CSC 503 Homework Assignment 3

Out: September 10, 2018
Due: September 17, 2018
Unity ID: zzha

1. **[30 points]** Using the method described in lecture, construct a formula ϕ in **DNF** to match the following truth table. Show and explain any intermediate steps.

α	β	γ	ϕ
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

   Answer

   1. Find lines with ϕ true:

α	β	γ	ϕ
T	T	T	T
T	T	F	T
T	F	F	T
F	T	T	T
F	F	F	T

   2. Conjoin true literals:
   $(\alpha \wedge \beta \wedge \gamma)$
   $(\alpha \wedge \beta \wedge \neg\gamma)$
   $(\alpha \wedge \neg\beta \wedge \neg\gamma)$
   $(\neg\alpha \wedge \beta \wedge \gamma)$
   $(\neg\alpha \wedge \neg\beta \wedge \neg\gamma)$

   3. Disjoin conjunctions:
   $(\alpha \wedge \beta \wedge \gamma) \vee (\alpha \wedge \beta \wedge \neg\gamma) \vee (\alpha \wedge \neg\beta \wedge \neg\gamma) \vee (\neg\alpha \wedge \beta \wedge \gamma) \vee (\neg\alpha \wedge \neg\beta \wedge \neg\gamma)$

2. **[30 points]** Using the method described in lecture, construct a formula ψ in **CNF** to match the following truth table. Show and explain any intermediate steps.

α	β	γ	ψ
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

   Answer

   1. Find lines with ϕ false:

α	β	γ	ϕ
T	T	F	F
T	F	T	F
F	F	T	F

2. All these must be false:
$\neg(\alpha \wedge \beta \wedge \neg\gamma)$
$\neg(\alpha \wedge \neg\beta \wedge \gamma)$
$\neg(\neg\alpha \wedge \neg\beta \wedge \gamma)$
3. So disjoin false literals:
$(\neg\alpha \vee \neg\beta \vee \gamma)$
$(\neg\alpha \vee \beta \vee \neg\gamma)$
$(\alpha \vee \beta \vee \neg\gamma)$
4. Conjoin disjunctions:
$(\neg\alpha \vee \neg\beta \vee \gamma) \wedge (\neg\alpha \vee \beta \vee \neg\gamma) \wedge (\alpha \vee \beta \vee \neg\gamma)$

3. **[30 points total]** Apply the following version of the algorithm HORN from pages 66–67 of the textbook to the following Horn formula φ.

   (a) Mark each occurrence of $\top$ in each conjunct of φ with a "1" to indicate $\top$ is marked on the first pass.

   (b) Search the list of conjuncts of φ in order until either

      i. the end of the list is reached, or

      ii. a "markable" conjunct is found, that is, one with each of its antecedent propositions marked and its consequent proposition unmarked.

   (c) If the end of the list was reached,

      i. If $\perp$ is not marked, return "satisfiable" and halt.

      ii. If $\perp$ is marked, return "unsatisfiable" and halt.

   (d) Otherwise, mark every occurrence of the consequent proposition of the first markable conjunct with the pass number (2, 3, ...) in each conjunct of φ.

   (e) Continue with step (b) again.

$$\varphi = \left|\begin{array}{lll} 1. & (\top \to w) & \wedge \\ 2. & (w \to q) & \wedge \\ 3. & (x \wedge t \to \perp) & \wedge \\ 4. & (q \wedge r \to p) & \wedge \\ 5. & (v \to s) & \wedge \\ 6. & (w \to r) & \wedge \\ 7. & (r \wedge s \to x) & \wedge \\ 8. & (\top \to v) & \wedge \\ 9. & (v \wedge q \to u) & \wedge \\ 10. & (p \wedge r \wedge s \to u) & \wedge \\ 11. & (u \to v) & \end{array}\right.$$

Your answer should list propositional letters in the order in which they are marked and indicate the returned value.

Answer

1, Mark each occurrence of $\top$ in each conjunct on the first pass and the others on the following passes:

$$\varphi = \left| \begin{array}{lll} 1. & (\top^1 \to w^2) & \wedge \\ 2. & (w^2 \to q^3) & \wedge \\ 3. & (x^4 \wedge t \to \bot) & \wedge \\ 4. & (q^3 \wedge r^3 \to p^4) & \wedge \\ 5. & (v^2 \to s^3) & \wedge \\ 6. & (w^2 \to r^3) & \wedge \\ 7. & (r^3 \wedge s^3 \to x^4) & \wedge \\ 8. & (\top^1 \to v^2) & \wedge \\ 9. & (v^2 \wedge q^3 \to u^4) & \wedge \\ 10. & (p^4 \wedge r^3 \wedge s^3 \to u^4) & \wedge \\ 11. & (u^4 \to v^2) & \end{array} \right.$$

The formula is satisfiable because $\bot$ is not marked. The order of marking of variables is w, v, q, s, r, p, x, u. The variable t is not marked.

4. **[10 points]** Can one determine whether the following formula ψ is satisfiable more quickly than by applying the HORN algorithm? Explain your answer.

$$\psi = \left| \begin{array}{ll} (\top \to p) & \wedge \\ (\bot \to q) & \wedge \\ (p \wedge q \to v) & \wedge \\ (q \to r) & \wedge \\ (v \to s) & \wedge \\ (\top \to r) & \wedge \\ (\top \to s) & \wedge \\ (v \wedge t \to u) & \wedge \\ (p \wedge r \wedge s \to u) & \end{array} \right.$$

Answer

Yes. Using the HORN algorithm: there will be 3 steps to determine the formula is satisfiable(Please see the following steps), but it is noticed that $\bot$ is not at the right side of a clause, which means it will never get marked, and we can conclude that the formula is satisfiable immediately).

$$\psi = \left| \begin{array}{ll} (\top^1 \to p^2) & \wedge \\ (\bot \to q) & \wedge \\ (p^2 \wedge q \to v) & \wedge \\ (q \to r^2) & \wedge \\ (v \to s^2) & \wedge \\ (\top^1 \to r^2) & \wedge \\ (\top^1 \to s^2) & \wedge \\ (v \wedge t \to u^2) & \wedge \\ (p^2 \wedge r^2 \wedge s^2 \to u^3) & \end{array} \right.$$