STA Homework 6 2.

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2.

(a) Find the distribution.

$$\begin{aligned} P(X_{\max} < x) &= \prod_{i=1}^{n} P(X_i < x) \\ &= (x/\theta)^n \\ f_{X_{\max}}(x) &= \frac{nx^{n-1}}{\theta^n} \end{aligned}$$

so that we get the pdf of $X_{\max}$

(b) Derive the analytic expression for the variance.

$$\begin{aligned} E(\hat{\theta}) &= \int_0^\theta x \frac{nx^{n-1}}{\theta^n} dx \\ &= \frac{n}{\theta^n} \times \frac{1}{n+1} x^{n+1}|_0^\theta \\ &= \frac{n}{n+1}\theta \\ E(\hat{\theta}^2) &= \int_0^\theta x^2 \frac{nx^{n-1}}{\theta^n} dx \\ &= \frac{n}{\theta^n} \times \frac{1}{n+2} x^{n+2}|_0^\theta \\ &= \frac{n}{n+2}\theta^2 \\ Var(\hat{\theta}) &= E(\hat{\theta}^2) - E(\hat{\theta})^2 \\ &= \frac{n}{n+2}\theta^2 - (\frac{n}{n+1}\theta)^2 \\ &= \frac{n\theta^2}{(n+1)^2(n+2)} \end{aligned}$$

(c) Generate a data set of size $n = 50$ and $\theta = 3$. Then generate $B = 5000$ bootstrap samples using parametric bootstrap. Use the bootstrap samples to approximate $Var_{F_\theta}(\hat{\theta})$. Compare your answer to (b).

Take the maximum of the sample as the $\hat{\theta}$, simulate bootstrap samples from unif(0,$\hat{\theta}$)

the parametric bootstrap result is

```
## [1] 0.003174573
```

the answer to (b) is

```
## [1] 0.003327123
```

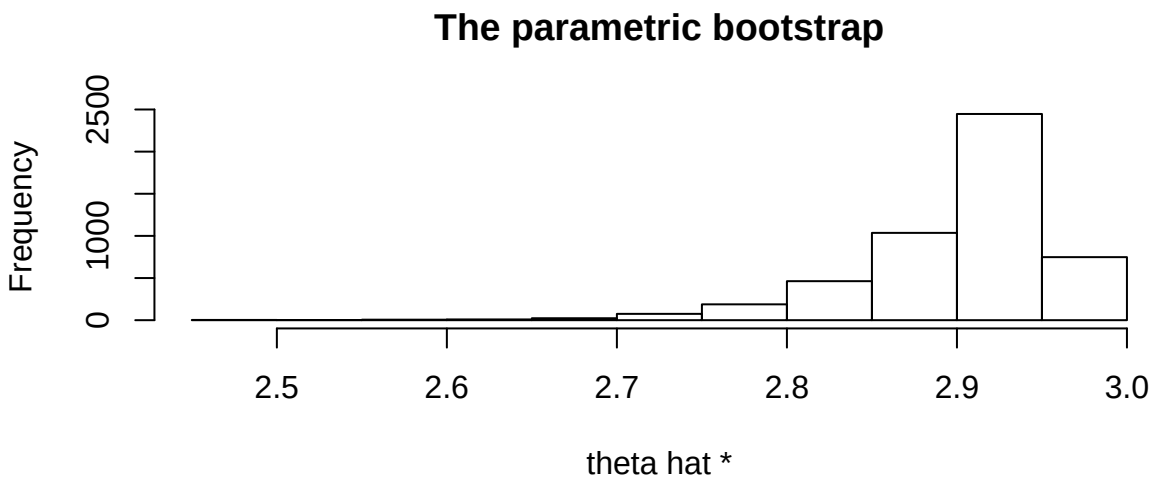
They are close.

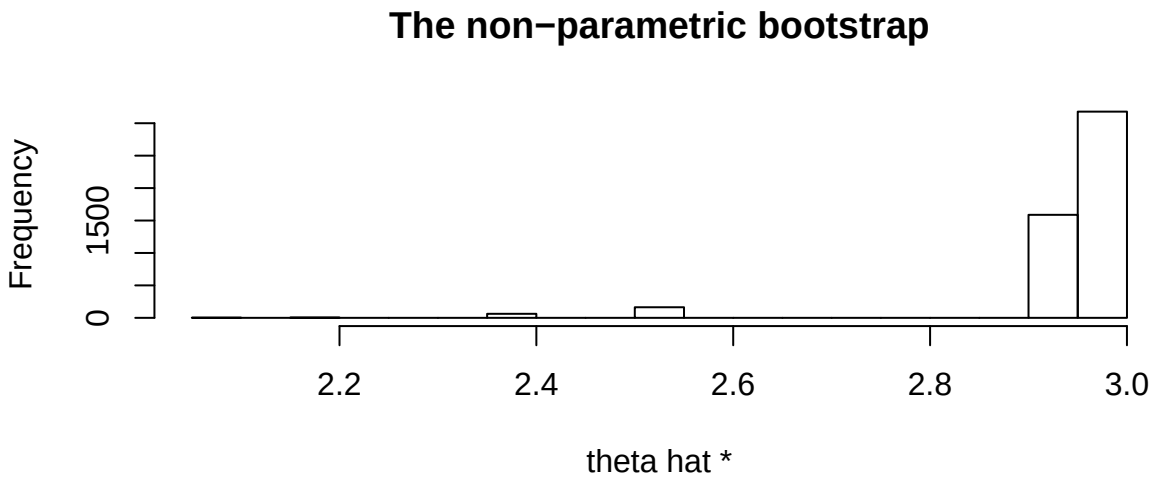
(d)

the nonparametric bootstrap samples results is

```
## [1] 0.01081648
```

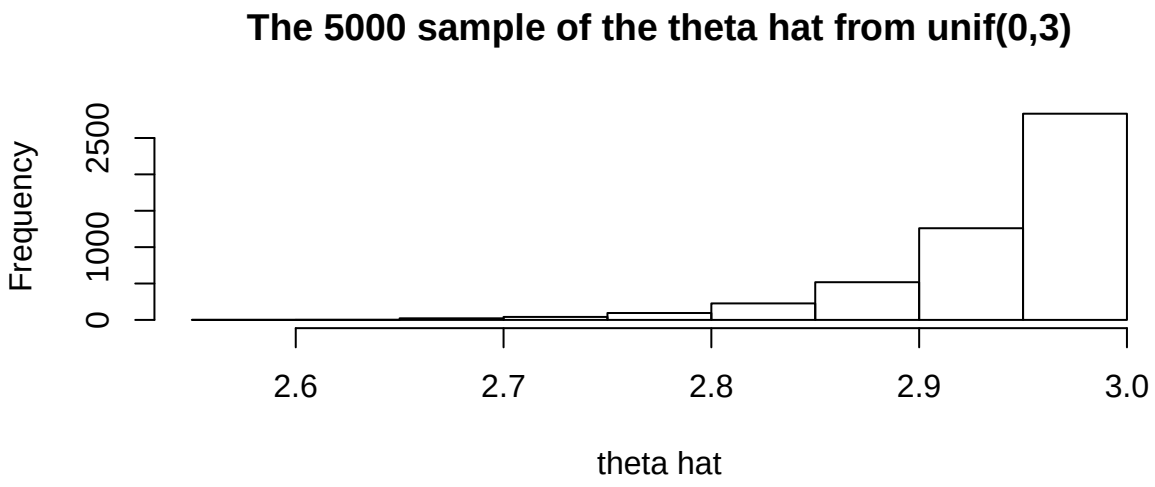
(e)





(f)

the true distribution of $\hat{\theta}$


The 5000 sample of the theta hat from unif(0,3)
Frequency
0
1000
2500
2.6
2.7
2.8
2.9
3.0
theta hat