Question 2: How Do My Plots Compare?

1. 0-2 Gyr: For the first 1-2 Gyr, the predicted orbit and the simulation orbit align relatively well in both distance and velocity. This suggests that the initial conditions (t = 0) match closely.

2. After ~2 Gyr: After 2 Gyr, the analytic orbit's distance increases to over 200 kpc, peaking around 400 kpc, while the simulation stays below 200 kpc and shows multiple pericenter passages. Velocity differences grow to 200-300 km/s by 5-6 Gyr. On average, there's a ~192 kpc distance difference and approximately 80 km/s velocity difference. The static potential model for M31 can't capture key physical processes in the simulation.

3. Late Times: Beyond 5 Gyr, the simulation orbit shows repeated close encounters with M31 ("bouncing"), while the analytic orbit flings out and returns slowly, following a near-Keplerian path. The divergence indicates additional physics present in the simulation.

Question 3: Missing Physics

1. Dynamical Friction: The model uses a static potential for M31, ignoring dynamical friction. In reality, M33 loses orbital energy due to friction from M31's dark matter halo, causing tighter orbits. My model can't replicate that, resulting in an overly extended trajectory.

2. Tidal Stripping: M33's halo is stripped over time by M31 in the simulation, reducing its mass and altering its path. My code treats M33 as a point mass and doesn't include this.

3. M31's Response: The simulation allows M31 to respond gravitationally to M33, but my analytic model pins M31 at the center, omitting any back-reaction or motion.

4. Gas and Baryonic Effects: If M33 contains gas, processes like star formation and supernova feedback will shift its mass distribution-again, ignored in the static model.

Question 4: The Role of the Milky Way

Currently, the model includes only M31's gravitational influence, but over 10+ Gyr, the Milky Way (MW) should be considered too.

* Three-Body Approach: Simultaneously modeling M31, MW, and M33 via full integration would be more accurate, though computationally more complex.

* Adding MW as Fixed Potential: A simpler approach would be adding MW's potential like M31's:

r.._M33 = -grad(Phi_M31(r_M33-M31)) - grad(Phi_MW(r_M33-MW)).

This still requires assumptions or modeling of M31's motion relative to the MW, so it's not entirely trivial.