HW2. real space.

$$\underline{\underline{\hat{V}}} = \frac{1}{2}\sum_{\sigma\sigma'}\int d^d r \int d^d r' \; a^{+}_{\sigma}(r)\, a^{+}_{\sigma'}(r')\, \underline{\underline{V(r,r')}}\, a_{\sigma'}(r')\, a_{\sigma}(r)$$

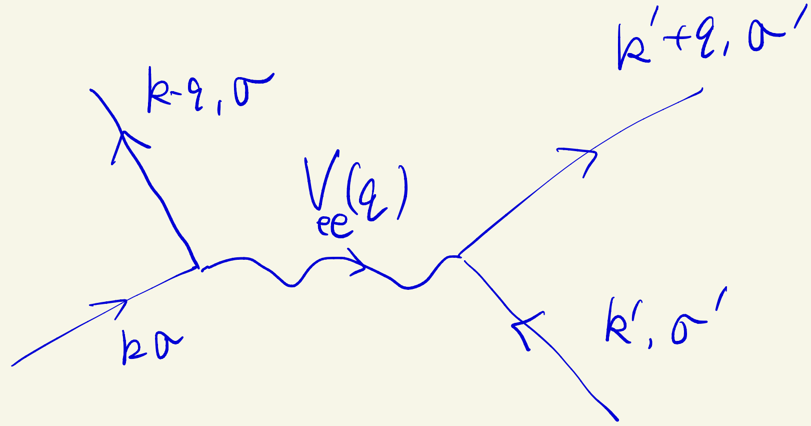
$\boxed{d=3}$

3-D real space.
$$V(r,r') = \frac{e^2}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|}$$

$$\hat{V} = \frac{1}{2V}\sum_{kk'q} V_{ee}(q)\, a^{+}_{k-q\,\sigma}\, a^{+}_{k'+q,\sigma'}\, a_{k',\sigma'}\, a_{k\sigma}$$

<u>3D</u>
$$V^{3D}_{ee}(q) = \frac{e^2}{\varepsilon_0 q^2}$$

2D:
$$V^{2D}_{ee} = \frac{e^2}{2\varepsilon_0 q}$$



Lesson 4

non-interacting fermion system.

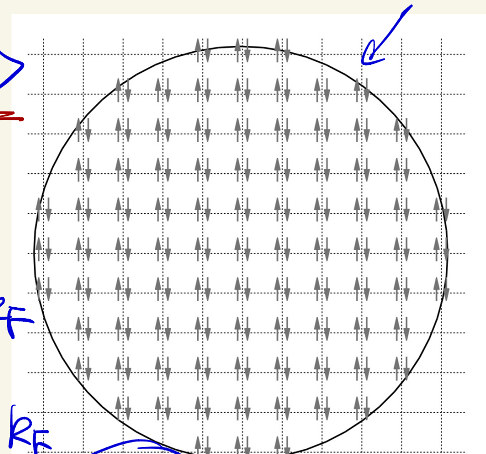
$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}, \qquad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m};$$

Ground state: $\underline{\underline{|\Omega\rangle}} = \mathcal{N}\cdot \prod_{|k|\le k_F,\sigma} \underline{\underline{a^{+}_{k\sigma}|0\rangle}}$

redefine: $c_{k\sigma}|\Omega\rangle = 0$

$$c_{k\sigma} = \begin{cases} a_{k\sigma} & k>k_F \\ a^{+}_{k\sigma} & k\le k_F \end{cases}, \qquad c^{+}_{k\sigma} = \begin{cases} a^{+}_{k\sigma} & k>k_F \\ a_{k\sigma} & k\le k_F \end{cases}$$

ε_F



2.20

Particle density operator

$\rho_\sigma(\vec{r}) = \delta(\vec{r}' - \vec{r})$

$$\rho_\sigma(\vec{r}) = \int d\vec{r}\; \psi_\sigma^\dagger(\vec{r}')\, \delta(\vec{r}' - \vec{r})\, \psi_\sigma(\vec{r}')$$

$$= \psi_\sigma^\dagger(r)\, \psi_\sigma(r)$$

$$\rho_\sigma(\vec{r}) = \frac{1}{V} \sum_{\vec{k}\vec{k}'} e^{i(\vec{k} - \vec{k}')\cdot\vec{r}}\, a_{k'\sigma}^\dagger a_{k\sigma}$$

$$= \frac{1}{V} \sum_{q} e^{i\vec{q}\cdot\vec{r}} \underbrace{\sum_{k'} a_{k'\sigma}^\dagger a_{k'+q,\sigma}}_{\rho_\sigma(\vec{q})}$$

$$\rho_\sigma(\vec{r}) = \frac{1}{V} \sum_{q} e^{i\vec{q}\cdot\vec{r}}\, \rho_\sigma(\vec{q})$$

$$\rho_\sigma(\vec{q}) = \sum_{k'} a_{k'\sigma}^\dagger a_{k'+q,\sigma}$$

Current operator

$$H_{kinetic} = \frac{1}{2m} \sum_\sigma \int d\vec{r}\; \psi_\sigma^\dagger(\vec{r}) \underline{\left(\frac{\hbar}{i}\nabla_r - q\vec{A}\right)^2} \psi_\sigma(\vec{r})$$

$$\hat{\vec{J}} = \hat{\vec{V}}$$

$$* \quad \mathcal{L} = \frac{1}{2} m v^2 - q\phi(\vec{r}, r) + \underline{q\vec{v}}\cdot\vec{A}$$

$$\mathcal{H} = \vec{v}\cdot\vec{p} - \mathcal{L} = \vec{v}\cdot\vec{p} - \frac{1}{2} m v^2 + q\phi$$
$$\underline{- q\vec{v}\cdot\vec{A}}$$

$$* \quad \delta \mathcal{H} = -q \int d\vec{r} \cdot \vec{J} \cdot \delta \vec{A} \quad *$$

$$\delta H_{kinetic} = -q \int d\vec{r} \cdot \underline{\underline{\hat{\vec{J}}}} \cdot \delta \vec{A}$$

paramagnetic diamagnetic

$$J_\sigma(\vec{r}) = J_\sigma^P(\vec{r}) + J_\sigma^A(\vec{r})$$

$$J_\sigma^P(\vec{r}) = \frac{\hbar}{2mi}\left[\Psi_\sigma^+(r)\left(\nabla \Psi_\sigma(r)\right) - \left(\nabla \Psi_\sigma^+(r)\right) \Psi_\sigma(r) \right]$$

(2.21)

$$J_\sigma^A(\vec{r}) = -\frac{q}{m} A(r)\, \Psi_\sigma^+(r)\, \Psi_\sigma(r)$$

Tight-binding systems

$$|\psi_k\rangle = \frac{1}{\sqrt{N}} \sum_{R_n} e^{ik\cdot R_n} |\psi_{R_n}\rangle \qquad (2.22)$$

$$|\psi_{R_n}\rangle = \frac{1}{\sqrt{N}} \sum_{k}^{B.Z.} e^{-ik R_n} |\psi_k\rangle$$

$$\psi_{R_n}(r) = \langle r | \psi_{R_n}\rangle \qquad R_n \sim r$$

(*) $|\psi_{R_n}\rangle$ form an orthogonal basis.

$$|r\rangle = \sum_R |\psi_R\rangle\langle\psi_R|r\rangle = \sum_R \psi_R^*(r)\,|\psi_R\rangle$$

$$a_\sigma^+(r) = \sum_R \psi_R^*(r)\, a_{R\sigma}^+ \qquad R_n \to R$$

$$= \sum_i \psi_{R_i}^*(r)\, a_{i\sigma}^+ \qquad 2.23$$

$$a_{k\sigma}^+ = \frac{1}{\sqrt{N}} \sum_i e^{i\vec{k}\cdot\vec{R}_i}\, a_{i\sigma}^+$$

$$a_{i\sigma}^+ = \frac{1}{\sqrt{N}} \sum_k^{B.Z.} e^{-i\vec{k}\cdot\vec{R}_i}\, a_{k\sigma}^+ \qquad 2.24$$

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

$$H_0 = \sum_k \varepsilon_k\, a_{k\sigma}^+ a_{k\sigma} = \underline{\frac{1}{N}} \sum_{ii'} \underbrace{\sum_k e^{ik(R_i - R_{i'})}\varepsilon_k}_{t_{ii'}}\, a_{i\sigma}^+ a_{i'\sigma}$$

$$= \sum_{ii'} t_{ii'}\, a_{i\sigma}^+ a_{i'\sigma} \simeq -t \sum_{\langle ij\rangle} a_{i\sigma}^+ a_{j\sigma}$$

$$t_{ii'} = \frac{1}{N} \sum_{k\in BZ} e^{ik(R_i - R_{i'})}\cdot \underline{\varepsilon_k}$$

if ε_k is constant; $\sum_{k\in BZ} e^{ik(R_i - R_{i'})} \propto \delta_{ii'}$

nearest neighbour N.N. $\langle i,i'\rangle$ $t_{ii'} \equiv -t$

2D: square lattice: $\varepsilon_k = -2t\,(\cos k_x a + \cos k_y a)$

Suggest reading:

Li zheng zhong : chapter 3, 4, 6

Altland & Simons : chapter 2.

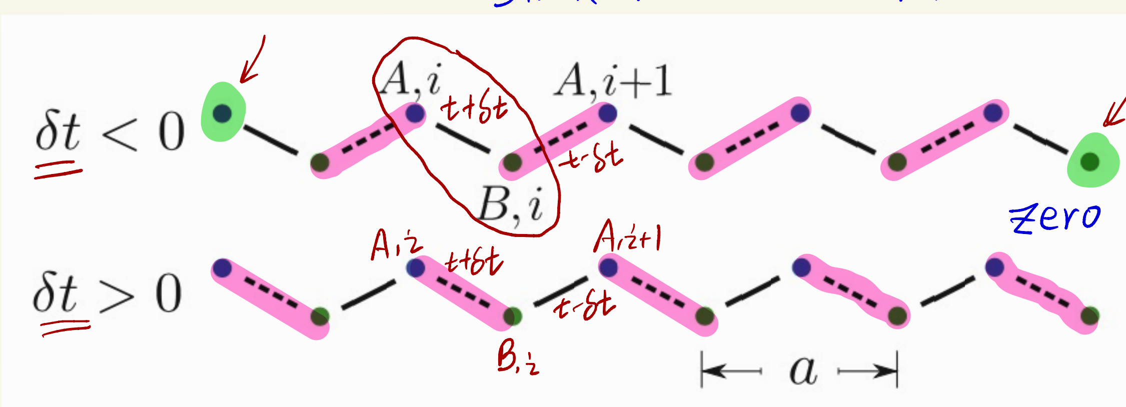
(*) Su-Shrieffer-Heeger model

W.P. Su, J.R. Schrieffer, A.J. Heeger, Phys. Rev. Lett. **42**, 1698 (1979)

A.J. Heeger, S. Kivelson, J.R. Schrieffer, W.P. Su, Rev. Mod. Phys. **60**, 781 (1988)

SSH model:

Similar to 1D Kitaev chain



zero energy mode

$$\mathcal{H} = \sum_{n=1}^{N} (t+\delta t)\, C^{\dagger}_{An} C_{Bn} + \sum_{n=1}^{N-1} (t-\delta t)\, C^{\dagger}_{A,n+1} C_{Bn} + h.c.$$

Fourier transformation:

$$a_k = \frac{1}{\sqrt{N}} \sum_n e^{-ik\cdot na}\, C_{An}$$

$$b_k = \frac{1}{\sqrt{N}} \sum_n e^{-ik\cdot na}\, C_{B.n} \qquad k \in [-\pi, \pi]$$

$$\mathcal{H} = (t+\delta t) \sum_{k \in B.Z.} (a_k^+ b_k + b_k^+ a_k)$$

$$+ (t-\delta t) \sum_k (e^{ik} a_k^+ b_k + e^{-ik} b_k^+ a_k)$$

$\cos k + i \sin k$

$$\psi_k = \begin{pmatrix} a_k \\ b_k \end{pmatrix} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\mathcal{H} = \sum_k \psi_k^+ \begin{bmatrix} 0 & [(t+\delta t) + (t-\delta t)\cos k] - i \sin k (t-\delta t) \\ * & 0 \end{bmatrix} \psi_k$$

$$= \sum_k \psi_k^+ \{ [(t+\delta t) + (t-\delta t)\cos k]\sigma_x + (t-\delta t)\sin k\, \sigma_y \} \psi_k$$

$\sigma_x \to \sigma_z$, $\sigma_y \to \sigma_x$, $\sigma_z \to \sigma_y$; $\underline{k \to k+\pi}$

1D massive Dirac equation

$$\mathcal{H} = \sum_k \psi_k^+ \{ \underbrace{[2\delta t + 2(t-\delta t)\sin^2 \tfrac{k}{2}]}_{\text{mass term}} \sigma_z - (t-\delta t)\sin k\, \sigma_x \} \psi_k$$

Jakiw - Rebbi model

1970

$$d_x = -(t-\delta t)\sin k$$

$$d_z \simeq \underbrace{2\delta t + 2(t-\delta t)\sin^2 \tfrac{k}{2}}_{\text{mass term}} \longrightarrow \begin{cases} d_z = 2 \cdot \delta t & k=0 \\ d_z = 2 \cdot t & k=\pi \end{cases}$$

$$E_\pm = \pm \sqrt{d_x^2 + d_z^2}$$

$d_x = d_z = 0$, Gap closing condition.

$$\begin{cases} k = 0, \\ \delta t = 0 \end{cases}$$

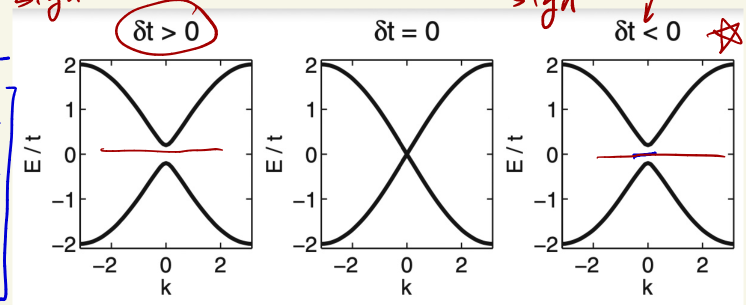
$\underline{\delta t \cdot t < 0}$ or $\underline{\delta t \cdot t > 0}$

mass change sign — mass donot change sign mass donot change sign

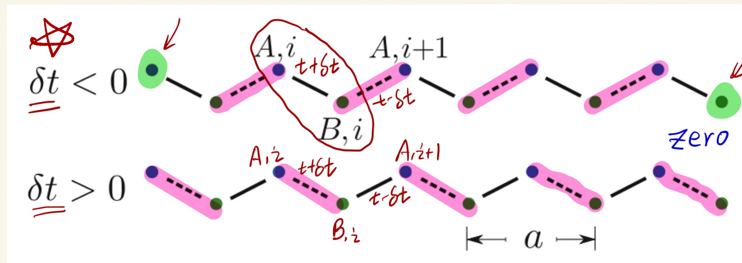
mass change sign

for negative energy band

$$|\psi_-\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathrm{sgn}(d_x)\sqrt{1 - \frac{d_z}{\sqrt{d_x^2+d_z^2}}} \\ \sqrt{1 + \frac{d_z}{\sqrt{d_x^2+d_z^2}}} \end{bmatrix}$$



Similar to 1D kitaev chain



zero energy mode

Zak phase

$$\gamma = \int_{-\pi}^{\pi} dk \, \langle \psi | i\partial_k | \psi \rangle = \frac{\pi}{2}\left[\mathrm{sgn}(t-\delta t) - \mathrm{sgn}(\delta t)\right]$$

$\boxed{t>0}$ if $\delta t < 0$; $\gamma = \pi$; topologically nontrivial.

if $\delta t > 0$; $\gamma = 0$;

Magnetic systems. symmetry + dimension

* Heisenberg model $SU(2)$
* XY model $U(1)$ [planar magnetic ..]
* Ising model Z_2

* symmetry

Interaction effect in tight-binding system

$$V = \frac{1}{2} \sum_{\sigma\sigma'} \int dr\, dr' \, \frac{e^2}{|r-r'|} a_\sigma^\dagger(r)\, a_{\sigma'}^\dagger(r')\, a_{\sigma'}(r')\, a_\sigma(r)$$

$$a_\sigma^+(r) = \sum_i \psi^*_{R_i}(r)\, a^+_{i\sigma}$$

(2.27)

$$\hat{V}_{ee} = \sum_{\sigma,\sigma' i i' j j'} \underline{\underline{U_{ii'jj'}}}\, a^+_{i\sigma} a^+_{i'\sigma'} a_{j'\sigma'} a_{j\sigma}$$

$$U_{ii'jj'} = \frac{1}{2}\int dr \int dr'\, \psi^*_{R_i}(r)\psi_{R_j}(r)\, V(r-r')\, \psi^*_{R_{i'}}(r')\psi_{R_{j'}}(r')$$

(A) direct term:

$$U_{ii'ii'} \equiv V_{ii'} \qquad i \neq i'$$

when $i \neq i'$ $$\sum_{\sigma\sigma'} \sum_{i\neq i'} V_{ii'}\, a^+_{i\sigma} a^+_{i'\sigma'} a_{i'\sigma'} a_{i\sigma}$$

$$= \sum_{\sigma\sigma'} \sum_{i\neq i'} V_{ii'}\, \hat{n}_{i\sigma}\cdot \hat{n}_{i'\sigma'} \qquad (2.28)$$

$$= \sum_{i\neq i'} \hat{n}_i \cdot \hat{n}_{i'}\, V_{ii'} \qquad \hat{n}_i = \sum_\sigma \hat{n}_{i\sigma}$$

Induce charge charge density wave ..

(B) exchange term. $U_{ii'jj'}\, a^+_{i\sigma} a^+_{i'\sigma'} a_{j'\sigma'} a_{j\sigma}$

$$i \neq j \qquad J^E_{ij} = U_{ijji}$$

$\underline{\underline{\hbar = 1}}$

$$\sum_{i\neq j} \overset{J^E_{ij}}{U_{ijji}}\, \underline{a^+_{i\sigma} a^+_{j\sigma'}}\, \underline{a_{i\sigma'} a_{j\sigma}}$$

Heisenberg model

(2.29)

spin $\frac{1}{2}$ $$= -2 \sum_{i\neq j} J^E_{ij} \left(\hat{S}_i \cdot \hat{S}_j + \frac{1}{4} \hat{n}_j \cdot \hat{n}_j \right)$$

$*$ $\hat{S}_i \equiv \frac{1}{2} \hat{a}^+_{i\alpha} \cdot \vec{\sigma}_{\alpha\beta} \cdot \hat{a}_{i\beta}$

$\hbar = 1$

$\hat{S}_j \equiv \frac{1}{2} \hat{a}^+_{j\cdot\gamma} \cdot \vec{\sigma}_{\gamma\delta} \cdot a_{j\delta}$

$$\vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} = 2 \cdot \delta_{\alpha\delta} \delta_{\beta\gamma} - \delta_{\alpha\beta} \cdot \delta_{\gamma\delta}$$

ETC:

$$\hat{S}_i \cdot \hat{S}_j = \frac{-1}{2} \underline{a^+_{i\alpha} a^+_{j\beta} a_{i\beta} a_{j\alpha}} - \frac{1}{4} \hat{n}_j \hat{n}_j \qquad (2.30)$$

(C) Atomic Limit (on-site term)

$\sigma\,\sigma', \uparrow \downarrow$

$\sigma\,\sigma' \downarrow, \uparrow$

$$U_{iiii} = \frac{U}{2}$$

$$\sum_{i\sigma\sigma'} U_{iiii}\, a^+_{i\sigma} a^+_{i\sigma'} a_{i\sigma'} a_{i\sigma}$$

$$= U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} \qquad (2.31)$$

single-band Hubbard model

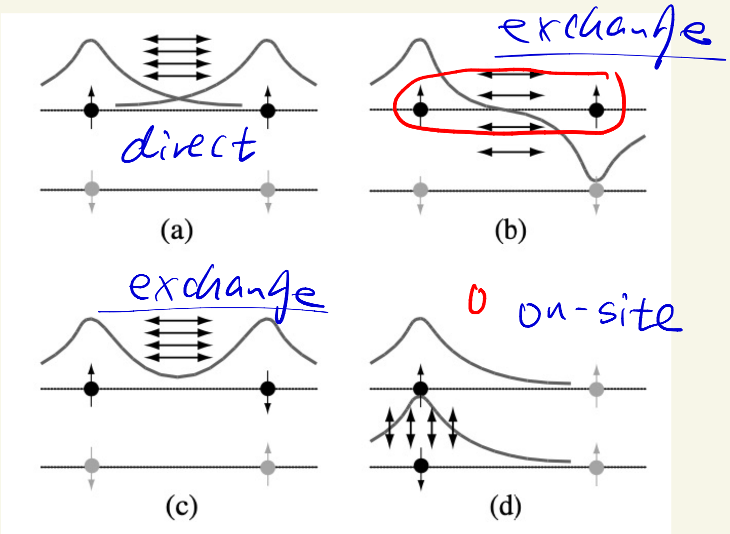
$$\hat{H} = -t \sum_{\langle ij \rangle} a^+_{i\sigma} a_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

Coulomb interaction U

Half-filled

Hubbard model.

occupancy is 1



(a) (b) (c) (d)

$U \ll t$ metal

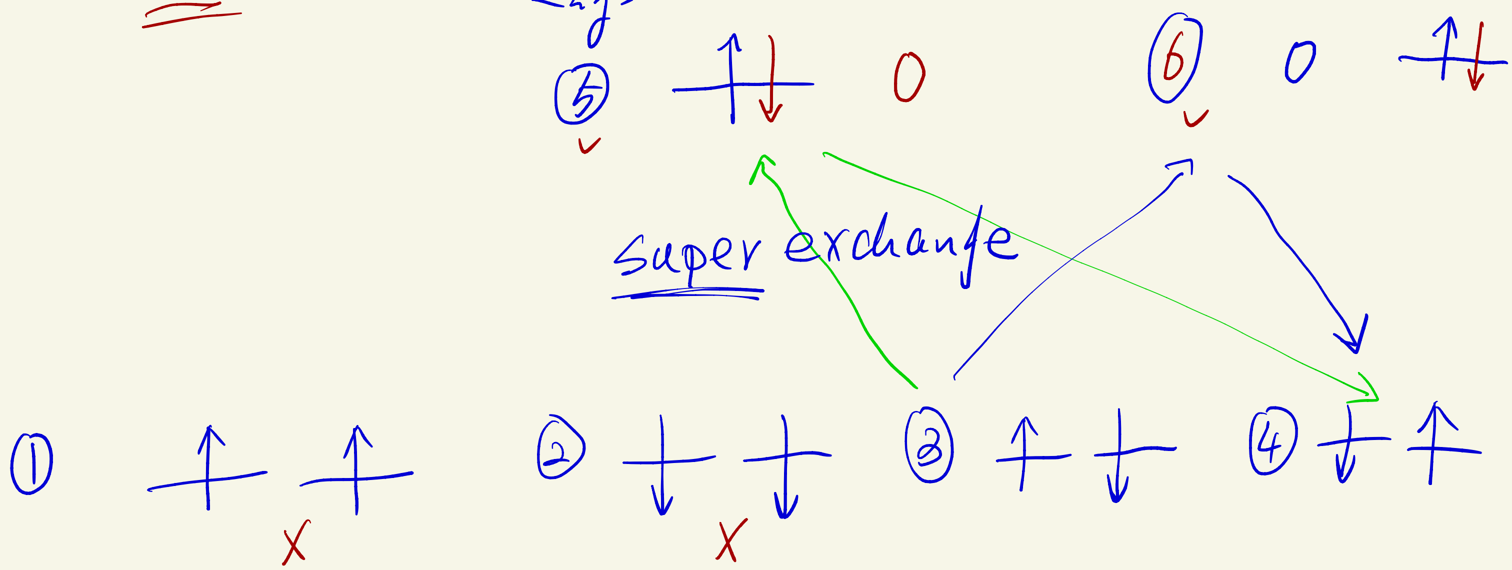
$U \gg t$ Mott insulator

strong coupling limit.

$$\hat{H} = -t \sum_{\langle ij \rangle} a^{\dagger}_{i\sigma} a_{j\sigma} + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

$$H_0 = U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

$$\underline{H_I} = -t \sum_{\langle ij \rangle} a^{\dagger}_{i\sigma} a_{j\sigma} \quad ✓$$



$$(*) \quad |\psi_0\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle \right)$$

$$\Delta E = \underline{-\frac{4t^2}{U}} = \frac{|\langle ⑤ | T | \psi_0 \rangle|^2 + |\langle ⑥ | T | \psi_0 \rangle|^2}{-U} \qquad (2.32)$$

$$H = J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j \qquad J = \frac{4t^2}{U} \qquad (2.33)$$

Large-U Hubbard model $\Rightarrow$ AF Heisenberg model

Ferromagentic Spin Wave

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j \qquad J>0, \quad FM$$

Spin: S Sakurai MQM chapter 3.

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \left(\hat{S}_i^z \hat{S}_j^z + \hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y \right) \quad \begin{cases} \hat{S}_i^+ = \hat{S}_i^x + i \hat{S}_i^y \\ \hat{S}_i^- = \hat{S}_i^x - i S_i^y \end{cases}$$

$$H = -J \sum_{\langle ij \rangle} \left(\hat{S}_i^z \hat{S}_j^z + \frac{S_i^+ S_j^- + S_i^- S_j^+}{2} \right) \qquad (2\text{-}34)$$

Ground state

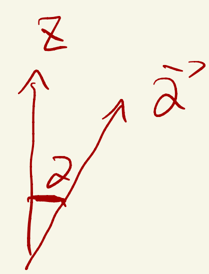
$$|SS\rangle \equiv |\underbrace{\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \cdots \uparrow}_{N}\rangle \qquad (*)$$

&

$$|\downarrow \downarrow \downarrow \downarrow \cdots \cdots \downarrow\rangle$$

ground state degeneracy $2S+1$

$$\hat{S}^- \equiv \sum_{j=1}^{N} S_j^- \qquad [H, S^-] = 0$$



$S^- |SS\rangle$ ground state.

(2.35)

$\cancel{X \; (*) \; |\alpha\rangle = \exp[i\alpha \cdot \hat{S}^-] |s,s\rangle \; X}$

ground state energy. $E_0 = -J \cdot S^2 \cdot Z \cdot N$

Z ↑ number of N.N.

$$H = -J \sum_{\langle ij \rangle} \left(\hat{S}_i^z \hat{S}_j^z + \frac{S_i^+ S_j^- + S_i^- S_j^+}{2} \right) \quad (2.34)$$

$$\begin{cases} [S_m^+, S_n^-] = 2 S_m^z \delta_{mn} \\ [S_m^-, S_n^z] = S_m^- \delta_{mn} \\ [S_m^+, S_n^z] = - S_m^+ \delta_{mn} \end{cases} \quad (2.36)$$

$$\Phi_m = S_m^- \,|\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \cdots \uparrow\rangle$$

$$= |\uparrow \uparrow \uparrow \cdots \underset{m}{\downarrow} \uparrow \cdots \uparrow\rangle \qquad \delta .\ N.N.$$

ETC.

$$\boxed{H \Phi_m = E_0 \Phi_m + 2J \cdot S \sum_\delta \left(\Phi_m - \Phi_{m+\delta} \right)} \quad (2.37)$$

$$\Phi_k = \sum_m e^{ik \cdot R_m} \Phi_m \Leftarrow$$

$$H \Phi_k = \left(E_0 + 2JS \cdot Z \left[1 - \frac{1}{Z} \sum_\delta e^{ik \cdot R_\delta} \right] \right) \Phi_k$$

$$\Phi_k = \sum_m e^{ik R_m} \cdot S_m^- \,|S, S\rangle \quad (2.38)$$

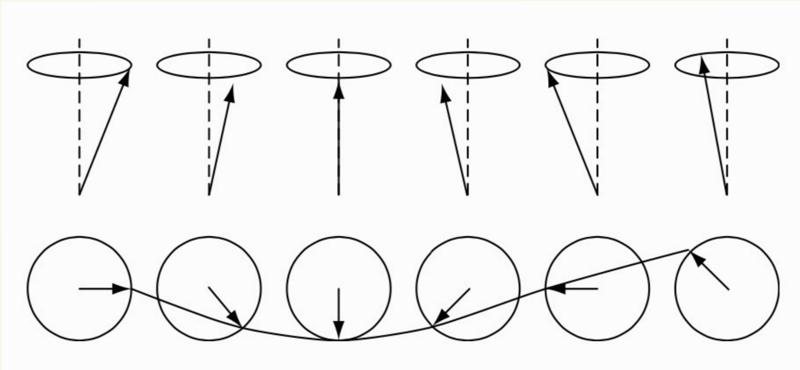
$$E_k = E_0 + 2JS \cdot Z \, [1 - \gamma_k]$$

$$\gamma_k \equiv \frac{1}{Z} \sum_\delta e^{i\vec{k}\cdot\vec{R}_\delta} \quad (2.39)$$

$$\Delta E = E_k - E_0 \propto k^2$$

FM spin wave $\underline{\underline{\Delta E \propto k^2}}$

Particle number representation



High spin $\xrightarrow[\text{mapping}]{}$ boson model

Holstein-Primakoff transformation

[High S spin]

Ground state $S_z = S$, magnon

$$n_i = S - S_i^z$$

$$|n_1, n_2, \cdots, n_N\rangle = |0, 0, 0, 0, \cdots, 0\rangle$$

$$\begin{cases} \hat{S}^+ |S, S_z\rangle = \sqrt{(S-S_z)(S+S_z+1)} \, |S, S_z+1\rangle \\ \hat{S}^- |S, S_z\rangle = \sqrt{(S+S_z)(S-S_z+1)} \, |S, S_z-1\rangle \\ \hat{S}_z |S, S_z\rangle = S_z |S, S_z\rangle \end{cases}$$

Guess

Spin $\quad$ Boson

$|S_z\rangle \Longrightarrow |n\rangle$

$|S_z \pm 1\rangle \Longrightarrow |n \mp 1\rangle$

$$\begin{cases} \hat{S}^+|S_z\rangle = \sqrt{(S-S_z)(S+S_z+1)}\,|S_z+1\rangle \\ \hat{S}^-|S_z\rangle = \sqrt{(S+S_z)(S-S_z+1)}\,|S_z-1\rangle \\ \hat{S}_z|S_z\rangle = S_z\,|S_z\rangle \end{cases}$$

$\swarrow n \qquad \swarrow |n-1\rangle \qquad |n+1\rangle$

$a|n\rangle$

$$\begin{cases} \hat{S}^+|n\rangle = \sqrt{2S+1-n}\cdot\sqrt{n}\,|n-1\rangle \\ \hat{S}^-|n\rangle = \sqrt{2S-n}\cdot\sqrt{n+1}\cdot|n+1\rangle \\ S_z|n\rangle = (S-n)\,|n\rangle \end{cases}$$

$a^+|n\rangle$

$$\begin{cases} a|n\rangle = \sqrt{n}\,|n-1\rangle \\ a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle \end{cases}$$

(2.40) $\quad n = S - S_z$

$$\begin{cases} [a_m, a_{m'}^+] = \delta_{mm'} \\ [a_m, a_{m'}] = [a_m^+, a_{m'}^+] = 0 \end{cases}$$

HP transformation.

large S

$$\begin{cases} S_m^+ = \sqrt{2S - a_m^+ a_m}\cdot a_m \\ S_m^- = a_m^+\cdot\sqrt{2S - a_m^+ a_m} \\ S_m^z = S - \hat{a}_m^+\hat{a}_m \end{cases} \quad (2.41) \qquad \simeq \begin{cases} S_m^+ \simeq \sqrt{2S}\,a_m \\ S_m^- \simeq \sqrt{2S}\,a_m^+ \\ S_m^z = S - a_m^+ a_m \end{cases} \quad (2.42)$$

$$\hat{H} = -J\sum_{m,\delta}\left[\hat{S}_m^z\hat{S}_{m+\delta}^z + \frac{1}{2}\left(\hat{S}_m^+\hat{S}_{m+\delta}^- + \hat{S}_m^-\hat{S}_{m+\delta}^+\right)\right]$$

$$= E_0 + JS\sum_m \left(a_{m+\delta}^+ - a_m^+\right)\left(a_{m+\delta} - a_m\right) + \cdots O(S^0)$$

$\uparrow$ $\qquad \downarrow$ FoF. $\qquad$ 4-operator

$$= E_0 + 2JZS \sum_k{}' (1-\gamma_k)\, a_k^+ a_k \quad (2.43)$$

$$+ \frac{ZJ}{2N} \sum_{k,k',q}{}' (\gamma_{k-q} + \gamma_{k'} - 2\gamma_{k-q-k'}) \cdot a_{k-q}^+ a_{k'+q}^+ a_{k'} a_k + \cdots$$

magnon-magnon interaction

2nd quantization FM Heisenberg model

Ground state + spin wave excitation

gapless excitation $\delta E \propto k^2$ (2.44)

Remarks:

① High spin HP transformation ✓

Spin 1/2, HP fails.

Matsubara, 1956, Prog Theor. phys, 16 569 ✓

② High spin, semi-classical arguments.

⇒ ③ spin wave excitation can destroy long-range order, if $D \leq 2$.

* Hohenberg - Mermin - Wagner theorem *

magnetization

$$M^z = N\cdot S - \sum_m a_m^+ a_m$$

$$= N\cdot S - \sum_k a_k^+ a_k = NS - \sum_k n(\varepsilon_k)$$

$$M^z = N\cdot S - \frac{V}{(2\pi)^d}\int_0^\infty \frac{C_\pi \cdot k^{d-1}\, dk}{e^{\frac{Ak^2}{k_BT}} - 1}$$

T > 0 when $k \to 0$, $e^{\frac{Ak^2}{k_BT}} - 1 \propto \frac{Ak^2}{k_BT}$

① $T>0$; $d \le 2$; $\int_0^\infty \frac{k^{d-1}\cdot dk}{e^{\frac{Ak^2}{k_BT}} - 1}$ diverge.

$\langle M^z \rangle = 0$; (2.45)

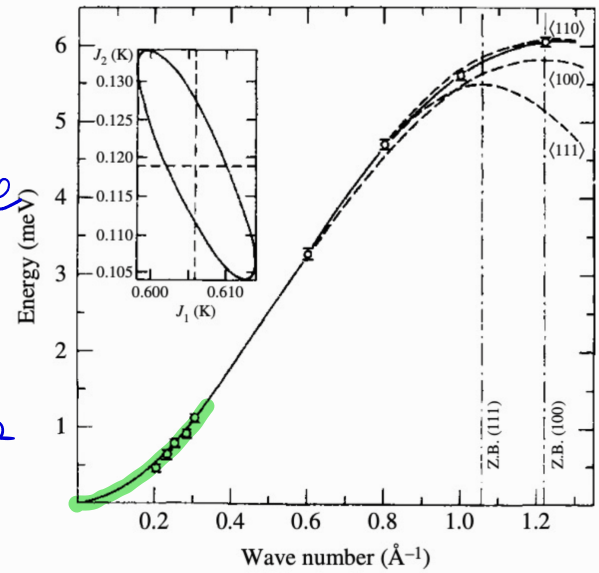
② $T>0$, $d=3$,

$$M = NS - \frac{V}{(2\pi)^3}\int_0^\infty \frac{4\pi k^2\, dk}{e^{\frac{Ak^2}{k_BT}} - 1} \quad (2.46)$$

$$= NS\left[1 - \left(\frac{T}{T_0}\right)^{3/2}\right]; \quad T_0 \sim J/k_B$$

T_0: magnon BEC temprature.

FM spin wave



$\Delta E \propto k^2$

Figure 2.11 Spin-wave spectrum of europium oxide as measured by inelastic neutron scattering a a reference temperature of 5.5 K. Note that, at low values of momenta q, the dispersion is quadratic in agreement with the low-energy theory. (Exercise: A closer inspection of the data shows th existence of a small gap in the spectrum at $\mathbf{q} = 0$. To what may this gap be attributed?) Figur reprinted with permission from L. Passell, O. W. Dietrich, and J. Als-Nielser, Neutron scattering from the Heisenberg ferromagnets EuO and EuS I: the exchange interaction, *Phys. Rev. B* **1** (1976), 4897–907. Copyright (1976) by the American Physical Society.

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Lesson 05

spin 1/2 FM Heisenberg model $\quad \frac{\text{HP failed}}{1940}$

Matsubara 1956 Prog. Theor. Phys. 16 569

$$S_j^{\pm} = S_j^x \pm i S_j^y \ ; \quad S_j^z$$

Commute relation:

$$i \neq j \quad [S_i^+, S_j^+]_- = [S_i^-, S_j^-]_- = [S_i^-, S_j^+]_- = 0 ;$$

anti-commute $\qquad$ spin 1/2 ; $S_i^z = 1/2 - S_i^- S_i^+$

$$i = j \qquad [S_i^-, S_i^+]_+ = 1 ; \qquad (2.48)$$

$$[S_i^+, S_i^+]_+ = 0 ; \quad [S_i^-, S_i^-]_+ = 0 ;$$