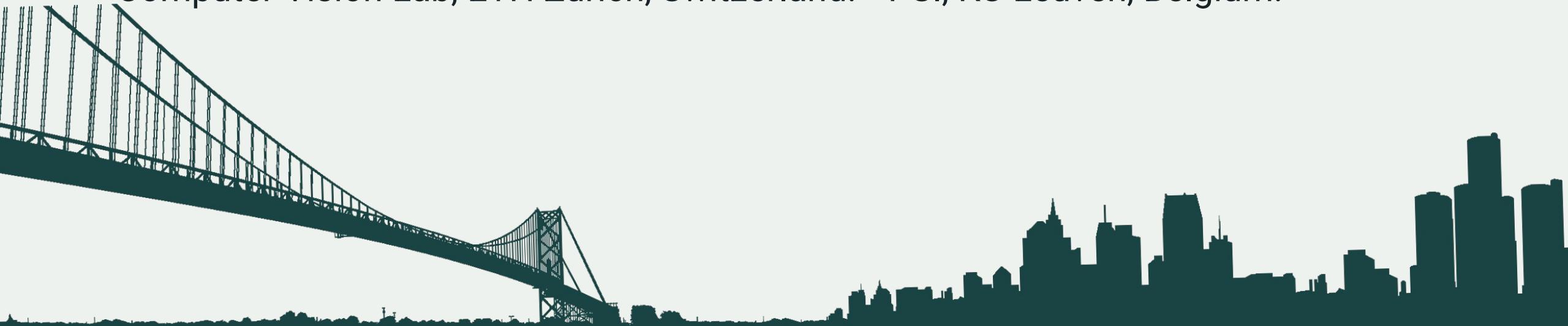


# A Multiplicative Value Function for Safe and Efficient Reinforcement Learning

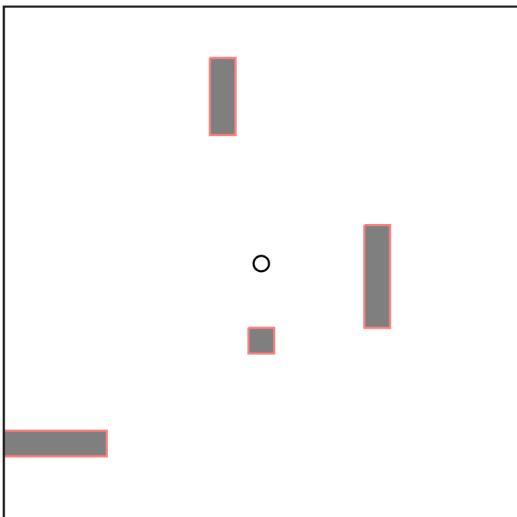


Nick Bührer<sup>1</sup>, Zhejun Zhang<sup>1</sup>, Alexander Liniger<sup>1</sup>,  
Fisher Yu<sup>1</sup>, Luc Van Gool<sup>1,2</sup>.

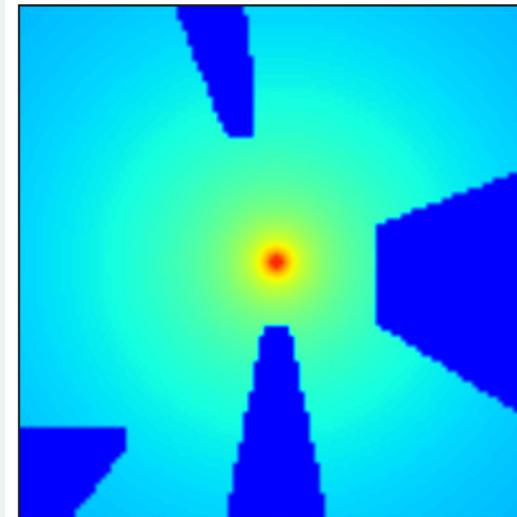
<sup>1</sup> Computer Vision Lab, ETH Zurich, Switzerland. <sup>2</sup> PSI, KU Leuven, Belgium.



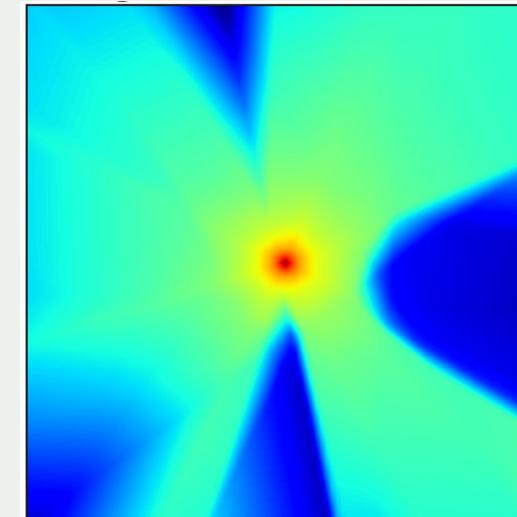
# Motivation



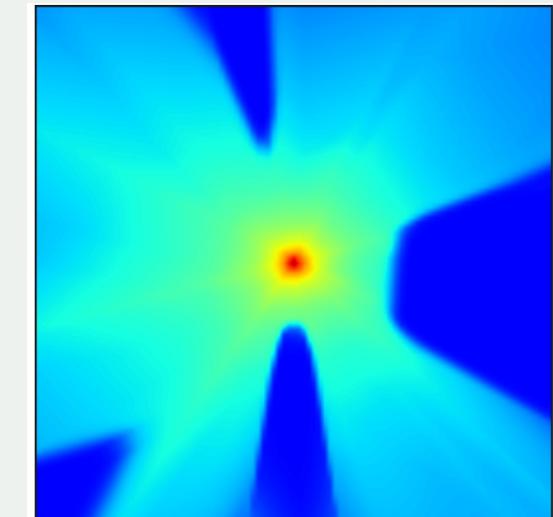
(A) Environment



(B) Ground-Truth  
Value



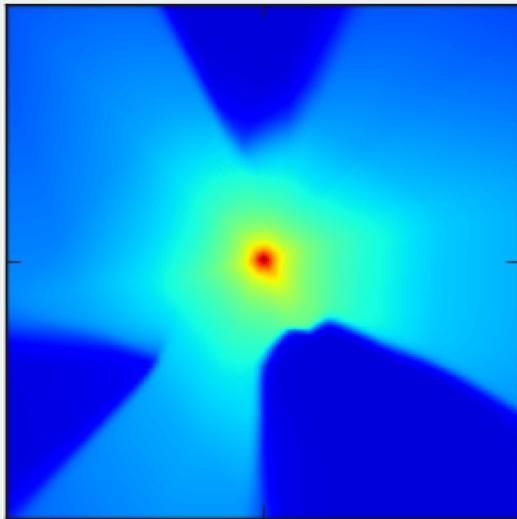
(C) Regular  
Value Function



(D) Multiplicative  
Value Function

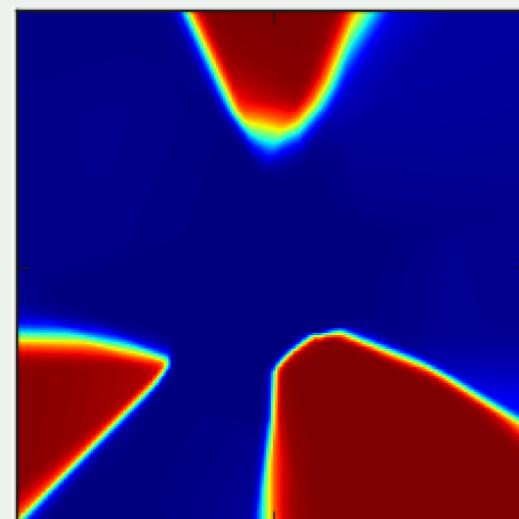
# Multiplicative Value Function

Multiplicative Value  
Function  $V_{\text{mult}}^{\pi}$

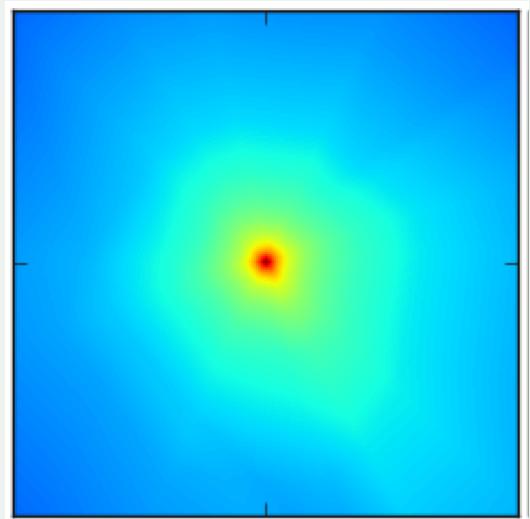


=

Probabilistic  
Safety Critic  $\Psi^{\pi}$



Constraint-Free  
Reward Critic  $\bar{V}^{\pi}$



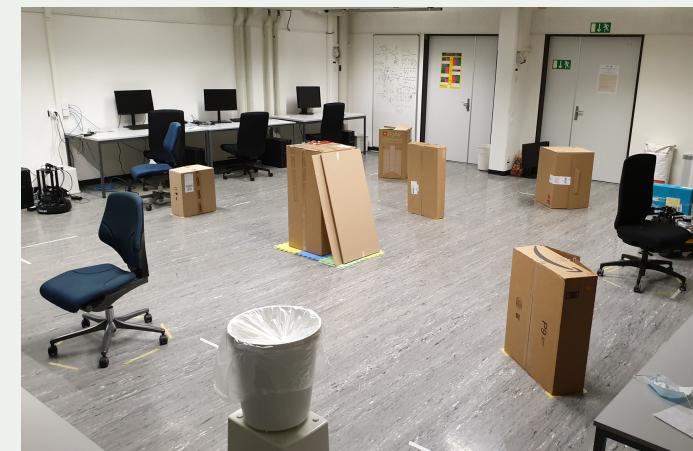
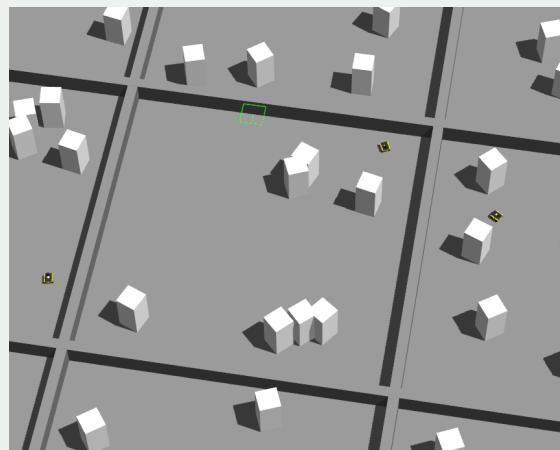
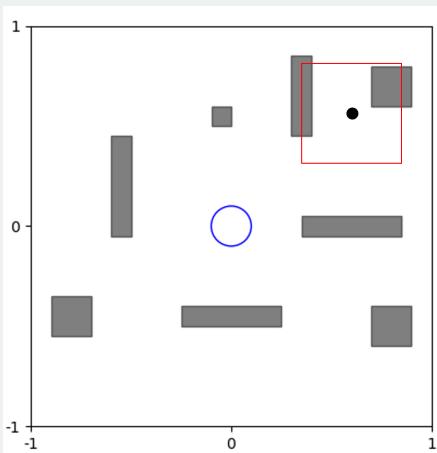
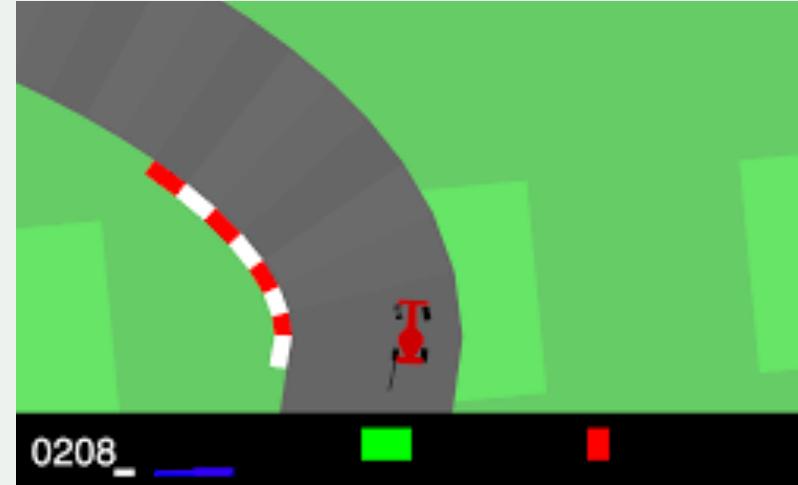
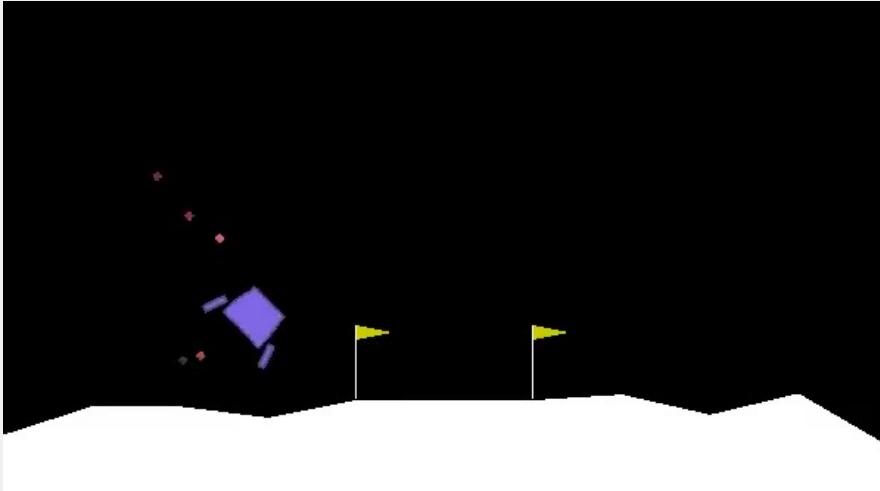
$$V_{\text{mult}}^{\pi}(s) = (\bar{V}^{\pi}(s) - \bar{v}_{\min}) \cdot (1 - \Psi^{\pi}(s)) + \bar{v}_{\min}, \quad \bar{v}_{\min} := \min_s \bar{V}^{\pi}(s)$$

# Apply to SAC and PPO

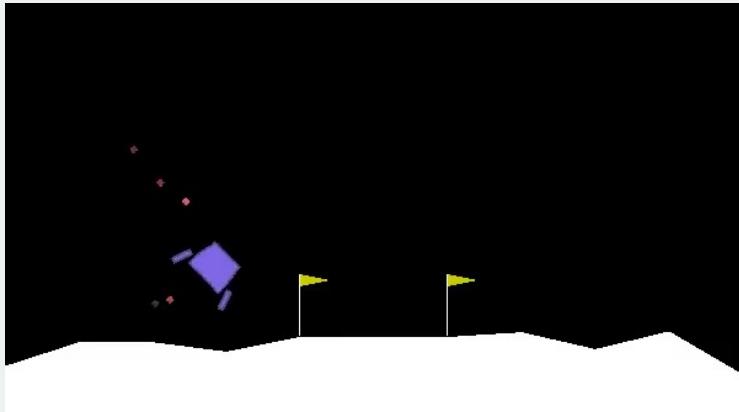
- **Q Function:**  $Q_{\text{mult}}^{\pi}(s, a) = (\bar{Q}^{\pi}(s, a) - \bar{q}_{min}) \cdot (1 - \Psi^{\pi}(s, a)) + \bar{q}_{min}$
- **Advantage**
  - V1 bootstrap Q:  $A_{\text{mult}}^{\pi}(s_t, a_t) = [\bar{r}_t + \gamma V_{\text{mult}}^{\pi}(s_{t+1})] - V_{\text{mult}}^{\pi}(s_t)$
  - V2 without bootstrap:  $A_{\text{mult}}^{\pi}(s_t, a_t) = Q_{\text{mult}}^{\pi}(s_t, a_t) - V_{\text{mult}}^{\pi}(s_t)$
  - V3 bootstrap the safety critic inside  $Q_{\text{mult}}^{\pi}(s_t, a_t)$  of V2
- **SAC:**  $\max_{\theta} \mathbb{E}_{a_{\theta} \sim \pi_{\theta}} [Q_{\text{mult}}^{\pi_{\theta}}(s, a_{\theta}) - \alpha \log \pi_{\theta}(s_{\theta} | x)]$
- **PPO:**  $\max_{\theta} \mathbb{E}_{a \sim \pi_{\theta}} \left[ \min \left\{ \frac{\pi_{\theta}(a|s)}{\pi_{\theta_k}(a|s)} A_{\text{mult}}^{\pi_{\theta_k}}(s, a), g(\epsilon, A_{\text{mult}}^{\pi_{\theta_k}}(s, a)) \right\} \right]$



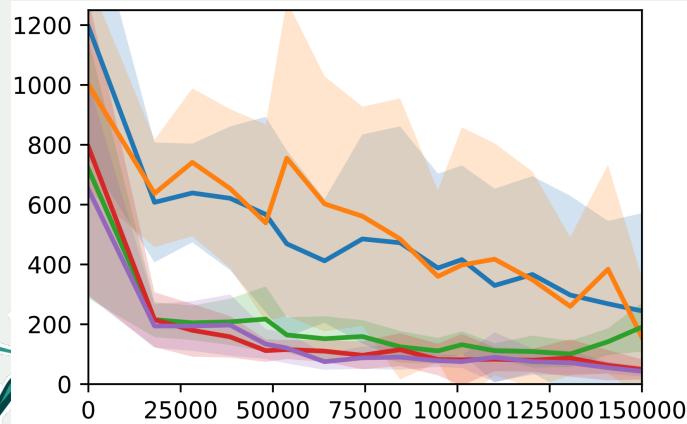
# Safety-Focused Environments



# Quantitative Results



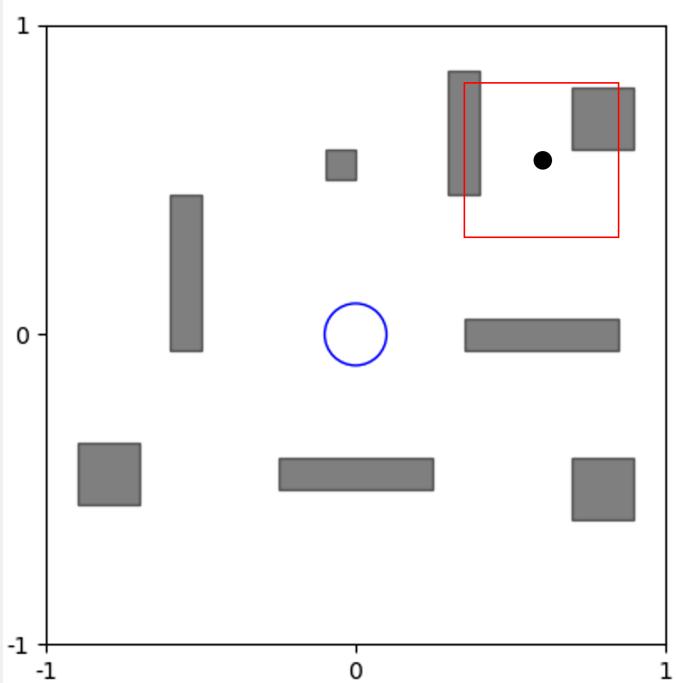
**Value Loss of Lunar Lander Safe**



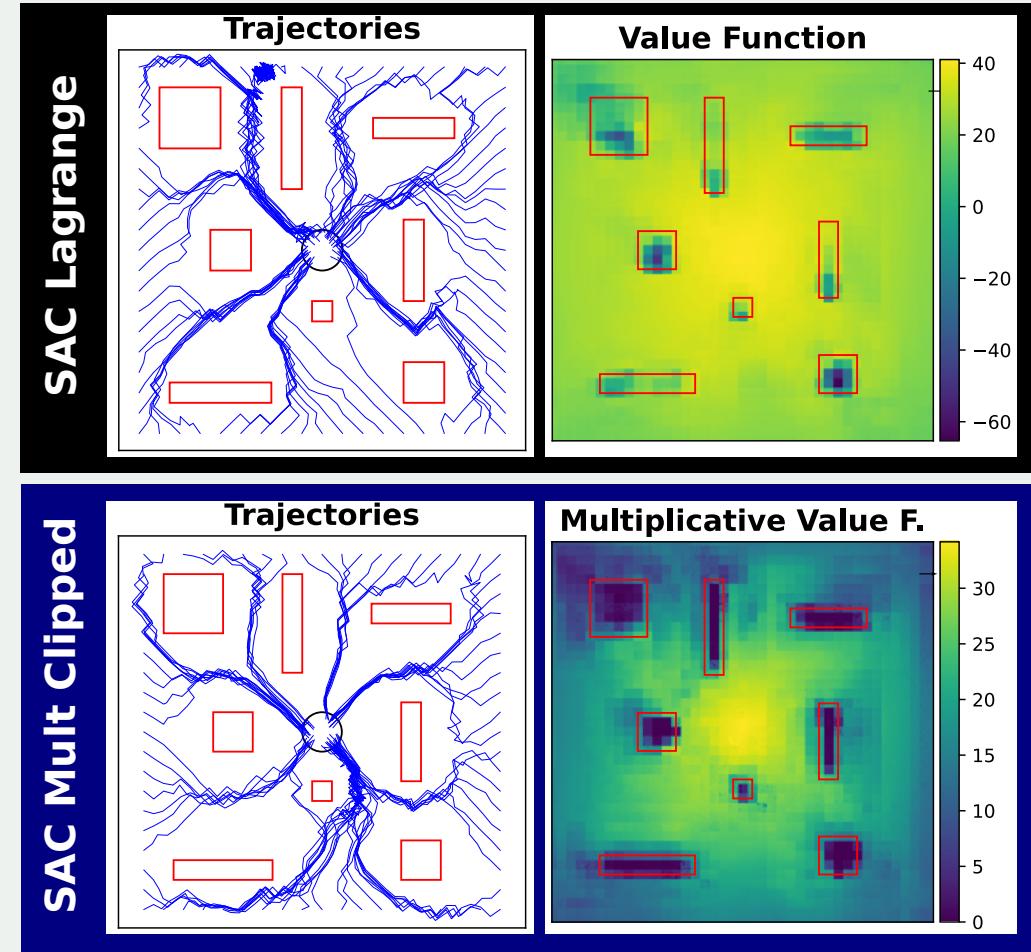
**Blue:** PPO base  
**Orange:** Lagrange  
**Green:** V1  
**Violet:** V2  
**Red:** V3  
**Grey:** FOCOPS

	Reward ↑	% Constraint violations ↓	Reward ↑	% Constraint violations ↓
Lunar Lander Safe				
<b>SAC</b>		<b>50k</b>		<b>150k</b>
SAC base	$90 \pm 108$	$10 \pm 16$	$181 \pm 117$	$3 \pm 6$
Lagrange	$111 \pm 105$	$17 \pm 13$	$184 \pm 128$	$2 \pm 3$
Mult	$-35 \pm 27$	$3 \pm 5$	$-34 \pm 22$	$3 \pm 4$
Mult Clipped	<b><math>134 \pm 94</math></b>	$14 \pm 13$	$243 \pm 49$	$8 \pm 15$
Mult Lagrange	$125 \pm 59$	$29 \pm 15$	<b><math>251 \pm 20</math></b>	$2 \pm 2$
<b>PPO</b>		<b>50k</b>		<b>150k</b>
PPO base	$-126 \pm 158$	$77 \pm 29$	$225 \pm 100$	$10 \pm 30$
Lagrange	$-24 \pm 146$	$54 \pm 39$	$204 \pm 116$	$12 \pm 24$
V1	$101 \pm 84$	$41 \pm 19$	$205 \pm 78$	$7 \pm 16$
V2	$89 \pm 122$	$44 \pm 34$	$251 \pm 28$	$5 \pm 9$
V3	<b><math>144 \pm 4</math></b>	<b><math>26 \pm 22</math></b>	<b><math>264 \pm 5</math></b>	<b><math>1 \pm 2</math></b>
FOCOPS	$-129 \pm 21$	$64 \pm 24$	$117 \pm 80$	$30 \pm 19$

# Qualitative Results



**Point Robot Navigation**  
Via local occupancy grid and  
a vector pointing to the goal.



# Simulation and Real-world

- Differential drive robot with 1D-Lidar
- Gazebo simulation
- Zero-shot sim-to-real



# Summary

- Multiplicative value function
  - Constraint-free reward critic  $\otimes$  Probabilistic safety critic
- Integration into SAC and PPO
  - Increased sample efficiency and learning stability.
- Experiments
  - Safety-focused RL environments and real-world robot navigation.
- Future works
  - Theoretical justification.

