STA 243 Assignment 4.5

5.

(a) Generate a random sample of size n = 100 for the ZIP model using parameters $p = 0.3$ and $\lambda = 2$.

According to the question, $X_i = R_iY_i$ where $Y_i's$ have a Poisson(λ) distribution and the $R_i's$ have a Bernoulli(p) distribution, all independent of each other.

first generate 100 $R_i \sim Bernoulli(p)$ and 100 $Y_i \sim Poisson(\lambda)$ and let $X_i = R_iY_i$

(b)

i. $(\lambda|p, r, x)$ treating p, r, x as fixed in $f(p, \lambda, r, x)$

$$
\begin{aligned}
f(\lambda|p, r, x) &\propto \lambda^{a-1}e^{-b\lambda}\prod_{i=1}^{n} e^{-\lambda r_i}\lambda^{x_i} \\
&\propto \lambda^{a+\sum_i x_i-1}e^{-(b+\sum_i r_i)\lambda}
\end{aligned}
$$

which is the the pdf of Gamma($a + \sum_i x_i, b + \sum_i r_i$) with shape and rate parameters.

ii. $(p|\lambda, r, x)$ treating λ, r, x as fixed in $f(p, \lambda, r, x)$

$$
\begin{aligned}
f(p|\lambda, r, x) &\propto \prod_{i=1}^{n} p^{r_i}(1-p)^{1-r_i} \\
&\propto p^{(\sum_i r_i+1)-1}(1-p)^{(n-\sum_i r_i+1)-1}
\end{aligned}
$$

which is the the pdf of Beta($\sum_i r_i + 1, n - \sum_i r_i + 1$).

iii.

$(r_i|\lambda, p) \sim Bernoulli(p)$ and $(x_i|r, \lambda, p) \sim Poisson(\lambda r_i)$ and etc.

r_i's are independent.x_i's are independent.

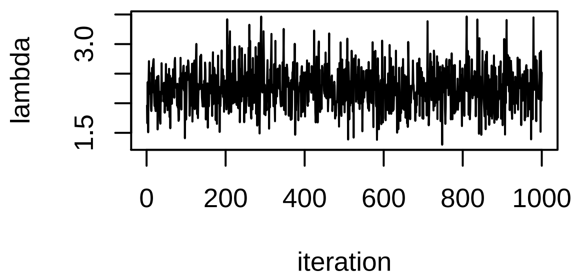
$$
\begin{aligned}
f(r_i|\lambda, p, x) &= \frac{f(r_i, x|\lambda, p)}{f(x_i|\lambda, p)} \\
&= \frac{f(r_i, x, \lambda|p)}{f(x_i|\lambda, p)f(\lambda|p)} \\
&= \frac{f(r_i, x, \lambda, p)}{f(x_i|\lambda, p)f(\lambda|p)f(p)} \\
&= \frac{e^{-\lambda r_i}(\lambda r_i)^{x_i}p^{r_i}(1-p)^{1-r_i}}{x_i!(f(x|\lambda, p)} \\
&= \frac{e^{-\lambda r_i}(\lambda r_i)^{x_i}p^{r_i}(1-p)^{1-r_i}}{x_i!(f(x|r_i = 1, \lambda, p)p + f(x|r_i = 0, \lambda, p))(1-p)}
\end{aligned}
$$

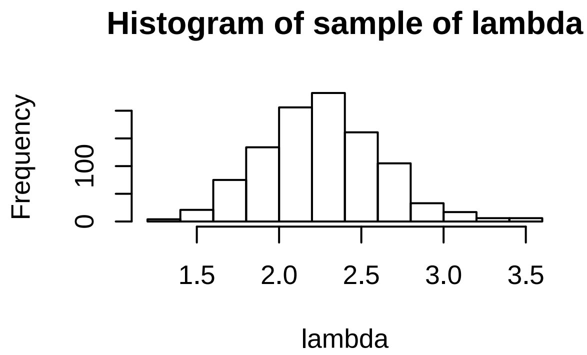
so that

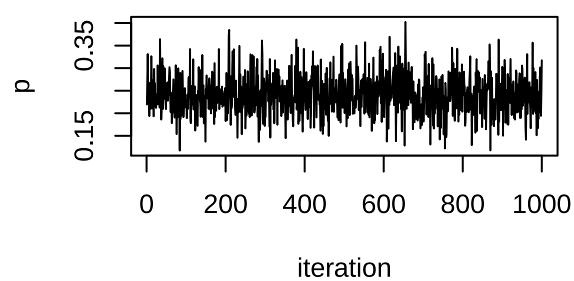
$$
\begin{aligned}
P(r_i = 1|\lambda, p, x) &= \frac{e^{-\lambda}\lambda^{x_i}p}{x_i!(\frac{(\lambda r_i)^{x_i}e^{-\lambda r_i}}{x_i!}|_{r_i=0}(1-p) + \frac{(\lambda r_i)^{x_i}e^{-\lambda r_i}}{x_i!}|_{r_i=1}p)} \\
&= \frac{e^{-\lambda}p}{(r_i)^{x_i}e^{-\lambda r_i}|_{r_i=0}(1-p) + (r_i)^{x_i}e^{-\lambda r_i}|_{r_i=1}p)} \\
&= \frac{e^{-\lambda}p}{I_{\{x_i=0\}}(1-p) + e^{-\lambda}p}
\end{aligned}
$$

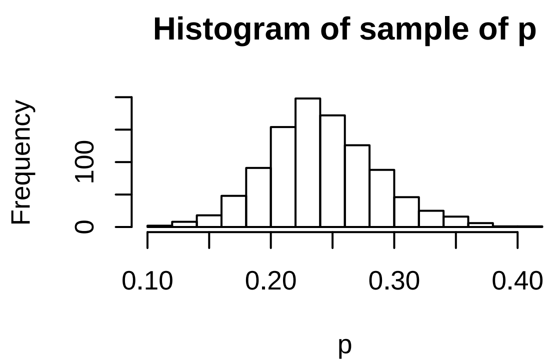
so that $(r_i|\lambda, p, x) \sim Bernoulli(\frac{e^{-\lambda}p}{I_{\{x_i=0\}}(1-p)+e^{-\lambda}p})$

(c).









bayesian confidence intervals,

for p

```
##                lower     upper
## a=1,b=1 0.1644786 0.3293876
## a=2,b=1 0.1644616 0.3293660
## a=1,b=2 0.1688772 0.3349578
## a=2,b=2 0.1654792 0.3306579
```

for λ

```
##               lower    upper
## a=1,b=1 1.712693 2.898500
## a=2,b=1 1.748036 2.944514
## a=1,b=2 1.613526 2.730673
## a=2,b=2 1.672202 2.816774
```

We can see that the true value for $p = 0.3$ and $\lambda = 2$ are included in the confidence intervals.