Harmonic Oscillator and Perturbation Theory

```
ClearAll[CircleDot];
Attributes[CircleDot] = {Flat, OneIdentity};
a___ ⊙ (b_ c_) ⊙ d___ := b (a ⊙ c ⊙ d) /; NumericQ[b]
a___ ⊙ (b_ + c_) ⊙ d___ := a ⊙ b ⊙ d + a ⊙ c ⊙ d
a__ ⊙ b_ ⊙ d___ := b (a ⊙ d) /; NumericQ[b]
a___ ⊙ b_ ⊙ d__ := b (a ⊙ d) /; NumericQ[b]

a ⊙ ad := ad ⊙ a + 1;
c___ ⊙ ad ⊙ k[m_] := √(m + 1) c ⊙ k[m + 1]
c___ ⊙ a ⊙ k[m_] := √m c ⊙ k[m - 1]
b[n_] ⊙ k[m_] := Boole[n == m]

H0 = ad ⊙ a;
V = λ (a + ad) ⊙ (a + ad) ⊙ (a + ad) ⊙ (a + ad);
NumericQ[λ] = True;

E0[n_] = b[n] ⊙ H0 ⊙ k[n];
𝕍[n_, m_] = b[n] ⊙ V ⊙ k[m] // FullSimplify;

inf = 40;
E1[n_] = Simplify[𝕍[n, n]];
E2[n_] := ∑_{k=0}^{inf} If[k == n, 0, 𝕍[n, k] 𝕍[k, n] / (E0[n] - E0[k])] // FullSimplify

E0[4] + E1[4] + E2[4]
```

$4 + 123\,\lambda - 6498\,\lambda^2$

Trash

```
CircleDot[d] + a ⊙ b /. CircleDot[c_] → Ahhh
CircleDot[d] + CircleDot[a, b] /. CircleDot[c_] → Ahhh
```

2 Ahhh

2 Ahhh

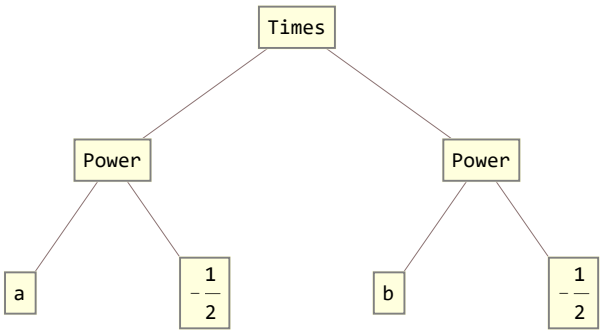
```
Sqrt[a] Sqrt[b] + 1 /. Sqrt[a_] Sqrt[b_] → Sqrt[a b]
```

```
Sqrt[a] Sqrt[b] + 1/(Sqrt[c] Sqrt[d]) /. {Sqrt[a_] Sqrt[b_] → Sqrt[a b], 1/(Sqrt[a_] Sqrt[b_]) → 1/Sqrt[a b]}
```

$$1 + \sqrt{a\,b}$$

$$\sqrt{a\,b} + \frac{1}{\sqrt{c\,d}}$$

```
1/(Sqrt[a] Sqrt[b]) // TreeForm
```




```
Denominator[1/(Sqrt[a] Sqrt[b])]
```

$$\sqrt{a}\ \sqrt{b}$$

```
(IdentityMatrix[2^14];) // AbsoluteTiming
```

```
{0.977244, Null}
```

```
(IdentityMatrix[2^14] // SparseArray;) // AbsoluteTiming
```

```
{1.29278, Null}
```

```
(Table[{i, i} → 1, {i, 2^14}];) // AbsoluteTiming
```

```
{0.0090063, Null}
```

```
(SparseArray@Table[{i, i} → 1, {i, 2^14}];) // AbsoluteTiming
```

```
{0.0161921, Null}
```

```
𝕀[16]; // AbsoluteTiming
```

$$\{9. \times 10^{-7}, \text{Null}\}$$

```
(IdentityMatrix[2^16, SparseArray];) // AbsoluteTiming
```

```
{0.0003114, Null}
```