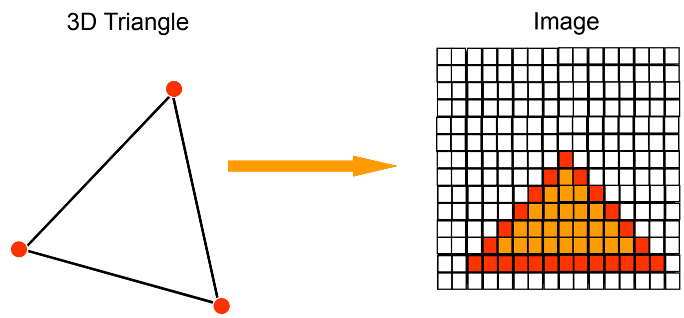
Rasterization



Line Rasterization
Reminder: Line Rendering Algorithm

Compute $\mathbf{M} = \mathbf{M}_{vp}\,\mathbf{M}_{proj}\,\mathbf{M}^{-1}{}_{cam}\,\mathbf{M}_{mod}$

for each line segment *i* between points P_i and Q_i **do**

  $P = \mathbf{M}P_i$; $Q = \mathbf{M}Q_i$ // w_P, w_Q are 4th coords of P, Q

  drawline(P_x/w_P, P_y/w_P, Q_x/w_Q, Q_y/w_Q)

end for

Line Rasterization

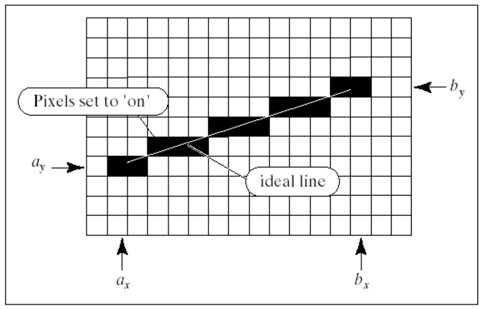


FIGURE 10.23 Drawing a straight-line-segment.

Line Rasterization

Desired properties

- Straight
- Pass through end points
- Smooth
- Independent of end point order
- Uniform brightness
- Brightness independent of slope
- Efficient!

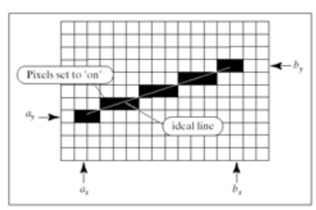


FIGURE 10.23 Drawing a straight-line-segment.

from: **Computer Graphics Using OpenGL, 2e,** by F. S. Hill
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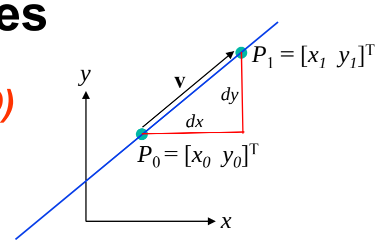
Reminder: Lines



Representations of a line (in 2D)

- Explicit $y = \alpha x + \beta$

  $$y = m(x - x_0) + y_0; \quad m = \frac{dy}{dx} = \frac{y_1 - y_0}{x_1 - x_0}$$

  $P_1 = [x_1 \ y_1]^T$, $P_0 = [x_0 \ y_0]^T$

- Implicit $f(x,y) = (x - x_0)dy - (y - y_0)dx$

  if $f(x,y) = 0$ then (x,y) is **on** the line

  $f(x,y) > 0$ then (x,y) is **below** the line

  $f(x,y) < 0$ then (x,y) is **above** the line

- Parametric $x(t) = x_0 + t(x_1 - x_0)$

  $y(t) = y_0 + t(y_1 - y_0)$

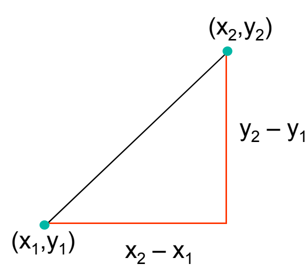
  $t \in [0,1]$ for line segment, or $t \in [-\infty, \infty]$ for infinite line

  $P(t) = P_0 + t(P_1 - P_0)$ or $P(t) = P_0 + t\,\mathbf{v}$

  $P(t) = (1-t)P_0 + tP_1$

Straightforward Implementation

Line between two points




```
DrawLine(int x1,int y1, int x2,int y2)
   {
       float y;
       int x;
       for (x=x1; x<=x2; x++) {
               y = y1 +  (x-x1)*(y2-y1)/(x2-x1)
               SetPixel(x, Round(y) );
       }
   }
```

Better Implementation

How can we improve this algorithm?

```
DrawLine(int x1,int y1, int x2,int y2)
  {
       float y;
       int x;
       for (x=x1; x<=x2; x++) {
              y = y1 +  (x-x1)*(y2-y1)/(x2-x1)
              SetPixel(x, Round(y) );
       }
  }
```

Better Implementation

```
DrawLine(int x1,int y1, int x2,int y2)
  {
       float y,m;
       int x;
       int dx = x2-x1 ;
       int dy = y2-y1 ;
       m = dy/(float)dx ;
       for (x=x1; x<=x2; x++) {
              y = y1 + m*(x-x1) ;
              SetPixel(x, Round(y) );
       }
  }
```

Even Better Implementation

```
DrawLine(int x1,int y1, int x2,int y2)
  {
      float y,m;
      int x;
      dx = x2-x1 ;
      dy = y2-y1 ;
      m = dy/(float)dx ;
      y = y1 + 0.5 ;
      for (x=x1; x<=x2; x++) {
            SetPixel(x, Floor(y) );
            y = y + m ;
      }
  }
```

Midpoint Algorithm (Bresenham)

Line in the first quadrant (0 < slope < 45 deg)

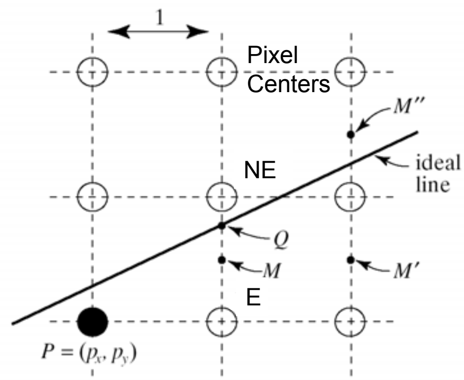
Implicit form of line:

F(x,y) = x dy – y dx + c,

Note: dx = x2 – x1; dy = y2 – y1

dx, dy > 0 and dy/dx ≤ 1.0 ;

- Current choice P = (x,y)
- How do we choose next pixel, P'= (x+1,y') ?

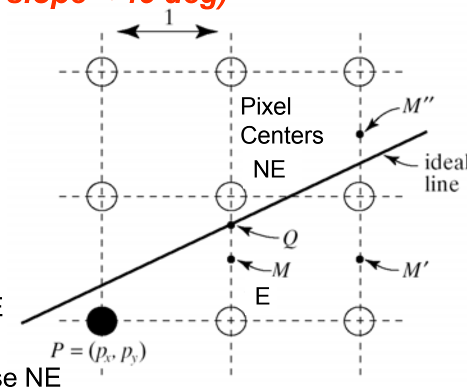


Midpoint Algorithm (Bresenham)

Line in the first quadrant (0 < slope < 45 deg)

Implicit form of line:
F(x,y) = x dy – y dx + c

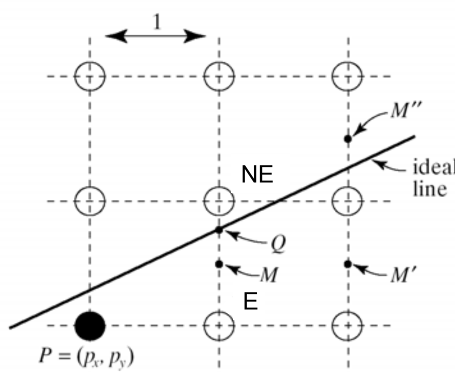
- Current choice P = (x,y)
- How do we choose next pixel,
  P'= (x+1,y') ?
  If(F(M) = F(x+1,y+0.5) < 0)
  M is above line, so choose E
  else
  M on or below line, so choose NE



Midpoint Algorithm (Bresenham)

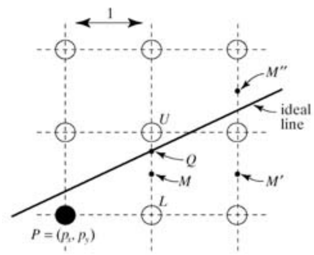
```
DrawLine(int x1, int y1, int x2, int y2,)
  {
      int x, y ;

      y = y1;
      for (x=x1; x<=x2; x++) {
              SetPixel(x, y);
              if (F(x+1,y+0.5) > 0) {
                      y = y + 1 ;
              }
      }
  }
```



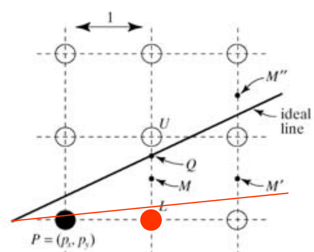
Can We Compute F in a Smart Way?

- We are at pixel (x,y) we evaluate F at M = (x+1,y+0.5) and choose E = (x+1,y) or NE = (x+1,y+1) accordingly
- Reminder: F(x,y) = x dy – y dx + c



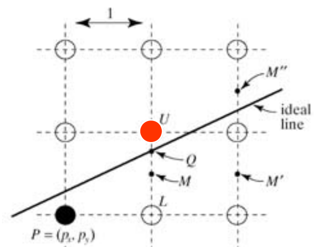
Can We Compute F in a Smart Way?

- We are at pixel (x,y) we evaluate F at M = (x+1,y+0.5) and choose E = (x+1,y) or NE = (x+1,y+1) accordingly
- Reminder: F(x,y) = x dy – y dx + c
- If we choose E for x+1, then the next test will be at M':
  F(x+2,y+0.5) = [(x+1)dy + 1dy] - (y+0.5)dx + c →
  F(x+2,y+0.5) = F(x+1,y+0.5) + dy →
  F_E = F + dy



Can We Compute F in a Smart Way?

- We are at pixel (x,y) we evaluate F at M = (x+1,y+0.5) and choose E = (x+1,y) or NE = (x+1,y+1) accordingly
- Reminder: F(x,y) = x dy – y dx + c
- If we choose E for x+1, the next test will be at M':
  F(x+2,y+0.5) = [(x+1)dy + dy] - (y+0.5)dx + c →
  F(x+2,y+0.5) = F(x+1,y+0.5) + dy →
  F_E = F + dy
- If we chose NE, then the next test will be at M'':
  F(x+2,y+1+0.5) =
  F(x+1,y+0.5) + dy – dx →
  F_{NE} = F + dy - dx

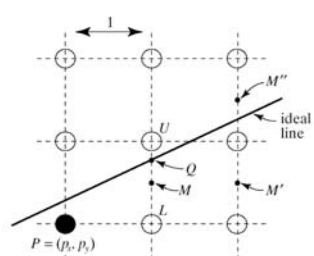


Can We Compute F in a Smart Way?

- We are at pixel (x,y) we evaluate F at M = (x+1,y+0.5) and E = (x+1,y) or NE = (x+1,y+1) accordingly
- Reminder: F(x,y) = x dy – y dx + c
- If we chose E for x+1, then the next test will be at M':

  F_E = F + dy
- If we chose NE, then the next test will be at M'':

  F_{NE} = F + dy – dx

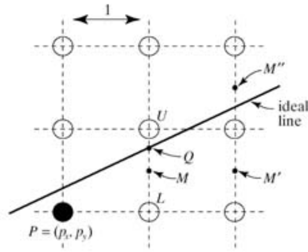


Test Update

Update

$F_E = F + dy = F + dF_E$ $\quad$ $(dF_E = dy)$

$F_{NE} = F + dy – dx = F + dF_{NE}$ $\quad$ $(dF_{NE} = dy – dx)$



Starting value?

Line equation: $F(x,y) = x\,dy – y\,dx + c$

Assume line starts at pixel (x_0,y_0)

$F_{start} = F(x_0+1,y_0+0.5) = (x_0+1)dy - (y_0+0.5)dx + c =$

$= (x_0dy - y_0dx + c) + dy - 0.5dx = F(x_0,y_0) + dy - 0.5dx.$

(x_0,y_0) belongs on the line, so: $F(x_0,y_0) = 0$

Therefore:

$F_{start} = dy - 0.5dx$

Test Update (Integer Version)

Update

$F_{start} = dy – 0.5dx$

$F_E = F + dy = F + dF_E$

$F_{NE} = F + dy – dx = F + dF_{NE}$

Everything is integer except F_{start}

Multiply by 2 → $\boxed{\begin{array}{l} F_{start} = 2dy – dx \\ dF_E = 2dy \\ dF_{NE} = 2(dy – dx) \end{array}}$

Midpoint Algorithm (Bresenham)

```
DrawLine(int x1, int y1, int x2, int y2)
    {
        int x, y, dx, dy, d, dE, dNE;
        dx = x2-x1 ;
        dy = y2-y1 ;
        d = 2*dy-dx ; // initialize d
        dE = 2*dy ;
        dNE = 2*(dy-dx) ;
        y = y1 ;
        for (x=x1; x<=x2; x++) {
                SetPixel(x, y);
                if (d>0) {      // choose NE
                        d = d + dNE ;
                        y = y + 1 ;
                } else {        // choose E
                        d = d + dE ;
                }
        }
    }
```

Midpoint Algorithm (Bresenham)

```
DrawLine(int x1, int y1, int x2, int y2, int color)
    {
        …

        for (x=x1; x<=x2; x++) {
                SetPixel(x, y, color );
                if (d>0) {      // choose NE
                        …
                } else {        // choose E
                        …
                }
        }
    }
```

Other Incremental Rasterization Algorithms

The Bresenham incremental approach works for more complex geometries

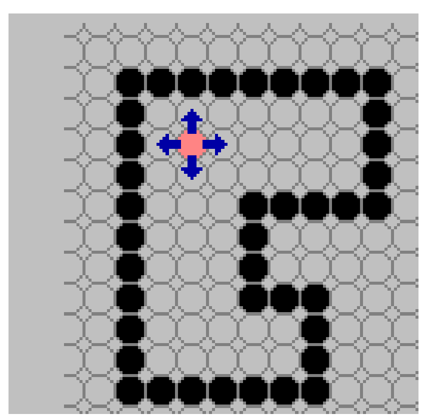
- Circles
- Polynomials

Pixel Region Filling Algorithms

Scan convert boundary

Fill in regions

2D paint programs

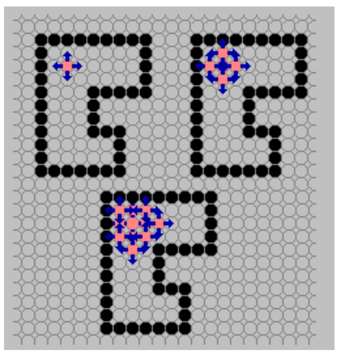


Flood Fill

```
public void floodFill(int x, int y, int fill, int old)
{
    if ((x < 0) || (x >= width)) return;
    if ((y < 0) || (y >= height))  return;
    if (getPixel(x, y) == old) {
        setPixel(x, y, fill);
        floodFill(x+1, y, fill, old);
        floodFill(x, y+1, fill, old);
        floodFill(x-1, y, fill, old);
        floodFill(x, y-1, fill, old);
    }
}
```

Boundary Fill

```
boundaryFill(int x, int y, int fill, int boundary) {
        if ((x < 0) || (x >= width)) return;
        if ((y < 0) || (y >= height)) return;
        int current = getPixel(x, y);
        if ((current != boundary) & (current != fill)) {
            setPixel(x, y, fill);
            boundaryFill(x+1, y, fill,boundary);
            boundaryFill(x, y+1, fill, boundary);
            boundaryFill(x-1, y, fill, boundary);
            boundaryFill(x, y-1, fill, boundary);
        }
}
```

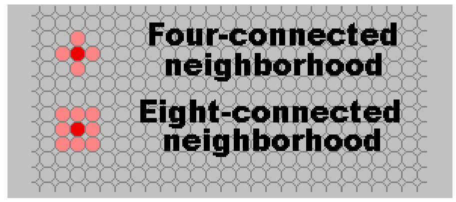


Adjacency

4-connected

8-connected

- Will leak through diagonal boundaries
- Can be used to color boundaries



Polygon Rasterization

Scan conversion

Shade pixels lying within a closed polygon **efficiently**

Algorithm

- For each row of pixels define a *scanline* through their centers
- Intersect each scanline with all edges
- Sort intersections in x
- Calculate parity of intersections to determine ‘interior’ / ‘exterior’
- Fill the ‘interior’ pixels
- Exploit coherence of intersections between scanlines