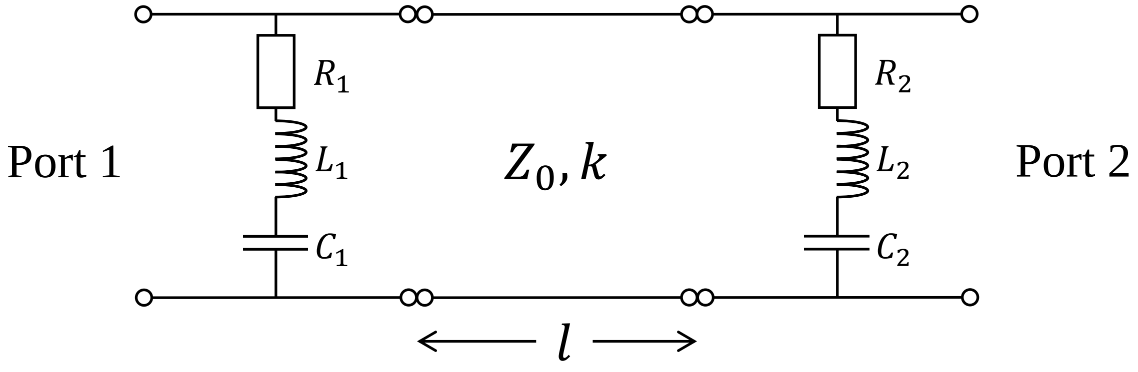


$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}} \qquad \omega_2 = \frac{1}{\sqrt{L_2 C_2}}$$

$$\beta = \frac{R_1}{2\,L_1} \qquad \alpha = \frac{R_2}{2\,L_2}$$

$$\kappa_{ext1} = \frac{Z_o}{4\,L_1} \qquad \kappa_{ext2} = \frac{Z_o}{4\,L_2}$$

Transmission (ABCD) matrices

$$M_1 = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix}$$

$$Z_1 = R_1 - i\omega L_1 - \frac{1}{i\omega C_1} = -i\,\frac{L_1}{\omega}\left(\omega^2 - \omega_1^2 + 2i\omega\beta\right)$$

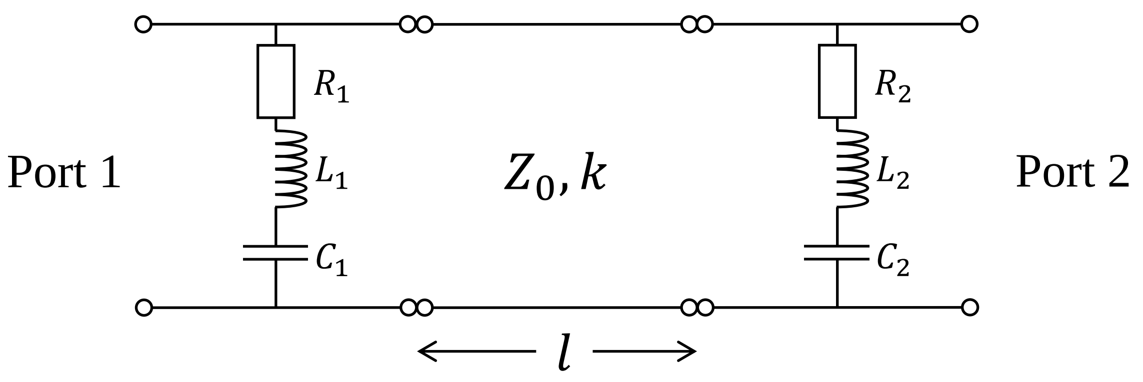
$$Z_2 = R_2 - i\omega L_2 - \frac{1}{i\omega C_2} = -i\,\frac{L_2}{\omega}\left(\omega^2 - \omega_2^2 + 2i\omega\alpha\right)$$

$$M_t = \begin{bmatrix} \cos kl & -i\,Z_0 \sin kl \\ -iZ_0^{-1} \sin kl & \cos kl \end{bmatrix}$$

$$M = M_1 \cdot M_t \cdot M_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D}$$

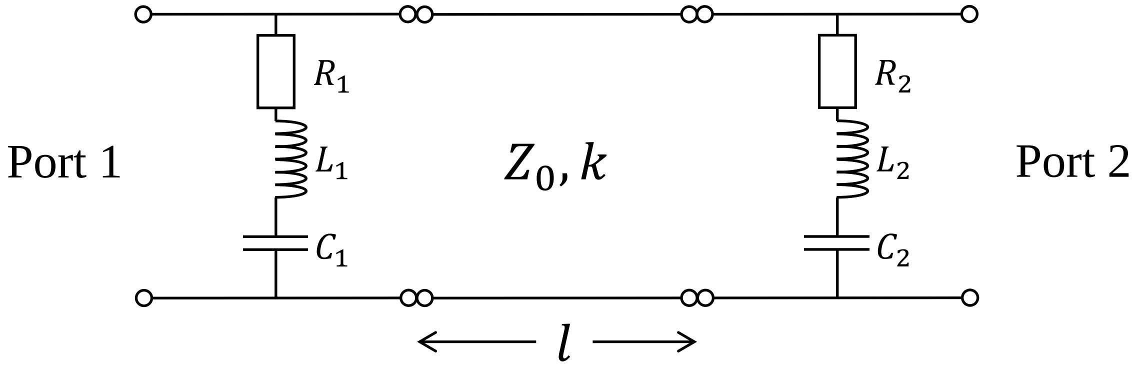


$$A = \cos(kl) + \frac{\omega Z_0 \sin(kl)}{L_2\,(\omega^2 - \omega_2^2 + 2i\omega\alpha)} \qquad B = -iZ_0 \sin(kl)$$

$$C = \frac{i\omega\cos(kl)}{L_1(\omega^2 - \omega_1^2 + 2i\omega\beta)} - \frac{i\sin(kl)}{Z_0} + \frac{i\omega\left(\cos(kl) + \frac{\omega Z_0 \sin(kl)}{L_1(\omega^2 - \omega_1^2 + 2i\omega\beta)}\right)}{L_2(\omega^2 - \omega_2^2 + 2i\omega\alpha)} \qquad D = \cos(kl) + \frac{\omega Z_0 \sin(kl)}{L_1\,(\omega^2 - \omega_1^2 + 2i\omega\beta)}$$

精确解：

$$S_{21} = 1 + \frac{\overset{\substack{\textbf{\textit{negligible}} \\ \text{Driving force act on the second oscillator}}}{\boxed{\left(e^{ikl} - 1\right)(\omega^2 - \omega_1^2 + 2i\omega\beta)}} + i\omega\left(-2\kappa_{ext1} - \overset{\textbf{\textit{negligible}}}{\boxed{\overset{\kappa_{ext1} \gg \kappa_{ext2}}{2\kappa_{ext2}e^{ikl}}\;\overset{\approx 1}{\frac{(\omega^2 - \omega_1^2 + i\omega\beta) - 2i\omega\kappa_{ext1}e^{ikl}}{(\omega^2 - \omega_2^2 + i\omega\alpha) + 2i\omega\kappa_{ext2}}}}}\right)}{(\omega^2 - \omega_1^2 + 2i\omega\beta) + 2i\omega\kappa_{ext1} + \frac{e^{2ikl}\,4\omega^2\kappa_{ext1}\kappa_{ext2}}{(\omega^2 - \omega_2^2 + 2i\omega\alpha) + 2i\omega\kappa_{ext2}}}$$

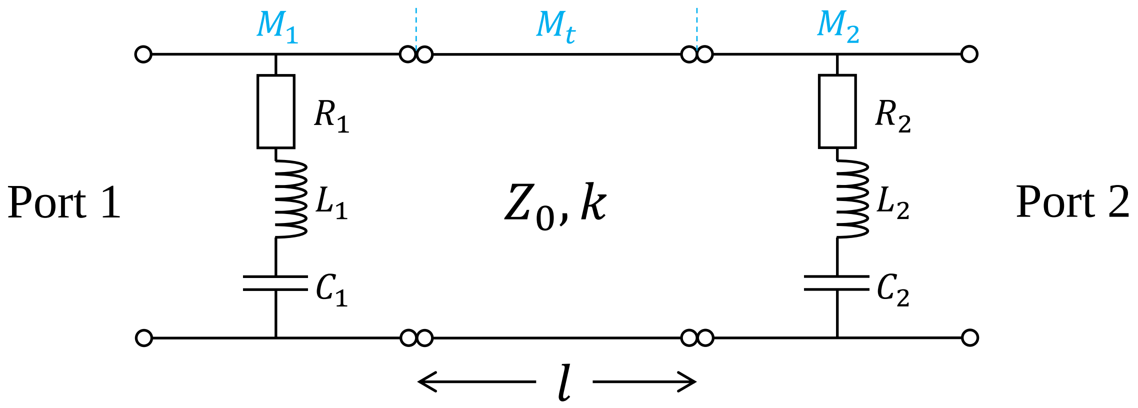


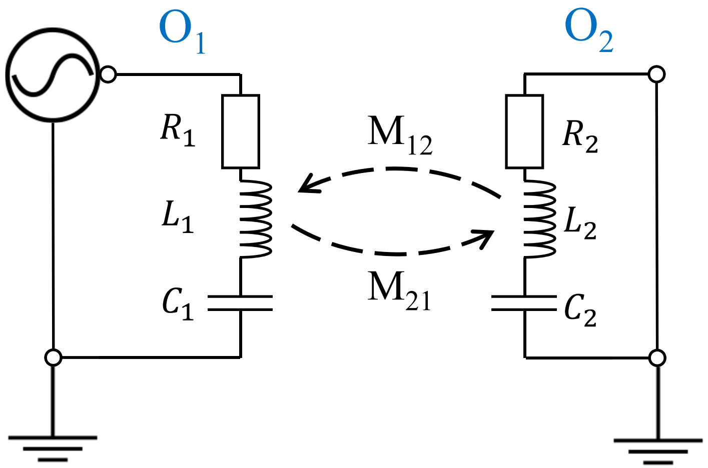
近似解：

$$S_{21} = 1 + \frac{\kappa_{ext1}}{i(\omega - \omega_1) - \beta - \kappa_{ext1} + \frac{e^{2ikl+\pi(0)}\kappa_{ext1}\kappa_{ext2}}{i(\omega-\omega_2) - \alpha - \kappa_{ext2}}}$$

→ Same as we derived from quantum model, this also explains the interference of coherent and dissipative coupling.

Different phase generates the nonreciprocity, this may relate to different local curl field.


M_1
M_t
M_2
R_1
R_2
Port 1
L_1
Z_0, k
L_2
Port 2
C_1
C_2
l


O1
O2
R1
L1
C1
M12
M21
R2
L2
C2