STA Homework 6 1.

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1.

(a)

$$\rho_{Y,Z} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

Use Pearson correlation coefficient as the estimate of the correlation coefficient.

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

(b)

Estimate the standard error of the Pearson correlation coefficient.

using jackknife

$$\hat{se}_{jack} = \{\frac{n-1}{n}\sum[\hat{r}_{(i)} - \hat{r}_{(.)}]^2\}^{1/2}$$

we can get $\hat{se}_{jack}$ is

```
## [1] 0.2547034
```

using bootstrap

1. select B independent bootstrap samples
2. calculate the bootstrap replication corresponding to each bootstrap samples.
3. estimate the S.E. by the sample standard deviation of the B replicates.

we can get $\hat{se}_B$ is

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## [1] 0.1978107
```

(c)

1. using "normal theory"

$$\hat{\theta}_{(.)} \pm \Phi(0.975)\hat{se}_B$$

the C.I is

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## [1] 0.2097442 0.9851479
```

2. using bootstrap t-interval approaches.

1. generate B bootstrap samples
2. for each sample, calculate the Pearson correlation coefficient and estimated standard error (using bootstrap in the bootstrap, as in (b) to get the estimated standard error).
3. calculate $z^*(b) = \frac{\hat{\theta}(b) - \hat{\theta}}{\hat{se}(b)}$ for each b.
4. get the α percentile of $z^*(b)$ is estimated by the value $\hat{t}^{(\alpha)}$ such that $\#\{z^*(b) < \hat{t}^{(\alpha)}\}/B = \alpha$

5. the bootstrap-t confidence interval is $(\hat{\theta} - \hat{t}^{(1-\alpha)}\hat{se}, (\hat{\theta} - \hat{t}^{(\alpha)}\hat{se})$,where $\hat{se}$ is the standard deviation of $\hat{\theta}^*(b)$'s.

The C.I is

```
## [1] -1.3782126  0.9002225
```