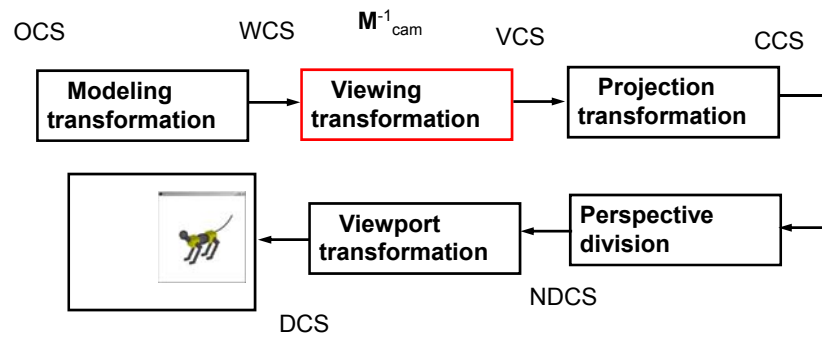
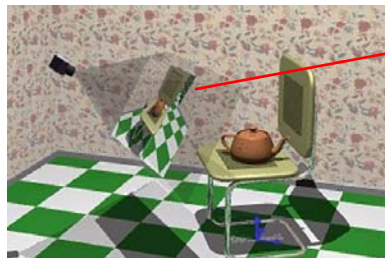


Graphics Pipeline

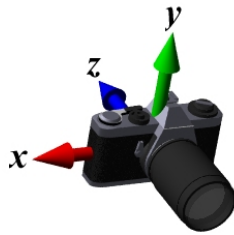


Rendering a 3D Scene From the Point of View of a Virtual Camera



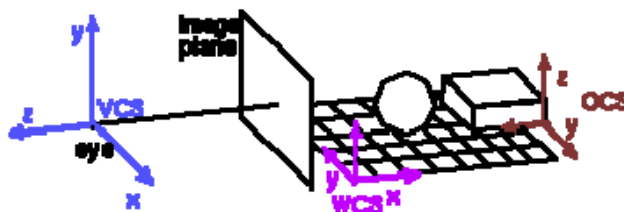
OpenGL Convention

In world coordinates, the camera system is defined as follows:



Camera Transformation

Transforms objects to camera coordinates



$$\left. \begin{aligned} P_{wcs} &= M_{cam} P_{vcs} \rightarrow P_{vcs} = M_{cam}^{-1} P_{wcs} \\ P_{wcs} &= M_{mod} P_{ocs} \end{aligned} \right\} \rightarrow$$

$$P_{vcs} = \underbrace{M_{cam}^{-1} M_{mod}}_{\text{Modelview Transformation}} P_{ocs}$$

Defining M_{cam}

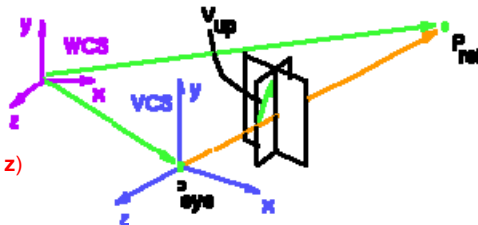
Given:

Eye point P_{eye}

Reference point P_{ref}

Up vector \mathbf{v}_{up}

(\mathbf{v}_{up} is not necessarily orthogonal to \mathbf{z})



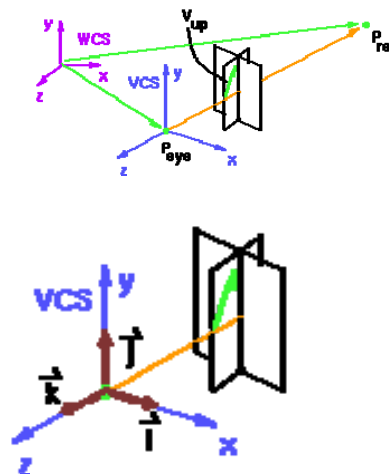
To build M_{cam} we need to define a camera coordinate system $[i \ j \ k \ O]$

Camera Coordinate System

$$\mathbf{k} = \frac{P_{\text{eye}} - P_{\text{ref}}}{|P_{\text{eye}} - P_{\text{ref}}|}$$

$$\mathbf{i} = \frac{\mathbf{v}_{\text{up}} \times \mathbf{k}}{|\mathbf{v}_{\text{up}} \times \mathbf{k}|}$$

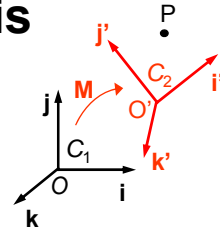
$$\mathbf{j} = \mathbf{k} \times \mathbf{i}$$



Reminder: Change of Basis

$$P_{C_1} = M P_{C_2}$$

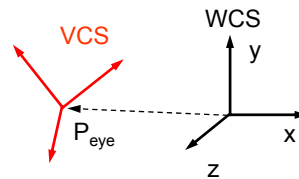
$$P_{C_1} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} i'_x & j'_x & k'_x & O'_x \\ i'_y & j'_y & k'_y & O'_y \\ i'_z & j'_z & k'_z & O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = M P_{C_2}$$



Building M_{cam}

Change of basis

Our reference system is WCS,
we know the camera parameters with
respect to the world



Align WCS with VCS

$$M_{\text{cam}} = \begin{bmatrix} 1 & 0 & 0 & P_{\text{eye}_x} \\ 0 & 1 & 0 & P_{\text{eye}_y} \\ 0 & 0 & 1 & P_{\text{eye}_z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{\text{WCS}} = M_{\text{cam}} P_{\text{VCS}}$$

Building M_{cam} Inverse

Invert the smart way

$$\begin{aligned}
 M_{\text{cam}}^{-1} &= \left(\begin{bmatrix} 1 & 0 & 0 & P_{\text{eye}_x} \\ 0 & 1 & 0 & P_{\text{eye}_y} \\ 0 & 0 & 1 & P_{\text{eye}_z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1} \\
 &= \begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & P_{\text{eye}_x} \\ 0 & 1 & 0 & P_{\text{eye}_y} \\ 0 & 0 & 1 & P_{\text{eye}_z} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}
 \end{aligned}$$

Building M_{cam} Inverse

Invert the smart way

$$M_{\text{cam}}^{-1} = \begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & P_{\text{eye}_x} \\ 0 & 1 & 0 & P_{\text{eye}_y} \\ 0 & 0 & 1 & P_{\text{eye}_z} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} i_x & i_y & i_z & 0 \\ j_x & j_y & j_z & 0 \\ k_x & k_y & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -P_{\text{eye}_x} \\ 0 & 1 & 0 & -P_{\text{eye}_y} \\ 0 & 0 & 1 & -P_{\text{eye}_z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{\text{VCS}} = M_{\text{cam}}^{-1} P_{\text{WCS}}$$

Summary of the Modelview Transformation

- 1. An affine transformation composed of elementary affine transformations***
- 2. The camera transformation is a change of basis***
- 3. The modelview transformation preserves:***
 - lines and planes
 - parallelism of lines and planes
 - affine combinations of points and relative ratios