Algebra of γ matrices

```
ClearAll[CenterDot, δ];
SetAttributes[CenterDot, {Flat, OneIdentity}];
SetAttributes[δ, Orderless];
rule1 = γ[a_] · γ[b_] :→ 2 δ[a, b] - γ[b] · γ[a] /; Not@OrderedQ[{a, b}];
rule2 = γ[a_] · γ[a_] :→ δ[a, a];
CenterDot /: (a1_ + a2_) · a3_ := a1 · a3 + a2 · a3;
CenterDot /: a3_ · (a1_ + a2_) := a3 · a1 + a3 · a2;
NumQ[δ[_, _] | _?NumericQ] = True;
NumQ[_] = False;
CenterDot /: a1_ · a2_ := a1 a2 /; NumQ[a2] ⋁ NumQ[a1]
CenterDot /: (a3_ a1_) · a2_ := a3 (a1 · a2) /; NumQ[a3]
CenterDot /: a1_ · (a2_ a3_) := a3 (a1 · a2) /; NumQ[a3]
rule = {
    rule1,
    rule2,
    δ[a1_, a1_] → dim,
    δ[a1_, a2_]^2 → dim,
    δ[a1_, a2_] a3_ :→ (a3 /. a1 → a2) /; Not@FreeQ[a3, a1]
  };
```

```
δ[a, b]
δ[b, a]
```

```
δ[a, b]
```

```
δ[a, b]
```

```
Flatten@F[a, F[b, c]]
```

```
F[a, b, c]
```

```
CenterDot[A, B]
```

```
A · B
```

```
? CenterDot
```

CenterDot[x, y, …] displays as $x \cdot y \cdot \ldots$ ≫

```
CenterDot[c, CenterDot[a, b]] // FullForm
c · a · b // FullForm
```

```
CenterDot[c, a, b]
```

```
CenterDot[c, a, b]
```

```
example1 = γ[a] · γ[c] · γ[b] · γ[c];
```

```
? OrderedQ
```

OrderedQ[h[e_1, e_2, …]] gives True if the e_i are in canonical order, and False otherwise. ≫

```
example1 /. rule1
```

$$2\,\gamma[a]\cdot\gamma[c]\,\delta[c, b]-\gamma[a]\cdot\gamma[b]\,\delta[c, c]$$

```
example1 /. rule1
```

$$2\,\gamma[a]\cdot\gamma[c]\,\delta[c, b]-\gamma[a]\cdot\gamma[b]\,\delta[c, c]$$

```
NumericQ[δ[a, b]]
```

```
False
```

V↔||

```
dim
```

```
dim
```

```
example1 //. rule // Simplify
```

$$-\left(\gamma[a]\cdot\gamma[b]\right)\left(-2+\delta[c, c]\right)$$

```
example2 = γ[d] · γ[a] · γ[c] · γ[b] · γ[d] · γ[c] · γ[a];
example2 //. rule // Simplify
```

$$-\left(-16+24\,\text{dim}-10\,\text{dim}^2+\text{dim}^3\right)\gamma[b]$$

Lorentz algebra

```
[Jμν, Jρσ] = i (ημσ Jνρ − ημρ Jνσ − ηνσ Jμρ + ηνρ Jμσ);
J[a_, b_] = −(I/4) (γ[a] · γ[b] − γ[b] · γ[a]);
lhs = J[μ, ν] · J[α, β] − J[α, β] · J[μ, ν] //. rule;
rhs =
  I δ[μ, α] · J[ν, β] − I δ[μ, β] · J[ν, α] − I δ[ν, α] · J[μ, β] + I δ[ν, β] · J[μ, α] //. rule;
lhs − rhs // Simplify
```

```
0
```

Trash

```
g /: f[g[x_]] = fg[x];(*I want this to be a property of g!! not f!!*)
? f
```

```
Global`f
```

```
? g
```

```
Global`g
```

```
f[g[x_]] ^= fg[x]
```