Iterative Projector Calibration using Multi-frequency Phase-shifting Method

Xu Zhang[1,2,*], Zhaorui Zhang[1], and Wei Cheng[1]

Abstract— **In this paper, an iterative projector calibration method is proposed using multi-frequency phase-shifting methods. With the help of the multi-frequency phase shifting, the projector can 'see' the 3D control point from the correspondence between the camera image and the projector image. The 3D control point information is directly adopted instead of computing them from the calibrated camera. Thus, the error propagation from the camera calibration is avoided. To diminish the influence of the lens distortion and the projective transformation, an iterative refining method is adopted to approach the real canonical fronto-parallel image from the camera image instead of the projector image. Because the canonical fronto-parallel image is determined by the virtual camera and the pose of calibration board, they are same from the camera image and the projector image to generate the canonical fronto-parallel image. However, it is complicated and time-consuming to obtain it from the projector image, because the projector image has to be created from the correspondence at first. The proposed method is conducted in our portable structured light system. It is compared with the non-iterative method. The results confirm that the proposed method has higher accuracy.**

I. Introduction

The structured light is a widely used active method in the field of optical 3D measurement, and it is an important problem to precisely calibrate the structured light system, in which even small improvements are beneficial for tasks such as 3D reconstruction, robot navigation, etc. This often includes two separate primary stages: camera calibration and projector calibration.

The camera calibration procedure typically consists of either localizing the calibration pattern control points and then determining the camera parameters, or using some geometric property of the pattern itself to obtain the camera parameters directly [1]–[7].

On the other hand, Because of the projector can be regarded as the inverse of a camera, the model is the same as the camera model. However, a projector cannot capture images directly, causing the difficulty of determining the correspondence between the projector image and the 3D control points. Generally, a calibrated camera is adopted to bridge the gap. A specific pattern image is projected onto a calibration board by the projector and the pattern modulated by the 3D calibration object is captured by the camera. With the pattern codification strategies such as Gray code, De Bruijn, M-array, or others, the correspondence between the camera image and the projector image is determined. Further, the 3D control point is computed after detecting the 2D control point in the camera image. Like camera calibration, the projector can be calibrated using the observations, including both the projector image feature points and the 3D point coordinates [8]–[16].

The methods above are simple, convenient, and very practical, but the accuracy of the projector calibration may not meet the high demand of application. The projector images suffer from the lens distortion and the projective transformation, which deteriorate localizing the control points or determining geometric properties of the pattern. Even the small error may lead to imprecise projector calibration. What's worse, the error of the camera calibration would propagate to 3D control points and reduce the precision of the projector calibration.

In this paper, a high precision method for the projector calibration is proposed. In order to avoid the error propagation, the 3D geometric information is directly adopted instead of computing them from the calibrated camera. With the help of the multi-frequency phase shifting, the projector can 'see' the 3D control point from the correspondence between the camera image and the projector image. To diminish the influence of the lens distortion and the projective transform, the control point should be detected in the canonical fronto-parallel image for the calibration board. The dilemma is that generating the canonical fronto-parallel image should know the projector parameters in ahead, which actually is not available. In this paper, an iterative refining method is adopted to approach the real canonical fronto-parallel image.

II. Related Work

The structured light system calibration is a necessary step to extract accurate metric information from 2D images. This often includes two separate stages: camera calibration and projector calibration. Camera calibration has been extensively studied in the computer vision and photogrammetric communities [6], [7], [17]. Zhang in [17] noted the difficulty of localizing square control points in distorted non-fronto parallel images due to camera optics, however, no steps were presented to rectify it. Datta et al. in [18] introduced an accurate camera calibration method using iterative refinement of control points. They proved that the distortion and the projective transformation degrade the precision of determining the control point. Their method greatly enhance the camera calibration.

The primary difficulty of calibrating the structured light system is the projector calibration, because the projector

1 School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 200072

2 State Key Laboratory of Mechanical System and Vibration, Shanghai 200240,China

∗ Corresponding author, E-mail: xuzhang@shu.edu.cn

cannot 'see' images like a camera and is just able to project images. Zhang et al. in [19] adopts M-array to determine the correspondence between the projector image points and 3D points. The colorful pattern code with square patterns is projected onto calibration board, and the captured image by the camera is decoded and the corner information is adopted to computing 3D points in the calibration board. Researches in [8], [9] developed a method to calibrating projector using the method of camera calibration. However, the method relies on the accuracy of the camera, since reprojecting the 2D image points in camera image to 3D calibration board should know parameters of calibrated camera. Researches in [11] proposed a novel algorithm of projector calibration, which make it is easy to calibrate the projector through determining the correspondence between the camera images and the projector images. However, they do not consider the influence of lens distortion and projective transformation in locating the control points. It would make the correspondence between the projector image points and the 3D points imprecise, and lead to the imprecise calibration of the projector. The goal of our work is accurate calibration by addressing the problem of precise localization of control points and avoiding error propagation.

III. Proposed Calibration Method

A. Principle

1) Model of The Projector: The model of the projector is the same as the camera model, since the projector can be conceptually regarded as a camera acting in reverse. Through the following transformations, the projector can be modeled as a pinhole imaging model without regard to the lens distortion.

$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & 0 & u_0 & 0 \\ 0 & \alpha_y & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \quad (1)$$

where s is an unknown scalar, $\alpha_x = f/dX$ and $\alpha_y = f/dY$ denote the normalized focal distance in u and v axes, and $[u_0,v_0]$ are the coordinates of the principal point.

Due to the error in processing and assembling of camera optical systems, there is distortion error in camera imaging relative to the ideal case. The lens distortion includes the radial distortion and tangential distortion. It is modeled as

$$\begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} = (k_1 r^2 + k_2 r^4) \begin{bmatrix} X' - u_0 \\ Y' - v_0 \end{bmatrix} \quad (2)$$

where δ_x and δ_y are the nonlinear distortion values, (X', Y') is the realistic point affected by lens distortion, and $r^2 = (X' - u_0)^2 + (Y' - v_0)^2$.

2) Obtaining Images for Projector Calibration: In order to make the projector 'see' the scene, phase measuring profilometry (PMP) is adopted to determine the correspondence between the camera images and projector images. Based on this correspondence, the captured image of the projector can be created from the camera image. The great advantage of PMP is that the dense correspondence for all pixels is highly accurate and robust. The position information is encoded into the phase of the multiple patterns, which always has a phase shift. With these different patterns projected, the light value corresponding to the pixel (x^c, y^c) can be formulated as:

$$I_n^c(x^c, y^c) = A^c + B^c cos(2\pi f y^p - \frac{2\pi n}{N}) \quad (3)$$

Here, A^c is the average light intensity, B^c is the intensity modulation, and n is index of N steps. The term $2\pi f y^p$ denotes the absolute phase, where the f is the frequency of cosine, the y^p denotes the coordinates of projector images, and ϕ is the wrapped phase. According to N steps of phase shifting, three parameters can be calculated with the method of least squares.

$$\begin{cases} A^c = \dfrac{1}{N} \sum\limits_{n=0}^{N-1} I_n^c \\ B^c = \dfrac{2}{N} \left\{ \left[\sum\limits_{n=0}^{N-1} I_n^c sin\left(\dfrac{2\pi n}{N}\right) \right]^2 + \left[\sum\limits_{n=0}^{N-1} I_n^c cos\left(\dfrac{2\pi n}{N}\right) \right]^2 \right\}^{\frac{1}{2}} \\ \phi = \arctan \left[\dfrac{\sum\limits_{n=0}^{N-1} I_n^c sin\left(\dfrac{2\pi n}{N}\right)}{\sum\limits_{n=0}^{N-1} I_n^c cos\left(\dfrac{2\pi n}{N}\right)} \right] \end{cases} \quad (4)$$

With these L wrapped phase values $\phi_1, \phi_2, ..., \phi_L$ obtained, which correspond to the sinusoidal grating stripes with frequencies of $0 < f_1 < f_2 < ... < f_L$, the phase unwrapping problem can be formulated as:

$$2\pi f_i y^p = 2k_i\pi + \phi_i, \qquad 0 \le i \le L \quad (5)$$

Where k_i is a unknown integer corresponding to the frequency f_i, called folding integer, and $2\pi f_i y^p$ denotes absolute phase. With both sides divided by $2\pi f_i$, Eq. (5) can be transformed into:

$$\begin{cases} y^p = k_i\lambda_i + r_i, \\ \lambda_i = \dfrac{1}{f_i}, \\ r_i = \dfrac{\phi_i}{2\pi f_i} \end{cases} \qquad 0 \le i \le L \quad (6)$$

Here, λ_i denotes wavelength of cosusoidal wave corresponding to the frequency f_i, and $r_i = y_p \ (mod\ \lambda_i)$, named remainder. From Eq. (6) we can find that, with remainder r_i already known and folding integer k_i unknown, the nature of solving for the integer y_p divided by different λ_i is the congruence problem in the number theory. According to Chinese Remainder Theorem (CRT), if λ_i relatively prime with each other, y_p can be computed by the equation as follows:

$$y_p = \sum_{i=1}^{L} \overline{F}_i F_i r_i \qquad (mod \quad \lambda) \quad (7)$$

Here, $\lambda = \prod\limits_{i=1}^{L} \lambda_i$, $F_i = \lambda/\lambda_i$, $\overline{F}_i F_i \equiv 1 \quad mod(\lambda_i)$. However, CRT is extremely sensitive to noise, Xia et al. in [20] generalize it to Robust Chinese Remainder Theorem, which is described as: with the assumption that $\lambda = M\Gamma_i$, where M is a positive real known number, Γ_i relatively prime from each other and $0 \leq y^p < M\Gamma_1\Gamma_2...\Gamma_L$, the true value of the folding integer k_i can be estimated precisely only if the maximum of remainder noise less than $M/4$.

The correspondence between the projector image points and the 3D image points can be calculated precisely through the robust CRT in both x and y directions, as shown in Fig.2 (g) and (i). From these correspondence, the projector 'see' the calibration board and the geometric information of the ring is also demonstrated.

B. Choice of control point types

In the calibration, the control point is important. Generally, there are three kinds, square, circle, and ring. Because of the great advantages for the ring pattern, it is selected as the control point. First, the correspondence is more robust. The square provides the geometric information in the contour, specifically the corners. Because the reflective property has sharp change, the unwrapped phase in PMP will be incorrect, as shown in Fig.1. The center of the circle is the control point. The black area, as shown in Fig.1 (c), (d), (e) and (f), make the noise-signal ratio of PMP very low. The correspondence may be unreliable. However, the ring has white area for determining the robust correspondence. Second, the ring can achieve highest precision in location [18].

C. Iterative Calibration of The Projector

With the 'captured' images, 2D control points can be located, and the projector can be calibrated. Because the camera parameters are not required, the projector calibration is influenced by the camera calibration and the error propagation is avoided. Although the 'captured' images for the projector have been obtained, they are non-fronto parallel images that suffer from nonlinear distortion due to optics distortion. Therefore, locating the control points in the non-fronto parallel images will introduce errors, which may lead to imprecise projector calibration.

In order to obtain higher precision, an iterative calibration method is proposed to diminish the error from the lens distortion and the projective transformation. The canonical fronto-parallel image is generated from the camera image, not the 'captured' image of the projector. The calibration board is captured by the camera and projector at the same time. Its canonical fronto-parallel image is captured from a virtual camera, which perpendicularly directs the calibration board and has no lens distortion. Based on the definition, it can be generated from the projector image or the camera image. The result is the same, when the virtual camera has been determined. It is complicated and time-consuming to obtain it from the projector image, because the projector image has to be created from the correspondence at first. Thus, the camera image is adopted. Besides of time efficiency, another advantage is that the control point information can be adopted for camera calibration and the stereo calibration of the camera and the projector. The framework of the projector calibration is shown in Fig.3.

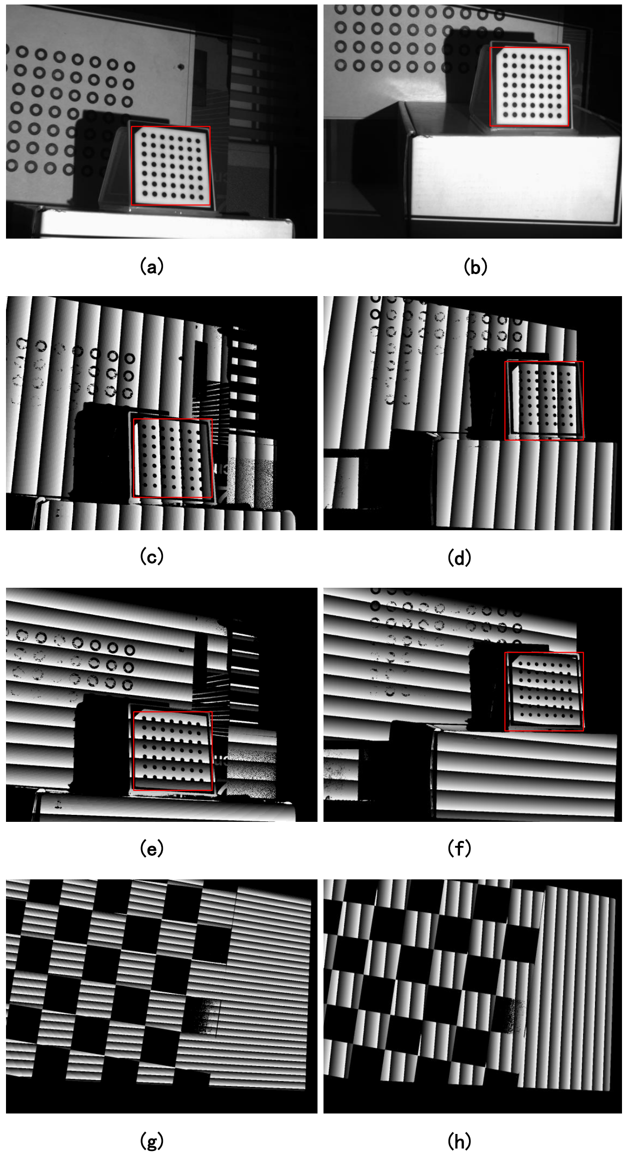

Fig. 1. The correspondence of circle control point images. (a) and (b) are calibration pattern input images. (c) and (d) show variations of the wrapped phase in X direction, where the black area denotes wrong phase values, and (e) and (f) show variations of the wrapped phase in Y direction. (g) and (h) indicate the correspondence between the projector image points and the camera image points is unreliable and imprecise because we can not computer the phase values in black area.

In the procedure of the proposed calibration, there are six steps. First, a series of sinusoidal grating images are projected onto the calibration board, which has the ring patterns. Three frequencies $1/60, 1/80$ and $1/100$ of the phase patterns are adopted for x, y, respectively. The patterns of each frequency has four grating images whose have $\pi/2$ of phase difference. After capturing these twenty-four images, another image is obtained by the camera, when the projector projects the all white light. Second, the ring center points [21] is extracted from the last image, as shown in Fig.2 (a). Third, the correspondence is determined from the absolute phase, as shown in Fig.2 (g) and (i). The 3D control points

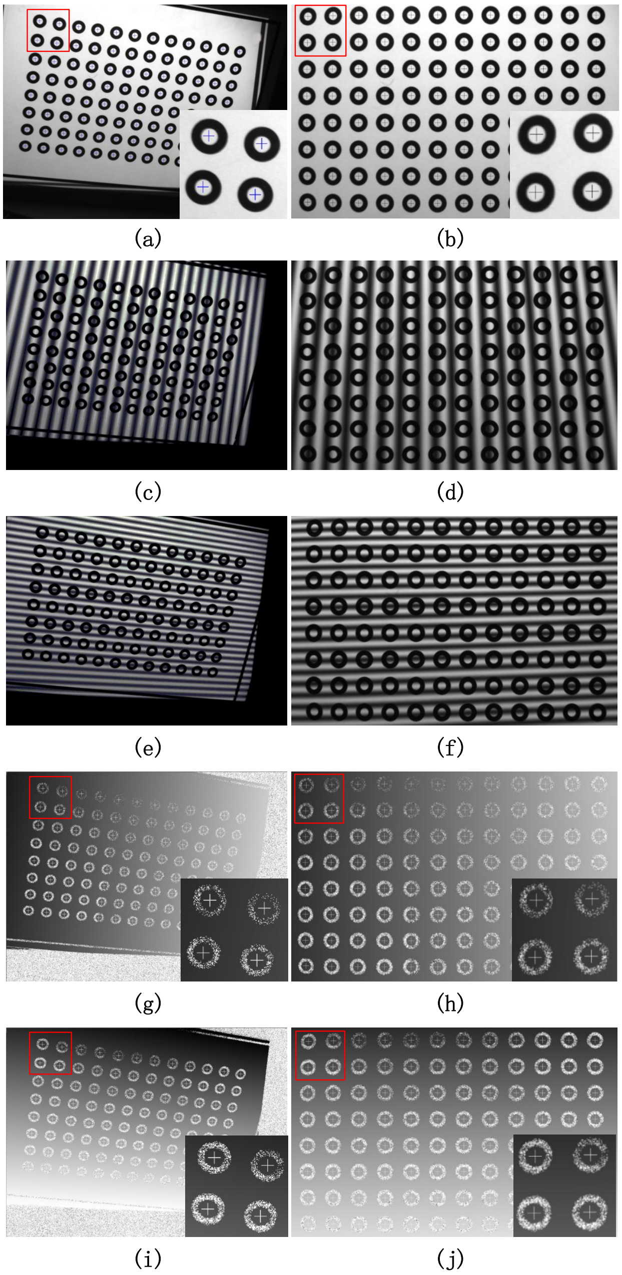

Fig. 2. The primary steps of projector iterative calibration.

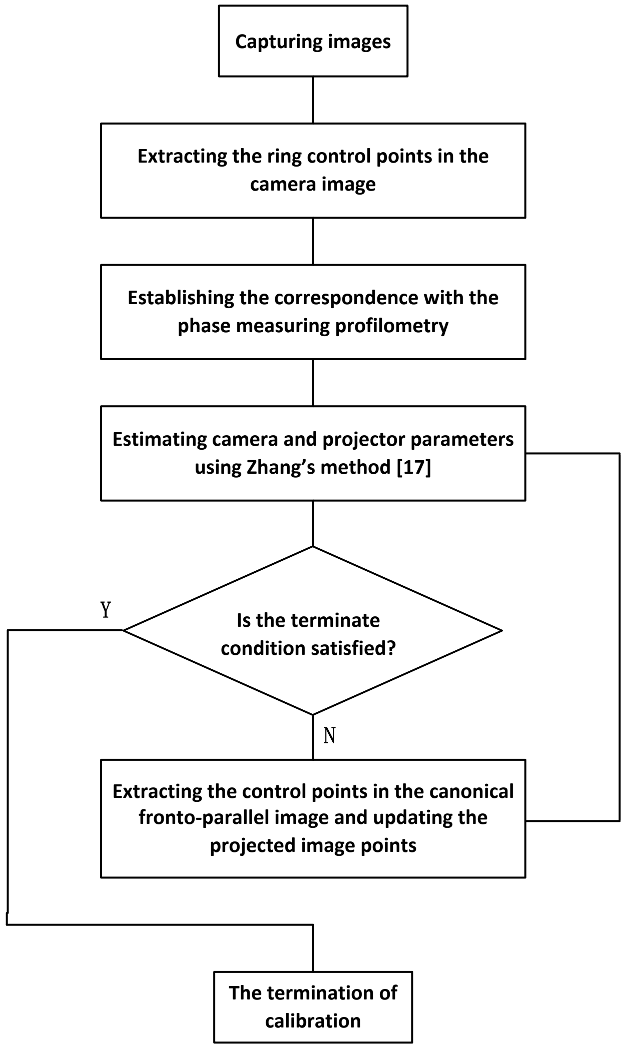

Fig. 3. The flowchart of the structured light system iterative calibration.

and 2D information in the projector and camera images are input into the calibration algorithm. If the calibration is not converged or the iterated times is not larger than the defined value, the control point will be refined in the canonical fronto-parallel image. Otherwise, the method is terminated. The camera calibration images are been undistorted and projected to canonical fronto-parallel images with no distortion, as shown in Fig.2 (b). After locating the control points in it, the control points can be re-projected to the original image and the 2D control point information for the camera calibration is refined. Based on the correspondence between the camera image and the projector image, the 2D control point information for the projector calibration can also be refined, as shown in Fig.2 (h) and (j). After calibrating the camera and the projector, the structured light system is determined.

IV. Experiments and Results

In order to verify the proposed method, three experiments were conducted in a structured light system, as shown in Fig.4 It is composed of a DLP projector with 912×1280 resolution and a camera with 1280×960 resolution.

A. *Results of the projector calibration*

In this experiment, the proposed projector calibration is compared with the non-iterative method. These two methods adopt the same calibration board, projector camera system, and captured images. The ellipse detection method [21] finds the control points in the original image. These data, combined with the 3D control data information, are input into the non-iterative calibration method [17]. The estimated parameters are the results. On the other hand, the proposed method adopts these parameters to generate the canonical

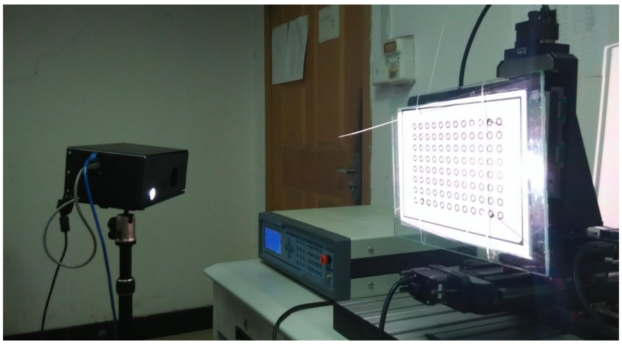

Fig. 4. the portable structured light system and The Servo Platform.

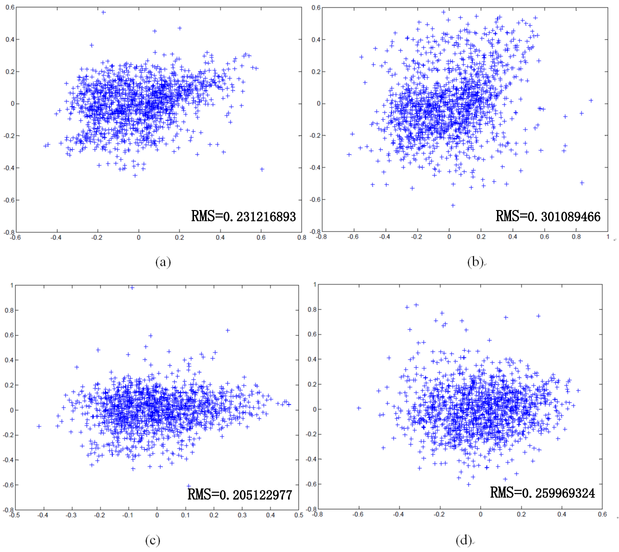

Fig. 5. Scatter plots for reprojection error between the detected ring center points and the reprojected ring centet points using the estimated calibration parameters. (a) The scatter plot for error of camera in iterative approach. (b) The scatter plot for error of camera in traditional approach. (c) The scatter plot for error of projector in iterative approach. (d) The scatter plot for error of projector in traditional approach.

fronto-parallel image and the circle is detected from it. Then, the 2D control point data in the original image is computed from the control data in the canonical fronto-parallel image. The projector is recalibrated using these new data. Until the termination condition is satisfied, the iterated refining process is conducted.

After conducting these two methods, the root mean square (RMS) error is computed. It is the standard variance of the differences between the detected ring center points and the reprojected ring center points using the estimated calibration parameters. If the projector is calibrated with higher precision, the RMS generally is smaller. Fig.5 shows the scatter plot for the error between actual ring center point locations and that of reprojected from the projector calibration result using the different approaches. We can find that the proposed iterative method reduces the RMS more than 25%. On the other hand, the camera calibration is also enhanced. This result accords with the conclusion in [18].

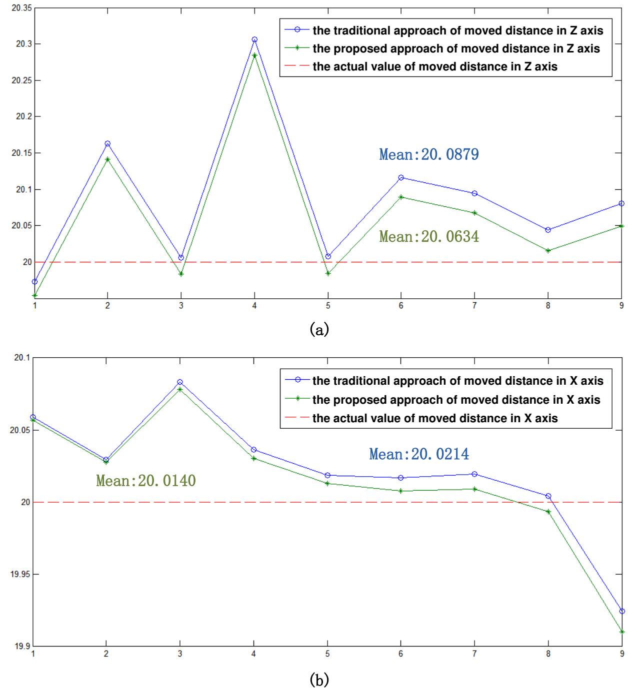

Fig. 6. The distance moved of the ring center points in the calibration board fixed in the servo platform using the estimated calibration parameters with and without iterative refinement.

B. *Comparing Precision in Servo Platform Experiment*

In the second experiment, a four-axis high-precision servo platform is adopted to verify the precision. The calibration board is fixed on it, and moved to some specific positions. The positions in the coordinate of the servo platform are (0,0,0), (20,0,0) and (20,0,20). In each position, the structured light system measures these control points. In this experiment, two calibration parameters from the proposed projector calibration and the non-iterative method are adopted. Then, the transformations between these positions are computed using the measured 3D points. The accuracy of these transformation demonstrate the performance of the calibration methods.

Fig.6 shows the distance moved of the ring center points in the calibration board fixed in the servo platform using the estimated calibration parameters with and without iterative refinement. Compared to the traditional approach, the proposed approach leads to 25~35% improvement in accuracy of calibration 3D reconstruction results.

C. *Results in 3D Reconstruction*

In the last experiment, the calibrated system is adopted to measure objects. The reconstruction results are shown in Fig.7. We can see that the quality of surface is as expected.

V. Conclusion

We proposed a method to calibrate the projector in the structured light system using iterative refinement of control points. With the help of the multi-frequency phase shifting, the projector can 'see' the 3D control point from the correspondence between the camera image and the projector image. The 3D control point information is directly adopted instead of computing them from the calibrated camera. Thus, the error propagation from the camera calibration is avoided. To diminish the influence of the lens distortion and the projective transformation, an iterative refining method is adopted to approach the real canonical fronto-parallel image from the camera image instead of the projector image. The comparative experiments are conducted in our developed structured system. First,the reprojection error is compared. The iterative method decreased 25% in RMS and achieves higher estimation precision. In addition, the result in our servo platform experiment testified the more accuracy reconstruction results using proposed method than traditional one. Finally, the reconstruction results of 3D objects suggestion that the projector calibration parameters with proposed method has high quality.

VI. ACKNOWLEDGMENTS

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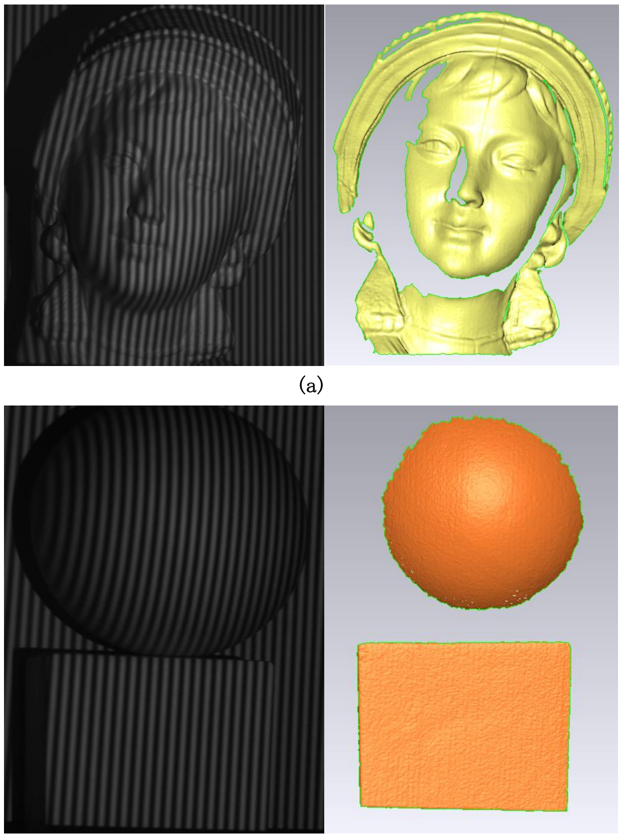

(a)

(b)

Fig. 7. the objects and the reconstructed surface of them.

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