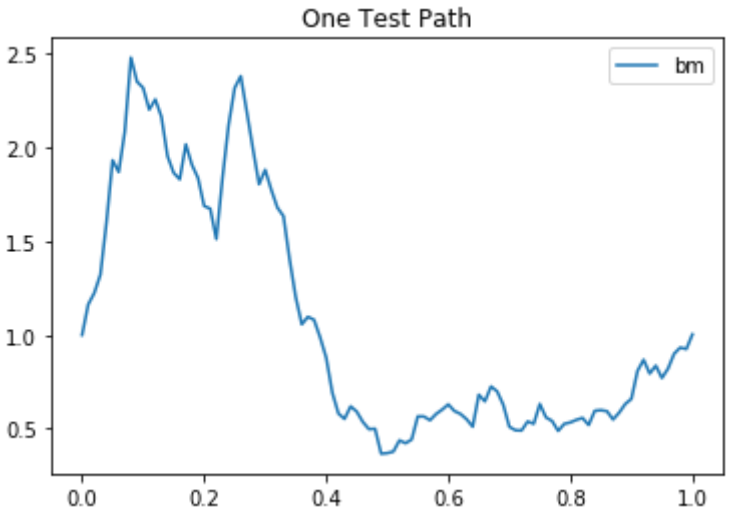

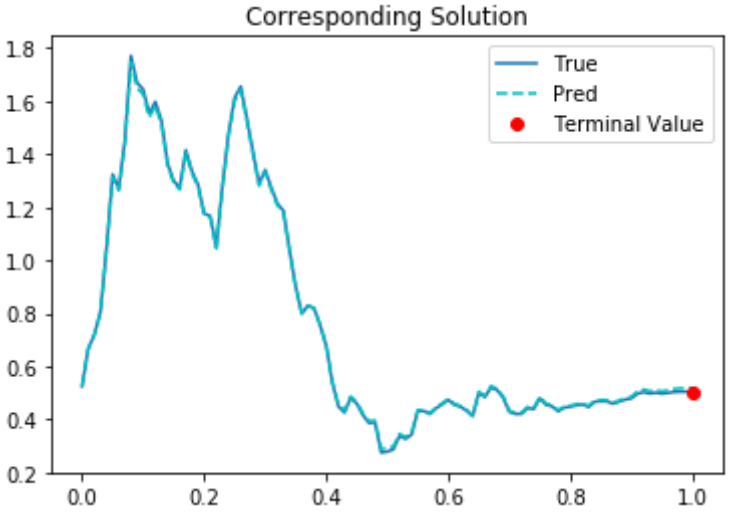

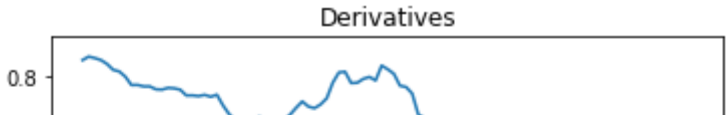

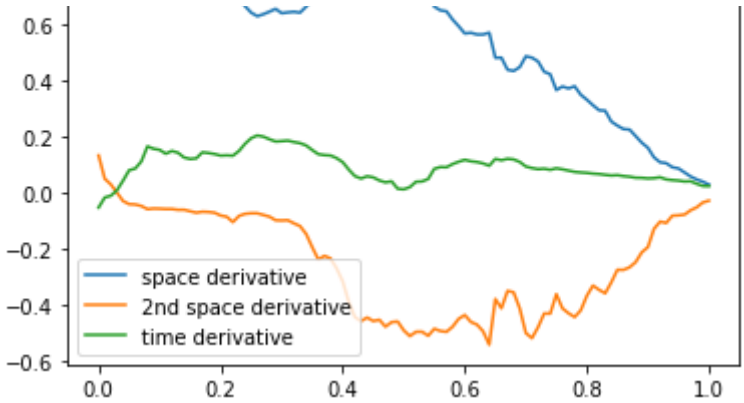


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6311th Epoch:
traing loss is 0.0006214542081579566, test loss is 0.00043746046139858663, and learning rate

6321th Epoch:
traing loss is 0.0006188023835420609, test loss is 0.0005292523419484496, and learning rate i

6331th Epoch:
traing loss is 0.0003018898132722825, test loss is 0.00038236513501033187, and learning rate

6341th Epoch:
traing loss is 0.0002710344851948321, test loss is 0.0002506547898519784, and learning rate i

6351th Epoch:
traing loss is 0.00015716876077931374, test loss is 0.00016291534120682627, and learning rate

6361th Epoch:
traing loss is 0.0005681232432834804, test loss is 0.0002961080754175782, and learning rate i

6371th Epoch:
traing loss is 0.00040899726445786655, test loss is 0.00039057957474142313, and learning rate

6381th Epoch:
traing loss is 0.0002364122774451971, test loss is 0.00019317578698974103, and learning rate

6391th Epoch:
traing loss is 0.00022203610569704324, test loss is 0.00014977698447182775, and learning rate

6401th Epoch:
traing loss is 0.0007064485107548535, test loss is 0.00019169818551745266, and learning rate
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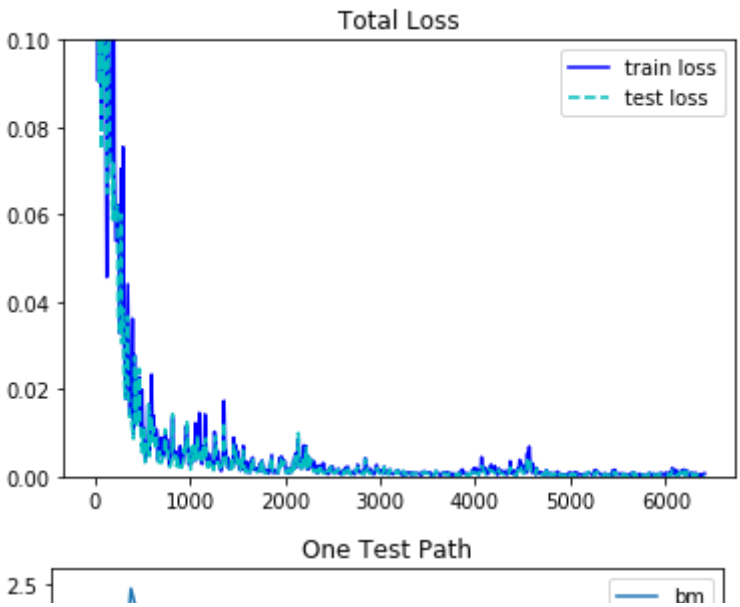

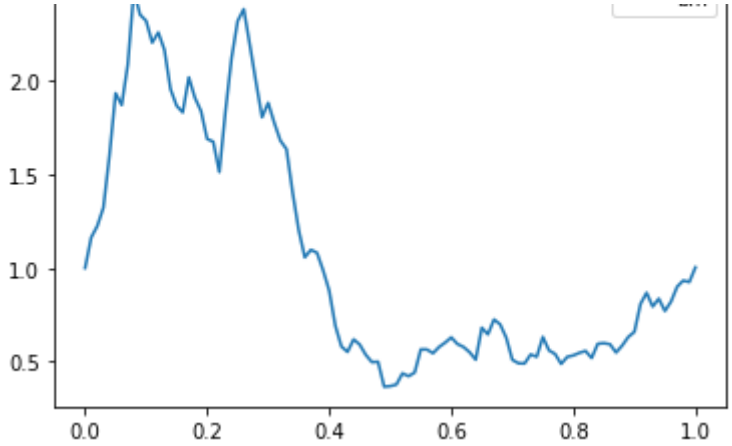

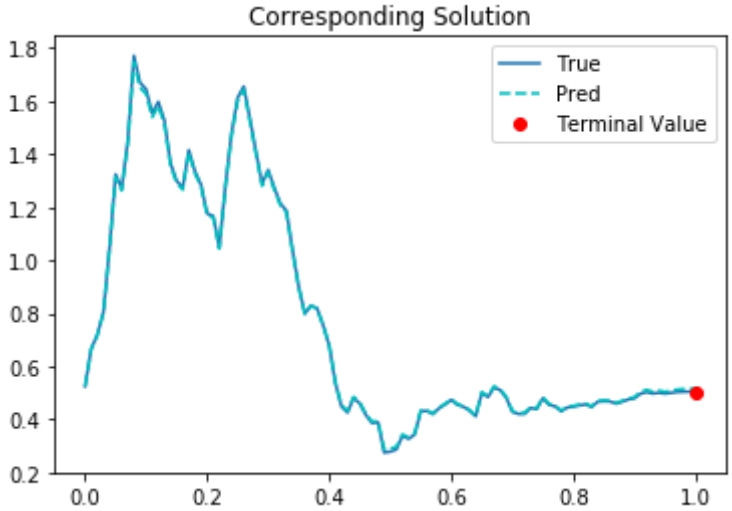

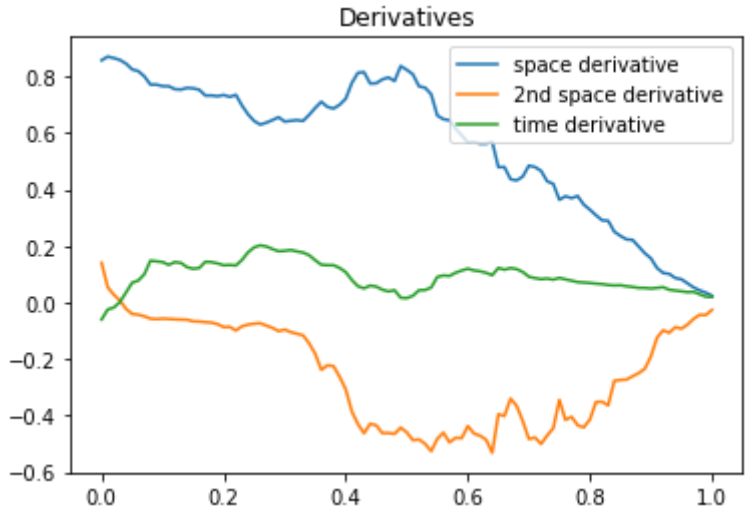


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6411th Epoch:
traing loss is 0.0006236918270587921, test loss is 0.00043782504508271813, and learning rate

6421th Epoch:
traing loss is 0.0005947939353063703, test loss is 0.0005231521208770573, and learning rate i

6431th Epoch:
traing loss is 0.0003010891086887568, test loss is 0.0003841368015855551, and learning rate i

6441th Epoch:
traing loss is 0.00019877591694239527, test loss is 0.00019440852338448167, and learning rate

6451th Epoch:
traing loss is 0.00011141558934468776, test loss is 0.00012998143211007118, and learning rate

6461th Epoch:
```

```
traing loss is 0.0003625878307502717, test loss is 0.00019366703054402024, and learning rate

6471th Epoch:
traing loss is 0.0001988523727050051, test loss is 0.00019215658539906144, and learning rate

6481th Epoch:
traing loss is 0.00019507396791595966, test loss is 0.0001374740240862593, and learning rate

6491th Epoch:
traing loss is 0.0001898319460451603, test loss is 0.00012402902939356863, and learning rate

6501th Epoch:
traing loss is 0.0006732785841450095, test loss is 0.0001778451114660129, and learning rate i
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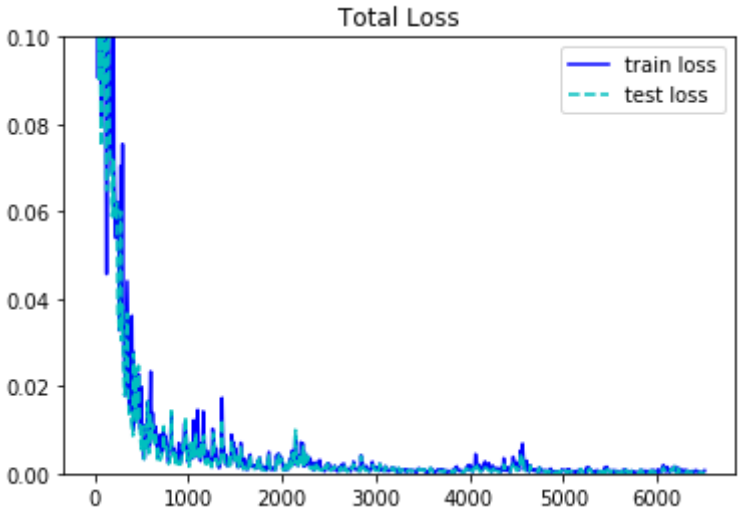

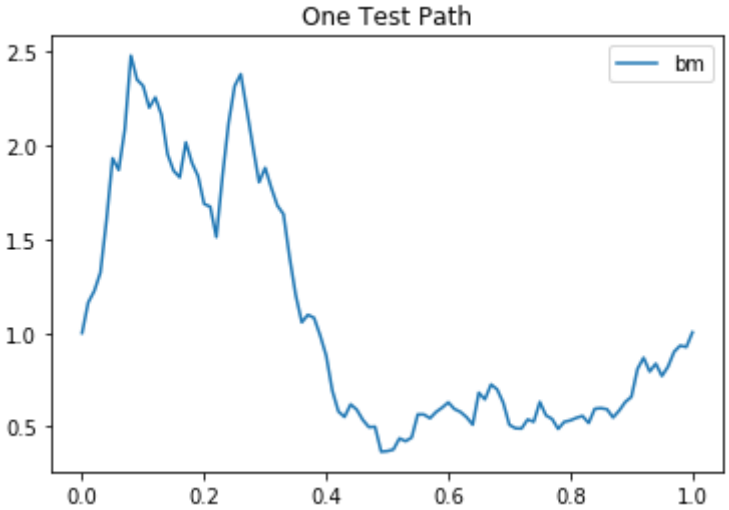

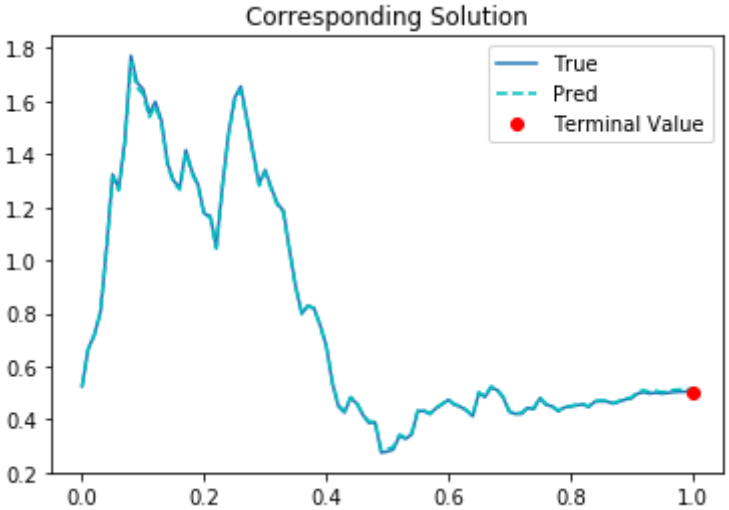

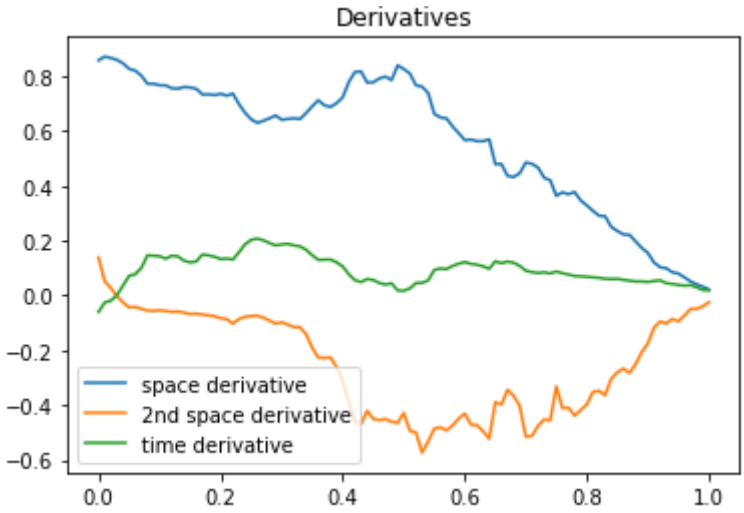


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6511th Epoch:
traing loss is 0.0006294178892858326, test loss is 0.00043330935295671225, and learning rate

6521th Epoch:
traing loss is 0.0005056805675849319, test loss is 0.00044734569382853806, and learning rate

6531th Epoch:
traing loss is 0.00027726287953555584, test loss is 0.0003602973010856658, and learning rate

6541th Epoch:
traing loss is 0.00017545055015943944, test loss is 0.00017630941874813288, and learning rate

6551th Epoch:
traing loss is 9.915581904351711e-05, test loss is 0.00011514843936311081, and learning rate

6561th Epoch:
traing loss is 0.00028662869590334594, test loss is 0.00014812506560701877, and learning rate

6571th Epoch:
traing loss is 0.00013069715350866318, test loss is 0.00012832430365961045, and learning rate

6581th Epoch:
traing loss is 0.00018369454483035952, test loss is 0.00012502921163104475, and learning rate

6591th Epoch:
traing loss is 0.0001758656435413286, test loss is 0.00010745915642473847, and learning rate

6601th Epoch:
traing loss is 0.0005748402909375727, test loss is 0.00017641714657656848, and learning rate
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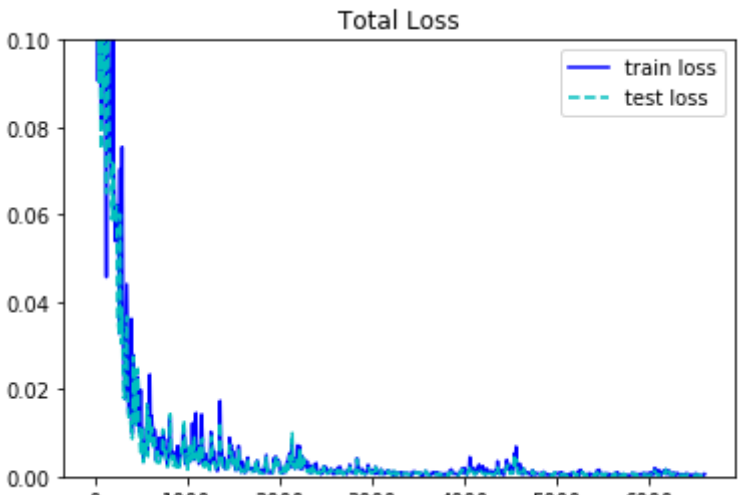

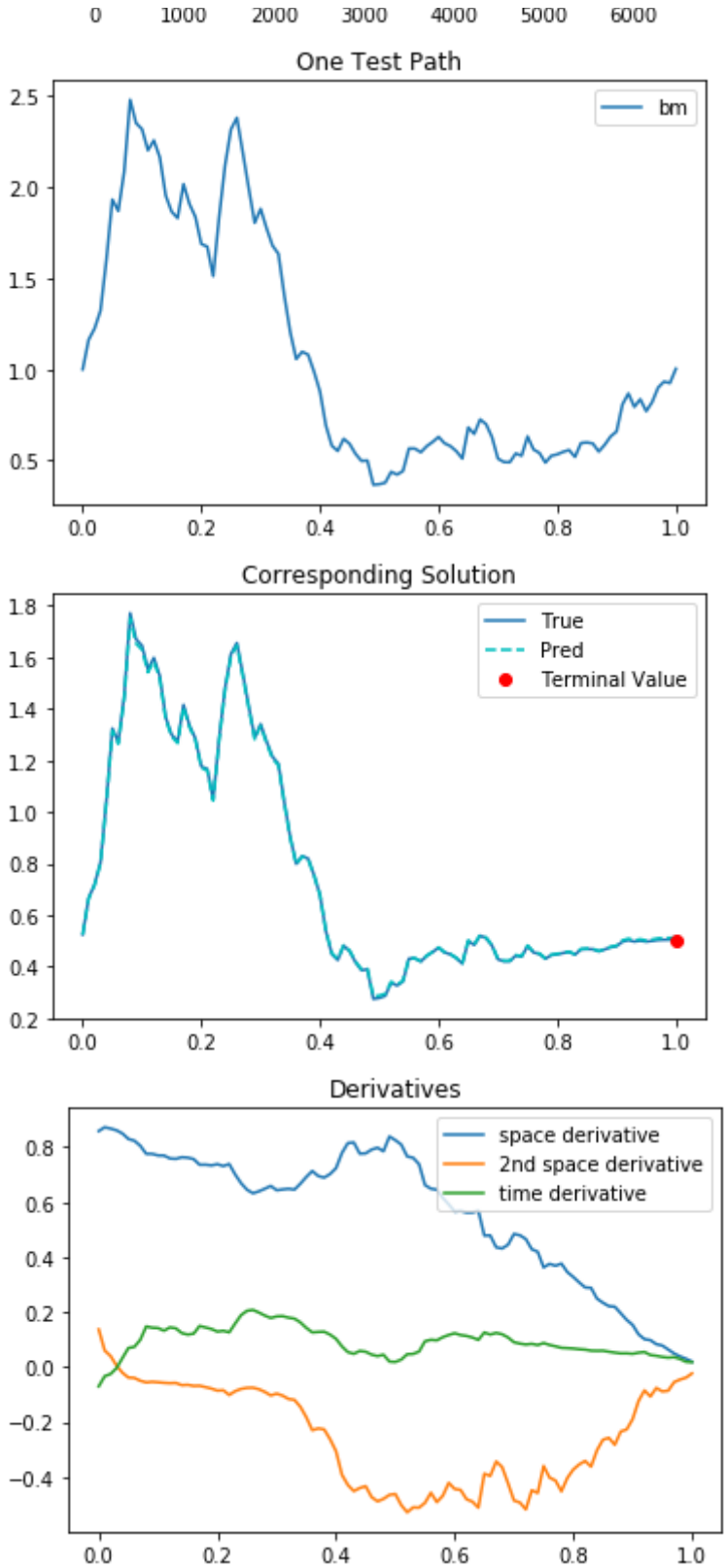


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6611th Epoch:
traing loss is 0.0005613456014543772, test loss is 0.0004113492905162275, and learning rate i

6621th Epoch:
traing loss is 0.000446427526185289, test loss is 0.00039166214992292225, and learning rate i

6631th Epoch:
traing loss is 0.00025747373001649976, test loss is 0.00033492344664409757, and learning rate

6641th Epoch:
traing loss is 0.00017192590166814625, test loss is 0.00017389390268363059, and learning rate

6651th Epoch:
```

```
traing loss is 9.562439663568512e-05, test loss is 0.00011000595986843109, and learning rate

6661th Epoch:
traing loss is 0.00022552258451469243, test loss is 0.00012009093188680708, and learning rate

6671th Epoch:
traing loss is 0.00011642832396319136, test loss is 0.00011586904292926192, and learning rate

6681th Epoch:
traing loss is 0.00023322738707065582, test loss is 0.00014640908921137452, and learning rate

6691th Epoch:
traing loss is 0.00018838248797692358, test loss is 0.00012047067139064893, and learning rate

6701th Epoch:
traing loss is 0.0004874873557128012, test loss is 0.00016397051513195038, and learning rate
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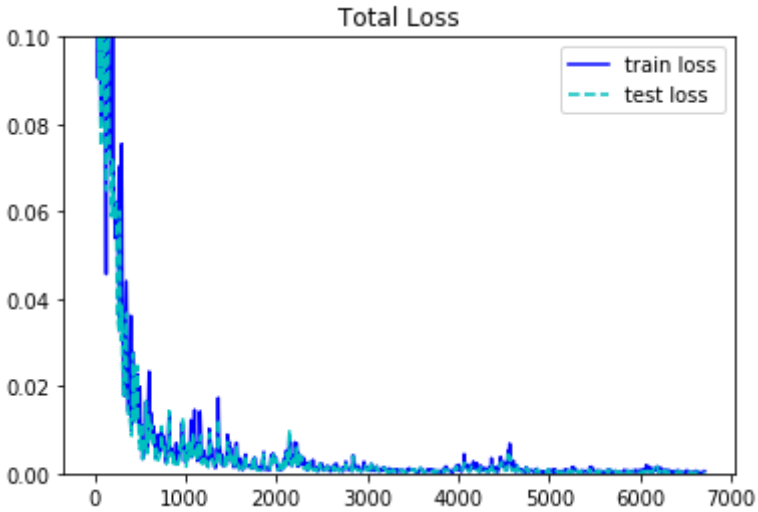

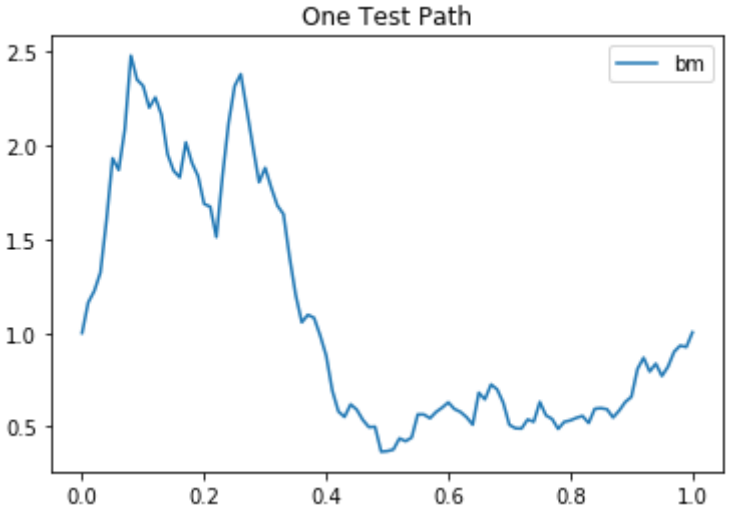

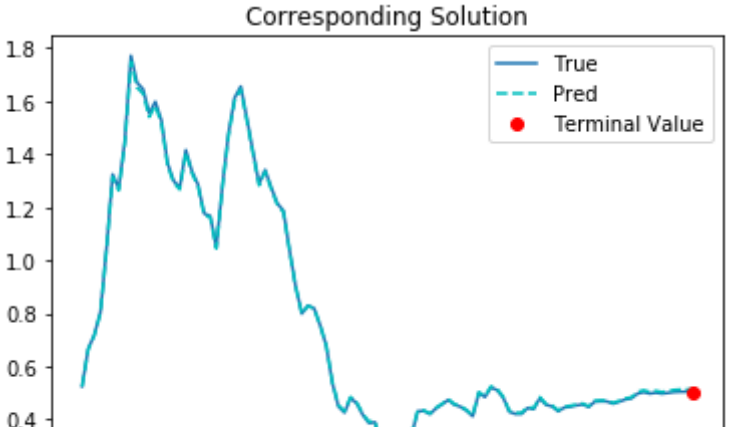

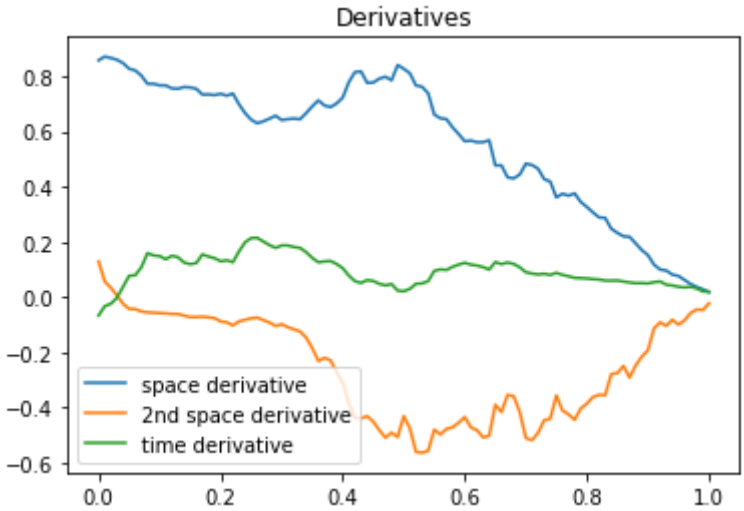


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6711th Epoch:
traing loss is 0.0005004355916753411, test loss is 0.00033934772363863885, and learning rate

6721th Epoch:
traing loss is 0.00026122306007891893, test loss is 0.00022994533355813473, and learning rate

6731th Epoch:
traing loss is 0.00017141035641543567, test loss is 0.00023438695643562824, and learning rate

6741th Epoch:
traing loss is 0.00014253755216486752, test loss is 0.000121943601698149, and learning rate i

6751th Epoch:
traing loss is 8.817749767331406e-05, test loss is 0.0001039635535562411, and learning rate i

6761th Epoch:
traing loss is 0.00019585550762712955, test loss is 0.00011917493975488469, and learning rate

6771th Epoch:
traing loss is 0.00011241279571549967, test loss is 0.00011113328946521506, and learning rate

6781th Epoch:
traing loss is 0.00022818631259724498, test loss is 0.00013931913417764008, and learning rate

6791th Epoch:
traing loss is 0.0001973681792151183, test loss is 0.00012240660726092756, and learning rate

6801th Epoch:
traing loss is 0.00045178033178672194, test loss is 0.00013186786964070052, and learning rate
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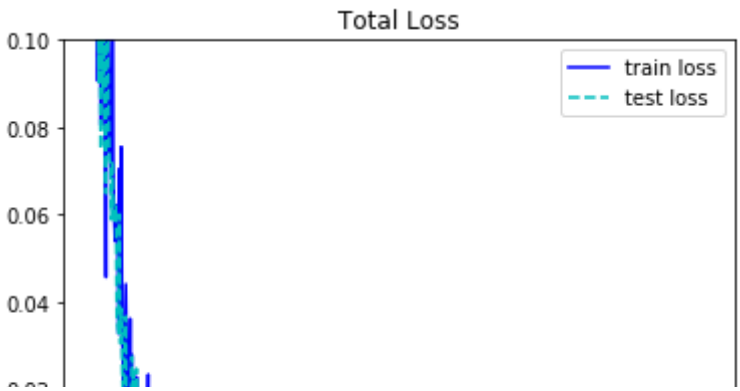

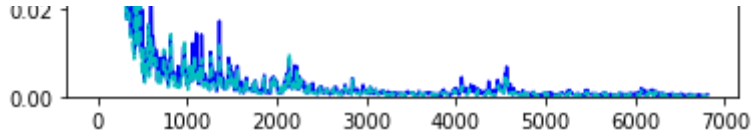

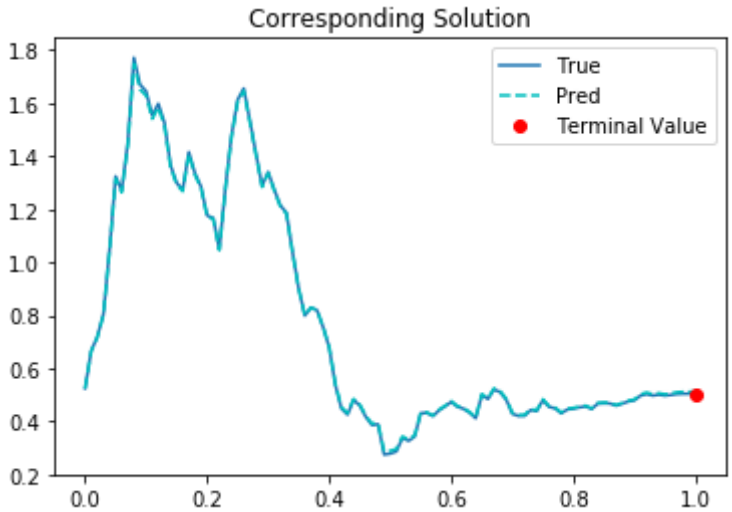

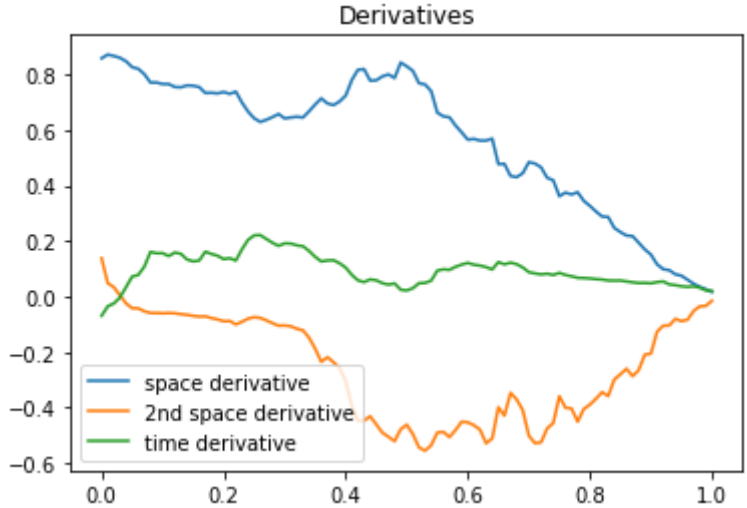


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6811th Epoch:
traing loss is 0.0004262322327122092, test loss is 0.00029408623231574893, and learning rate

6821th Epoch:
traing loss is 0.0002817675704136491, test loss is 0.0002474302309565246, and learning rate i

6831th Epoch:
traing loss is 0.0001684363669482991, test loss is 0.0002262673806399107, and learning rate i

6841th Epoch:
```

```
traing loss is 0.00026062608230859 04, test loss is 0.00023560722183901817, and learning rate

6851th Epoch:
traing loss is 0.00012218821211718023, test loss is 0.0001383976632496342, and learning rate

6861th Epoch:
traing loss is 0.0002787228731904179, test loss is 0.0001432394637959078, and learning rate i

6871th Epoch:
traing loss is 0.0001801239122869447, test loss is 0.00018509173241909593, and learning rate

6881th Epoch:
traing loss is 0.0002893982455134392, test loss is 0.00021382853446993977, and learning rate

6891th Epoch:
traing loss is 0.00021980419114697725, test loss is 0.00015868748596403748, and learning rate

6901th Epoch:
traing loss is 0.0003724244888871908, test loss is 0.00013664746074937284, and learning rate
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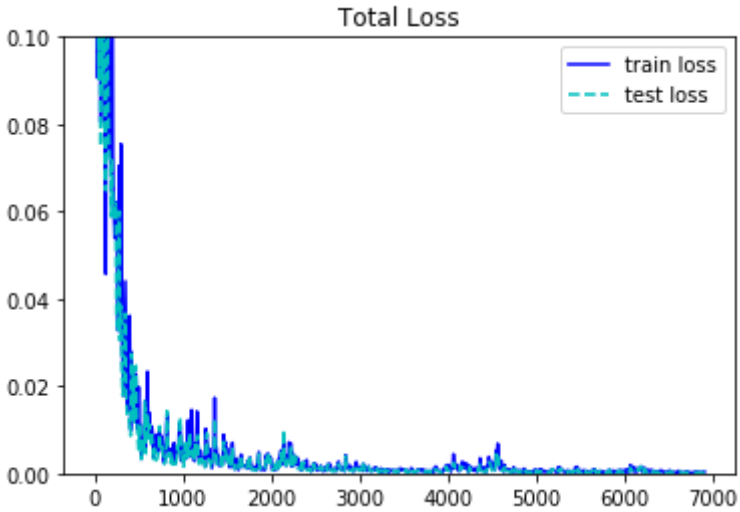

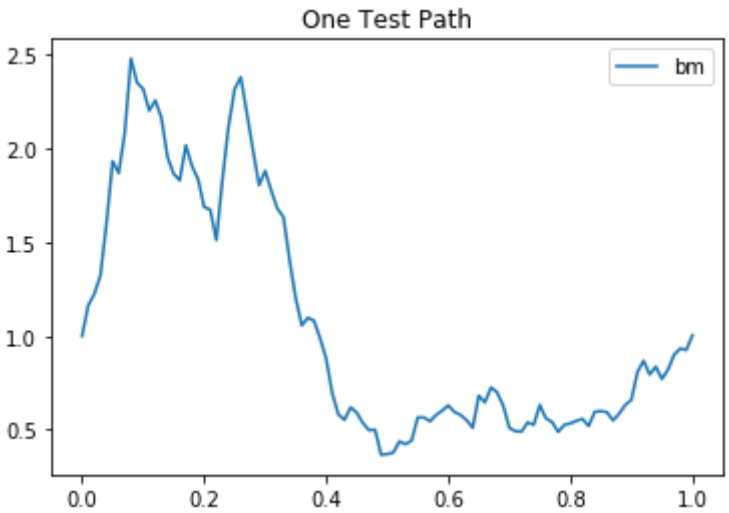

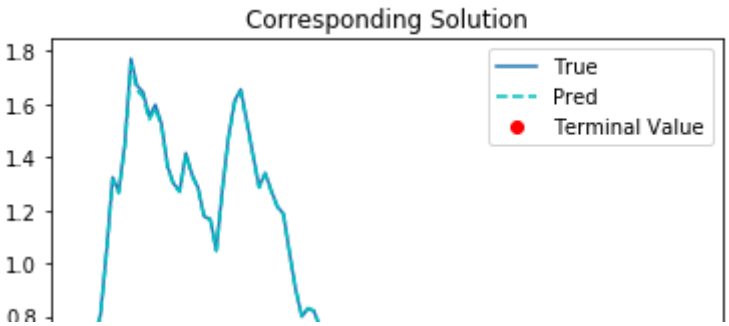

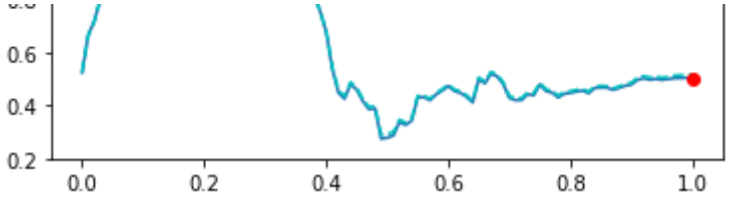

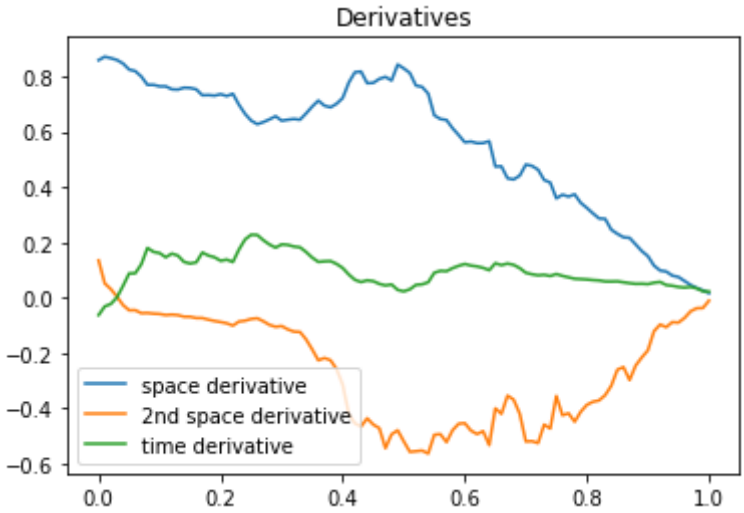


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6911th Epoch:
traing loss is 0.00038407029933296144, test loss is 0.00022561581863556057, and learning rate

6921th Epoch:
traing loss is 0.00015281751984730363, test loss is 0.00013612749171443284, and learning rate

6931th Epoch:
traing loss is 0.00010848657984752208, test loss is 0.0001584605051903054, and learning rate

6941th Epoch:
traing loss is 0.001230924273841083, test loss is 0.0006937059806659818, and learning rate is

6951th Epoch:
traing loss is 0.0005082066054455936, test loss is 0.0004150292952544987, and learning rate i

6961th Epoch:
traing loss is 0.0008701531332917511, test loss is 0.00035874167224392295, and learning rate

6971th Epoch:
traing loss is 0.00039298649062402546, test loss is 0.00034578298800624907, and learning rate

6981th Epoch:
traing loss is 0.000569833442568779, test loss is 0.00031808248604647815, and learning rate i

6991th Epoch:
traing loss is 0.00041262872400693595, test loss is 0.0002963602892123163, and learning rate

7001th Epoch:
traing loss is 0.00095884979236871, test loss is 0.00016955639875959605, and learning rate is
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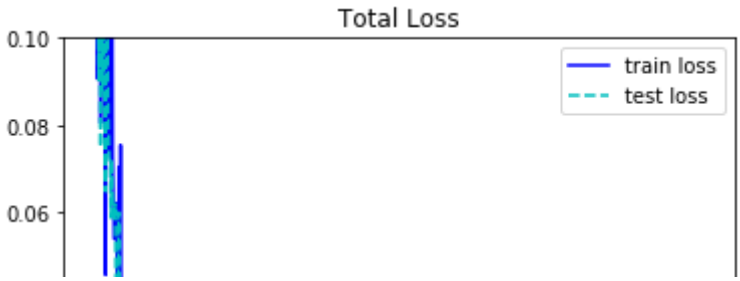

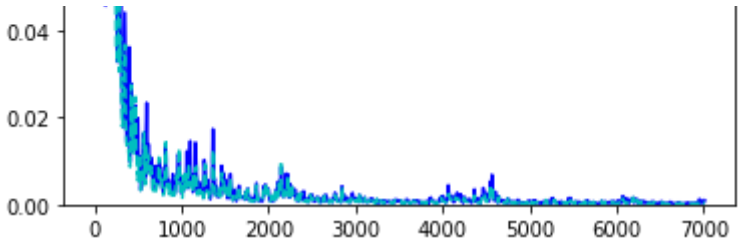

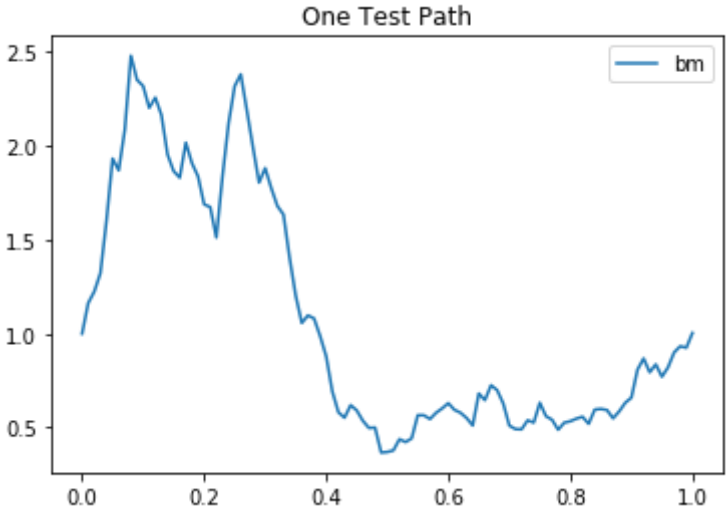

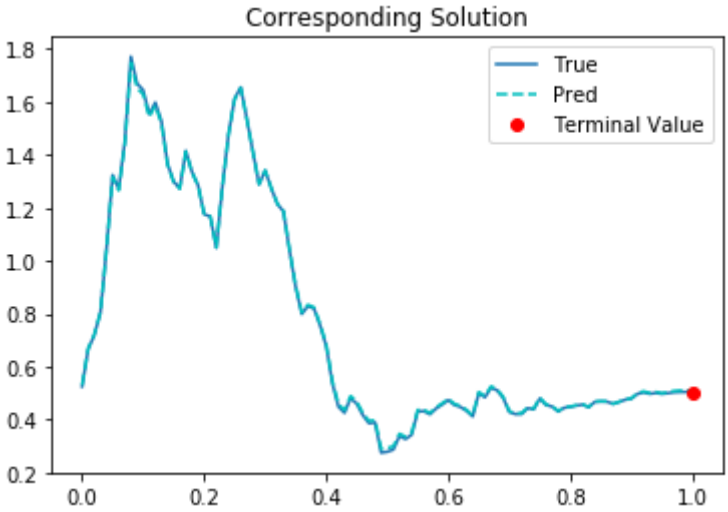

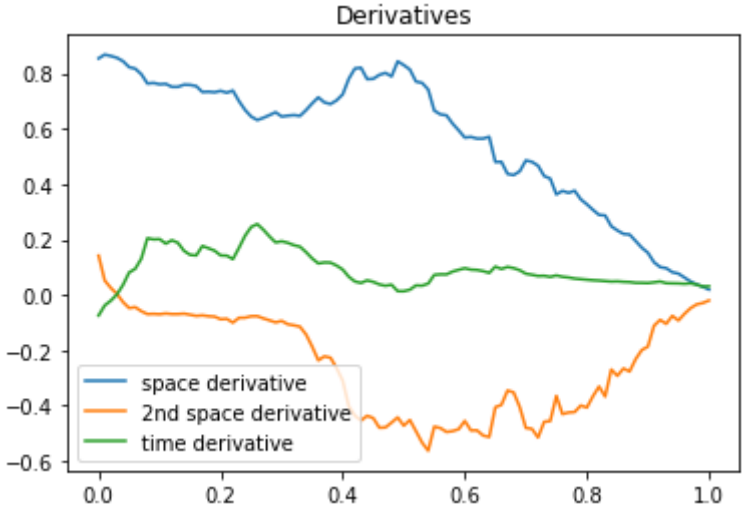


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7011th Epoch:
traing loss is 0.00039353102329187095, test loss is 0.00028381479205563664, and learning rate

7021th Epoch:
traing loss is 0.0006149520631879568, test loss is 0.0005099732079543173, and learning rate i

7031th Epoch:
```

```
traing loss is 0.0002854216727428138, test loss is 0.0003531184047460556, and learning rate i

7041th Epoch:
traing loss is 0.0009129404788836837, test loss is 0.0004098141798749566, and learning rate i

7051th Epoch:
traing loss is 0.0003086569777224213, test loss is 0.00031231908360496163, and learning rate

7061th Epoch:
traing loss is 0.0005459141102619469, test loss is 0.0004133573092985898, and learning rate i

7071th Epoch:
traing loss is 0.000615299039054662, test loss is 0.0006210516439750791, and learning rate is

7081th Epoch:
traing loss is 0.0002960941637866199, test loss is 0.0002561953733675182, and learning rate i

7091th Epoch:
traing loss is 0.0002887319424189627, test loss is 0.00019696715753525496, and learning rate

7101th Epoch:
traing loss is 0.00043372364598326385, test loss is 0.00018445098248776048, and learning rate
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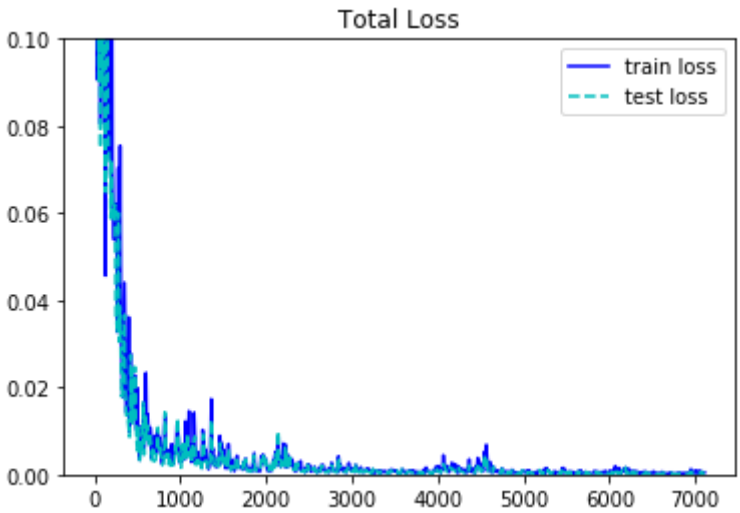

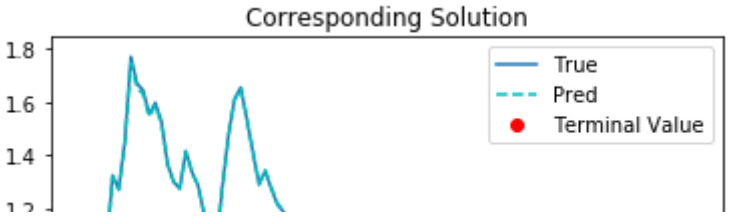

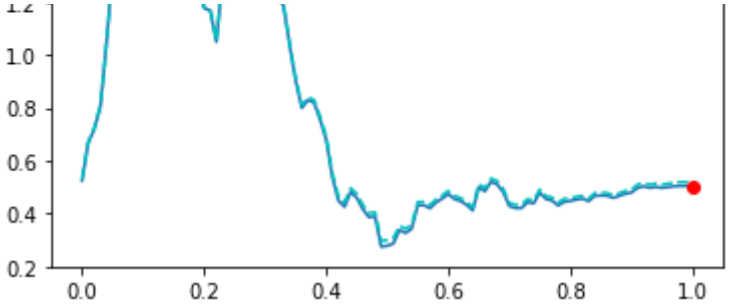

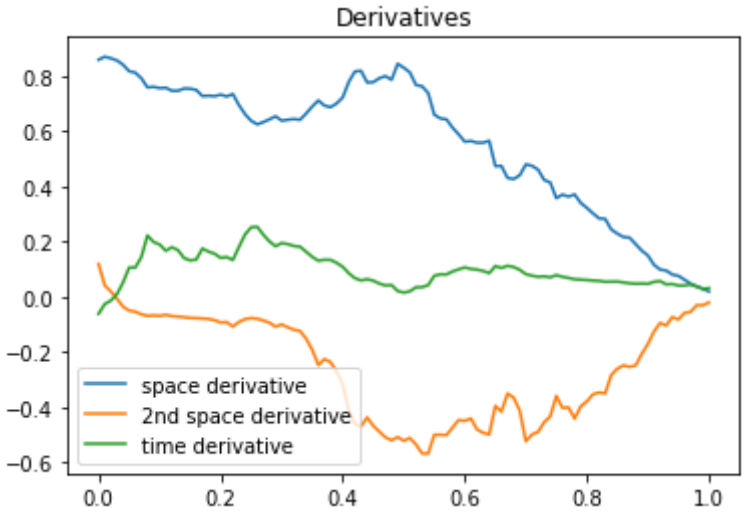


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7111th Epoch:
traing loss is 0.00029638383421115577, test loss is 0.0001699893909972161, and learning rate

7121th Epoch:
traing loss is 0.0001526705891592428, test loss is 0.00016255053924396634, and learning rate

7131th Epoch:
traing loss is 9.424702875548974e-05, test loss is 0.00012898299610242248, and learning rate

7141th Epoch:
traing loss is 0.0005909505998715758, test loss is 0.0004773658874910325, and learning rate i

7151th Epoch:
traing loss is 0.00028039352037012577, test loss is 0.00025060801999643445, and learning rate

7161th Epoch:
traing loss is 0.00039432651828974485, test loss is 0.0002817124768625945, and learning rate

7171th Epoch:
traing loss is 0.00035138195380568504, test loss is 0.00034705005236901343, and learning rate

7181th Epoch:
traing loss is 0.00022079174232203513, test loss is 0.0001957650383701548, and learning rate

7191th Epoch:
traing loss is 0.0001620738476049155, test loss is 0.0001301343145314604, and learning rate i

7201th Epoch:
traing loss is 0.00034914849675260484, test loss is 0.00016086507821455598, and learning rate
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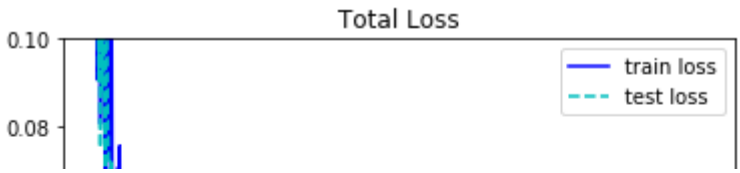

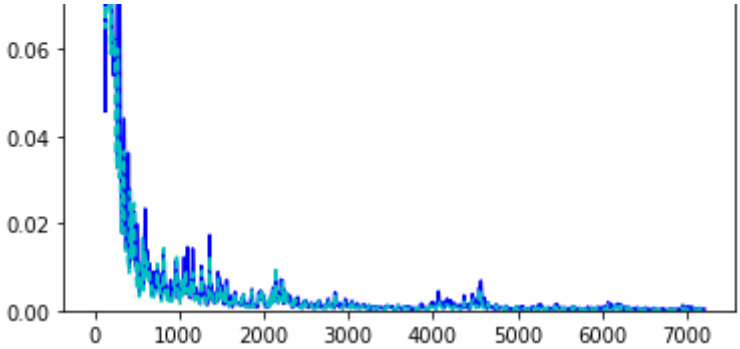

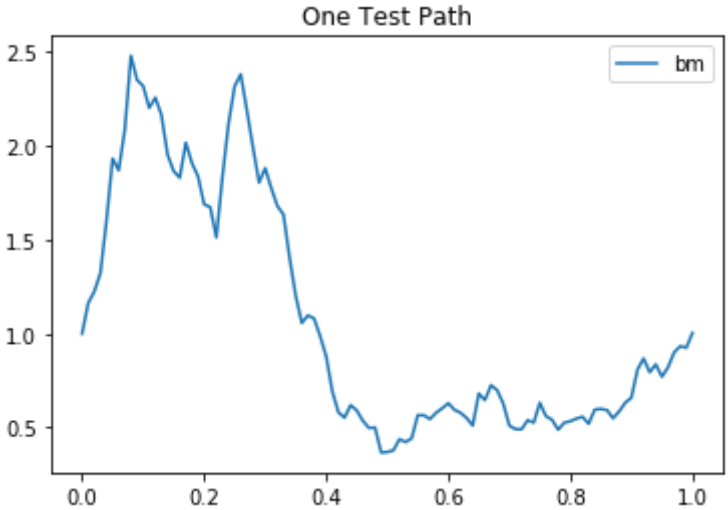

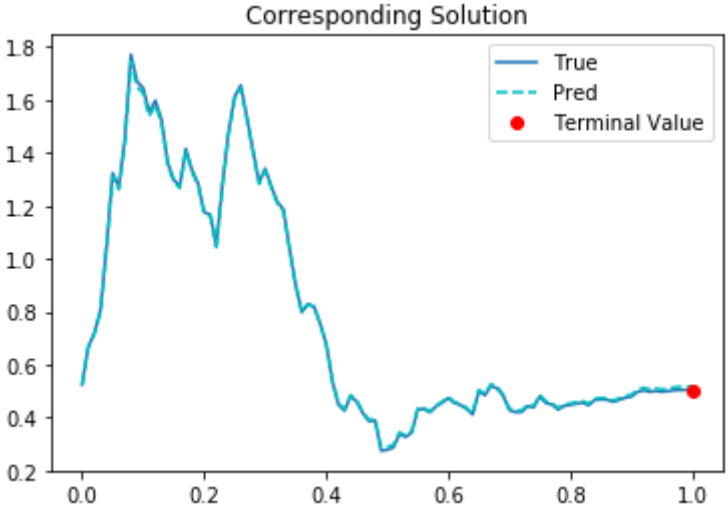

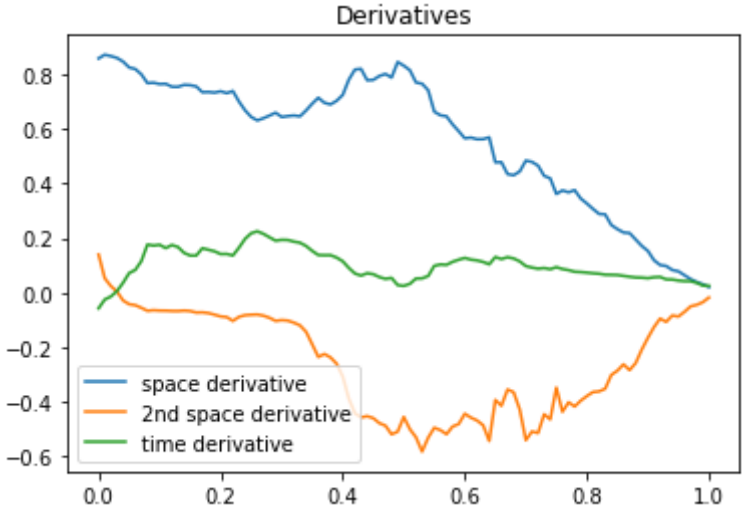

7211th Epoch:
traing loss is 0.00046409328933805227, test loss is 0.00032671133521944284, and learning rate

7221th Epoch:

```
traing loss is 0.00016632411279715598, test loss is 0.0001640603441046551, and learning rate

7231th Epoch:
traing loss is 0.00012649352720472962, test loss is 0.0001841574121499434, and learning rate

7241th Epoch:
traing loss is 0.000322631181916222, test loss is 0.00025233582709915936, and learning rate i

7251th Epoch:
traing loss is 0.00013425422366708517, test loss is 0.00014371763973031193, and learning rate

7261th Epoch:
traing loss is 0.00023221361334435642, test loss is 0.00015593752323184162, and learning rate

7271th Epoch:
traing loss is 0.00014668938820250332, test loss is 0.0001531451562186703, and learning rate

7281th Epoch:
traing loss is 0.00014146137982606888, test loss is 0.00011831473238999024, and learning rate

7291th Epoch:
traing loss is 0.00012901212903670967, test loss is 9.807053720578551e-05, and learning rate

7301th Epoch:
traing loss is 0.0003064556512981653, test loss is 0.0001414637954439968, and learning rate i
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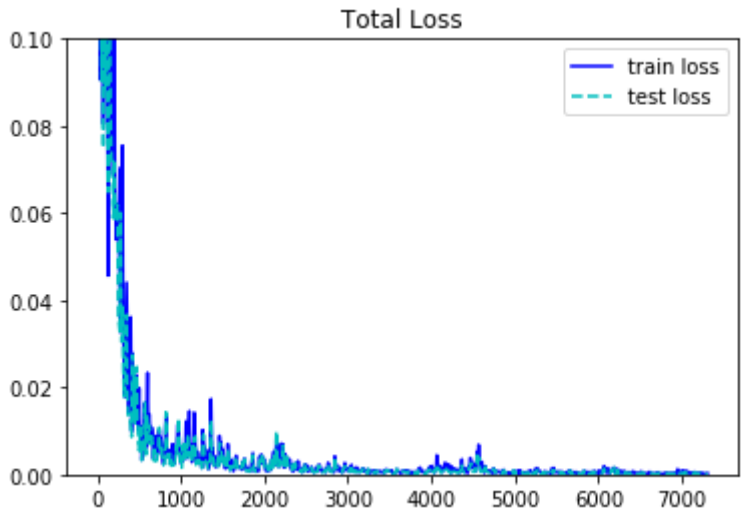

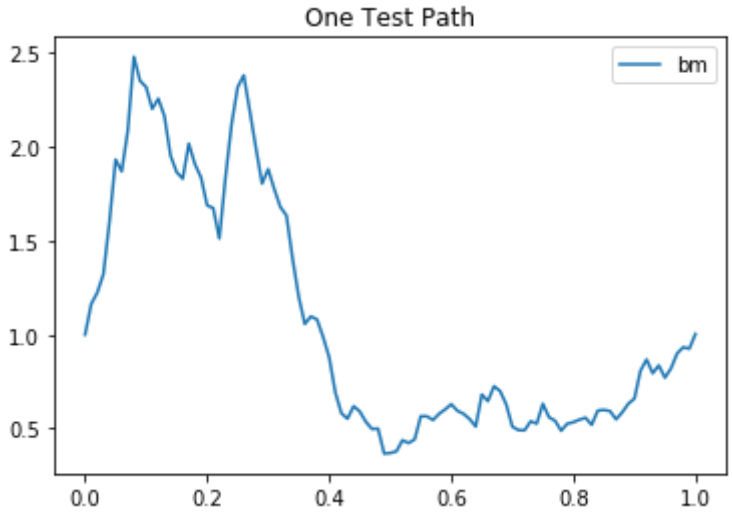

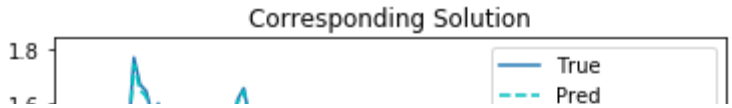

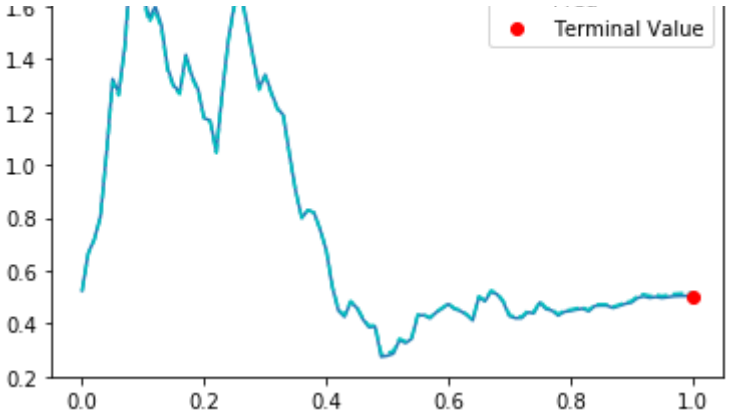

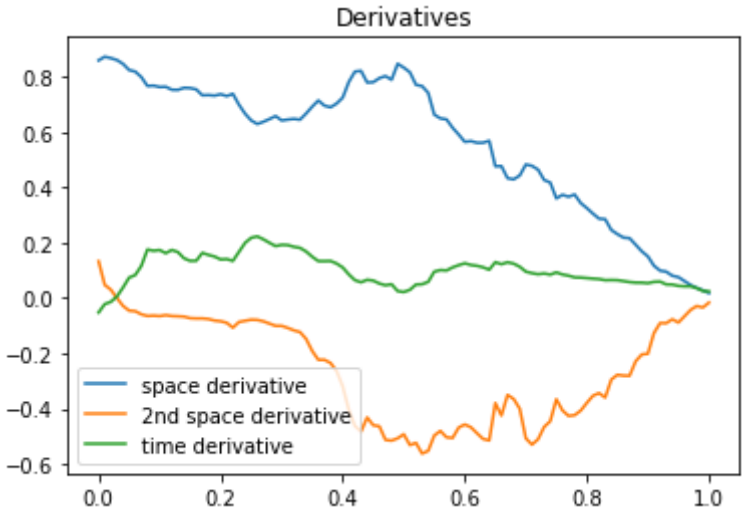


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7311th Epoch:
traing loss is 0.0004818718880414963, test loss is 0.00032983021810650826, and learning rate

7321th Epoch:
traing loss is 0.0001639488327782601, test loss is 0.0001532646274426952, and learning rate i

7331th Epoch:
traing loss is 0.00012793188216164708, test loss is 0.00018620173796080053, and learning rate

7341th Epoch:
traing loss is 0.00021075397671665996, test loss is 0.0001626096200197935, and learning rate

7351th Epoch:
traing loss is 0.00010068552364828065, test loss is 0.0001112191894208081, and learning rate

7361th Epoch:
traing loss is 0.00017813508748076856, test loss is 0.00011348498810548335, and learning rate

7371th Epoch:
traing loss is 9.975453576771542e-05, test loss is 0.00010699228732846677, and learning rate

7381th Epoch:
traing loss is 0.0001210855771205388, test loss is 9.964283526642248e-05, and learning rate i

7391th Epoch:
traing loss is 0.0001221998973051086, test loss is 9.268694702768698e-05, and learning rate i

7401th Epoch:
traing loss is 0.0002457782975398004, test loss is 0.00011805116082541645, and learning rate
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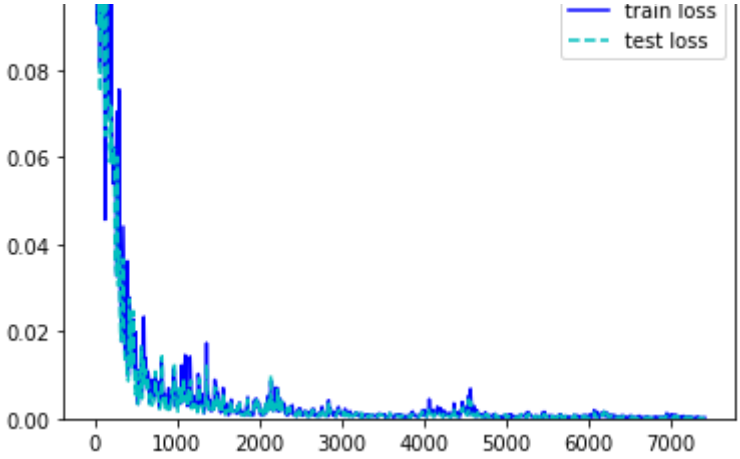

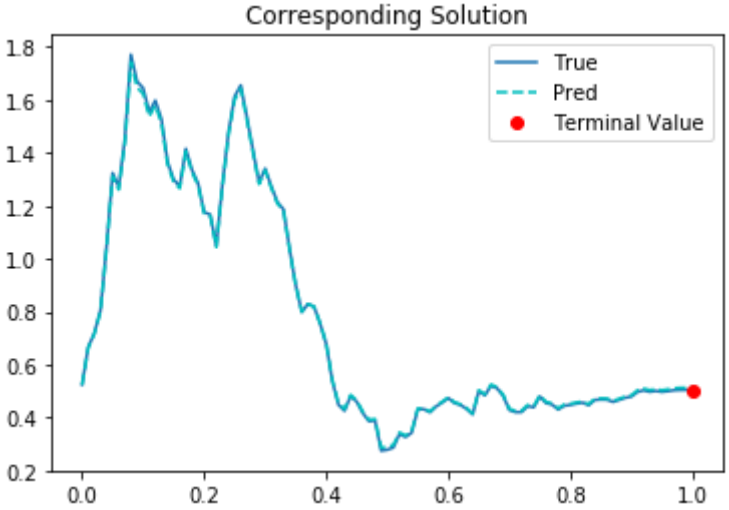

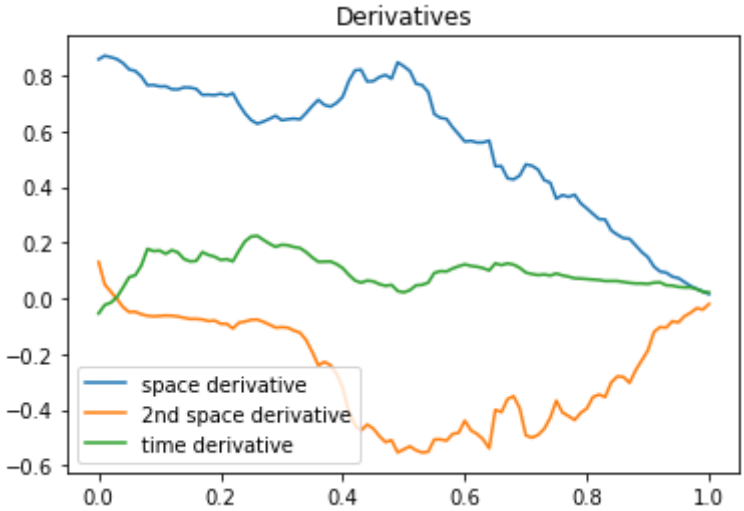

7411th Epoch:

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traing loss is 0.00041307520586997 27, test loss is 0.00027030732599087 06, and learning rate i

7421th Epoch:
traing loss is 0.0001335618580924347, test loss is 0.00012389078619889915, and learning rate

7431th Epoch:
traing loss is 0.00010681153071345761, test loss is 0.0001567427534610033, and learning rate

7441th Epoch:
traing loss is 0.0001295447291340679, test loss is 0.00011563571752049029, and learning rate

7451th Epoch:
traing loss is 8.169066131813452e-05, test loss is 9.285323176300153e-05, and learning rate i

7461th Epoch:
traing loss is 0.0001624805445317179, test loss is 9.826226596487686e-05, and learning rate i

7471th Epoch:
traing loss is 8.559108391636983e-05, test loss is 9.1178560978733e-05, and learning rate is

7481th Epoch:
traing loss is 0.00010851625120267272, test loss is 9.056418639374897e-05, and learning rate

7491th Epoch:
traing loss is 0.0001151976321125403, test loss is 8.477817755192518e-05, and learning rate i

7501th Epoch:
traing loss is 0.00021479761926457286, test loss is 0.00010663358989404514, and learning rate
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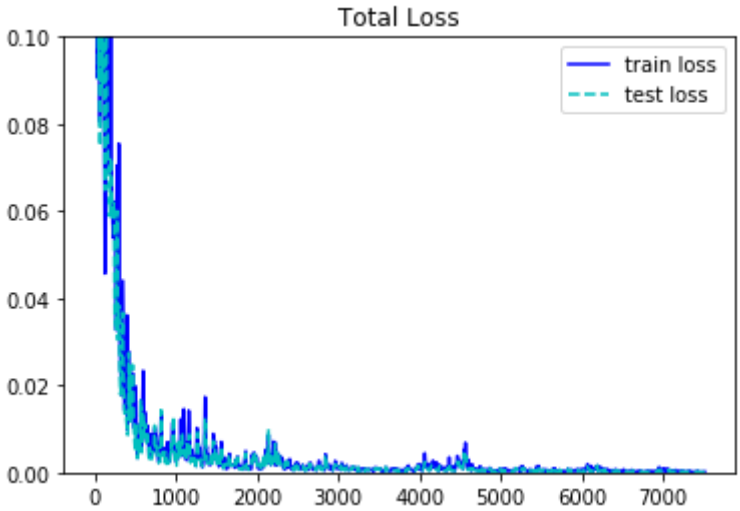

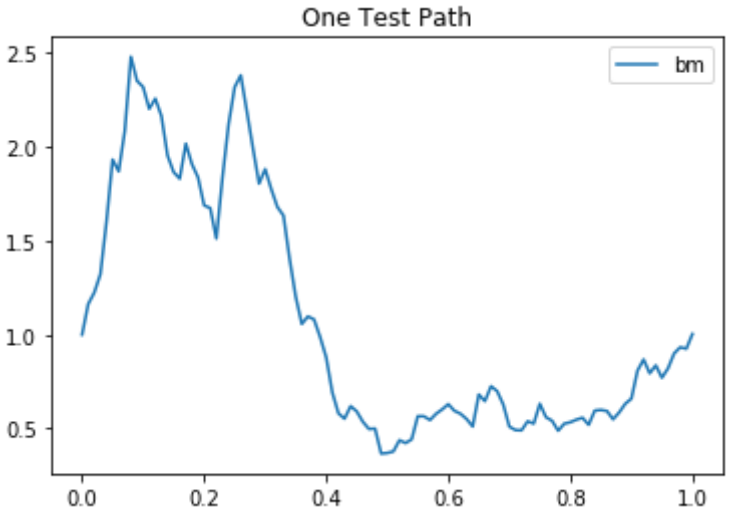

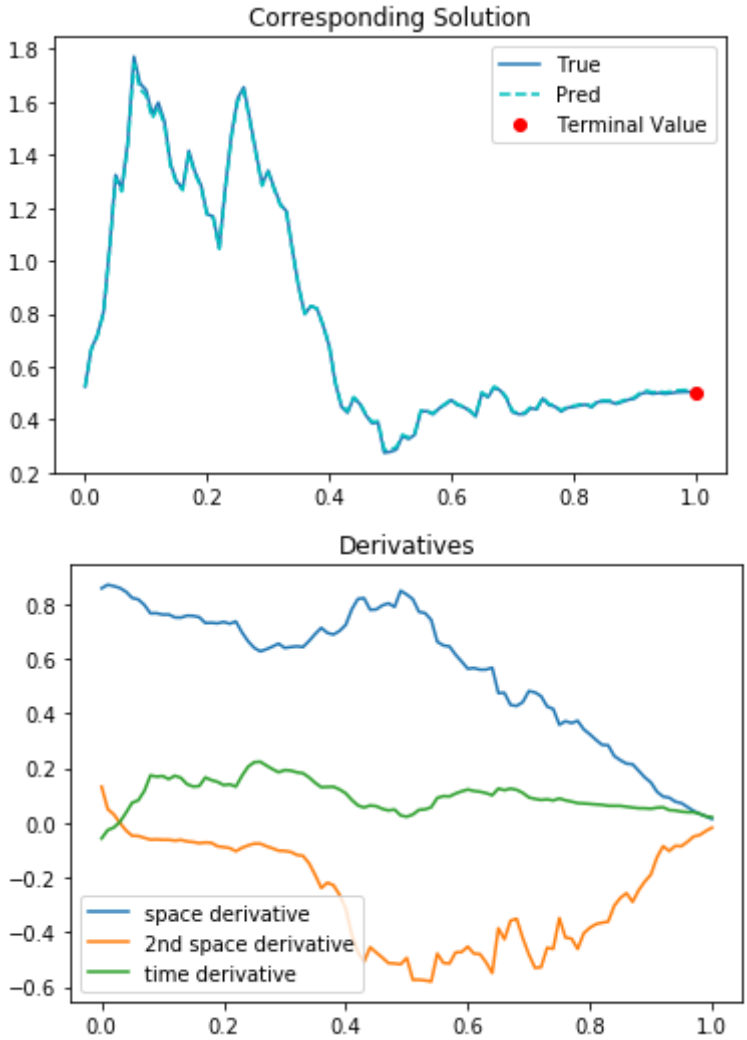


```
7511th Epoch:
traing loss is 0.0003690563316922635, test loss is 0.00024119803856592625, and learning rate

7521th Epoch:
traing loss is 0.0001326190249528736, test loss is 0.00011930034088436514, and learning rate

7531th Epoch:
traing loss is 9.958224836736917e-05, test loss is 0.00014633229875471443, and learning rate

7541th Epoch:
traing loss is 9.900875011226162e-05, test loss is 9.618177864467725e-05, and learning rate i

7551th Epoch:
traing loss is 7.331877714022994e-05, test loss is 8.362714288523421e-05, and learning rate i

7561th Epoch:
traing loss is 0.00015029640053398907, test loss is 9.015073737828061e-05, and learning rate

7571th Epoch:
traing loss is 8.387534035136923e-05, test loss is 8.915673970477656e-05, and learning rate i

7581th Epoch:
traing loss is 0.0001032518848660402, test loss is 8.564102608943358e-05, and learning rate i

7591th Epoch:
traing loss is 0.00011279904720140621, test loss is 8.119870472000912e-05, and learning rate

7601th Epoch:
traing loss is 0.0001971871533896774, test loss is 9.84211583272554e-05, and learning rate is
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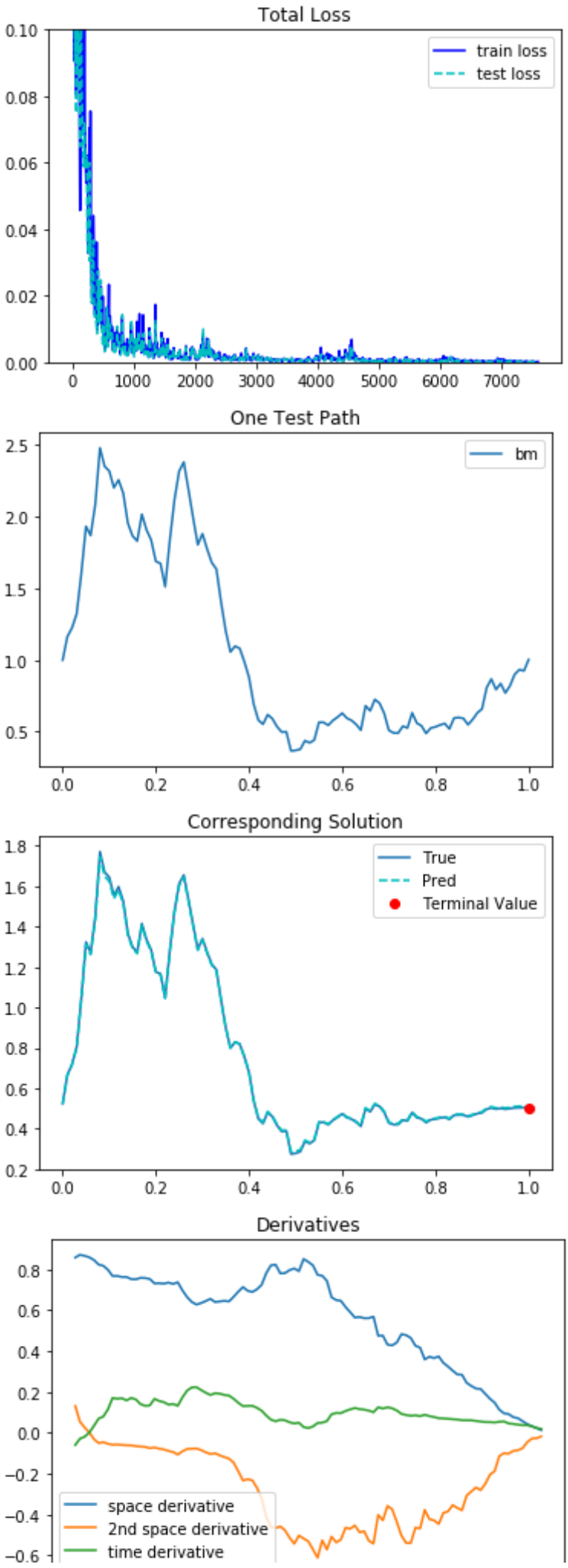
Total Loss
train loss
test loss
0.10
0.08
0.06
0.04
0.02
0.00
0
1000
2000
3000
4000
5000
6000
7000
One Test Path
bm
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0
Corresponding Solution
True
Pred
Terminal Value
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
Derivatives
space derivative
2nd space derivative
time derivative
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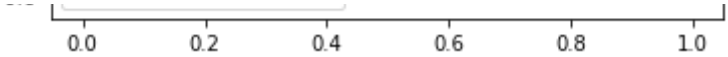
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6




```
7611th Epoch:
traing loss is 0.000330625131027773, test loss is 0.00021823096903972328, and learning rate i

7621th Epoch:
traing loss is 0.00012920834706164896, test loss is 0.00011350202839821577, and learning rate

7631th Epoch:
traing loss is 9.511003008810803e-05, test loss is 0.00013817766739521176, and learning rate

7641th Epoch:
traing loss is 8.80044317455031e-05, test loss is 8.640791202196851e-05, and learning rate is

7651th Epoch:
traing loss is 6.821528950240463e-05, test loss is 7.72708299336955e-05, and learning rate is

7661th Epoch:
traing loss is 0.00014373657177202404, test loss is 8.519148104824126e-05, and learning rate

7671th Epoch:
traing loss is 7.978947542142123e-05, test loss is 8.400627848459408e-05, and learning rate i

7681th Epoch:
traing loss is 9.738043445395306e-05, test loss is 8.117867400869727e-05, and learning rate i

7691th Epoch:
traing loss is 0.0001083179158740677, test loss is 7.799160812282935e-05, and learning rate i

7701th Epoch:
traing loss is 0.00018291943706572056, test loss is 9.315458737546578e-05, and learning rate
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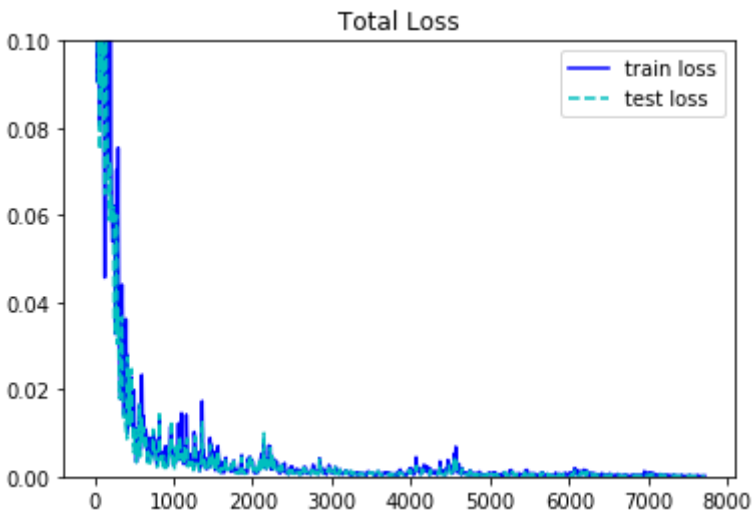

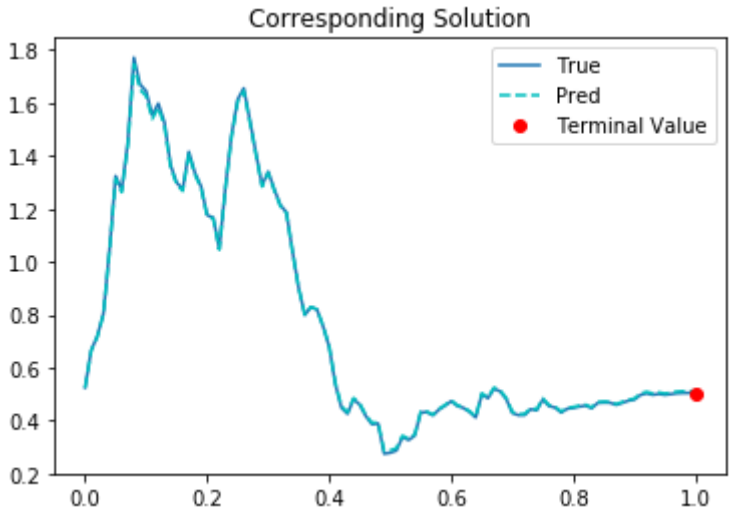

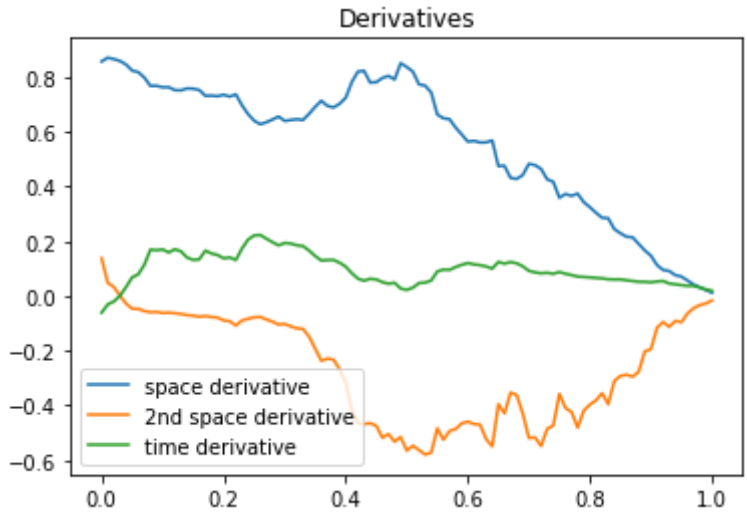


```
7711th Epoch:
traing loss is 0.00030511085060425103, test loss is 0.0002016777143580839, and learning rate

7721th Epoch:
traing loss is 0.00012831538333557546, test loss is 0.0001113159378292039, and learning rate

7731th Epoch:
traing loss is 9.273439354728907e-05, test loss is 0.00013280048733577132, and learning rate

7741th Epoch:
traing loss is 8.50413998705335e-05, test loss is 8.252510451711714e-05, and learning rate is

7751th Epoch:
traing loss is 6.665570253971964e-05, test loss is 7.506081601604819e-05, and learning rate i

7761th Epoch:
traing loss is 0.00013541399675887078, test loss is 8.129261550493538e-05, and learning rate

7771th Epoch:
traing loss is 7.907064718892798e-05, test loss is 8.271619299193844e-05, and learning rate i

7781th Epoch:
traing loss is 9.316017531091347e-05, test loss is 7.659939728910103e-05, and learning rate i

7791th Epoch:
traing loss is 0.00010516517795622349, test loss is 7.542636740254238e-05, and learning rate
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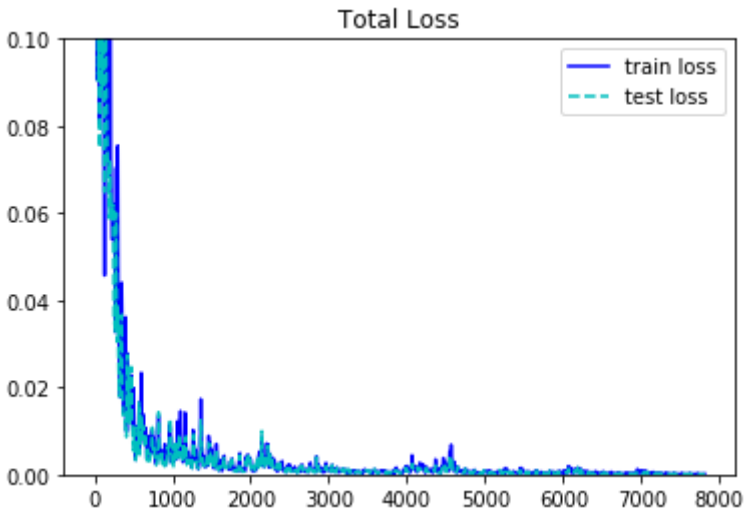
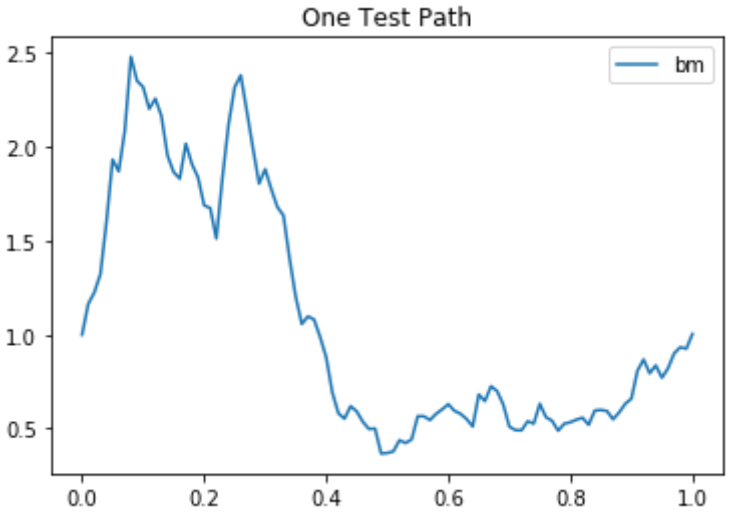
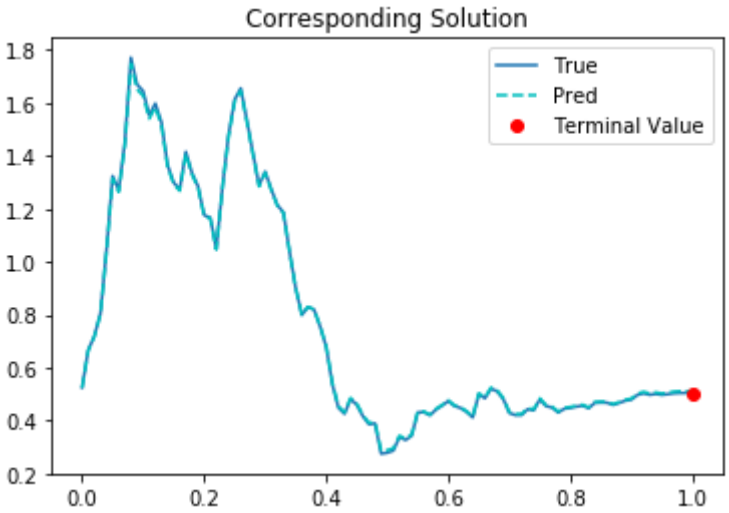
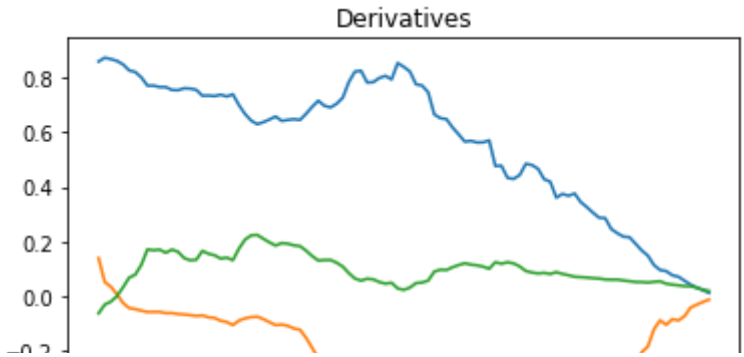
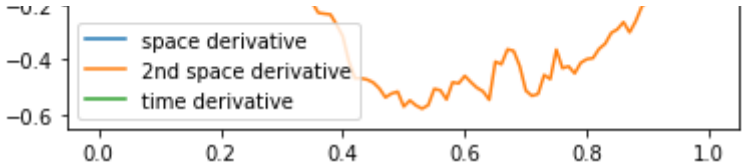
7801th Epoch:
traing loss is 0.00017060645041055977, test loss is 8.784148667473346e-05, and learning rate












```
7811th Epoch:
traing loss is 0.0002877700899261981, test loss is 0.00019147360580973327, and learning rate

7821th Epoch:
traing loss is 0.00012524261546786875, test loss is 0.00010684819426387548, and learning rate

7831th Epoch:
traing loss is 8.802219235803932e-05, test loss is 0.0001261322759091854, and learning rate i

7841th Epoch:
traing loss is 8.508839528076351e-05, test loss is 7.991606980795041e-05, and learning rate i

7851th Epoch:
traing loss is 6.354677316267043e-05, test loss is 7.108994759619236e-05, and learning rate i

7861th Epoch:
traing loss is 0.00012677082850132138, test loss is 7.658399408683181e-05, and learning rate

7871th Epoch:
traing loss is 7.433172868331894e-05, test loss is 7.804072083672509e-05, and learning rate i

7881th Epoch:
traing loss is 9.260782826459035e-05, test loss is 7.376304711215198e-05, and learning rate i

7891th Epoch:
traing loss is 0.00010313042002962902, test loss is 7.23330958862789e-05, and learning rate i

7901th Epoch:
traing loss is 0.00015482233720831573, test loss is 8.438417717115954e-05, and learning rate
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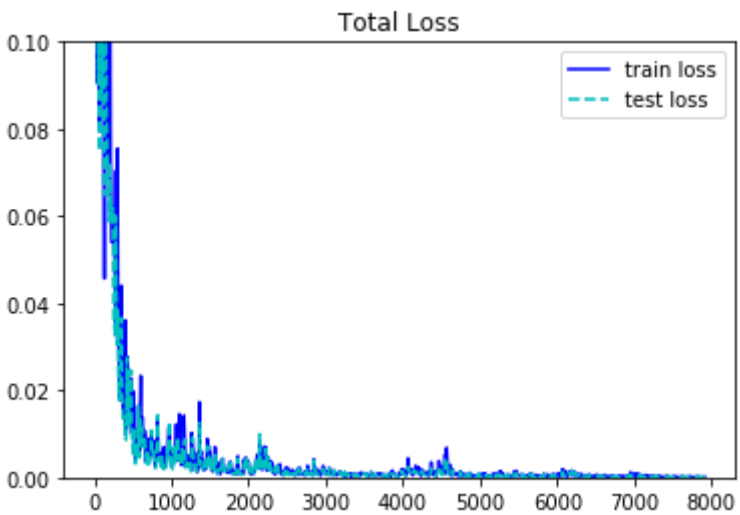

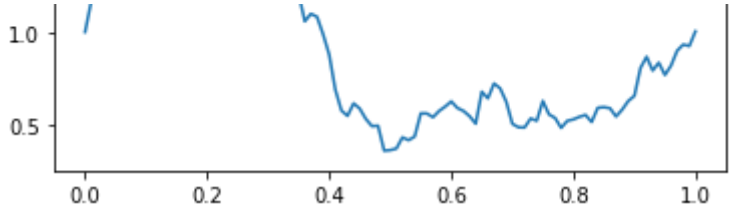

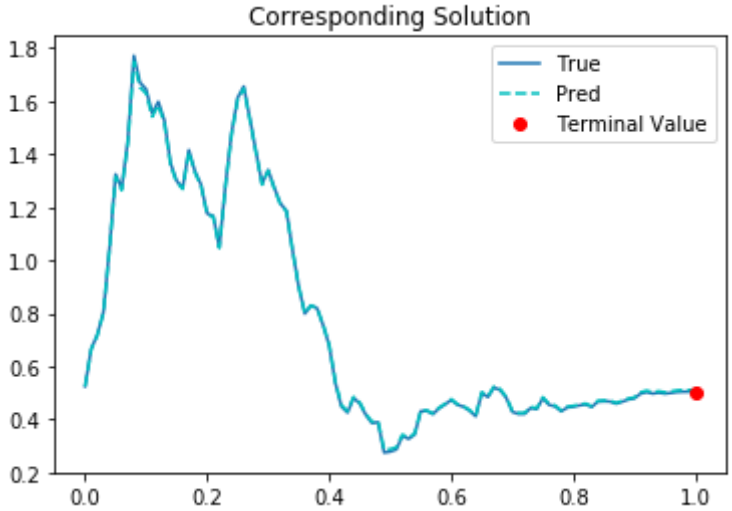

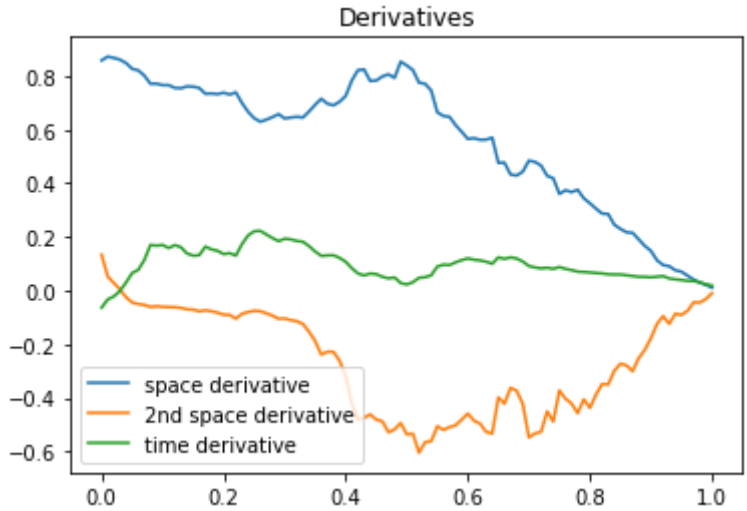


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7911th Epoch:
traing loss is 0.00027690856950357556, test loss is 0.00018367629672866315, and learning rate

7921th Epoch:
traing loss is 0.00012301756942179054, test loss is 0.00010479710181243718, and learning rate

7931th Epoch:
traing loss is 8.702065679244697e-05, test loss is 0.00012271481682546437, and learning rate

7941th Epoch:
traing loss is 8.368209091713652e-05, test loss is 7.837351586204022e-05, and learning rate i

7951th Epoch:
traing loss is 6.023605237714946e-05, test loss is 6.78792130202055e-05, and learning rate is

7961th Epoch:
traing loss is 0.00012013460946036503, test loss is 7.323454337893054e-05, and learning rate

7971th Epoch:
traing loss is 7.413472485495731e-05, test loss is 7.869828550610691e-05, and learning rate i

7981th Epoch:
traing loss is 8.95345292519778e-05, test loss is 7.294979877769947e-05, and learning rate is
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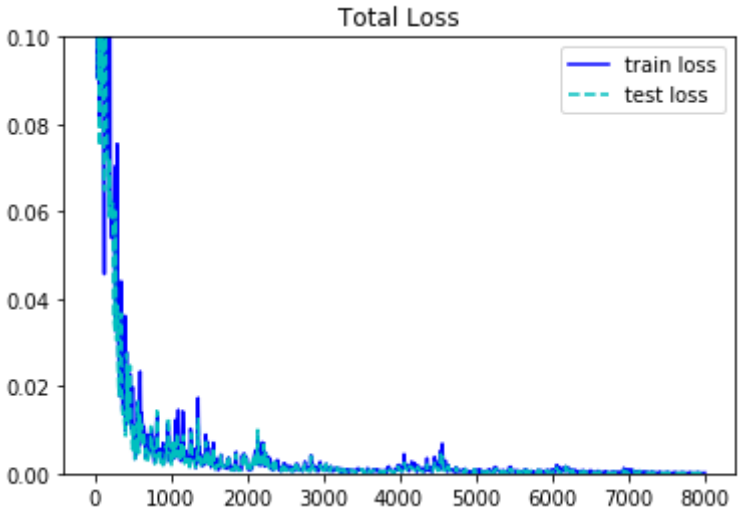
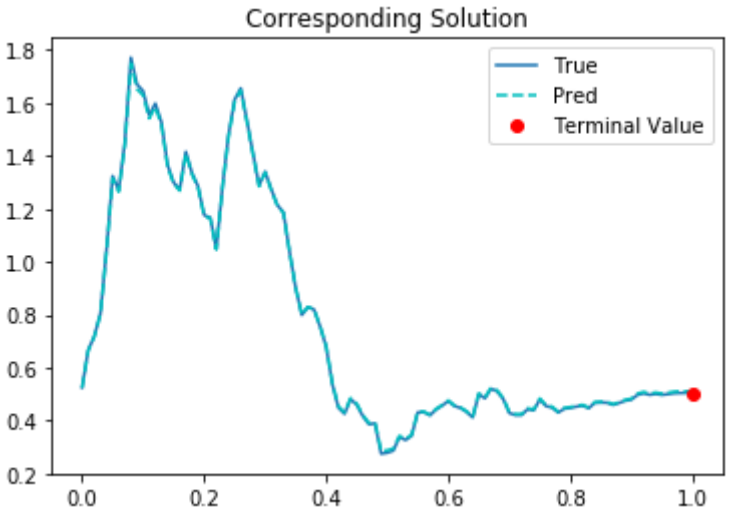
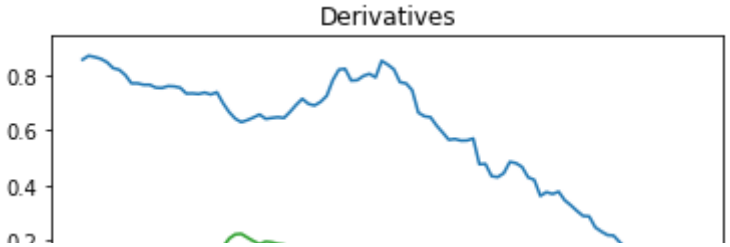
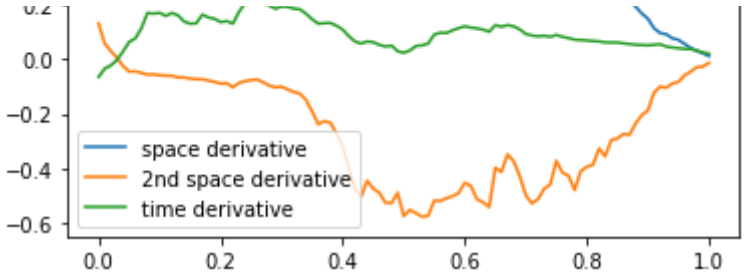
7991th Epoch:
traing loss is 9.985631186282262e-05, test loss is 6.957671575946733e-05, and learning rate i

8001th Epoch:
traing loss is 0.00015089675434865057, test loss is 8.328027615789324e-05, and learning rate







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8011th Epoch:
traing loss is 0.0002678825694601983, test loss is 0.00017931938054971397, and learning rate

8021th Epoch:
traing loss is 0.00011823813110822812, test loss is 9.985271753976122e-05, and learning rate

8031th Epoch:
traing loss is 8.283302304334939e-05, test loss is 0.00011764784721890464, and learning rate

8041th Epoch:
traing loss is 8.302293281303719e-05, test loss is 7.595613715238869e-05, and learning rate i

8051th Epoch:
traing loss is 5.89376941206865e-05, test loss is 6.649353599641472e-05, and learning rate is

8061th Epoch:
traing loss is 0.00011423173418734223, test loss is 7.108444697223604e-05, and learning rate

8071th Epoch:
traing loss is 7.12906985427253e-05, test loss is 7.537670171586797e-05, and learning rate is

8081th Epoch:
traing loss is 8.701807382749394e-05, test loss is 7.073481538100168e-05, and learning rate i

8091th Epoch:
traing loss is 9.683676762506366e-05, test loss is 6.751160981366411e-05, and learning rate i

8101th Epoch:
traing loss is 0.0001397446176270023, test loss is 7.854112482164055e-05, and learning rate i
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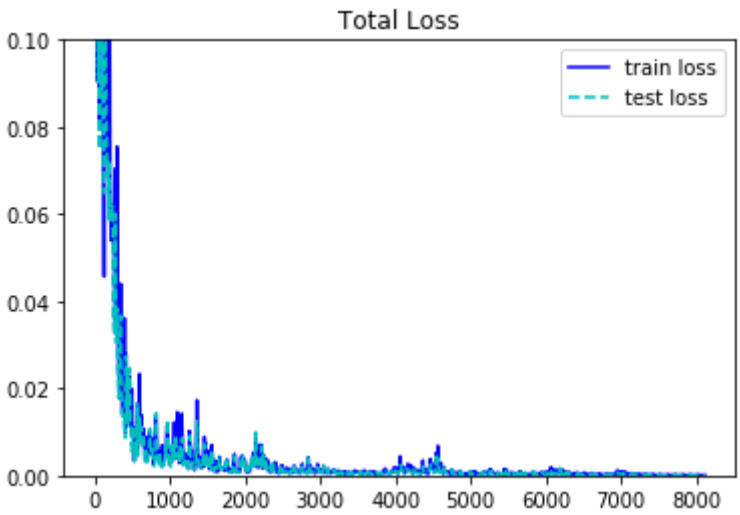

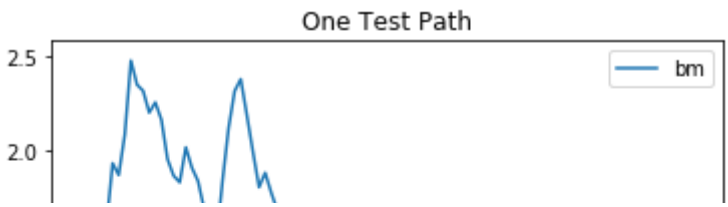

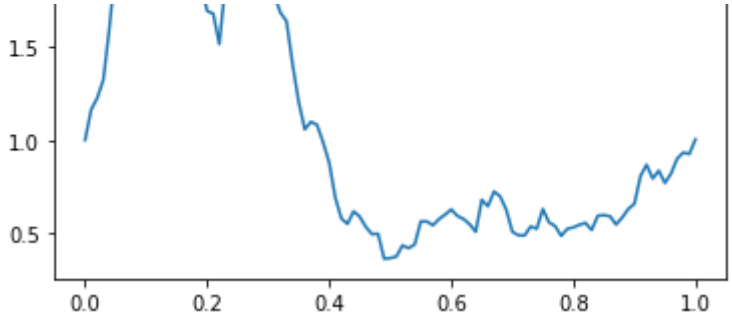

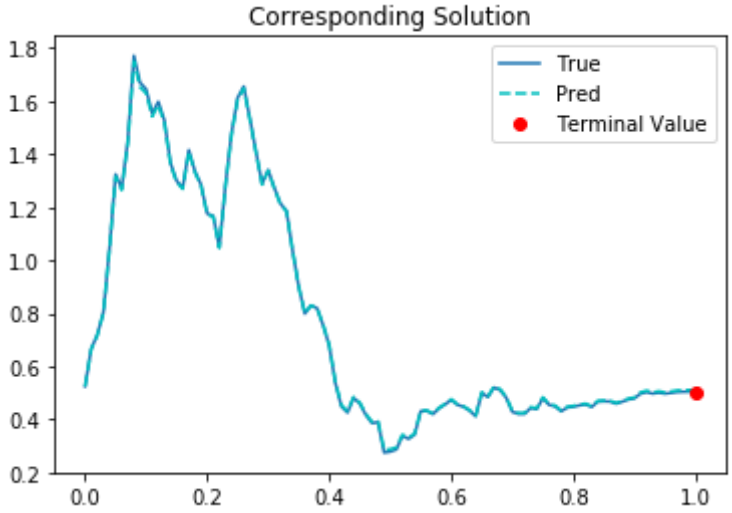

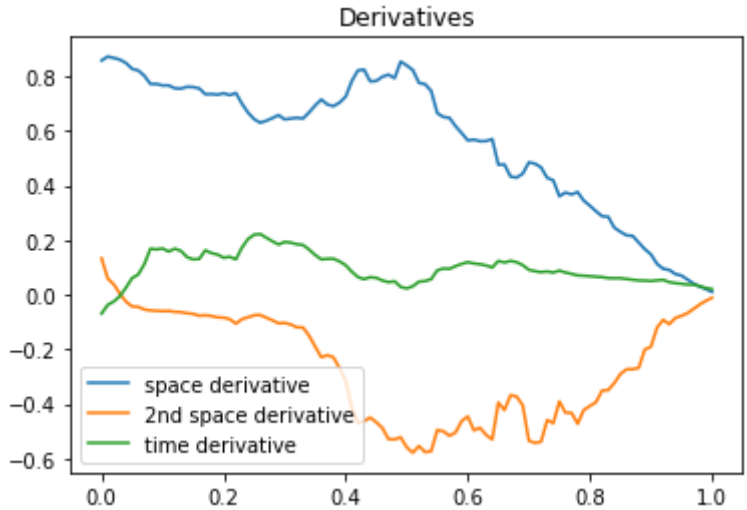


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8111th Epoch:
traing loss is 0.00025565168471075594, test loss is 0.00017268533702008426, and learning rate

8121th Epoch:
traing loss is 0.00011181024456163868, test loss is 9.496849088463932e-05, and learning rate

8131th Epoch:
traing loss is 7.886173989390954e-05, test loss is 0.00011181434092577547, and learning rate

8141th Epoch:
traing loss is 8.359803905477747e-05, test loss is 7.738178101135418e-05, and learning rate i

8151th Epoch:
traing loss is 5.776345278718509e-05, test loss is 6.595873855985701e-05, and learning rate i

8161th Epoch:
traing loss is 0.00011306280794087797, test loss is 6.894723628647625e-05, and learning rate

8171th Epoch:
traing loss is 6.777277303626761e-05, test loss is 7.28982558939606e-05, and learning rate is
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```
8181th Epoch:
traing loss is 8.365709072677419e-05, test loss is 6.936721183592454e-05, and learning rate i

8191th Epoch:
traing loss is 9.728290751809254e-05, test loss is 6.68026550556533e-05, and learning rate is

8201th Epoch:
traing loss is 0.00013301405124366283, test loss is 7.625699072377756e-05, and learning rate
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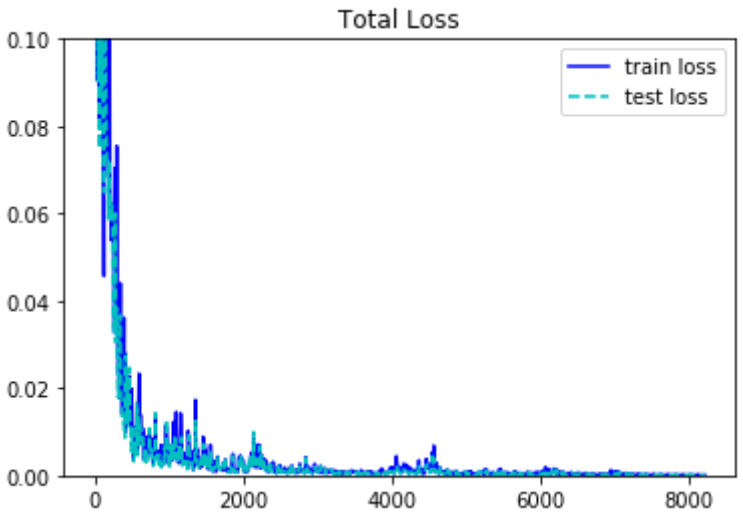

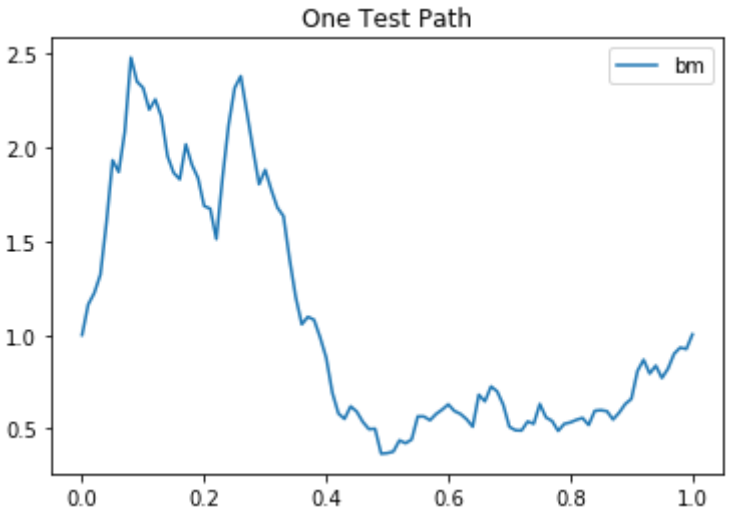

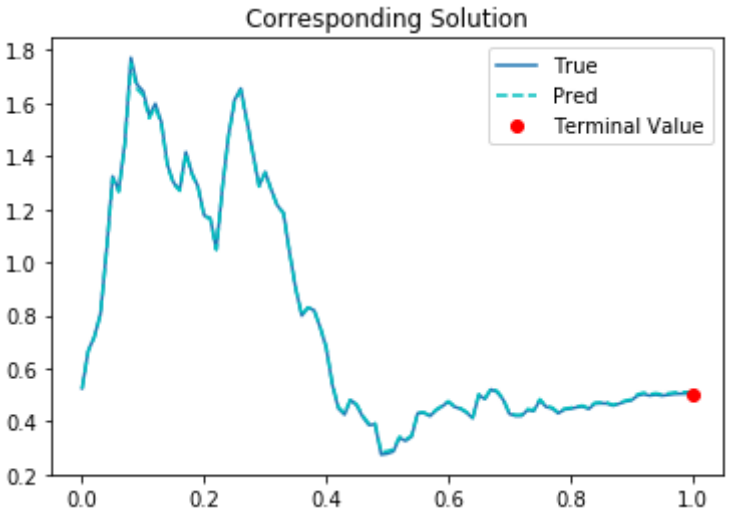

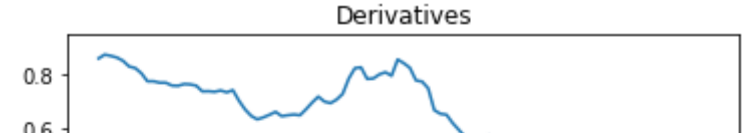

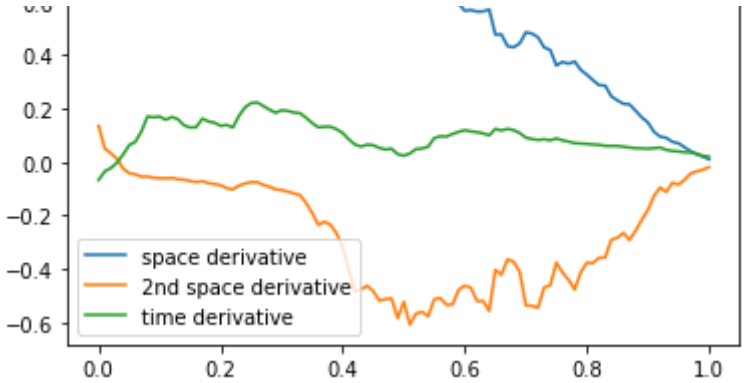


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8211th Epoch:
traing loss is 0.00024590411339886487, test loss is 0.00016750914801377803, and learning rate

8221th Epoch:
traing loss is 0.00010402098268968984, test loss is 8.850869198795408e-05, and learning rate

8231th Epoch:
traing loss is 7.249778718687594e-05, test loss is 0.0001027532052830793, and learning rate i

8241th Epoch:
traing loss is 8.130559581331909e-05, test loss is 7.308941712835804e-05, and learning rate i

8251th Epoch:
traing loss is 5.512802817975171e-05, test loss is 6.384462903952226e-05, and learning rate i

8261th Epoch:
traing loss is 0.0001079303547157906, test loss is 6.638040213147178e-05, and learning rate i

8271th Epoch:
traing loss is 6.231047882465646e-05, test loss is 6.732314795954153e-05, and learning rate i

8281th Epoch:
traing loss is 8.199304284062237e-05, test loss is 6.766274600522593e-05, and learning rate i

8291th Epoch:
traing loss is 9.376938396599144e-05, test loss is 6.467237108154222e-05, and learning rate i

8301th Epoch:
traing loss is 0.00012590234109666198, test loss is 7.4077928729821e-05, and learning rate is
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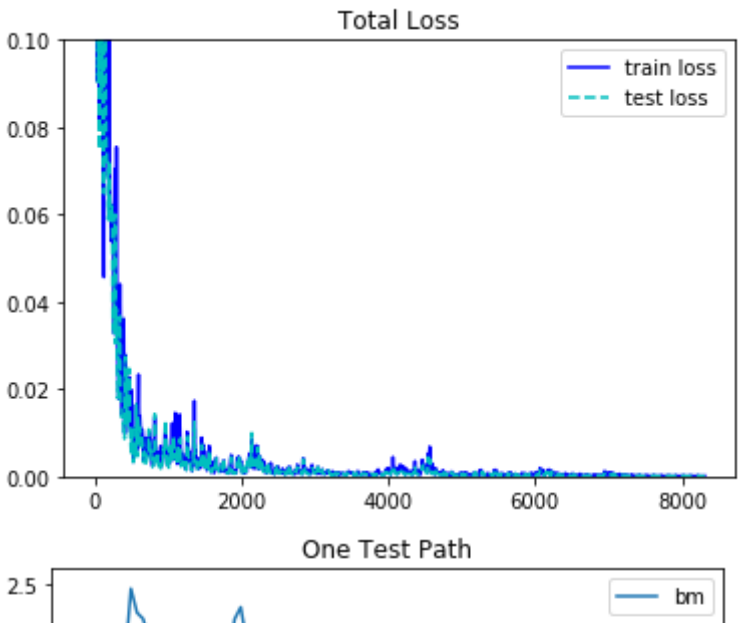

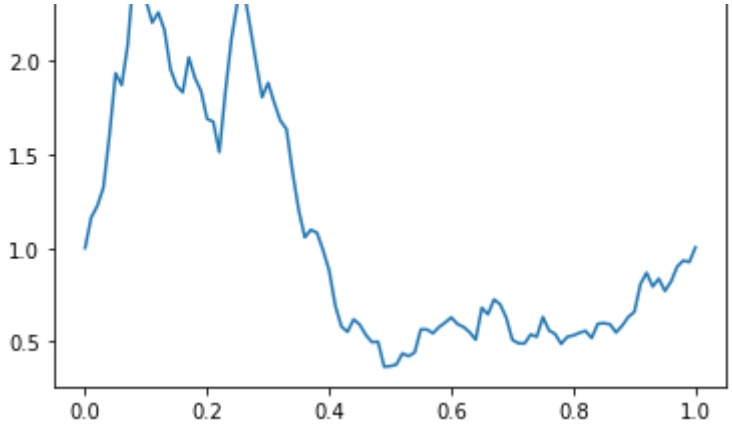

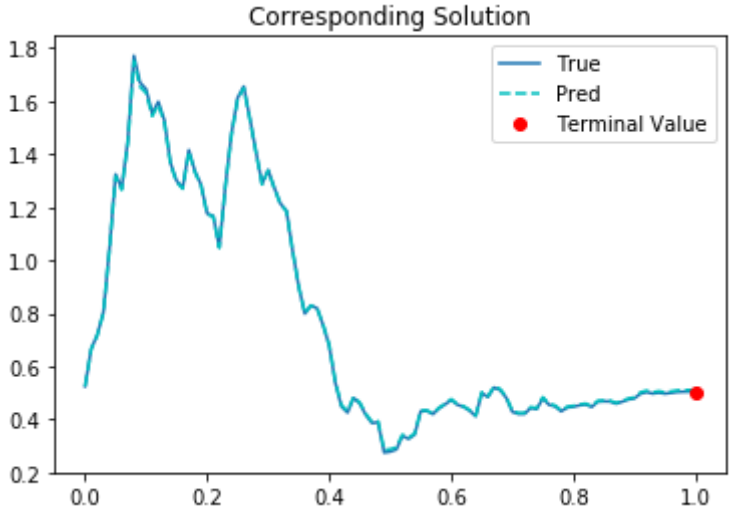

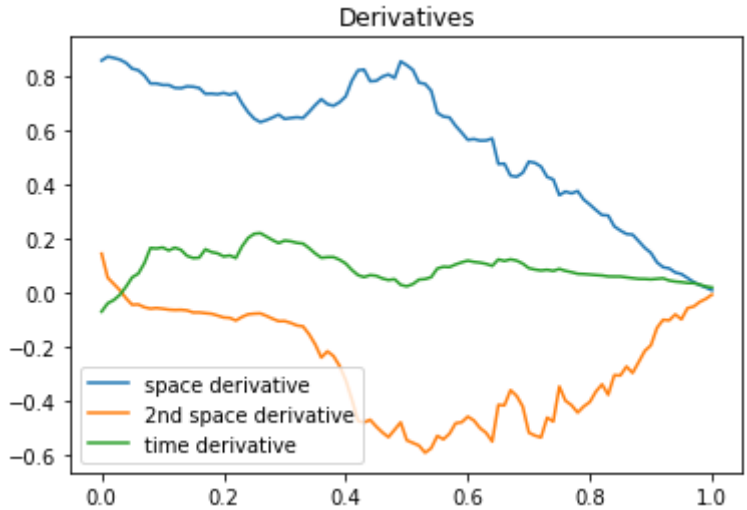


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8311th Epoch:
traing loss is 0.0002389932342339307, test loss is 0.0001648776524234563, and learning rate i

8321th Epoch:
traing loss is 0.00010128015128429979, test loss is 8.655276178615168e-05, and learning rate

8331th Epoch:
traing loss is 7.176500366767868e-05, test loss is 0.00010079755884362385, and learning rate

8341th Epoch:
traing loss is 7.996067870408297e-05, test loss is 7.465628004865721e-05, and learning rate i

8351th Epoch:
traing loss is 5.551573121920228e-05, test loss is 6.371827475959435e-05, and learning rate i

8361th Epoch:
traing loss is 0.00010323220340069383, test loss is 6.490620580734685e-05, and learning rate
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8371th Epoch:
traing loss is 6.241405208129436e-05, test loss is 6.782065611332655e-05, and learning rate i

8381th Epoch:
traing loss is 8.00627822172828e-05, test loss is 6.628785922657698e-05, and learning rate is

8391th Epoch:
traing loss is 9.18778168852441e-05, test loss is 6.334349018288776e-05, and learning rate is

8401th Epoch:
traing loss is 0.00011939723481191322, test loss is 7.171618926804513e-05, and learning rate
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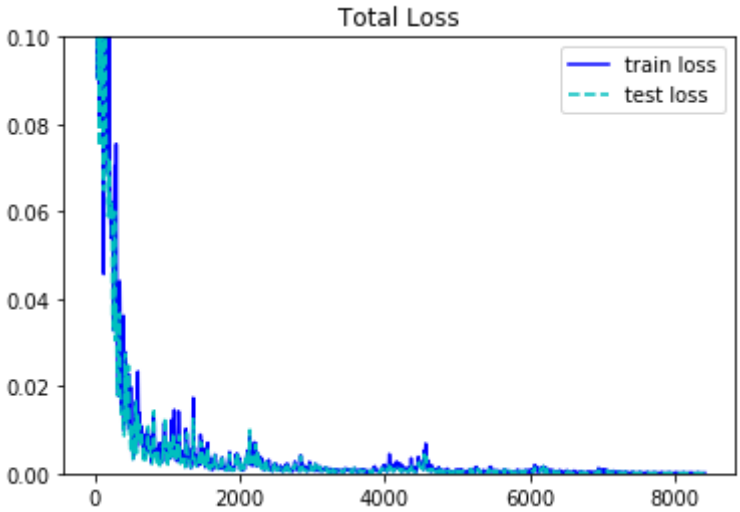

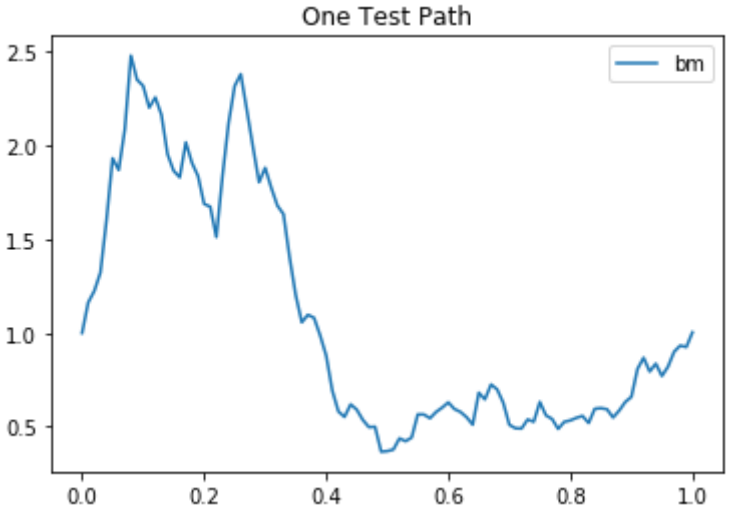

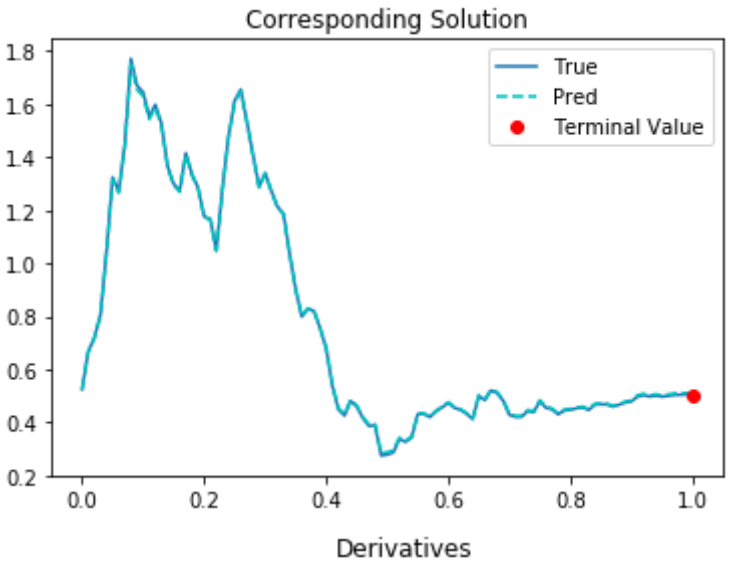

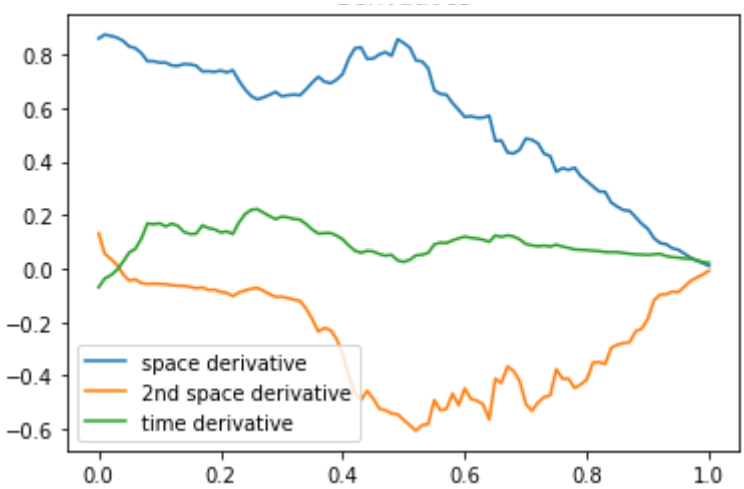


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8411th Epoch:
traing loss is 0.0002279627078678459, test loss is 0.00015523238107562065, and learning rate

8421th Epoch:
traing loss is 9.397578833159059e-05, test loss is 8.044604328460991e-05, and learning rate i

8431th Epoch:
traing loss is 6.51049631414935e-05, test loss is 9.193384903483093e-05, and learning rate is

8441th Epoch:
traing loss is 7.845282380003482e-05, test loss is 7.030640699667856e-05, and learning rate i

8451th Epoch:
traing loss is 5.5558168241987005e-05, test loss is 6.286687130341306e-05, and learning rate

8461th Epoch:
traing loss is 0.0001025973615469411, test loss is 6.241231312742457e-05, and learning rate i

8471th Epoch:
traing loss is 5.740009873989038e-05, test loss is 6.326138827716932e-05, and learning rate i

8481th Epoch:
traing loss is 7.800942694302648e-05, test loss is 6.468012725235894e-05, and learning rate i

8491th Epoch:
traing loss is 8.879666711436585e-05, test loss is 6.185551319504157e-05, and learning rate i

8501th Epoch:
traing loss is 0.00011184300819877535, test loss is 6.923486216692254e-05, and learning rate
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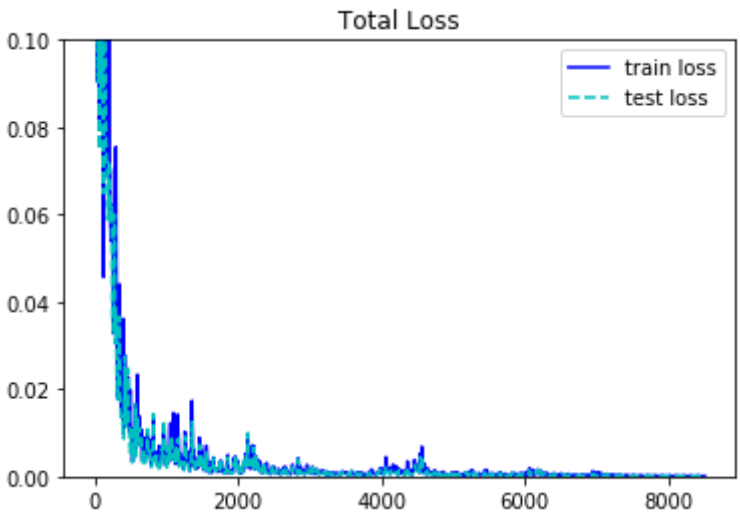

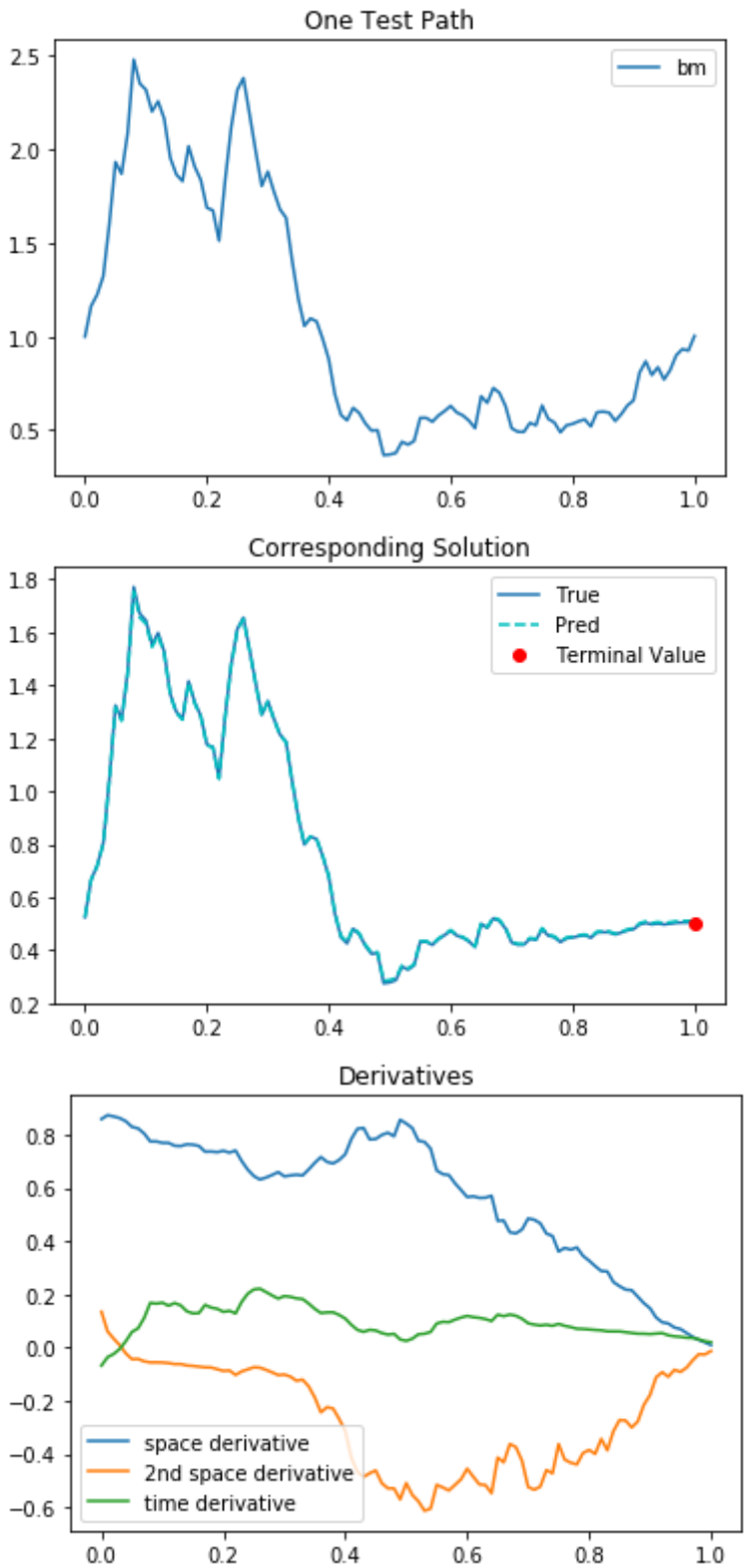


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8511th Epoch:
traing loss is 0.00022140963119454682, test loss is 0.00015229331620503217, and learning rate

8521th Epoch:
traing loss is 9.091461106436327e-05, test loss is 7.661408744752407e-05, and learning rate i

8531th Epoch:
traing loss is 6.501723692053929e-05, test loss is 9.073002001969144e-05, and learning rate i

8541th Epoch:
traing loss is 7.706587348366156e-05, test loss is 7.333189569180831e-05, and learning rate i

8551th Epoch:
traing loss is 5.5121239711297676e-05, test loss is 6.373725045705214e-05, and learning rate
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```
8561th Epoch:
traing loss is 9.689928992884234e-05, test loss is 6.167893297970295e-05, and learning rate i

8571th Epoch:
traing loss is 5.9127152781002223e-05, test loss is 6.44255051156506e-05, and learning rate i

8581th Epoch:
traing loss is 7.775699486956e-05, test loss is 6.597739411517978e-05, and learning rate is 0

8591th Epoch:
traing loss is 9.004350431496277e-05, test loss is 6.263083923840895e-05, and learning rate i

8601th Epoch:
traing loss is 0.00011000943050021306, test loss is 6.723396654706448e-05, and learning rate
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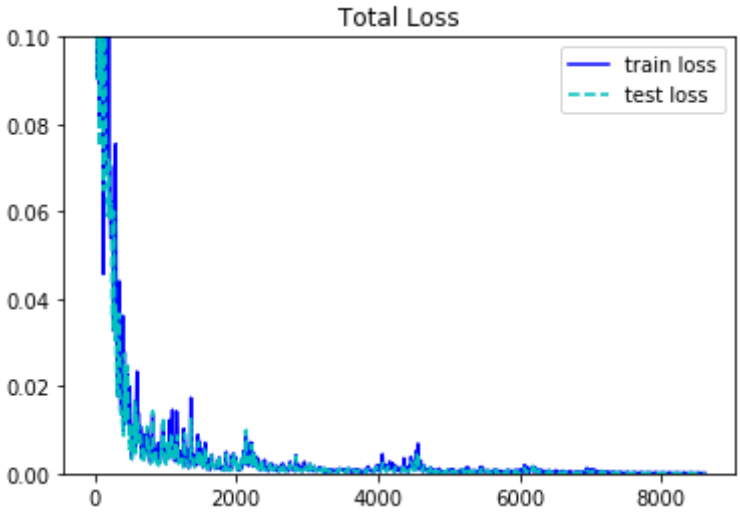

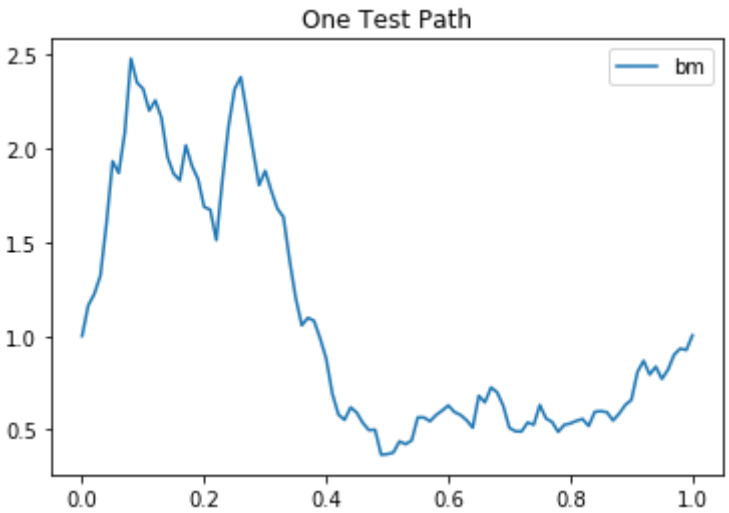

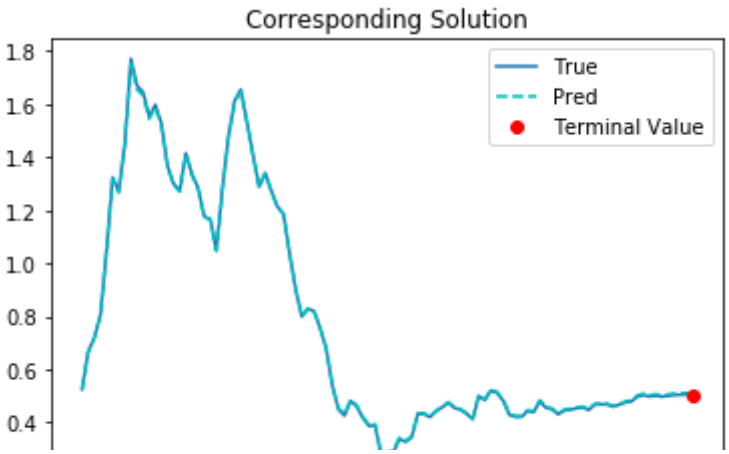

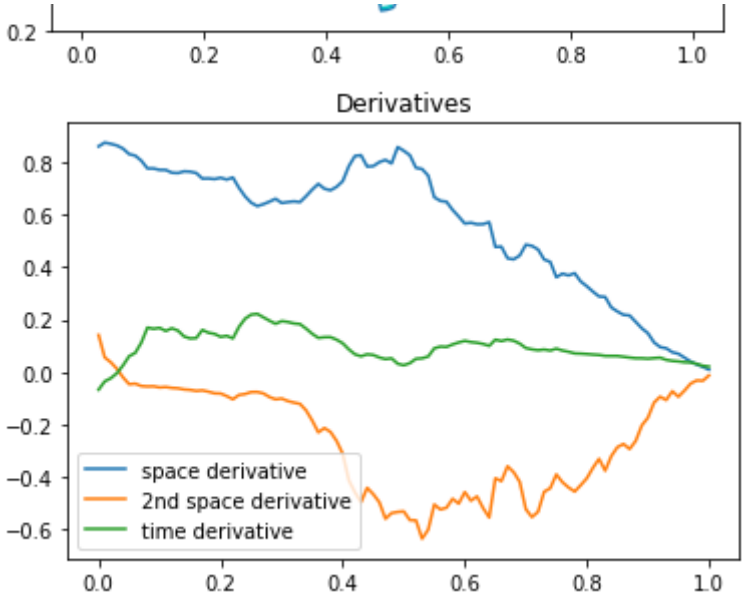


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8611th Epoch:
traing loss is 0.00021022804139647633, test loss is 0.00014359632041305304, and learning rate

8621th Epoch:
traing loss is 8.468054147670045e-05, test loss is 7.170917524490505e-05, and learning rate i

8631th Epoch:
traing loss is 5.6817661970853806e-05, test loss is 8.020860695978627e-05, and learning rate

8641th Epoch:
traing loss is 7.82420247560367e-05, test loss is 6.741559627698734e-05, and learning rate is

8651th Epoch:
traing loss is 5.4799020290374756e-05, test loss is 6.302235851762816e-05, and learning rate

8661th Epoch:
traing loss is 9.767695883056149e-05, test loss is 6.0722912166966125e-05, and learning rate

8671th Epoch:
traing loss is 5.344340024748817e-05, test loss is 5.953501022304408e-05, and learning rate i

8681th Epoch:
traing loss is 7.269612979143858e-05, test loss is 6.12556395935826e-05, and learning rate is

8691th Epoch:
traing loss is 8.550204802304506e-05, test loss is 6.0190555814187974e-05, and learning rate

8701th Epoch:
traing loss is 0.00010197205119766295, test loss is 6.58171993563883e-05, and learning rate i
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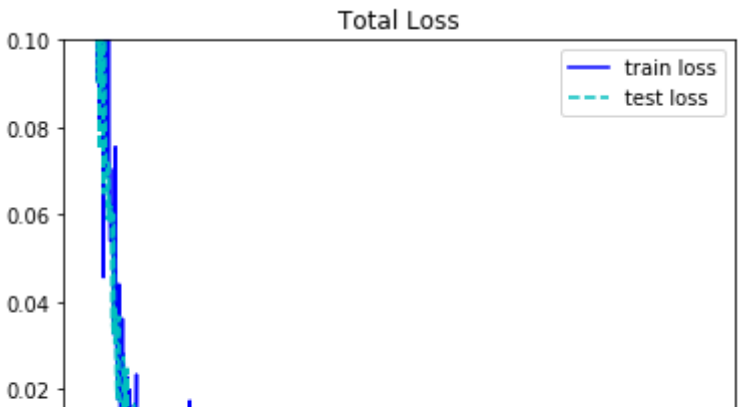

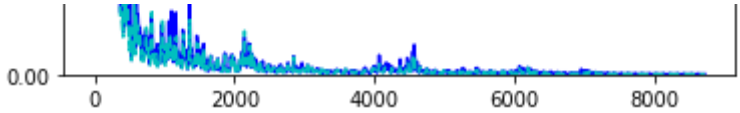

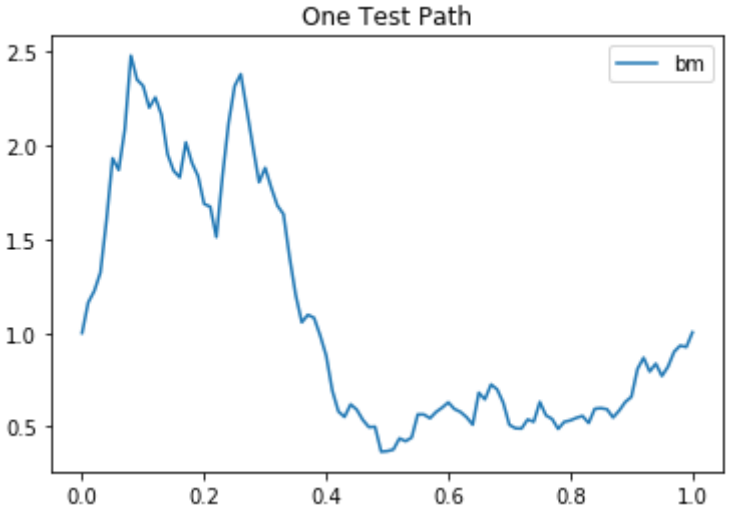

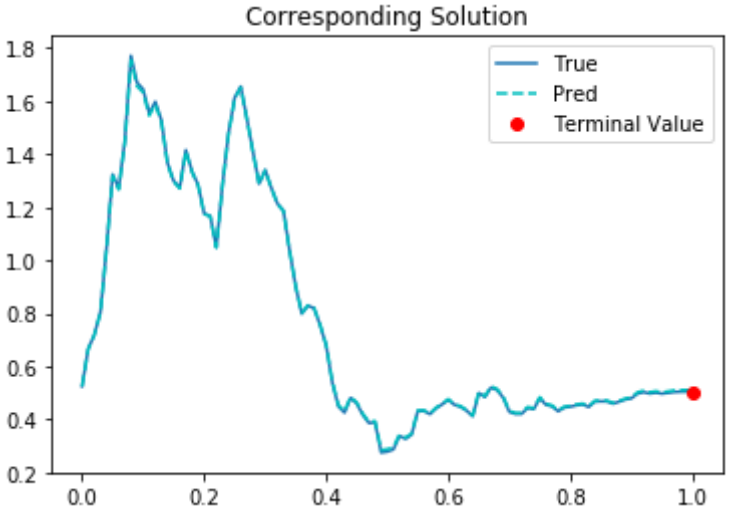

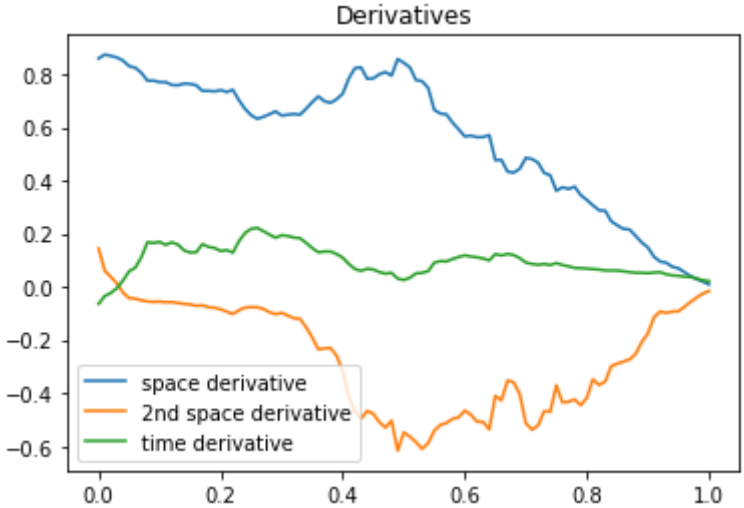

8711th Epoch:
traing loss is 0.00020645638869609684, test loss is 0.00014278781600296497, and learning rate

8721th Epoch:
traing loss is 8.405405969824642e-05, test loss is 6.98761796229519e-05, and learning rate is

8731th Epoch:
traing loss is 5.7203564210794866e-05, test loss is 7.943002128740773e-05, and learning rate

8741th Epoch:
traing loss is 7.343990728259087e-05, test loss is 7.157125946832821e-05, and learning rate i

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8751th Epoch:
traing loss is 5.5524695198982954e-05, test loss is 6.388029578374699e-05, and learning rate

8761th Epoch:
traing loss is 9.171897545456886e-05, test loss is 5.889042222406715e-05, and learning rate i

8771th Epoch:
traing loss is 5.9894551668548957e-05, test loss is 6.444958125939593e-05, and learning rate

8781th Epoch:
traing loss is 7.654885848751292e-05, test loss is 6.530975952045992e-05, and learning rate i

8791th Epoch:
traing loss is 8.758807234698907e-05, test loss is 6.199627387104556e-05, and learning rate i

8801th Epoch:
traing loss is 0.00010750396177172661, test loss is 6.3987827161327e-05, and learning rate is
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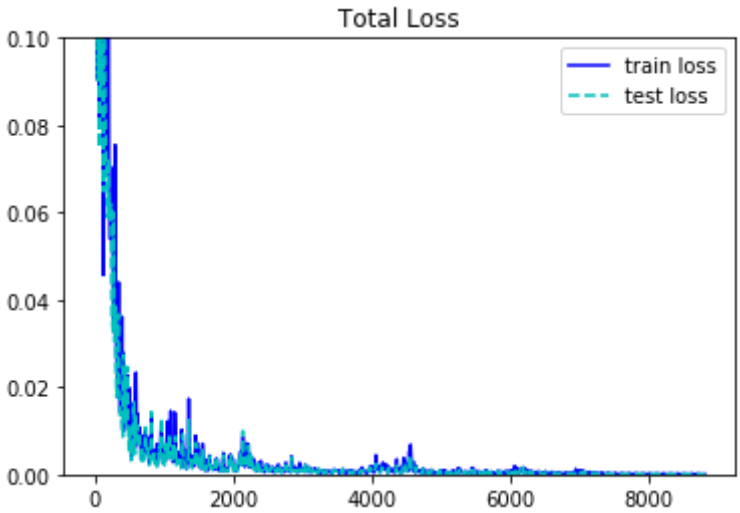

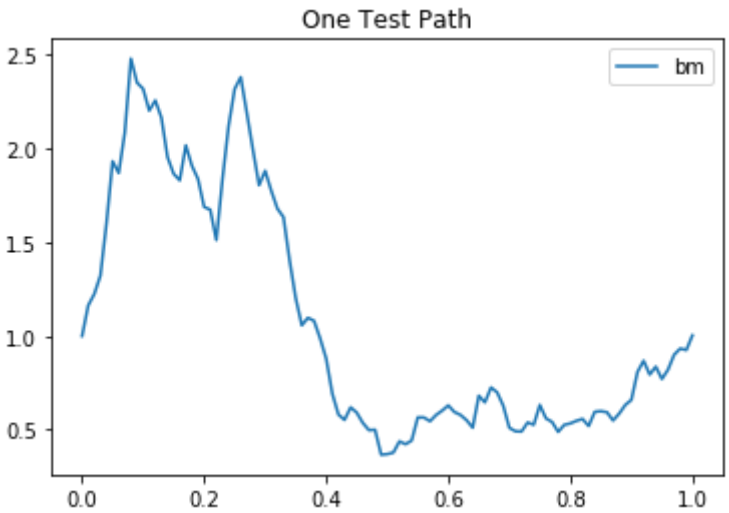

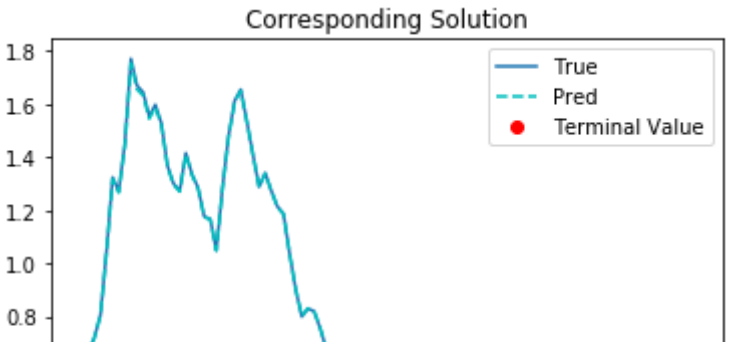

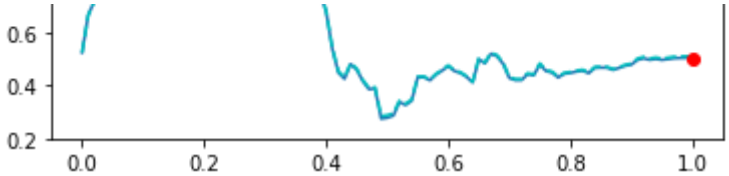

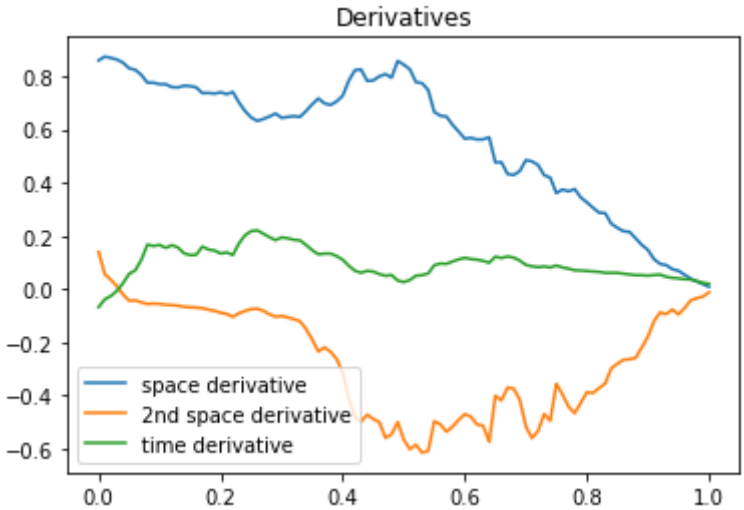


```
8811th Epoch:
traing loss is 0.00019734562374651432, test loss is 0.00013159906666260213, and learning rate

8821th Epoch:
traing loss is 7.577169162686914e-05, test loss is 6.469817890319973e-05, and learning rate i

8831th Epoch:
traing loss is 5.024289566790685e-05, test loss is 6.984989158809185e-05, and learning rate i

8841th Epoch:
traing loss is 8.237664587795734e-05, test loss is 6.505712372018024e-05, and learning rate i

8851th Epoch:
traing loss is 5.680464892066084e-05, test loss is 6.340417894534767e-05, and learning rate i

8861th Epoch:
traing loss is 0.00010176459909416735, test loss is 6.060425948817283e-05, and learning rate

8871th Epoch:
traing loss is 5.109237463329919e-05, test loss is 5.699438770534471e-05, and learning rate i

8881th Epoch:
traing loss is 7.219017425086349e-05, test loss is 5.874389535165392e-05, and learning rate i

8891th Epoch:
traing loss is 8.118804544210434e-05, test loss is 5.6476354075130075e-05, and learning rate

8901th Epoch:
traing loss is 9.663958189776167e-05, test loss is 6.111601396696642e-05, and learning rate i
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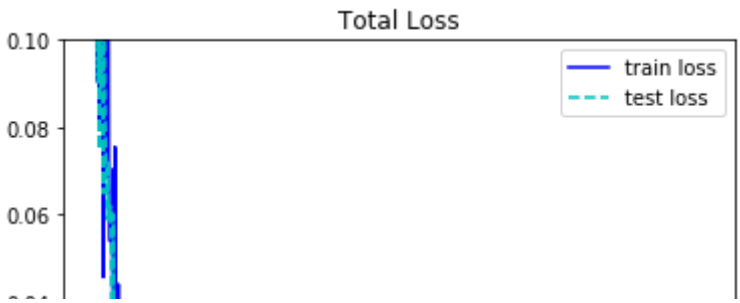

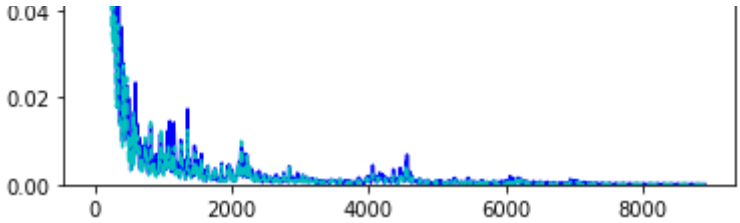

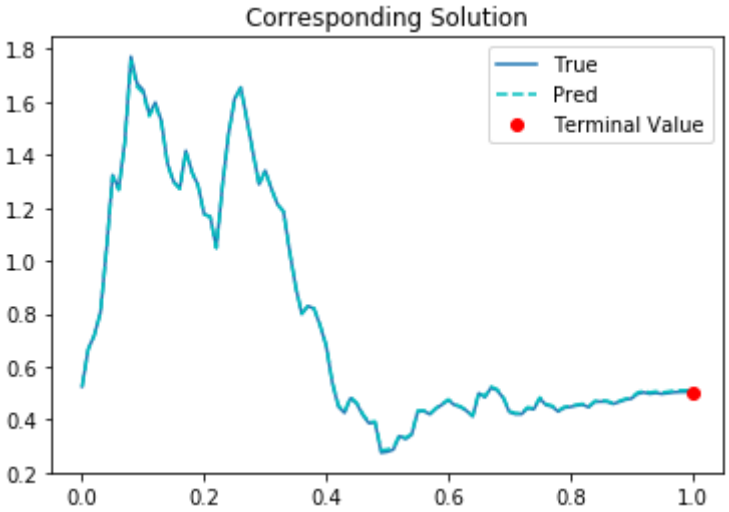

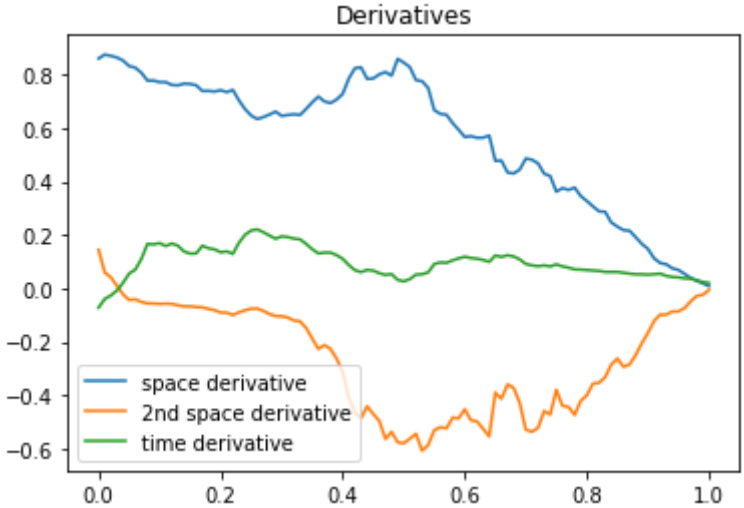


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8911th Epoch:
traing loss is 0.00019527060794644058, test loss is 0.0001364982163067907, and learning rate

8921th Epoch:
traing loss is 8.251488907262683e-05, test loss is 6.625366950174794e-05, and learning rate i

8931th Epoch:
traing loss is 5.144227179698646e-05, test loss is 7.102530071279034e-05, and learning rate i
```

```
8941th Epoch:
traing loss is 7.728138007223606e-05, test loss is 7.645935693290085e-05, and learning rate i

8951th Epoch:
traing loss is 6.033486715750769e-05, test loss is 6.658171332674101e-05, and learning rate i

8961th Epoch:
traing loss is 9.45327992667444e-05, test loss is 6.105928332544863e-05, and learning rate is

8971th Epoch:
traing loss is 6.463559839176014e-05, test loss is 6.904136535013095e-05, and learning rate i

8981th Epoch:
traing loss is 8.064262510742992e-05, test loss is 7.057467882987112e-05, and learning rate i

8991th Epoch:
traing loss is 9.406384924659505e-05, test loss is 6.798409594921395e-05, and learning rate i

9001th Epoch:
traing loss is 0.00012226747639942914, test loss is 6.160767225082964e-05, and learning rate
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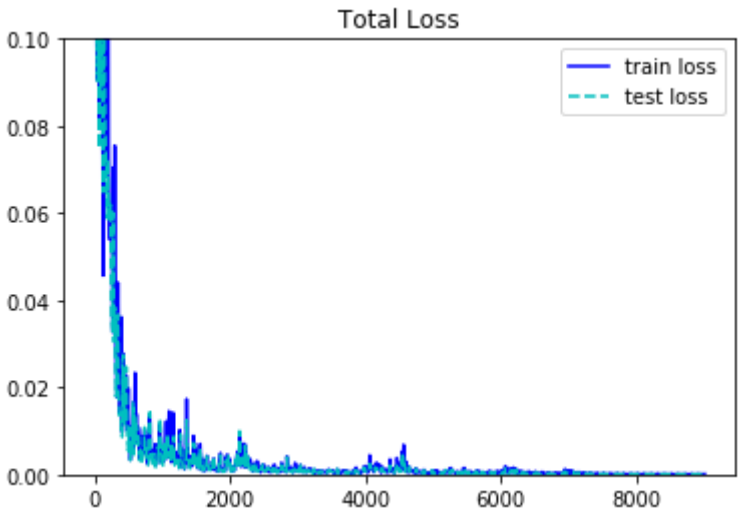

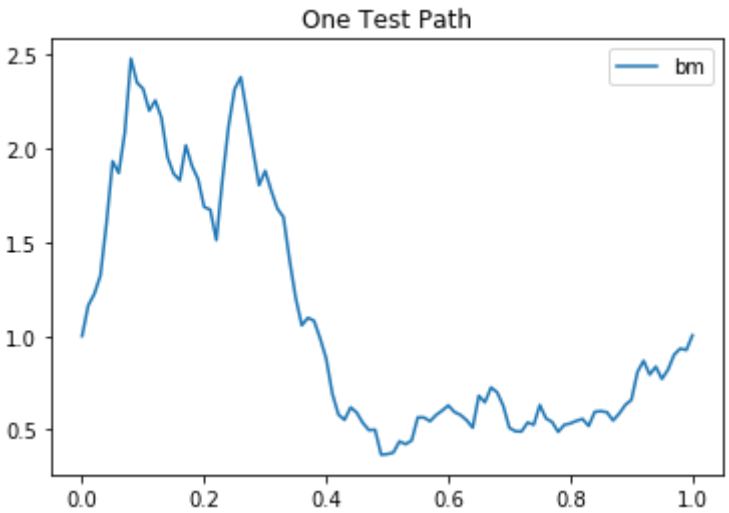

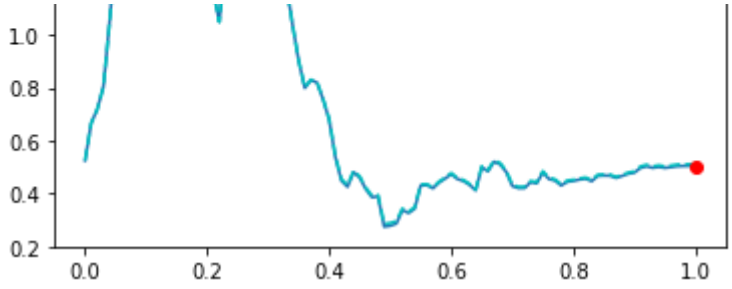

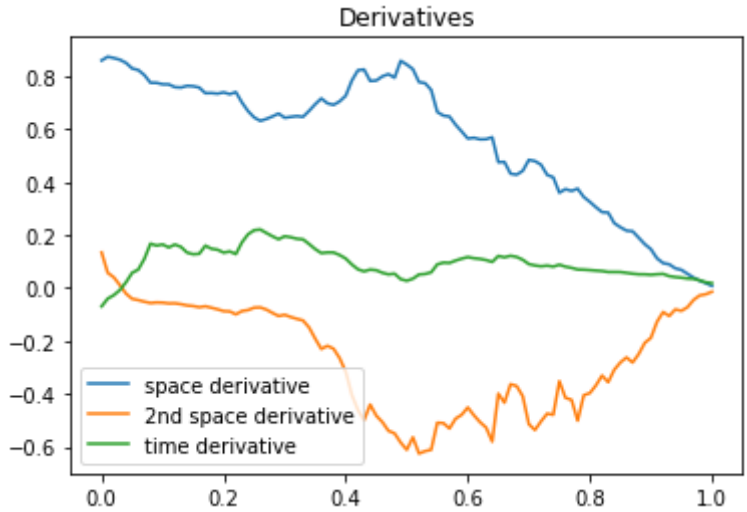


```
9011th Epoch:
traing loss is 0.00018366368021816015, test loss is 0.00011686496145557612, and learning rate

9021th Epoch:
traing loss is 7.074861059663817e-05, test loss is 6.298418884398416e-05, and learning rate i

9031th Epoch:
traing loss is 4.461496791918762e-05, test loss is 6.085784116294235e-05, and learning rate i

9041th Epoch:
traing loss is 0.00010796120477607474, test loss is 6.639125058427453e-05, and learning rate

9051th Epoch:
traing loss is 6.626315007451922e-05, test loss is 6.981602928135544e-05, and learning rate i

9061th Epoch:
traing loss is 0.00013312797818798572, test loss is 7.243218715302646e-05, and learning rate

9071th Epoch:
traing loss is 5.545741805690341e-05, test loss is 6.12786752753891e-05, and learning rate is

9081th Epoch:
traing loss is 7.635199290234596e-05, test loss is 6.089600719860755e-05, and learning rate i

9091th Epoch:
traing loss is 7.890071719884872e-05, test loss is 5.35739709448535e-05, and learning rate is

9101th Epoch:
traing loss is 0.00011982617434114218, test loss is 5.9517682529985905e-05, and learning rate
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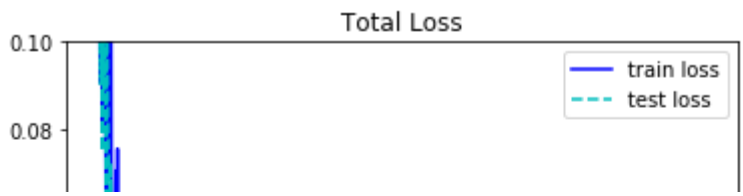

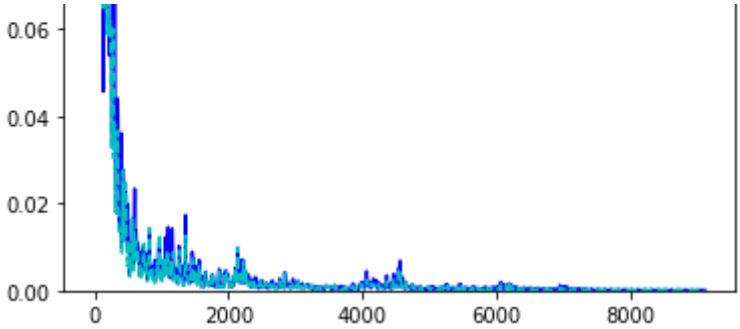

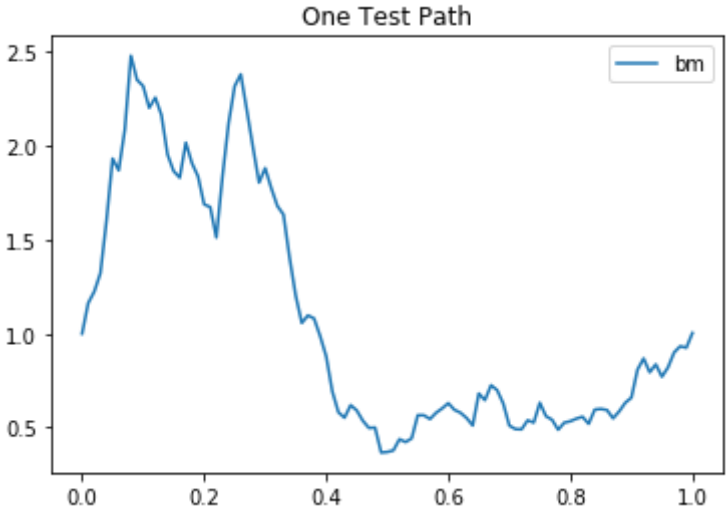

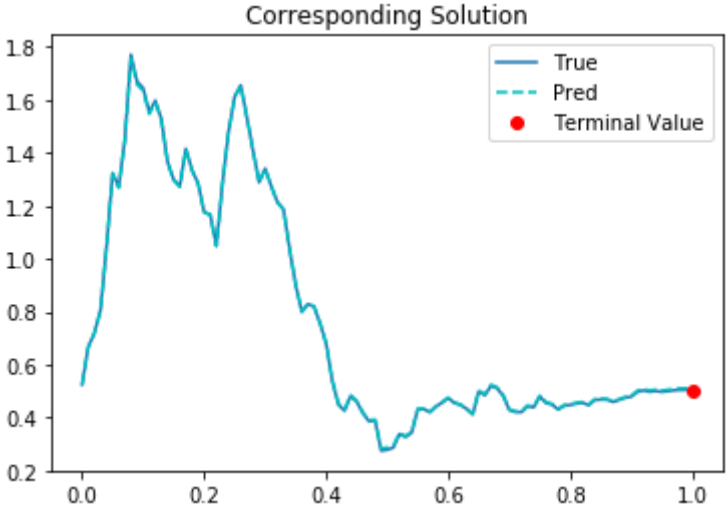

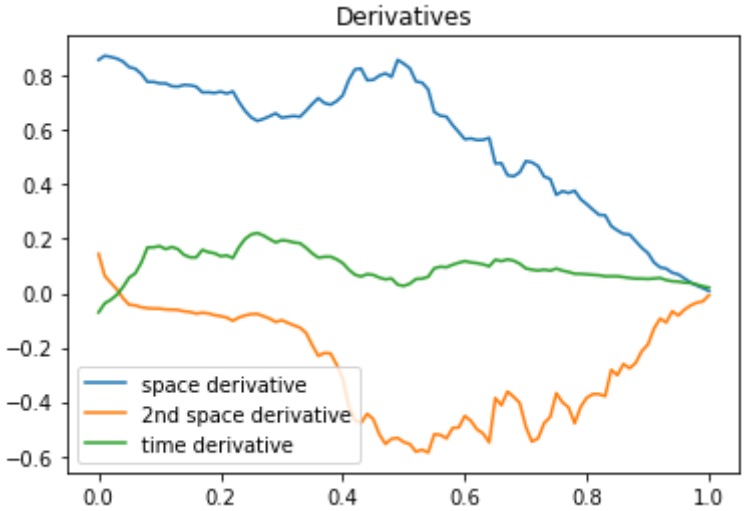


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9111th Epoch:
traing loss is 0.00019165209960192442, test loss is 0.00013124353426974267, and learning rate

9121th Epoch:
traing loss is 8.386358240386471e-05, test loss is 6.489768566098064e-05, and learning rate i
```

```
9131th Epoch:
traing loss is 4.9369984481018037e-05, test loss is 6.695442425552756e-05, and learning rate

9141th Epoch:
traing loss is 0.0001139998494181782, test loss is 9.609958215150982e-05, and learning rate i

9151th Epoch:
traing loss is 7.981538510648534e-05, test loss is 8.135262760333717e-05, and learning rate i

9161th Epoch:
traing loss is 0.00012849211634602398, test loss is 7.720461144344881e-05, and learning rate

9171th Epoch:
traing loss is 8.916721708374098e-05, test loss is 9.442887676414102e-05, and learning rate i

9181th Epoch:
traing loss is 0.00010128066060133278, test loss is 9.083033364731818e-05, and learning rate

9191th Epoch:
traing loss is 0.00012035098916385323, test loss is 9.238219354301691e-05, and learning rate

9201th Epoch:
traing loss is 0.00018794708012137562, test loss is 6.342864799080417e-05, and learning rate
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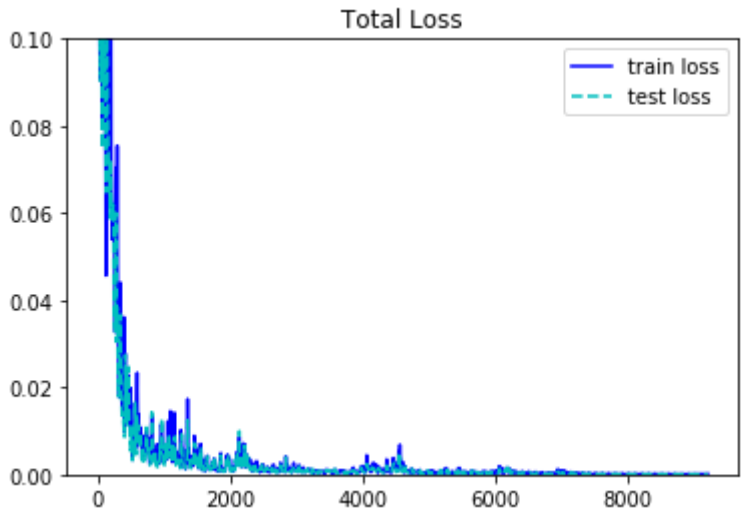

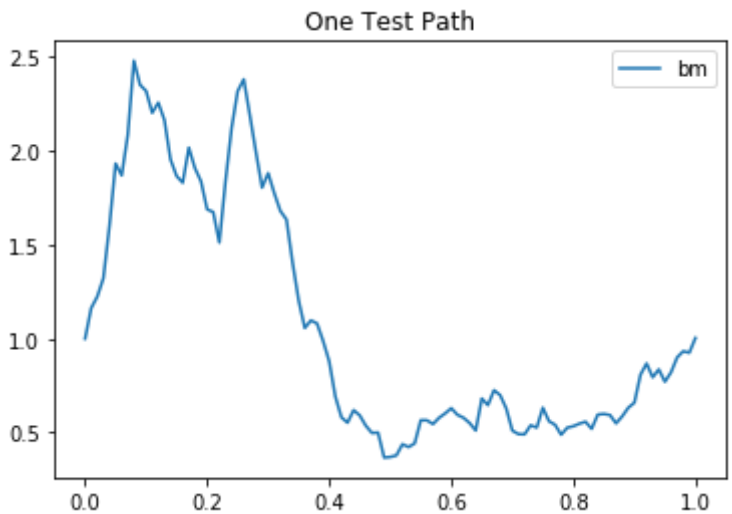

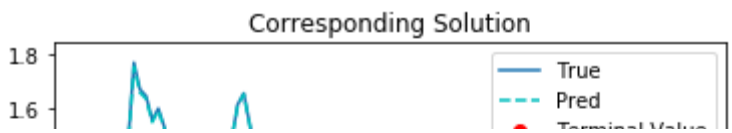

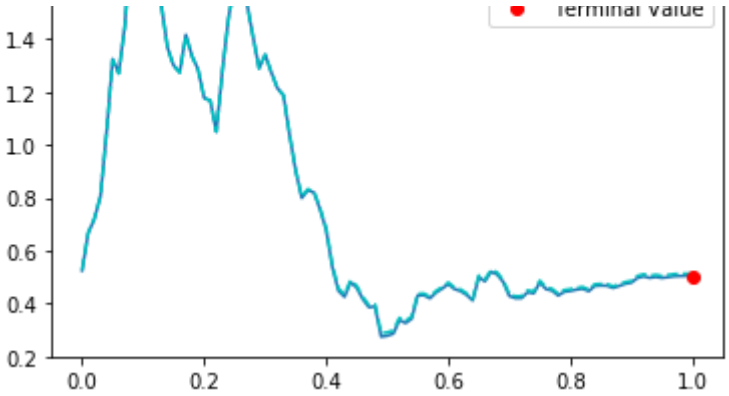

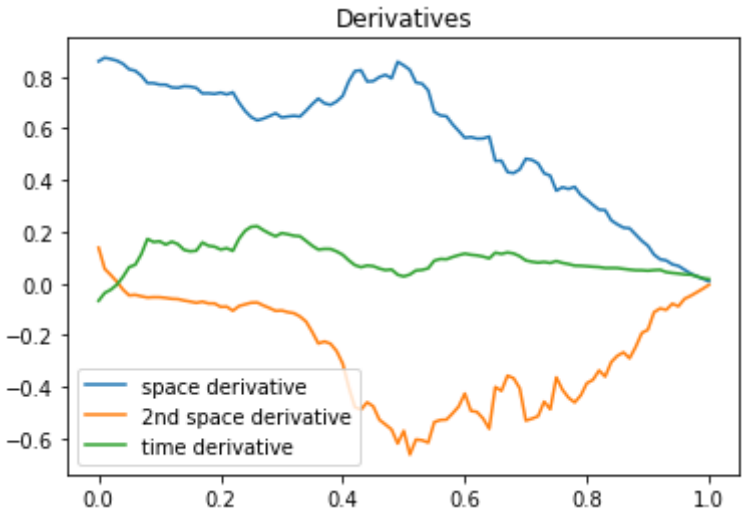


```
9211th Epoch:
traing loss is 0.00017918481898959726, test loss is 9.189919364871457e-05, and learning rate

9221th Epoch:
traing loss is 7.495830504922196e-05, test loss is 7.214760989882052e-05, and learning rate i

9231th Epoch:
traing loss is 5.108527693664655e-05, test loss is 6.511033279821277e-05, and learning rate i

9241th Epoch:
traing loss is 0.0001788178487913683, test loss is 8.869844168657437e-05, and learning rate i

9251th Epoch:
traing loss is 9.074151603272185e-05, test loss is 8.602198795415461e-05, and learning rate i

9261th Epoch:
traing loss is 0.00020914251217618585, test loss is 0.00010453449067426845, and learning rate

9271th Epoch:
traing loss is 7.510191790061072e-05, test loss is 7.996855129022151e-05, and learning rate i

9281th Epoch:
traing loss is 9.592676360625774e-05, test loss is 7.312616799026728e-05, and learning rate i

9291th Epoch:
traing loss is 8.673820411786437e-05, test loss is 5.58105566597078e-05, and learning rate is

9301th Epoch:
traing loss is 0.00021478203416336328, test loss is 6.000735083944164e-05, and learning rate
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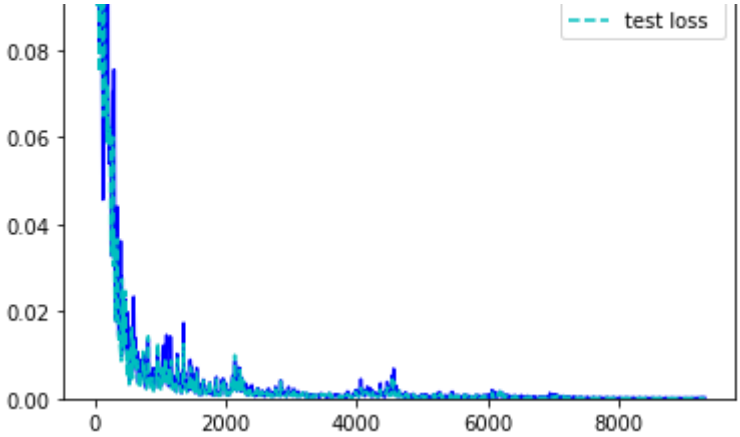

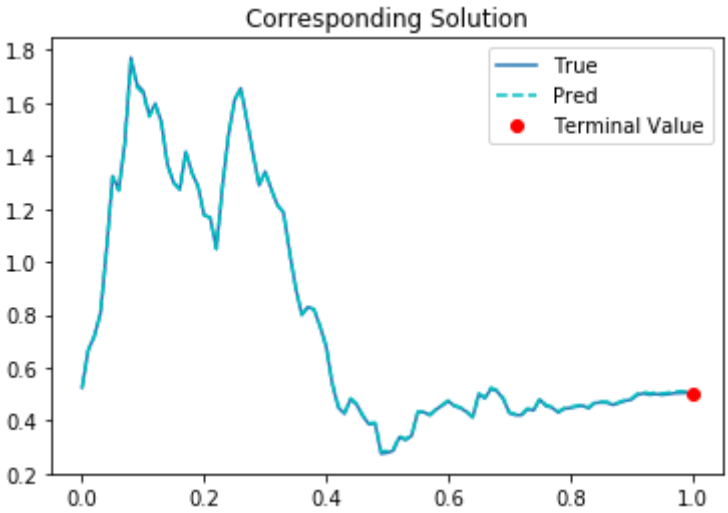

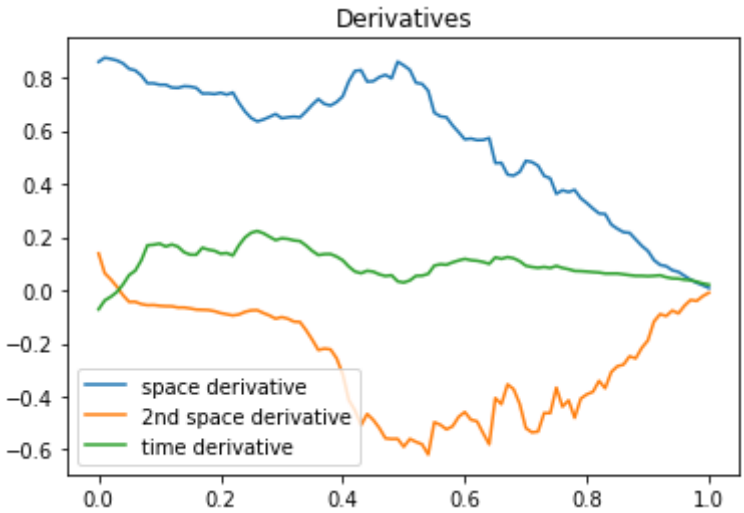

9311th Epoch:
traing loss is 0.00019878936291206628, test loss is 0.00012854310625698417, and learning rate

```
9321th Epoch:
traing loss is 8.8506392785348e-05, test loss is 6.756663788110018e-05, and learning rate is

9331th Epoch:
traing loss is 5.1959988923044875e-05, test loss is 6.58773205941543e-05, and learning rate i

9341th Epoch:
traing loss is 0.0001740598672768101, test loss is 0.00011758386244764552, and learning rate

9351th Epoch:
traing loss is 9.470792429056019e-05, test loss is 9.311077155871317e-05, and learning rate i

9361th Epoch:
traing loss is 0.00016758327546995133, test loss is 0.00010423209459986538, and learning rate

9371th Epoch:
traing loss is 0.00010927733092103153, test loss is 0.00011616700794547796, and learning rate

9381th Epoch:
traing loss is 0.00011900014214916155, test loss is 0.00010446363739902154, and learning rate

9391th Epoch:
traing loss is 0.00014037299843039364, test loss is 0.00011361193901393563, and learning rate

9401th Epoch:
traing loss is 0.00022186993737705052, test loss is 7.056328468024731e-05, and learning rate
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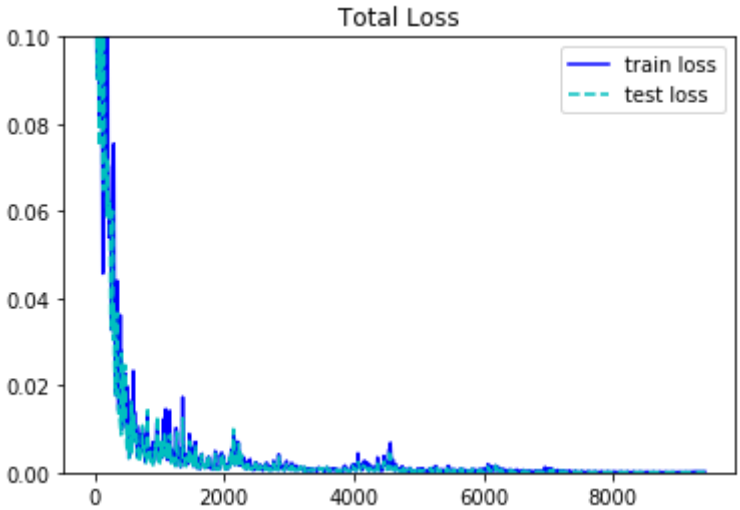

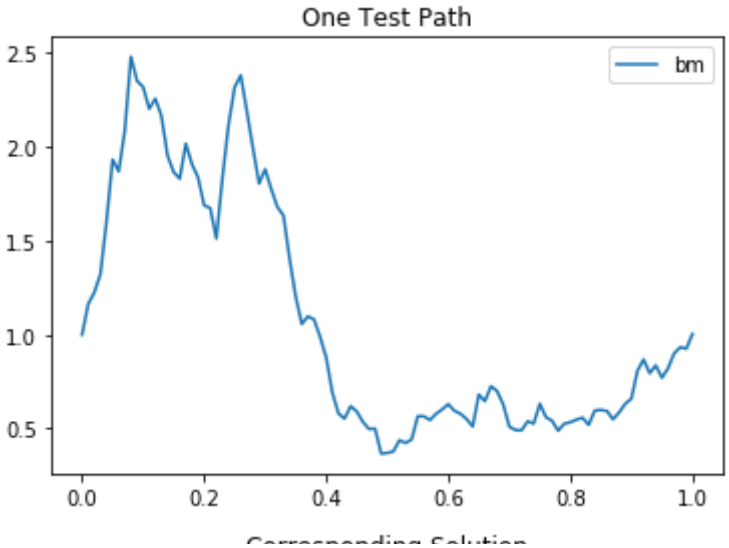

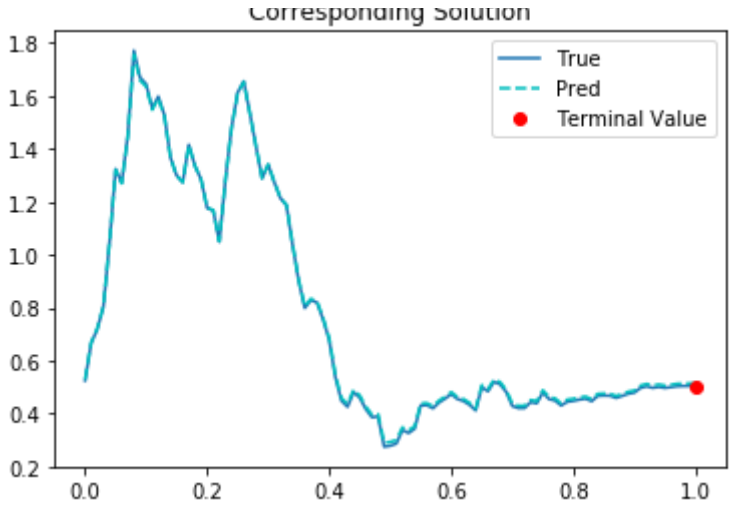

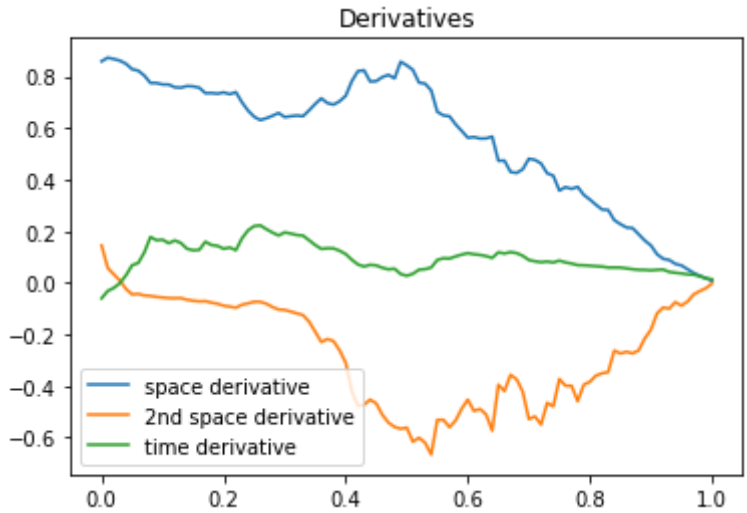


```
9411th Epoch:
traing loss is 0.0001716797414701432, test loss is 7.688317418796942e-05, and learning rate i

9421th Epoch:
traing loss is 7.324977923417464e-05, test loss is 7.146754796849564e-05, and learning rate i

9431th Epoch:
traing loss is 5.555665848078206e-05, test loss is 6.963428313611075e-05, and learning rate i

9441th Epoch:
traing loss is 0.00016460896586067975, test loss is 8.858425280777737e-05, and learning rate

9451th Epoch:
traing loss is 8.981689461506903e-05, test loss is 8.554278610972688e-05, and learning rate i

9461th Epoch:
traing loss is 0.00019128287385683507, test loss is 9.518981096334755e-05, and learning rate

9471th Epoch:
traing loss is 7.164069393184036e-05, test loss is 7.921568612800911e-05, and learning rate i

9481th Epoch:
traing loss is 8.971769420895725e-05, test loss is 7.169286254793406e-05, and learning rate i

9491th Epoch:
traing loss is 8.451492612948641e-05, test loss is 5.374467218643986e-05, and learning rate i

9501th Epoch:
traing loss is 0.00021239904162939638, test loss is 6.118875171523541e-05, and learning rate
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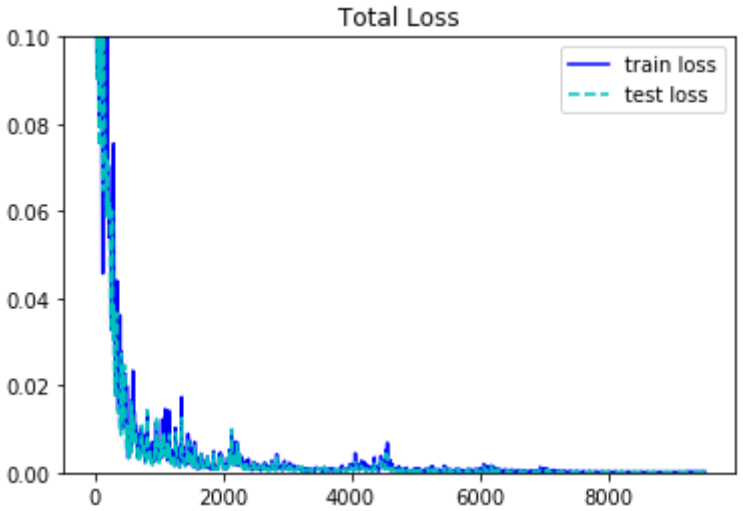
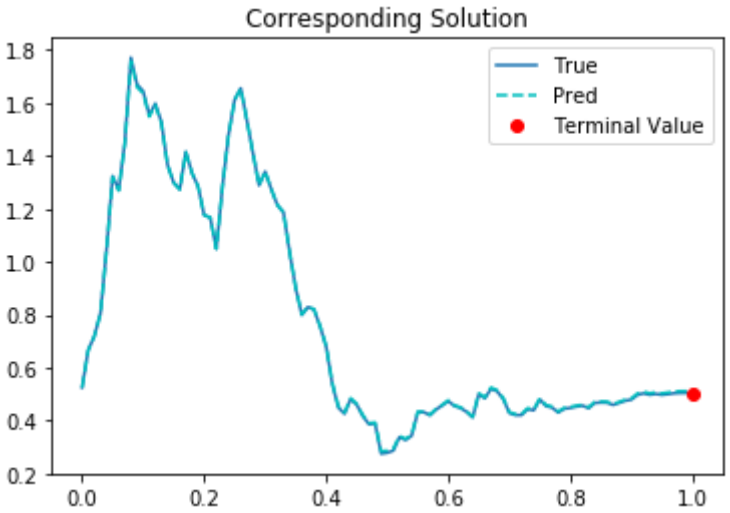
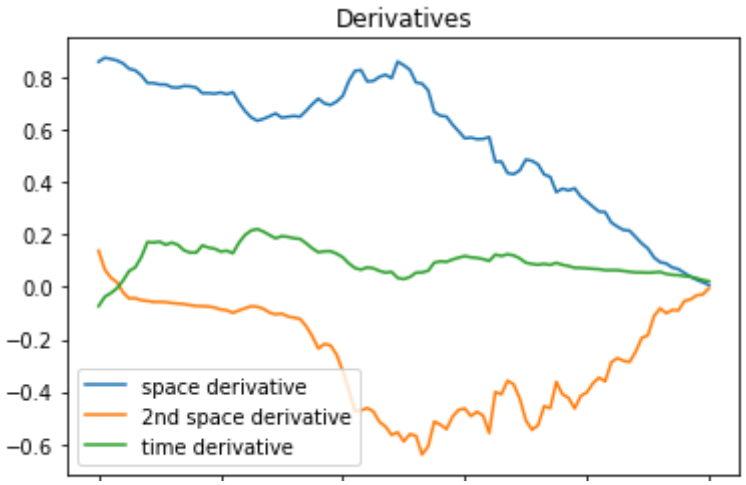

Total Loss
train loss
test loss
0.10
0.08
0.06
0.04
0.02
0.00
0
2000
4000
6000
8000

One Test Path
bm
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0


Corresponding Solution
True
Pred
Terminal Value
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


Derivatives
space derivative
2nd space derivative
time derivative
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6

```
0.0        0.2        0.4        0.6        0.8        1.0
9511th Epoch:
traing loss is 0.0001859287149272859, test loss is 0.0001129294978454709, and learning rate i

9521th Epoch:
traing loss is 8.916531805880368e-05, test loss is 7.090827421052381e-05, and learning rate i

9531th Epoch:
traing loss is 5.5725424317643046e-05, test loss is 6.840449350420386e-05, and learning rate

9541th Epoch:
traing loss is 0.00012886626063846052, test loss is 9.722851245896891e-05, and learning rate

9551th Epoch:
traing loss is 6.415443203877658e-05, test loss is 6.973154086153954e-05, and learning rate i

9561th Epoch:
traing loss is 0.00013387018407229334, test loss is 9.403901640325785e-05, and learning rate

9571th Epoch:
traing loss is 7.17778384569101e-05, test loss is 8.057167724473402e-05, and learning rate is

9581th Epoch:
traing loss is 8.174843969754875e-05, test loss is 6.884767935844138e-05, and learning rate i

9591th Epoch:
traing loss is 9.722275717649609e-05, test loss is 7.589317829115316e-05, and learning rate i

9601th Epoch:
traing loss is 0.0001390441757394001, test loss is 5.864326885784976e-05, and learning rate i
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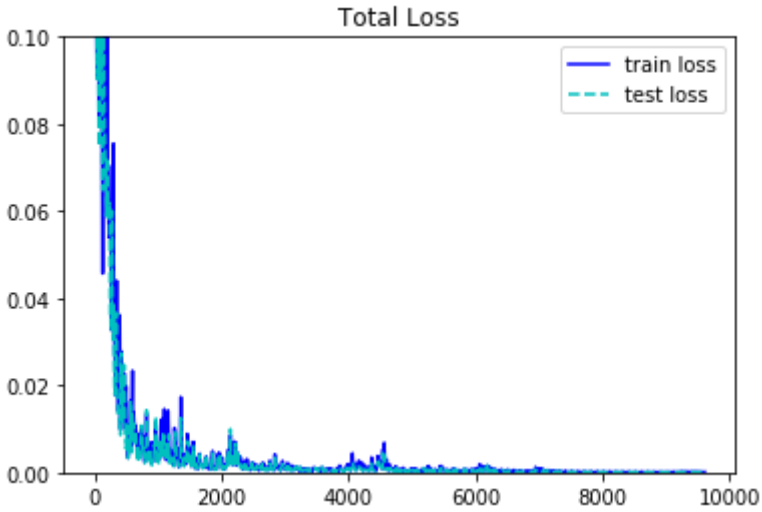

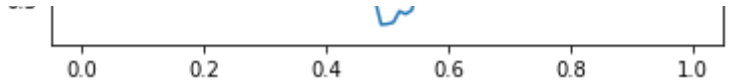

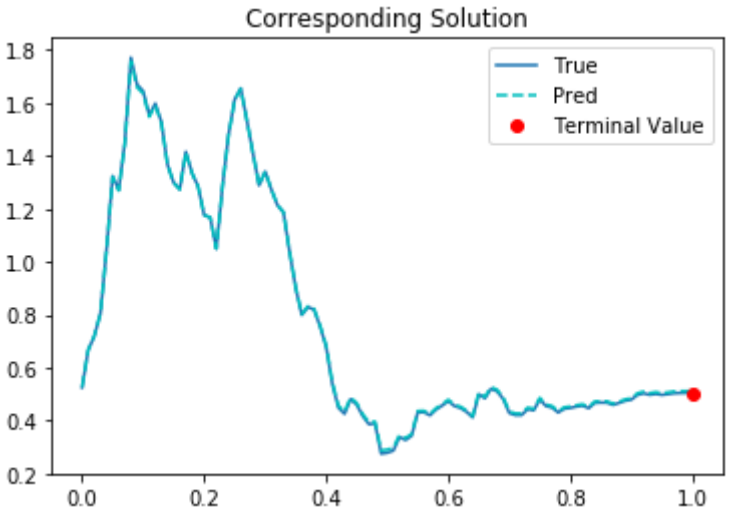

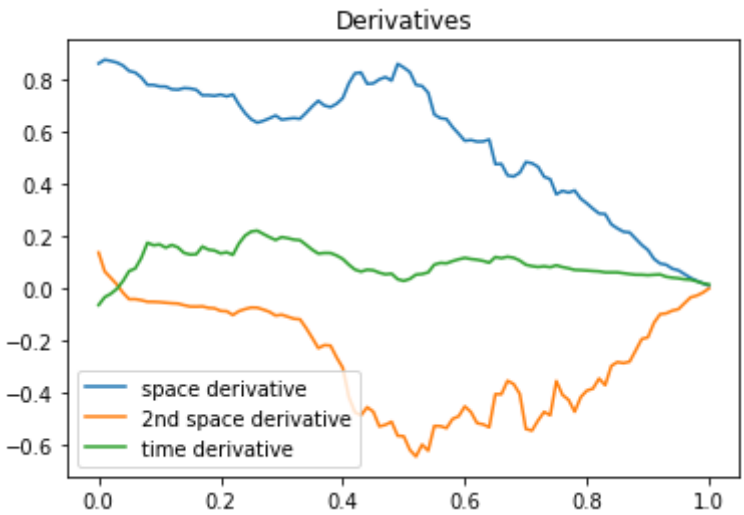


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9611th Epoch:
traing loss is 0.0001541158853797242, test loss is 8.01584028522484e-05, and learning rate is

9621th Epoch:
traing loss is 7.553549949079752e-05, test loss is 7.460507185896859e-05, and learning rate i

9631th Epoch:
traing loss is 5.526704990188591e-05, test loss is 6.856150139356032e-05, and learning rate i

9641th Epoch:
traing loss is 0.00010811904212459922, test loss is 6.646272231591865e-05, and learning rate

9651th Epoch:
traing loss is 5.7831442973110825e-05, test loss is 6.081908941268921e-05, and learning rate

9661th Epoch:
traing loss is 0.00011677208385663107, test loss is 6.220640352694318e-05, and learning rate

9671th Epoch:
traing loss is 5.045412763138302e-05, test loss is 5.684424104401842e-05, and learning rate i

9681th Epoch:
traing loss is 7.07244107616134e-05, test loss is 5.85684429097455e-05, and learning rate is

9691th Epoch:
traing loss is 7.472982542822137e-05, test loss is 4.971845555701293e-05, and learning rate i
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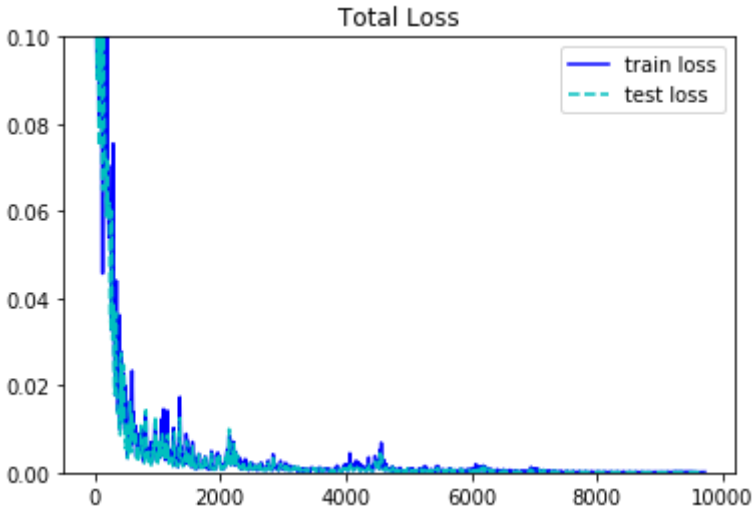
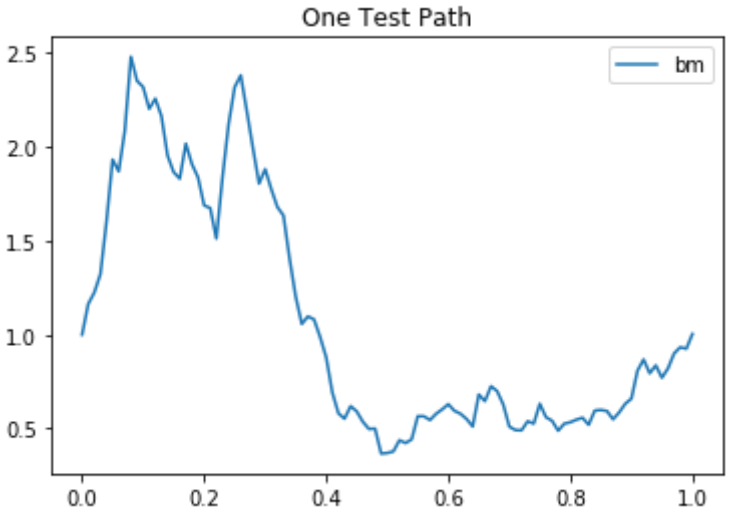
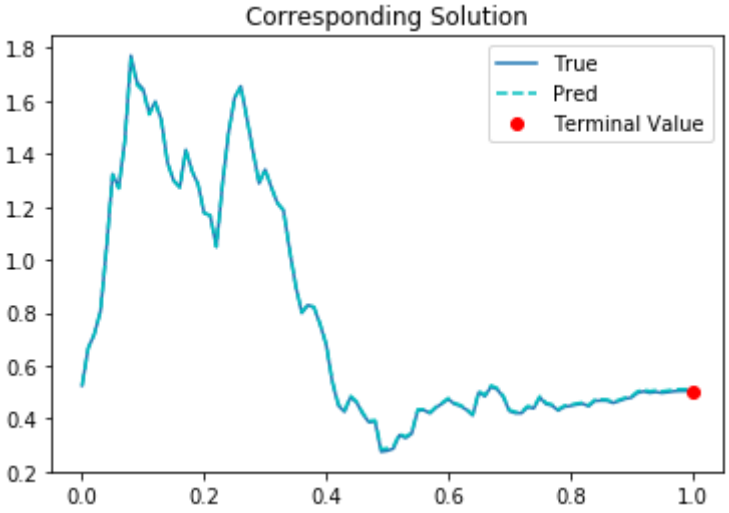
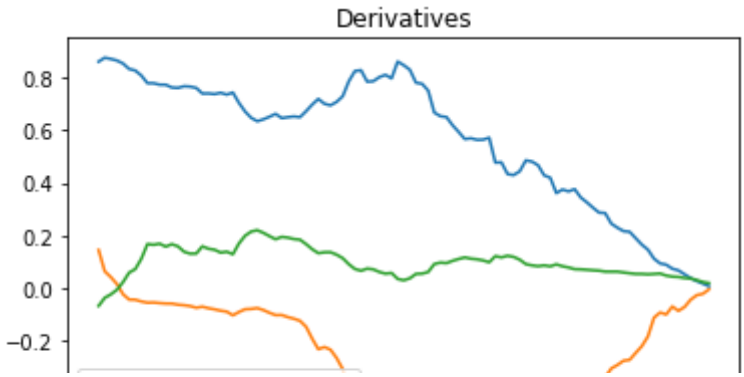
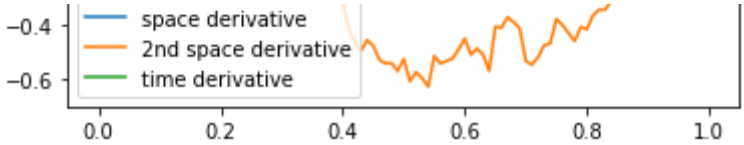
9701th Epoch:
traing loss is 0.0001313283428316936, test loss is 5.403676914284006e-05, and learning rate i









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9711th Epoch:
traing loss is 0.0001661917194724083, test loss is 9.53673297772184e-05, and learning rate is

9721th Epoch:
traing loss is 8.533694926882163e-05, test loss is 7.11709726601839e-05, and learning rate is

9731th Epoch:
traing loss is 5.238855010247789e-05, test loss is 6.50483270874247e-05, and learning rate is

9741th Epoch:
traing loss is 8.010426972759888e-05, test loss is 7.256718527060002e-05, and learning rate i

9751th Epoch:
traing loss is 4.4796441216021776e-05, test loss is 5.4583928431384265e-05, and learning rate

9761th Epoch:
traing loss is 0.00010040240158559754, test loss is 7.529178401455283e-05, and learning rate

9771th Epoch:
traing loss is 5.427723590401001e-05, test loss is 6.25626853434369e-05, and learning rate is

9781th Epoch:
traing loss is 6.581631896551698e-05, test loss is 5.69002513657324e-05, and learning rate is

9791th Epoch:
traing loss is 8.020008681342006e-05, test loss is 6.028383722878061e-05, and learning rate i

9801th Epoch:
traing loss is 9.869885252555832e-05, test loss is 5.398237408371642e-05, and learning rate i
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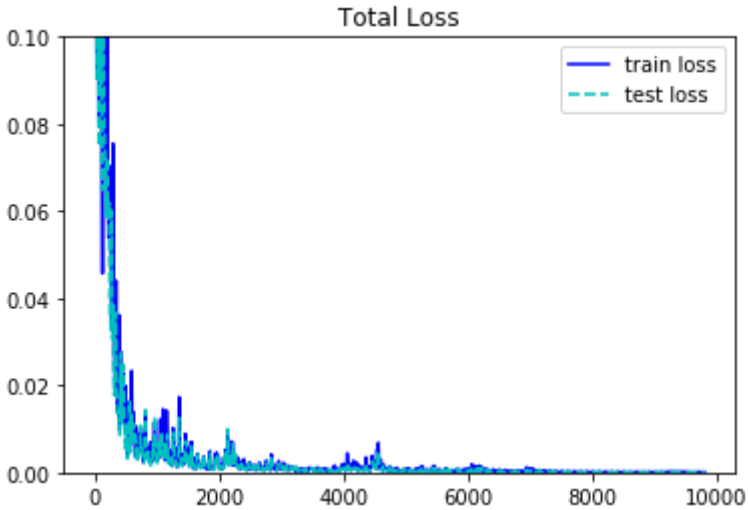

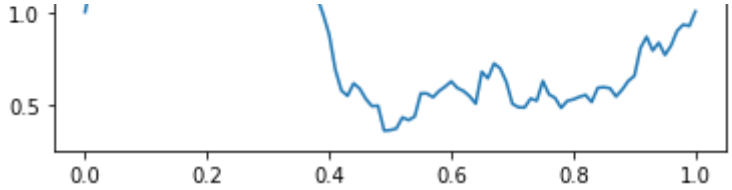

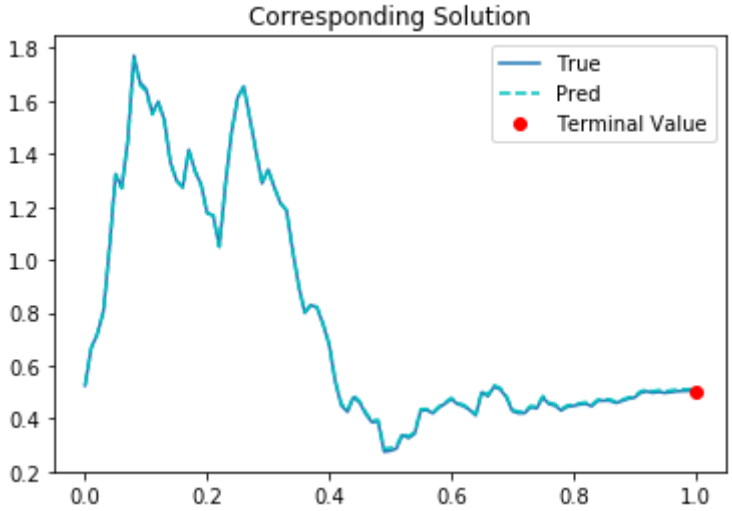

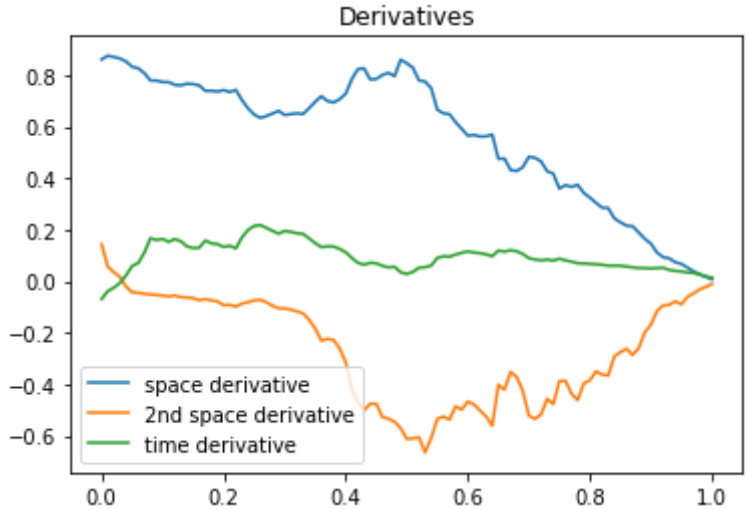


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9811th Epoch:
traing loss is 0.00014143859152682126, test loss is 7.640587136847898e-05, and learning rate

9821th Epoch:
traing loss is 7.609232852701098e-05, test loss is 7.07121507730335e-05, and learning rate is

9831th Epoch:
traing loss is 4.980119410902262e-05, test loss is 6.189751002239063e-05, and learning rate i

9841th Epoch:
traing loss is 8.429711306234822e-05, test loss is 5.779157436336391e-05, and learning rate i

9851th Epoch:
traing loss is 4.566803545458242e-05, test loss is 5.216551653575152e-05, and learning rate i

9861th Epoch:
traing loss is 8.995246753329411e-05, test loss is 5.286583473207429e-05, and learning rate i

9871th Epoch:
traing loss is 4.35138754255604e-05, test loss is 5.0232320063514635e-05, and learning rate i

9881th Epoch:
traing loss is 6.525249045807868e-05, test loss is 5.528856854652986e-05, and learning rate i
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```
9891th Epoch:
traing loss is 7.207448652479798e-05, test loss is 4.835284562432207e-05, and learning rate i

9901th Epoch:
traing loss is 9.580452024238184e-05, test loss is 5.080971823190339e-05, and learning rate i
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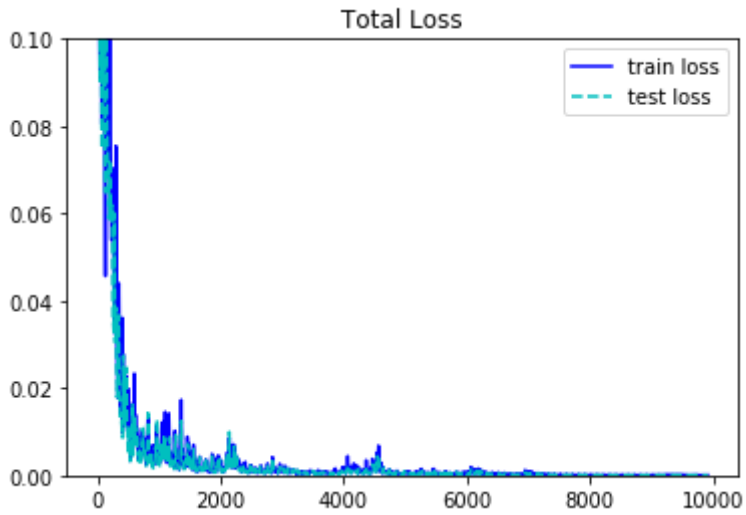

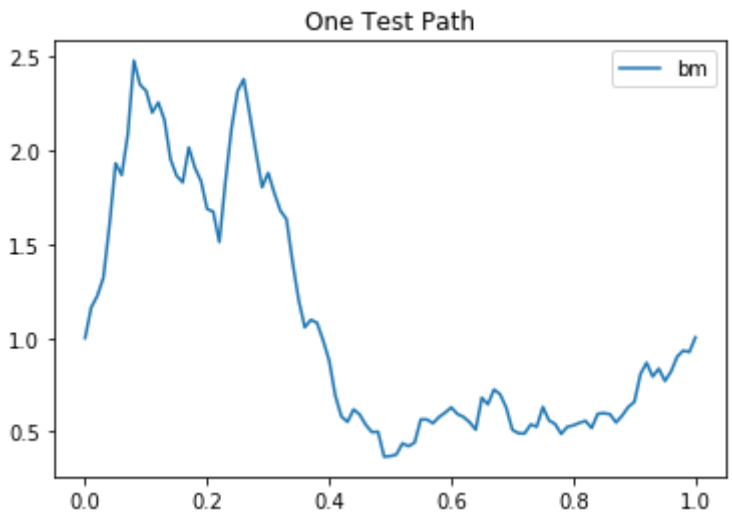

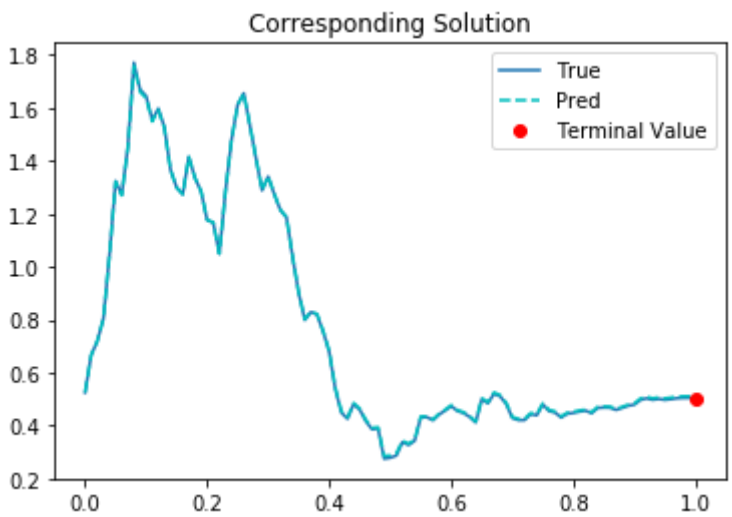

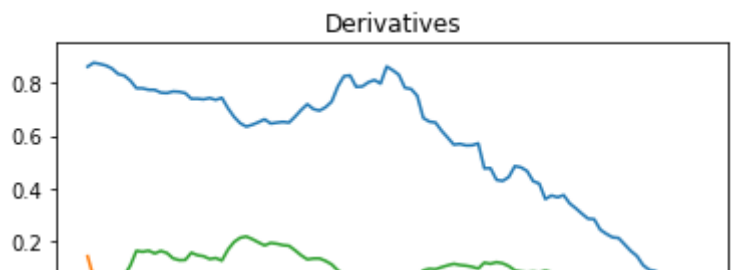

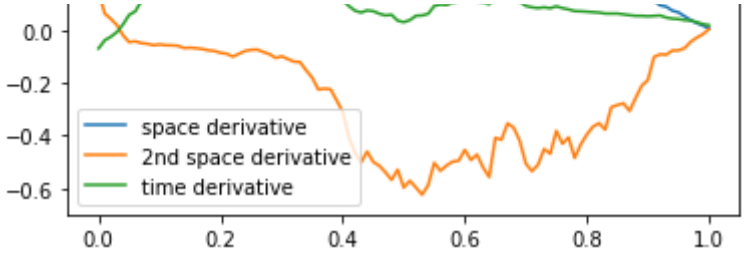


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9911th Epoch:
traing loss is 0.00014524796279147267, test loss is 8.318354957737029e-05, and learning rate

9921th Epoch:
traing loss is 8.006974530871958e-05, test loss is 7.009984983596951e-05, and learning rate i

9931th Epoch:
traing loss is 4.97693290526513e-05, test loss is 6.230081635294482e-05, and learning rate is

9941th Epoch:
traing loss is 6.246473640203476e-05, test loss is 6.114807911217213e-05, and learning rate i

9951th Epoch:
traing loss is 3.747391019714996e-05, test loss is 4.876363163930364e-05, and learning rate i

9961th Epoch:
traing loss is 8.560971764381975e-05, test loss is 6.163447687868029e-05, and learning rate i

9971th Epoch:
traing loss is 4.6357567043742165e-05, test loss is 5.438723383122124e-05, and learning rate

9981th Epoch:
traing loss is 6.199241033755243e-05, test loss is 5.3365441999631e-05, and learning rate is

9991th Epoch:
traing loss is 7.37561858841218e-05, test loss is 5.427379801403731e-05, and learning rate is

10001th Epoch:
traing loss is 8.541139686712995e-05, test loss is 5.210990275372751e-05, and learning rate i
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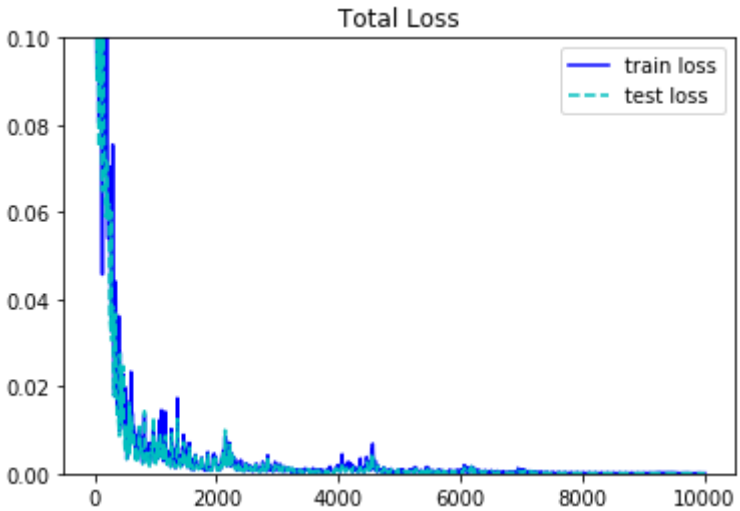

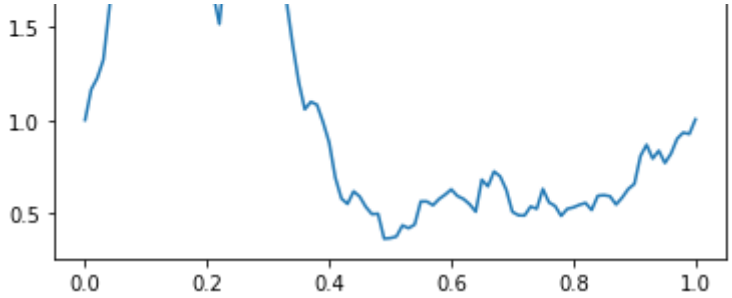

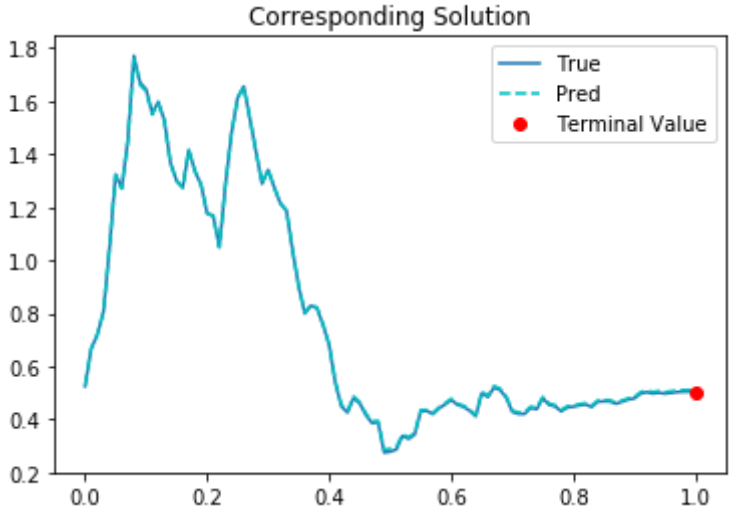

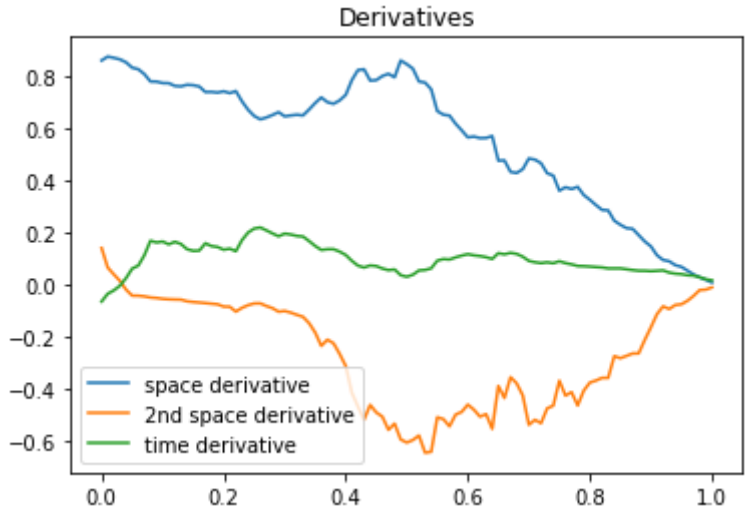


```
10011th Epoch:
traing loss is 0.00013366196071729064, test loss is 7.411180558847263e-05, and learning rate

10021th Epoch:
traing loss is 7.648839527973905e-05, test loss is 6.817581743234769e-05, and learning rate i

10031th Epoch:
traing loss is 4.675514719565399e-05, test loss is 5.884270649403334e-05, and learning rate i

10041th Epoch:
traing loss is 7.216705125756562e-05, test loss is 5.380938819143921e-05, and learning rate i

10051th Epoch:
traing loss is 4.1360966861248016e-05, test loss is 4.8556401452515274e-05, and learning rate

10061th Epoch:
traing loss is 8.125197928166017e-05, test loss is 4.9023710744222626e-05, and learning rate

10071th Epoch:
traing loss is 4.075212200405076e-05, test loss is 4.83333169540856e-05, and learning rate is
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```
10081th Epoch:
traing loss is 6.21360304648988e-05, test loss is 5.282504207571037e-05, and learning rate is

10091th Epoch:
traing loss is 6.92109897499904e-05, test loss is 4.838808308704756e-05, and learning rate is

10101th Epoch:
traing loss is 8.561693539377302e-05, test loss is 4.9018897698260844e-05, and learning rate
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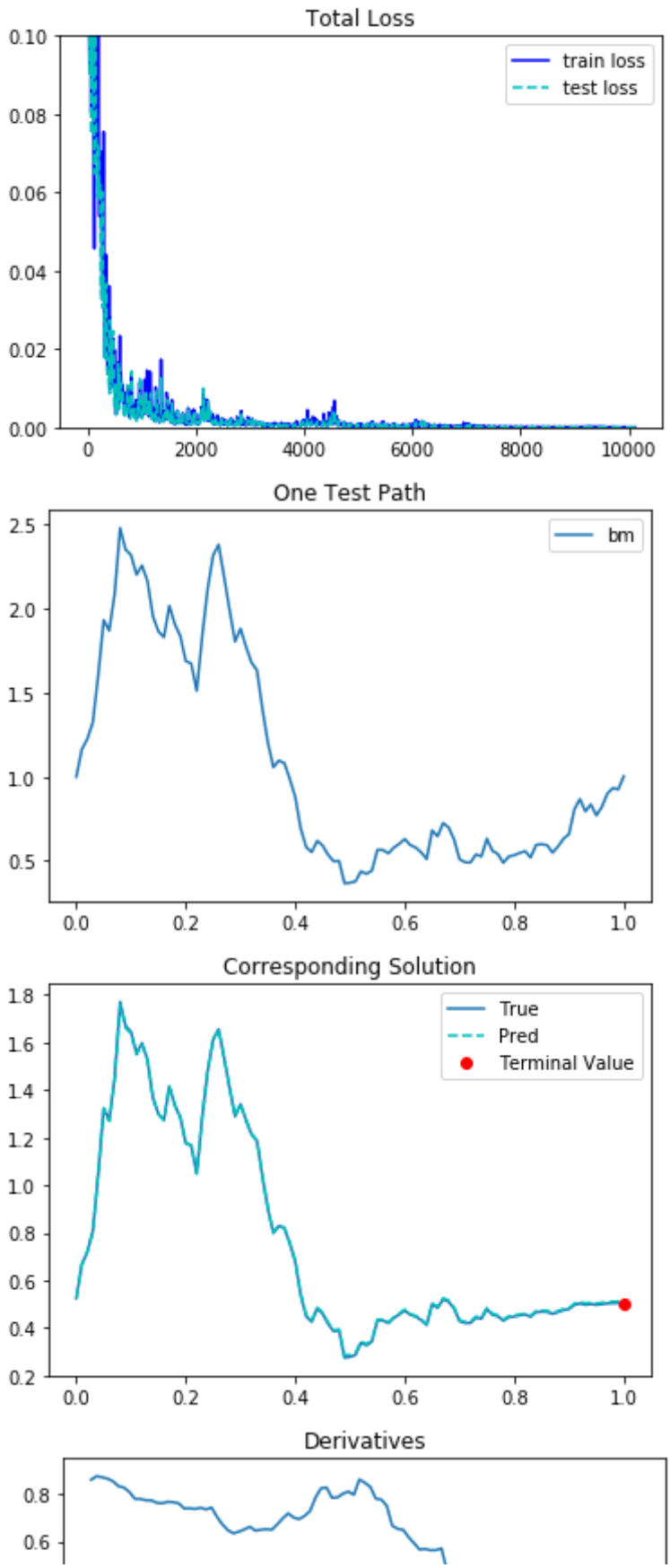

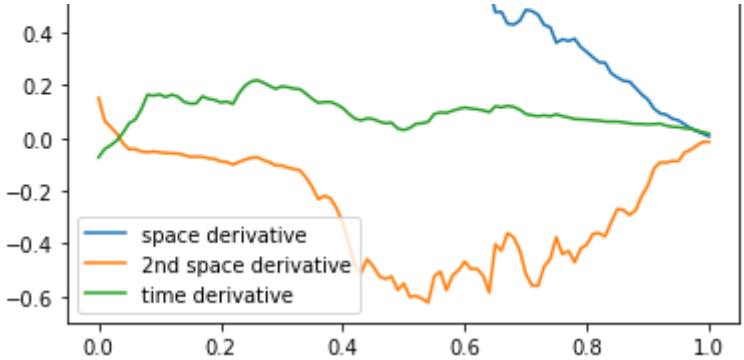


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10111th Epoch:
traing loss is 0.00013309552741702646, test loss is 7.669875776628032e-05, and learning rate

10121th Epoch:
traing loss is 8.134671952575445e-05, test loss is 6.803251744713634e-05, and learning rate i

10131th Epoch:
traing loss is 4.7669811465311795e-05, test loss is 5.911002881475724e-05, and learning rate

10141th Epoch:
traing loss is 5.630637315334752e-05, test loss is 5.47590825590305e-05, and learning rate is

10151th Epoch:
traing loss is 3.582471254048869e-05, test loss is 4.632871423382312e-05, and learning rate i

10161th Epoch:
traing loss is 7.65697768656537e-05, test loss is 5.5406308092642576e-05, and learning rate i

10171th Epoch:
traing loss is 4.372627881821245e-05, test loss is 5.1686791266547516e-05, and learning rate

10181th Epoch:
traing loss is 5.965422678855248e-05, test loss is 5.097219764138572e-05, and learning rate i

10191th Epoch:
traing loss is 7.141205423977226e-05, test loss is 5.200915984460153e-05, and learning rate i

10201th Epoch:
traing loss is 8.251715189544484e-05, test loss is 5.022913683205843e-05, and learning rate i
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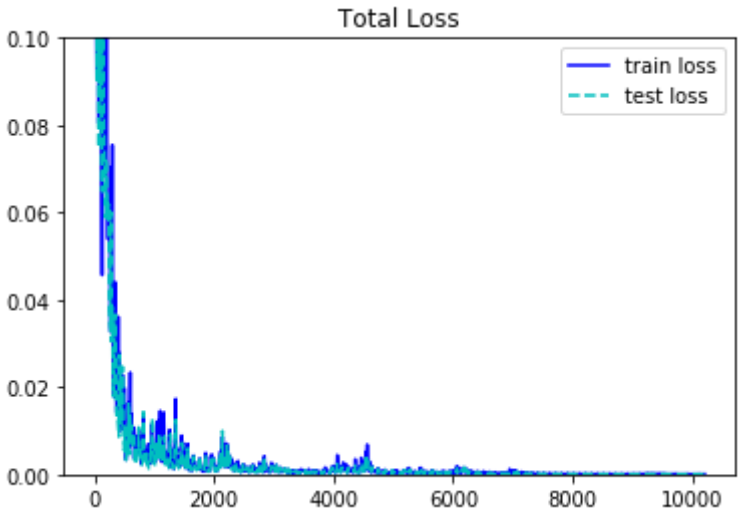

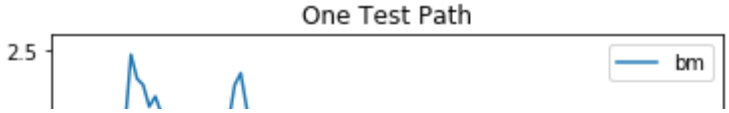

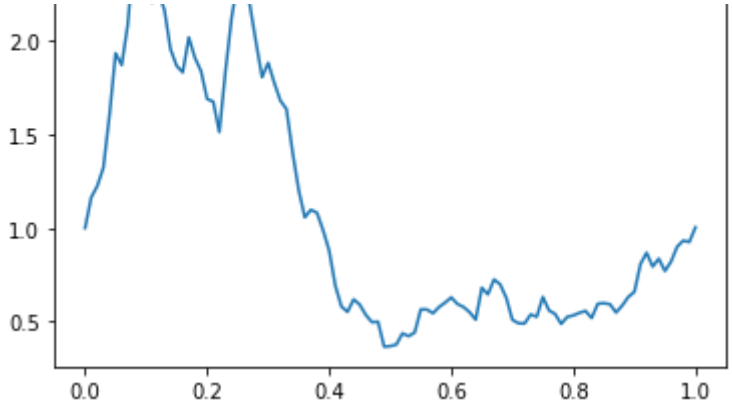

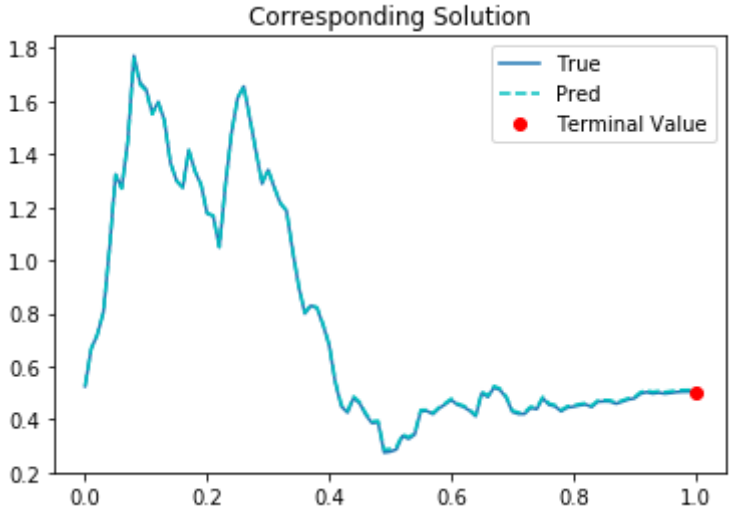

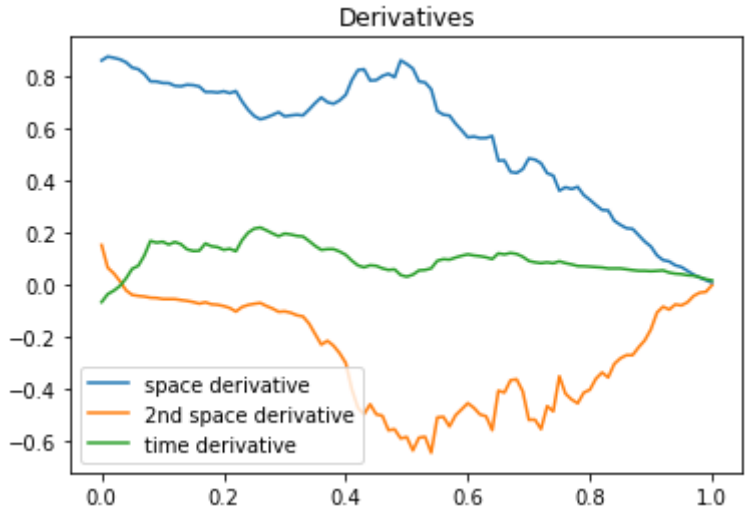


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10211th Epoch:
traing loss is 0.0001275902905035764, test loss is 7.00667078490369e-05, and learning rate is

10221th Epoch:
traing loss is 7.478066254407167e-05, test loss is 6.465874321293086e-05, and learning rate i

10231th Epoch:
traing loss is 4.50624902441632e-05, test loss is 5.683148265234195e-05, and learning rate is

10241th Epoch:
traing loss is 6.48119457764551e-05, test loss is 5.011143002775498e-05, and learning rate is

10251th Epoch:
traing loss is 3.896116322721355e-05, test loss is 4.7196310333674774e-05, and learning rate

10261th Epoch:
traing loss is 7.604843995068222e-05, test loss is 4.817179433302954e-05, and learning rate i
```

```
10271th Epoch:
traing loss is 3.995134466094896e-05, test loss is 4.684716259362176e-05, and learning rate i

10281th Epoch:
traing loss is 6.092208423069678e-05, test loss is 5.192303069634363e-05, and learning rate i

10291th Epoch:
traing loss is 6.72683963784948e-05, test loss is 4.6877579734427854e-05, and learning rate i

10301th Epoch:
traing loss is 8.020823588594794e-05, test loss is 4.7659155825385824e-05, and learning rate
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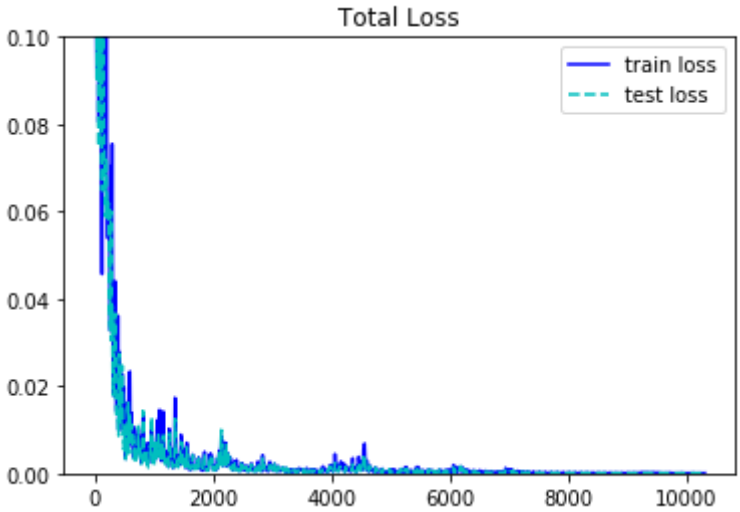
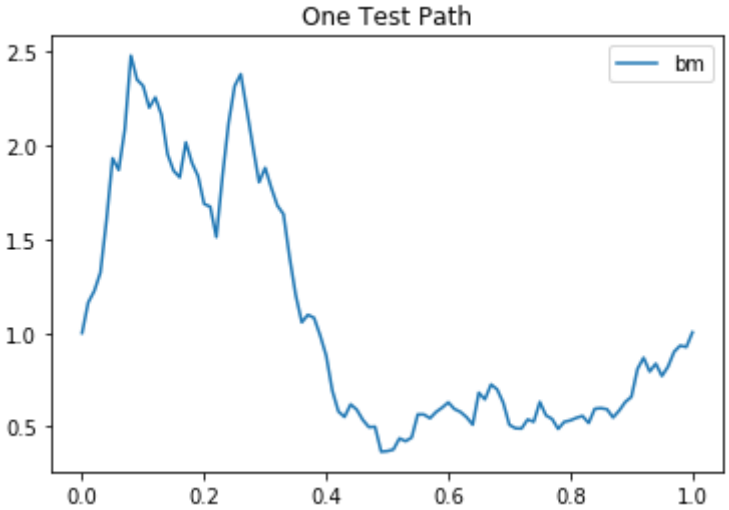
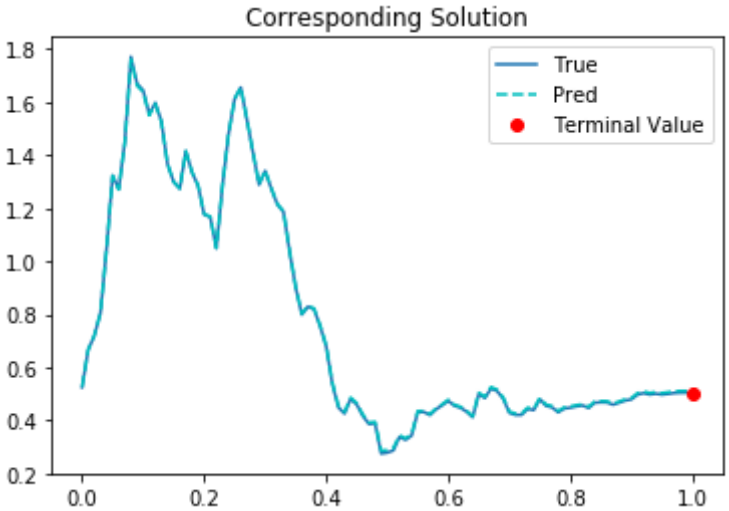
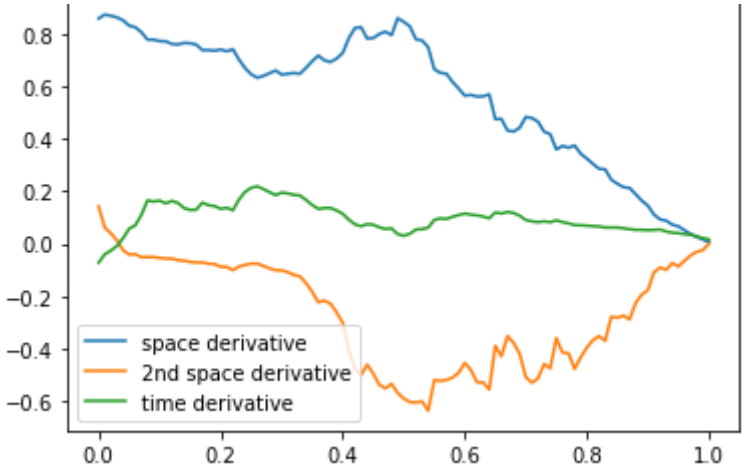






Derivatives

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10311th Epoch:
traing loss is 0.00012603816867340356, test loss is 7.24258425179869e-05, and learning rate i

10321th Epoch:
traing loss is 7.994630868779495e-05, test loss is 6.672069866908714e-05, and learning rate i

10331th Epoch:
traing loss is 4.5259603211889043e-05, test loss is 5.716504529118538e-05, and learning rate

10341th Epoch:
traing loss is 5.2979699830757454e-05, test loss is 5.11159269080963e-05, and learning rate i

10351th Epoch:
traing loss is 3.4136704925913364e-05, test loss is 4.437021925696172e-05, and learning rate

10361th Epoch:
traing loss is 7.403493509627879e-05, test loss is 5.248927845968865e-05, and learning rate i

10371th Epoch:
traing loss is 4.169492603978142e-05, test loss is 4.906503818347119e-05, and learning rate i

10381th Epoch:
traing loss is 5.7288212701678276e-05, test loss is 5.0564896810101345e-05, and learning rate

10391th Epoch:
traing loss is 6.671992741758004e-05, test loss is 4.951351729687303e-05, and learning rate i

10401th Epoch:
traing loss is 7.900151103967801e-05, test loss is 4.881467611994594e-05, and learning rate i
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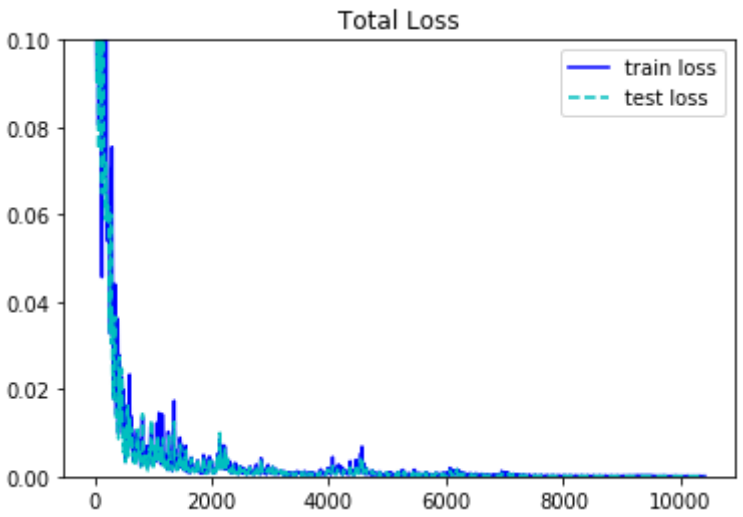

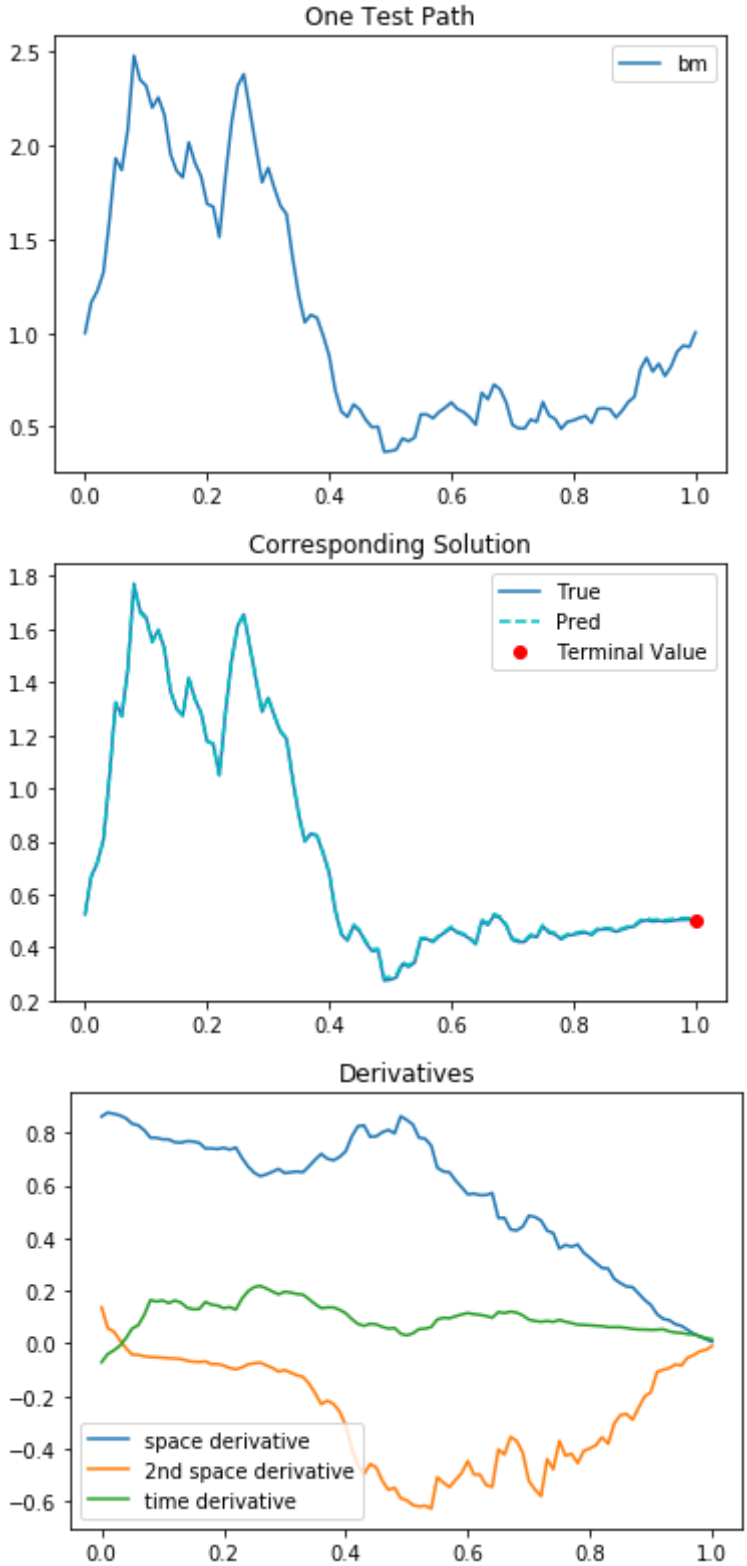


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10411th Epoch:
traing loss is 0.00011953570356126875, test loss is 6.671761366305873e-05, and learning rate

10421th Epoch:
traing loss is 7.572382310172543e-05, test loss is 6.375498924171552e-05, and learning rate i

10431th Epoch:
traing loss is 4.203652497380972e-05, test loss is 5.430042801890522e-05, and learning rate i

10441th Epoch:
traing loss is 6.129823304945603e-05, test loss is 4.826448275707662e-05, and learning rate i

10451th Epoch:
traing loss is 3.837784970528446e-05, test loss is 4.597864972311072e-05, and learning rate i
```

```
10461th Epoch:
traing loss is 7.414270658046007e-05, test loss is 4.716304465546273e-05, and learning rate i

10471th Epoch:
traing loss is 3.90512213925831e-05, test loss is 4.595789869199507e-05, and learning rate is

10481th Epoch:
traing loss is 5.969586709397845e-05, test loss is 5.1324557716725394e-05, and learning rate

10491th Epoch:
traing loss is 6.583720096386969e-05, test loss is 4.709632048616186e-05, and learning rate i

10501th Epoch:
traing loss is 7.750328222755343e-05, test loss is 4.8364086978835985e-05, and learning rate
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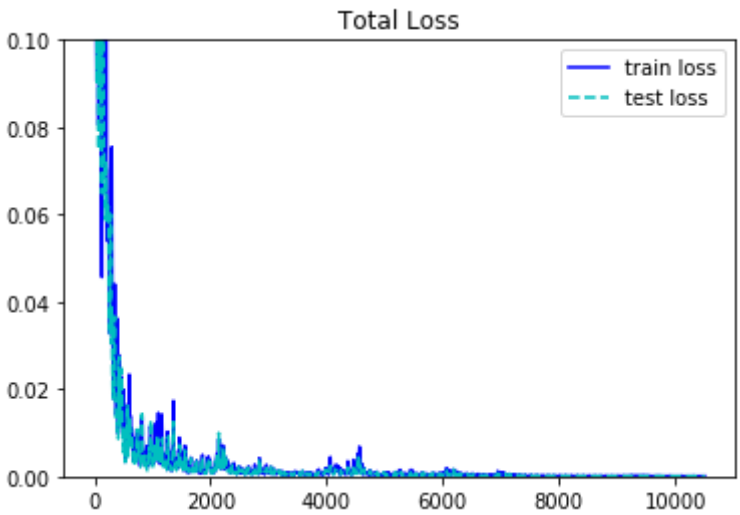

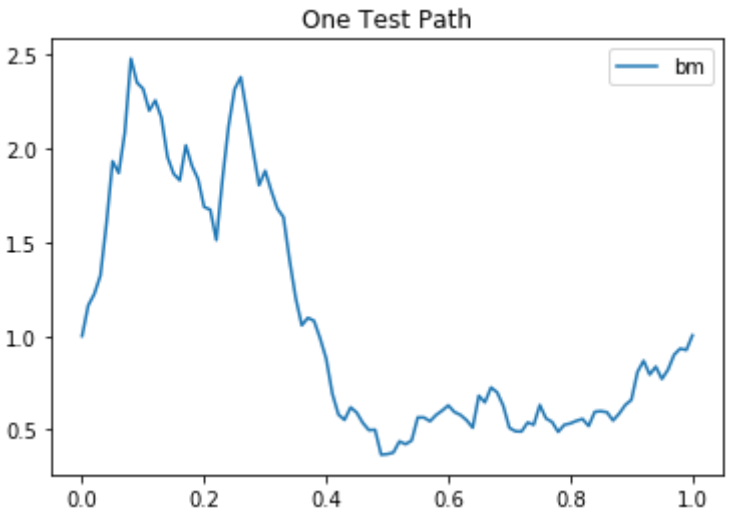

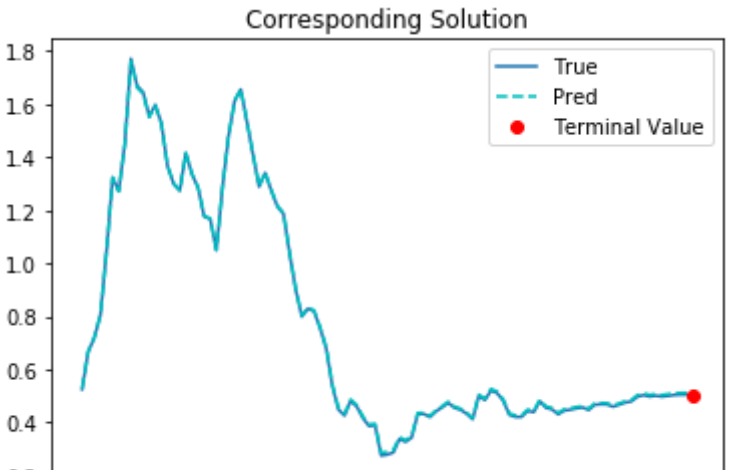

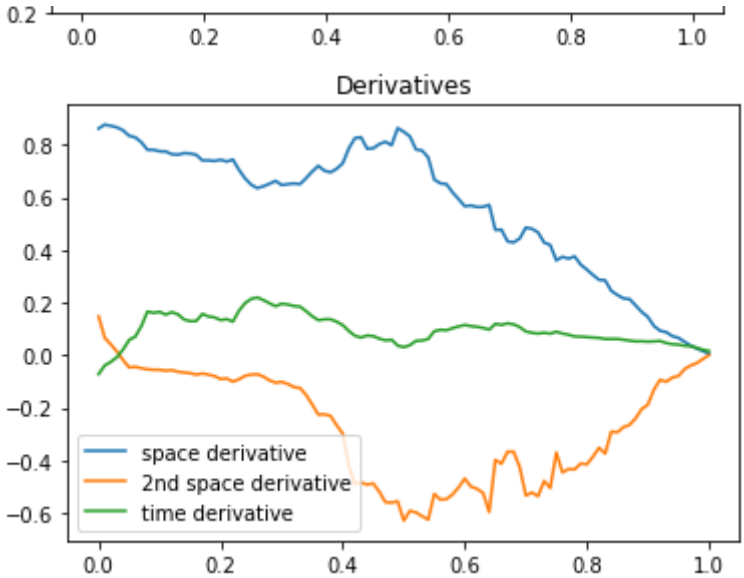


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10511th Epoch:
traing loss is 0.00012067038915120065, test loss is 6.856727850390598e-05, and learning rate

10521th Epoch:
traing loss is 7.848865789128467e-05, test loss is 6.361374835250899e-05, and learning rate i

10531th Epoch:
traing loss is 4.2938791011692956e-05, test loss is 5.466560833156109e-05, and learning rate

10541th Epoch:
traing loss is 5.099551708553918e-05, test loss is 4.872606950812042e-05, and learning rate i

10551th Epoch:
traing loss is 3.290560198365711e-05, test loss is 4.352822725195438e-05, and learning rate i

10561th Epoch:
traing loss is 7.239461410790682e-05, test loss is 5.134168895892799e-05, and learning rate i

10571th Epoch:
traing loss is 4.099289799341932e-05, test loss is 4.847030504606664e-05, and learning rate i

10581th Epoch:
traing loss is 5.6667940953047946e-05, test loss is 4.936307595926337e-05, and learning rate

10591th Epoch:
traing loss is 6.607577233808115e-05, test loss is 4.813490158994682e-05, and learning rate i

10601th Epoch:
traing loss is 7.535524491686374e-05, test loss is 4.875747254118323e-05, and learning rate i
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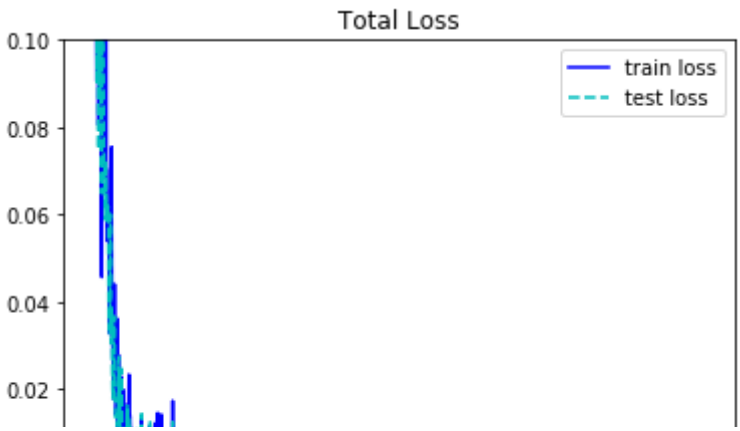

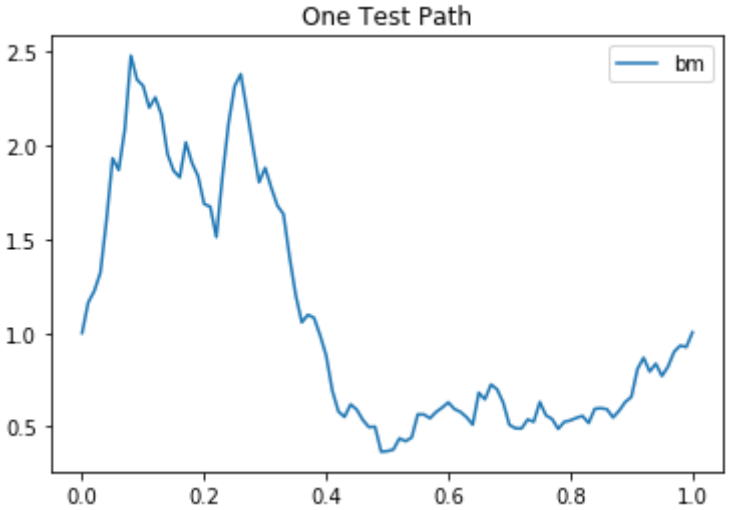

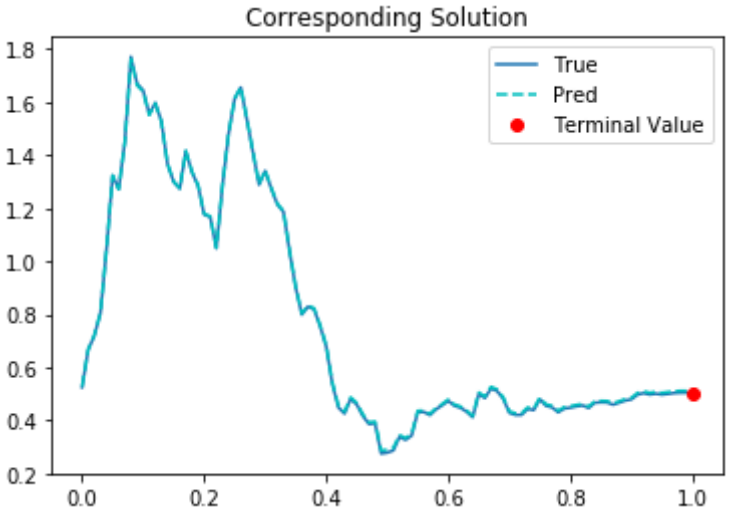

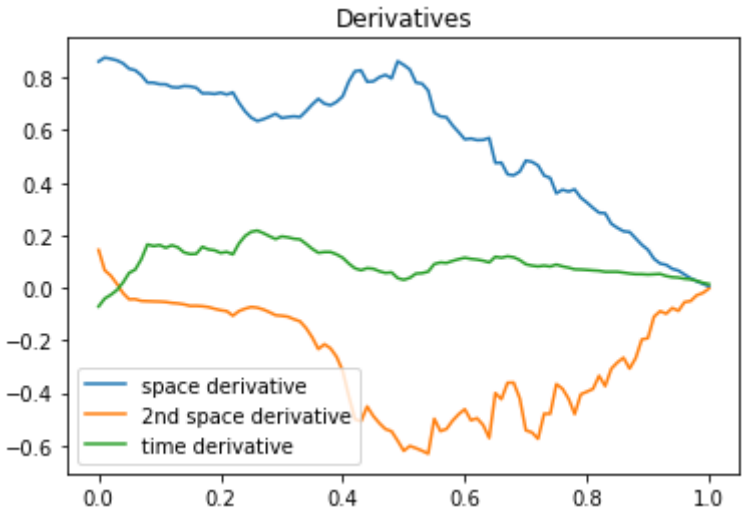


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10611th Epoch:
traing loss is 0.00011796332546509802, test loss is 6.4906693296507e-05, and learning rate is

10621th Epoch:
traing loss is 7.516353070968762e-05, test loss is 6.228869460755959e-05, and learning rate i

10631th Epoch:
traing loss is 4.0234557673102245e-05, test loss is 5.251467155176215e-05, and learning rate

10641th Epoch:
traing loss is 5.877353032701649e-05, test loss is 4.6374629164347425e-05, and learning rate
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10651th Epoch:
traing loss is 3.671647937153466e-05, test loss is 4.493195592658594e-05, and learning rate i

10661th Epoch:
traing loss is 7.350149098783731e-05, test loss is 4.739443102153018e-05, and learning rate i

10671th Epoch:
traing loss is 3.915769775630906e-05, test loss is 4.6846231271047145e-05, and learning rate

10681th Epoch:
traing loss is 5.8856381656369194e-05, test loss is 5.089694604976103e-05, and learning rate

10691th Epoch:
traing loss is 6.3993880758062e-05, test loss is 4.5738059270661324e-05, and learning rate is

10701th Epoch:
traing loss is 7.490134885301813e-05, test loss is 4.8404628614662215e-05, and learning rate
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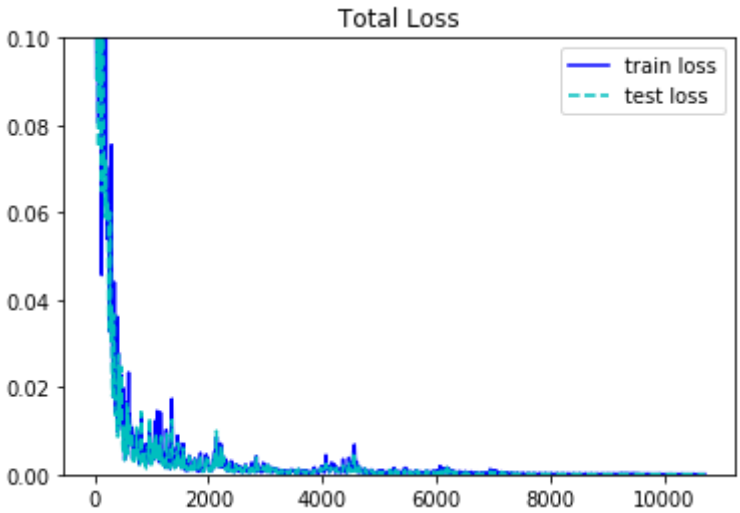

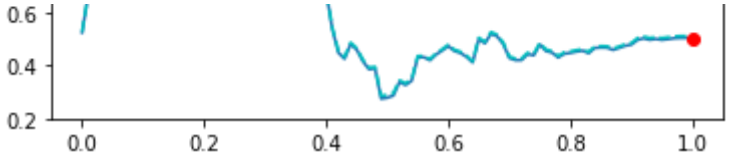

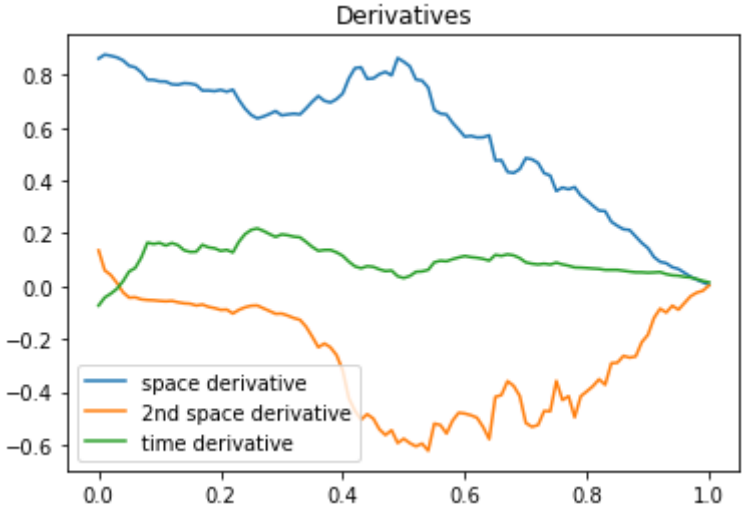


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10711th Epoch:
traing loss is 0.0001182516643893905, test loss is 6.673571624560282e-05, and learning rate i

10721th Epoch:
traing loss is 7.983682735357434e-05, test loss is 6.565210060216486e-05, and learning rate i

10731th Epoch:
traing loss is 4.031277057947591e-05, test loss is 5.183521716389805e-05, and learning rate i

10741th Epoch:
traing loss is 4.877568062511273e-05, test loss is 4.5597058488056064e-05, and learning rate

10751th Epoch:
traing loss is 3.300779280834831e-05, test loss is 4.358178921393119e-05, and learning rate i

10761th Epoch:
traing loss is 7.175686914706603e-05, test loss is 5.115720341564156e-05, and learning rate i

10771th Epoch:
traing loss is 4.148614971200004e-05, test loss is 4.86252065456938e-05, and learning rate is

10781th Epoch:
traing loss is 5.510896153282374e-05, test loss is 4.822785922442563e-05, and learning rate i

10791th Epoch:
traing loss is 6.484289770014584e-05, test loss is 4.757745409733616e-05, and learning rate i

10801th Epoch:
traing loss is 7.746851042611524e-05, test loss is 4.94372688990552e-05, and learning rate is
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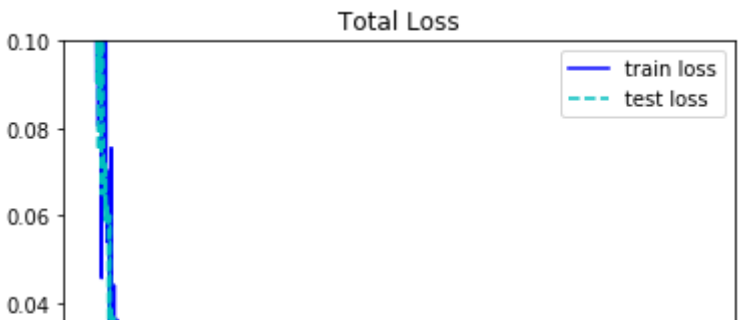

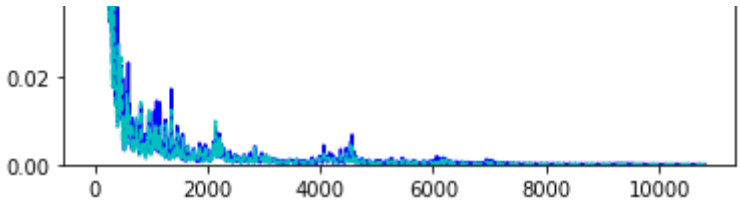

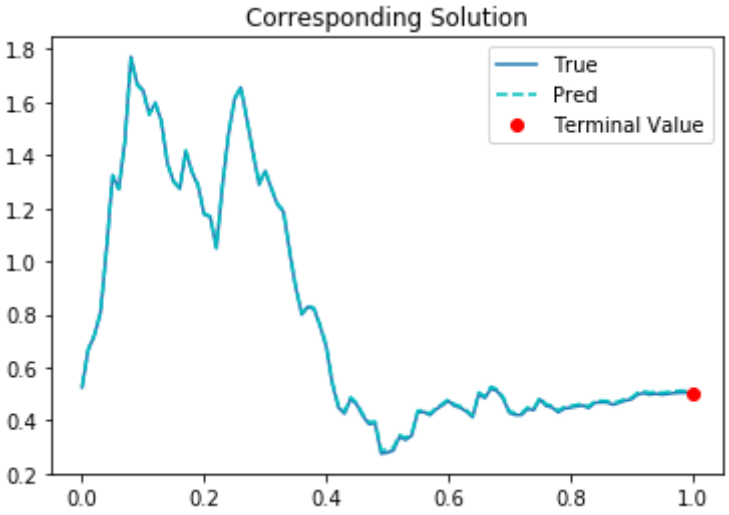

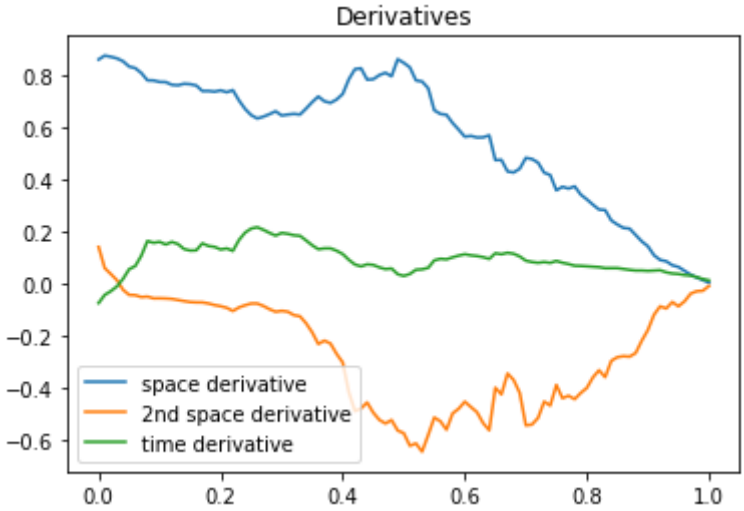


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10811th Epoch:
traing loss is 0.00011568454647203907, test loss is 6.368514732457697e-05, and learning rate

10821th Epoch:
traing loss is 7.588672451674938e-05, test loss is 6.492743705166504e-05, and learning rate i

10831th Epoch:
traing loss is 3.728806404978968e-05, test loss is 4.894384983344935e-05, and learning rate i
```

```
10841th Epoch:
traing loss is 5.724186485167593e-05, test loss is 4.647297828341834e-05, and learning rate i

10851th Epoch:
traing loss is 3.6589328374248e-05, test loss is 4.38061251770705e-05, and learning rate is 0

10861th Epoch:
traing loss is 7.565796113340184e-05, test loss is 5.0246268074261025e-05, and learning rate

10871th Epoch:
traing loss is 3.911510430043563e-05, test loss is 4.7332581743830815e-05, and learning rate

10881th Epoch:
traing loss is 5.6712047808105126e-05, test loss is 4.86837889184244e-05, and learning rate i

10891th Epoch:
traing loss is 6.339853280223906e-05, test loss is 4.548323340713978e-05, and learning rate i

10901th Epoch:
traing loss is 7.712769001955166e-05, test loss is 5.080572736915201e-05, and learning rate i
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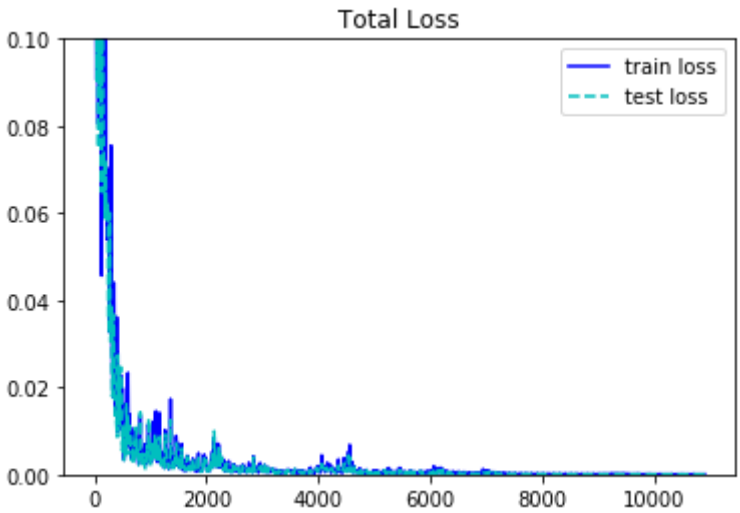

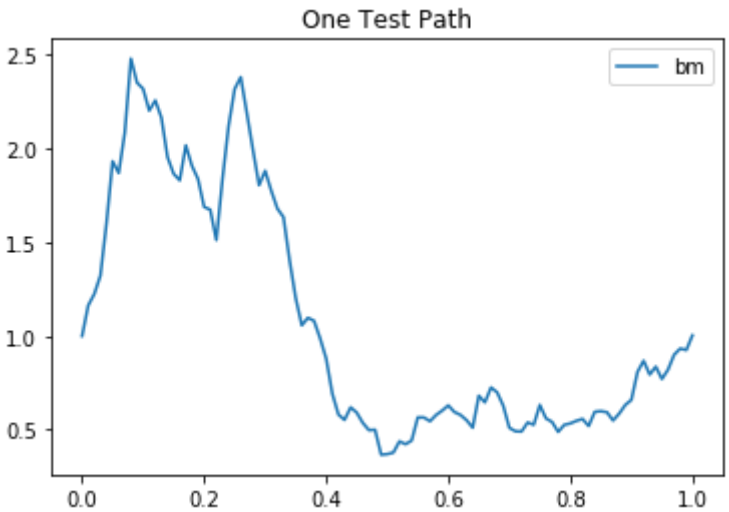

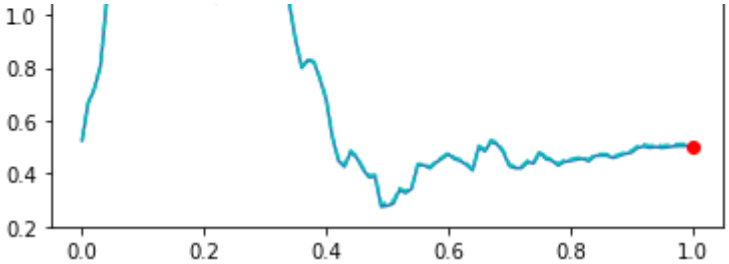

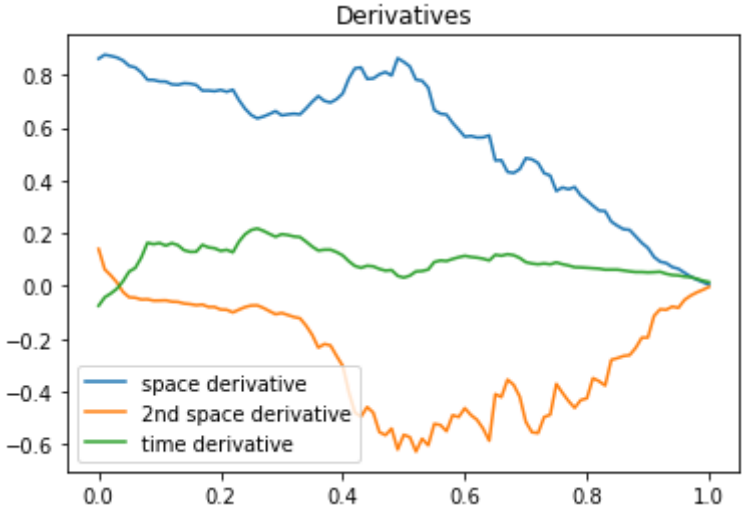


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10911th Epoch:
traing loss is 0.00011224541231058538, test loss is 6.312820187304169e-05, and learning rate

10921th Epoch:
traing loss is 8.276908192783594e-05, test loss is 7.03255645930767e-05, and learning rate is

10931th Epoch:
traing loss is 3.6361620004754514e-05, test loss is 4.796466600964777e-05, and learning rate

10941th Epoch:
traing loss is 4.975360570824705e-05, test loss is 4.6303815906867385e-05, and learning rate

10951th Epoch:
traing loss is 3.195329918526113e-05, test loss is 4.293231540941633e-05, and learning rate i

10961th Epoch:
traing loss is 7.419975008815527e-05, test loss is 5.4925800213823095e-05, and learning rate

10971th Epoch:
traing loss is 4.0011283999774605e-05, test loss is 4.674293813877739e-05, and learning rate

10981th Epoch:
traing loss is 5.3721330914413556e-05, test loss is 4.612578050000593e-05, and learning rate

10991th Epoch:
traing loss is 6.520924216601998e-05, test loss is 4.7715806431369856e-05, and learning rate

11001th Epoch:
traing loss is 7.588410517200828e-05, test loss is 5.006667925044894e-05, and learning rate i
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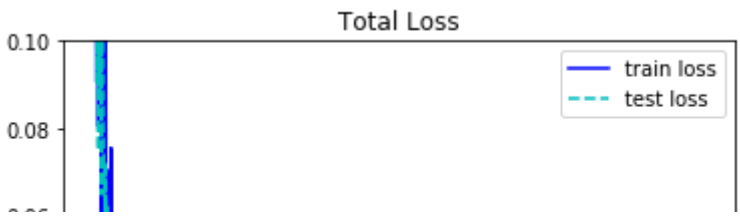

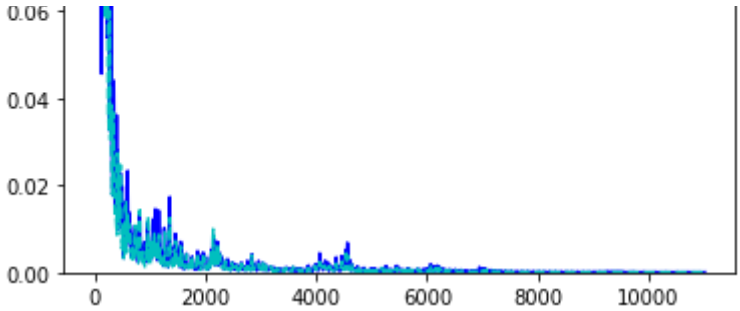

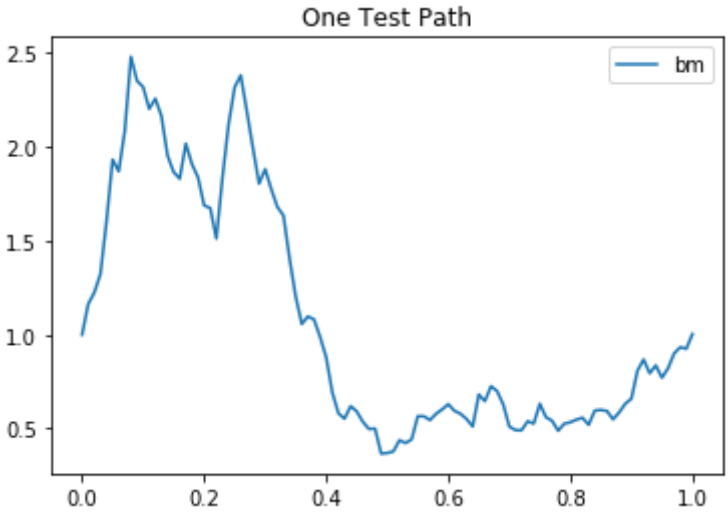

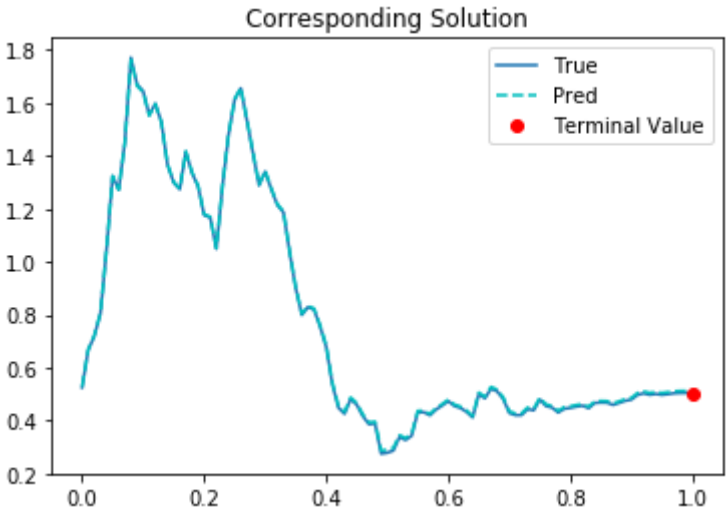

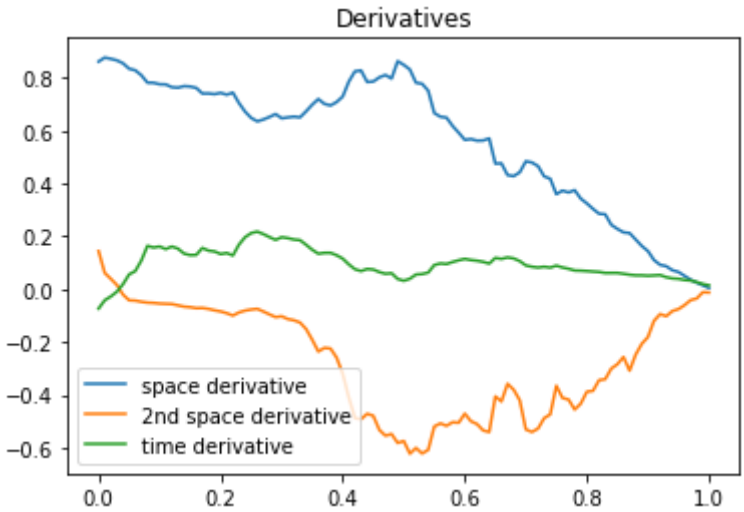

11011th Epoch:
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11021th Epoch:
traing loss is 7.859555626055226e-05, test loss is 6.808545003877953e-05, and learning rate i

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11031th Epoch:
traing loss is 3.2973577617667615e-05, test loss is 4.432184505276382e-05, and learning rate

11041th Epoch:
traing loss is 6.027485869708471e-05, test loss is 5.0215956434840336e-05, and learning rate

11051th Epoch:
traing loss is 3.456439662841149e-05, test loss is 4.235763844917528e-05, and learning rate i

11061th Epoch:
traing loss is 8.17029140307568e-05, test loss is 5.9294532547937706e-05, and learning rate i

11071th Epoch:
traing loss is 3.804712468991056e-05, test loss is 4.509026985033415e-05, and learning rate i

11081th Epoch:
traing loss is 5.093883737572469e-05, test loss is 4.4344531488604844e-05, and learning rate

11091th Epoch:
traing loss is 6.49638895993121e-05, test loss is 4.6840952563798055e-05, and learning rate i

11101th Epoch:
traing loss is 7.167818694142625e-05, test loss is 4.8388817958766595e-05, and learning rate
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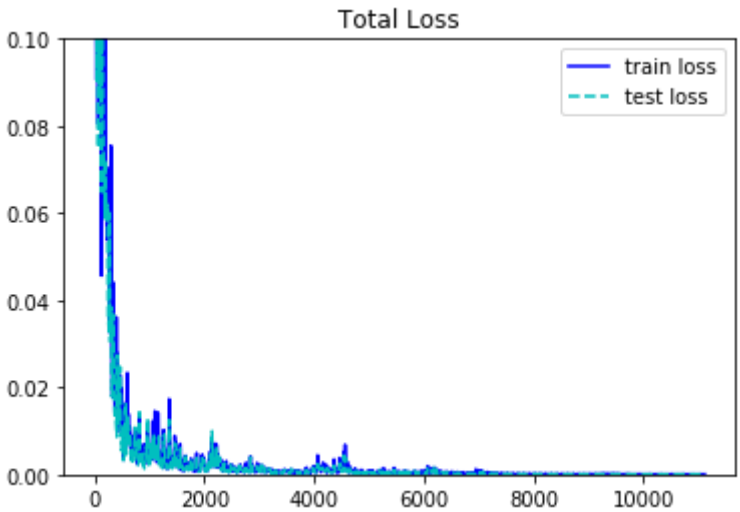

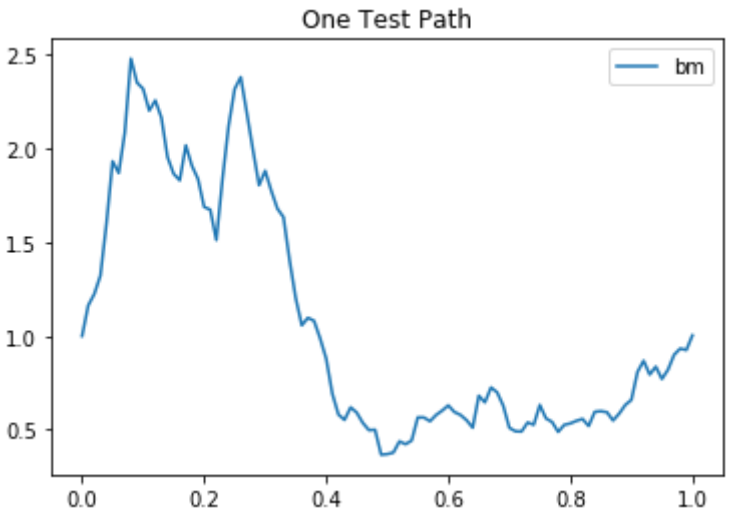

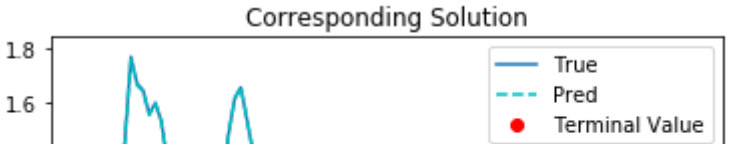

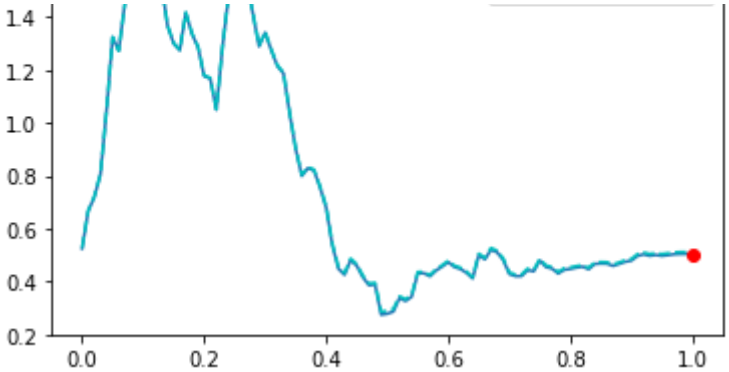

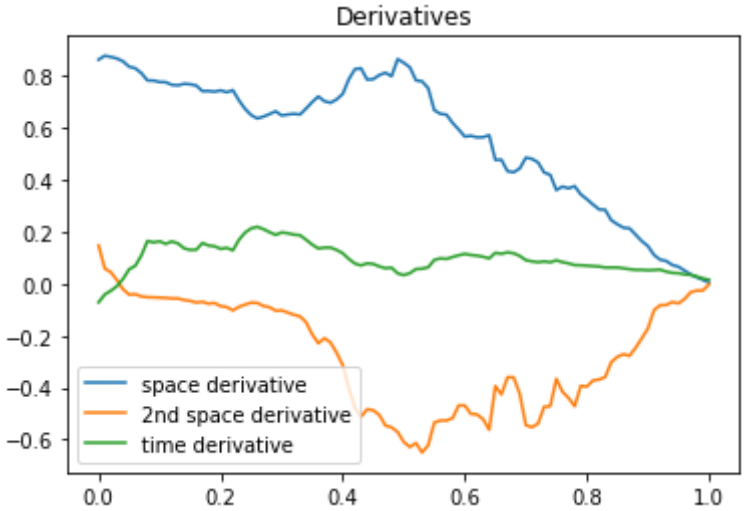


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11111th Epoch:
traing loss is 0.00010483233927516267, test loss is 6.191765714902431e-05, and learning rate

11121th Epoch:
traing loss is 8.3482067566365e-05, test loss is 7.15265196049586e-05, and learning rate is 0

11131th Epoch:
traing loss is 3.4440749004716054e-05, test loss is 4.5961020077811554e-05, and learning rate

11141th Epoch:
traing loss is 5.130043064127676e-05, test loss is 4.737034032586962e-05, and learning rate i

11151th Epoch:
traing loss is 3.3772666938602924e-05, test loss is 4.4845401134807616e-05, and learning rate

11161th Epoch:
traing loss is 7.840653415769339e-05, test loss is 5.864691411261447e-05, and learning rate i

11171th Epoch:
traing loss is 3.621478390414268e-05, test loss is 4.285958857508376e-05, and learning rate i

11181th Epoch:
traing loss is 4.988149521523155e-05, test loss is 4.374288982944563e-05, and learning rate i

11191th Epoch:
traing loss is 6.374604708980769e-05, test loss is 4.6156270400388166e-05, and learning rate

11201th Epoch:
traing loss is 6.850533100077882e-05, test loss is 4.634532888303511e-05, and learning rate i
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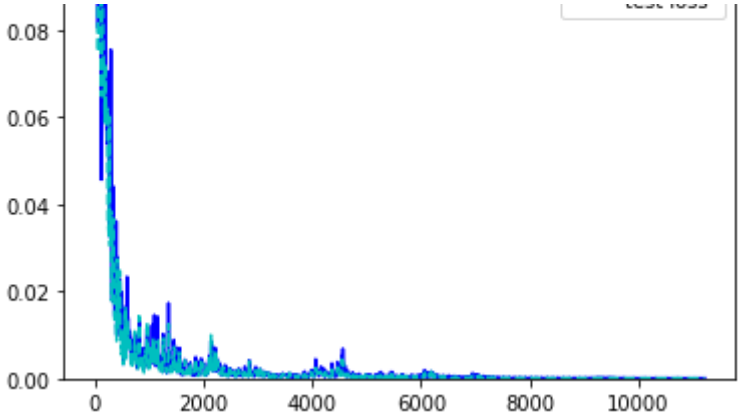
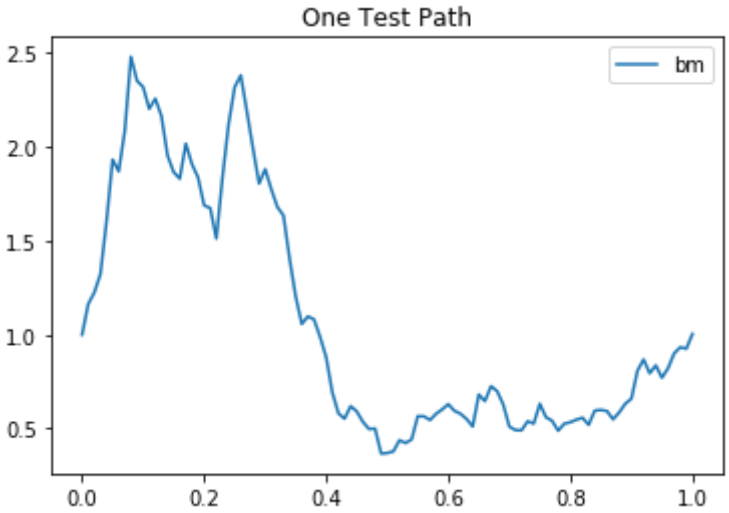
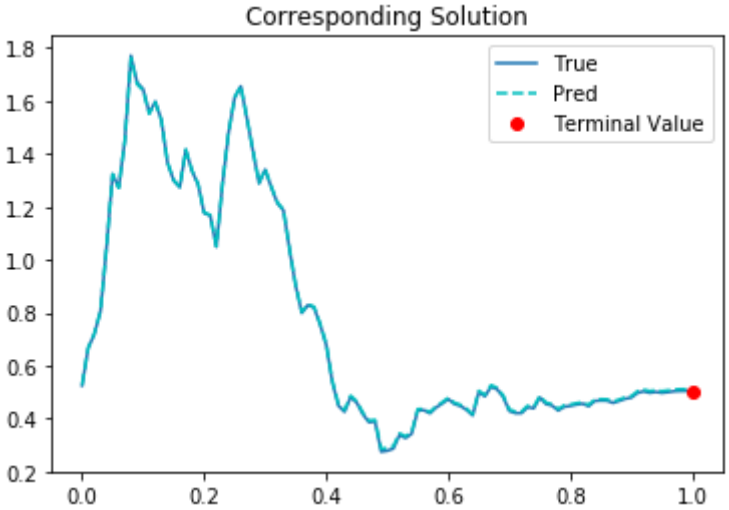
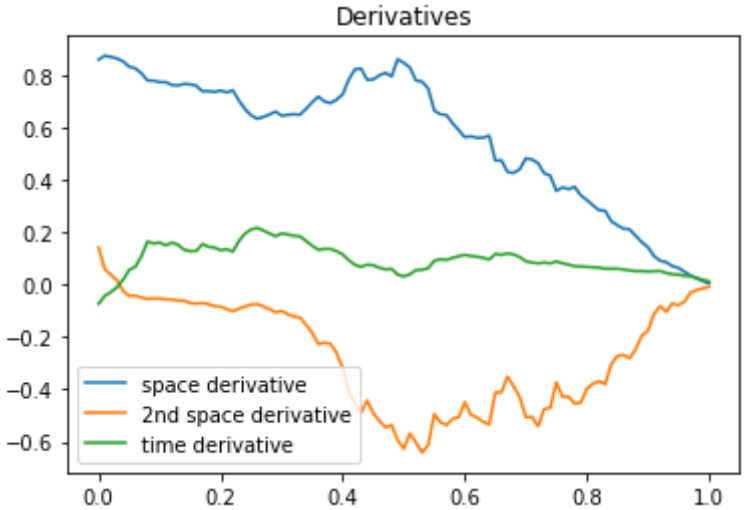








11211th Epoch:
traing loss is 0.00010479961929377168, test loss is 5.9066358517156914e-05, and learning rate

```
11221th Epoch:
traing loss is 7.319256837945431e-05, test loss is 6.319375825114548e-05, and learning rate i

11231th Epoch:
traing loss is 3.5486824344843626e-05, test loss is 4.747318962472491e-05, and learning rate

11241th Epoch:
traing loss is 5.869599408470094e-05, test loss is 4.828049713978544e-05, and learning rate i

11251th Epoch:
traing loss is 3.7967161915730685e-05, test loss is 4.667474786401726e-05, and learning rate

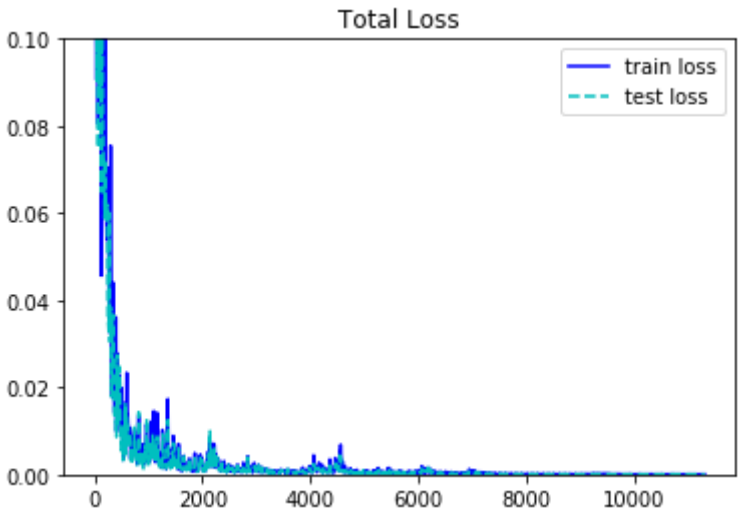
11261th Epoch:
traing loss is 7.806202484061942e-05, test loss is 5.4037547670304775e-05, and learning rate

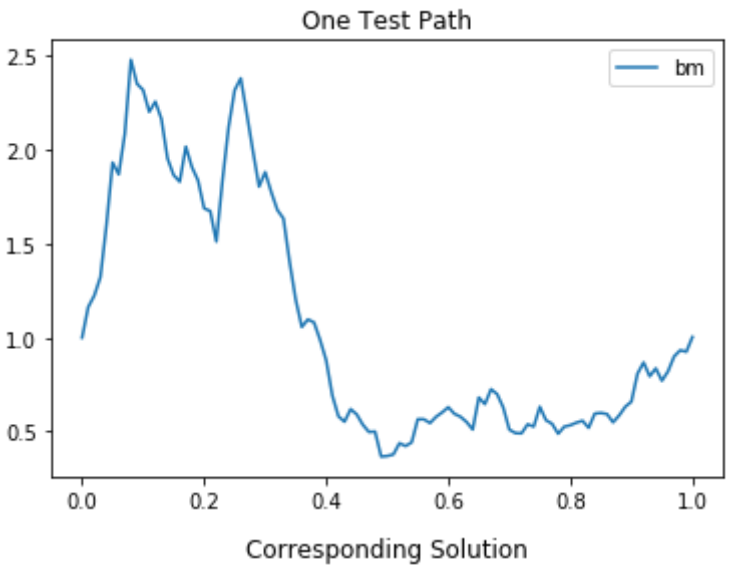
11271th Epoch:
traing loss is 3.7034427805338055e-05, test loss is 4.230197373544797e-05, and learning rate

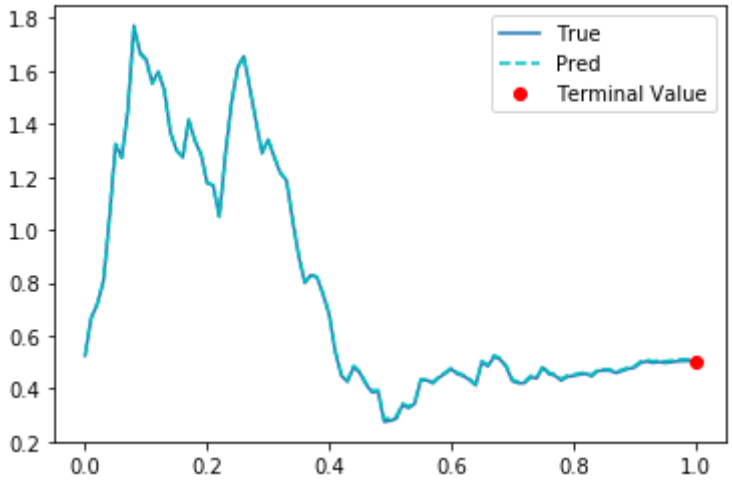
11281th Epoch:
traing loss is 4.915355020784773e-05, test loss is 4.28632483817637e-05, and learning rate is

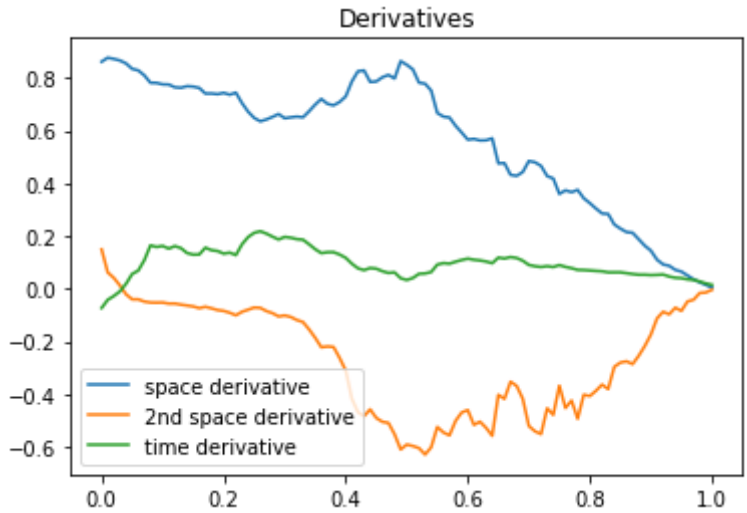
11291th Epoch:
traing loss is 6.355840741889551e-05, test loss is 4.530019941739738e-05, and learning rate i

11301th Epoch:
traing loss is 6.68202992528677e-05, test loss is 4.478745177038945e-05, and learning rate is
```










```
11311th Epoch:
traing loss is 0.00010151362948818132, test loss is 6.008079435559921e-05, and learning rate

11321th Epoch:
traing loss is 7.249927148222923e-05, test loss is 6.044641122571193e-05, and learning rate i

11331th Epoch:
traing loss is 3.911254680133425e-05, test loss is 5.13788872922305e-05, and learning rate is

11341th Epoch:
traing loss is 4.568683652905747e-05, test loss is 4.3886051571462303e-05, and learning rate

11351th Epoch:
traing loss is 3.780868064495735e-05, test loss is 4.889288902631961e-05, and learning rate i

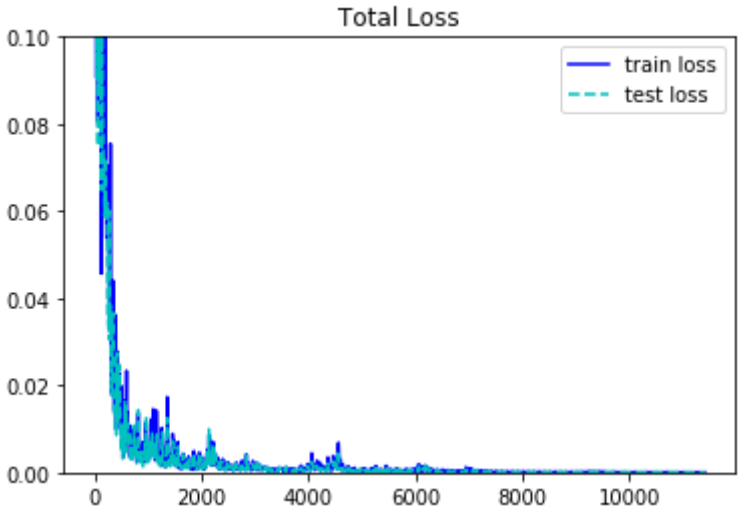
11361th Epoch:
traing loss is 6.857763219159096e-05, test loss is 4.528859426500276e-05, and learning rate i

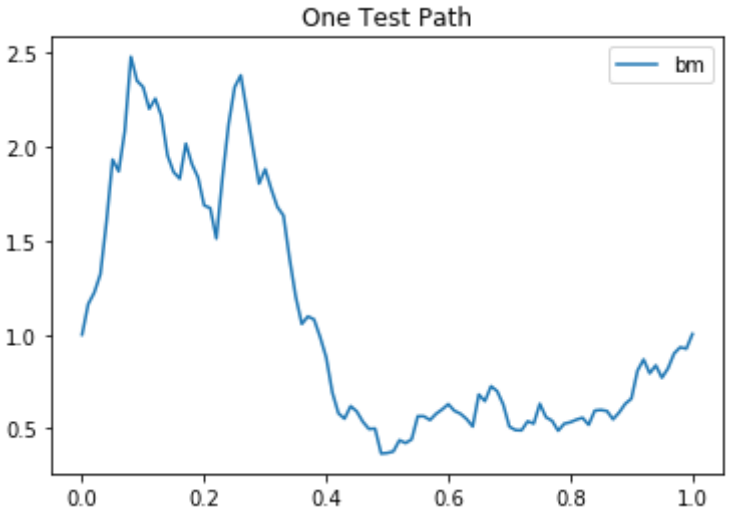
11371th Epoch:
traing loss is 3.5686407500179484e-05, test loss is 4.155762508162297e-05, and learning rate

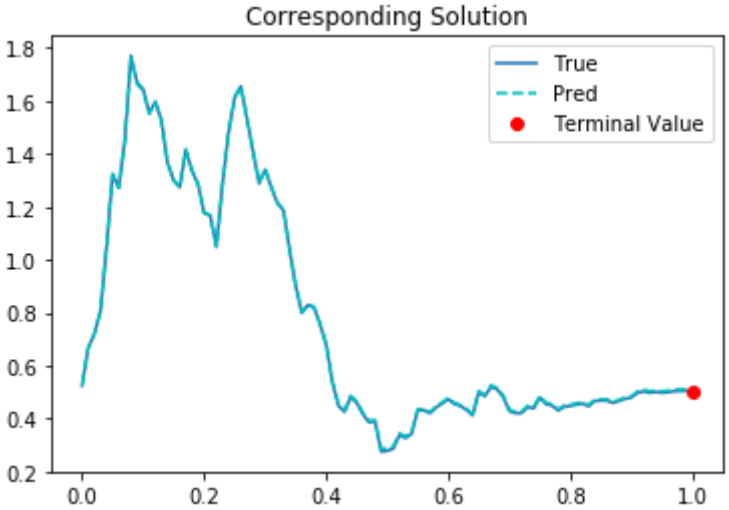
11381th Epoch:
traing loss is 4.942201485391706e-05, test loss is 4.3254494812572375e-05, and learning rate

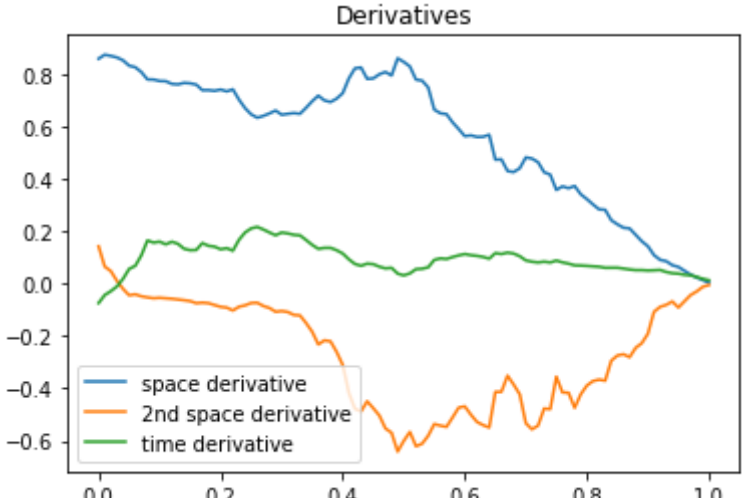
11391th Epoch:
traing loss is 6.32013616268523e-05, test loss is 4.649464972317219e-05, and learning rate is

11401th Epoch:
traing loss is 6.473702524090186e-05, test loss is 4.419549441081472e-05, and learning rate i
```


Total Loss
train loss
test loss
0.10
0.08
0.06
0.04
0.02
0.00
0
2000
4000
6000
8000
10000


One Test Path
bm
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0


Corresponding Solution
True
Pred
Terminal Value
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


Derivatives
space derivative
2nd space derivative
time derivative
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6

```
11411th Epoch:
traing loss is 0.00010123995889443904, test loss is 5.9071124269394204e-05, and learning rate

11421th Epoch:
traing loss is 6.695279444102198e-05, test loss is 5.438657899503596e-05, and learning rate i

11431th Epoch:
traing loss is 3.7975867599016055e-05, test loss is 4.987733336747624e-05, and learning rate

11441th Epoch:
traing loss is 5.4012256441637874e-05, test loss is 4.2947223846567795e-05, and learning rate

11451th Epoch:
traing loss is 3.9093087252695113e-05, test loss is 4.7255874960683286e-05, and learning rate

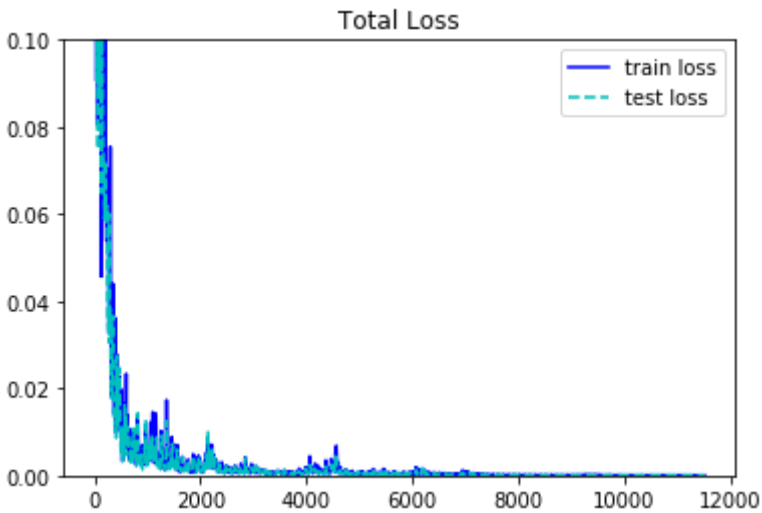
11461th Epoch:
traing loss is 6.915989797562361e-05, test loss is 4.4086118577979505e-05, and learning rate

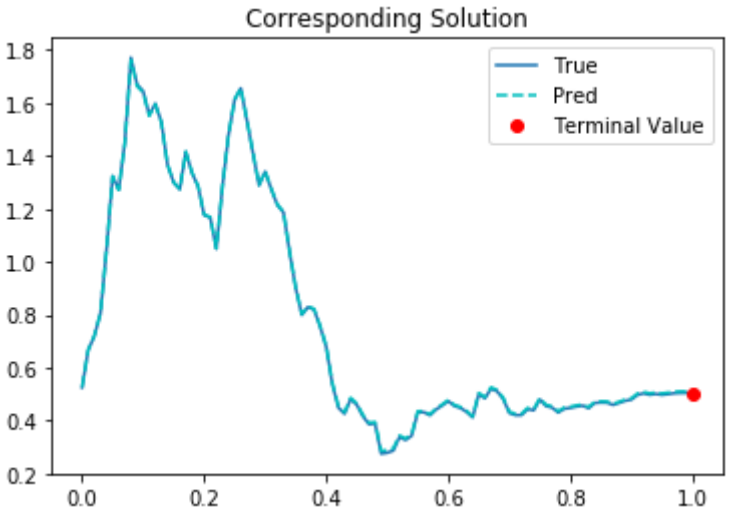
11471th Epoch:
traing loss is 3.486444256850518e-05, test loss is 4.1696035623317584e-05, and learning rate

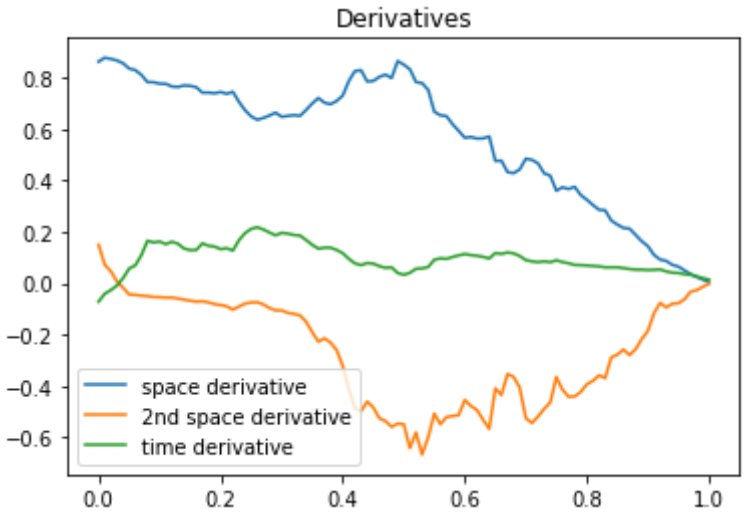
11481th Epoch:
traing loss is 4.8999096179613844e-05, test loss is 4.317162529332563e-05, and learning rate

11491th Epoch:
traing loss is 6.324644346022978e-05, test loss is 4.584437920129858e-05, and learning rate i

11501th Epoch:
traing loss is 6.400509300874546e-05, test loss is 4.3267304135952145e-05, and learning rate
```








```
11511th Epoch:
traing loss is 9.926381608238444e-05, test loss is 5.9082845837110654e-05, and learning rate

11521th Epoch:
traing loss is 6.594839942408726e-05, test loss is 5.238974335952662e-05, and learning rate i

11531th Epoch:
traing loss is 3.7470439565368e-05, test loss is 4.9249472795054317e-05, and learning rate is

11541th Epoch:
traing loss is 4.5678614696953446e-05, test loss is 4.374208583612926e-05, and learning rate

11551th Epoch:
traing loss is 3.365635348018259e-05, test loss is 4.389690002426505e-05, and learning rate i

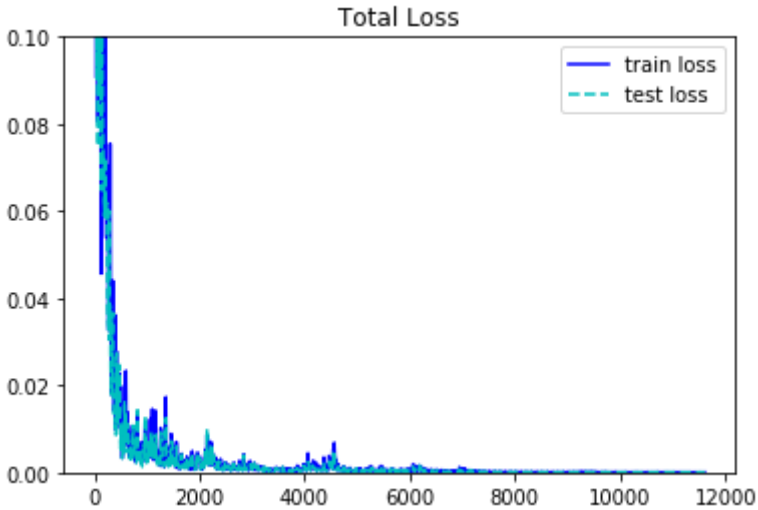
11561th Epoch:
traing loss is 6.365917943185195e-05, test loss is 4.2020979890367016e-05, and learning rate

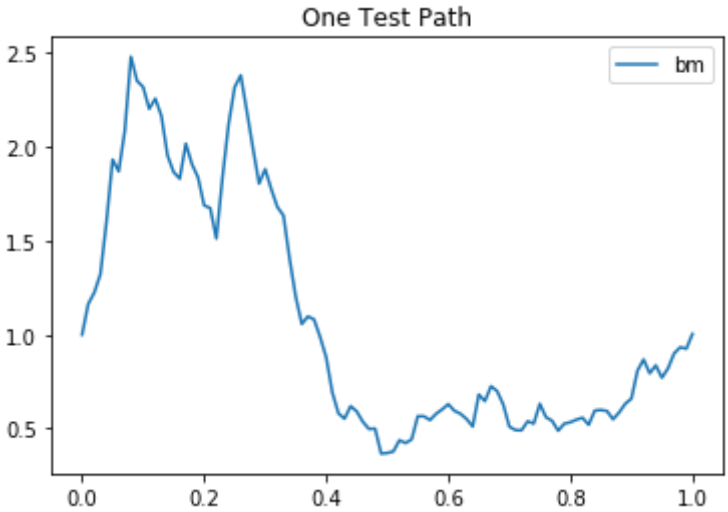
11571th Epoch:
traing loss is 3.399888737476431e-05, test loss is 4.004372749477625e-05, and learning rate i

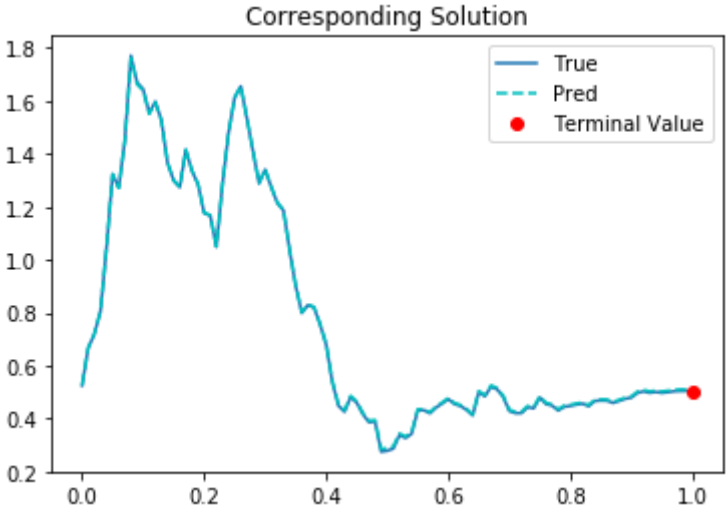
11581th Epoch:
traing loss is 4.9100217438535765e-05, test loss is 4.246653770678677e-05, and learning rate

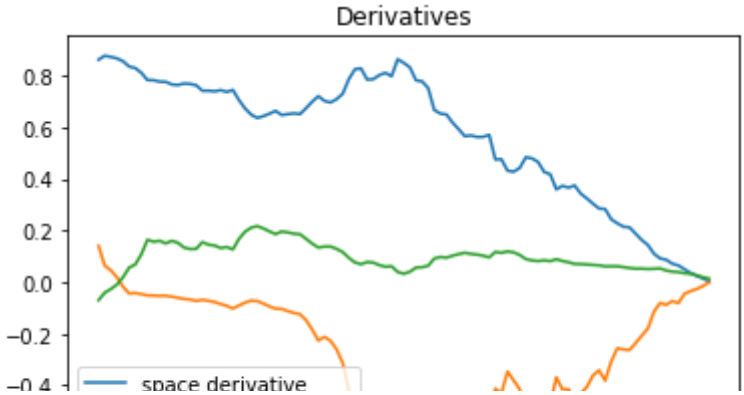
11591th Epoch:
traing loss is 6.347510498017073e-05, test loss is 4.654705116990954e-05, and learning rate i
```

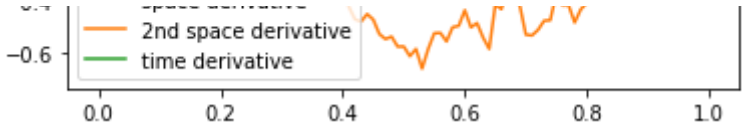
11601th Epoch:
traing loss is 6.259598740143701e-05, test loss is 4.343155524111353e-05, and learning rate i












```
11611th Epoch:
traing loss is 9.778298408491537e-05, test loss is 5.536031676456332e-05, and learning rate i

11621th Epoch:
traing loss is 6.304584530880675e-05, test loss is 4.933689706376754e-05, and learning rate i

11631th Epoch:
traing loss is 3.503748666844331e-05, test loss is 4.634388460544869e-05, and learning rate i

11641th Epoch:
traing loss is 5.352212974685244e-05, test loss is 4.281074507161975e-05, and learning rate i

11651th Epoch:
traing loss is 3.576912058633752e-05, test loss is 4.3957101297564805e-05, and learning rate

11661th Epoch:
traing loss is 6.79873555782251e-05, test loss is 4.256670217728242e-05, and learning rate is

11671th Epoch:
traing loss is 3.413265585550107e-05, test loss is 4.0241338865598664e-05, and learning rate

11681th Epoch:
traing loss is 4.755545160151087e-05, test loss is 4.1933366446755826e-05, and learning rate

11691th Epoch:
traing loss is 6.313050835160539e-05, test loss is 4.634058859664947e-05, and learning rate i

11701th Epoch:
traing loss is 6.137639138614759e-05, test loss is 4.25647413067054e-05, and learning rate is
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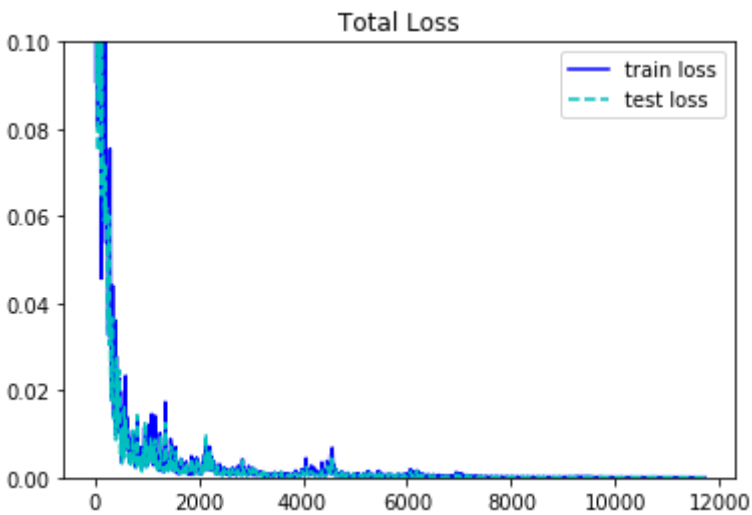

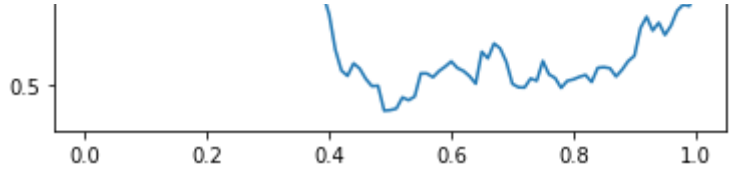

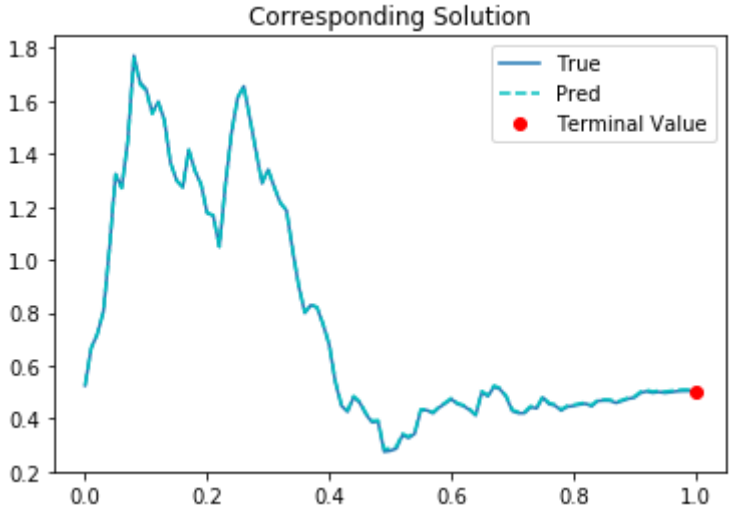

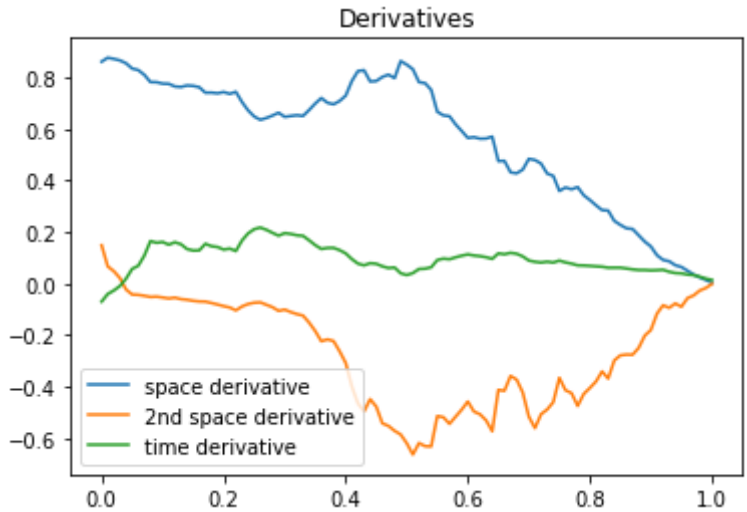


```
11711th Epoch:
traing loss is 9.610684355720878e-05, test loss is 5.523666550288908e-05, and learning rate i

11721th Epoch:
traing loss is 6.476711860159412e-05, test loss is 4.911160431220196e-05, and learning rate i

11731th Epoch:
traing loss is 3.5252320230938494e-05, test loss is 4.641880514100194e-05, and learning rate

11741th Epoch:
traing loss is 4.603328852681443e-05, test loss is 4.502896626945585e-05, and learning rate i

11751th Epoch:
traing loss is 3.086691503995098e-05, test loss is 4.0986247768159956e-05, and learning rate

11761th Epoch:
traing loss is 6.342582491924986e-05, test loss is 4.177688606432639e-05, and learning rate i

11771th Epoch:
traing loss is 3.31246715348375e-05, test loss is 3.9488295442424715e-05, and learning rate

11781th Epoch:
traing loss is 4.7486206312896684e-05, test loss is 4.1591010813135654e-05, and learning rate
```

```
11791th Epoch:
traing loss is 6.153806316433474e-05, test loss is 4.546349009615369e-05, and learning rate i

11801th Epoch:
traing loss is 6.0338086768751964e-05, test loss is 4.235318920109421e-05, and learning rate
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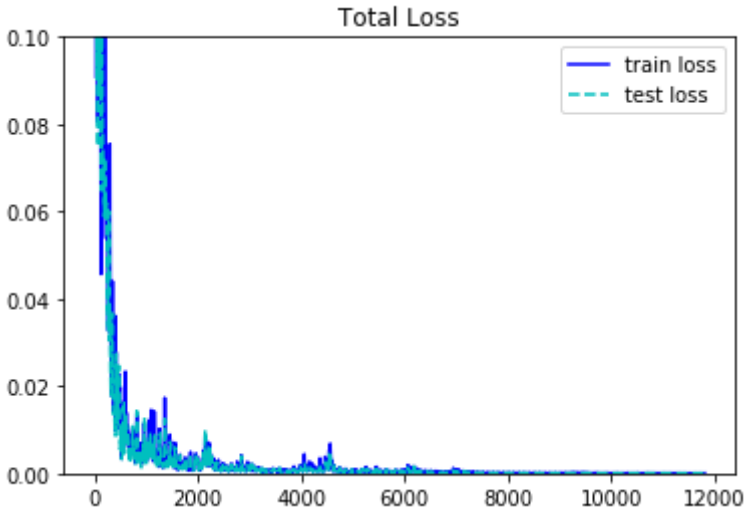

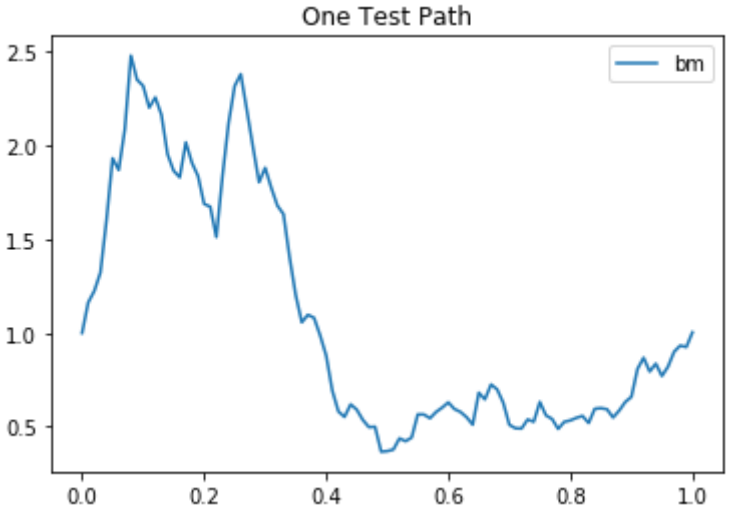

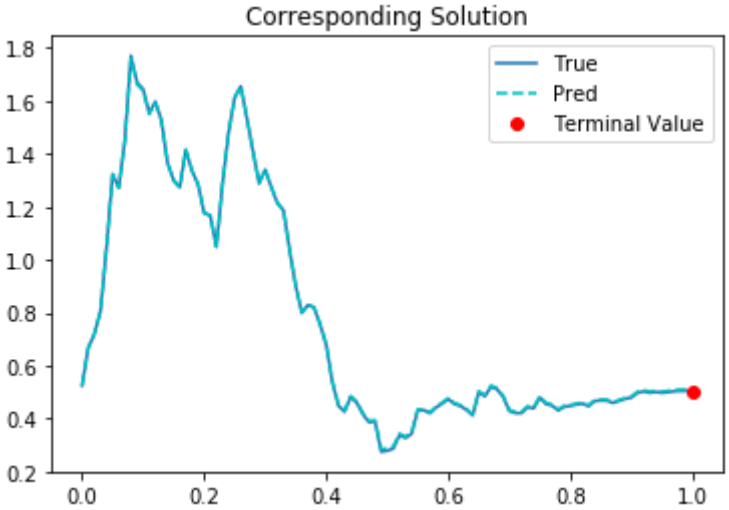

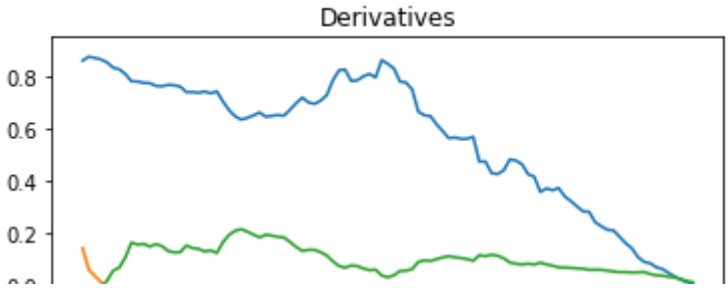

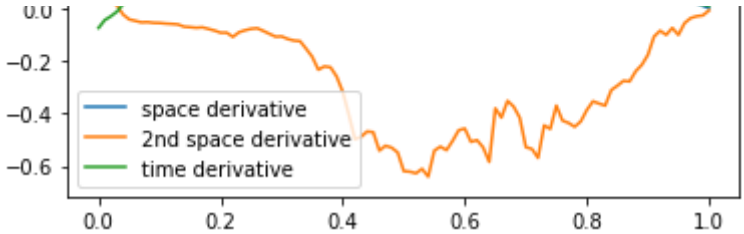


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11811th Epoch:
traing loss is 9.614697773940861e-05, test loss is 5.322877041180618e-05, and learning rate i

11821th Epoch:
traing loss is 6.146477244328707e-05, test loss is 4.681975769926794e-05, and learning rate i

11831th Epoch:
traing loss is 3.30541588482447e-05, test loss is 4.3793763325084e-05, and learning rate is 8

11841th Epoch:
traing loss is 5.326789687387645e-05, test loss is 4.254710802342743e-05, and learning rate i

11851th Epoch:
traing loss is 3.4493346902308986e-05, test loss is 4.164918573223986e-05, and learning rate

11861th Epoch:
traing loss is 6.813826621510088e-05, test loss is 4.156852810410783e-05, and learning rate i

11871th Epoch:
traing loss is 3.3708569390000775e-05, test loss is 3.969415774918161e-05, and learning rate

11881th Epoch:
traing loss is 4.71925268357154e-05, test loss is 4.138176882406697e-05, and learning rate is

11891th Epoch:
traing loss is 6.14043019595556e-05, test loss is 4.445172817213461e-05, and learning rate is

11901th Epoch:
traing loss is 5.950928607489914e-05, test loss is 4.140398232266307e-05, and learning rate i
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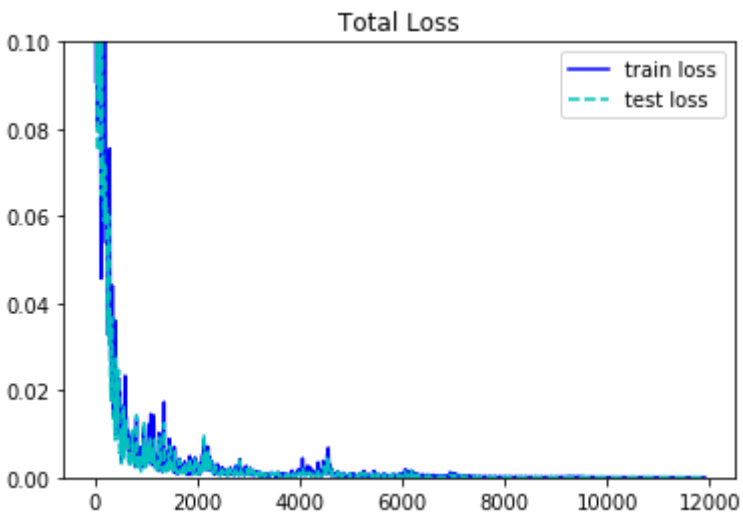

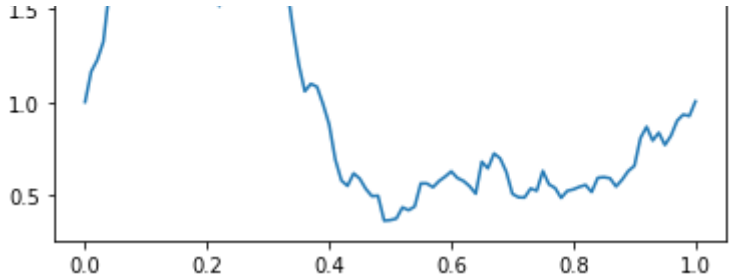

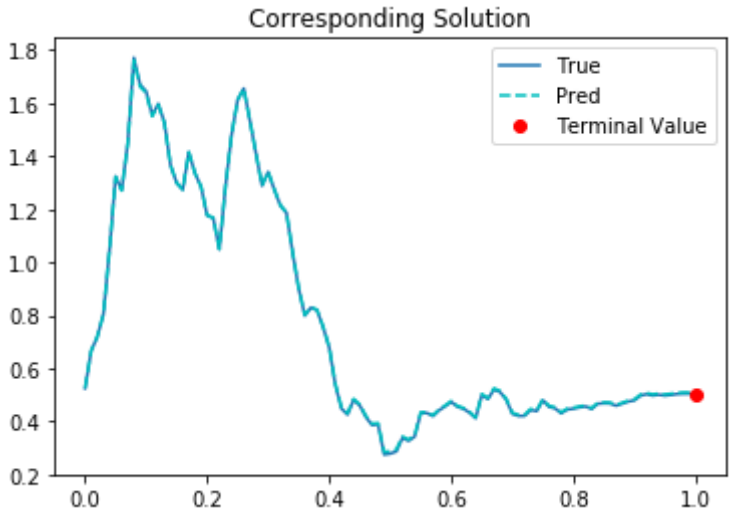

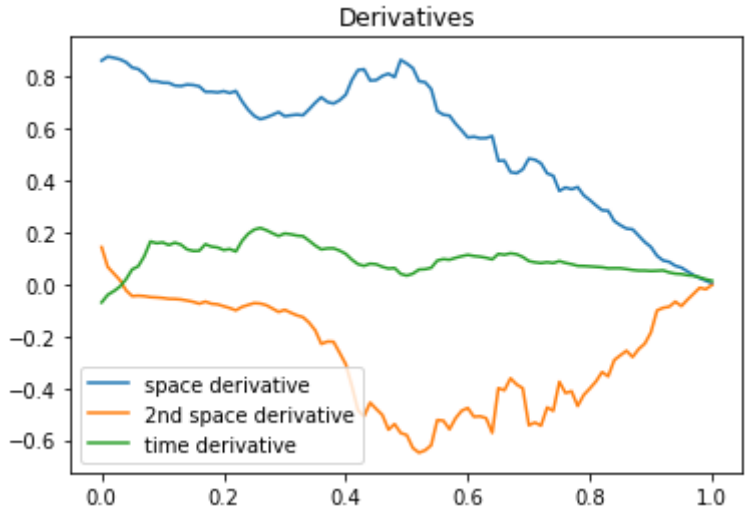


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11911th Epoch:
traing loss is 9.30053138290532e-05, test loss is 5.188387876842171e-05, and learning rate is

11921th Epoch:
traing loss is 6.01846695644781e-05, test loss is 4.658672696677968e-05, and learning rate is

11931th Epoch:
traing loss is 3.317504524602555e-05, test loss is 4.3896896386286244e-05, and learning rate

11941th Epoch:
traing loss is 4.4970180169912055e-05, test loss is 4.3503085180418566e-05, and learning rate

11951th Epoch:
traing loss is 2.9732176699326374e-05, test loss is 3.9675906009506434e-05, and learning rate

11961th Epoch:
traing loss is 6.272875907598063e-05, test loss is 4.171911859884858e-05, and learning rate i

11971th Epoch:
traing loss is 3.25180335494224e-05, test loss is 3.9229784306371585e-05, and learning rate i
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```
11981th Epoch:
traing loss is 4.67703357571736e-05, test loss is 4.1144601709675044e-05, and learning rate i

11991th Epoch:
traing loss is 6.063710316084325e-05, test loss is 4.524867108557373e-05, and learning rate i

12001th Epoch:
traing loss is 5.827183849760331e-05, test loss is 4.1770712414290756e-05, and learning rate
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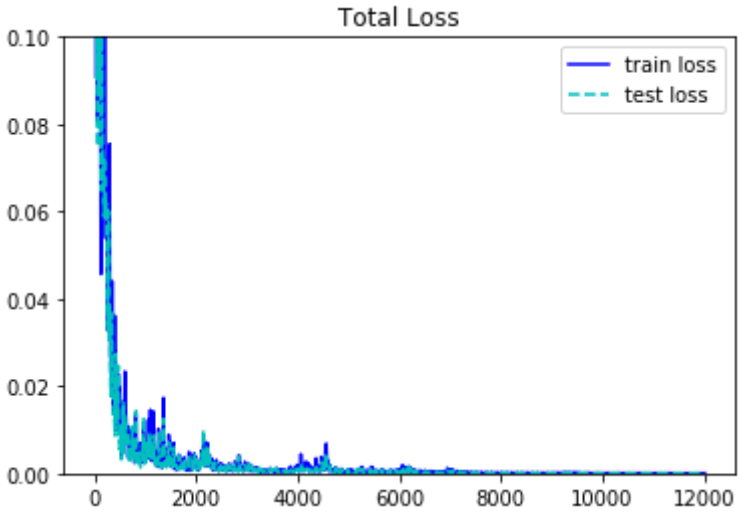

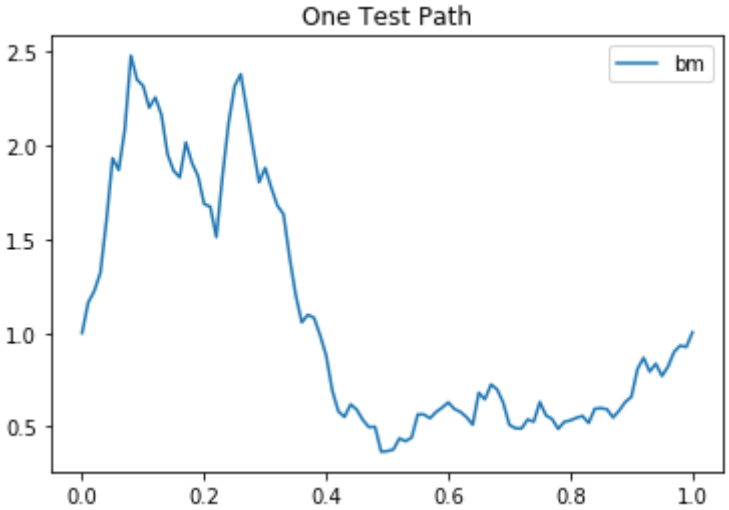

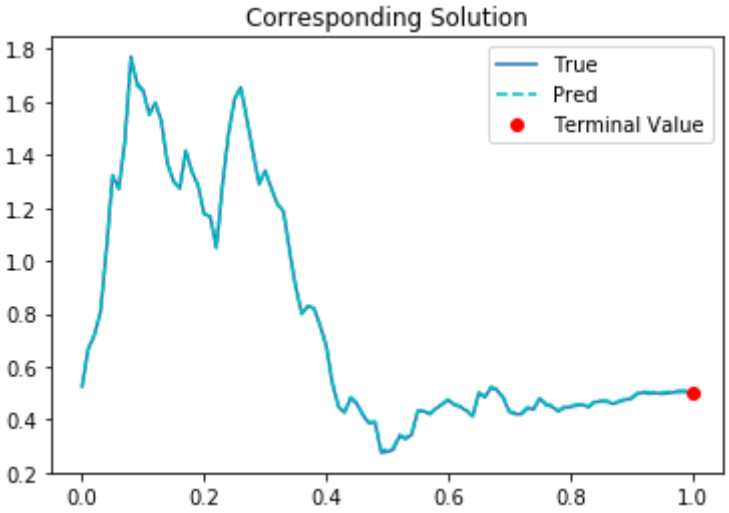

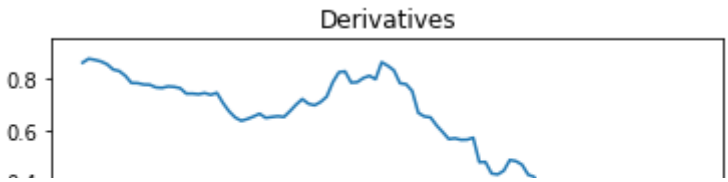

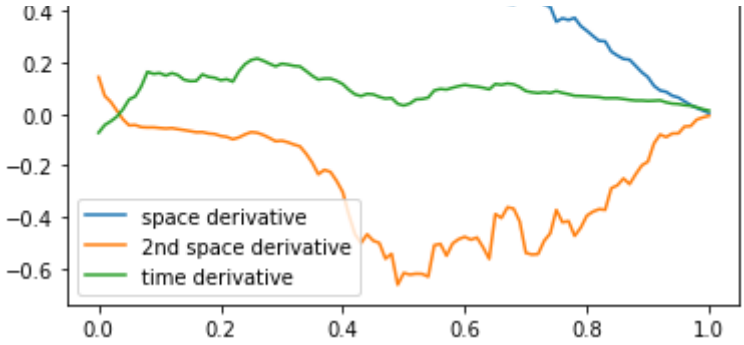


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12011th Epoch:
traing loss is 9.358344686916098e-05, test loss is 5.0436930905561894e-05, and learning rate

12021th Epoch:
traing loss is 5.9808204241562635e-05, test loss is 4.5507167669711635e-05, and learning rate

12031th Epoch:
traing loss is 3.139348700642586e-05, test loss is 4.2297393520129845e-05, and learning rate

12041th Epoch:
traing loss is 5.242606857791543e-05, test loss is 4.1662082367111e-05, and learning rate is

12051th Epoch:
traing loss is 3.3620519388932735e-05, test loss is 4.0813087252900004e-05, and learning rate

12061th Epoch:
traing loss is 6.479019793914631e-05, test loss is 4.111412272322923e-05, and learning rate i

12071th Epoch:
traing loss is 3.351550913066603e-05, test loss is 3.958287925343029e-05, and learning rate i

12081th Epoch:
traing loss is 4.669504050980322e-05, test loss is 4.069165152031928e-05, and learning rate i

12091th Epoch:
traing loss is 6.138398020993918e-05, test loss is 4.442046702024527e-05, and learning rate i

12101th Epoch:
traing loss is 5.7021396060008556e-05, test loss is 4.047926267958246e-05, and learning rate
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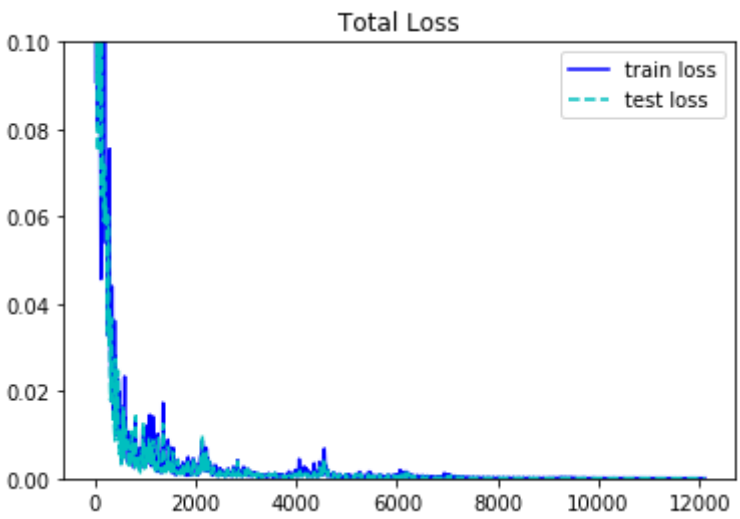

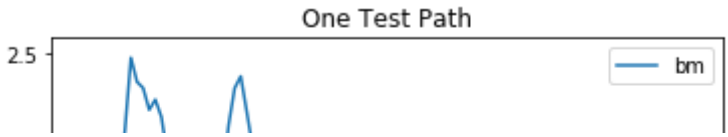

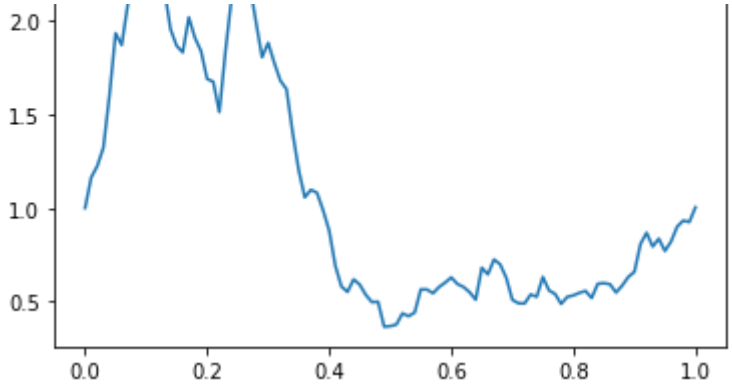

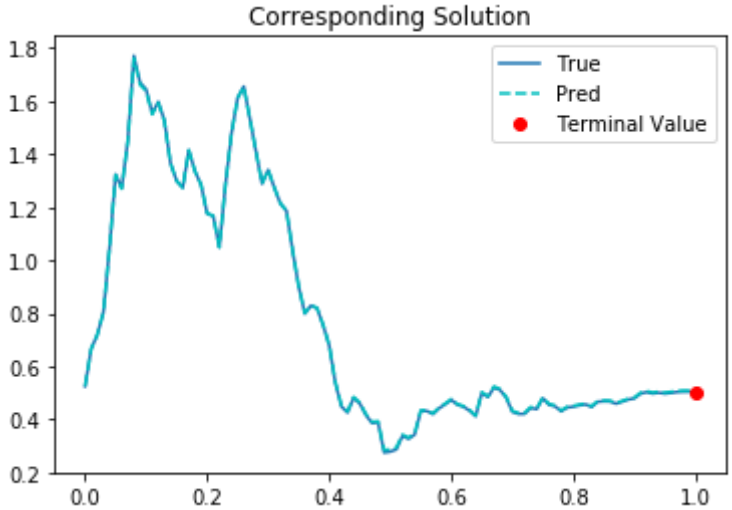

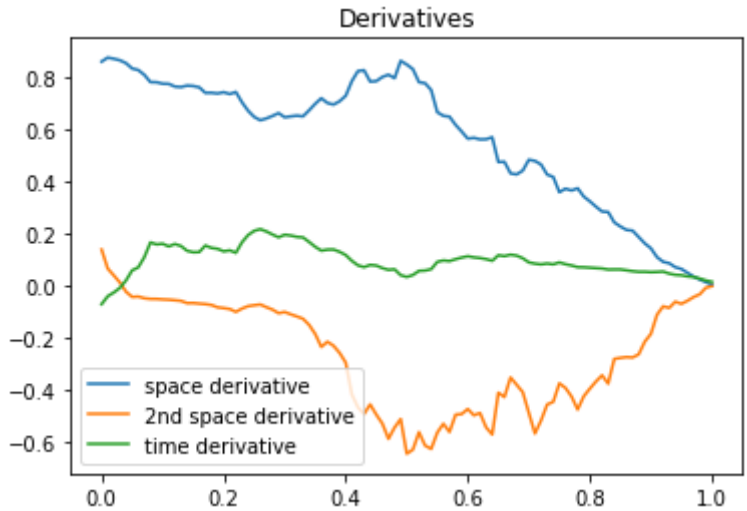


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12111th Epoch:
traing loss is 8.977613470051438e-05, test loss is 5.077431706013158e-05, and learning rate i

12121th Epoch:
traing loss is 5.961265924270265e-05, test loss is 4.4349570089252666e-05, and learning rate

12131th Epoch:
traing loss is 3.164281224599108e-05, test loss is 4.156617433181964e-05, and learning rate i

12141th Epoch:
traing loss is 4.369580710772425e-05, test loss is 4.303184323362075e-05, and learning rate i

12151th Epoch:
traing loss is 2.9552096748375334e-05, test loss is 3.942571856896393e-05, and learning rate

12161th Epoch:
traing loss is 6.196874164743349e-05, test loss is 4.131440800847486e-05, and learning rate i
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12171th Epoch:
traing loss is 3.2393887522630394e-05, test loss is 3.8660484278807417e-05, and learning rate

12181th Epoch:
traing loss is 4.6022789319977164e-05, test loss is 4.056448597111739e-05, and learning rate

12191th Epoch:
traing loss is 6.0278165619820356e-05, test loss is 4.441446435521357e-05, and learning rate

12201th Epoch:
traing loss is 5.6582943216199055e-05, test loss is 4.17235387430992e-05, and learning rate i
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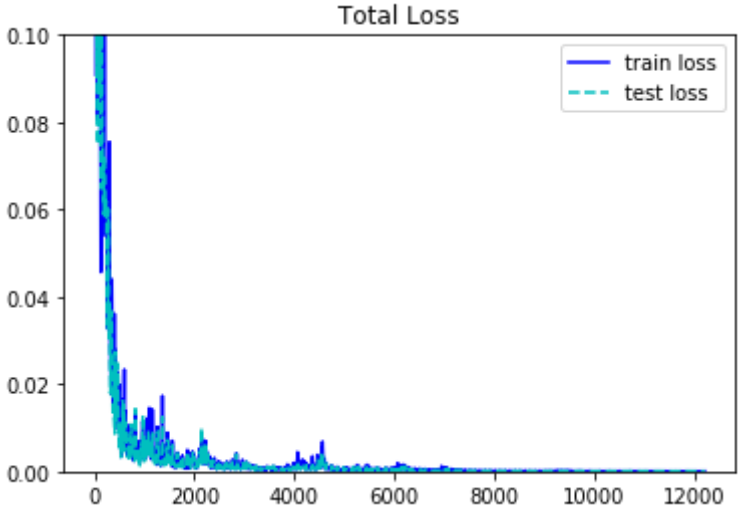

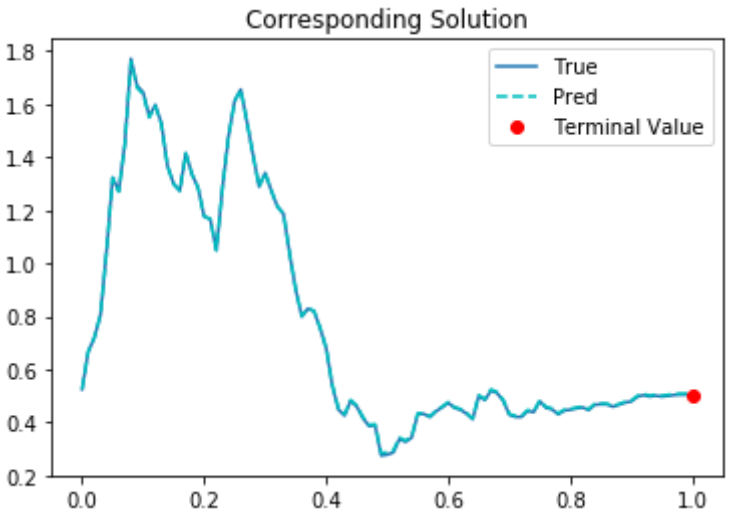

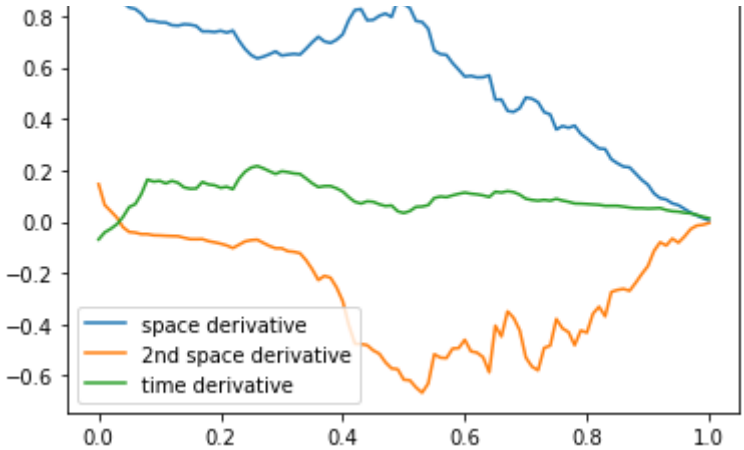


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12211th Epoch:
traing loss is 9.043341560754925e-05, test loss is 4.8623260227032006e-05, and learning rate

12221th Epoch:
traing loss is 5.8527606597635895e-05, test loss is 4.385917418403551e-05, and learning rate

12231th Epoch:
traing loss is 3.0357159630511887e-05, test loss is 4.102907041669823e-05, and learning rate

12241th Epoch:
traing loss is 5.0826023652916774e-05, test loss is 4.067205736646429e-05, and learning rate

12251th Epoch:
traing loss is 3.2941017707344145e-05, test loss is 4.009383701486513e-05, and learning rate

12261th Epoch:
traing loss is 6.582924834219739e-05, test loss is 3.99216114601586e-05, and learning rate is

12271th Epoch:
traing loss is 3.278241274529137e-05, test loss is 3.872716843034141e-05, and learning rate i

12281th Epoch:
traing loss is 4.599056046572514e-05, test loss is 4.039279883727431e-05, and learning rate i

12291th Epoch:
traing loss is 5.926564335823059e-05, test loss is 4.303556852391921e-05, and learning rate i

12301th Epoch:
traing loss is 5.643070471705869e-05, test loss is 4.0246744902106e-05, and learning rate is
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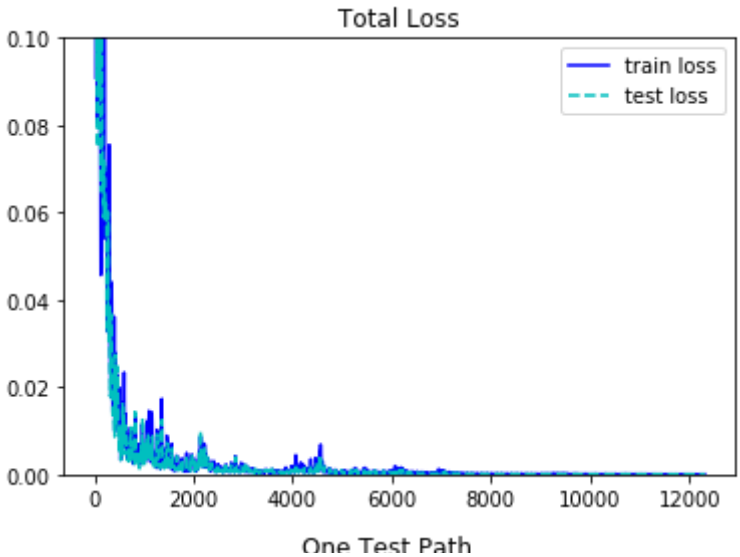

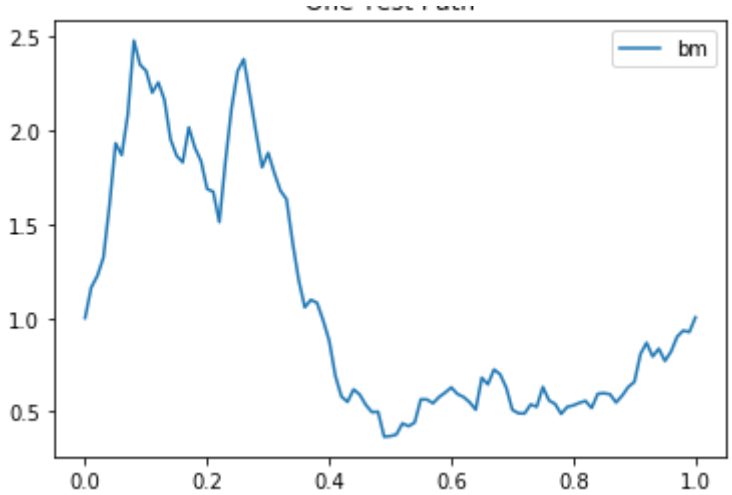

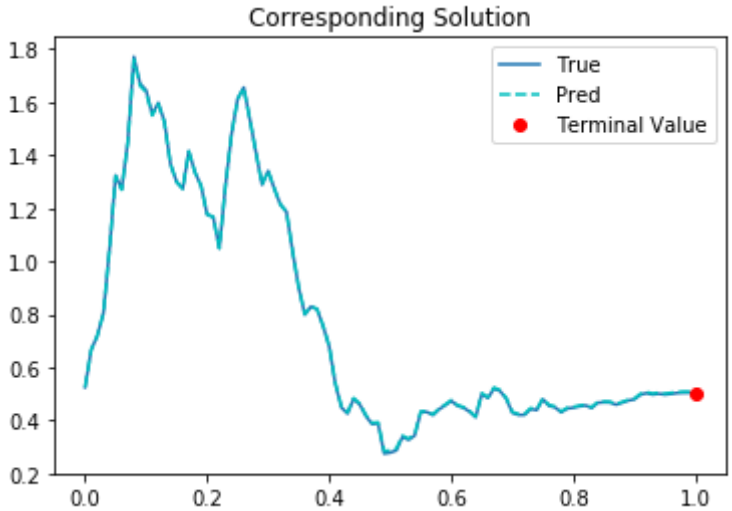

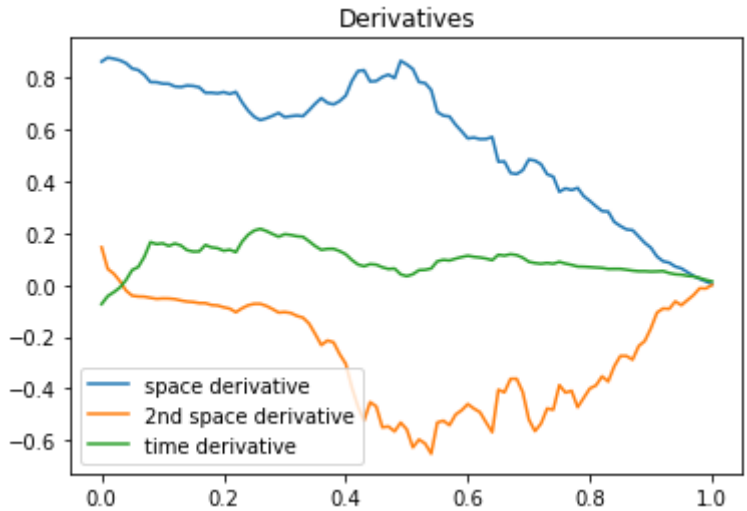


```
12311th Epoch:
traing loss is 8.59915598994121e-05, test loss is 4.8309248086297885e-05, and learning rate i

12321th Epoch:
traing loss is 5.776277248514816e-05, test loss is 4.325140980654396e-05, and learning rate i

12331th Epoch:
traing loss is 3.0207007512217388e-05, test loss is 4.0552386053605005e-05, and learning rate

12341th Epoch:
traing loss is 4.22001066908706e-05, test loss is 4.12686713389121e-05, and learning rate is

12351th Epoch:
traing loss is 2.8359176212688908e-05, test loss is 3.847150583169423e-05, and learning rate
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```
12361th Epoch:
traing loss is 6.146746454760432e-05, test loss is 4.1129402234219015e-05, and learning rate

12371th Epoch:
traing loss is 3.222956365789287e-05, test loss is 3.816422395175323e-05, and learning rate i

12381th Epoch:
traing loss is 4.530171645455994e-05, test loss is 4.0082435589283705e-05, and learning rate

12391th Epoch:
traing loss is 5.910384425078519e-05, test loss is 4.386870205053128e-05, and learning rate i

12401th Epoch:
traing loss is 5.488645183504559e-05, test loss is 4.0685095882508904e-05, and learning rate
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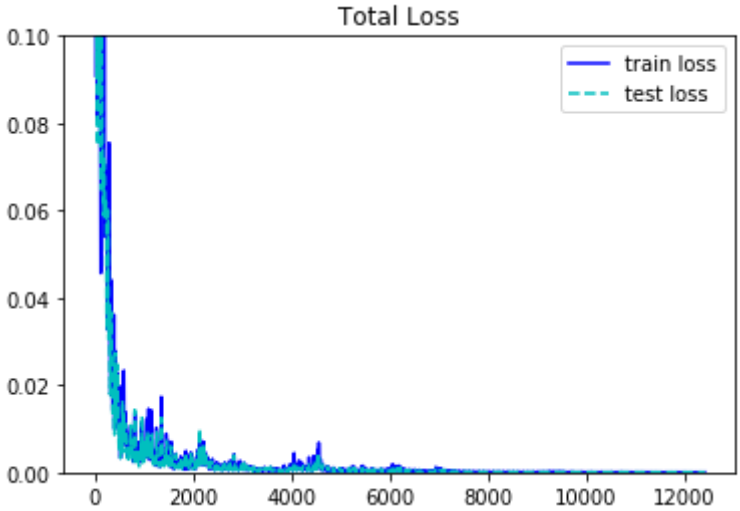

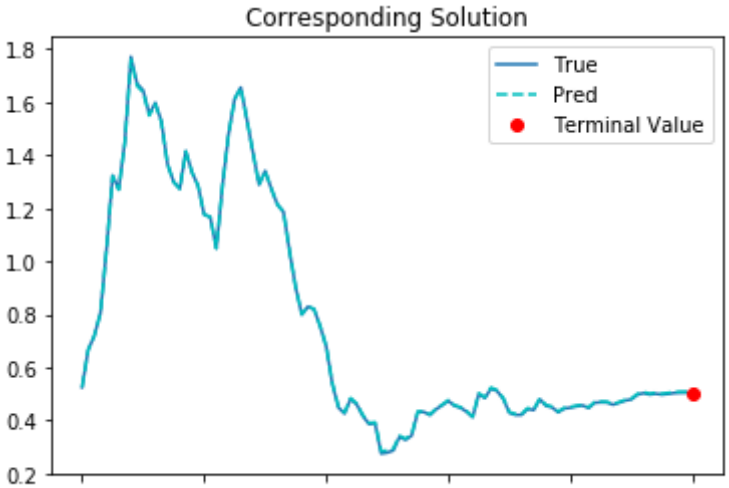

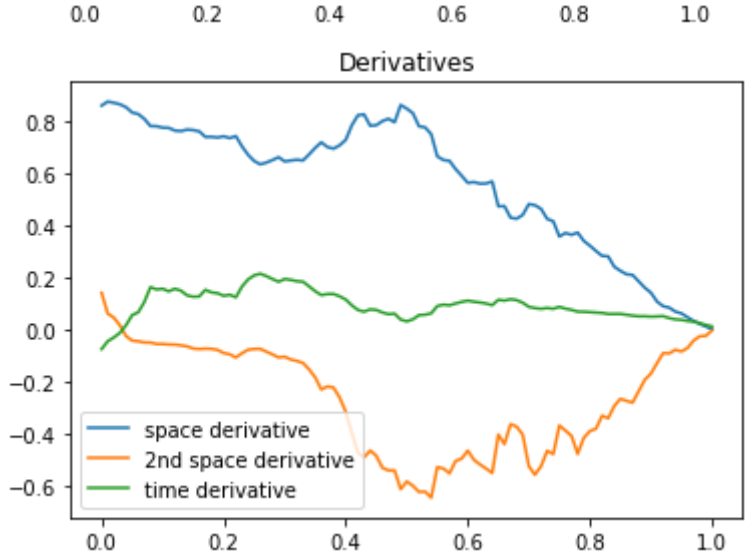


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12411th Epoch:
traing loss is 9.038187272381037e-05, test loss is 4.748612627736293e-05, and learning rate i

12421th Epoch:
traing loss is 5.754737867391668e-05, test loss is 4.2879113607341424e-05, and learning rate

12431th Epoch:
traing loss is 2.931426752184052e-05, test loss is 3.9711467252345756e-05, and learning rate

12441th Epoch:
traing loss is 5.012270776205696e-05, test loss is 4.009139593108557e-05, and learning rate i

12451th Epoch:
traing loss is 3.261179881519638e-05, test loss is 3.981300687883049e-05, and learning rate i

12461th Epoch:
traing loss is 6.28988491371274e-05, test loss is 3.928922888007946e-05, and learning rate is

12471th Epoch:
traing loss is 3.2499137887498364e-05, test loss is 3.847102198051289e-05, and learning rate

12481th Epoch:
traing loss is 4.398795499582775e-05, test loss is 3.897069473168813e-05, and learning rate i

12491th Epoch:
traing loss is 5.887545194127597e-05, test loss is 4.267823896952905e-05, and learning rate i

12501th Epoch:
traing loss is 5.531442729989067e-05, test loss is 4.063528831466101e-05, and learning rate i
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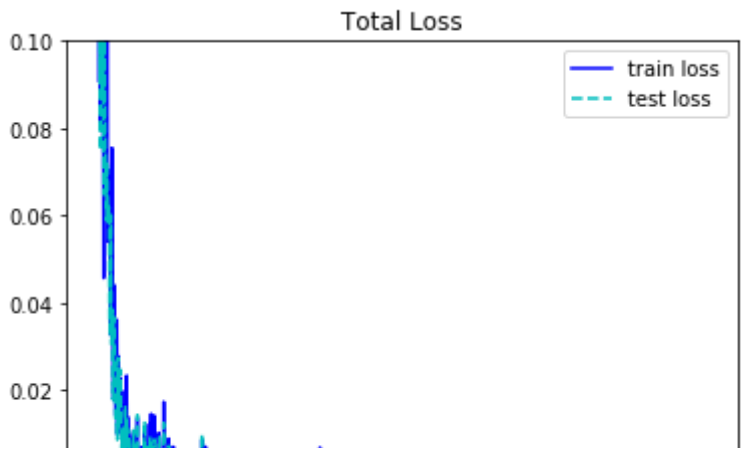

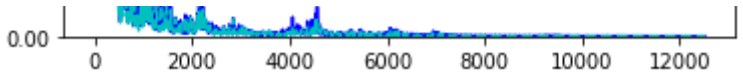

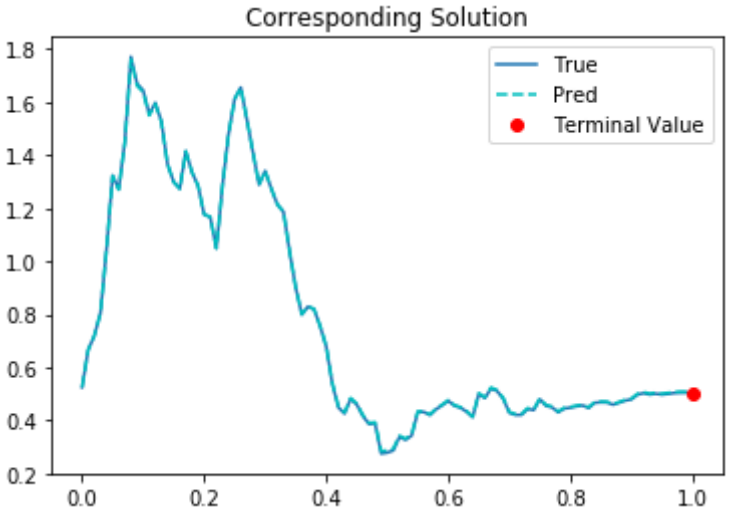

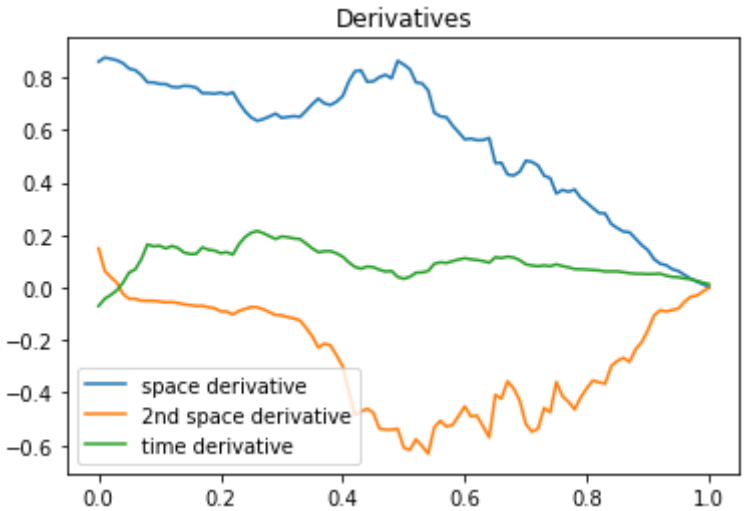


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12511th Epoch:
traing loss is 8.443419937975705e-05, test loss is 4.6689543523825705e-05, and learning rate

12521th Epoch:
traing loss is 5.735809827456251e-05, test loss is 4.233375875628553e-05, and learning rate i

12531th Epoch:
traing loss is 2.9501219614758156e-05, test loss is 3.994957296526991e-05, and learning rate

12541th Epoch:
traing loss is 4.055059253005311e-05, test loss is 3.965641371905804e-05, and learning rate i
```

```
12551th Epoch:
traing loss is 2.8393509637680836e-05, test loss is 3.817178730969317e-05, and learning rate

12561th Epoch:
traing loss is 6.0104590374976397e-05, test loss is 3.971974365413189e-05, and learning rate

12571th Epoch:
traing loss is 3.139295949949883e-05, test loss is 3.67671818821691e-05, and learning rate is

12581th Epoch:
traing loss is 4.5970889914315194e-05, test loss is 3.9167600334621966e-05, and learning rate

12591th Epoch:
traing loss is 5.811063601868227e-05, test loss is 4.2288749682484195e-05, and learning rate

12601th Epoch:
traing loss is 5.441965549835004e-05, test loss is 4.059005368617363e-05, and learning rate i
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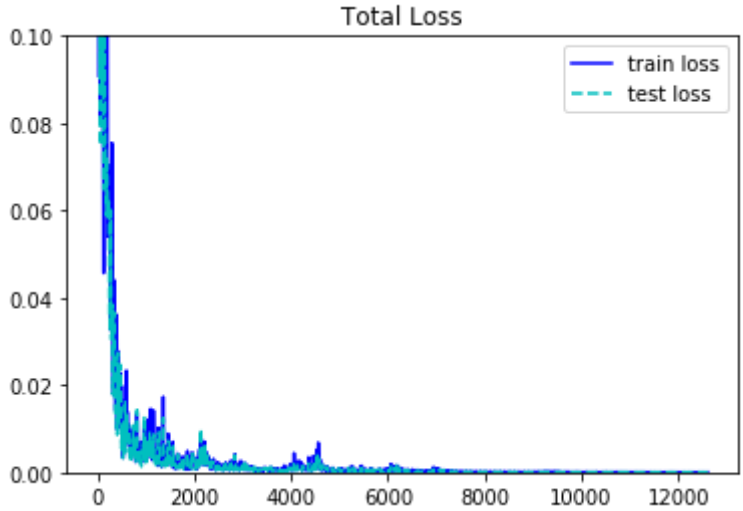

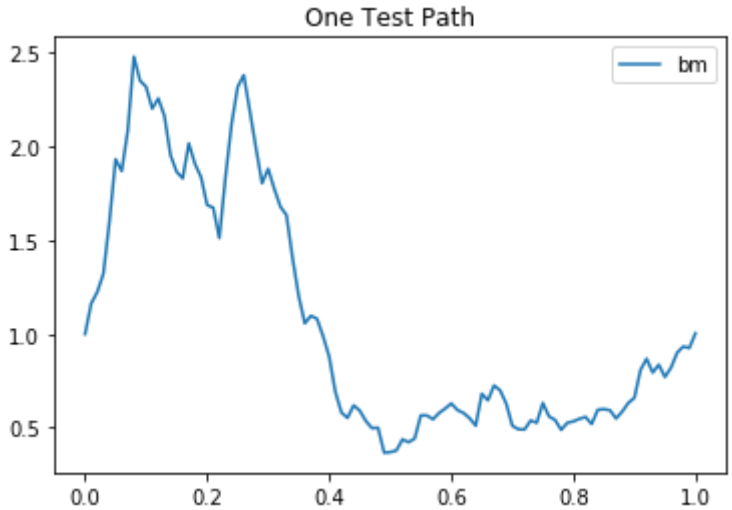

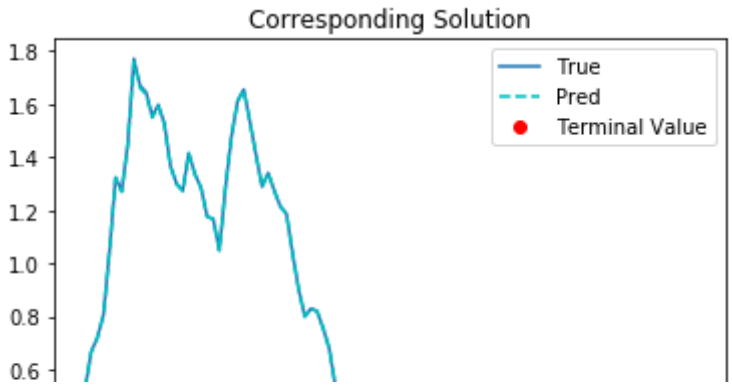

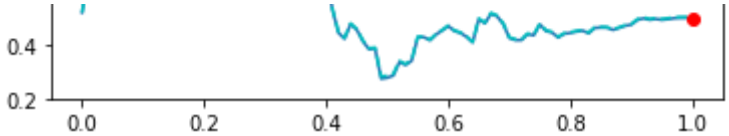

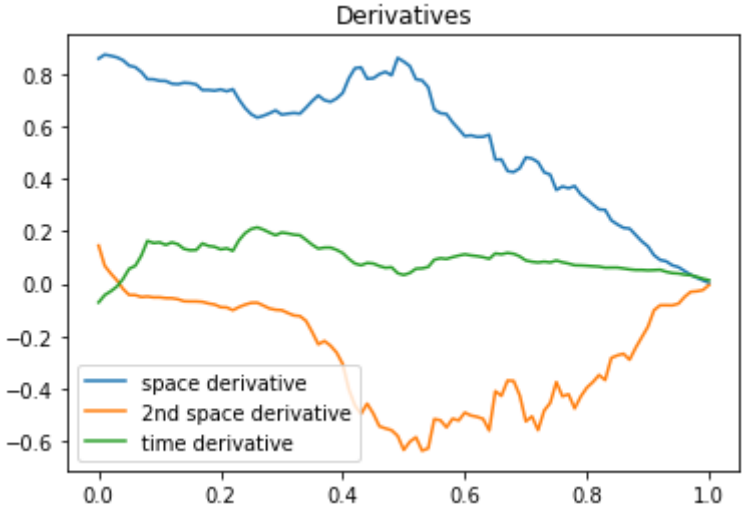


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12611th Epoch:
traing loss is 8.63101813592948e-05, test loss is 4.5919350668555126e-05, and learning rate i

12621th Epoch:
traing loss is 5.649301965604536e-05, test loss is 4.195821384200826e-05, and learning rate i

12631th Epoch:
traing loss is 2.8827422283939086e-05, test loss is 3.9681784983258694e-05, and learning rate

12641th Epoch:
traing loss is 4.727535633719526e-05, test loss is 3.926235149265267e-05, and learning rate i

12651th Epoch:
traing loss is 3.2039992220234126e-05, test loss is 3.954549174522981e-05, and learning rate

12661th Epoch:
traing loss is 6.25863904133439e-05, test loss is 3.793881114688702e-05, and learning rate is

12671th Epoch:
traing loss is 3.1810002838028595e-05, test loss is 3.7499386962736025e-05, and learning rate

12681th Epoch:
traing loss is 4.391609036247246e-05, test loss is 3.8682916056131944e-05, and learning rate

12691th Epoch:
traing loss is 5.712726851925254e-05, test loss is 4.174399145995267e-05, and learning rate i

12701th Epoch:
traing loss is 5.3417537856148556e-05, test loss is 4.02507466787938e-05, and learning rate i
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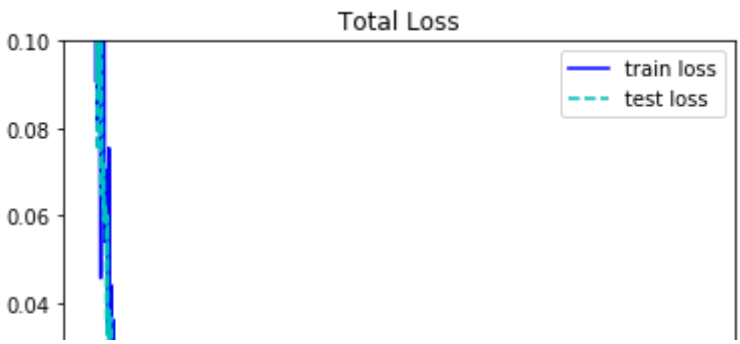

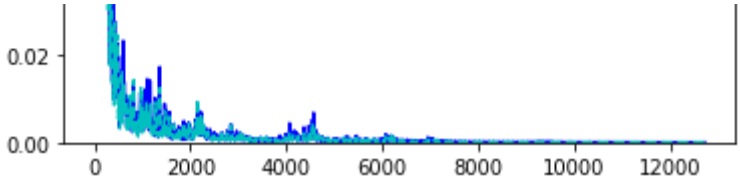

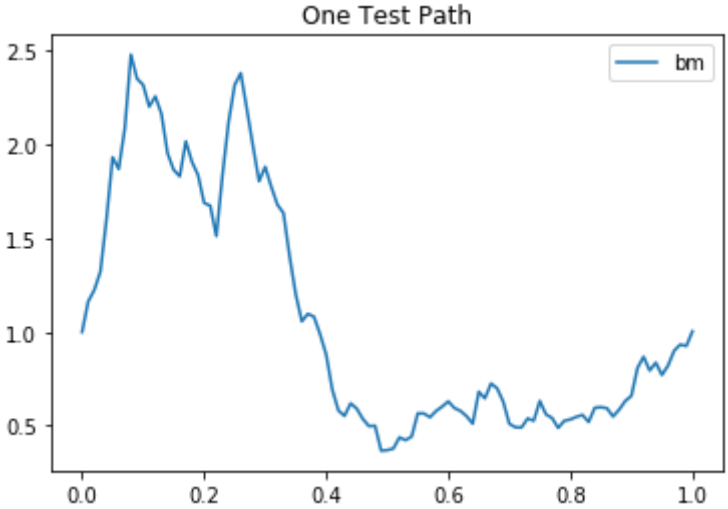

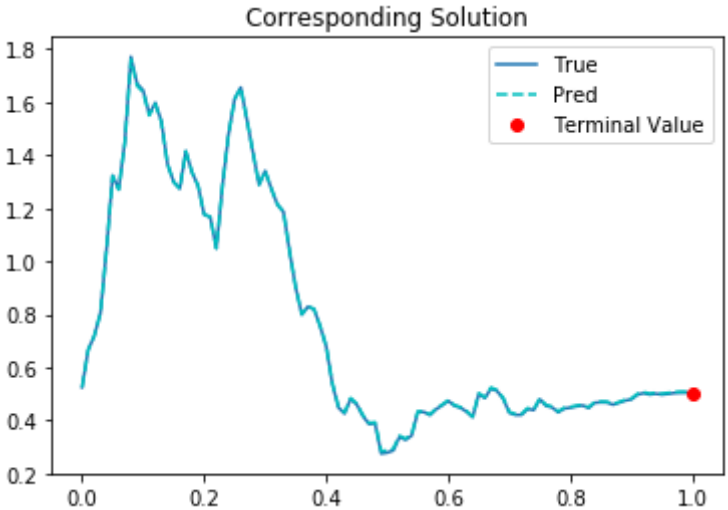

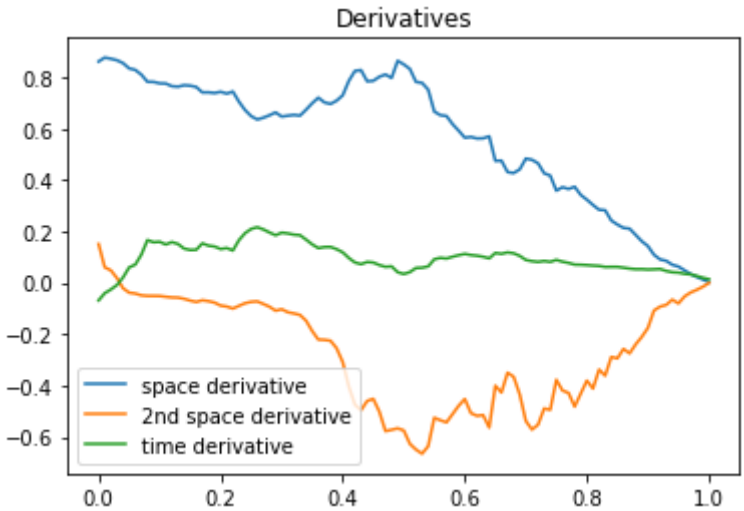


```
12711th Epoch:
traing loss is 8.46154143800959e-05, test loss is 4.749200525111519e-05, and learning rate is

12721th Epoch:
traing loss is 5.6082219089148566e-05, test loss is 4.168488158029504e-05, and learning rate

12731th Epoch:
traing loss is 2.9251712476252578e-05, test loss is 4.039872874272987e-05, and learning rate
```

```
12741th Epoch:
traing loss is 3.907395148416981e-05, test loss is 3.84473278245423e-05, and learning rate is

12751th Epoch:
traing loss is 2.8492859200923704e-05, test loss is 3.859696153085679e-05, and learning rate

12761th Epoch:
traing loss is 5.796354162157513e-05, test loss is 3.8555885112145916e-05, and learning rate

12771th Epoch:
traing loss is 3.100532558164559e-05, test loss is 3.697873398778029e-05, and learning rate i

12781th Epoch:
traing loss is 4.4146123400423676e-05, test loss is 3.774021752178669e-05, and learning rate

12791th Epoch:
traing loss is 5.546420652535744e-05, test loss is 4.111285670660436e-05, and learning rate i

12801th Epoch:
traing loss is 5.2864139433950186e-05, test loss is 3.943731280742213e-05, and learning rate
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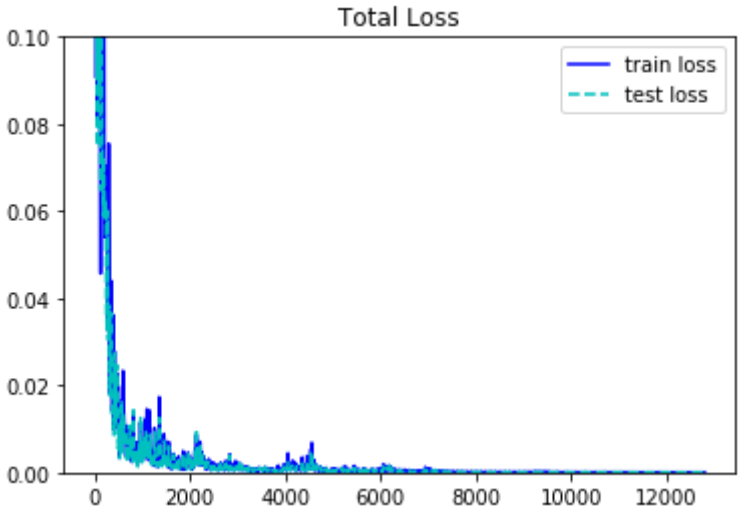

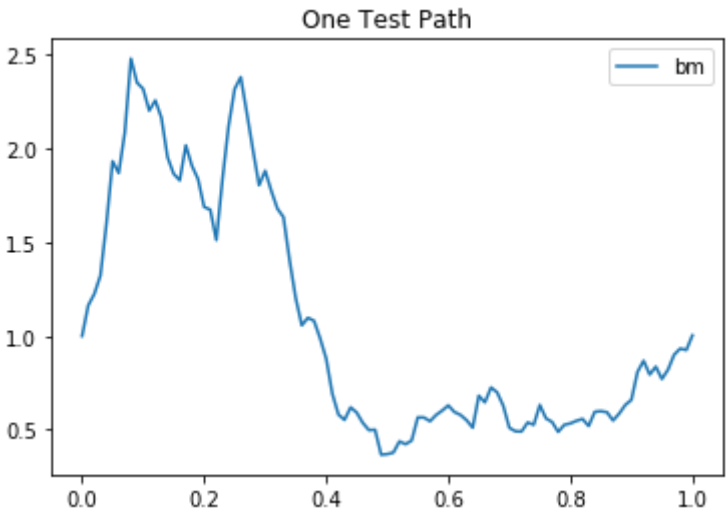

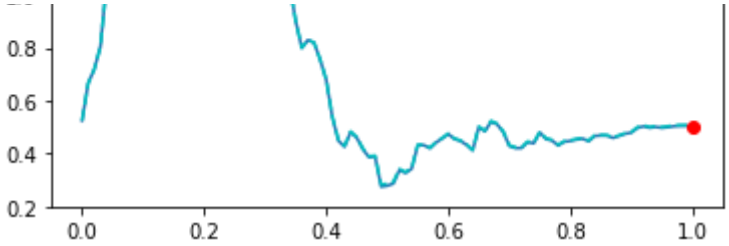

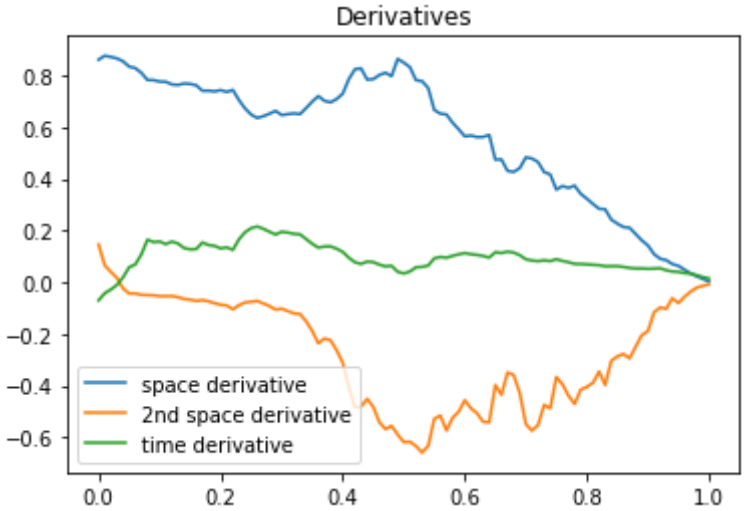


```
12811th Epoch:
traing loss is 8.343627268914133e-05, test loss is 4.640756014850922e-05, and learning rate i

12821th Epoch:
traing loss is 5.63896719540935e-05, test loss is 4.0477207221556455e-05, and learning rate i

12831th Epoch:
traing loss is 2.8324522645561956e-05, test loss is 3.882716191583313e-05, and learning rate

12841th Epoch:
traing loss is 4.5576580305350944e-05, test loss is 3.9026188460411504e-05, and learning rate

12851th Epoch:
traing loss is 3.2336931326426566e-05, test loss is 3.986683441326022e-05, and learning rate

12861th Epoch:
traing loss is 6.138952448964119e-05, test loss is 3.7671325117116794e-05, and learning rate

12871th Epoch:
traing loss is 3.062076575588435e-05, test loss is 3.682680107885972e-05, and learning rate i

12881th Epoch:
traing loss is 4.333648757892661e-05, test loss is 3.8031994336051866e-05, and learning rate

12891th Epoch:
traing loss is 5.528023393708281e-05, test loss is 4.001946945209056e-05, and learning rate i

12901th Epoch:
traing loss is 5.190171214053407e-05, test loss is 3.8341855542967096e-05, and learning rate
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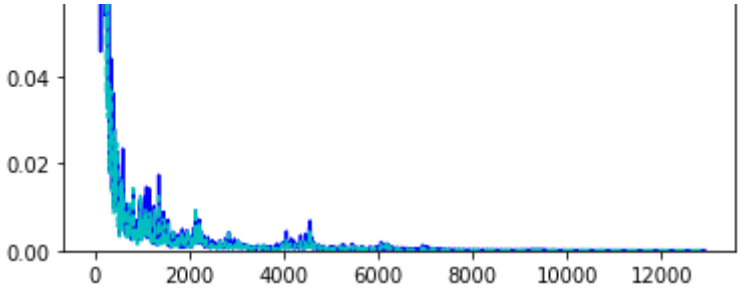

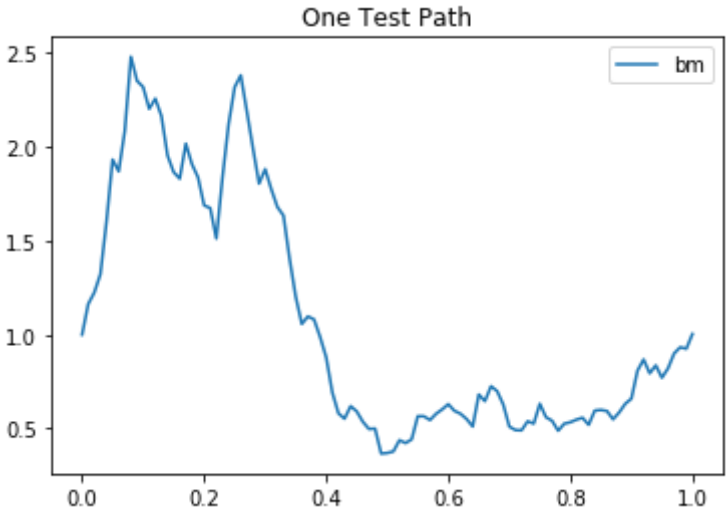

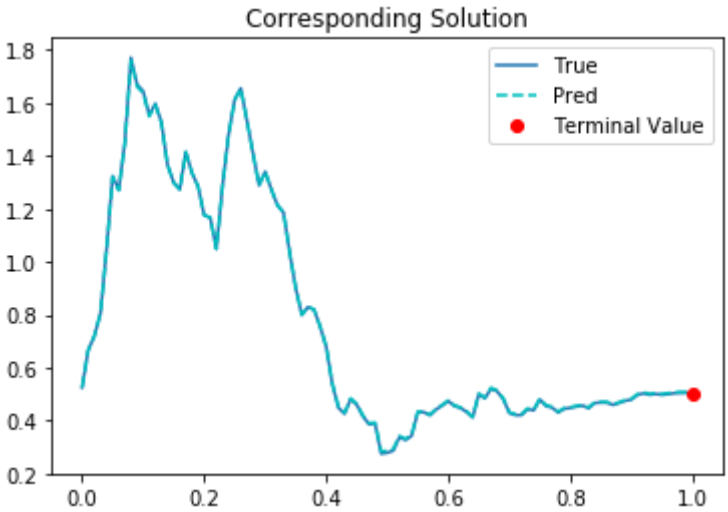

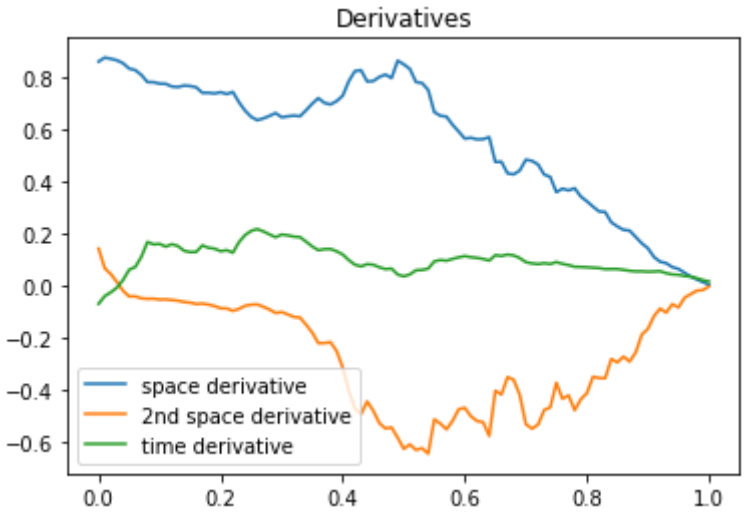

12911th Epoch:
traing loss is 8.268125384347513e-05, test loss is 4.73337531730067e-05, and learning rate is

12921th Epoch:
traing loss is 5.6471799325663596e-05, test loss is 4.177687151241116e-05, and learning rate

```
12931th Epoch:
traing loss is 2.9286480639711954e-05, test loss is 4.07498846470844e-05, and learning rate i

12941th Epoch:
traing loss is 3.8563968701055273e-05, test loss is 3.742661283467896e-05, and learning rate

12951th Epoch:
traing loss is 2.8268928872421384e-05, test loss is 3.767746238736436e-05, and learning rate

12961th Epoch:
traing loss is 5.869810047443025e-05, test loss is 3.8070811569923535e-05, and learning rate

12971th Epoch:
traing loss is 3.031647611351218e-05, test loss is 3.645044125732966e-05, and learning rate i

12981th Epoch:
traing loss is 4.3778272811323404e-05, test loss is 3.7522790080402046e-05, and learning rate

12991th Epoch:
traing loss is 5.4998927225824445e-05, test loss is 3.957722583436407e-05, and learning rate

13001th Epoch:
traing loss is 5.106061144033447e-05, test loss is 3.913703403668478e-05, and learning rate i
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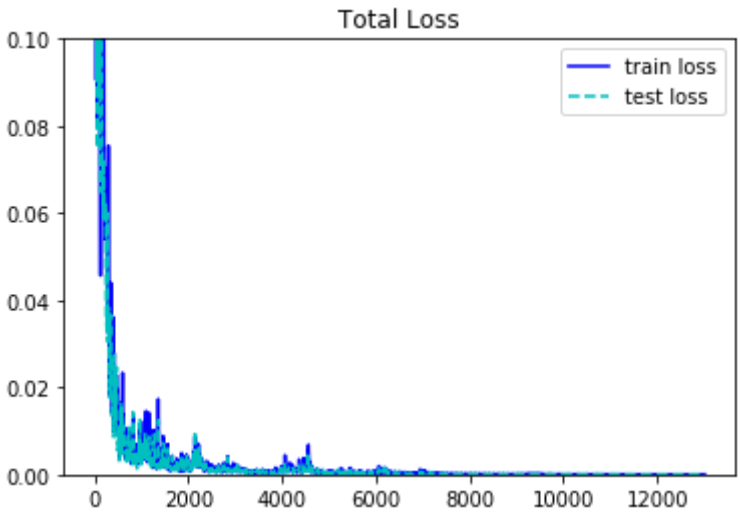

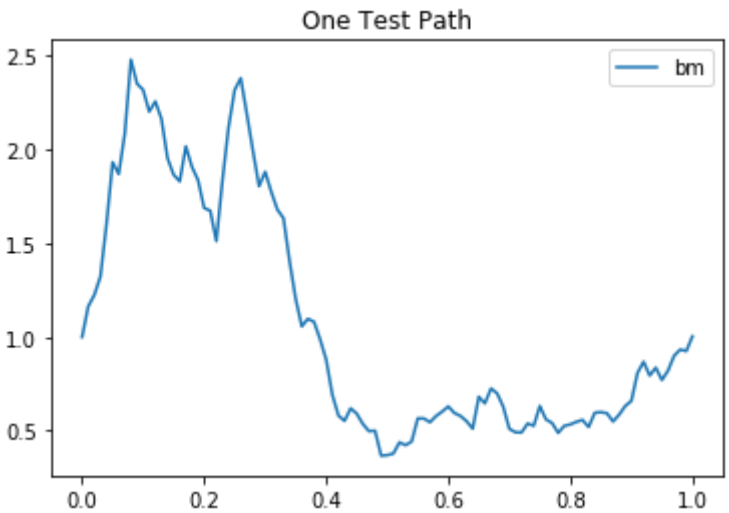

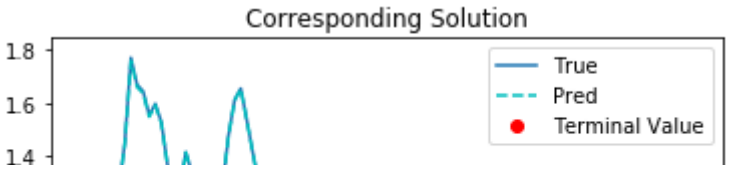

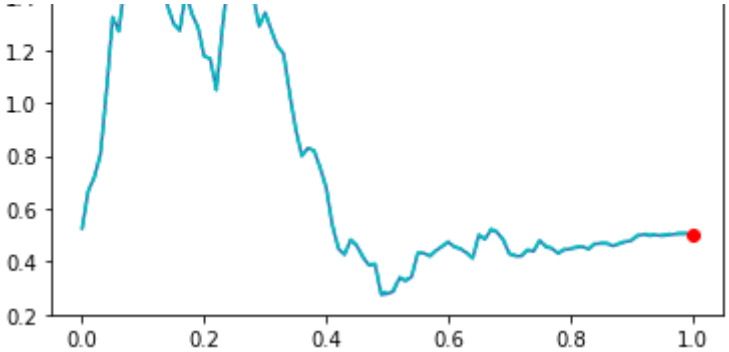

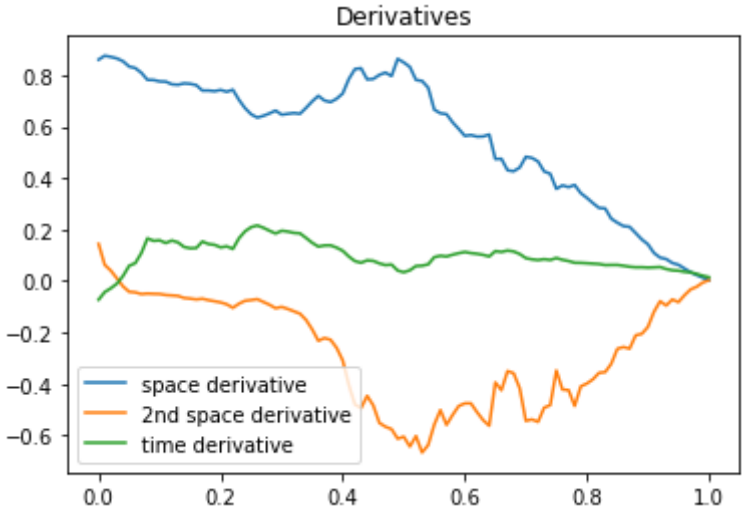


```
13011th Epoch:
traing loss is 8.228069054894149e-05, test loss is 4.6639983338536695e-05, and learning rate

13021th Epoch:
traing loss is 5.765404421254061e-05, test loss is 4.1558421798981726e-05, and learning rate

13031th Epoch:
traing loss is 2.907264388340991e-05, test loss is 4.008375617559068e-05, and learning rate i

13041th Epoch:
traing loss is 4.395356882014312e-05, test loss is 3.856172043015249e-05, and learning rate i

13051th Epoch:
traing loss is 3.170337367919274e-05, test loss is 3.954006388084963e-05, and learning rate i

13061th Epoch:
traing loss is 5.961233182461001e-05, test loss is 3.651200677268207e-05, and learning rate i

13071th Epoch:
traing loss is 3.076277062064037e-05, test loss is 3.649108111858368e-05, and learning rate i

13081th Epoch:
traing loss is 4.248972254572436e-05, test loss is 3.728620140464045e-05, and learning rate i

13091th Epoch:
traing loss is 5.4210384405450895e-05, test loss is 3.940890746889636e-05, and learning rate

13101th Epoch:
traing loss is 5.1630300731630996e-05, test loss is 3.896869020536542e-05, and learning rate
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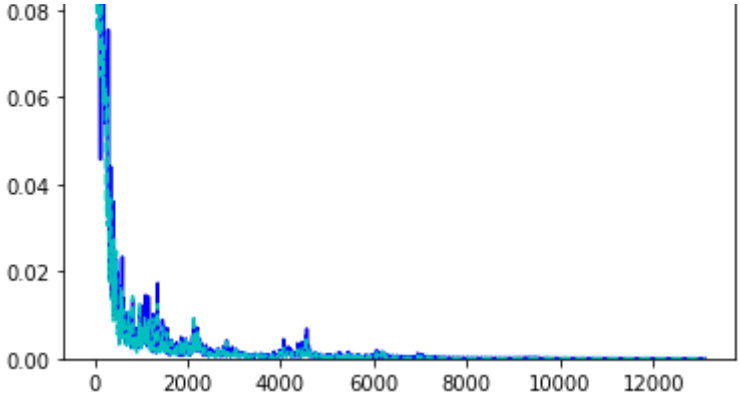
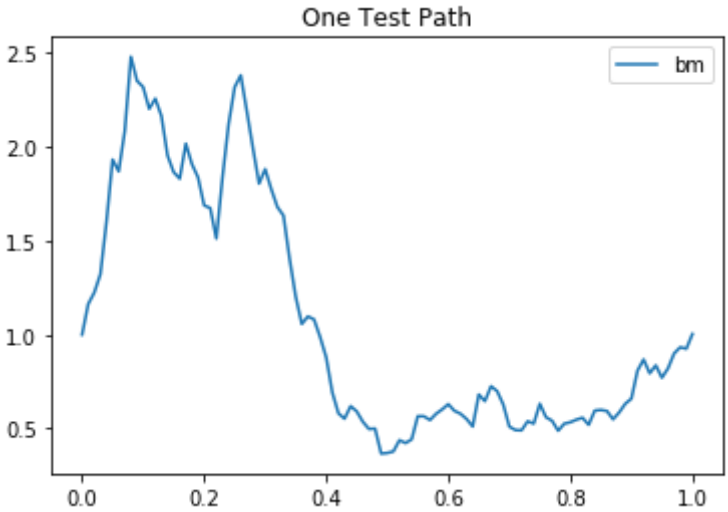
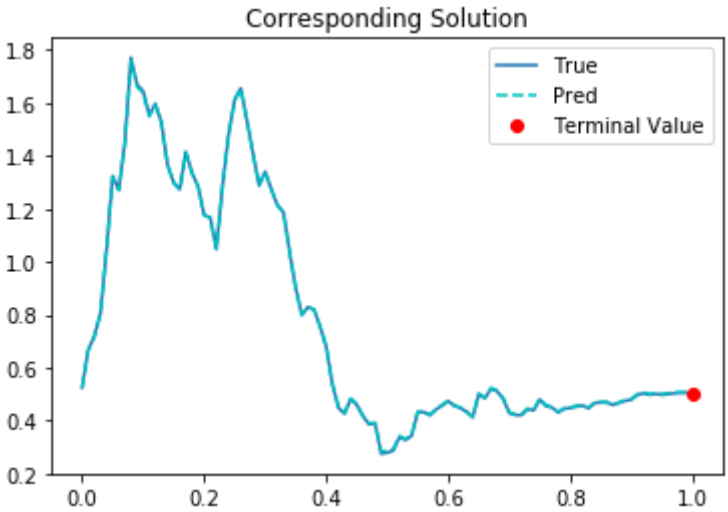
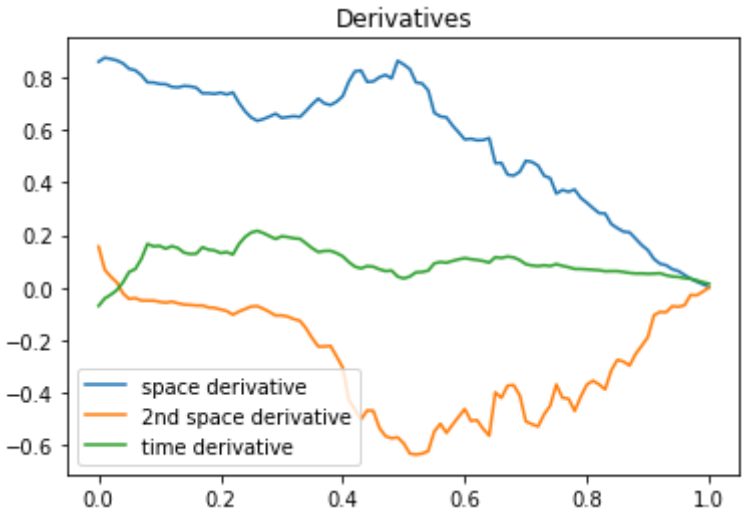








13111th Epoch:
traing loss is 8.249565144069493e-05, test loss is 4.728407657239586e-05, and learning rate i

```
13121th Epoch:
traing loss is 5.769680137746036e-05, test loss is 4.1955132473958656e-05, and learning rate

13131th Epoch:
traing loss is 2.9413498850772157e-05, test loss is 4.036344398627989e-05, and learning rate

13141th Epoch:
traing loss is 3.797909084823914e-05, test loss is 3.686308264150284e-05, and learning rate i

13151th Epoch:
traing loss is 2.726792445173487e-05, test loss is 3.7096597225172445e-05, and learning rate

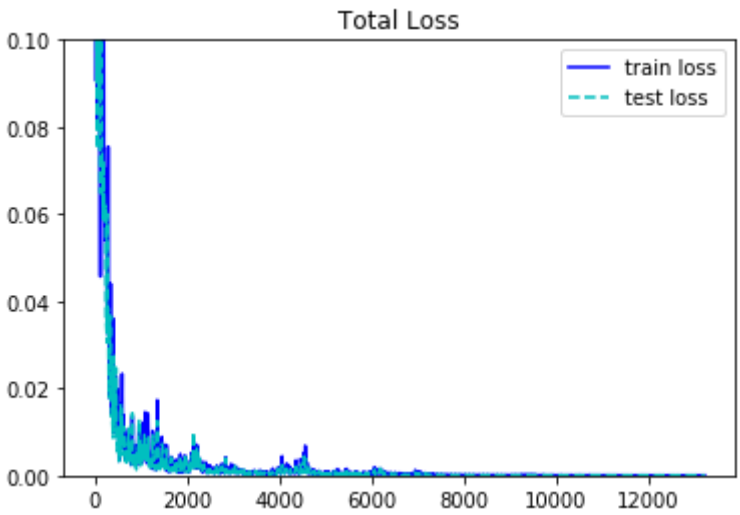
13161th Epoch:
traing loss is 5.763828448834829e-05, test loss is 3.673366518341936e-05, and learning rate i

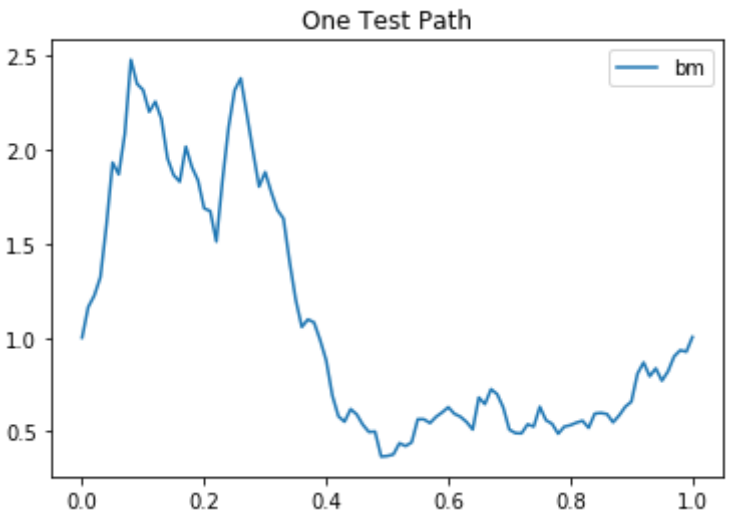
13171th Epoch:
traing loss is 3.064230622840114e-05, test loss is 3.577673851395957e-05, and learning rate i

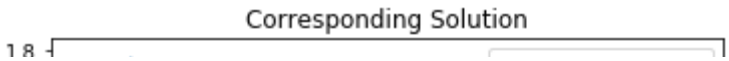
13181th Epoch:
traing loss is 4.3849679059349e-05, test loss is 3.6959423596272245e-05, and learning rate is

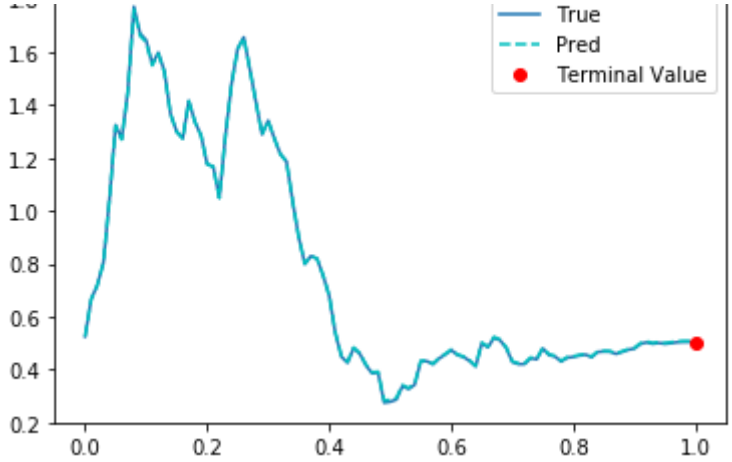
13191th Epoch:
traing loss is 5.298339601722546e-05, test loss is 3.830575951724313e-05, and learning rate i

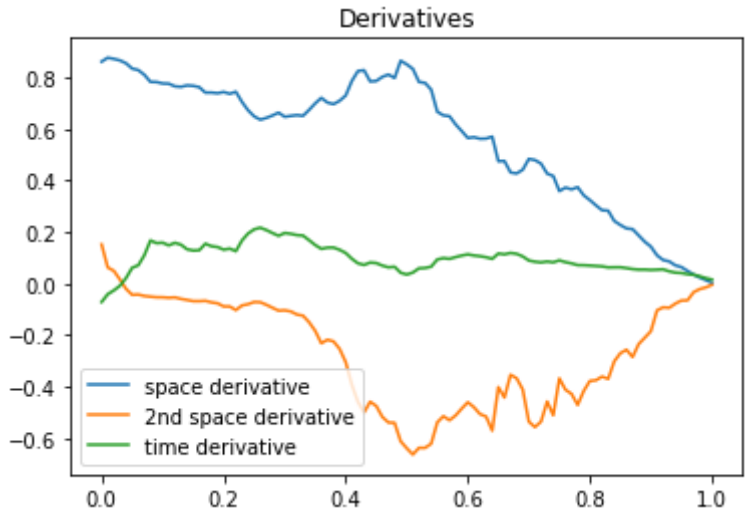
13201th Epoch:
traing loss is 5.054269786342047e-05, test loss is 3.822324288194068e-05, and learning rate i
```












```
13211th Epoch:
traing loss is 8.350482676178217e-05, test loss is 4.762989192386158e-05, and learning rate i

13221th Epoch:
traing loss is 5.929542749072425e-05, test loss is 4.290904325898737e-05, and learning rate i

13231th Epoch:
traing loss is 2.8273741918383166e-05, test loss is 3.990043842350133e-05, and learning rate

13241th Epoch:
traing loss is 4.2820269300136715e-05, test loss is 3.743431807379238e-05, and learning rate

13251th Epoch:
traing loss is 2.9907840144005604e-05, test loss is 3.755529905902222e-05, and learning rate

13261th Epoch:
traing loss is 5.866531137144193e-05, test loss is 3.602551805670373e-05, and learning rate i

13271th Epoch:
traing loss is 3.0280789360404015e-05, test loss is 3.58208708348684e-05, and learning rate i

13281th Epoch:
traing loss is 4.259212801116519e-05, test loss is 3.6748810089193285e-05, and learning rate

13291th Epoch:
traing loss is 5.2930576202925295e-05, test loss is 3.7808036722708493e-05, and learning rate

13301th Epoch:
traing loss is 4.951613300363533e-05, test loss is 3.7415175029309466e-05, and learning rate
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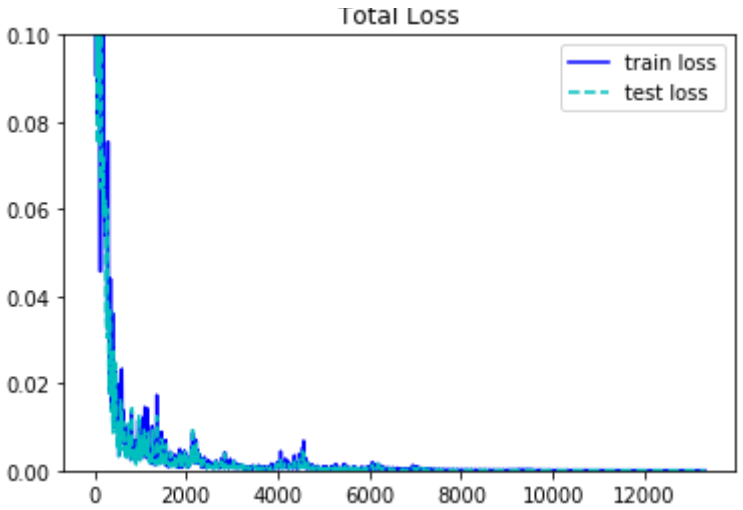
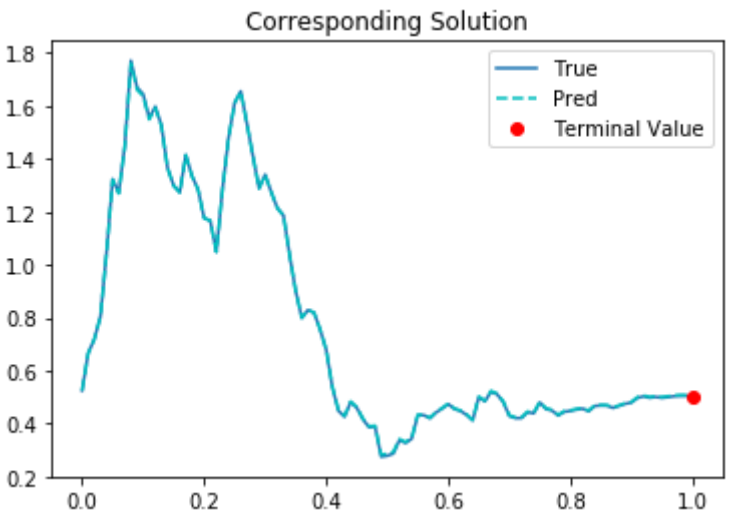
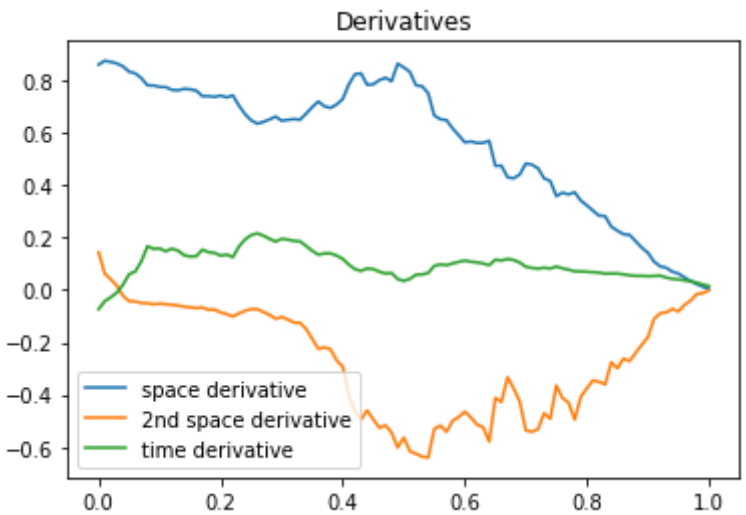

Total Loss
train loss
test loss
0.10
0.08
0.06
0.04
0.02
0.00
0
2000
4000
6000
8000
10000
12000

One Test Path
bm
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0


Corresponding Solution
True
Pred
Terminal Value
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


Derivatives
space derivative
2nd space derivative
time derivative
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.0
0.2
0.4
0.6
0.8
1.0

```
13311th Epoch:
traing loss is 8.120683924062178e-05, test loss is 4.798312147613615e-05, and learning rate i

13321th Epoch:
traing loss is 5.841992242494598e-05, test loss is 4.2898362153209746e-05, and learning rate

13331th Epoch:
traing loss is 2.8578335331985727e-05, test loss is 3.982029011240229e-05, and learning rate

13341th Epoch:
traing loss is 3.8196503737708554e-05, test loss is 3.6233839637134224e-05, and learning rate

13351th Epoch:
traing loss is 2.7512083761394024e-05, test loss is 3.687158823595382e-05, and learning rate

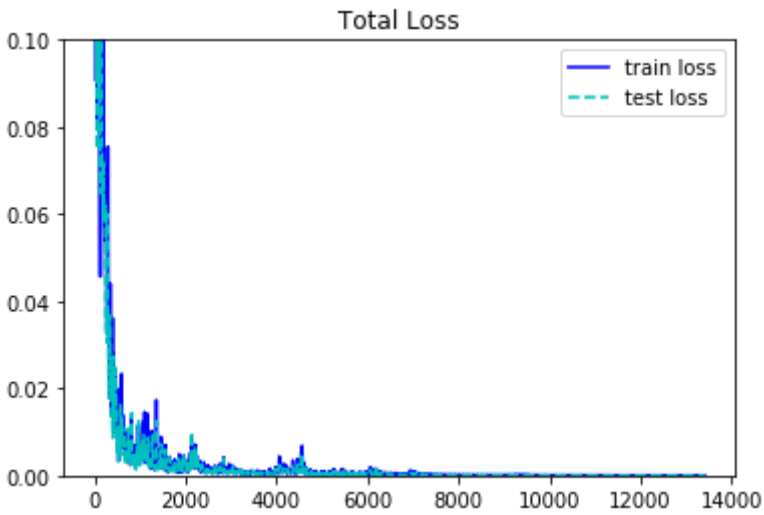
13361th Epoch:
traing loss is 5.718345710192807e-05, test loss is 3.626105171861127e-05, and learning rate i

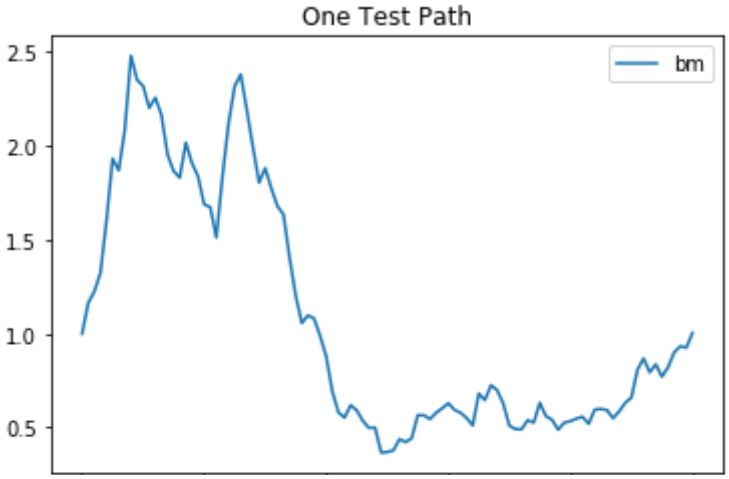
13371th Epoch:
traing loss is 3.0439934562309645e-05, test loss is 3.5798588214674965e-05, and learning rate

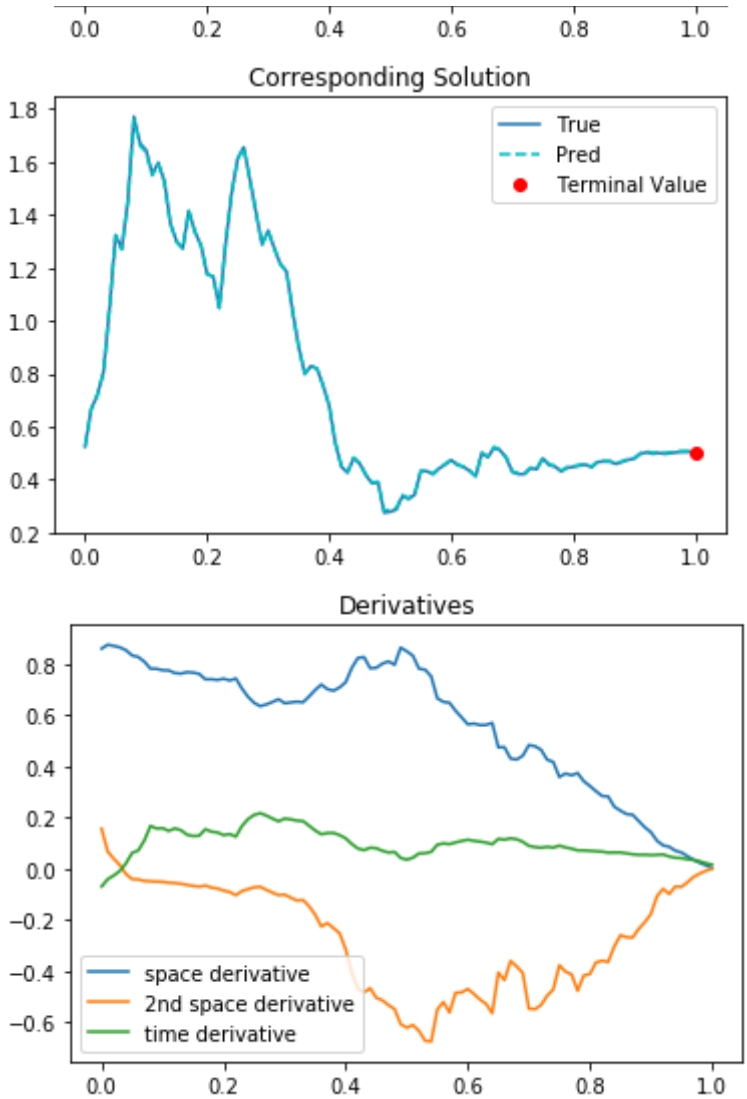
13381th Epoch:
traing loss is 4.274964157957584e-05, test loss is 3.660637230495922e-05, and learning rate i

13391th Epoch:
traing loss is 5.170136137166992e-05, test loss is 3.76078205590602e-05, and learning rate is

13401th Epoch:
traing loss is 5.050865365774371e-05, test loss is 3.818018376477994e-05, and learning rate i
```





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13411th Epoch:
traing loss is 8.231298852479085e-05, test loss is 4.847363379667513e-05, and learning rate i

13421th Epoch:
traing loss is 5.794558455818333e-05, test loss is 4.2301184294046834e-05, and learning rate

13431th Epoch:
traing loss is 2.8067128369002603e-05, test loss is 3.875889888149686e-05, and learning rate

13441th Epoch:
traing loss is 4.0228915167972445e-05, test loss is 3.673652099678293e-05, and learning rate

13451th Epoch:
traing loss is 2.9047987482044846e-05, test loss is 3.671505328384228e-05, and learning rate

13461th Epoch:
traing loss is 5.8063622418558225e-05, test loss is 3.581607961677946e-05, and learning rate

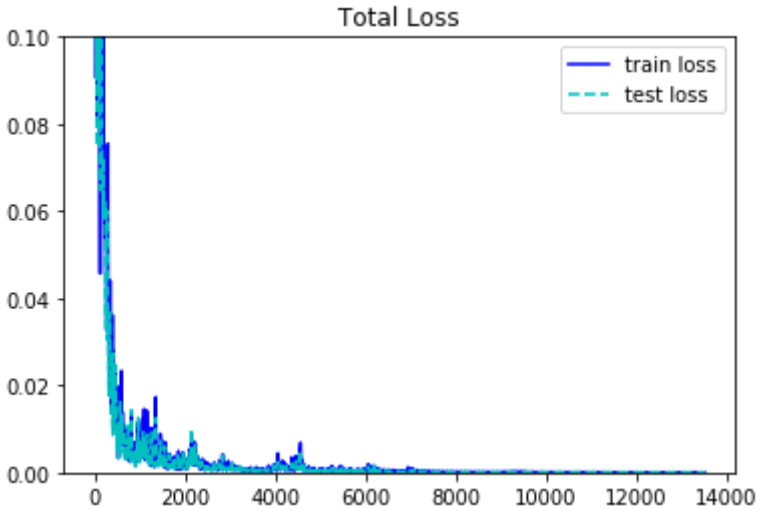
13471th Epoch:
traing loss is 3.006159204232972e-05, test loss is 3.577673851395957e-05, and learning rate i

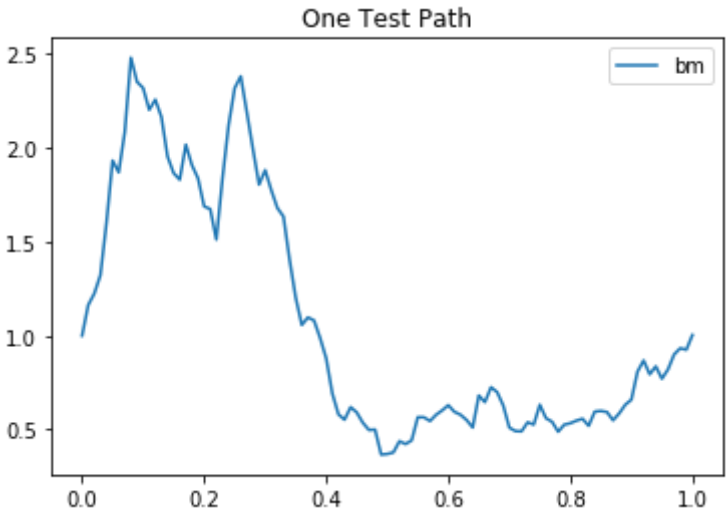
13481th Epoch:
traing loss is 4.235686719766818e-05, test loss is 3.6324017855804414e-05, and learning rate

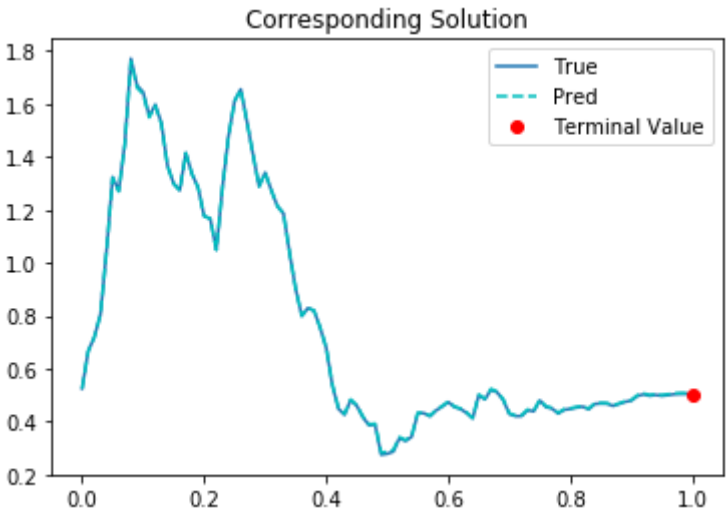
13491th Epoch:
traing loss is 5.242096449364908e-05, test loss is 3.652176383184269e-05, and learning rate i

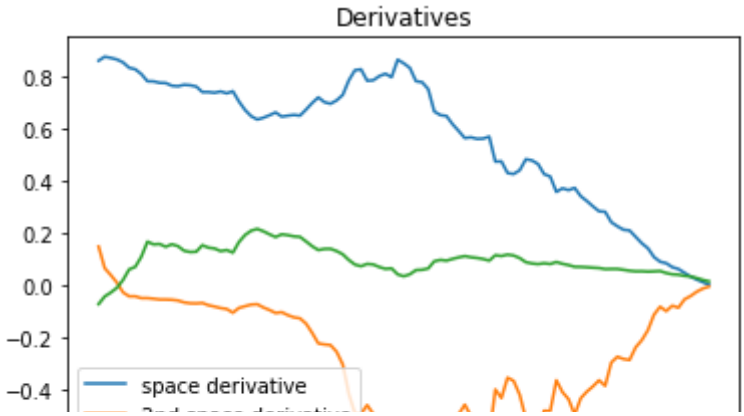
13501th Epoch:
```

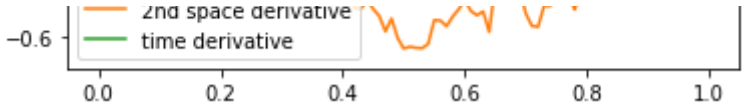
13501th Epoch:
traing loss is 4.953120878781192e-05, test loss is 3.7075882573844865e-05, and learning rate












```
13511th Epoch:
traing loss is 8.137393888318911e-05, test loss is 4.729127249447629e-05, and learning rate i

13521th Epoch:
traing loss is 5.71473574382253e-05, test loss is 4.157680814387277e-05, and learning rate is

13531th Epoch:
traing loss is 2.7427831810200587e-05, test loss is 3.774149081436917e-05, and learning rate

13541th Epoch:
traing loss is 3.833224764093757e-05, test loss is 3.6279907362768427e-05, and learning rate

13551th Epoch:
traing loss is 2.7772714020102285e-05, test loss is 3.6290155549068004e-05, and learning rate

13561th Epoch:
traing loss is 5.54399739485234e-05, test loss is 3.557105810614303e-05, and learning rate is

13571th Epoch:
traing loss is 2.9144799555069767e-05, test loss is 3.490886228973977e-05, and learning rate

13581th Epoch:
traing loss is 4.254670784575865e-05, test loss is 3.589497646316886e-05, and learning rate i

13591th Epoch:
traing loss is 5.1496717787813395e-05, test loss is 3.62764039891772e-05, and learning rate i

13601th Epoch:
traing loss is 4.938182246405631e-05, test loss is 3.7239373341435567e-05, and learning rate
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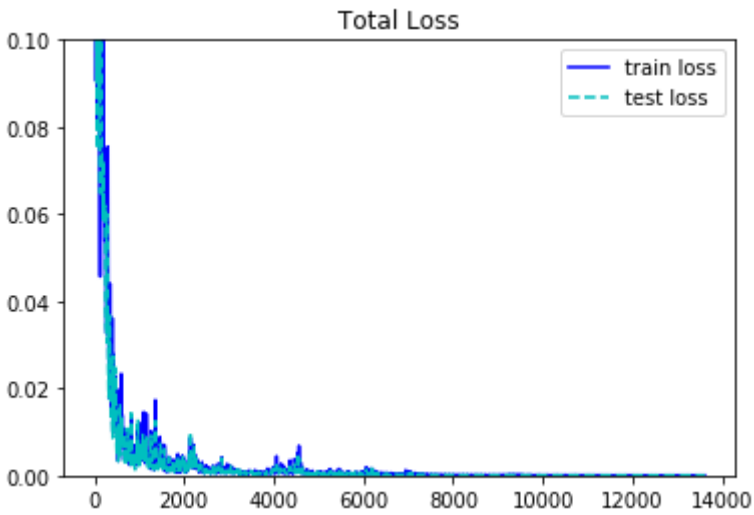
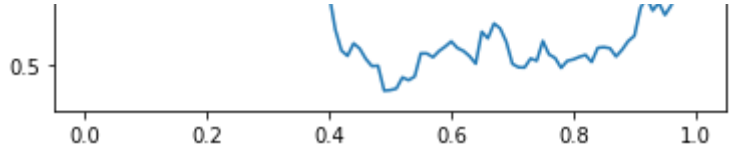
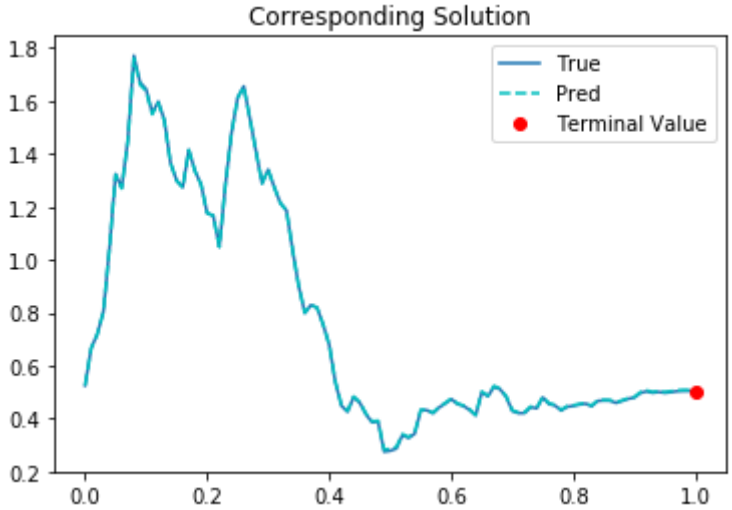
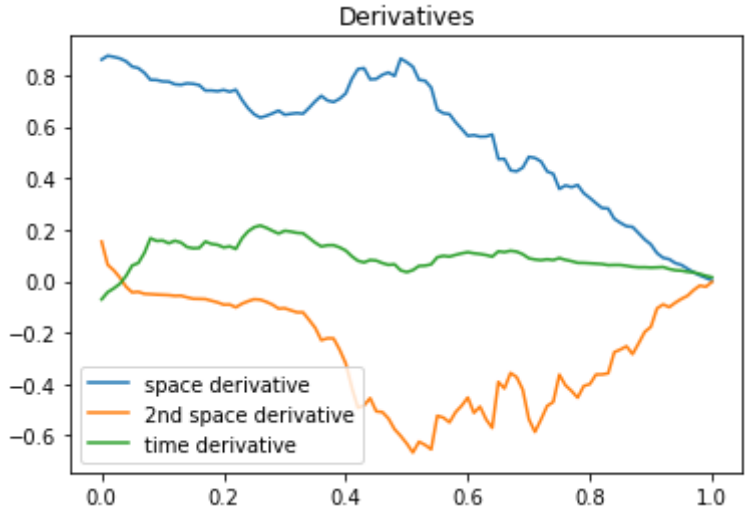









```
13611th Epoch:
traing loss is 8.074784273048863e-05, test loss is 4.748570790980011e-05, and learning rate i

13621th Epoch:
traing loss is 5.68702416785527e-05, test loss is 4.123806502320804e-05, and learning rate is

13631th Epoch:
traing loss is 2.6974656066158786e-05, test loss is 3.734236452146433e-05, and learning rate

13641th Epoch:
traing loss is 3.96655268559698e-05, test loss is 3.605834717745893e-05, and learning rate is

13651th Epoch:
traing loss is 2.929887705249712e-05, test loss is 3.661265145638026e-05, and learning rate i

13661th Epoch:
traing loss is 5.687610246241093e-05, test loss is 3.525167630868964e-05, and learning rate i

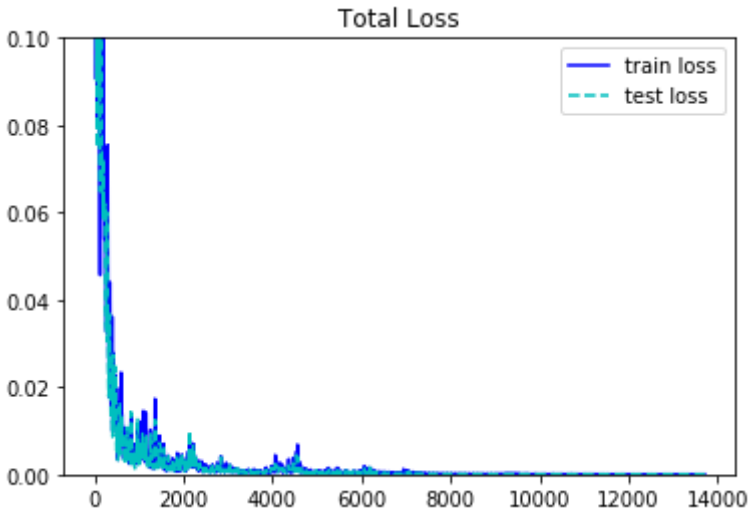
13671th Epoch:
traing loss is 2.910751754825469e-05, test loss is 3.541852856869809e-05, and learning rate i

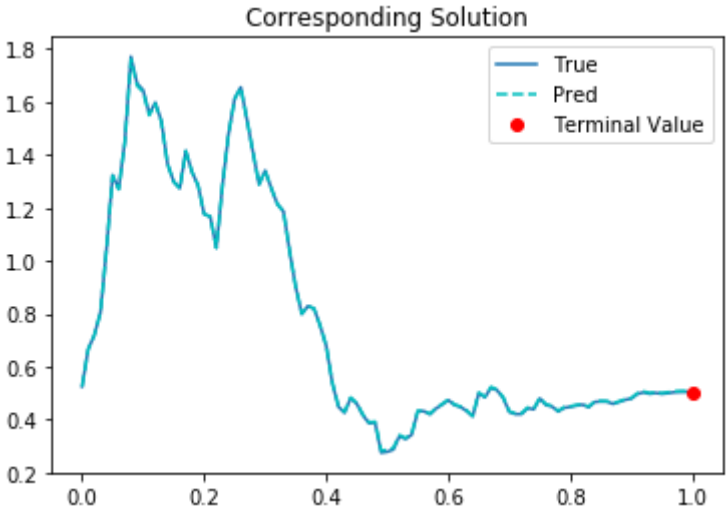
13681th Epoch:
traing loss is 4.21617369283922e-05, test loss is 3.591978020267561e-05, and learning rate is

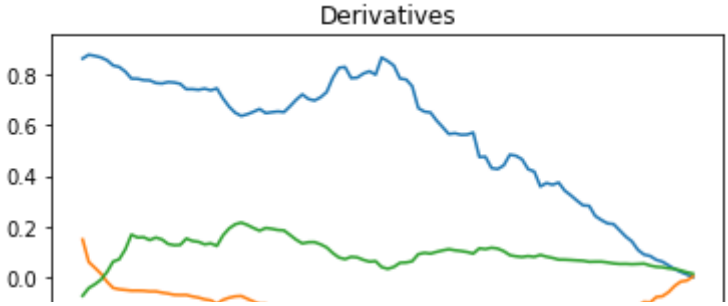
13691th Epoch:
```

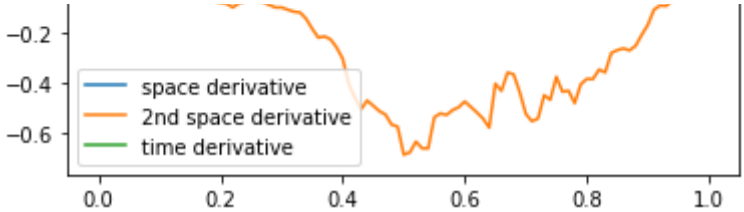
13691th Epoch:
traing loss is 5.1209572120569646e-05, test loss is 3.625985482358374e-05, and learning rate

13701th Epoch:
traing loss is 4.8650479584466666e-05, test loss is 3.640256909420714e-05, and learning rate










```
13711th Epoch:
traing loss is 7.809535600244999e-05, test loss is 4.7544697736157104e-05, and learning rate

13721th Epoch:
traing loss is 5.563786908169277e-05, test loss is 4.051140422234312e-05, and learning rate i

13731th Epoch:
traing loss is 2.6552424969850108e-05, test loss is 3.7134901504032314e-05, and learning rate

13741th Epoch:
traing loss is 3.811940041487105e-05, test loss is 3.6050481867277995e-05, and learning rate

13751th Epoch:
traing loss is 2.7585288989939727e-05, test loss is 3.6229572287993506e-05, and learning rate

13761th Epoch:
traing loss is 5.641951065626927e-05, test loss is 3.468694558250718e-05, and learning rate i

13771th Epoch:
traing loss is 2.897883859986905e-05, test loss is 3.457215643720701e-05, and learning rate i

13781th Epoch:
traing loss is 4.1782372136367485e-05, test loss is 3.5333039704710245e-05, and learning rate

13791th Epoch:
traing loss is 5.011557004763745e-05, test loss is 3.5669170756591484e-05, and learning rate

13801th Epoch:
traing loss is 4.8343157686758786e-05, test loss is 3.630404171417467e-05, and learning rate
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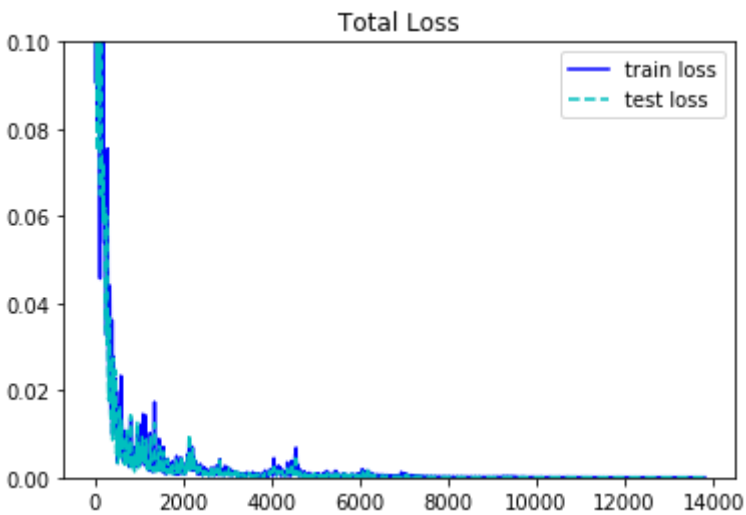
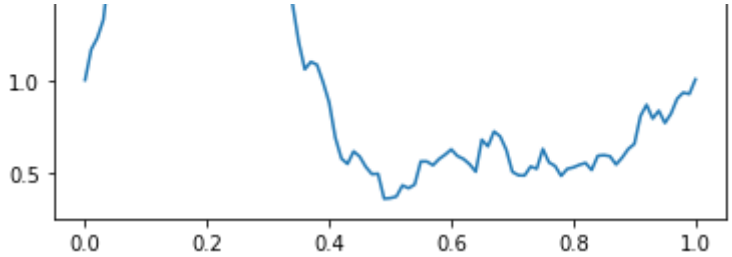
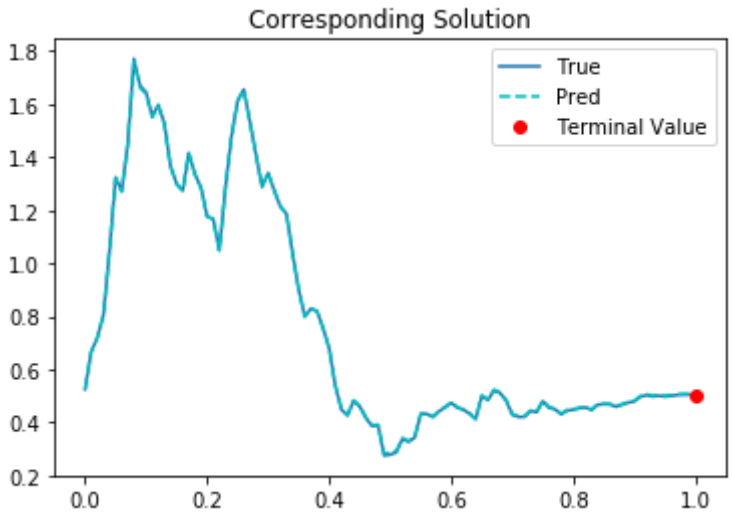
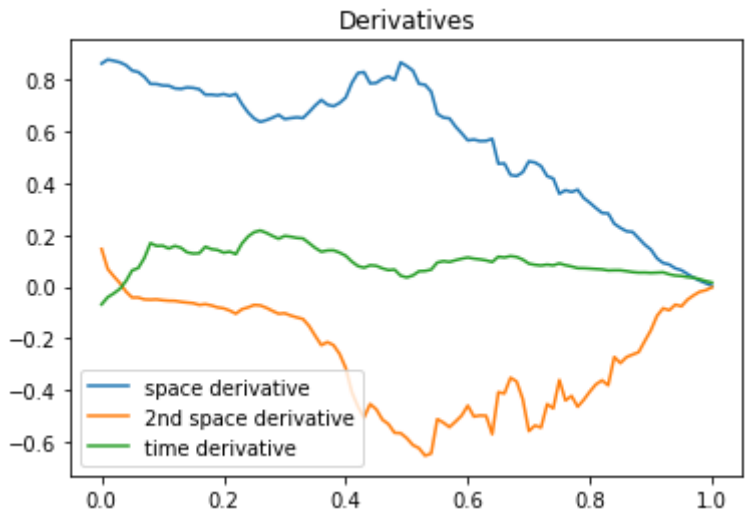









```
13811th Epoch:
traing loss is 8.215440902858973e-05, test loss is 4.771922976942733e-05, and learning rate i

13821th Epoch:
traing loss is 5.480180698214099e-05, test loss is 3.950098471250385e-05, and learning rate i

13831th Epoch:
traing loss is 2.6407580662635155e-05, test loss is 3.7086752854520455e-05, and learning rate

13841th Epoch:
traing loss is 3.912896863766946e-05, test loss is 3.6105771869188175e-05, and learning rate

13851th Epoch:
traing loss is 2.9341001209104434e-05, test loss is 3.668030694825575e-05, and learning rate

13861th Epoch:
traing loss is 5.780323772341944e-05, test loss is 3.467143324087374e-05, and learning rate i

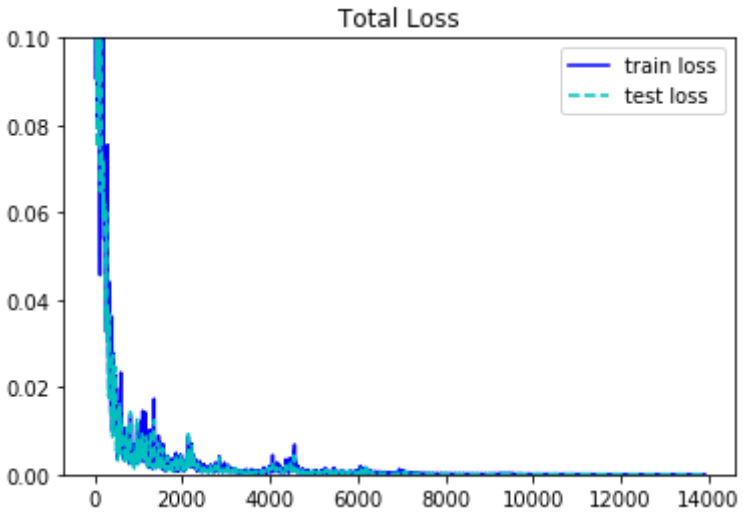
13871th Epoch:
traing loss is 2.87641269096639e-05, test loss is 3.4110194974346086e-05, and learning rate i

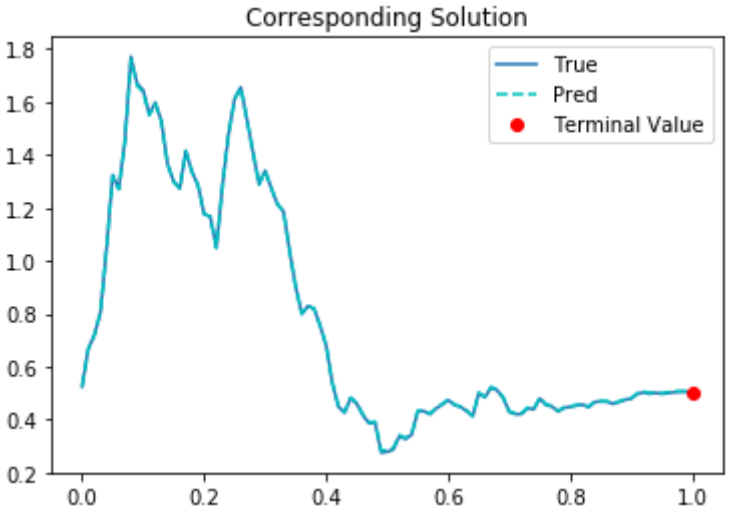
13881th Epoch:
```

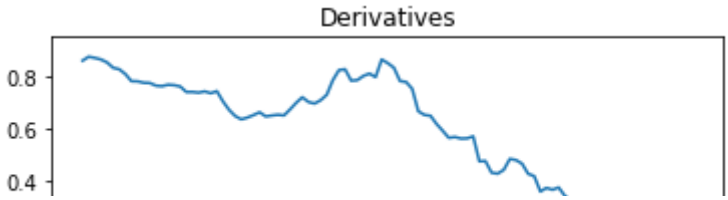
13881th Epoch:
traing loss is 4.188153980067e-05, test loss is 3.541212572599761e-05, and learning rate is 3

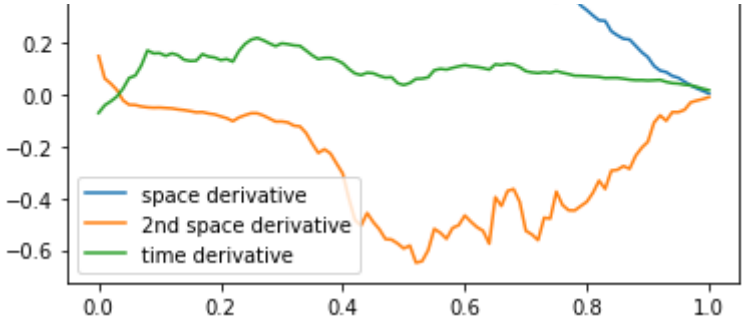
13891th Epoch:
traing loss is 4.994997652829625e-05, test loss is 3.487550566205755e-05, and learning rate i

13901th Epoch:
traing loss is 4.824719871976413e-05, test loss is 3.58560610038694e-05, and learning rate is










```
13911th Epoch:
traing loss is 7.927192928036675e-05, test loss is 4.7031266149133444e-05, and learning rate

13921th Epoch:
traing loss is 5.367191988625564e-05, test loss is 3.8904825487406924e-05, and learning rate

13931th Epoch:
traing loss is 2.6294639610568993e-05, test loss is 3.6481418646872044e-05, and learning rate

13941th Epoch:
traing loss is 3.773705611820333e-05, test loss is 3.596944588935003e-05, and learning rate i

13951th Epoch:
traing loss is 2.8682503398158588e-05, test loss is 3.694060796988197e-05, and learning rate

13961th Epoch:
traing loss is 5.506119123310782e-05, test loss is 3.467737042228691e-05, and learning rate i

13971th Epoch:
traing loss is 2.9258466383907944e-05, test loss is 3.4631997550604865e-05, and learning rate

13981th Epoch:
traing loss is 4.2235336877638474e-05, test loss is 3.505521090119146e-05, and learning rate

13991th Epoch:
traing loss is 4.9945439968723804e-05, test loss is 3.5213590308558196e-05, and learning rate

14001th Epoch:
traing loss is 4.750570224132389e-05, test loss is 3.514022318995558e-05, and learning rate i
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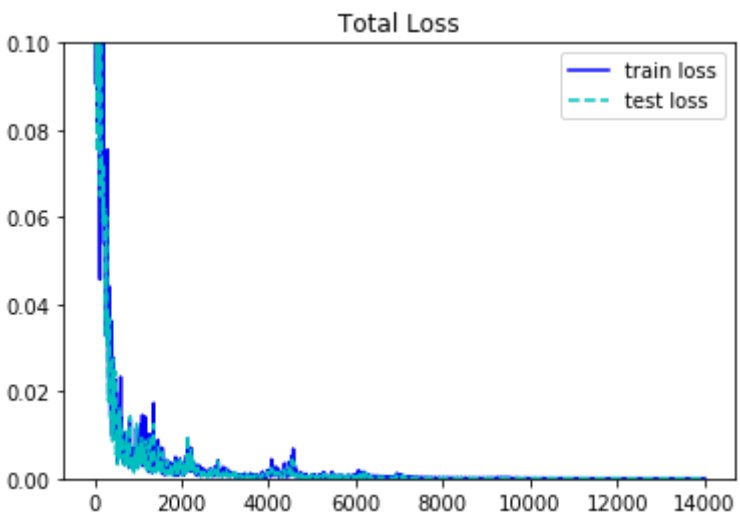

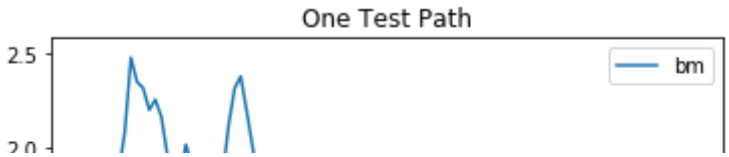

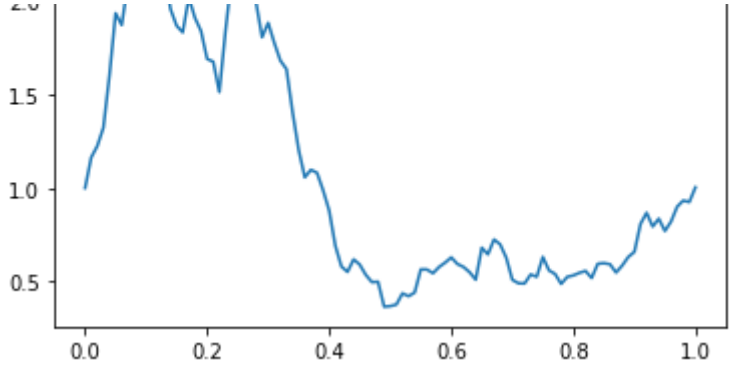

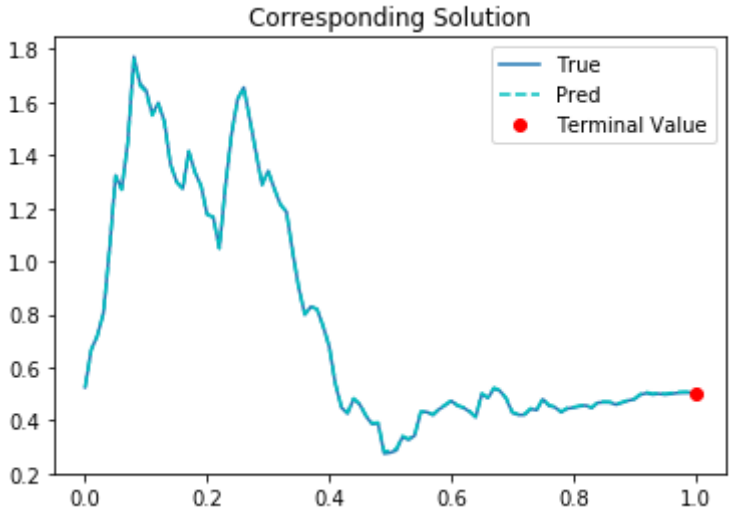

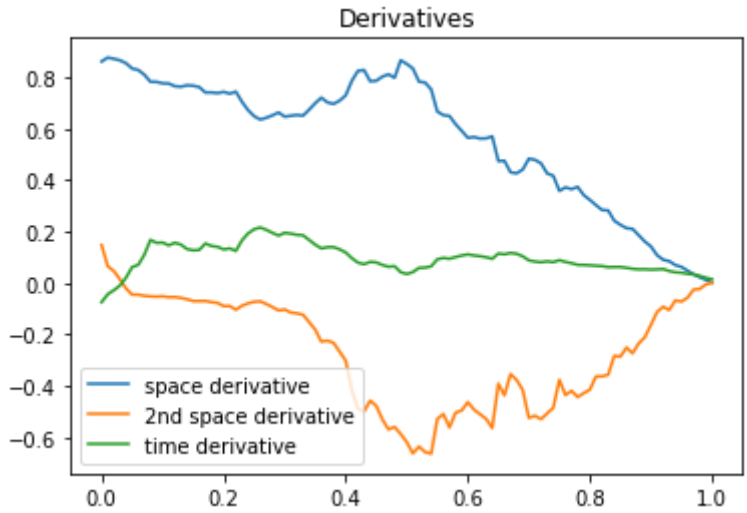


```
14011th Epoch:
traing loss is 7.924445526441559e-05, test loss is 4.629747854778543e-05, and learning rate i

14021th Epoch:
traing loss is 5.2925795898772776e-05, test loss is 3.778073005378246e-05, and learning rate

14031th Epoch:
traing loss is 2.603497341624461e-05, test loss is 3.6573372199200094e-05, and learning rate

14041th Epoch:
traing loss is 3.8245510950218886e-05, test loss is 3.534511415637098e-05, and learning rate

14051th Epoch:
traing loss is 2.943798608612269e-05, test loss is 3.69239533029031e-05, and learning rate is

14061th Epoch:
traing loss is 5.6842218327801675e-05, test loss is 3.473385004326701e-05, and learning rate

14071th Epoch:
```

```
14071th Epoch:
traing loss is 2.863173358491622e-05, test loss is 3.467273199930787e-05, and learning rate i

14081th Epoch:
traing loss is 4.1167091694660485e-05, test loss is 3.4882163163274527e-05, and learning rate

14091th Epoch:
traing loss is 4.9519519961904734e-05, test loss is 3.4511485864641145e-05, and learning rate

14101th Epoch:
traing loss is 4.827660814044066e-05, test loss is 3.522648330545053e-05, and learning rate i
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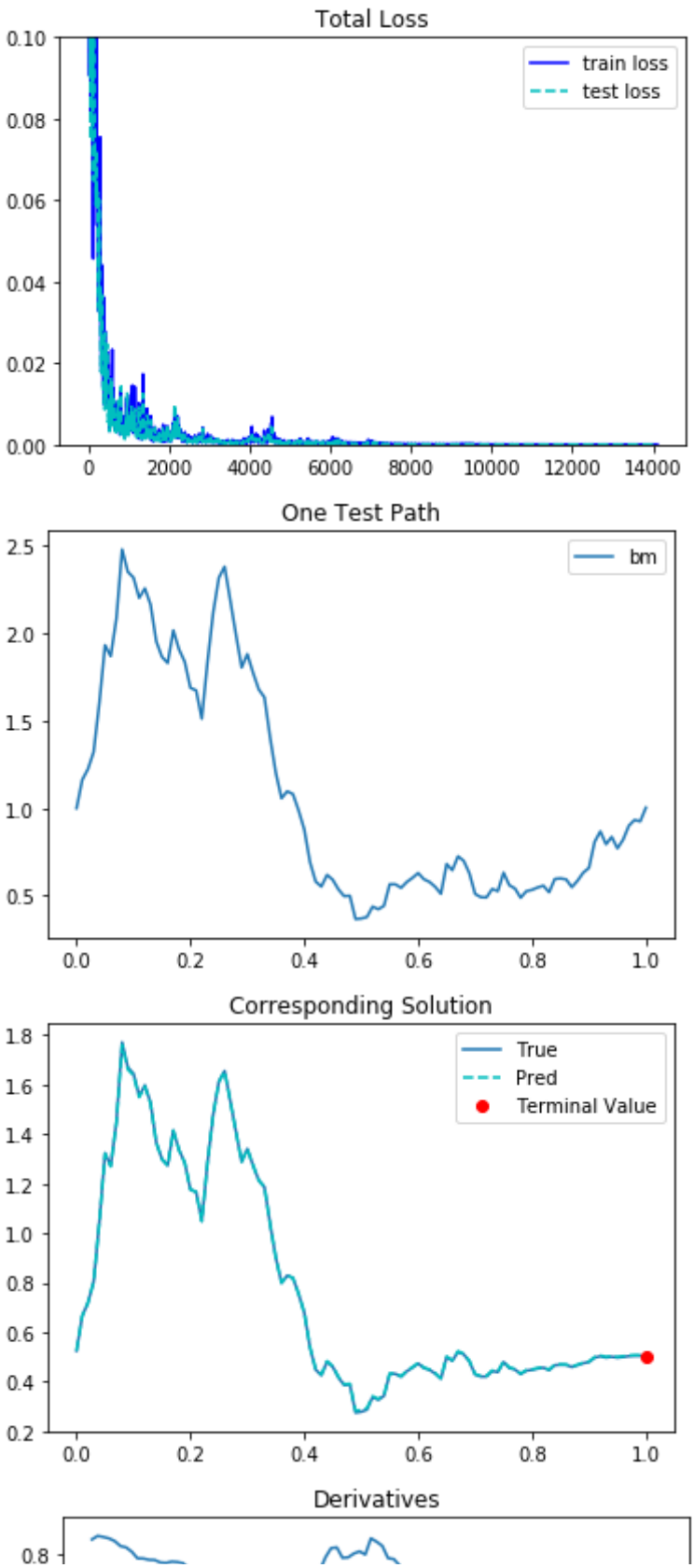

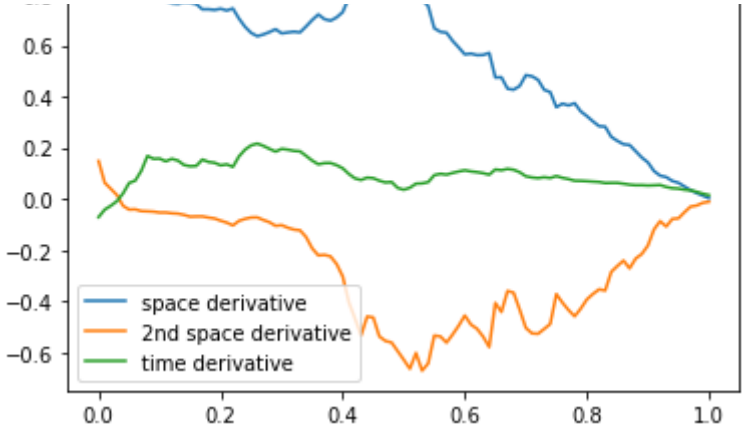


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14111th Epoch:
traing loss is 7.988409925019369e-05, test loss is 4.7129884478636086e-05, and learning rate

14121th Epoch:
traing loss is 5.101010901853442e-05, test loss is 3.7359175621531904e-05, and learning rate

14131th Epoch:
traing loss is 2.633392250572797e-05, test loss is 3.643312811618671e-05, and learning rate i

14141th Epoch:
traing loss is 3.718692823895253e-05, test loss is 3.5347689845366403e-05, and learning rate

14151th Epoch:
traing loss is 2.9076792998239398e-05, test loss is 3.721114990185015e-05, and learning rate

14161th Epoch:
traing loss is 5.610123116639443e-05, test loss is 3.4170665458077565e-05, and learning rate

14171th Epoch:
traing loss is 2.9119610189809464e-05, test loss is 3.440603904891759e-05, and learning rate

14181th Epoch:
traing loss is 4.1697010601637885e-05, test loss is 3.512587500154041e-05, and learning rate

14191th Epoch:
traing loss is 4.9738788220565766e-05, test loss is 3.438828207436018e-05, and learning rate

14201th Epoch:
traing loss is 4.753826215164736e-05, test loss is 3.49516594724264e-05, and learning rate is
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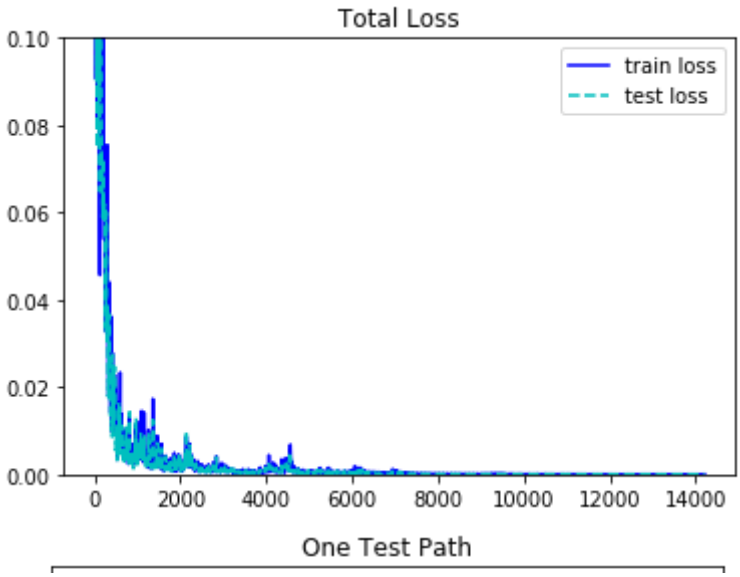

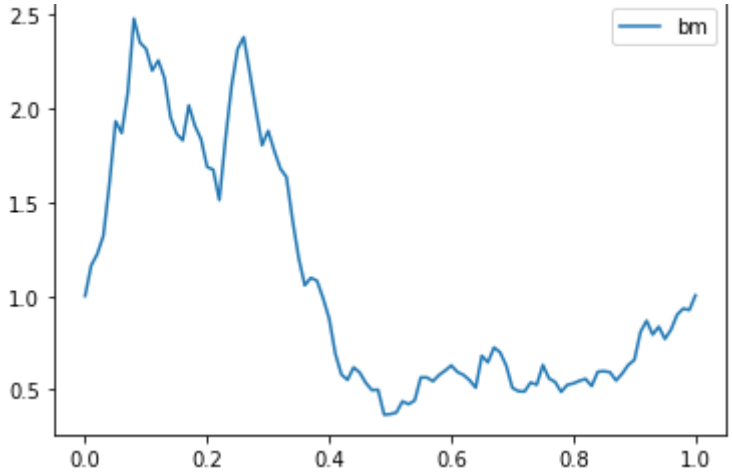

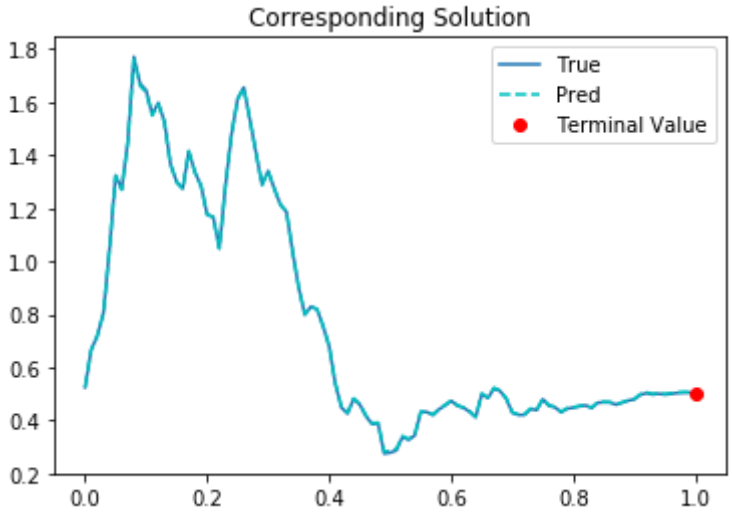

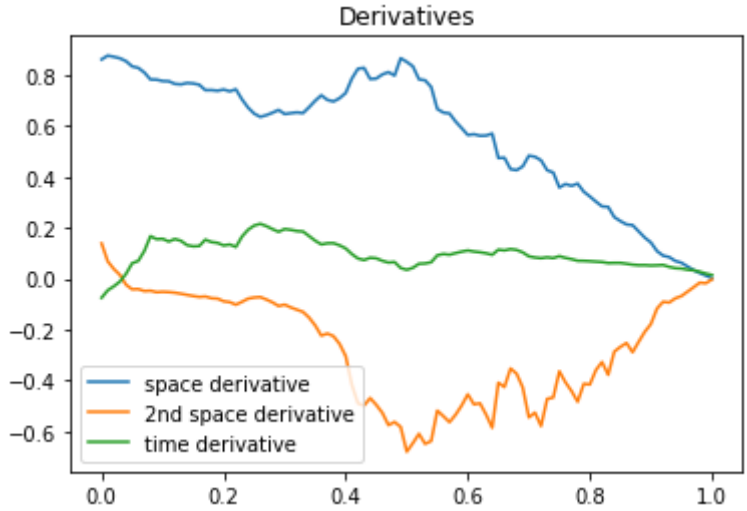


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14211th Epoch:
traing loss is 7.906118844402954e-05, test loss is 4.70948543807026e-05, and learning rate is

14221th Epoch:
traing loss is 5.111034988658503e-05, test loss is 3.683456088765524e-05, and learning rate i

14231th Epoch:
traing loss is 2.5888897653203458e-05, test loss is 3.645422839326784e-05, and learning rate

14241th Epoch:
traing loss is 3.7841022276552394e-05, test loss is 3.533695780788548e-05, and learning rate

14251th Epoch:
traing loss is 2.957478864118457e-05, test loss is 3.734799611265771e-05, and learning rate i

14261th Epoch:
```

```
14261th Epoch:
traing loss is 5.6242810387630016e-05, test loss is 3.5060467780567706e-05, and learning rate

14271th Epoch:
traing loss is 2.856309038179461e-05, test loss is 3.387176911928691e-05, and learning rate i

14281th Epoch:
traing loss is 4.224909207550809e-05, test loss is 3.532831397023983e-05, and learning rate i

14291th Epoch:
traing loss is 4.93273873871658e-05, test loss is 3.4472406696295366e-05, and learning rate i

14301th Epoch:
traing loss is 4.635515870177187e-05, test loss is 3.4784232411766425e-05, and learning rate
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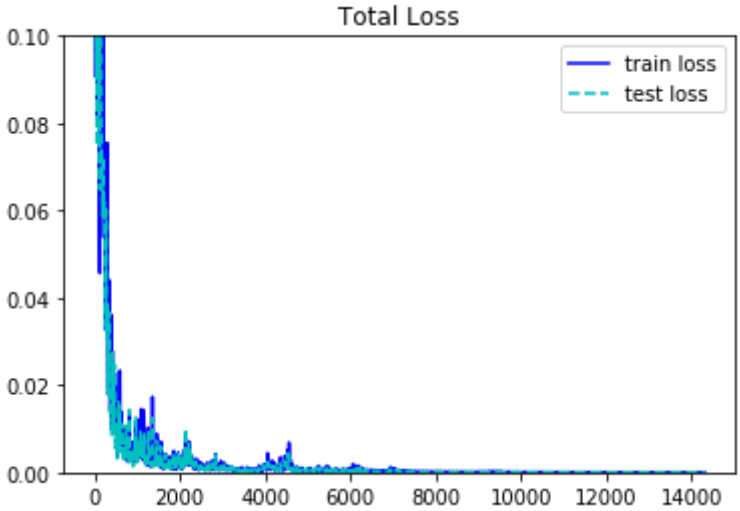

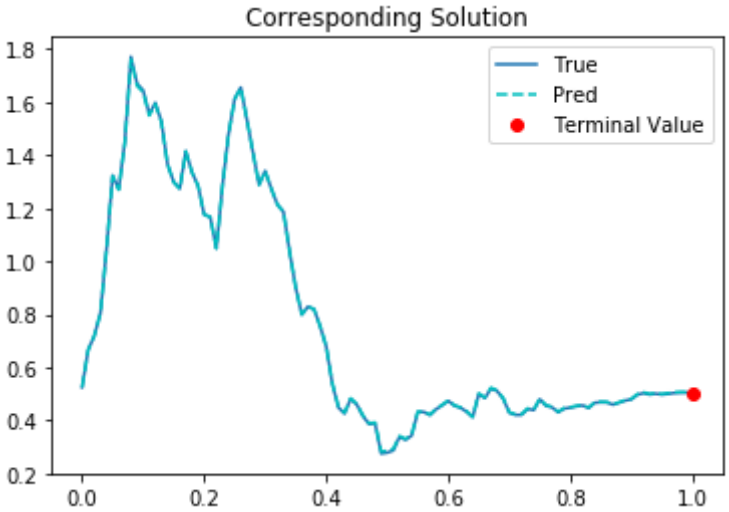

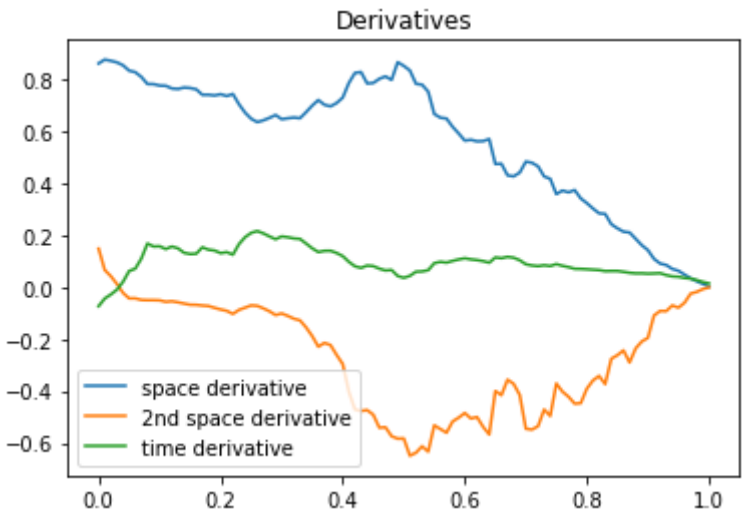


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14311th Epoch:
traing loss is 7.874769653426483e-05, test loss is 4.739113865070976e-05, and learning rate i

14321th Epoch:
traing loss is 4.948817513650283e-05, test loss is 3.598231342039071e-05, and learning rate i

14331th Epoch:
traing loss is 2.5227791411452927e-05, test loss is 3.6152167012915015e-05, and learning rate

14341th Epoch:
traing loss is 3.699308217619546e-05, test loss is 3.533007838996127e-05, and learning rate i

14351th Epoch:
traing loss is 2.8582782761077397e-05, test loss is 3.576845847419463e-05, and learning rate

14361th Epoch:
traing loss is 5.6503347877878696e-05, test loss is 3.529079185682349e-05, and learning rate

14371th Epoch:
traing loss is 2.8392247259034775e-05, test loss is 3.376958193257451e-05, and learning rate

14381th Epoch:
traing loss is 4.235893356963061e-05, test loss is 3.5306122299516574e-05, and learning rate

14391th Epoch:
traing loss is 4.947167690261267e-05, test loss is 3.4052991395583376e-05, and learning rate

14401th Epoch:
traing loss is 4.666335735237226e-05, test loss is 3.474468394415453e-05, and learning rate i
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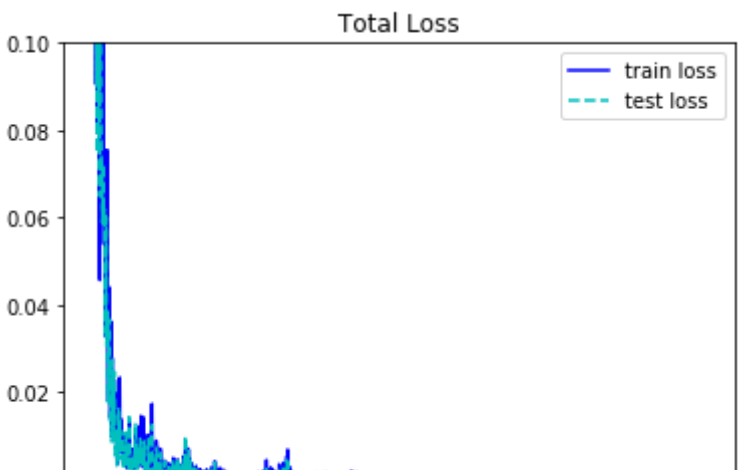

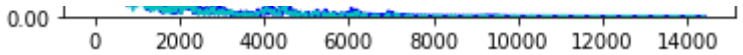

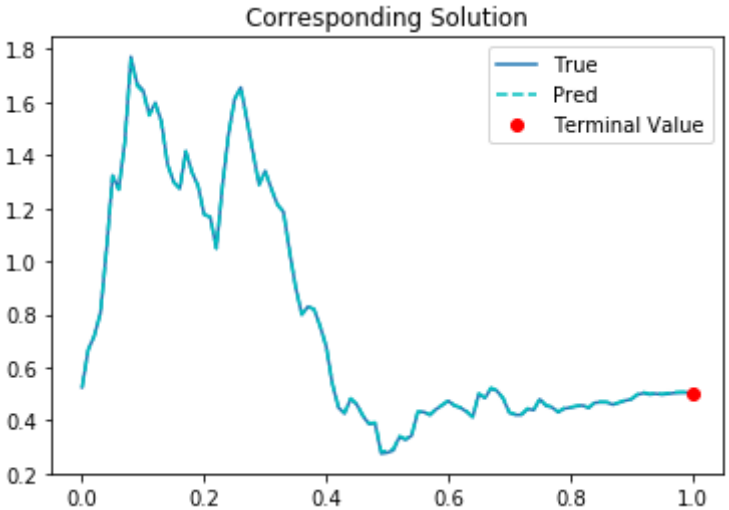

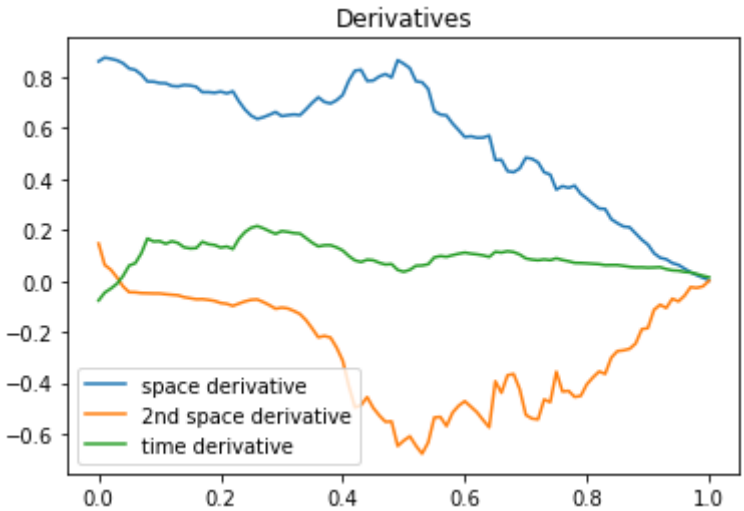


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14411th Epoch:
traing loss is 7.814656419213861e-05, test loss is 4.792027539224364e-05, and learning rate i

14421th Epoch:
traing loss is 4.772176544065587e-05, test loss is 3.5084896808257326e-05, and learning rate

14431th Epoch:
traing loss is 2.5032397388713434e-05, test loss is 3.507741348585114e-05, and learning rate

14441th Epoch:
traing loss is 3.6831479519605637e-05, test loss is 3.515490243444219e-05, and learning rate

14451th Epoch:
```

```
14451th Epoch:
traing loss is 2.8297163225943223e-05, test loss is 3.5020515497308224e-05, and learning rate

14461th Epoch:
traing loss is 5.6925451644929126e-05, test loss is 3.502595063764602e-05, and learning rate

14471th Epoch:
traing loss is 2.802029666781891e-05, test loss is 3.333788117743097e-05, and learning rate i

14481th Epoch:
traing loss is 4.183263808954507e-05, test loss is 3.522311453707516e-05, and learning rate i

14491th Epoch:
traing loss is 4.980995072401129e-05, test loss is 3.3962678571697325e-05, and learning rate

14501th Epoch:
traing loss is 4.655836528399959e-05, test loss is 3.427090996410698e-05, and learning rate i
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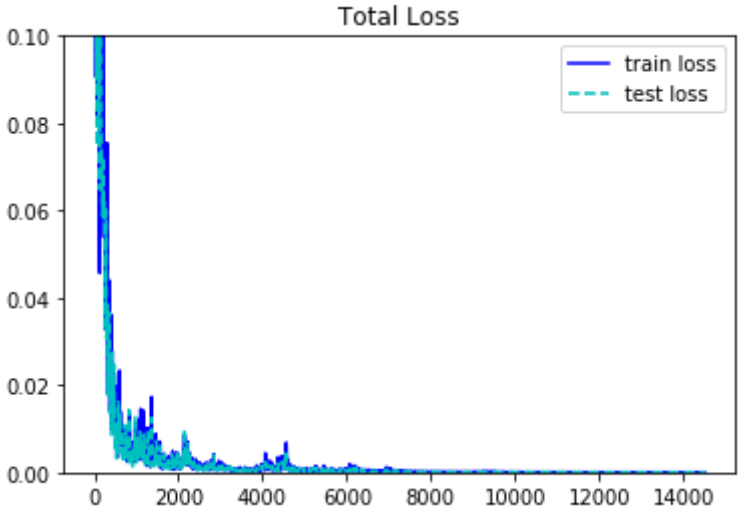

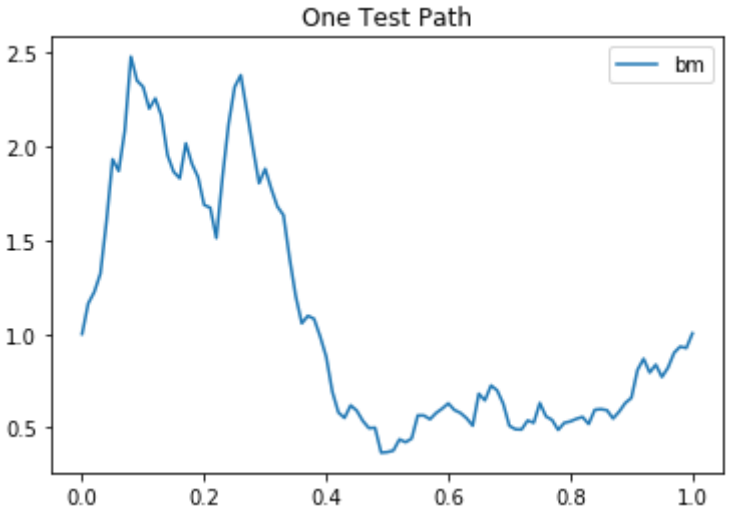

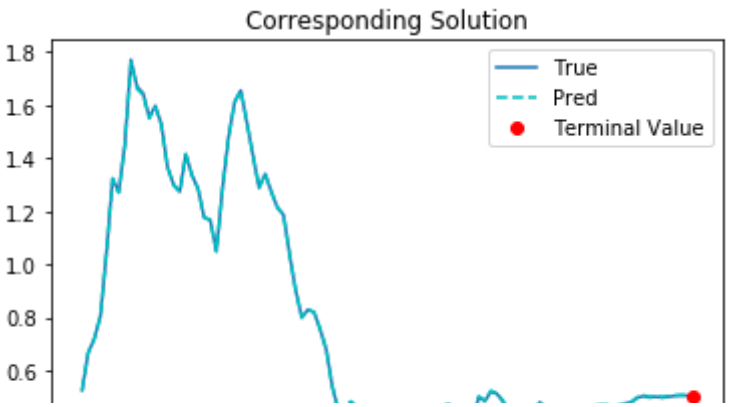

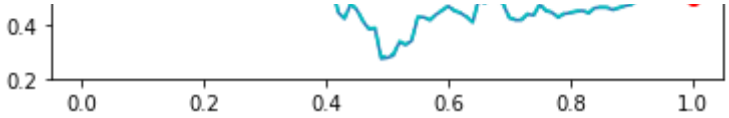

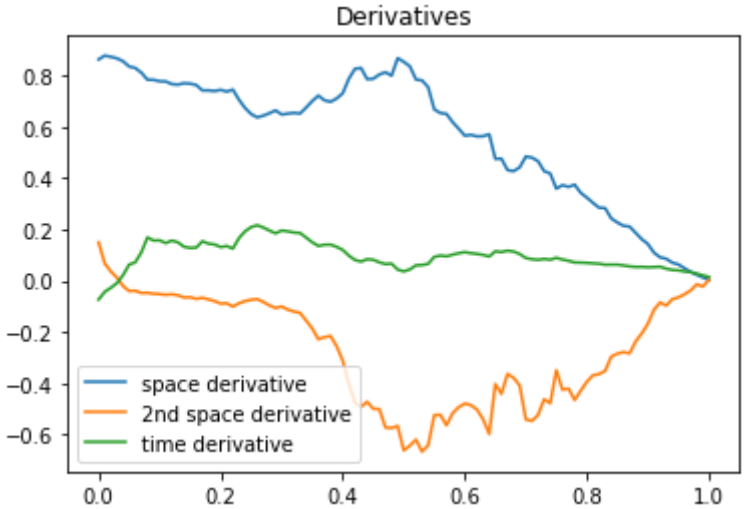


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14511th Epoch:
traing loss is 7.859025936340913e-05, test loss is 4.7721274313516915e-05, and learning rate

14521th Epoch:
traing loss is 4.850823461310938e-05, test loss is 3.547820961102843e-05, and learning rate i

14531th Epoch:
traing loss is 2.4630542611703277e-05, test loss is 3.5219909477746114e-05, and learning rate

14541th Epoch:
traing loss is 3.676052438095212e-05, test loss is 3.558461321517825e-05, and learning rate i

14551th Epoch:
traing loss is 2.7294674509903416e-05, test loss is 3.467039641691372e-05, and learning rate

14561th Epoch:
traing loss is 5.581617369898595e-05, test loss is 3.459959043539129e-05, and learning rate i

14571th Epoch:
traing loss is 2.745086203503888e-05, test loss is 3.324080171296373e-05, and learning rate i

14581th Epoch:
traing loss is 4.1338102164445445e-05, test loss is 3.456381455180235e-05, and learning rate

14591th Epoch:
traing loss is 4.9246948037762195e-05, test loss is 3.387549804756418e-05, and learning rate

14601th Epoch:
traing loss is 4.787571742781438e-05, test loss is 3.373619256308302e-05, and learning rate i
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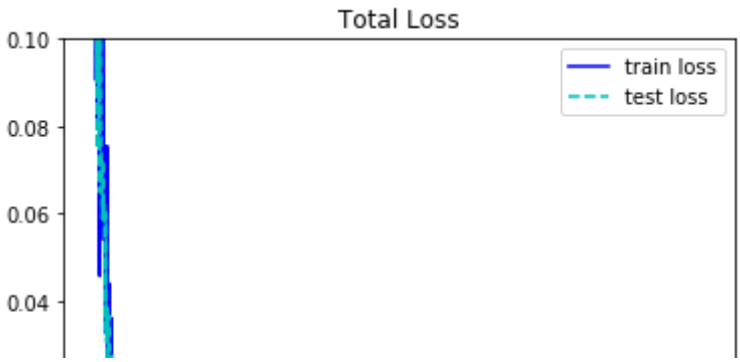

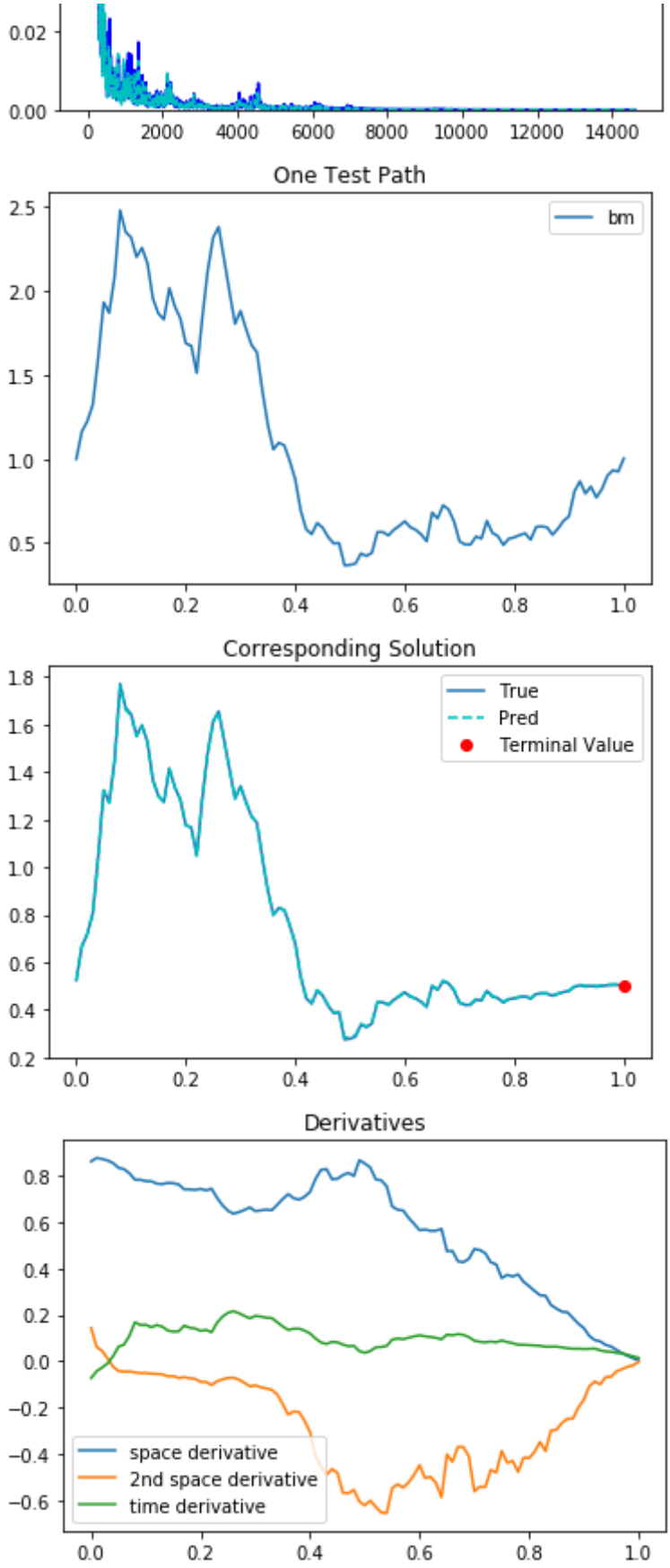

14611th Epoch:
traing loss is 7.846755033824593e-05, test loss is 4.5647393562830985e-05, and learning rate

14621th Epoch:
traing loss is 4.755704867420718e-05, test loss is 3.5196655517211184e-05, and learning rate

14631th Epoch:
traing loss is 2.4031169232330285e-05, test loss is 3.409431883483194e-05, and learning rate

14641th Epoch:

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14641th Epoch:
traing loss is 3.627007390605286e-05, test loss is 3.4325090382480994e-05, and learning rate

14651th Epoch:
traing loss is 2.7117739591631107e-05, test loss is 3.399702472961508e-05, and learning rate

14661th Epoch:
traing loss is 5.545153180719353e-05, test loss is 3.4198510547867045e-05, and learning rate

14671th Epoch:
traing loss is 2.7410707843955606e-05, test loss is 3.3361957321176305e-05, and learning rate

14681th Epoch:
traing loss is 4.125104533159174e-05, test loss is 3.441734952502884e-05, and learning rate i

14691th Epoch:
traing loss is 4.863793219556101e-05, test loss is 3.361331619089469e-05, and learning rate i

14701th Epoch:
traing loss is 4.788132355315611e-05, test loss is 3.4422260796418414e-05, and learning rate
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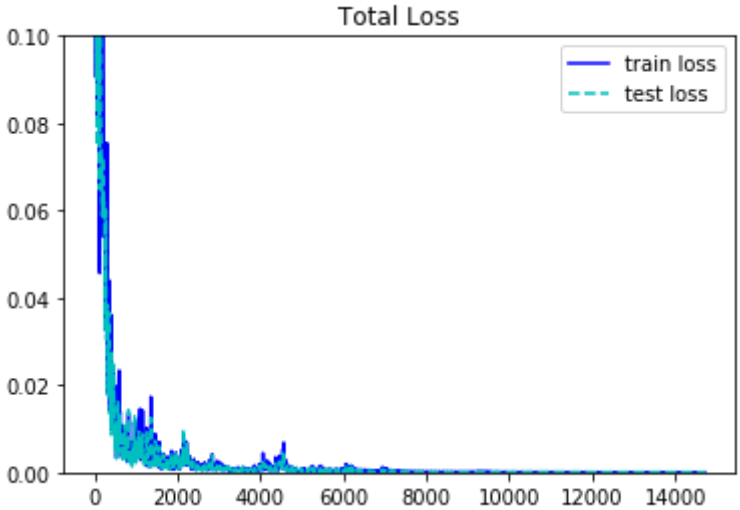

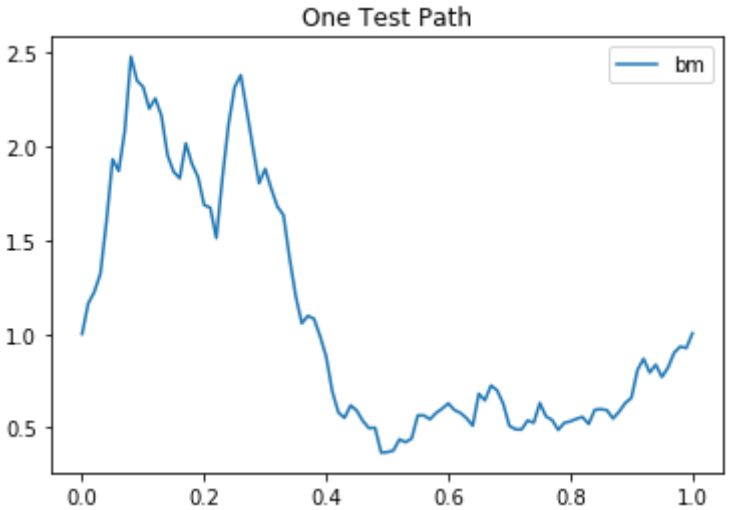

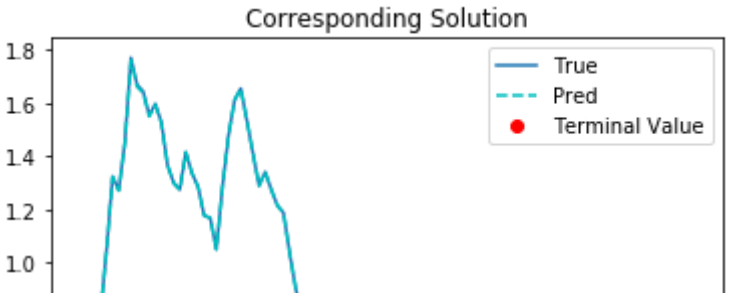

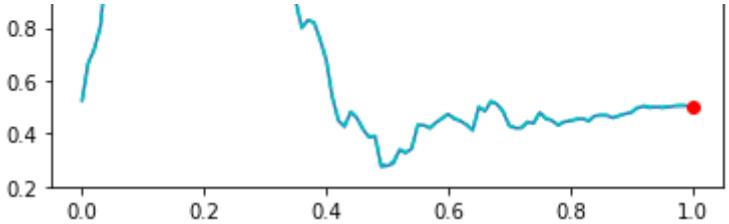

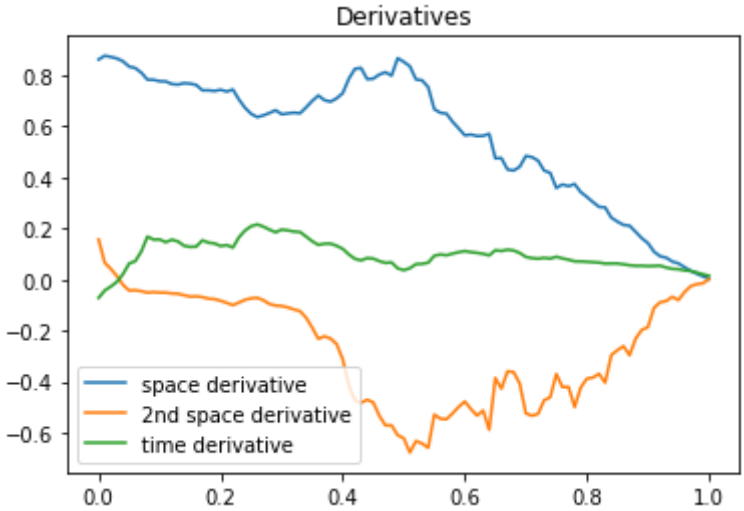


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14711th Epoch:
traing loss is 7.850684050936252e-05, test loss is 4.541284215520136e-05, and learning rate i

14721th Epoch:
traing loss is 4.864981383434497e-05, test loss is 3.5336503060534596e-05, and learning rate

14731th Epoch:
traing loss is 2.3835000320104882e-05, test loss is 3.4373661037534475e-05, and learning rate

14741th Epoch:
traing loss is 3.5324472264619544e-05, test loss is 3.422958616283722e-05, and learning rate

14751th Epoch:
traing loss is 2.6984032956534065e-05, test loss is 3.353572537889704e-05, and learning rate

14761th Epoch:
traing loss is 5.506634261109866e-05, test loss is 3.401594585739076e-05, and learning rate i

14771th Epoch:
traing loss is 2.71902736130869e-05, test loss is 3.3397995139239356e-05, and learning rate i

14781th Epoch:
traing loss is 4.120135054108687e-05, test loss is 3.4799457353074104e-05, and learning rate

14791th Epoch:
traing loss is 4.94619453093037e-05, test loss is 3.3434949727961794e-05, and learning rate i

14801th Epoch:
traing loss is 4.756894850288518e-05, test loss is 3.383171133464202e-05, and learning rate i
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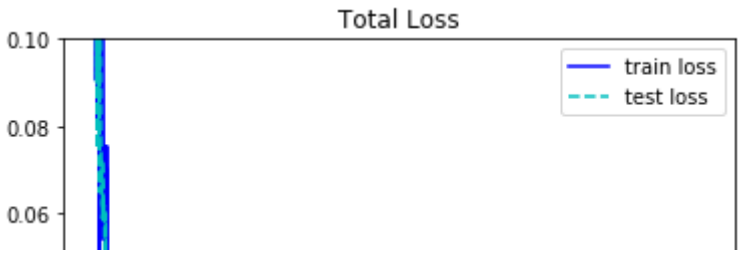

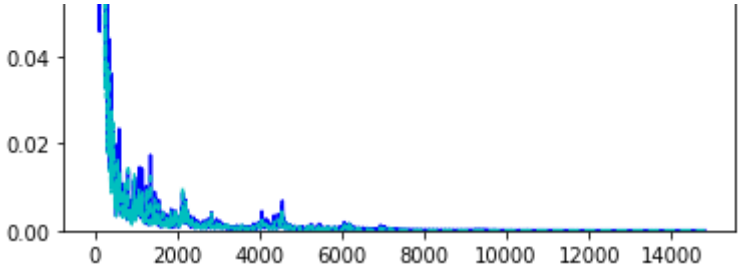

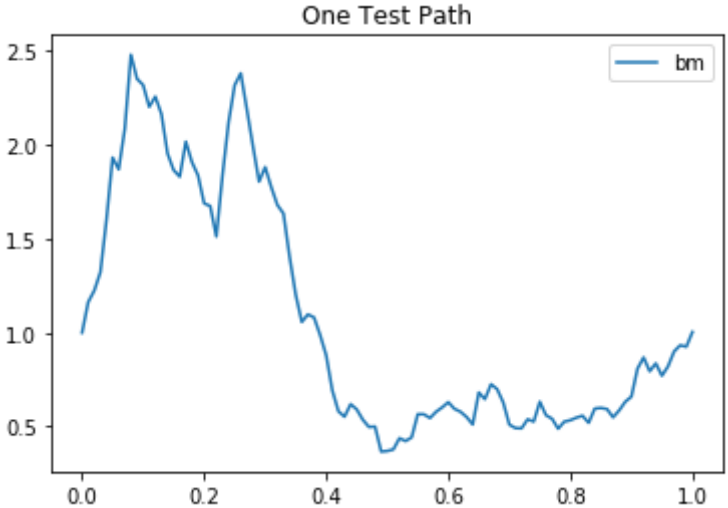

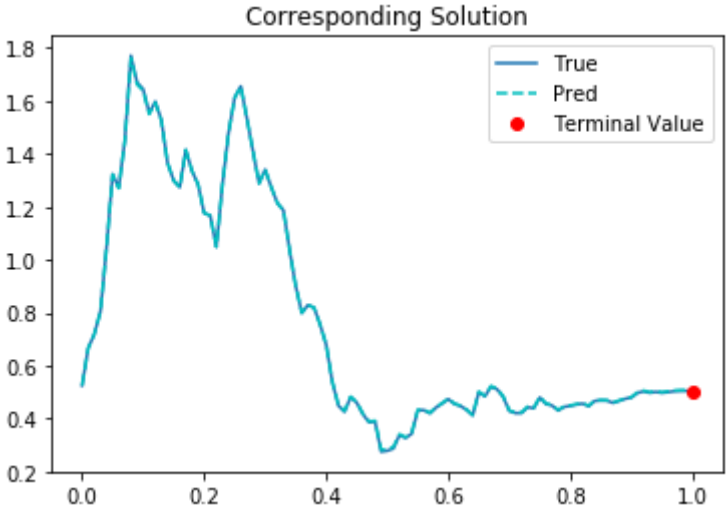

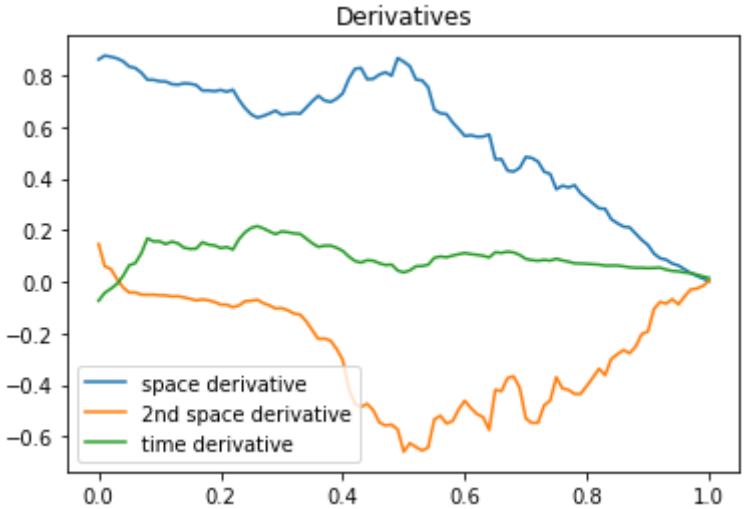


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14811th Epoch:
traing loss is 7.624960562679917e-05, test loss is 4.472339423955418e-05, and learning rate i

14821th Epoch:
traing loss is 4.833248385693878e-05, test loss is 3.587448372854851e-05, and learning rate i

14831th Epoch:
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14831th Epoch:
traing loss is 2.4548042347305454e-05, test loss is 3.44420550391078e-05, and learning rate i

14841th Epoch:
traing loss is 3.54525946022477e-05, test loss is 3.419584027142264e-05, and learning rate is

14851th Epoch:
traing loss is 2.6848516426980495e-05, test loss is 3.382395880180411e-05, and learning rate

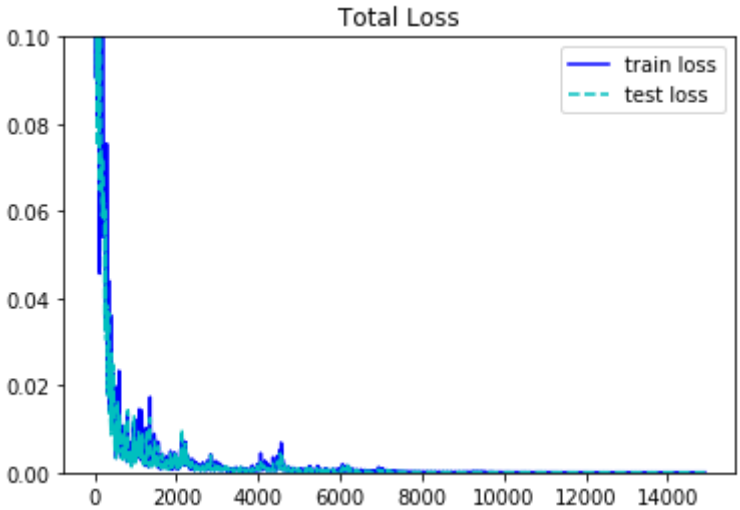
14861th Epoch:
traing loss is 5.597427298198454e-05, test loss is 3.397714681341313e-05, and learning rate i

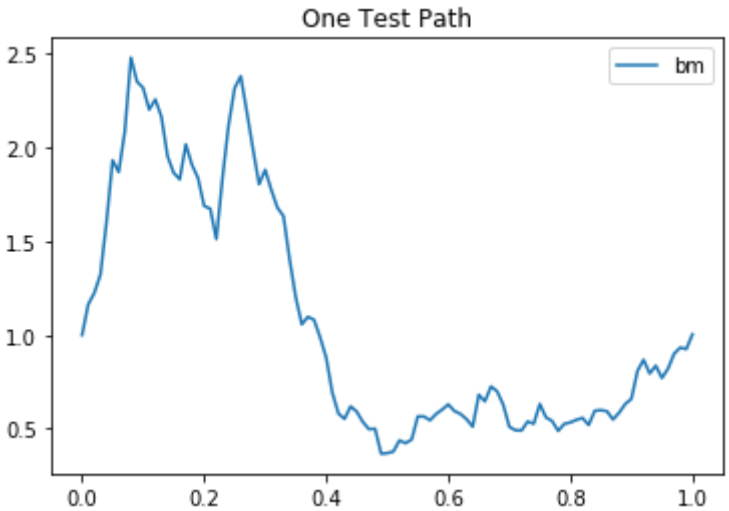
14871th Epoch:
traing loss is 2.7150277674081735e-05, test loss is 3.280298915342428e-05, and learning rate

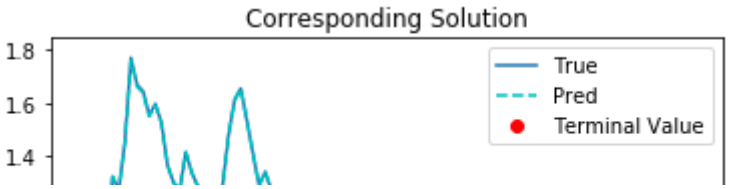
14881th Epoch:
traing loss is 4.119390359846875e-05, test loss is 3.425711111049168e-05, and learning rate i

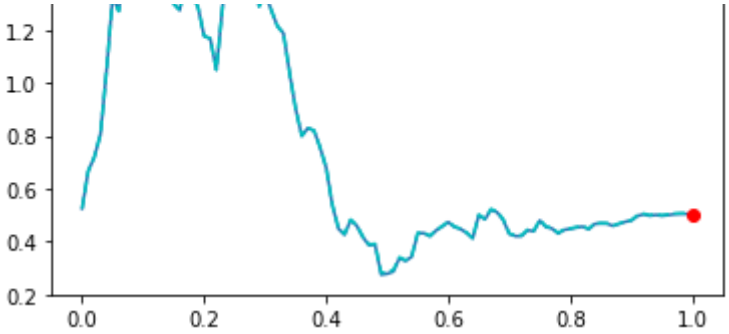
14891th Epoch:
traing loss is 4.931880175718106e-05, test loss is 3.339211252750829e-05, and learning rate i

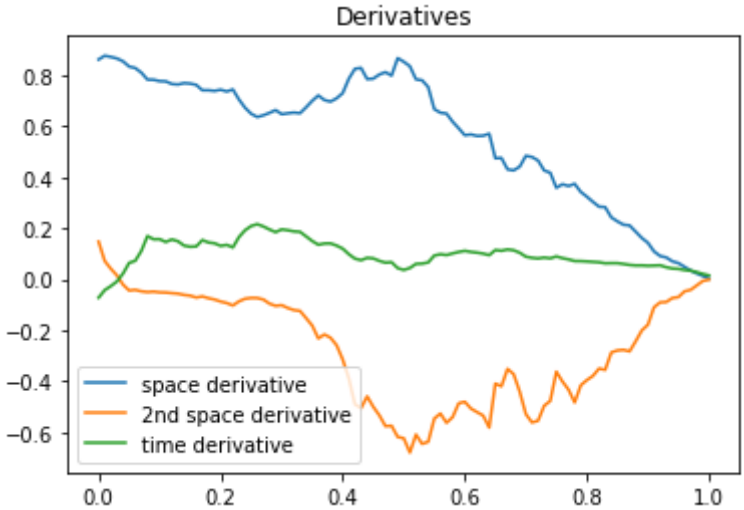
14901th Epoch:
traing loss is 4.71945277240593e-05, test loss is 3.319245661259629e-05, and learning rate is












```
14911th Epoch:
traing loss is 7.678731344640255e-05, test loss is 4.373414412839338e-05, and learning rate i

14921th Epoch:
traing loss is 4.768012877320871e-05, test loss is 3.500786260701716e-05, and learning rate i

14931th Epoch:
traing loss is 2.3742251869407482e-05, test loss is 3.434140307945199e-05, and learning rate

14941th Epoch:
traing loss is 3.431126242503524e-05, test loss is 3.353240026626736e-05, and learning rate i

14951th Epoch:
traing loss is 2.6669256840250455e-05, test loss is 3.350338010932319e-05, and learning rate

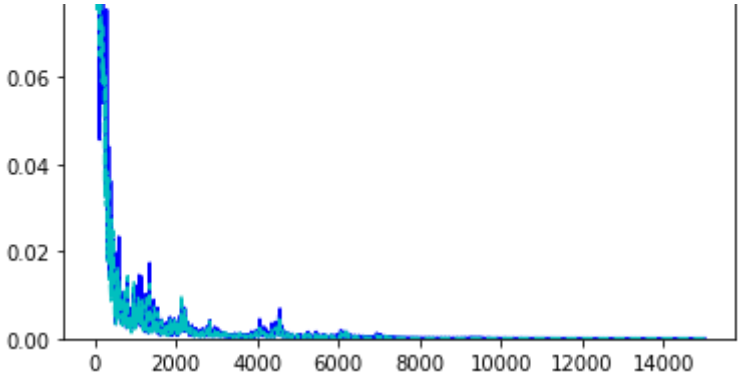
14961th Epoch:
traing loss is 5.611500455415808e-05, test loss is 3.398048647795804e-05, and learning rate i

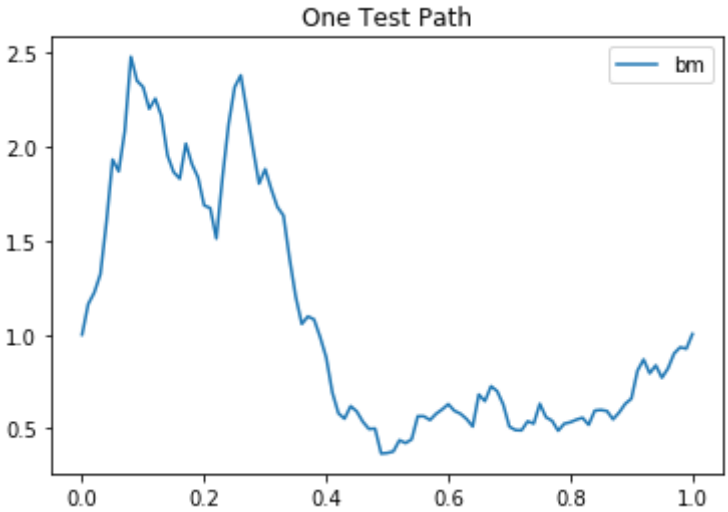
14971th Epoch:
traing loss is 2.6799294573720545e-05, test loss is 3.26168374158442e-05, and learning rate i

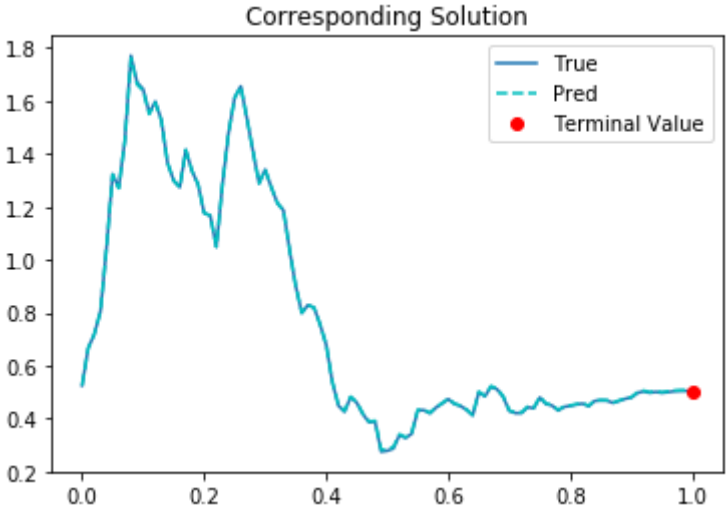
14981th Epoch:
traing loss is 4.121520396438427e-05, test loss is 3.417850530240685e-05, and learning rate i

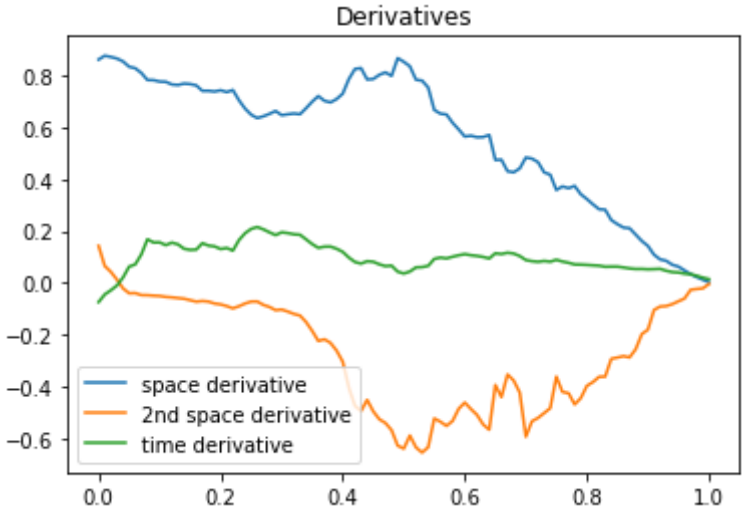
14991th Epoch:
traing loss is 4.926890323986299e-05, test loss is 3.329558239784092e-05, and learning rate i

15001th Epoch:
traing loss is 4.797623478225432e-05, test loss is 3.322788325021975e-05, and learning rate i
```


0.06
0.04
0.02
0.00
0
2000
4000
6000
8000
10000
12000
14000


One Test Path
bm
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0


Corresponding Solution
True
Pred
Terminal Value
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


Derivatives
space derivative
2nd space derivative
time derivative
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.0
0.2
0.4
0.6
0.8
1.0

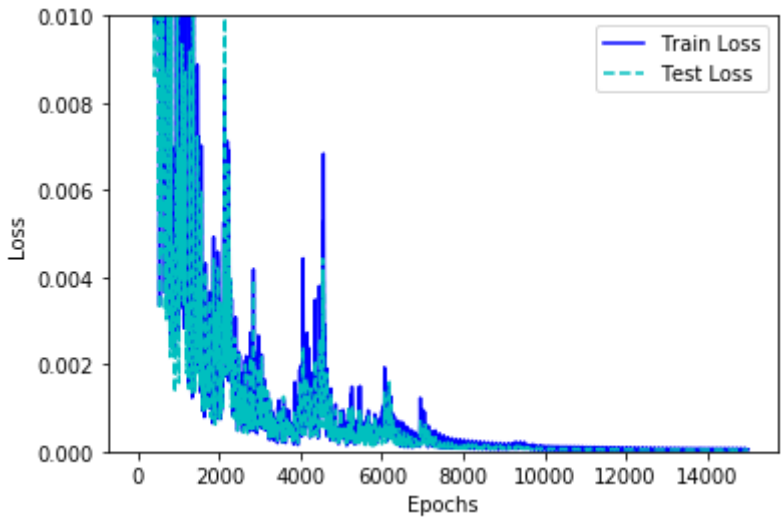
Evaluation

```
tf.gfile.MkDir("pics")  #  create  new  folder  called  pics
```

```
np.save('asian_train_loss_list.npy',  train_loss_list)
np.save('asian_test_loss_list.npy',  test_loss_list)
np.save('asian_solution_pred.npy',  solution_pred)
np.save('asian_space_derivative_pred.npy',  space_derivative_pred)
np.save('asian_space_2nd_derivative_pred.npy',  space_derivative_pred)
np.save('asian_time_derivative_pred.npy',  time_derivative_pred)
```

```
num_epoches  =  len(train_loss_list)
x_axis_epoches  =  np.linspace(0,  num_epoches*10,  num_epoches)
plt.plot(x_axis_epoches,  train_loss_list,  'b-',  label  =  "Train  Loss")
plt.plot(x_axis_epoches,  test_loss_list,  'c--',  label  =  "Test  Loss")
plt.legend()
plt.ylim([0,  0.01])
plt.xlabel("Epochs")
plt.ylabel("Loss")

plt.savefig("./pics/geo_asian_loss.pdf")
plt.show()
```



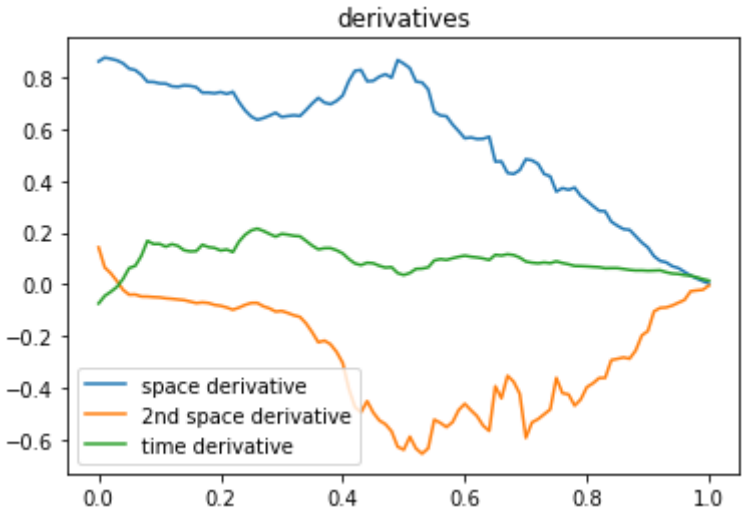

```
solution_pred,  time_derivative_pred,  space_derivative_pred  =  sess.run([solution,  time_derivat
```

```
print(train_loss_list[-5:])
print(test_loss_list[-5:])
```

```
[5.6115005e-05,  2.6799295e-05,  4.1215204e-05,  4.9268903e-05,  4.7976235e-05]
[3.3980486e-05,  3.2616837e-05,  3.4178505e-05,  3.3295582e-05,  3.3227883e-05]
```

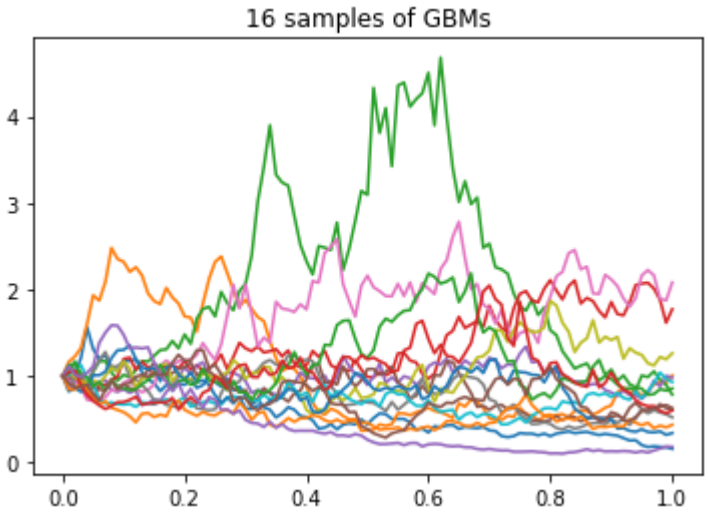
```
#  plot  derivatives
plt.plot(t,  space_derivative_pred[1],  label  =  "space  derivative")
plt.plot(t,  space_2nd_derivative_pred[1],  label  =  "2nd  space  derivative")
plt.plot(t,  time_derivative_pred[1],  label  =  "time  derivative")
```

```
plt.legend()
plt.title("derivatives")
plt.show()
```

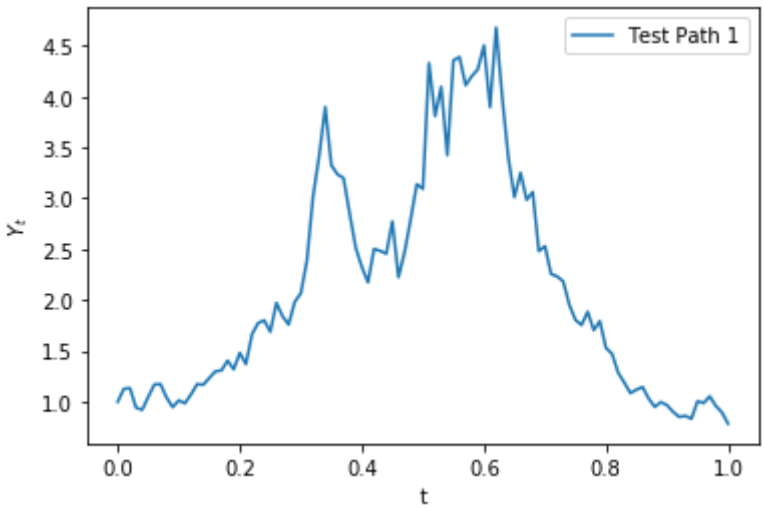



```
# 16 test paths
for i in range(16):
        plt.plot(t, path_test[i])
plt.title("16 samples of GBMs")
```

```
Text(0.5, 1.0, '16 samples of GBMs')
```

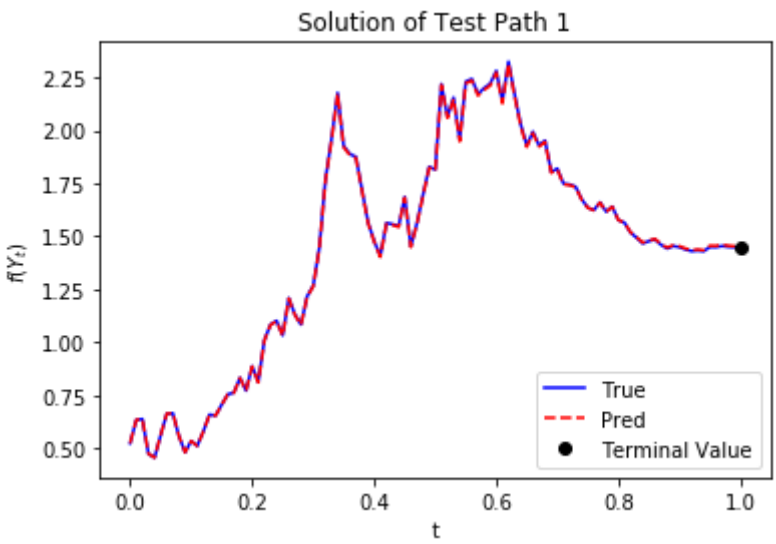



```
# A sample test path
plt.plot(t, path_test[2], label = "Test Path 1")
plt.xlabel("t")
plt.ylabel(r"$Y_t$")
plt.legend()
plt.savefig("./pics/geo_asian_test1.pdf")
```




```
# comparison of true vs predicted solution for the above GBM path
plt.plot(t, true_solution(path_test[2]), "b", label = "True")
plt.plot(t, solution_pred[2], "r--", label = "Pred")
plt.plot(T, terminal_condition(path_test[2]), "ko", label = "Terminal Value")

plt.title(r"Solution of Test Path 1")
plt.xlabel("t")
plt.ylabel(r"$f(Y_t)$")
plt.legend()
plt.savefig("./pics/geo_asian_sol1.pdf")
```



Test Another Path

```
path_test2 = np.tile(np.linspace(1,1e-6, steps+1), (M,1))**2
pred_dict2 = {path: path_test2, time: time_feed}

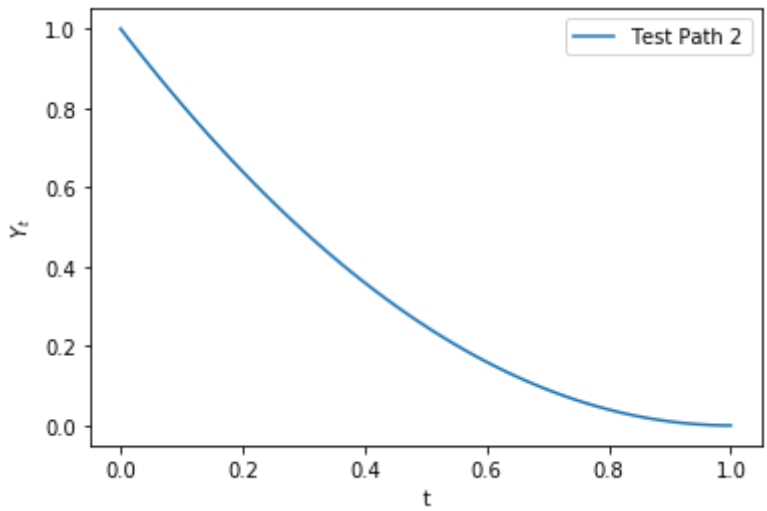
solution_2, space_derivative_2, time_derivative_2 = sess.run([solution, space_derivative, ti

loss_2 = sess.run(loss, pred_dict2)
loss_2
```

3.781876e-05

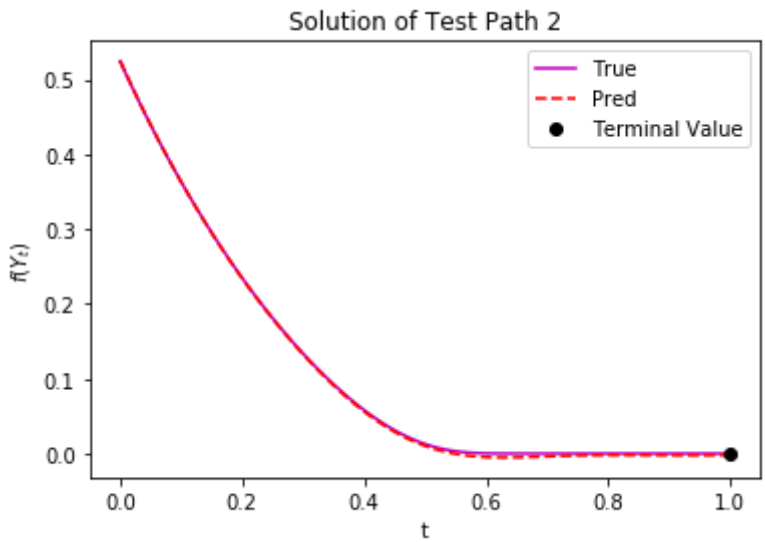
```
plt.plot(t, path_test2[1], label = "Test Path 2")
plt.legend()
# plt.title("Test Path 2")
plt.xlabel("t")
plt.ylabel(r"$Y_t$")

plt.savefig("./pics/geo_asian_test2.pdf")
plt.show()
```




```
# comparison of true vs predicted solution of this path

plt.plot(t, true_solution(path_test2[1]), "m", label = "True")
plt.plot(t, solution_2[1], "r--", label = "Pred")
plt.plot(T, terminal_condition(path_test2[1]), "ko", label = "Terminal Value")
plt.legend()
plt.xlabel("t")
plt.ylabel(r"$f(Y_t)$")
plt.title("Solution of Test Path 2")
plt.savefig("./pics/geo_asian_sol2.pdf")
```



```
## Test Another Path (Random Uniform)

np.random.seed(5)

path_test4 = np.tile(np.random.uniform(1, 3, steps+1), (M,1))
pred_dict4 = {path: path_test4, time: time_feed}

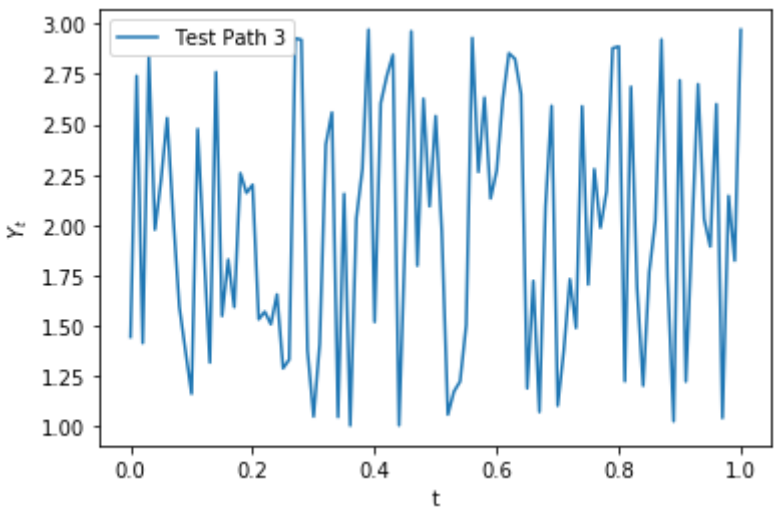
solution_4, space_derivative_4, time_derivative_4 = sess.run([solution, space_derivative, ti

loss_4 = sess.run(loss, pred_dict4)
loss_4
```

0.0046384605

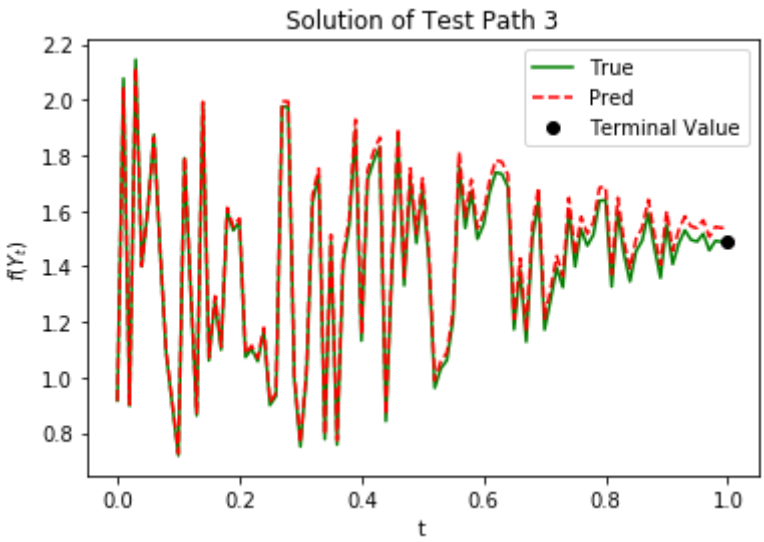
```
plt.plot(t, path_test4[1], label = "Test Path 3")
plt.legend()
plt.xlabel("t")
plt.ylabel(r"$Y_t$")

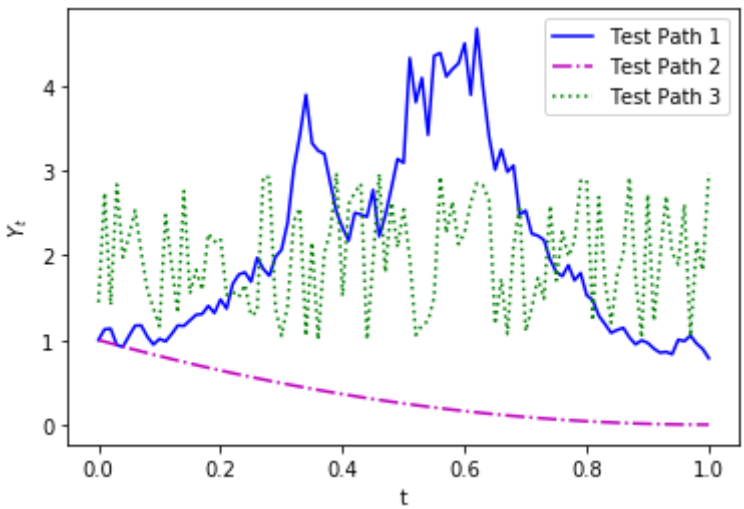

plt.savefig("./pics/geo_asian_test3.pdf")
plt.show()
```




```
# comparison of true vs predicted solution of this path

plt.plot(t, true_solution(path_test4[1]),"g", label = "True")
plt.plot(t, solution_4[1], "r--", label = "Pred")
plt.plot(T, terminal_condition(path_test4[1]), "ko", label = "Terminal Value")
plt.legend()
plt.xlabel("t")
plt.ylabel(r"$f(Y_t)$")
plt.title("Solution of Test Path 3")
plt.savefig("./pics/geo_asian_sol3.pdf")
```




```
plt.plot(t, path_test[2], "b", label = "Test Path 1")
plt.plot(t, path_test2[1],"m-.", label = "Test Path 2")
plt.plot(t, path_test4[1], "g:", label = "Test Path 3")

plt.xlabel("t")
plt.ylabel("$Y_t$")
plt.legend()
plt.savefig("./pics/geo_asian_paths.pdf")
```