

## STA 243 Assignment 4.3

3.

(a) following the instruction and here is the result.

```
## [1] "the estimating integral value is 0.687521922445303"
```

The exact value for I is  $\ln 2 \approx 0.6931472$

```
## [1] "the sample standard error of the estimating integral value is 0.00361487622969483"
```

(b) and (c)

$$E(1+U) = 1 + E(U) = 1 + 0.5 = 1.5.$$

$$\text{Var}[c(U)] = \text{Var}(1+U) = \frac{1}{12} \text{ and } \text{Cov}[h(U), c(U)] = 1 - \frac{3\ln 2}{2}$$

then we can get the best value for b,  $b^* = \frac{\text{Cov}[h(U), c(U)]}{\text{Var}[c(U)]} \approx -0.4767$

```
## [1] "the estimating integral value is 0.693510867294138"
```

```
## [1] "the sample standard error of the estimating integral value is 0.000638837689698595"
```

The variance of the CV version is less which implies the CV version is better.

(d) Here we use  $d(x)\sqrt{x}$  instead of  $c(x) = 1+x$ .

```
## [1] "the estimating integral value is 0.693074509597988"
```

```
## [1] "the sample standard error of the estimating integral value is 0.000208229736888388"
```

the sample stanard error is even smaller than the previous one.