Heisenberg xxx spin chain

Spin operators

In[135]:=
```
S[a_] = 1/2 PauliMatrix[a];
```

In[101]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2;
check := And[
   S[1].S[2] - S[2].S[1] == I S[3],
   S[2].S[3] - S[3].S[2] == I S[1],
   S[3].S[1] - S[1].S[3] == I S[2]
  ]
(* case: spin 1 *)
```

In[131]:=
```
(*ϵ[a__]:=Signature[{a}];
S[i_]:=Table[-I ϵ[i,j,k],{j,3},{k,3}]*)
```

In[115]:=
```
ϵ[1, 2, 2]
```

Out[115]= 0

In[136]:=
```
S[3] // MatrixForm
```

Out[136]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

In[137]:=
```
spin
dim
check
```

Out[137]= $\frac{1}{2}$

Out[138]= 2

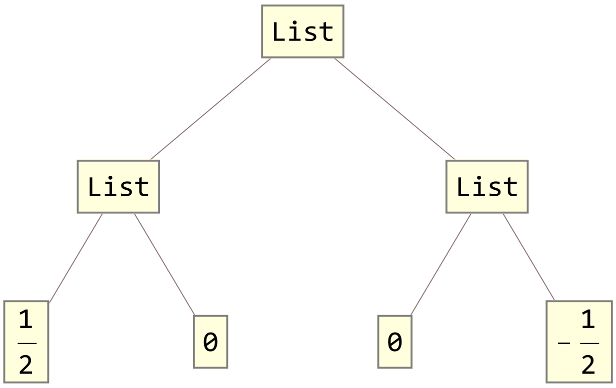
Out[139]= True

In[18]:=
```
PauliMatrix[a]/2
```

Out[18]= $\frac{\text{PauliMatrix}[a]}{2}$

In[22]:=
```
S[3] // TreeForm
```

Out[22]//TreeForm=



In[25]:=
```
S[3][[1]]
```

Out[25]= $\left\{\frac{1}{2}, 0\right\}$

In[28]:=
```
S[3]⟦1, 1⟧
```

Out[28]= $\frac{1}{2}$

Tensor products

In[37]:=
```
CircleTimes = KroneckerProduct;
```

In[29]:=
```
m1 = {{a, b}, {c, d}};
m2 = {{aa, bb}, {cc, dd}};
```

In[38]:=
```
m1 ⊗ m2 // MatrixForm
```

Out[38]//MatrixForm=

$$\begin{pmatrix} a\,aa & a\,bb & aa\,b & b\,bb \\ a\,cc & a\,dd & b\,cc & b\,dd \\ aa\,c & bb\,c & aa\,d & bb\,d \\ c\,cc & c\,dd & cc\,d & d\,dd \end{pmatrix}$$

In[31]:=
```
MatrixForm[m1]
```

Out[31]//MatrixForm=

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

In[33]:=
```
KroneckerProduct[m1, m2] // MatrixForm
```

Out[33]//MatrixForm=

$$\begin{pmatrix} a\,aa & a\,bb & aa\,b & b\,bb \\ a\,cc & a\,dd & b\,cc & b\,dd \\ aa\,c & bb\,c & aa\,d & bb\,d \\ c\,cc & c\,dd & cc\,d & d\,dd \end{pmatrix}$$

In[34]:=
```
? CircleTimes
```

CircleTimes[x] displays as $\bigotimes x$.
CircleTimes[x, y, …] displays as $x \otimes y \otimes \ldots$. >>

In[35]:=
```
CircleTimes[a, b]
```

Out[35]= $a \otimes b$

In[36]:=
```
a ⊗ b
```

Out[36]= $a \otimes b$

In[42]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[42]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

$S_n \,.\, S_{n+1} = \left(\left((S_n + S_{n+1})\ ^2 - S_n^2 - S_{n+1}^2\right)/2\right) = \frac{1}{2}(j(j+1) - 2\ \frac{1}{2}\ \ (\frac{1}{2}+1))$

In[46]:=
```
ClearAll[j];
```

In[47]:= $\frac{1}{2}\left(j\ (j+1) - 2\ \ \frac{1}{2}\left(\frac{1}{2}+1\right)\right)$ /. j → {0, 1}

Out[47]= $\left\{-\frac{3}{4}, \frac{1}{4}\right\}$

Everything else

In[51]:= $\mathbb{i}$[n_] := IdentityMatrix[dim^n]

In[57]:=
```
max = 10;
```

In[151]:=
```
ClearAll[h];
```

In[149]:= h[L_] := h[L] = $\frac{1}{L}$ Sum[$\mathbb{i}$[n] ⊗ S[a] ⊗ S[a] ⊗ $\mathbb{i}$[L - n - 2], {n, 0, L - 2}, {a, 3}]

$$h[L\_] := \frac{1}{L}\sum_{n=0}^{L-2}\sum_{a}^{3} \mathbb{i}(n) \otimes S(a) \otimes S(a) \otimes \mathbb{i}(L-n-2)$$

In[150]:=
```
h[10] // MatrixForm // N
```

Out[150]//MatrixForm=

(… 1 …)

large output | show less | show more | show all | set size limit...

Memorizer Exercise:

In[144]:=
```
f[n_] := f[n] = f[n - 1] + 1 / f[n - 2]
f[1] = f[2] = 1.2;
```

In[146]:=
```
f[5]
```

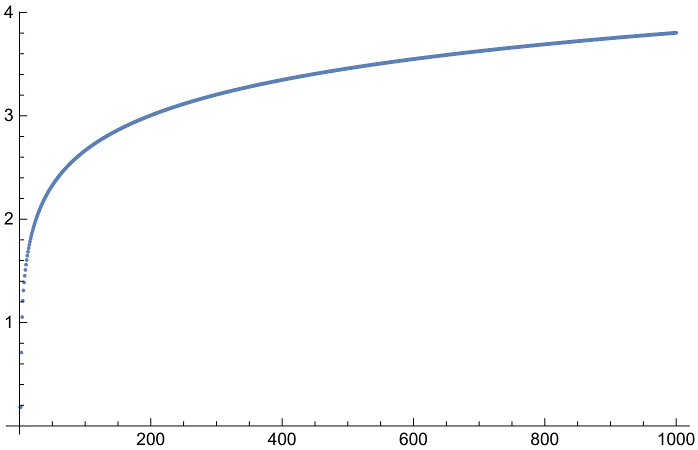
Out[146]= 3.35847

In[147]:=
```
? f
```

```
Global`f

f[1] = 1.2

f[2] = 1.2

f[3] = 2.03333

f[4] = 2.86667

f[5] = 3.35847
```

$f[n\_] := f[n] = f[n-1] + \frac{1}{f[n-2]}$

In[148]:=
```
Table[f[j] // Log, {j, 1, 1000}] // ListPlot
```

Out[148]=


In[58]:=
```
Hs = Table[h[L] // N, {L, 2, max}];
```

In[157]:=
```
Hs[[3]] // SparseArray // Eigenvalues
```

Out[157]=
```
{-0.404006, -0.239277, -0.239277, -0.239277, 0.1875, 0.1875, 0.1875, 0.1875,
 0.1875, 0.114277, 0.114277, 0.114277, -0.0625, -0.0625, -0.0625, 0.0290064}
```

In[153]:=
```
Eigenvalues[Hs[[3]], 1][[1]]
```

Out[153]=
```
-0.404006
```

In[160]:=
```
Hs2 = SparseArray /@ Hs;
```

In[161]:=
```
Hs2
```

Out[161]=
```
{SparseArray[ Specified elements: 6 Dimensions: {4, 4} ],
 SparseArray[ Specified elements: 12 Dimensions: {8, 8} ],
 SparseArray[ Specified elements: 40 Dimensions: {16, 16} ],
 SparseArray[ Specified elements: 84 Dimensions: {32, 32} ],
 SparseArray[ Specified elements: 224 Dimensions: {64, 64} ],
 SparseArray[ Specified elements: 472 Dimensions: {128, 128} ],
 SparseArray[ Specified elements: 1152 Dimensions: {256, 256} ],
 SparseArray[ Specified elements: 2420 Dimensions: {512, 512} ],
 SparseArray[ Specified elements: 5632 Dimensions: {1024, 1024} ]}
```

In[164]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) // Timing
```

Out[164]=
```
{0., {{2, -0.375}, {3, -0.333333}, {4, -0.404006}, {5, -0.385577}, {6, -0.415596},
  {7, -0.405177}, {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[165]:=
```
tb
```

Out[165]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006}, {5, -0.385577}, {6, -0.415596},
 {7, -0.405177}, {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[170]:=
```
odd = Table[tb[[j]], {j, 2, max - 1, 2(*step*)}] // Fit[#, {1, 1/x, 1/x^2, 1/x^3}, x] &;
even = Table[tb[[j]], {j, 1, max, 2(*step*)}] // Fit[#, {1, 1/x, 1/x^2, 1/x^3}, x] &;
fit = Fit[tb, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x];
```

In[168]:=
```
odd
```

Out[168]=

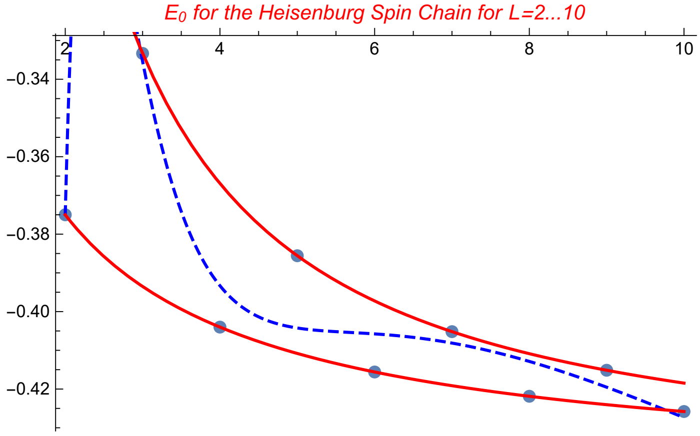
$$-0.443881 - \frac{0.311931}{x^3} + \frac{0.467293}{x^2} + \frac{0.210538}{x}$$

In[169]:=
```
even
```

Out[169]= $-0.442932+\frac{0.0615692}{x^3}-\frac{0.12553}{x^2}+\frac{0.183238}{x}$

In[180]:=
```
pl = Plot[{fit, odd, even}, {x, 2, max}, PlotStyle → {{Blue, Dashed}, {Red}, {Red}}];
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
   PlotLabel → Style[
     "E0 for the Heisenburg Spin Chain for L=2..." <> ToString[max], {Red, Italic}]];
Show[
 lp,
 pl]
```

Out[182]=



In[77]:=
```
prediction = fit /. x → ∞
```

Out[77]= −0.697758

We should split into even and odd L to fit.

In[187]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[187]= −0.443407

In[188]:=
```
-Log[2] + 1/4 // N
```

Out[188]= −0.443147

In[189]:=
```
(% - %%)/%
```

Out[189]= −0.000585792

In[194]:=
```
Rule[PointSize, 0.02]
```

Out[194]= PointSize → 0.02

In[195]:=
```
lp /. RGBColor[1, 0, 0] → Blue /. Rule[PointSize, a_] :→ Rule[PointSize, 0.1]
```

Out[195]=