Control of Mobile Robots: Glue Lectures

Instructor:

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Purpose

- To connect lectures to quizzes
- To give helpful hints about the quizzes
- To clarify and repeat key concepts

There will be one Glue Lecture every week

Last year's instructor

Amy LaViers
Assistant Professor
University of Virginia

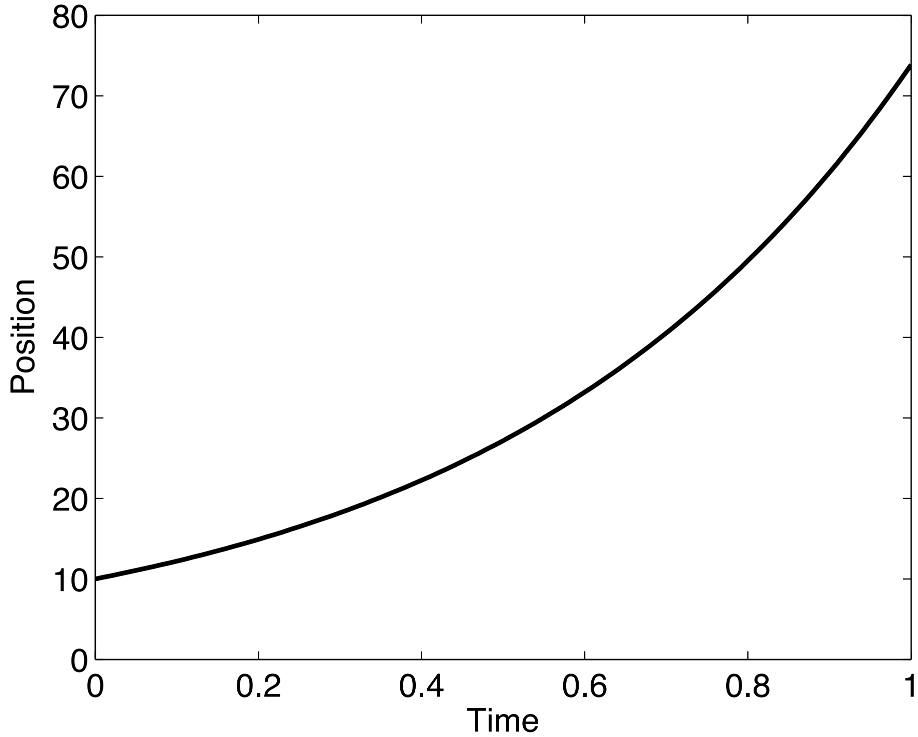
Glue Lecture 1: Dynamical Models

Pay attention, this lecture will help you all with Quiz 1!

An example: Position…

$$x(t) = x_0\, e^{a(t-t_0)}$$

$$t_0 = 0,\ \ x_0 = 10\ a = 2$$

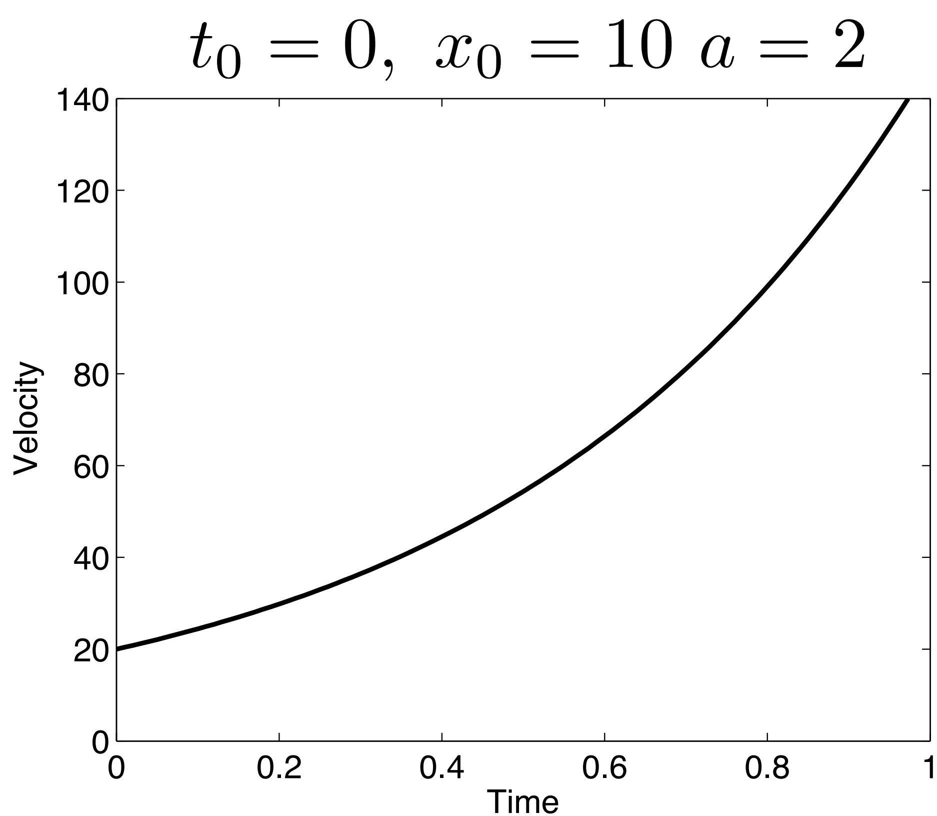


Velocity…

$$x(t) = x_0\, e^{a(t-t_0)}$$

$$\dot{x}(t) = a x_0\, e^{a(t-t_0)}$$

$$= a x(t)$$



And further on…

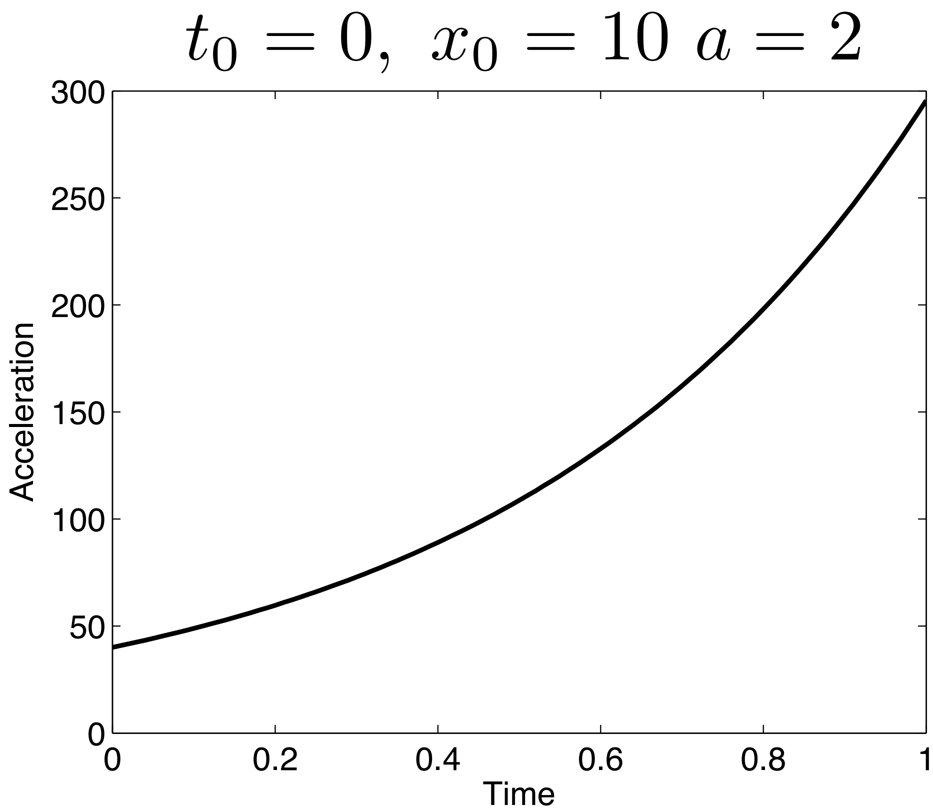
$$x(t) = x_0\, e^{a(t-t_0)}$$

$$\dot{x}(t) = a x_0\, e^{a(t-t_0)}$$

$$= a x(t)$$

$$\ddot{x}(t) = a^2 x_0\, e^{a(t-t_0)}$$

$$= a^2 x(t)$$



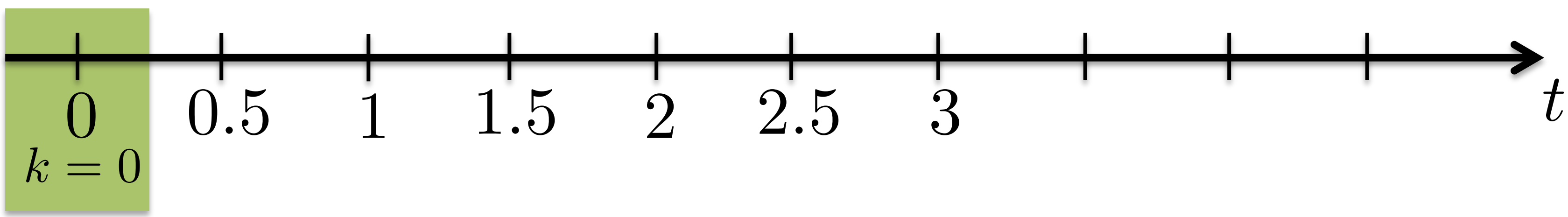
In Action…

$$x(t) = x_0\, e^{2(t-t_0)}$$
$$= 10e^{2t}$$

Using the differential equation …

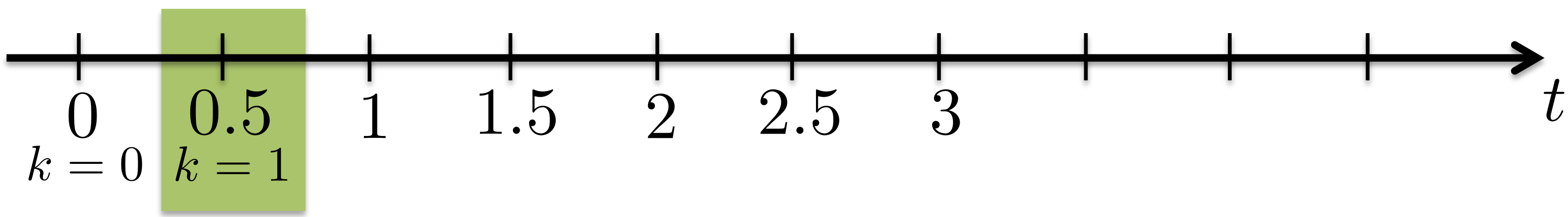
$$\dot{x}(t) = 2x$$
$$x(0) = 10$$

Let's discretize the world…



$$\delta t = 0.5$$
$$t = k\delta t$$

Let's discretize the world…



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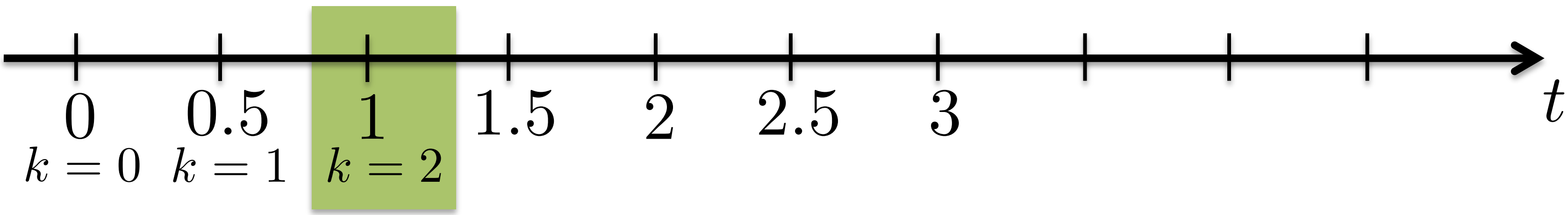
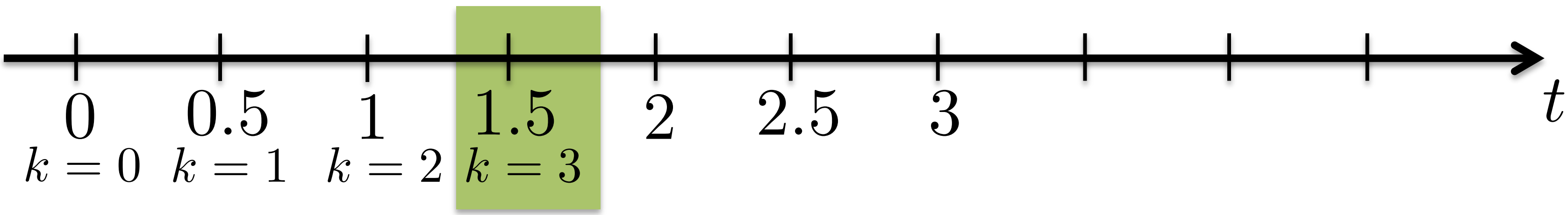


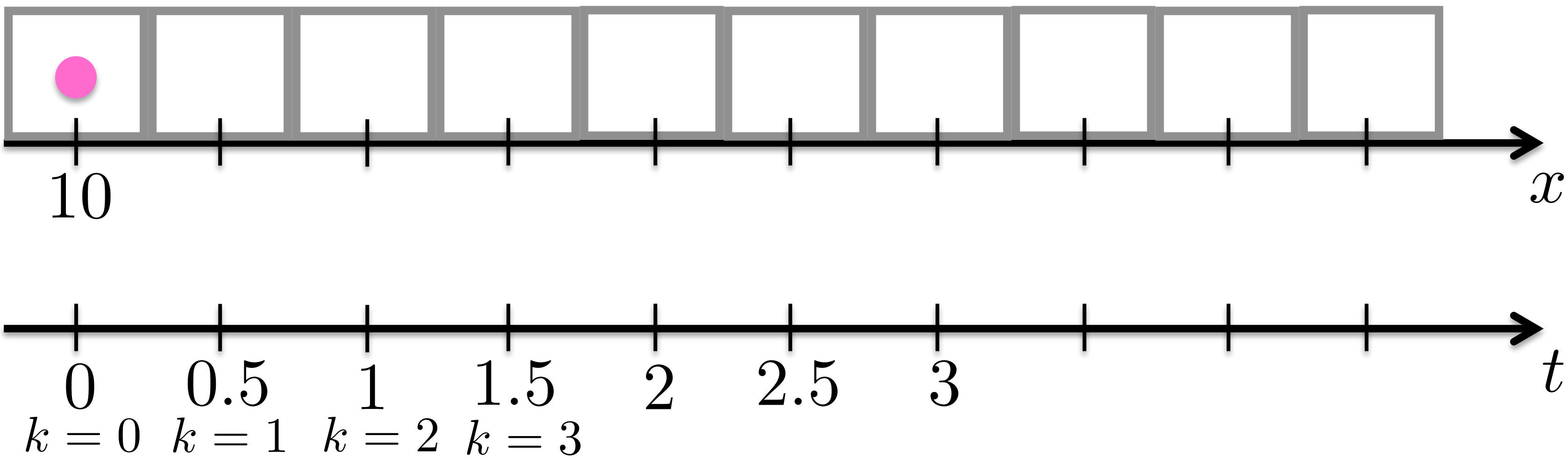
$$\delta t = 0.5$$
$$t = k\delta t$$

Let's discretize the world…



$$\delta t = 0.5$$
$$t = k\delta t$$

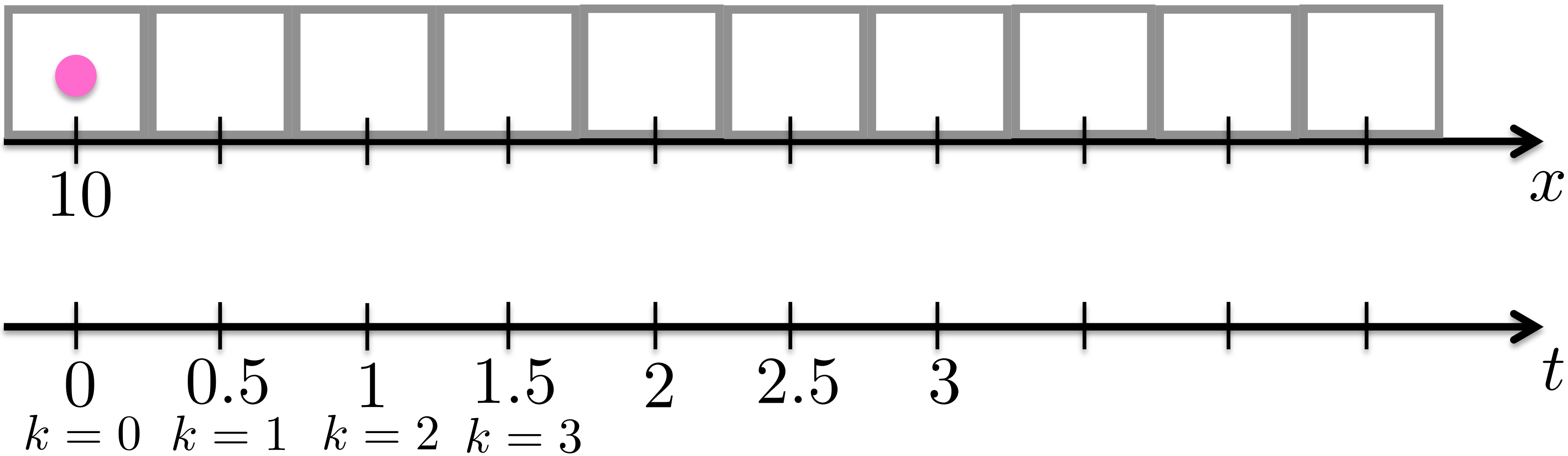
Now… how's the ball moving?



$\dot{x}(t) = 2x$ continuous time

$x_{k+1} = x_k + \delta t \dot{x}_k + \frac{\delta t^2}{2!}\ddot{x}_k + ..$ discrete time

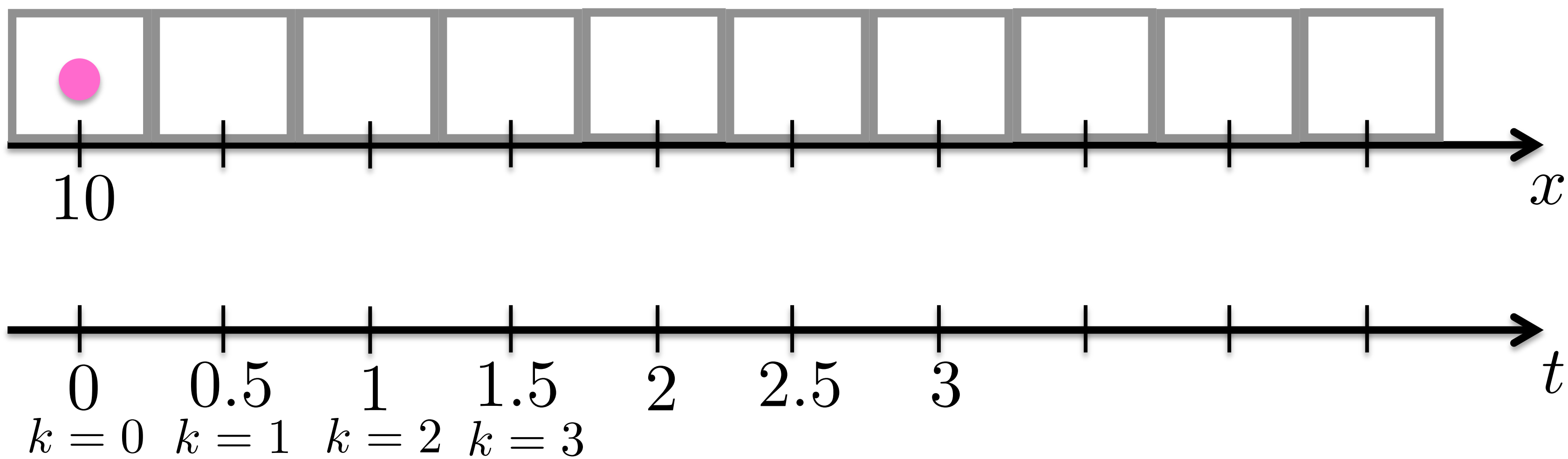
Now… how's the ball moving?



$$\dot{x}(t) = 2x \quad \delta t = 0.5 \quad t = k\delta t$$

$$x_{k+1} = x_k + \delta t \dot{x}_k + \frac{\delta t^2}{2!}\ddot{x}_k + .. \quad \text{discrete time}$$

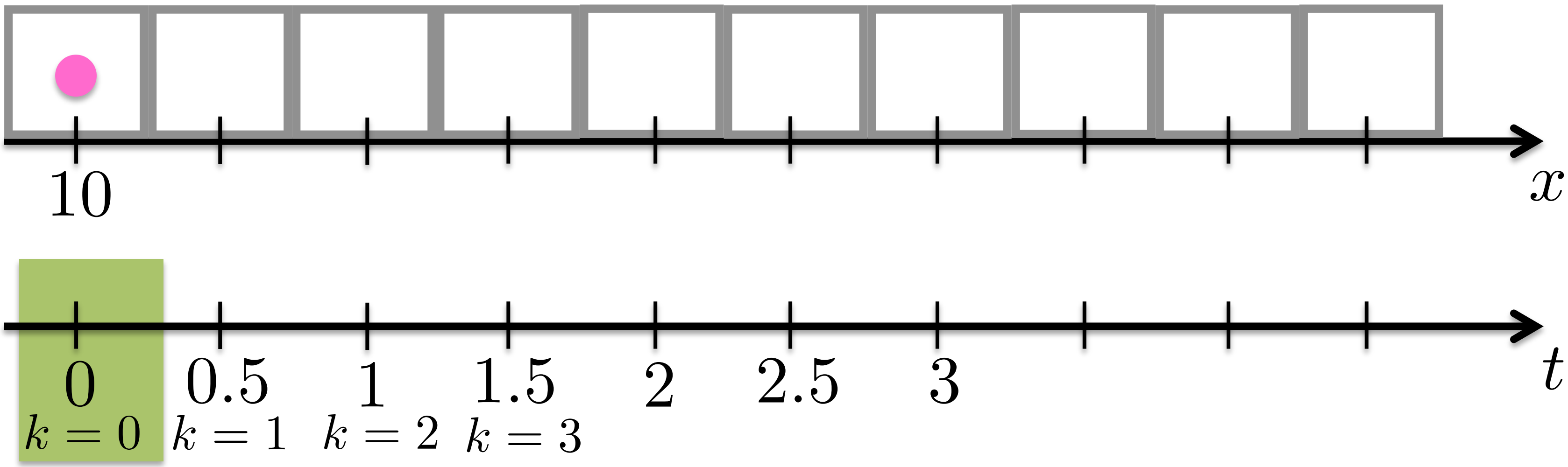
Now… how's the ball moving?



$$\dot{x}(t) = 2x \quad \delta t = 0.5 \quad t = k\delta t$$

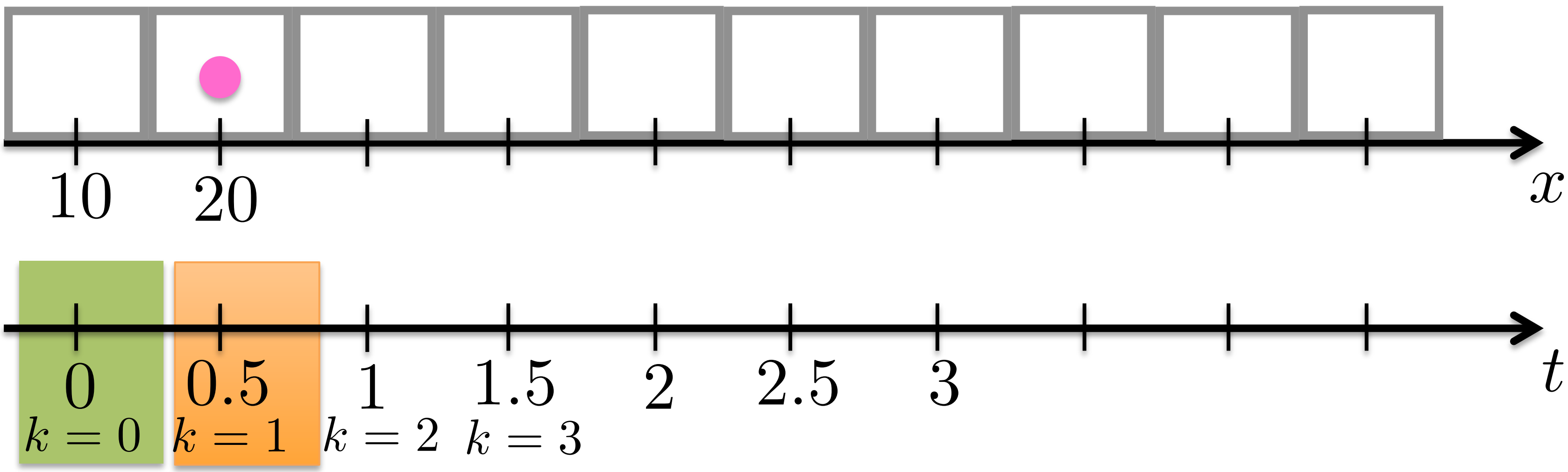
$$x_{k+1} = 2x_k$$

Now… how's the ball moving?



$$x_0 = 10$$
$$x_{k+1} = 2x_k$$
$$x_1 = 2x_0$$

Now… how's the ball moving?

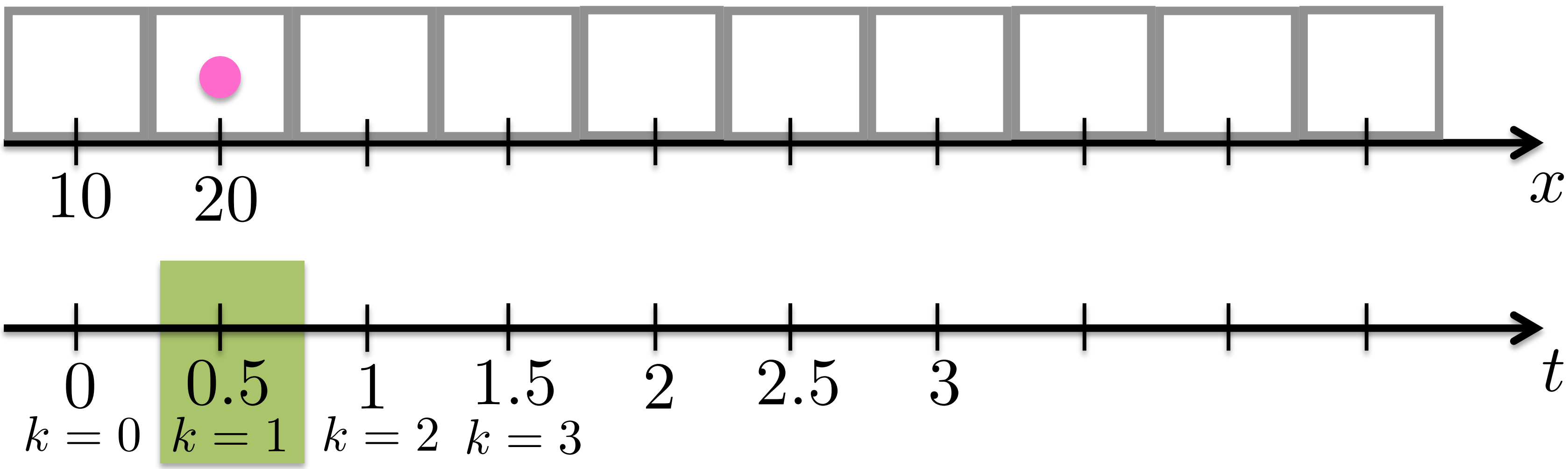


$$x_0 = 10$$

$$x_{k+1} = 2x_k$$

$$\boxed{x_1 = 20}$$

Now… how's the ball moving?

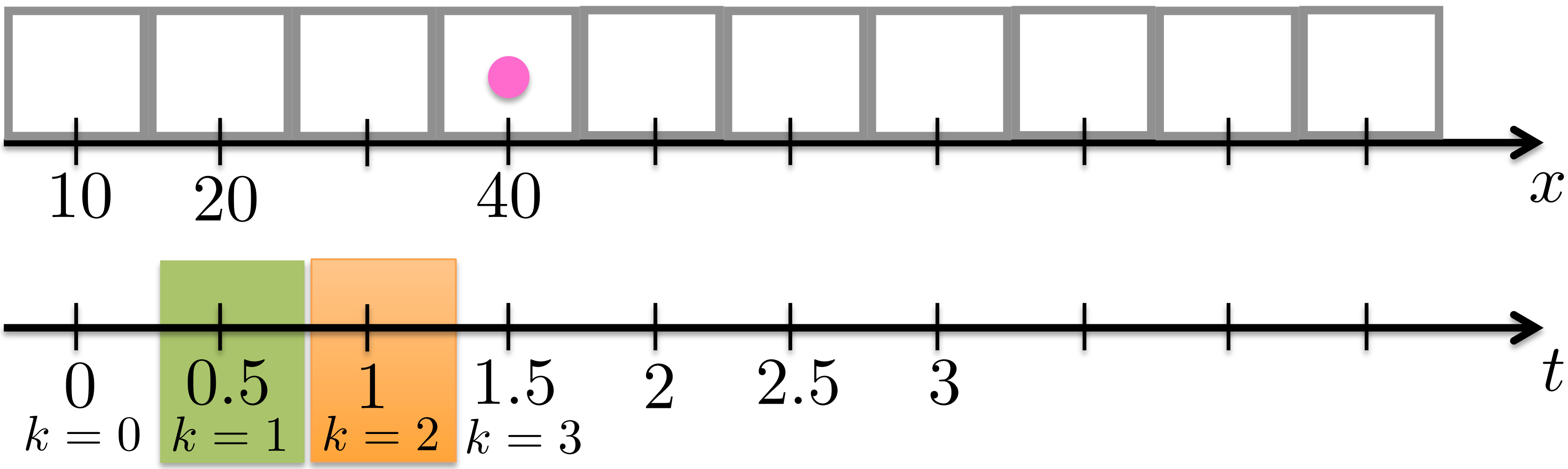


$$x_1 = 20$$

$$x_{k+1} = 2x_k$$

$$x_2 = 2x_1$$

Now… how's the ball moving?

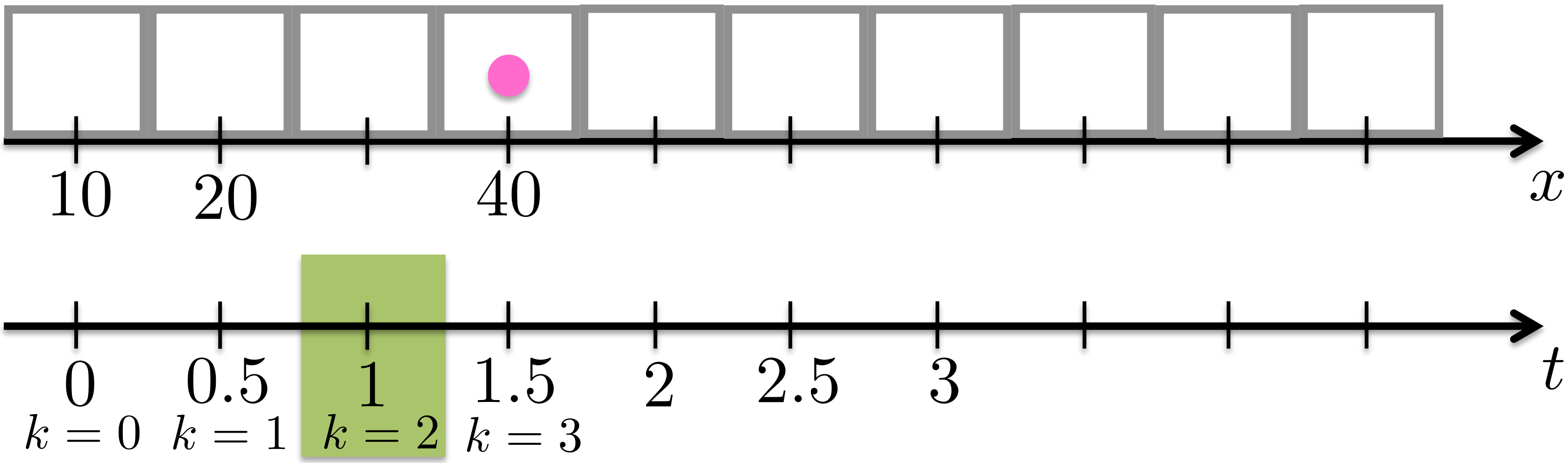


$$x_1 = 20$$

$$x_{k+1} = 2x_k$$

$$\boxed{x_2 = 40}$$

Now… how's the ball moving?

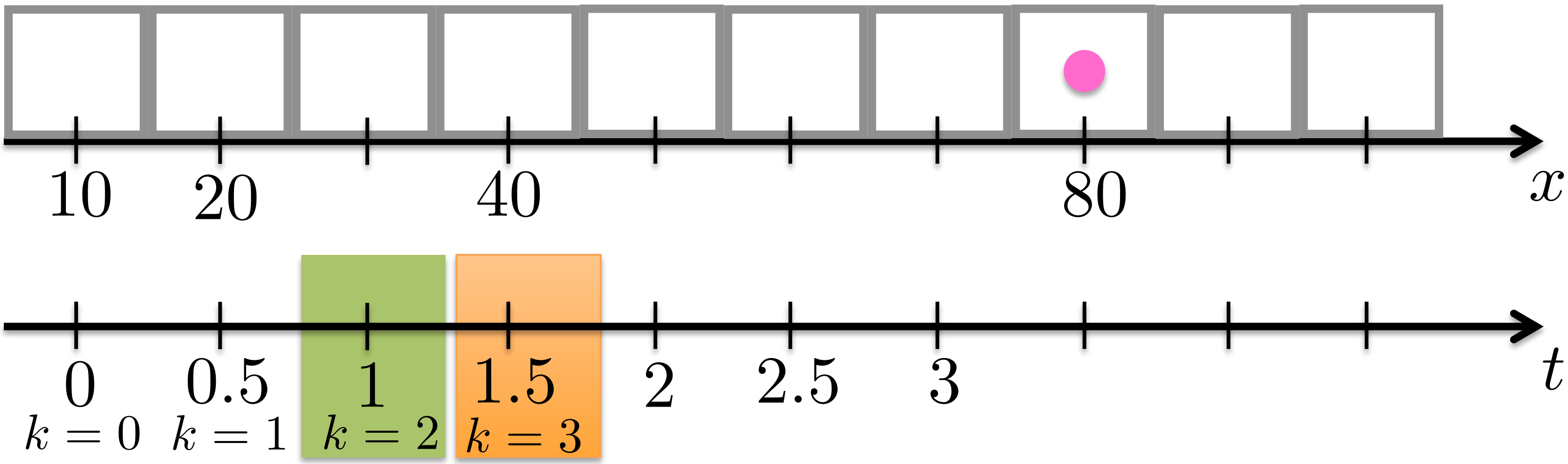


$$x_2 = 40$$

$$x_{k+1} = 2x_k$$

$$x_3 = 2x_2$$

Now… how's the ball moving?



$$x_2 = 40$$

$$x_{k+1} = 2x_k$$

$$\boxed{x_3 = 80}$$

And …. that's a dynamical model!

$$\dot{x}(t) = 2x$$
$$x(0) = 10$$

In general,

$$\dot{x}(t) = f(x, t)$$
$$x(t^*) = x^*$$

Punchline: Given the following dynamical model ….

$$\dot{x}(t) = 2x$$
$$x(0) = 10$$

… you know how x evolves ☺

… numerically

Punchline: Given the following dynamical model ….

$$\dot{x}(t) = 2x$$
$$x(0) = 10$$

To get the mathematical expression for $x(t)$, we integrate!

Hint : we already know the solution

$$x(t) = x_0\, e^{2(t-t_0)} = 10e^{2t}$$

Integration

$$\dot{x}(t) = 2x$$
$$x(0) = 10$$

woohoo !!

"Candidate" solution…

$$\dot{x}(t) = 2x$$

$$x(0) = 10$$

$$\boxed{x(t) = 10e^{2t}}$$

perform the following two checks to confirm candidate is indeed the solution …

(a) Initial condition
(b) Differential equation

Take home message :

1) Dynamical models

$$\dot{x}(t) = 9x + 3$$

$$x(3) = 5$$

2) Solving them

(i) Numerically

(ii) Analytically (integration)

3) Checking a candidate solution

$$x(t) = \frac{16}{3}e^{9(t-3)} - \frac{1}{3}$$

(a) initial condition

(b) differential equation

4) Equilibrium point

$$\dot{x}(t) = 0$$

$$x = -1/3$$

Check the forums, and good luck with Quiz 1!