1. Let
$$x_{n+1} = \frac{1-(1-x_n^3)^{1/3}}{1+(1-x_n^3)^{1/3}}, \quad x_0 = 1/2.$$
Assuming $x_n \to 0$, determine the order of convergence of the sequence.

2. Prove Taylor's Remainder Theorem (Lagrange form) for the case $n = 3$.

3. Let $F \in C^1(\mathbf{R})$ and $s_1 < s_2 < s_3$ be such that
$$F(s_1) = s_1, \quad F(s_2) = s_2, \quad F(s_3) = s_3, \quad \text{and} \quad F(x) \neq x \text{ for } x \in \mathbf{R} \setminus \{s_1, s_2, s_3\}.$$
If
$$|F'(s_1)| = |F'(s_3)| = \frac{3}{4}$$
what bounds exist on $|F'(s_2)|$? With these derivative conditions on F, if
$$x_{n+1} = F(x_n)$$
converges which of the 3 fixed points will it converge to?

4. Let x_0 and x_1 be close to a root r of some function f. Provide conditions on f such that
$$x_{n+1} = x_n - f(x_n)\left(\frac{x_n - x_0}{f(x_n) - f(x_0)}\right)$$
converges linearly to r.