CSC 503 Homework Assignment 7

Out: November 1, 2018
Due: November 7, 2018
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For your reference, the algorithm to calculate the most general unifier σ of a set S of predicate expressions consists of the following steps.

- Step 0:
  1. Let $S_0 = S$
  2. Let $\sigma_0 = \epsilon$
- Step $k+1$:
  1. If S_k has exactly one element, return the product substitution $\sigma_0 \cdots \sigma_k$ as the value of σ
  2. If the disagreement set $D(S_k)$ contains both a variable v and a term t in which v *does not occur*, then
     (a) Choose least such pair (alphabetical order by variable, then by term)
     (b) Let $\sigma_{k+1} = \{t/v\}$
     (c) Let $S_{k+1} = S_k \sigma_{k+1}$
     (d) Proceed to step $k+2$
  3. Otherwise, announce that S has no unifier

When showing the steps of a unification calculation, use the form given in the lecture notes on slides 338-341. That is, for each step k, show

- the disagreement set $D(S_k)$ of S_k,
- the substitution σ_k if there is one, or an explanation why there is no unifying substitution, and
- the result S_{k+1} of applying σ_k to S_k, if there is one.

If the set unifies, at the end of the steps show

- the overall substitution $\sigma_0 \ldots \sigma_k$ expressed as a single substitution.

That is, compute and display the complete product of the stepwise substitutions.

Consider now a language in which a and b are constant symbols, f is a binary function symbol, g is a unary function symbol, P is a ternary predicate symbol, and u, v, w, x, y, z are variable symbols.

1. **[20 points]** $S = \{P(f(b), y, f(y)), P(x, x, z)\}$
2. **[20 points]** $S = \{P(x, x), P(y, g(h(y)))\}$
3. **[20 points]** $S = \{P(f(w, a), h(g(v), u)), P(f(w, w), h(x, g(y))), P(f(v, a), g(v))\}$

The following is an example of a propositional resolution refutation proof of Q from premises P and $P \to Q$, with the proof written in linear form.

Line	Clause	Justification
1.	$\{P\}$	Given
2.	$\{\neg P, Q\}$	Given
3.	$\{\neg Q\}$	Refutation clause, negation of goal Q
4.	$\{\neg P\}$	Resolving (2) with (3) on Q
5.	$\{\}$	Resolving (4) with (1) on P

If the negation of the goal had consisted of several clauses, each of those would have been listed on a separate line as a refutation clause (as is done for the single clause on line 3). The order in which lines are mentioned in lines 4 and 5 does not matter.

The following is an example of a predicate resolution refutation proof of $Q(a)$ from premises $P(a)$ and $\forall x\, [P(x) \to Q(x)]$, again with the proof written in linear form.

Line	Clause	Justification	Unifier
1.	$\{P(a)\}$	Given	
2.	$\{\neg P(x_2), Q(x_2)\}$	Given	
3.	$\{\neg Q(a)\}$	Refutation clause, negation of goal $Q(a)$	
4.	$\{\neg P(a)\}$	Resolving (2) with (3) on Q	$\{a/x_2\}$
5.	$\{\}$	Resolving (4) with (1) on P	$\{\}$

Here the variables in line 2 have been standardized apart, which was not really necessary for this proof as line 2 represents the only given involving variables. If the resolution steps in lines 4 and 5 had involved multiple literals in one of the clauses, the justification would have needed to indicate the specific literals involved.

In the following two problems concerning resolution proofs, write the proofs in either linear or tree form.

4. **[20 points]** Let P, Q, R, S, T, U, V be atomic propositional symbols, and give a resolution refutation proof of V.

Line	Clause	Justification
1.	$\{\neg P, Q\}$	Given
2.	$\{\neg P, R\}$	Given
3.	$\{\neg S, \neg T, U\}$	Given
4.	$\{\neg U, \neg Q, V\}$	Given
5.	$\{P\}$	Given
6.	$\{S\}$	Given
7.	$\{T\}$	Given

5. **[20 points]** Let a and b be constant symbols, f and h unary function symbols, P a ternary predicate symbol, Q and R binary prdicate symbols, and u, v, w, x, y, z variables. Rewrite the folllwing five clauses with the variables standardized apart, and then give a resolution refutation of the set of rewritten clauses. At each step of the refutation, indicate the literals being resolved together and the most general unifying substitution used in the proof step. If there is only one possible choice of complementary literals, indicate only the predicate symbol being resoleve on, not the arguments of the literals. Note: you may need to resolve multiple instances of the same literal in a single step.

Line	Clause	Justification	Substitution
1.	$\{\neg P(w, a, f(h(b))), Q(a, x)\}$	Given	
2.	$\{\neg Q(a, v)\}$	Given	
3.	$\{P(u, a, f(h(u))), Q(a, u), R(h(b), b)\}$	Given	
4.	$\{\neg R(h(b), w), Q(a, w)\}$	Given	
5.	$\{P(x, a, f(y)), P(z, a, f(h(b))), \neg R(y, z)\}$	Given	