CSC 503 Homework Assignment 3 Sample Solutions

Out: September 10, 2018
Due: September 17, 2018
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1. **[30 points]** Using the method described in lecture, construct a formula ϕ in **DNF** to match the following truth table. Show and explain any intermediate steps.

α	β	γ	ϕ
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

Answer

To construct a DNF formula from a truth table, one conjoins the formulas in each line labeled T and then disjoins these conjunctions. In this case there are five lines labeled T, so one DNF formula is

$$(\alpha \wedge \beta \wedge \gamma) \vee (\alpha \wedge \beta \wedge \neg\gamma) \vee (\alpha \wedge \neg\beta \wedge \neg\gamma) \vee (\neg\alpha \wedge \beta \wedge \gamma) \vee (\neg\alpha \wedge \neg\beta \wedge \neg\gamma).$$

There are other equivalent DNF formulas. One can replace the first and second disjuncts just given by $(\alpha \wedge \beta)$.

2. **[30 points]** Using the method described in lecture, construct a formula ψ in **CNF** to match the following truth table. Show and explain any intermediate steps.

α	β	γ	ψ
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

Answer

To construct a CNF formula from a truth table, one conjoins the formulas in each line labeled F, negates each such conjunction to form a disjunction, and then conjoins these disjunction, basically finding a DNF representation of the negation of the formula and negating it to obtain a CNF representation of the un-negated formula. In this case there are three lines labeled F, so the equivalent formula is

$$(\neg\alpha \vee \beta \vee \neg\gamma) \wedge (\neg\alpha \vee \beta \vee \neg\gamma) \wedge (\alpha \vee \neg\beta \vee \gamma)$$

Each of these conjuncts ensures the conjunction is false in the interpretation corresponding to the corresponding F line of the truth table.

There are other correct answers. One can replace the first and fourth conjuncts just given by $(\neg\beta \vee \gamma)$, or the second and third conjuncts by $(\alpha \vee \neg\beta)$, or the second and fourth conjuncts by $(\alpha \vee \neg\gamma)$.

3. **[30 points total]** Apply the following version of the algorithm HORN from pages 66–67 of the textbook to the following Horn formula φ.

   (a) Mark each occurrence of $\top$ in each conjunct of φ with a "1" to indicate $\top$ is marked on the first pass.

   (b) Search the list of conjuncts of φ in order until either

      i. the end of the list is reached, or

      ii. a "markable" conjunct is found, that is, one with each of its antecedent propositions marked and its consequent proposition unmarked.

   (c) If the end of the list was reached,

      i. If $\bot$ is not marked, return "satisfiable" and halt.

      ii. If $\bot$ is marked, return "unsatisfiable" and halt.

   (d) Otherwise, mark every occurrence of the consequent proposition of the first markable conjunct with the pass number (2, 3, . . .) in each conjunct of φ.

   (e) Continue with step (b) again.

$$\varphi = \left| \begin{array}{lll} 1. & (\top \to w) & \wedge \\ 2. & (w \to q) & \wedge \\ 3. & (x \wedge t \to \bot) & \wedge \\ 4. & (q \wedge r \to p) & \wedge \\ 5. & (v \to s) & \wedge \\ 6. & (w \to r) & \wedge \\ 7. & (r \wedge s \to x) & \wedge \\ 8. & (\top \to v) & \wedge \\ 9. & (v \wedge q \to u) & \wedge \\ 10. & (p \wedge r \wedge s \to u) & \wedge \\ 11. & (u \to v) & \end{array} \right.$$

Your answer should list propositional letters in the order in which they are marked and indicate the returned value.

Answer

The algorithm begins by marking each occurrence of $\top$ and then marking the consequent of every implication all of whose antecedents are marked. In this case, we have

$$\begin{array}{lll} 1. & (\top^1 \to w^2) & \wedge \\ 2. & (w^2 \to q^3) & \wedge \\ 3. & (x^8 \wedge t \to \bot) & \wedge \\ 4. & (q^3 \wedge r^4 \to p^5) & \wedge \\ 5. & (v^6 \to s^7) & \wedge \\ 6. & (w^2 \to r^4) & \wedge \\ 7. & (r^4 \wedge s^7 \to x^8) & \wedge \\ 8. & (\top^1 \to v^6) & \wedge \\ 9. & (v^6 \wedge q^3 \to u^9) & \wedge \\ 10. & (p^5 \wedge r^4 \wedge s^7 \to u^9) & \wedge \\ 11. & (u^9 \to v^6) & \end{array}$$

The formula is satisfiable because $\bot$ is unmarked after all markable propositions have been marked. The order of marking of variables is w, q, r, p, v, s, x, u. The variable t is never marked.

4. **[10 points]** Can one determine whether the following subformula of φ is satisfiable more quickly than by

applying the HORN algorithm? Explain your answer.

$$\varphi = \left|\begin{array}{ll} (\top \to p) & \wedge \\ (\bot \to q) & \wedge \\ (p \wedge q \to v) & \wedge \\ (q \to r) & \wedge \\ (v \to s) & \wedge \\ (\top \to r) & \wedge \\ (\top \to s) & \wedge \\ (v \wedge t \to u) & \wedge \\ (p \wedge r \wedge s \to u) & \end{array}\right.$$

Answer

Yes. The subformula is satisfiable because $\bot$ does not appear as a consequent in any conjunct, and so never is marked.