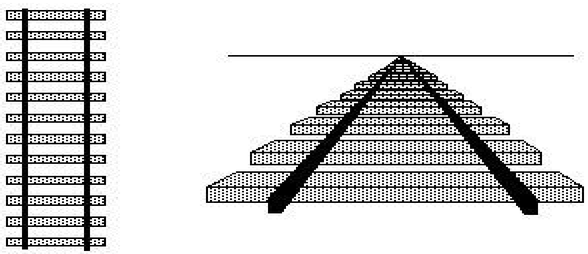
Projections

Online illustrative demos

http://threejs.org/examples/#webgl_camera

- Perspective vs orthographic - the difference between the two projection systems (and what they see) -- press O and P to switch between the two.
- Clipping planes

Perspective transforms



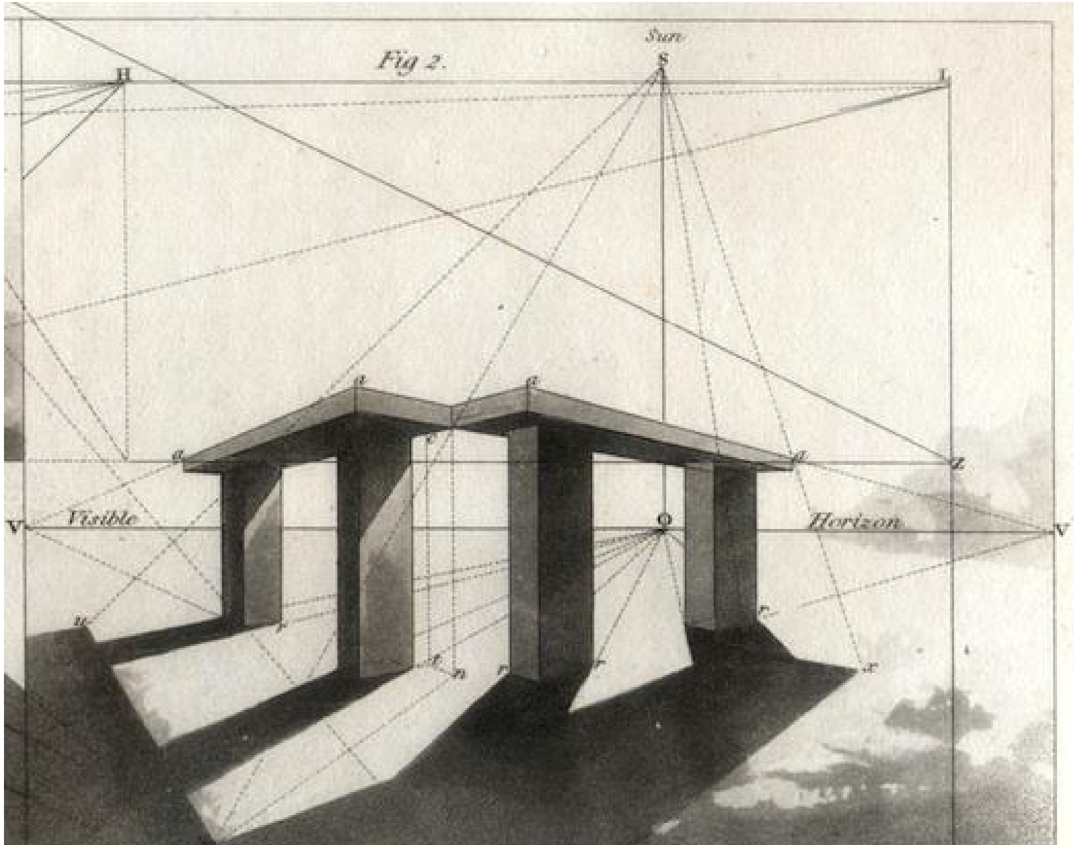
What happens to parallel lines during the transform?

What happens to ratios along straight lines?

Shadow Mapping with Projections

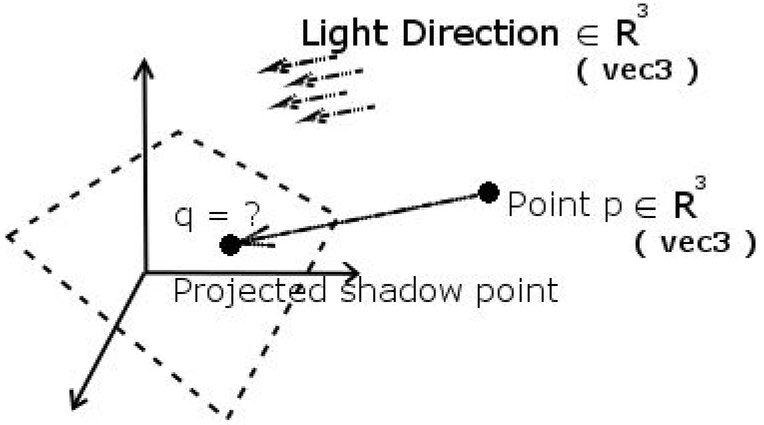
Render the scene using the light source as the camera.

Test any point by checking if it got obstructed from this vantage point.   If so, it's in shadow.



Projection of shadow point onto plane

- Problem from book
  - Our exam question on projection plane requires much simpler geometry



Projection of shadow point onto plane demo

- 2D version - project onto line
  - becomes 2D line-2D line intersection
- In 3D:
  - Becomes 3D line - 3D plane intersection
  - How to express 3D line? More constraints than plane; parametric is the simplest way to express that many constraints

How do we intersect two lines?

- We could:
  - Express the linear system as a matrix, and solve
  - (Equivalently) Plug one explicit line equation into the other in place of y
  - (Works in 3D too) Convert one line to parametric form, find parameter value that satisfies other line too

Forms of a line and their grade-school equivalents:

Explicit:

<u>The Slope-Intercept Form of the Equation of a Line</u>

The equation of a line with slope m and y-intercept (0, b) is given by,

$y = mx + b$

where m is the slope and (0, b) is the y-intercept

Implicit:

<u>The Standard Form of the Equation of a Line</u>

$Ax + By = C$

where A, B, and C are real numbers

Parametric (after some derivation)

<u>The Point-Slope Form of the Equation of a Line</u>

The equation of a line with slope m and passing through the point (x_1, y_1) is given by,

$y - y_1 = m (x - x_1)$

where m is the slope and (x_1, y_1) is the point given

Derivation of parametric form

If the two point form is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

We can write it as

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Since they are equal, we set them both equal to t:

$$y - y_1 = (y_2 - y_1)t \Rightarrow y = y_1 + (y_2 - y_1)t$$
$$x - x_1 = (x_2 - x_1)t \Rightarrow x = x_1 + (x_2 - x_1)t$$

This is then the parametric form.

Next slide: Solving for the projected point, q, using vector algebra

(Solve for t first, then look where the result is plugged into q’s equation)

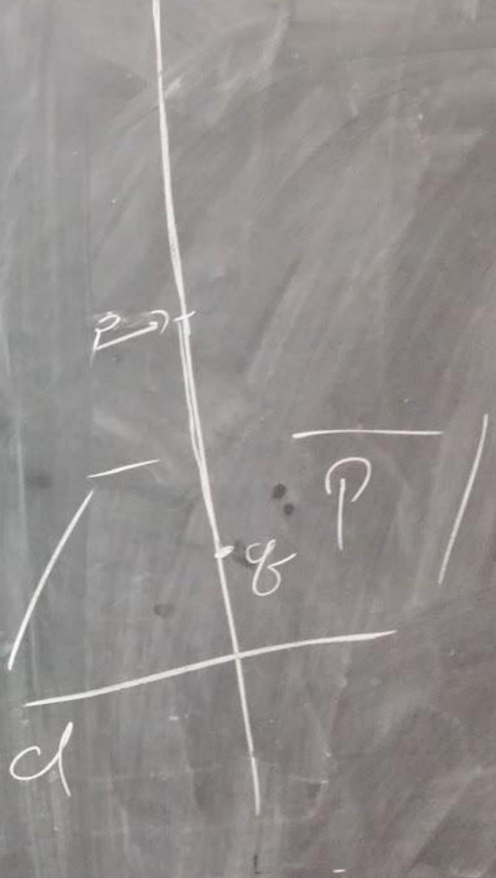
$v \cdot \bar{x} = d$ Plane P

$tu + w$

Project p onto P along

q’s equation:

$tu + p$ $\quad q = \frac{d - \langle p, v \rangle}{\langle u, v \rangle} \cdot u + p$



$v \cdot (tu_1 + p_1, tu_2 + p_2, tu_3 + p_3) = d$

$(tu_1 + p_1)v_1 + (tu_2 + p_2)v_2 + (tu_3 + p_3)v_3 = d$

$t(u_1 v_1 + u_2 v_2 + u_3 v_3) = d - p_1 v_1 - p_2 v_2 - p_3 v_3$

$t = \frac{d - \langle p, v \rangle}{\langle u, v \rangle}$