CSC 503 Homework Assignment 1

Out: August 28, 2018
Due: September 5, 2018
Unity ID: zzha

The formulas of propositional logic implicitly assume the binding priorities of the logical connectives put forward in Convention 1.3. In the first set of problems, make sure that you fully understand those conventions by reinserting all omitted parentheses in the following two abbreviated statements.

1. **[10 points]** $p \to \neg q \vee \neg r \to \neg\neg q \to p \vee r$

   Answer

   $p \to (((\neg q) \vee (\neg r)) \to ((\neg(\neg q)) \to (p \vee r)))$

2. **[10 points]** $r \vee p \to \neg\neg q \to \neg r \vee (q \to p)$

   Answer

   $(r \vee p) \to ((\neg(\neg q)) \to ((\neg r) \vee (q \to p)))$

3. **[10 points]** Why is the expression $p \wedge q \vee r$ problematic? Use truth tables or interpretations to justify your answer.

   Answer

   Since the binding precedence order of disjunction and conjunction are undefined, with parentheses $p \wedge q \vee r$ can be seen as $(p \wedge q) \vee r$ or $p \wedge (q \vee r)$, which have different truth value of F and T when p, q and r are assigned to be the value of F, T and T. Please see the Table 1 and Table 2

   Table1:

p	q	r	$(p \wedge q) \vee r$
F	T	T	T

   Table2:

p	q	r	$p \wedge (q \vee r)$
F	T	T	F

4. **[10 points]** List all subformulas of the formula $((p \wedge p) \vee q) \to (((\neg r) \to r) \to (p \wedge q))$.

   Answer

   $S = \{p, q, r, \neg r, (p \wedge q), (p \wedge q) \vee q, (\neg r) \to r, p \wedge q, ((\neg r) \to r) \to (p \wedge q), ((p \wedge p) \vee q) \to (((\neg r) \to r) \to (p \wedge q))\}$

5. **[10 points]** Compute and present the complete truth table of the formula $(\neg p \vee q) \to (p \to \neg q)$.

   Answer

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$p \to \neg q$	$(\neg p \vee q) \to (p \to \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Let ϕ_1 be the formula $(p \vee q) \to (q \to p)$.

6. **[5 points]** Is ϕ_1 satisfiable? Justify your answer.

   Answer

   Yes, one of the interpretation make the formula true, please see the following truth table:

p	q	$p \vee q$	$q \to p$	$(p \vee q) \to (q \to p)$
T	T	T	T	T

7. **[5 points]** Is ϕ_1 valid? Justify your answer.

   Answer

   No, one of the interpretation make the formula false, please see the following truth table:

p	q	$p \vee q$	$q \to p$	$(p \vee q) \to (q \to p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	F
F	F	F	T	T

Let ϕ_2 be the formula $(q \wedge \neg r) \wedge (q \to r)$.

8. **[5 points]** Is ϕ_2 falsifiable? Justify your answer.

   Answer

   Yes, one of the interpretation make the formula false, please see the following truth table:

q	r	$\neg r$	$q \wedge \neg r$	$q \to r$	$(q \wedge \neg r) \wedge (q \to r)$
T	T	F	F	T	F

9. **[5 points]** Is ϕ_2 unsatisfiable? Justify your answer.

   Answer

   Yes, every interpretation make the formula false, please see the following truth table:

q	r	$\neg r$	$q \wedge \neg r$	$q \to r$	$(q \wedge \neg r) \wedge (q \to r)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

10. **[10 points]** Show that the entailment claim $p \vee (q \to r), \neg p \vee r \models p \to q$ is not correct. Justify your answer in terms of truth value assignments to the propositions p, q, and r.

    Answer

    In the below condition, this entailment claim is not correct, because the truth values of the formulas to the left of $\models$ is true, the right is false.

p	q	r	$p \vee (q \to r)$	$\neg p \vee r$	$p \to q$
T	F	T	T	T	F

11. **[10 points]** Does $\models (p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ hold? Justify your answer.

    Answer

    This formula is hold, because there is no interpretation that assigns this formula false.

p	q	r	$(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	F	T
F	F	T	T

12. **[10 points]** Are the sentences (A) $q \to p$, (B) $p \to r$, and (C) $r \to (p \wedge q)$ logically independent? Justify your answer formally.

    Answer

Yes, A set is logically independent iff each statement is independent of the rest, from the below table we know for each statement, some interpretation makes it false and makes all the rest true.

$\neg(q \to p) \land (p \to r) \land (r \to (p \land q))$ is satisfiable:

p	q	r	$\neg(q \to p) \land (p \to r) \land (r \to (p \land q))$
F	T	F	T

$(q \to p) \land \neg(p \to r) \land (r \to (p \land q))$ is satisfiable:

p	q	r	$(q \to p) \land \neg(p \to r) \land (r \to (p \land q))$
T	F	F	T

$(q \to p) \land (p \to r) \land \neg(r \to (p \land q))$ is satisfiable:

p	q	r	$(q \to p) \land (p \to r) \land \neg(r \to (p \land q))$
F	F	T	T