CSC 503 Homework Assignment 1 Sample Solutions

Out: August 28, 2018
Due: September 5, 2018
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The formulas of propositional logic implicitly assume the binding priorities of the logical connectives put forward in Convention 1.3. In the first set of problems, make sure that you fully understand those conventions by reinserting all omitted parentheses in the following two abbreviated statements.

1. **[10 points]** $p \to \neg q \vee \neg r \to \neg\neg q \to p \vee r$

Answer
$p \to (((\neg q) \vee (\neg r)) \to ((\neg(\neg q)) \to (p \vee r)))$
(also accept outer parentheses)
Explanation:

$$\begin{array}{r|c} \neg & p \to (\neg q) \vee (\neg r) \to (\neg(\neg q)) \to p \vee r \\ \wedge/\vee & p \to ((\neg q) \vee (\neg r)) \to (\neg(\neg q)) \to (p \vee r) \\ \to & p \to (((\neg q) \vee (\neg r)) \to ((\neg(\neg q)) \to (p \vee r))) \end{array}$$

2. **[10 points]** $r \vee p \to \neg\neg q \to \neg r \vee (q \to p)$

Answer
$(r \vee p) \to ((\neg(\neg q)) \to ((\neg r) \vee (q \to p)))$
(also accept outer parentheses)
Explanation:

$$\begin{array}{r|c} \neg & r \vee p \to (\neg(\neg q)) \to (\neg r) \vee (q \to p) \\ \wedge/\vee & (r \vee p) \to (\neg(\neg q)) \to ((\neg r) \vee (q \to p)) \\ \to & (r \vee p) \to ((\neg(\neg q)) \to ((\neg r) \vee (q \to p))) \end{array}$$

3. **[10 points]** Why is the expression $p \wedge q \vee r$ problematic? Use truth tables or interpretations to justify your answer.

Answer
Neither conjunction nor disjunction have precedence over the other, so the convention allows the two readings of $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$. These readings are inequivalent, as seen in the following truth table.

p	q	r	$p \wedge q$	$q \vee r$	$(p \wedge q) \vee r$	$p \wedge (q \vee r)$	$((p \wedge q) \vee r) \leftrightarrow (p \wedge (q \vee r))$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	T
F	T	T	F	T	T	F	$\Rightarrow F \Leftarrow$
F	T	F	F	T	F	F	T
F	F	T	F	T	T	F	$\Rightarrow F \Leftarrow$
F	F	F	F	F	F	F	T

As seen in the final column, the interpretation in which p is false and q and r are true assigns different truth values to the two readings, as does the interpretation in which p and q are false and r is true.

4. **[10 points]** List all subformulas of the formula $((p \wedge p) \vee q) \to (((\neg r) \to r) \to (p \wedge q))$.

Answer
The order in which subformulas are listed does not matter. Accept lists with repetitions.

$$
\begin{array}{l}
((p \wedge p) \vee q) \to (((\neg r) \to r) \to (p \wedge q)) \\
(p \wedge p) \vee q \\
p \wedge p \\
p \\
q \\
((\neg r) \to r) \to (p \wedge q) \\
(\neg r) \to r \\
\neg r \\
r \\
p \wedge q
\end{array}
$$

5. **[10 points]** Compute and present the complete truth table of the formula $(\neg p \vee q) \to (p \to \neg q)$.

Answer

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$p \to \neg q$	$(\neg p \vee q) \to (p \to \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Let ϕ_1 be the formula $(p \vee q) \to (q \to p)$.

6. **[5 points]** Is ϕ_1 satisfiable? Justify your answer.

Answer
The formula is satisfiable because it is true if both p and q are false, which makes the antecedent of the main implication false, and hence the whole implication true.

7. **[5 points]** Is ϕ_1 valid? Justify your answer.

Answer
The formula is not valid because it is false if q is true and p is false. That would make the main consequent false and the main antecedent true, making the overall implication false.

Let ϕ_2 be the formula $(q \wedge \neg r) \wedge (q \to r)$.

8. **[5 points]** Is ϕ_2 falsifiable? Justify your answer.

Answer
The formula is falsifiable because it is false if either q is false or r is true, in which cases the first conjunction is false, making the overall conjunction false.

9. **[5 points]** Is ϕ_2 unsatisfiable? Justify your answer.

Answer
The formula is unsatisfiable because to be true the first conjunction must be true, which cannot happen unless q is true and r is false. That then makes the implication false, making the overall conjunction true.

10. **[10 points]** Show that the entailment claim $p \vee (q \to r), \neg p \vee r \models p \to q$ is not correct. Justify your answer in terms of truth value assignments to the propositions p, q, and r.

Answer

For the entailment claim to be incorrect, there must exist a truth value assignment that makes the formulas to the left of $\models$ true and the formula to the right of $\models$ false.

Any assignment making p true and r true makes both premises true. If the assignment also makes q false, it makes $p \to q$ false, thus making the entailment claim false.

11. **[10 points]** Does $\models (p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ hold? Justify your answer.

Answer

$\models (p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ means that the formula $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is valid, that is, true in all interpretations. This is the case, as one can see either by constructing a truth table for the formula, or by semantic reasoning.

Using a truth table, we see that every truth value assignment produces an interpretation in which the formula is true.

p	q	r	$p \vee q$	$\neg p$	$\neg p \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$	$(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$
T	T	T	T	F	T	T	T	T
T	T	F	T	F	F	F	T	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	F	F	T

Using semantic reasoning, we note that for the implication $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ to be false, q and r must both be false, and that the antecedent $(p \vee q) \wedge (\neg p \vee r)$ must be true. But in this case, $p \vee q$ is true only if p is true, and $\neg p \vee r$ is true only if $\neg p$ is true. Both of these cannot be true, so given the values for q and r, no assignment of a truth value to p makes the implication false. Thus every truth value assignment must make the formula true, ana so the formula is valid.

12. **[10 points]** Are the sentences (A) $q \to p$, (B) $p \to r$, and (C) $r \to (p \wedge q)$ logically independent? Justify your answer formally.

Answer

Three sentences are logically independent if no sentence is entailed by the other two. Formally, this means finding three assignments, such that each of these makes one statement false and the other two true.

The sentences (A), (B), and (C) are logically independent.

- To make (A) false though (B) and (C) are true, make p false, q true, and r false.
- To make (B) false though (A) and (C) are true, make p true, r false, and q either true or false.
- To make (C) false though (A) and (B) are true, make p false, q false, and r true.