To calculate the posterior, the following still holds:

$$P(\beta|Y,\alpha,\sigma_\eta^2,\sigma_\varepsilon^2) = \frac{P(Y,\alpha,\sigma_\eta^2,\sigma_\varepsilon^2|\beta)P(\beta)}{P(Y,\alpha,\sigma_\eta^2,\sigma_\varepsilon^2)}$$

and

$$\begin{aligned}
P(Y,\alpha,\sigma_\eta^2,\sigma_\varepsilon^2|\beta) =& P(y_1,...,y_J,\alpha_1,...,\alpha_J,\sigma_\eta^2,\sigma_\varepsilon^2|\beta) \\
=& P(y_J|y_1,...,y_{T-1},\alpha_1,...,\alpha_J,\sigma_\eta^2,\sigma_\varepsilon^2\beta) \\
& \times P(y_1,...,y_{T-1},\alpha_1,...,\alpha_J,\sigma_\eta^2,\sigma_\varepsilon^2|\beta) \\
=& P(y_J|\alpha_J,\sigma_\varepsilon^2\beta)P(\alpha_{T-1}|y_1,...,y_{T-1},\alpha_1,...,\alpha_{T-1},\sigma_\eta^2,\sigma_\varepsilon^2,\beta) \\
& \times P(y_1,...,y_{T-1},\alpha_1,...,\alpha_{T-1},\sigma_\eta^2,\sigma_\varepsilon^2|\beta) \\
=& P(y_J|\alpha_J,\sigma_\varepsilon^2\beta)P(\alpha_{T-1}|\alpha_{T-1},\sigma_\eta^2) \\
& \times P(y_1,...,y_{T-1},\alpha_1,...,\alpha_{T-1},\sigma_\eta^2,\sigma_\varepsilon^2|\beta) \\
=& P(\sigma_\varepsilon^2)P(\sigma_\eta^2)\left(\prod_{k=0}^{T} P(\alpha_k|\alpha_{k-1},\sigma_\eta^2)\right)\prod_{j=1}^{T} P(y_j|\alpha_j,\sigma_\varepsilon^2,\beta) \\
\propto & \prod_{j=1}^{T} P(y_j|\alpha_j,\sigma_\varepsilon^2,\beta)
\end{aligned}$$

For simplicity we can further write,

$$-2logP(Y,\alpha,\sigma_\eta^2,\sigma_\varepsilon^2|\beta) \propto \sum_{j=1}^{T}(y_j-\alpha_j-X_j\beta)'\Sigma_\varepsilon(y_j-\alpha_j-X_j\beta)$$

where,

$$y_j = \begin{bmatrix} y_{1j} \\ y_{2j} \end{bmatrix}, \quad \alpha_j = \begin{bmatrix} \alpha_{1j} \\ \alpha_{2j} \end{bmatrix}, \quad X_j\beta = \begin{bmatrix} X_j\beta_1 \\ X_j\beta_2 \end{bmatrix}, \quad \Sigma_\varepsilon = \begin{bmatrix} \sigma_{\varepsilon 1}^2 & & & \\ & \ddots & & \\ & & \sigma_{\varepsilon 2}^2 & \\ & & & \ddots \end{bmatrix}$$

Because Σ_ε is diagnol we can rewrite this portion as,

$$\begin{aligned}
-2logP(Y,\alpha,\sigma_\eta^2,\sigma_\varepsilon^2|\beta) \propto & \frac{\sum_{j=1}^{T}(y_{j1}-\alpha_{j1}-X_j\beta_1)^2}{\sigma_{\varepsilon 1}^2} + \frac{\sum_{j=1}^{T}(y_{j2}-\alpha_{j2}-X_j\beta_2)^2}{\sigma_{\varepsilon 2}^2} \\
\propto & \frac{\beta_1'(\sum_{j=1}^{T} X_j'X_j)\beta_1 - 2\sum_{j=1}^{T}(y_{j1}-\alpha_{j1})'X_j\beta_1}{\sigma_{\varepsilon 1}^2} + \frac{\beta_2'(\sum_{j=1}^{T} X_j'X_j)\beta_2 - 2\sum_{j=1}^{T}(y_{j2}-\alpha_{j2})'X_j\beta_2}{\sigma_{\varepsilon 2}^2}
\end{aligned}$$

Similarly for the prior we have,

$$-2logP(\beta) \propto \frac{(\beta_1-\theta_1)^2}{\sigma_\beta^2} + \frac{(\beta_2-\theta_2)^2}{\sigma_\beta^2}$$

So for the posterior of β,

$$\frac{\beta_1'(\sum_{j=1}^{T} X_j'X_j)\beta_1 - 2\sum_{j=1}^{T}(y_{j1}-\alpha_{j1})'X_j\beta_1}{\sigma_{\varepsilon 1}^2} + \frac{(\beta_1-\theta_1)^2}{\sigma_\beta^2}$$
$$+ \frac{\beta_2'(\sum_{j=1}^{T} X_j'X_j)\beta_2 - 2\sum_{j=1}^{T}(y_{j2}-\alpha_{j2})'X_j\beta_2}{\sigma_{\varepsilon 2}^2} + \frac{(\beta_2-\theta_2)^2}{\sigma_\beta^2}$$

which we have previously seen has the form

$$\frac{(\beta_1 - {\Sigma_1}^{-1}B_1)'\Sigma_1(\beta_1 - {\Sigma_1}^{-1}B_1)}{\sigma_{\varepsilon 1}^2\sigma_\beta^2} + \frac{(\beta_2 - {\Sigma_2}^{-1}B_2)'\Sigma_2(\beta_2 - {\Sigma_2}^{-1}B_2)}{\sigma_{\varepsilon 2}^2\sigma_\beta^2}$$

Where, $B_r = \sigma_\beta^2\big(\sum_{j=1}^{T} y_{jr} - \alpha_{jr}\big)'X_j - \sigma_{\varepsilon r}^2{\theta_r}'$ and $\Sigma_r = (\sigma_\beta^2\sum_{j=1}^{T} X_j'X_j) + \sigma_{\varepsilon r}^2 I_p$.

This means we have the following posterior distribution,

$$\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} \sim N(\begin{bmatrix}\Sigma_1^{-1}B_1\\ \Sigma_2^{-1}B_2\end{bmatrix}, \begin{bmatrix}\sigma_{\varepsilon 1}^2\sigma_\beta^2\Sigma_1^{-1} & 0\\ 0 & \sigma_{\varepsilon 1}^2\sigma_\beta^2\Sigma_2^{-1}\end{bmatrix})$$