

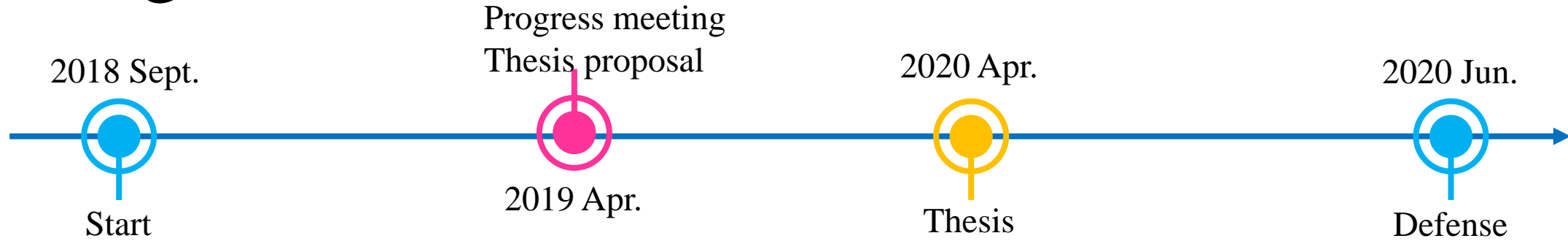


Progress meeting

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Department of Physics and Astronomy
April 25th 2019

Progress outline



- Coursework

- | | | | |
|----|-----------|--|---------------|
| 1. | PHYS 7720 | Quantum Mechanics 1 | (A+) |
| 2. | PHYS 7510 | Condensed Matter Physics 2 | (A) |
| 3. | PHYS 7590 | Electromagnetic Theory | (In progress) |
| 4. | ECE 7440 | Microwave Materials Measurement Techniques | (Fall 2019) |

- Experiment:

1. Level attraction in metamaterials
2. Level attraction in coupled mechanical oscillators

- Publications:

- 2 Coauthor: New Journal of Physics (under review), Nature Communication (under review)

Coherent coupling and Level repulsion

Equation of motion:


$$\ddot{x}_1 + \alpha \dot{x}_1 + \omega_1^2 x_1 + \kappa x_2 = F e^{i\omega t}$$

$$\ddot{x}_2 + \beta \dot{x}_2 + \omega_2^2 x_2 + \kappa x_1 = 0$$

Coupling terms

Matrix form:

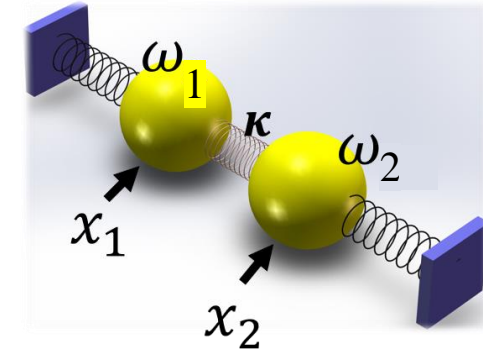
$$\begin{pmatrix} \omega_1^2 - \omega^2 + i\omega\alpha & \kappa \\ \kappa & \omega_2^2 - \omega^2 + i\omega\beta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}$$


 (Oscillator1 Coupling)
 (Coupling Oscillator2)

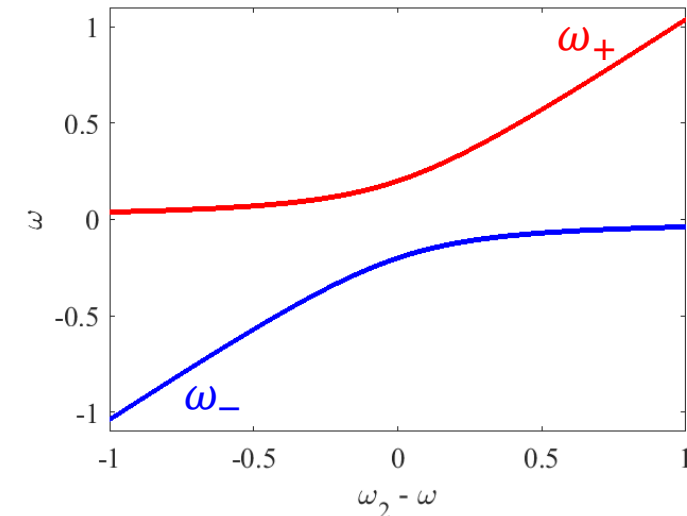
Neglecting the damping terms ($\alpha, \beta \approx 0$),
the eigenvalues of the coupled system:

$$\omega_{\pm} = \frac{1}{2} \left[(\omega_1 + \omega_2) \pm \sqrt{(\omega_1 - \omega_2)^2 + 4\kappa^2} \right]$$

Coupled mechanical oscillator



Dispersion relation



Repulsion behavior

Dissipative coupling and Level attraction

Equation of motion:

$$\ddot{x}_1 + \alpha \dot{x}_1 + \omega_1^2 x_1 + \kappa \dot{x}_2 = F e^{i\omega t}$$

$$\ddot{x}_2 + \beta \dot{x}_2 + \omega_2^2 x_2 + \kappa \dot{x}_1 = 0$$

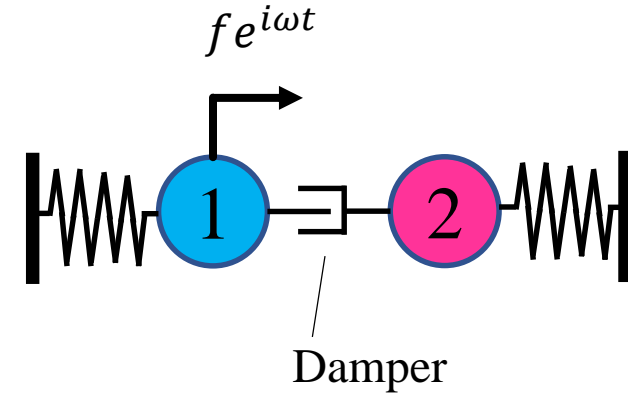
Dissipative

Matrix form:

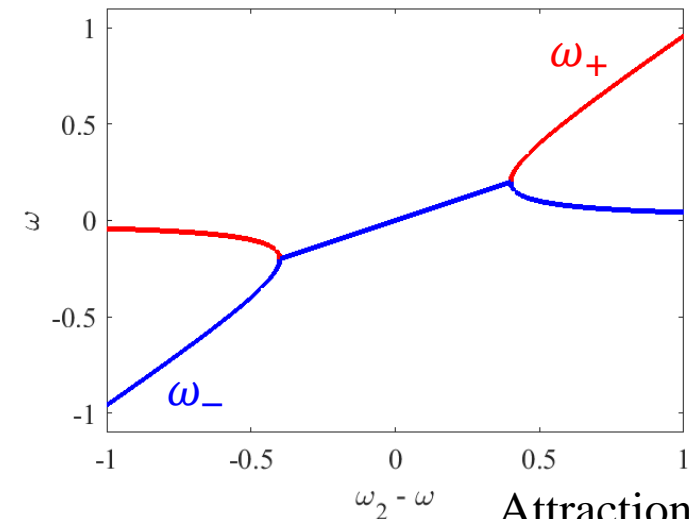
$$\begin{pmatrix} \omega_1^2 - \omega^2 + i\omega\alpha & i\kappa \\ i\kappa & \omega_2^2 - \omega^2 + i\omega\beta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}$$

Coupling is depend on the relative speed rather than the position of the oscillators!

Dissipative coupling of two oscillators



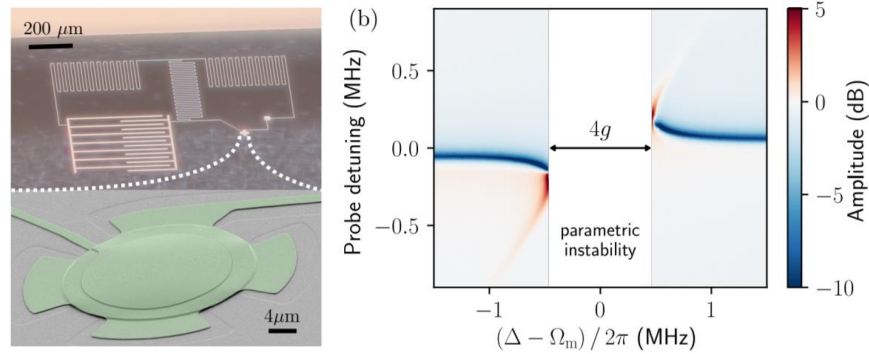
Dispersion relation



Attraction behavior

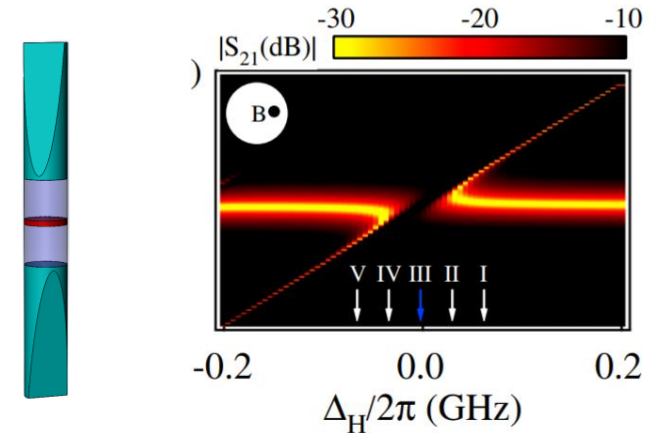
Emergence of level attraction

optomechanical circuit

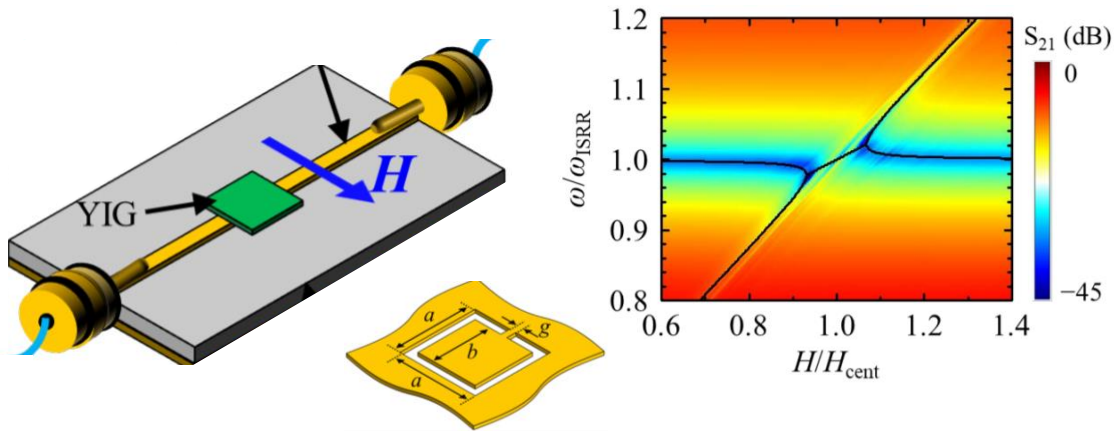


Bernier, N. R., et al. Physical Review A 98.2 (2018): 023841.

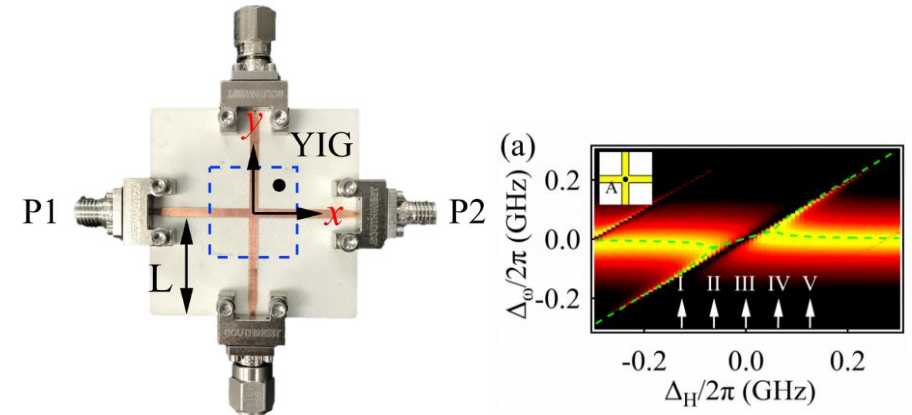
Cavity magnon system



Harder, M., et al. Physical review letters 121.13 (2018): 137203.



Bhoi, Biswanath, et al. arXiv preprint arXiv:1901.01729 (2019).



Yang, Y., et al. arXiv preprint arXiv:1901.07633 (2019).

Level attraction in metamaterials?

Metamaterials:

Material designed that have special properties.

Applications:

Negative refraction

Absorber

Cloaking devices

...

Now comes the question:

Level attraction in metamaterials?

Split-ring resonator

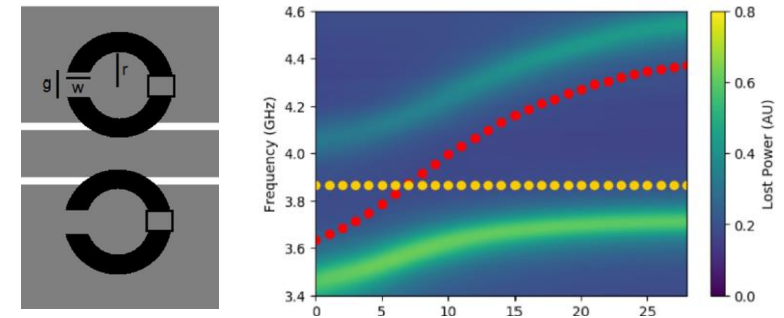


Cross resonator



Liu, Xianliang, et al. Physical review letters 107.4 (2011): 045901.
Shelby, Richard A., David R. Smith, and Seldon Schultz.
science 292.5514 (2001): 77-79.

Level repulsion in metamaterials



Baraclough, Milo, Ian R. Hooper, and William L. Barnes.
Physical Review B 98.8 (2018): 085146.

Characterize cross cavity resonator

RLC circuits model for the cross cavity

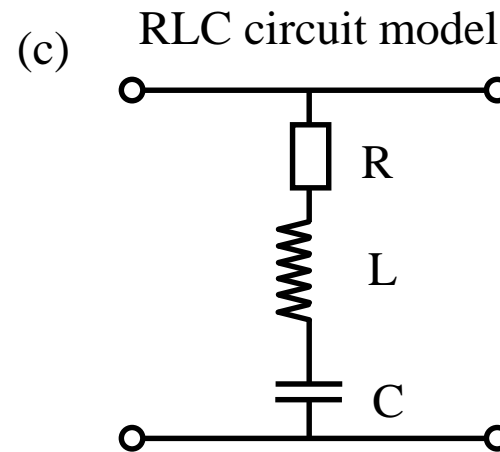
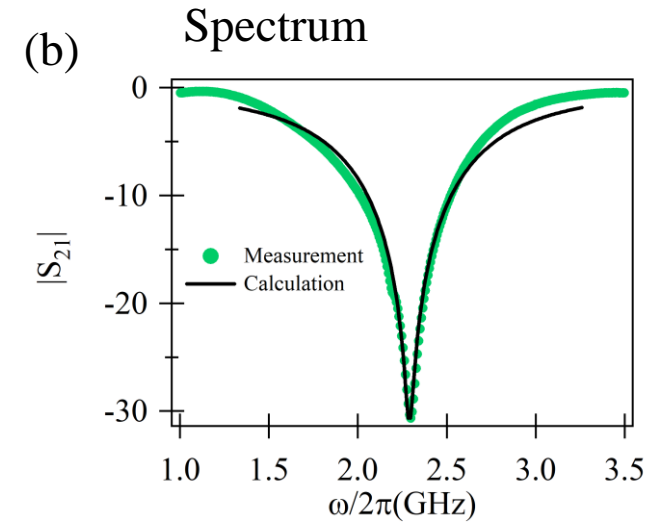
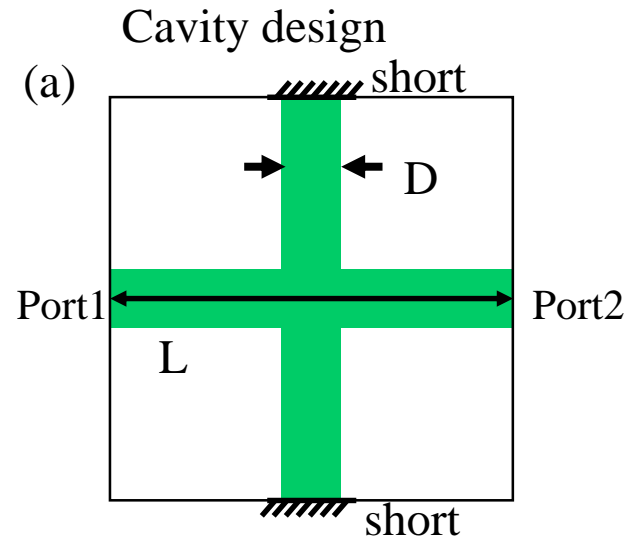
$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1/Z & 1 \end{pmatrix}$$

$$S_{21} = 1 - \frac{i\Delta\omega_e}{\omega - \omega_c + i(\Delta\omega_e + \Delta\omega_i)}$$

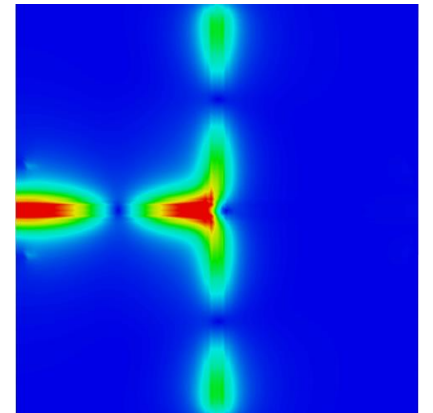
$\omega_c = 1/\sqrt{LC}$ - resonant frequency

$\Delta\omega_e = Z_0/2L$ - extrinsic damping

$\Delta\omega_i = R/2L$ - intrinsic damping



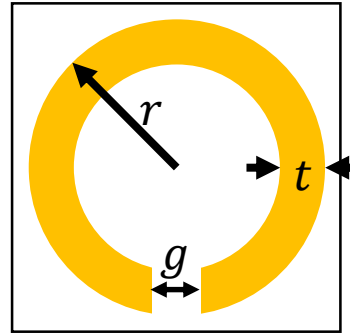
(d) Magnetic field distribution



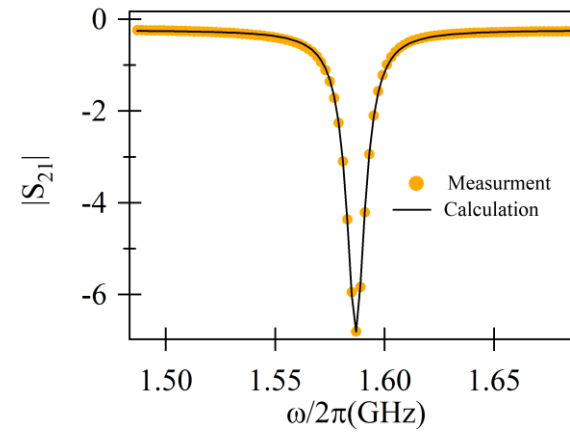
Characterize split ring resonator

The Split ring resonators

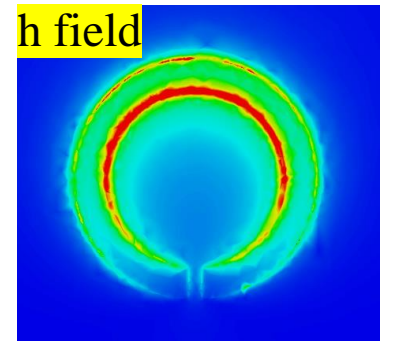
SSR design



spectrum



Field distribution

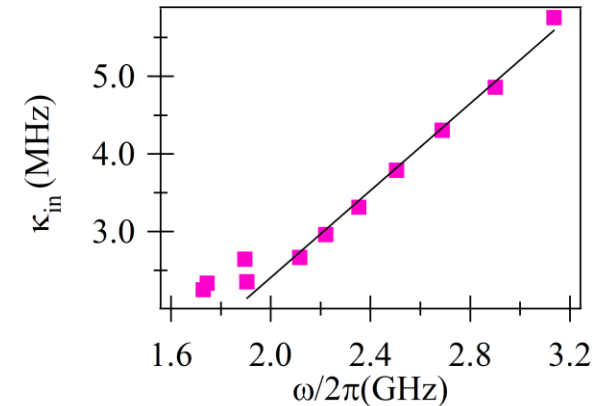
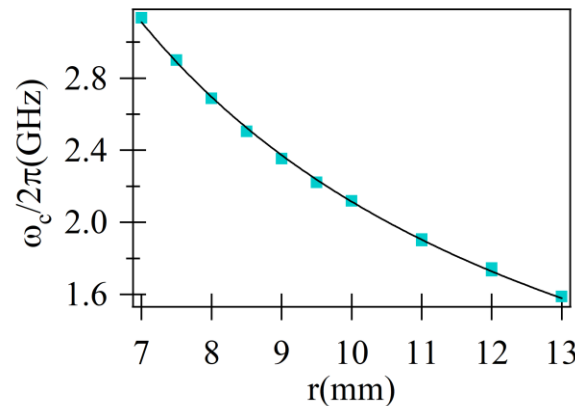


Resonant frequency

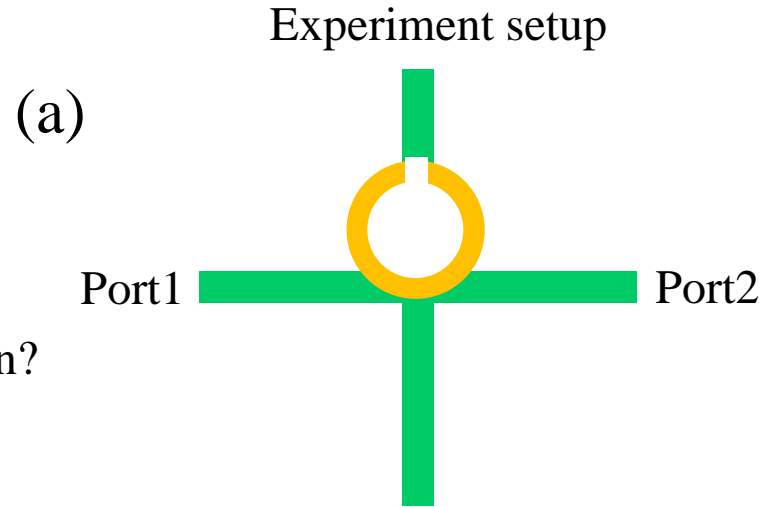
$$\omega_{SRR} = \frac{c'}{\lambda} = \frac{c'}{2\pi r} \propto \frac{a}{r}$$

Intrinsic damping

$$\Delta\omega_{in} = \beta\omega_c$$

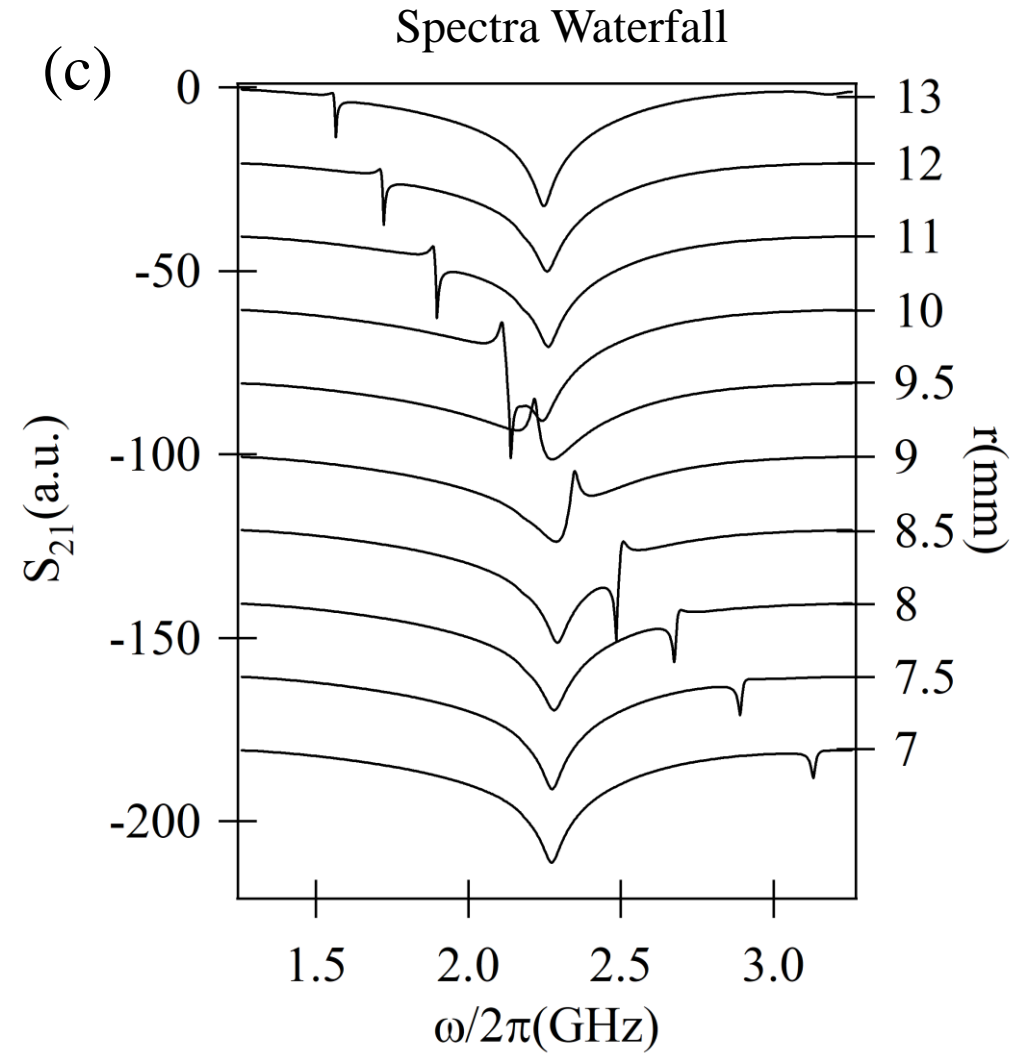
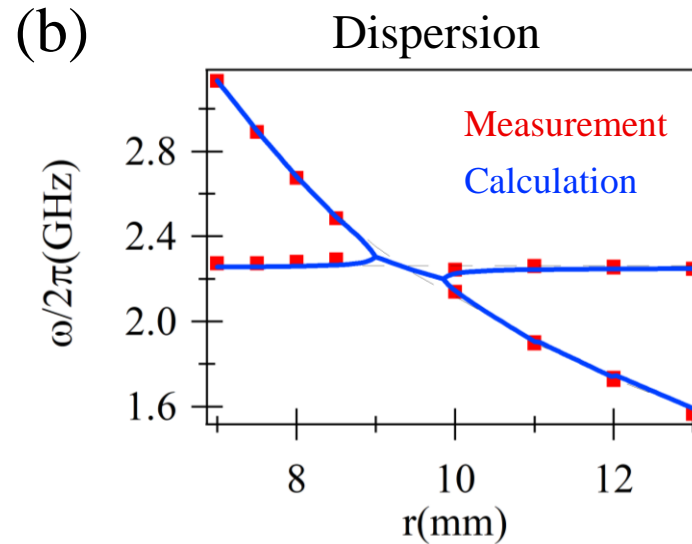


Level attraction with different SRRs



Why is this Level attraction?

- Dispersion analysis
- [Line shape analysis.](#)
- [Phase analysis](#)



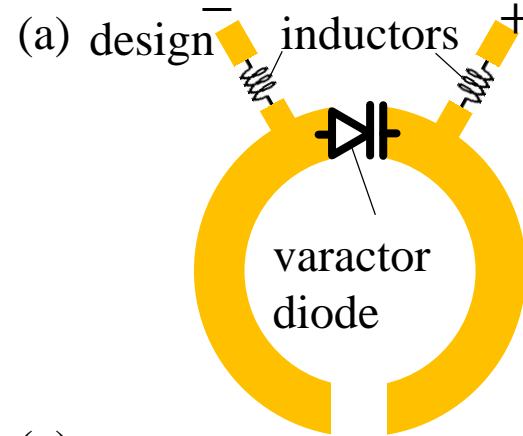
Level attraction with tunable SSRs

Tunable SSR can be made by a varactor.

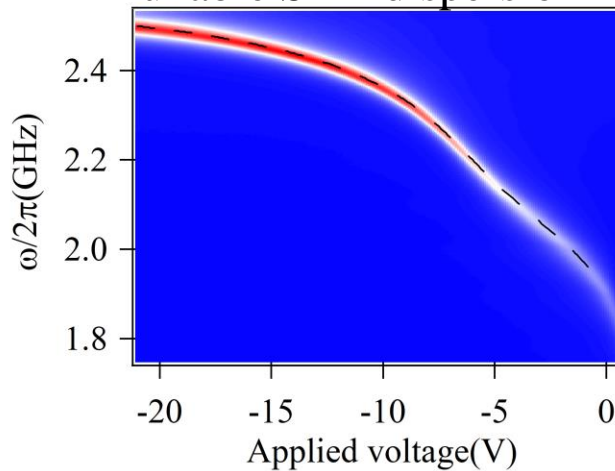
Capacitance : $C = C(V)$

V – applied voltage on the diode

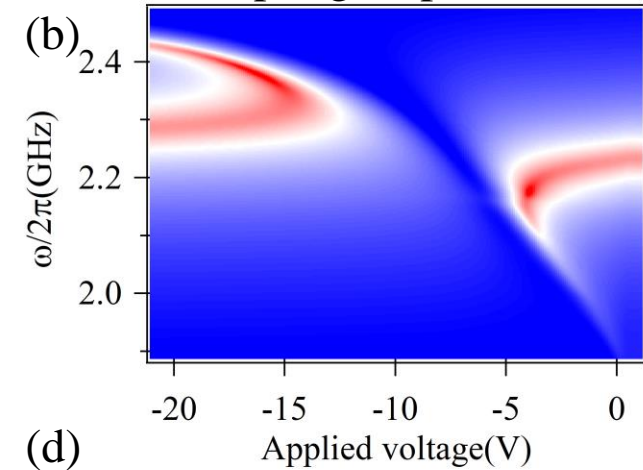
$$\omega = \frac{1}{\sqrt{LC}}$$



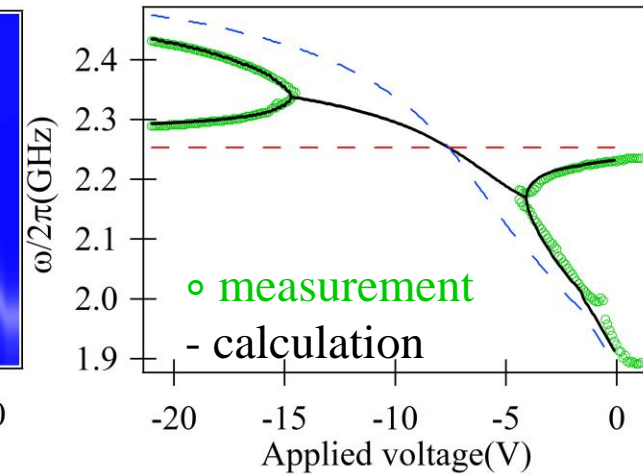
(c) Tunable SSR dispersion



Coupling dispersion



(d)

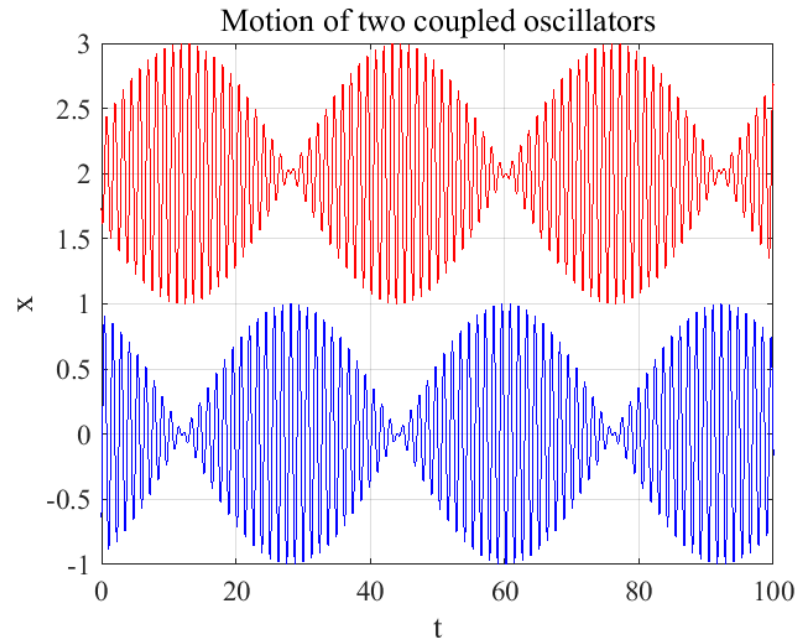


Proposed work

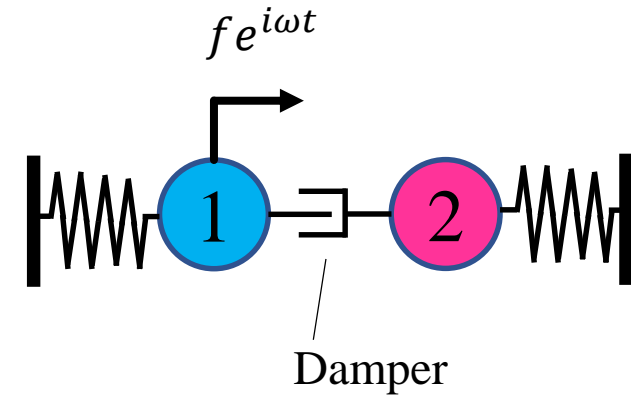
Level attraction in coupled mechanical oscillators

- Explore the level attraction in mechanical system.
- Study the time domain behavior of level attraction experimentally.

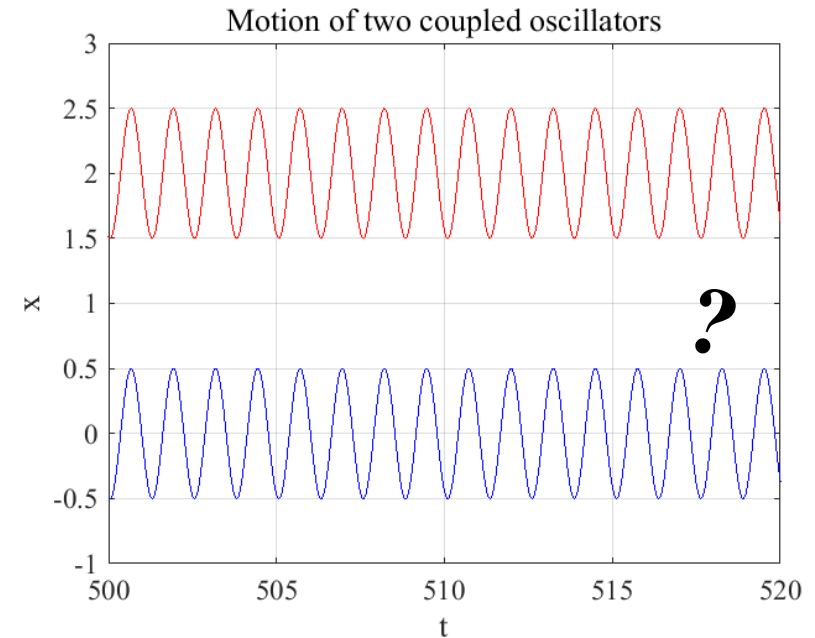
Coherent coupling



Dissipative coupling of two oscillators



Dissipative coupling



Summary

Two work for master program:

1. Level attraction in metamaterials.
 - Expand the level attraction to metamaterials.
2. Level attraction in mechanical oscillators.
 - Get a result for level attraction in time domain.