

CSC 503 Homework Assignment 2

Out: August 31, 2018
Due: September 10, 2018
Unity ID: zzha

Prove the validity of the following sequents. Use only the basic rules of natural deduction (no derived rules).

1. [10 points] $\vdash (p \rightarrow q) \rightarrow (p \rightarrow (p \rightarrow (p \wedge q)))$

Answer

1			Premise
2		$p \rightarrow q$	Assumption
3		p	Assumption
4		p	Assumption
5		q	$\rightarrow e, 4, 2$
6		$p \wedge q$	$\wedge i, 4, 5$
7		$p \rightarrow (p \wedge q)$	$\rightarrow i, 4, 6$
8		$p \rightarrow (p \rightarrow (p \wedge q))$	$\rightarrow i, 3, 7$
9		$(p \rightarrow q) \rightarrow (p \rightarrow (p \rightarrow (p \wedge q)))$	$\rightarrow i, 2, 8$

2. [15 points] $q \rightarrow p \vdash q \rightarrow (p \rightarrow q)$

Answer

1		$q \rightarrow p$	Premise
2		q	Assumption
3		p	Assumption
4		q	copy, 2
5		$p \rightarrow q$	$\rightarrow i, 3, 4$
6		$q \rightarrow (p \rightarrow q)$	$\rightarrow i, 2, 5$

3. [15 points] $\neg p \rightarrow (\neg q \rightarrow \neg r) \vdash (\neg p \rightarrow \neg q) \rightarrow (\neg p \rightarrow \neg r)$

Answer

1	$\neg p \rightarrow (\neg q \rightarrow \neg r)$	Premise
2	$\neg p \rightarrow \neg q$	Assumption
3	$\neg p$	Assumption
4	$\neg q \rightarrow \neg r$	$\rightarrow e, 3, 1$
5	$\neg q$	$\rightarrow e, 3, 2$
6	$\neg r$	$\rightarrow e, 5, 4$
7	$\neg p \rightarrow \neg r$	$\rightarrow i, 2, 6$
8	$(\neg p \rightarrow \neg q) \rightarrow (\neg p \rightarrow \neg r)$	$\rightarrow i, 1, 7$

4. [30 points] $p \vee q, p \rightarrow r \vee s, q \rightarrow r \vee s, r \rightarrow t, s \rightarrow t \vdash t$

Answer

1	$p \vee q$	Premise
2	$p \rightarrow r \vee s$	Premise
3	$q \rightarrow r \vee s$	Premise
4	$r \rightarrow t$	Premise
5	$s \rightarrow t$	Premise
6	p	Assumption
7	$r \vee s$	$\rightarrow e, 6, 2$
8	r	Assumption
9	t	$\rightarrow i, 8, 4$
10	s	Assumption
11	t	$\rightarrow i, 10, 5$
12	t	$\vee e, 7, 8-9, 10-11$
13	q	Assumption
14	$r \vee s$	$\rightarrow e, 13, 3$
15	r	Assumption
16	t	$\rightarrow i, 15, 4$
17	s	Assumption
18	t	$\rightarrow i, 17, 5$
19	t	$\vee e, 14, 15-16, 17-18$
20	t	$\vee e, 1, 6-12, 13-19$

5. [30 points] $r \rightarrow (p \vee q), p \rightarrow s, \neg q, \neg s \vdash \neg r$

Answer

1	$r \rightarrow (p \vee q)$	Premise
2	$p \rightarrow s$	Premise
3	$\neg q$	Premise
4	$\neg s$	Premise
5	r	Assumption
6	$p \vee q$	\rightarrow e, 5, 2
7	p	Assumption
8	s	\rightarrow e, 8, 2
9	$\neg s$	copy, 4
10	\perp	\neg e, 8, 9
11	q	Assumption
12	$\neg q$	copy, 3
13	\perp	\neg e, 11, 12
14	\perp	\vee e, 6, 7–10, 11–13
15	$\neg r$	\neg i, 5–14