CSC 503 Homework Assignment 2 Sample Solutions

Out: August 31, 2018
Due: September 10, 2018
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Prove the validity of the following sequents. Use only the basic rules of natural deduction (no derived rules).

1. **[10 points]** $\vdash (p \to q) \to (p \to (p \to (p \wedge q)))$

Answer
Note that line 4 could be produced from lines 1 and 2 instead of lines 1 and 3, and could appear right after line 2 instead of after it.

1	$p \to q$	Assumption
2	$\quad p$	Assumption
3	$\quad\quad p$	Assumption
4	$\quad\quad q$	$\to$e, 3, 1
5	$\quad\quad p \wedge q$	$\wedge$i, 3, 4
6	$\quad p \to (p \wedge q)$	$\to$i, 3–5
7	$p \to (p \to (p \wedge q))$	$\to$i, 2–6
8	$(p \to q) \to (p \to (p \to (p \wedge q)))$	$\to$i, 1–7

2. **[15 points]** $q \to p \vdash q \to (p \to q)$

Answer
Note that the proof does not actually use the premise in deriving the conclusion, so proof would be correct even if the first line were omitted.

1	$q \to p$	Premise
2	$\quad q$	Assumption
3	$\quad\quad p$	Assumption
4	$\quad\quad q$	copy, 2
5	$\quad p \to q$	$\to$i, 3–4
6	$q \to (p \to q)$	$\to$i, 2–5

3. **[15 points]** $\neg p \to (\neg q \to \neg r) \vdash (\neg p \to \neg q) \to (\neg p \to \neg r)$

Answer

1		$\neg p \to (\neg q \to \neg r)$	Premise
2		$\quad \neg p \to \neg q$	Assumption
3		$\quad\quad \neg p$	Assumption
4		$\quad\quad \neg q \to \neg r$	$\to$e, 3, 1
5		$\quad\quad \neg q$	$\to$e, 3, 2
6		$\quad\quad \neg r$	$\to$e, 5, 4
7		$\quad \neg p \to \neg r$	$\to$i, 3–6
8		$(\neg p \to \neg q) \to (\neg p \to \neg r)$	$\to$i, 2–7

4. **[30 points]** $p \vee q, p \to r \vee s, q \to r \vee s, r \to t, s \to t \vdash t$

Answer

Proof 1

1		$p \vee q$	Premise
2		$p \to r \vee s$	Premise
3		$q \to r \vee s$	Premise
4		$r \to t$	Premise
5		$s \to t$	Premise
6		$\quad p$	Assumption
7		$\quad r \vee s$	$\to$e, 6, 2
8		$\quad q$	Assumption
9		$\quad r \vee s$	$\to$e, 8, 3
10		$r \vee s$	$\vee$e, 1, 6–7, 8–9
11		$\quad r$	Assumption
12		$\quad t$	$\to$e, 11, 4
13		$\quad s$	Assumption
14		$\quad t$	$\to$e, 13, 5
15		t	$\vee$e, 10, 11–12, 13–14

Proof 2

1		$p \vee q$	Premise
2		$p \to r \vee s$	Premise
3		$q \to r \vee s$	Premise
4		$r \to t$	Premise
5		$s \to t$	Premise
6		$\quad p$	Assumption
7		$\quad r \vee s$	$\to$e, 6, 2
8		$\quad\quad r$	Assumption
9		$\quad\quad t$	$\to$e, 8, 4
10		$\quad\quad s$	Assumption
11		$\quad\quad t$	$\to$e, 10, 5
12		$\quad t$	$\vee$e, 7, 8–9, 10–11
13		$\quad q$	Assumption
14		$\quad r \vee s$	$\to$e, 13, 3
15		$\quad\quad r$	Assumption
16		$\quad\quad t$	$\to$e, 15, 4
17		$\quad\quad s$	Assumption
18		$\quad\quad t$	$\to$e, 17, 5
19		$\quad t$	$\vee$e, 14, 15–16, 17–18
20		t	$\vee$e, 1, 6–12, 13–19

5. **[30 points]** $r \to (p \vee q), p \to s, \neg q, \neg s \vdash \neg r$

Answer

1	$r \to (p \vee q)$	Premise
2	$p \to s$	Premise
3	$\neg q$	Premise
4	$\neg s$	Premise
5	$\quad r$	Assumption
6	$\quad p \vee q$	$\to$e, 5, 1
7	$\quad\quad p$	Assumption
8	$\quad\quad s$	$\to$e, 7, 2
9	$\quad\quad \bot$	$\neg$e, 8, 4
10	$\quad\quad \neg r$	$\bot$e, 9
11	$\quad\quad q$	Assumption
12	$\quad\quad \bot$	$\neg$e, 11, 3
13	$\quad\quad \neg r$	$\bot$e, 12
14	$\quad \neg r$	$\vee$e, 6, 7–10, 11–13
15	$\quad \bot$	$\neg$e, 5, 14
16	$\neg r$	$\neg$i, 5–15