Figure Skating Jumps

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Introduction

Figure skating is a highly competitive sport that requires a combination of strength, endurance, and finesse. There are multiple disciplines of figure skating - singles, pairs, and ice dancing - but perhaps the most intriguing and popularized parts of skating are the jumps. We are particularly fascinated by figure skaters' jumping abilities and by our own past experiences with the sport. In this report, we investigate the mechanics of the different numbered rotations in figure skating jumps.

Background

Figure skating jumps, in simple terms, are elements for which a skater leaves the ice on one foot, rotates a number of times, and lands on the ice on one foot. There are two main types of jumps -- toe pick jumps and edge jumps. These types refer to the take-off processes a skater performs. For a toe pick jump (ie. toe loop, flip jump, lutz jump), a backwards-skating individual will plant their toe pick from an extended, reaching leg, and use that leg in a vaulting motion to leave the ice as they continue on-ice rotation. For an edge jump (ie. loop jump in particular), a backwards-skating individual will be riding the outside edge of their dominant foot's blade, and as they begin rotation for the jump, they will push off their one leg to leave the ice.

As the level and training of a skater increases, the number of rotations in jumps to be learned and perfected increase. For example, beginner to intermediate skaters will work on single jumps while the highest level skaters, like Olympians, will perform triple and quadruple jumps (note: quadruple jumps are typically only able to be performed by male skaters). Usually, the last type of jump to be learned during basic training for skaters is the axel jump. The axel jump is a special type of edge jump. For an axel (fig. 1), a skater will already have taken a step on a forward outside edge in preparation for a jump.

In this report, we will take a closer look at axels and their jumping mechanics, as axels are ideal for mechanical analysis. Axels follow normal projectile motion, whereas the other type of edge jump, the loop jump, does not follow as much projectile motion due to a different take-off, where height is more important. Additionally, axels are easier to analyze than toe pick jumps in terms of the energy that goes into the jump, as it would be difficult to see how much the vaulting leg contributes to the overall jump. Axels are also easier to perform off-ice, which is how we recorded and measured the mechanics of figure skating jump techniques in this report.

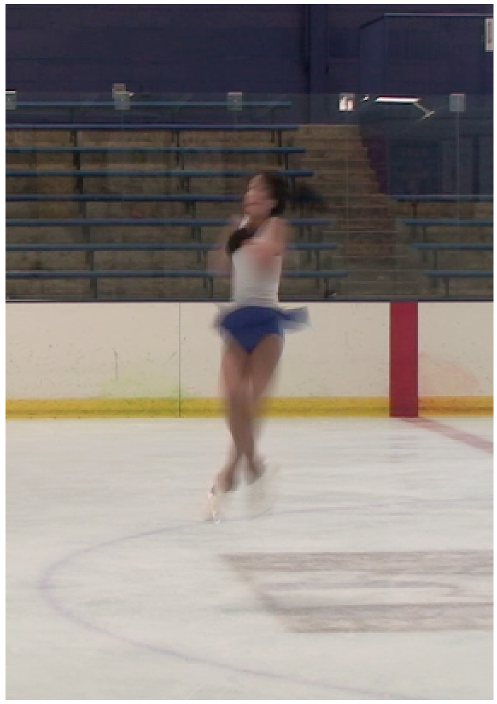

Figure 1: Kaitlyn performing a double axel on ice

Objective

A figure skating jump can vary in the take-off method and the number of rotations turned in the air. Our objective is to explore the mechanics behind how a figure skater includes rotations into his/her jump. We want to see if we can observe any trends between other jump parameters and the number of rotations of the jump. For example, theoretically, we know that to increase the number of rotations in a figure skating jump, a skater "must either jump higher or rotate faster or do both"[1]. Similarly, another paper states that the skater "must increase time in the air and/or rotate faster" to accommodate for more rotations [2]. These statements alone begs questions like "is it more one or the other, or is it equally both factors changing?" This "and/or" and "or do both" is ambiguous and is the focus of our study -- we would like to observe how much each jump parameter changes (or does not change) as the number of rotations change, which may give insight beyond the theoretical mechanics behind jumps.

Past Studies

Two studies guided the analysis of this study -- one by Johnson & King [1] and one by King, Arnold & Smith [2]. Johnson & King studied professional figure skaters at the National Championships in 2000. They analyzed triple and quadruple toe loops and salchows, with data acquired from skaters during competition and practice. King, Arnold & Smith studied elite skaters, analyzing single, double and triple axels. Their analysis included analysis of the skids, which is a mark left on the ice due to too much rotation having started before takeoff.

Both studies suggest early on that mechanically, we expect higher jump heights, longer hang times, and/or faster angular velocities with respect to more revolutions. Although we do not readily know of a formula to verify this (as it is the ultimate direction of our study - to find trends to a point where we could assemble a formula), these suggestions from past studies qualitatively make sense. More rotations in the air can only be accomplished if there was more time in the air (which correlates to jump height, see equation 1), and/or the skater rotates faster to complete the rotations within the given time frame.

$$t_h=2\ \mathrm{sqrt}(2gH)/g \qquad (1)$$

Methodology: Original Plan

Our original proposal for this experiment was to use Kaitlyn's videos (fig. 1) from her past figure skating career to manually analyze using Tracker software and compare the mechanics to vertical two-feet jumping videos. Instead of following this plan, we collected new data through recording new videos of axels from the ground. For comparison of off-ice axels, we took at least three videos of three types of jumps - one legged (0 rotations), waltz jump (0.5 rotation), and single axel (1.5 rotations). We altered our plans in order to gather new and better raw material to work with. The videos from Kaitlyn's past had a moving frame, which increased the difficulties and inaccuracy of manual tracking. In order to reach our goal of comparing similar jumps with different numbers of rotations, we changed the two-legged vertical jump to a one-legged jump that follows some sort of projectile motion. Additionally, bringing a camera to a skating rink was out of reach in our timeline, as this required us to work around the rink and camera schedule, as well as the possibility of recording in a less-controllable environment (ie. people, ice conditions, ability to adequately set up).

Methodology: Data Acquisition

We acquired our data by recording videos of Kaitlyn performing three different types of jumps on the ground in an empty classroom with a Casio High Speed X1-F1 camera and a tripod. The three jump types were one-legged, waltz, and single axel and each type of jump was recorded at least three times at 306 frames per second.The axel was recorded five times since there were errors and recording mistakes. Since we ended up with more videos of the single axel, we decided to include these videos in our analysis. A yardstick was placed in the background of the video recordings to act as a reference length and provide calibration of the tracking. The Tracker was used to manually track the Kaitlyn's center of mass as she performed the different types of jumps (fig. 2).

Methods for Analysis

After collecting our videos for data, we uploaded them into the Tracker software. In order to standardize some of the manual tracking of Kaitlyn's movement, the origin was set at the point when the countermovement begins for each jump. This set origin point can be seen as marked by the intersection of the pink lines in Fig. 2. From this origin, we tracked the position from the beginning of the countermovement through the depth of the countermovement, through the take-off, the highest point of the jump, and landing. Using this process, we were able to acquire the following parameters: time elapsed, position (x, y), velocity (horizontal x-direction, vertical y-direction, and composite), and angle of take-off. Other parameters that we calculated for can be found in Appendix B. The basic jumping parameters that we were interested in included contact and hang time, countermovement, jump height, force-to-weight (N/W) ratio, and vertical velocity. The long jump parameters that we were interested in were horizontal velocity and composite velocity, take-off angle, and distance. Additional metrics that we calculated for included angular velocity, and translational and rotational kinetic energy. During our data analysis and calculations, we included data from "axel3," despite it having a failed landing.

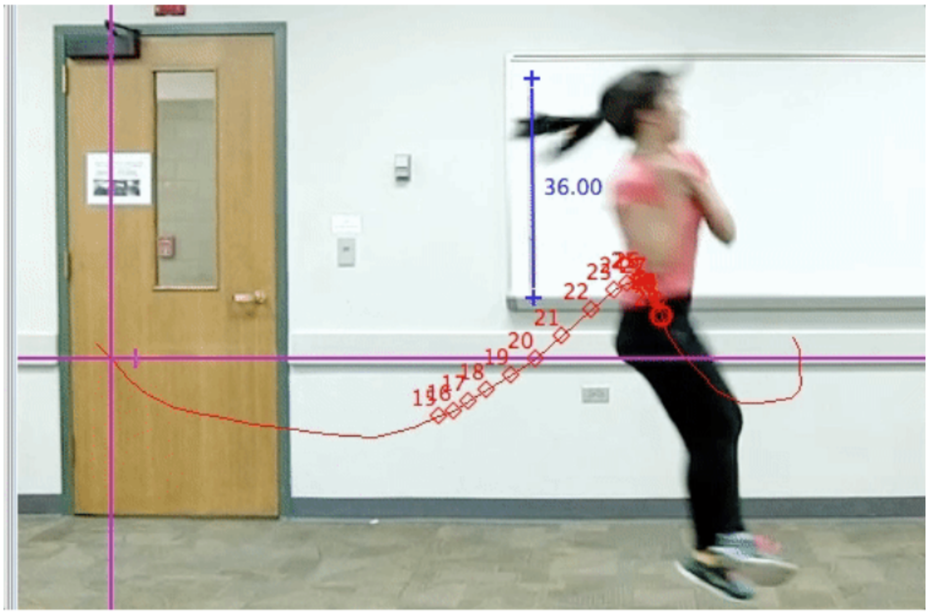

Figure 2: One of the axel trials loaded in tracker

Discussion of Results

The results of past studies suggested trends in some parameters such as in the horizontal frame (i.e x-direction velocities, distance, etc). Other parameters, however, either did not exhibit trends or were not explored in previous studies (ex. countermovement). Some of our findings matched while others did not, as seen in each parameter's results summarized in the following sections.

Contact Time & Hang Time

According to results from previous studies, there is a similar hang time in different axels [2] and loops [1] but an increased hang time in increased revolutions of salchows [1]. Our results show similar, constant contact times and hang times across the board of different numbered rotations, both among measured and calculated values (Table A-1). Mechanically, it is unclear how reasonable this may be without looking at other parameters -- a similar hang time may imply faster angular velocities in order to complete the target revolutions successfully before landing, which is possible (and also suggested, as mentioned earlier). If jump height gets taller (as we theoretically might expect), then the increased hang time of the salchows also make sense.

When we look at the variation in contact and hang time, we acknowledge that the frame rates may have affected the validity of our results. Due to the low frame rate, we could not locate some ideal frames such as landing and takeoff and instead had to find a frame that was directly before or after the event.

Countermovement Depth & Jump Height

Consistent with contact time & hang time, previous studies found similar jump heights for axel [2] and loops [1] but an increased jump height for salchows [1]. Previous studies did not explore countermovement parameters. Our data suggests similar jump heights between the 0 rotation (one-legged) and 0.5 rotation (waltz) jumps but a decrease from 0.5 to 1 rotation (axel) jumps (see Table A-2). Countermovement also did not change between waltz and axel, but did exhibit a decrease from one-legged to waltz. We also note at this point that the one-legged jump might also not be the best jump to look at when we isolate parameters such as countermovement depth and/or jump height against other jumps, because the skater isn’t trying to optimize the jump to complete any revolutions. However, looking at them in conjunction, we would expect that deeper countermovements would result in higher jump heights (eqn. 2), which we do not see in our results.

$$t_c=2s/\mathrm{sqrt}(2gH) \qquad (2)$$

Force to Weight (N/W) Ratio

Previous studies did not look into force-to-weight (N/W) ratio. Our study suggests an increase in N/W ratio from one-legged jump to waltz jump, but no change between waltz to axel (see Table A-3). Given that we calculated the N/W ratio from jump height and countermovement depth (eqn. 3), it is no surprise that this trend is consistent with our findings for countermovement depth decrease earlier. Generally speaking though, this implies that if an increase in jump height does occur with more rotations (as seen in the study with salchows [1]), then the N/W ratio would also increase.

$$N/W = (H/s)+1 \qquad (3)$$

Takeoff Velocity & Angle

To analyze takeoff, we looked at composite velocity in addition to separating it into the x and y components. We also looked at takeoff angle. In past studies, the vertical velocity either stayed the same or increased as rotations increased [1,2]. The horizontal velocities, however, all decreased in both previous studies [1,2]. The take off angle was also noted to grow steeper with increased rotations [2].

In our study, we found that the y-direction velocity did not suggest any trends such as in the previous studies referenced (Table A-4). This finding is also consistent with our finding of little to no change in jump height, especially between one-legged and waltz jumps (eqn. 4). However, we found that the horizontal and composite velocities increased as revolutions increased, contrary to previous studies (Table A-5). This suggests a greater change in horizontal parameters than vertical parameters when the number of rotations are increased. Additionally, our takeoff angle did not see any significant trends, also contrary to previous studies (Table A-6).

$$v_{TO}=\mathrm{sqrt}(2gH) \qquad (4)$$

We notice the difference between our study and the previous studies for horizontal velocity as one of the biggest discrepancies in our studies. We believe that our results are mechanically valid, as intuitively a skater will require more kinetic energy to complete her rotations -- an increased horizontal velocity can accomplish this. The decrease in horizontal velocity of previous studies may be due to the fact that rotations already start on the ground at takeoff, and thus some of the KE may have already been transferred to the rotational KE, leading to a slower translational horizontal velocity. Since our study was performed on the ground while past studies were taken off the ice, there are different frictional factors playing a role in the study and the point in the jump when rotations begin may be varied and explain our different results.

With respect to the difference in takeoff angle results, we believe that our study involving one-legged jumps and waltz jumps may not be the best to compare one-to-one with previous studies. The difference of methods in which the skater takes off for one-legged jumps and waltz jumps may be significantly different than the differences of methods of different rotation axels.

Distance Traveled

Our study of distance traveled seeks to juxtapose long jump parameters with a figure skating jump. The calculations for distance traveled uses projectile motion principles (eqn. 5) and knowingly do not include the effects of rotations in its mechanics. Our goal with using this equation is to compare our actual data from calculated projectile motion distances to make qualitative conclusions of how rotations may affect a jump.

$$x = v_o^2 \sin(2\theta)/g \qquad (5)$$

The distances calculated for the jumps (Table A-7) suggest varied implications when compared to the actual distances traveled (fig. 3, between distances ~30-70 where trajectory reaches a height 0), depending on the type of jump. For one-legged jumps, the actual distances are a slightly (< 10 inches) less than calculated. This can only be attributed to unaccounted for factors such as air drag, because the one-legged jump in nature should be the closest to projectile motion, since it has no rotations. For waltz jumps, the actual distance is about the same as calculated (which is surprising because it has half a rotation, compared to projectile motion). For the single axel, however, the actual distance was at least 20 inches less than calculated. The conclusions we made from these results are discussed in a later section.

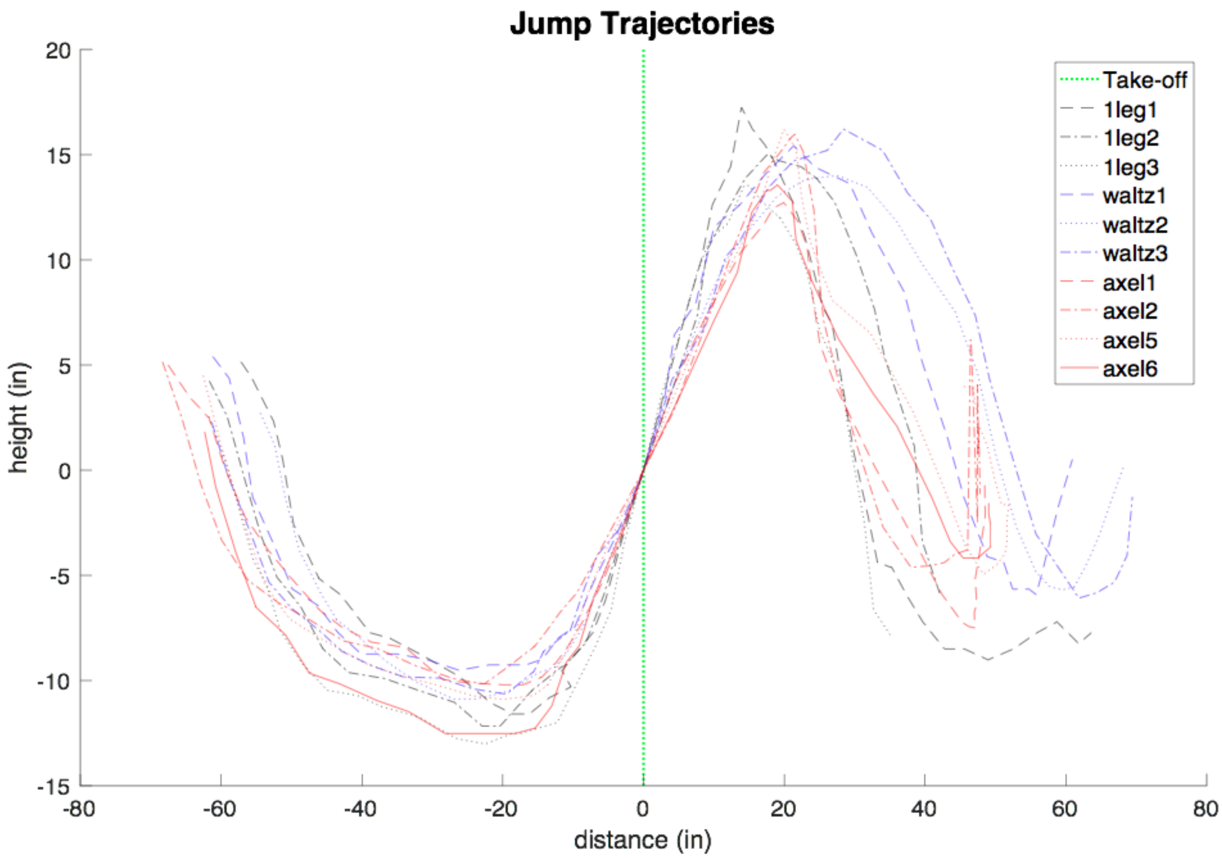

Figure 3: Jump trajectories of the center of mass for all trials, with green line denoting moment of takeoff, calibrated to be the location of (0 height, 0 distance).

Angular Velocity & Kinetic Energy

It became apparent that angular velocity was a significant parameter that needed to be quantified for further analysis. For example, to gain a sense of the energies present throughout different points of the jump, angular velocity is needed. Our estimated angular velocities displayed (to no surprise) an increase as the number of rotations in them jump increased (Table A-8). Using these estimated values, kinetic energies were calculated (Table A-8) (all values divided by mass). We were hoping the results may show more clear insights with how the energy is transferred to rotational, but the results implied that very little kinetic energy was rotational. With only estimates of angular velocities to use, there is not much we can say from our kinetic energy analysis, but it does give a blueprint for how further studies could be approached when paired with better data acquisition.

Further Discussion

Overall, the results showed no heavy trends with respect to one factor through the three types of jumps. However, during the analysis process, the possibility came up that comparing the one-legged jump to the waltz and single axel may not be the most conducive to getting relevant results with regards to the mechanics of the added rotation in the figure skating jumps.

In the vertical direction, the reasons for the slight decrease in jump height seems unclear, but the decrease is small such that we might need more trials to verify this trend. Later on, we also see similar (not decreased) vertical takeoff velocities, which also suggest that there may be some kind of energy transfer to complete the rotations, therefore resulting in a shorter jump height. However, we believe that the energy transfer occurring to accommodate for the rotations are heavily skewed towards only the horizontal direction, as the trends in the vertical direction in previous studies also suggested little to no change in the axel and loop jumps.

Our hypothesis is supported by the results we found about horizontal velocities as well as distance traveled. We observed that both horizontal velocity and distance parameters decrease, which may be explained by the fact that energy was transferred from horizontal velocity to rotational energy and thus resulted in a decreased distance traveled. Although the point at which angular velocity begins to be non-zero and the actual angular velocity can only be estimated in our analysis, we think qualitatively that the effects of different rotations in a figure skating jump can be seen more predominantly in the horizontal direction than the vertical.

Potential Sources of Error

Besides the potential sources of error mentioned in the individual discussion sections above, we observed three more overall sources of error. Our ability to manually track one location on Kaitlyn's body as she jumped was hindered by her rotation in the air since one location is not visible at all times. Additionally, we believe that after analysis, comparing the one-legged jump to the other two jumps might not be ideal, especially since the one-legged jump does not aim to maximize height or distance, whereas, for example, a skater may aim to maximize height for an axel so that the rotations can be fully completed at the time of landing. Another source of error is the frame rate. A higher frame rate will offer more accurate results.

Possible Future Work

From our experience with our study, we have identified many ways in which our study could be improved in the future to get potentially better results, both more accurate and/or more insightful.

Improvements in Execution

The following are methods we suggest and would like to make to collect better data in the future.

> *Colored belt* - We suggest that the subject performing the jumps in the videos wear a brightly colored belt. Manually tracking the same location on Kaitlyn as she rotated was difficult since a reference point easily seen in one frame is not easily seen in another (ie. if her hips were facing the camera vs if her hips were facing away from the camera). This increased the subjectivity with which the tracker points were chosen and therefore increased the error involved.
>
> *Shooting on Ice* - Another improvement would be to record individuals doing figure skating jumps while on ice. This would require a stationary frame and many controls to standardize each jump (ie. speed before take-off), but this would give us a more realistic analysis of figure skating jumps rather than doing analysis of off-ice jumping techniques.
>
> *Frame Rate* - A final improvement to the execution of our project would be to use a higher frame rate for recording videos. Having a higher frame rate would give us a better ability to locate exact moments, such as take-off and velocity (for better ability to locate moments such as takeoff/velocity), so that parameters would be more accurate and there would be less error.

Improvements in Methods

The following are suggestions for other analysis and experimental setups that could be done to gain different and potentially better insight on figure skating jumps.

Isolating the Vertical: Loop Jumps - In these analyses of axel jumps, Kaitlyn had to take a hop before beginning her axel take-off. An idea that we had to improvement the overall experiment would be to instead record and analyze vertical jumps with rotation that would emulate loop jumps. Therefore, there would be less variation in take-offs between jumps and the effect of rotation on jumps would be seen without the effect of projectile motion.

Measuring Angular Velocity - Another improvement in methods would be to place a camera to record from overhead so that the angular velocity can be accurately measured. In this project, we were only able to estimate this parameter by timing how long a half rotation took.

Suggestions for Analysis

The following are suggestions of deeper analysis that could be done to our current setup, and/or improved setups.

Investigating Work Done- With our basic kinetic energy analysis, we calculated work (divided by mass, Table A-8) and the next step would be to possibly explore the factors that go into work. We know that total work is related to force (which perhaps can be drawn from the N/W ratio), distance, torque, and angular displacement (which defines each jump). Further investigation of these factors contributing to total work might give better insight to the mechanics behind the rotations of figure skating jumps.

Investigating Rotational Kinetic Energy Through the Jump - Another observation from our kinetic energy analysis and analysis as a whole was that rotations do not begin at the point of takeoff, but may begin beforehand. If angular velocities could be more accurately measured, it might be insightful to take a look into the kinetic energies (specifically rotational) throughout the trajectory of the jump, and not just at the start of countermovement/takeoff.

Conclusions

In this project, we observed how the horizontal parameters decrease more than the vertical ones when the number of rotations increase. This is shown in the decrease in horizontal velocity and distance in our results. We believe this is due to energy transferred from horizontal velocity to rotational energy in order to accommodate the energy needed to complete rotations. While this data may have a lot of errors in estimation, we believe that the effects of different numbers of rotations in a figure skating jump can be seen more in the horizontal direction than the vertical. The fact that we performed our study on the ground rather than on ice, coupled with using one-legged jumps and waltz jumps rather than double and triple axels with single axels may have affected our results. Further studies are needed to give more support to our findings.

References

Johnson, M., & King, D. L. (2001). A kinematic analysis between triple and quadruple revolution figure skating jump. Knee, 40, 50.

King, D.L., Arnold, A.S., and Smith, S.L. (1994). A kinematic comparison of single, double, and triple axels. J. Appl. Biomech. 10: 51-60.

Appendix A: Data Tables for Selected Parameters

Table A-1: Contact time and hang time, both measured and calculated for all trials

Jump Name	measured contact time (s)	calculated contact time (s)	measured hang time (s)	calculated hang time (s)
1leg1	0.300	0.201	0.467	0.598
1leg2	0.267	0.212	0.467	0.552
1leg3	0.300	0.265	0.467	0.529
waltz1	0.267	0.198	0.434	0.565
waltz2	0.267	0.185	0.501	0.535
waltz3	0.234	0.186	0.501	0.580
axel1	0.267	0.163	0.467	0.508
axel2	0.300	0.250	0.400	0.524
axel3	*0.234*	*0.165*	*0.467*	*0.624*
axel5	0.200	0.151	0.467	0.580
axel6	0.267	0.229	0.467	0.530

Table A-2: Countermovement depth and jump height, measured for all trials

Jump Name	depth of CM (in)	avg depth of CM (in)	jump height (in)	avg jump height (in)
1leg1	11.590	12.145	16.741	14.993
1leg2	11.317		14.712	
1leg3	13.526		13.526	
waltz1	10.796	10.250	15.422	15.154
waltz2	9.565		13.820	
waltz3	10.390		16.219	
axel1	7.994	*10.152 or* 10.212 (w/o axel3)	12.449	*14.848 or* 13.865 (w/o axel3)
axel2	12.663		13.240	
axel3	*9.912*		*18.781*	
axel5	8.452		16.207	
axel6	11.738		13.564	

Table A-3: Force-to-weight ratio, calculated for all trials

Jump Name	N/W ratio	Average N/W ratio
1leg1	2.489	2.263
1leg2	2.300	
1leg3	2.000	
waltz1	2.429	2.478
waltz2	2.445	
waltz3	2.561	
axel1	2.557	*2.514 or* 2.419 (w/o axel3)
axel2	2.046	
axel3	*2.895*	
axel5	2.918	
axel6	2.156	

Table A-4:Takeoff y-direction velocity, both measured and calculated for all trials

Jump Name	measured take off velocity Y (in/s)	calculated take off velocity Y (in/s)
1leg1	108.066	115.434
1leg2	97.514	106.586
1leg3	107.084	102.200
waltz1	100.145	109.127
waltz2	103.547	103.303
waltz3	102.533	111.910
axel1	106.315	98.046
axel2	97.500	101.112
axel3	*101.627*	*120.424*
axel5	102.086	111.870
axel6	97.718	102.341

Table A-5:Takeoff x-direction velocity and composite velocity, measured for all trials

Jump Name	measured x TO-velocity (in/s)	measured composite take off velocity (in/s)
1leg1	92.628	142.331
1leg2	101.754	140.935
1leg3	80.313	133.855
waltz1	103.996	144.375
waltz2	114.113	154.090
waltz3	102.533	145.003
axel1	109.324	152.495
axel2	125.844	159.195
axel3	*82.083*	*130.636*
axel5	120.602	158.007
axel6	125.079	158.725

Table A-6:Takeoff angle, measured for all trials

Jump Name	Initial Takeoff angle (deg)	avg takeoff angle (deg)
1leg1	49.4	48.767
1leg2	43.8	
1leg3	53.1	
waltz1	43.9	42.700
waltz2	39.2	
waltz3	45	
axel1	52.6	*44.154 or* 42.41690532 (w/o axel3)
axel2	37.768	
axel3	*51.1*	
axel5	41.3	
axel6	38	

Table A-7:distance traveled, calculated for all trials using projectile motion equations

Jump Name	calc distance traveled (in) (projectile motion)	average calc distance (in) (projectile motion)
1leg1	51.85	49.27
1leg2	51.40	
1leg3	44.56	
waltz1	53.95	56.22
waltz2	60.24	
waltz3	54.46	
axel1	58.12	*58.47 or* 62.28 (w/o axel3)
axel2	63.56	
axel3	*43.20*	
axel5	64.13	
axel6	63.31	

Table A-8:angular velocity (estimated) and kinetic energies (calculated) and work (calculated) for all trials

Jump Name	est. angular velocity (rev/s)	KE at start of cm (divided by mass)	total KE at takeoff (divided by mass)	trans. KE at takeoff (divided by mass)	Work (difference of KE)
1leg1	0.000	3589.844	10129.02073	10129.021	6539.176618
1leg2	0.000	6192.499	9931.366716	9931.367	3738.867469
1leg3	0.000	3290.897	8958.552802	8958.553	5667.655854
waltz1	1.087	4843.866	10493.57931	10422.100	5649.71357
waltz2	1.000	4240.125	11932.42501	11871.925	7692.300495
waltz3	1.000	6669.756	10573.47741	10512.977	3903.721508
axel1	2.381	6101.132	11970.33021	11627.360	5869.198013
axel2	2.500	7165.834	13049.58583	12671.461	5883.751687

axel3	*2.381*	*8571.469*	*8875.801828*	*8532.831*	304.3327648
axel5	2.500	9157.241	12861.24863	12483.124	3704.007559
axel6	2.632	5904.994	13015.78691	12596.812	7110.792638

Appendix B: Summary of Equations

Variables Defined

h_0= starting hip height

h_c= countermovement height

h_j= height of jump

s = depth of countermovement

H = jump height

v_x = horizontal velocity

x = horizontal distance

v_{comp} = composite velocity

θ = initial take off angle

N = normal force

W = weight of jumper

v_{TO}= takeoff velocity

t_c= contact time

t_h= hang time

v_y = vertical velocity

KE = kinetic energy

I = rotational inertia

ω= angular velocity

Equations Used

$s = h_0 - h_c$

$H = h_j - h_0$

$N/W = (H/s)+1$

$x = v_o^2 \sin(2\theta)/g$

$KE_{rot}/m = 0.5r^2\omega$

$t_c=2s/\mathrm{sqrt}(2gH)$

$t_h=2v_{TO}/g$

$v_{comp} = \mathrm{sqrt}(v_y^2+v_x^2)$

$v_{TO}=\mathrm{sqrt}(2gH)$

$KE_{trans}/m = 0.5v_{comp}^2$