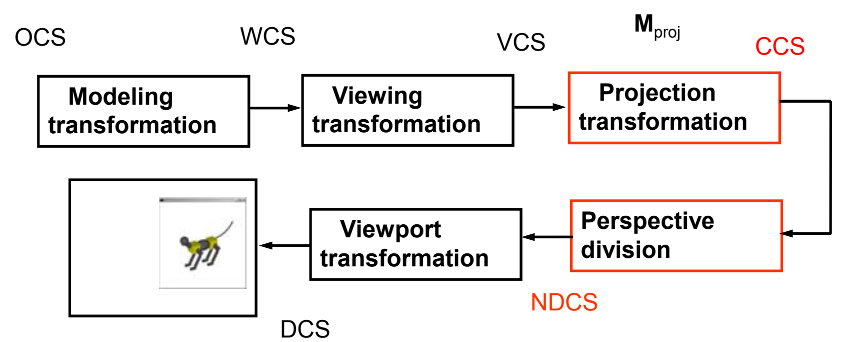
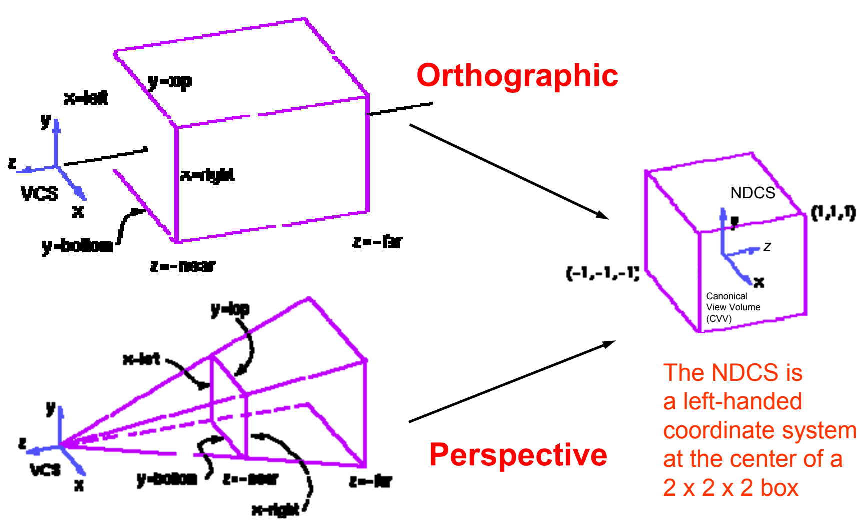
Graphics Pipeline



View Volumes

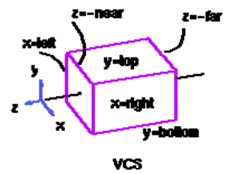
We display only the geometry in a view volume

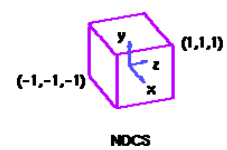


Derivation of the Orthographic Projection Matrix

Another coordinate system transformation

- Scale 2x2x2 cube to the rectangular cuboid and flip z then translate appropriately





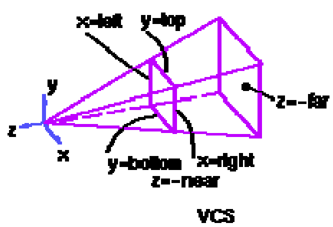
left: $x = l$ right: $x = r$
bottom: $y = b$ top: $y = t$
near: $z = -n$ far: $z = -f$

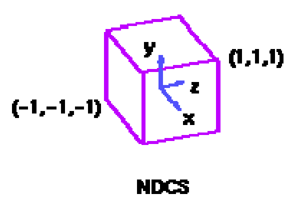
$$\mathbf{M}_O = \underbrace{\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & -\frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Scaling}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & +\frac{f+n}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Translation}} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling Translation

(note negation to flip z axis)

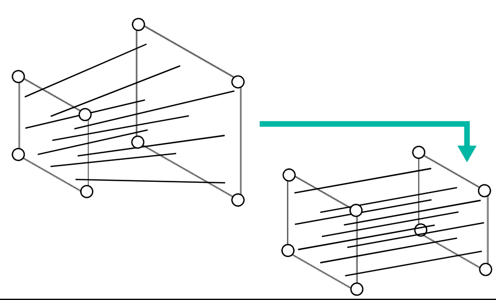
Derivation of the Perspective Transformation Matrix





Maps any line through the origin (eye) to a line parallel to the z axis

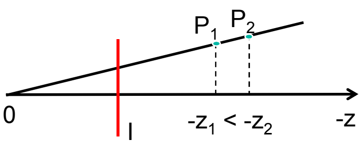
- *Without moving the point on the line at $z = -n$*
- *Leaves points on the $z = -f$ plane, while "squishing" them in x and y*

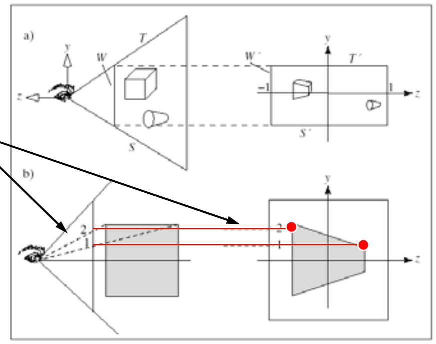


Derivation of the Perspective Transformation Matrix

This transformation warps the view volume and the objects in it

- Eye becomes a point at infinity, and the projection rays become **parallel lines** (i.e., orthographic projection)
- We also want to keep z
  - *Pseudodepth*





The Perspective Transformation Matrix

$$\mathbf{M}_P \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} nP_x \\ nP_y \\ P_z(n+f)+nf \\ -P_z \end{bmatrix} \xrightarrow[\div(h=-P_z)]{\text{homogenize}} \begin{bmatrix} -\frac{P_x n}{P_z} \\ -\frac{P_y n}{P_z} \\ -n-f-\frac{nf}{P_z} \\ 1 \end{bmatrix} = \begin{bmatrix} P'_x \\ P'_y \\ P'_z \\ 1 \end{bmatrix}$$

Therefore:

$$\mathbf{M}_P = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & nf \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Note: $P'_z = \begin{cases} -n, & \text{when } P_z = -n \\ -f, & \text{when } P_z = -f \end{cases}$

The Projection Matrix

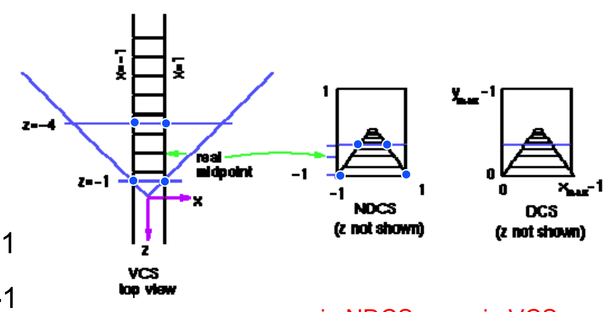
As defined by OpenGL

$$\mathbf{M}_{\text{proj}} = \mathbf{M}_O \mathbf{M}_P$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & nf \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{n+f}{n-f} & \frac{2nf}{n-f} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Nonlinearity of Perspective Transformation



Tracks:

z = -inf, +inf

Left track: x= -1, y = -1

Right track: x = 1, y = -1

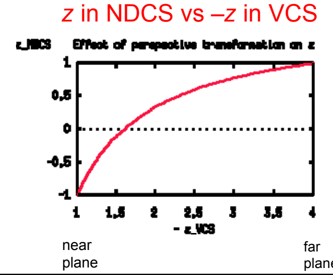
View volume:

Left = -1, Right = 1

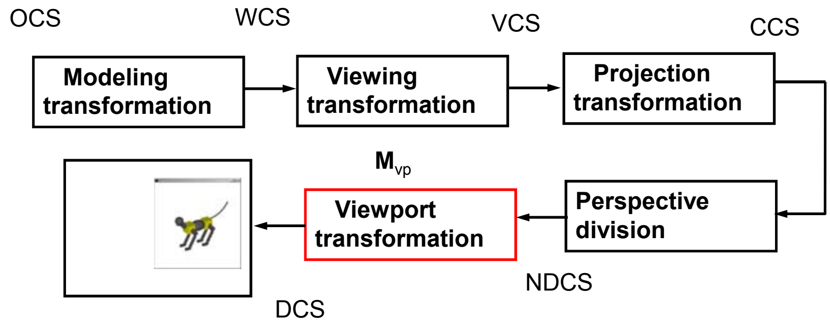
Bottom = -1, Top = 1

Near = 1, Far = 4

z in NDCS vs –*z* in VCS



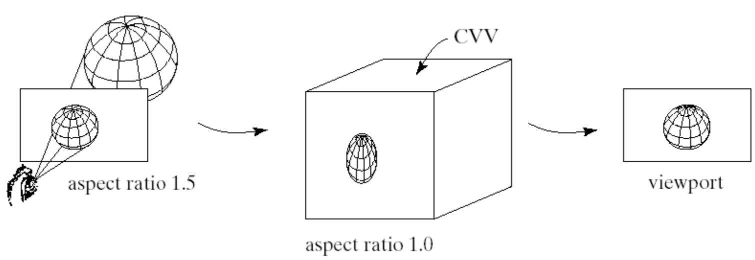
Viewport Transformation



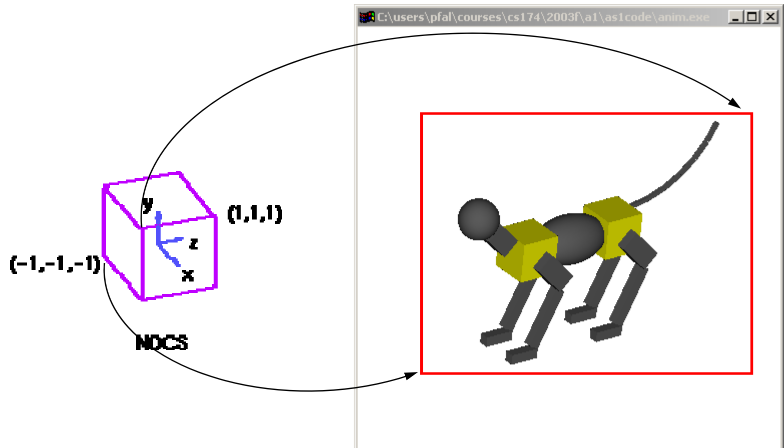
Why Viewports?

Undo the distortion of the projection transformation

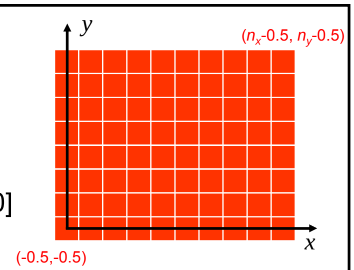
Map to pixel coordinates on screen



Viewport



Viewport Matrix

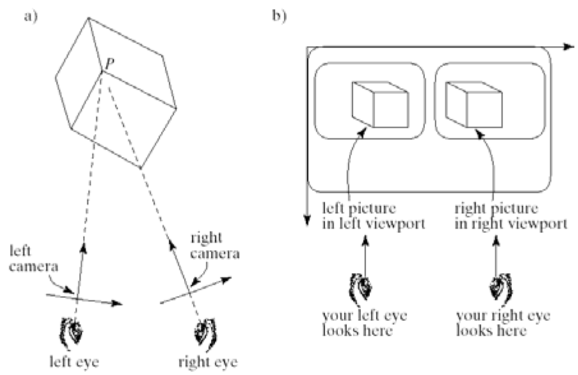


- Canonical image plane: [-1.0, -1.0] x [1.0, 1.0]
- Leave *z* coordinates unchanged
- Transform x,y coordinates to a rectangular viewport of size n_x x n_y pixels
  assume square pixels of size 1.0x1.0, from (0,0) at lower left;
  thus, viewport is [-0.5, n_x-0.5] x [-0.5, n_y-0.5]

$$\mathbf{M}_{VP} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \frac{n_x-1}{2} \\ 0 & 1 & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Translation}} \underbrace{\begin{bmatrix} \frac{n_x}{2} & 0 & 0 & 0 \\ 0 & \frac{n_y}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Scaling}} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

What would change if *y* were increasing downward?

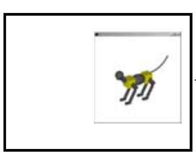
Stereo Views



OpenGL Functions in the Pipeline

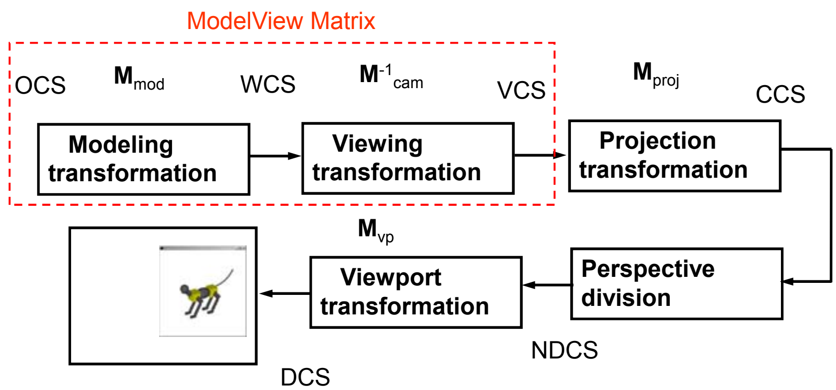
OCS
glTranslatef()
glRotate()
…
WCS
gluLookAt()
VCS
glOrtho()
glFrustrum()
gluPerspective()
CCS

Modeling transformation → **Viewing transformation** → **Projection transformation** → **Perspective division** → **Viewport transformation** →



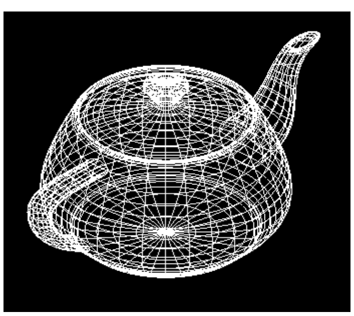
NDCS
glViewport()
DCS

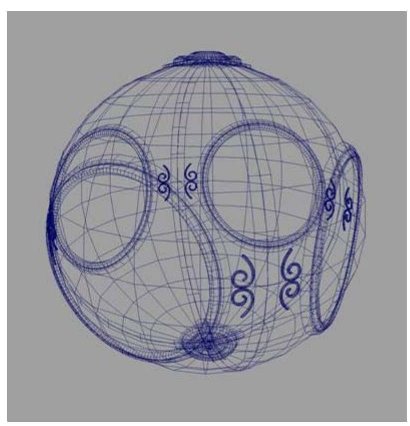
Transformations in the Pipeline



Wireframe Displays

No hidden line/surface removal





A Wireframe Rendering Algorithm

Compute $\mathbf{M}_{mod}$
Compute $\mathbf{M}^{-1}_{cam}$
Compute $\mathbf{M}_{modelview} = \mathbf{M}^{-1}_{cam}\ \mathbf{M}_{mod}$
Compute $\mathbf{M}_{O}$
Compute $\mathbf{M}_{P}$ // disregard $\mathbf{M}_{P}$ here and below for orthographic-only case
Compute $\mathbf{M}_{proj} = \mathbf{M}_{O}\ \mathbf{M}_{P}$
Compute $\mathbf{M}_{vp}$
Compute $\mathbf{M} = \mathbf{M}_{vp}\ \mathbf{M}_{proj}\ \mathbf{M}_{modelview}$
for each line segment *i* between points P_i and Q_i **do**
  $P = \mathbf{M}P_i$; $Q = \mathbf{M}Q_i$
  drawline(P_x/w_P, P_y/w_P, Q_x/w_Q, Q_y/w_Q) // w_P, w_Q are 4th coords of P, Q
end for

More Complex Wireframe Displays

No hidden line/surface removal