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Section 004

Problem 1

$(\frac{1}{3})^n < 6 < \log\log k < \boxed{\sqrt{n}} < \boxed{\log n \approx \ln m} < n^{\frac{1}{3}} + \log n < \frac{n}{\log n} < m < (\log n)^2 <$ (k)

$< n\log n < m^2 \approx k^2 + \log k < n^3 < n - n^3 + 7n^5 < (\frac{3}{2})^n < 2^n < n!$

Problem 2

~~Proof~~ We need to prove

$c_2 g(n) \le t(n) \le c_1 g(n)$ for all $n \ge n_0$.

First, $(n+a)^b \in O(n^b)$

$(n+a)^b \ge \frac{1}{a} n^b$ for all $n \ge 1$.

Second $(n+a)^b \in \Omega(n^b)$

$(n+a)^b \le a n^b$ for all $n \ge 1$

Hence, we can select $c_1 = a$, $c_2 = \frac{1}{a}$ and $n_0 = 2$.

Then, $(n+a)^b \in \Theta(n^b)$.

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Problem 3

1. $n(n+1)/2 \in O(n^3)$

$\frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}$ ~~~~ True

$\approx \frac{1}{2}n^2$ is quadratic

2. $n(n+1)/2 \in O(n^2)$. True

from 1. ~~that~~ then $\frac{1}{2}n^2$ is same order with n^2

3. $n(n+1)/2 \in \Theta(n^3)$

from 1. ~~that~~ then $\frac{1}{2}n^2$ is smaller order than n^3 False.

It's should be $n(n+1)/2 \notin \Theta(n^3)$

4. $n(n+1)/2 \in \Omega(n)$.

From 1 ~~that~~ then $\frac{1}{2}n^2$ is higher order than n True.

Problem 4

1. $S(0) = 0$

$S(1) = 0 + 1 \cdot 1$ $\quad S(n) = S(n-1) + n^2$.

$S(2) = S(1) + 2^2$

$S(3) = S(2) + 3^2$

$S(n) = 1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 + n^2 = \sum_{i=1}^{n} i^2$.

2. basic operation: multiplication & addition.

3. $C(n) = \sum_{i=1}^{n} 1 = n$.

4. $C(n) \in \Theta(n)$.

5. $S(n) = \sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(n+2)}{6}$ by make $n=1$ then $S(1) = 1$.

No better one.

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Problem 5

1. $1+3+5+\cdots+999$
$= (1+999)\cdot\frac{1}{2}\cdot\frac{1000}{2}$
$= 250{,}000$

2. $2+4+8+16+\cdots+1024$
$= \sum_{i=1}^{10} 2^i$
$= 2^{11}-1-1 = 2046$

3. $\sum_{i=3}^{n+1} 1$
$= n+1-3+1$
$= n-1$

4. $\sum_{i=3}^{n+1} i = 3+4+\cdots+n+1$
$= (n+1+3)\cdot\frac{(n+1-3+1)}{2}$
$= \frac{(n+4)(n-1)}{2}$

5. $\sum_{i=0}^{n-1} i(i+1)$
$= \sum_{i=0}^{n-1} i^2+i$
$= \sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} i$
$= \frac{n(n-1)(n-2)}{6} + \frac{n(n-1)}{2}$
$= \frac{(n^2-1)n}{3}$

6. $\sum_{j=1}^{n} 3^{j+1}$
$= 3\sum_{j=1}^{n} 3^{j+1}$
$= 3\left(\frac{3^{n+1}-1}{3-1}-1\right)$
$= \frac{3^{n+2}-9}{2}$

7. $\sum_{i=1}^{n}\sum_{j=1}^{n} ij$
$= \sum_{i=1}^{n} i \sum_{j=1}^{n} j$
$= \sum_{i=1}^{n} i \frac{n(n+1)}{2}$
$= \frac{n(n+1)}{2}\cdot\frac{n(n+1)}{2} = \left(\frac{n(n+1)}{2}\right)^2$

8. $\sum_{i=1}^{n} \frac{1}{i(i+1)}$
$= \sum_{i=1}^{n}\left(\frac{1}{i}-\frac{1}{i+1}\right)$
$= \frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{n-1}-\frac{1}{n}+\frac{1}{n}-\frac{1}{n+1}$
$= 1-\frac{1}{n+1}$
$= \frac{n}{n+1}$

Problem 6

1. $x(n) = x(n-1)+5$ for $n>1$, $x(1)=0$

$x(n) = x(n-1)+5$
$= x(n-2)+5+5$
$= x(n-3)+5+5+5$
$\ldots$
$= x(n-i)+5i$

Let $n-i=1$
then $i=n-1$
$x(n) = x(1)+5(n-1)$
$= 5(n-1)$

2. $x(n) = 3x(n-1)$ for $n>1$, $x(1)=4$.

$x(n) = 3x(n-1)$
$= 3\cdot 3x(n-2)$
$\ldots$
$= 3^i x(n-i)$

Let $n-i=1$
then $i=n-1$
$x(n) = 3^{n-1}\,x(1) = 3^{n-1}\cdot 4$

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B. 3. $x(n) = x(n-1) + n$ for $n > 0$, $x(0) = 0$.

$$
\begin{aligned}
x(n) &= x(n-1) + n \\
&= x(n-2) + n-1 + n \\
&= x(n-3) + (n-2) + (n-1) + n \\
&\dots \\
&= x(n-i) + (n-i+1) + (n-i+2) + \dots + (n-i+i)
\end{aligned}
$$

Let $n-i=0$.
then $n=i$.

$$x(n) = x(0) + 1+2+\dots+n = \frac{n(n+1)}{2}$$

4. $x(n) = x(\frac{n}{2}) + n$ for $n > 1$, $x(1) = 1$ (solve for $n = 2^k$)

$\cancel{x(n) = x(\frac{n}{2}) + n}$

$\cancel{= x(\frac{n}{4}) + \frac{n}{2} + n}$

$$
\begin{aligned}
x(2^k) &= x(2^{k-1}) + 2^k \\
&= x(2^{k-2}) + 2^{k-1} + 2^k \\
&\dots \\
&= x(2^{k-i}) + 2^{k-i+1} + \dots + 2^k
\end{aligned}
$$

Let $k-i=0$ (which $2^0 = 1$ 于 $x(1)$)
then $k=i$.

$$x(n) = x(1) + 2^1 + 2^2 + \dots + 2^k$$

$$= 2^{k+1} - 1 = 2 \cdot 2^k - 1 = 2n-1$$

5. $x(n) = x(\frac{n}{3}) + 1$ for $n > 1$, $x(1) = 1$ (solve for $n = 3^k$).

$$
\begin{aligned}
x(3^k) &= x(3^{k-1}) + 1 \\
&= x(3^{k-2}) + 1 + 1 \\
&\dots \\
&= x(3^{k-i}) + i
\end{aligned}
$$

Let $k-i=0$ (which $3^0 = 1$ 于 $x(1)$).
then $k=i$

$$x(n) = x(1) + k = 1 + \log_3 n$$