# STA 243 Assignment 4.4

4. ANOVA

(a)

Considering a model $y_{ij} = \mu + \alpha_i + e_{ij}$

$e_{ij}$ have independent and identical double exponential distribution centered on zero with pdf

$$f(x) = \frac{1}{2\theta}\exp(-\frac{|x|}{\theta}), -\infty < x < \infty$$

where $\theta > 0$ and have variance $2\theta^2$

the hypothesis:

- the null hypothesis is $H_0 : \forall\alpha_i = 0$ and $\theta > 0$.
- the alternative hypothesis is $H_1 : \exists\alpha_i \neq 0$ and $\theta > 0$

For this model, suppose we have k groups and for the $i$th group we have $n_i$ observations,totally n observations. According to the model, $e_{ij} = y_{ij} - \frac{1}{n}\sum_{j=1}^{n_i} y_{ij} = y_{ij} - \mu - \alpha_i$. Under $H_0$ we have $e_{ij} = y_{ij} - \mu$. use the sample variance $S^2$to estimate the variance $2\theta^2$ and we can get $\hat{\theta} = \sqrt{S^2/2}$

conduct the following test:

1. generate $x_1^*, x_2^*, ..., x_n^*$ from
$$f(x) = \frac{1}{2\hat{\theta}}\exp(-\frac{|x|}{\hat{\theta}}), -\infty < x < \infty$$
and calculate the mean $\bar{x}^*$
2. repeat this for 999 times to get 999 $\bar{x}^*$'s
3. for $i$th group $(i = 1, 2, ..., k)$, calculate $T_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij} - \bar{y}$ where $\bar{y}$ is the mean of $\{y_{ij}\}$
4. if any $T_i$ is amongst the smallest 2.5% or is amongst the largest 2.5% of the $\bar{x}^*$'s, we reject the $H_0$

(b)

Using the Permutation tests.

1. calculate $\mu$ and $\alpha_i$ for the model.
2. merge all the $y_{ij}$ to form a sample of n data points.
3. draw without replacement to from k groups with the number of sample as the same as the original groups. In other words, draw $n_i$ sample for the $i$th group.
4. calculate $\mu$ and $\alpha_i$ as $\mu^*$ and $\alpha_i^*$.
5. do the drawing and calculating procedure many times.
6. if any original $\alpha_i$ is outside of the middle 95% of the $\alpha_i^*$'s, reject $H_0$