A Belief-Driven Taylor Rule: Expectations As a Policy Tool *

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Abstract

Central banks do not observe economic fundamentals, and information dispersion affects all sectors. This paper employs a New Keynesian dynamic stochastic general equilibrium (DSGE) model with non-nested information to extend information dispersion to all sectors. We study monetary policies with the internalization of public expectations by including private sector information in taylor-rule based policies. Theoretically, we show that this internalization conveys information about economic fundamentals and affects total volatility. Numerically, we find increasing importance in private sectors' information in the context of anchoring public expectations. This paper aids in understanding the impacts of policy-rule information choices.

Keywords: Incomplete Information, Monetary Policy, Asymmetric Information, Multiple Endogenous Signals

JEL Codes: D82, E52

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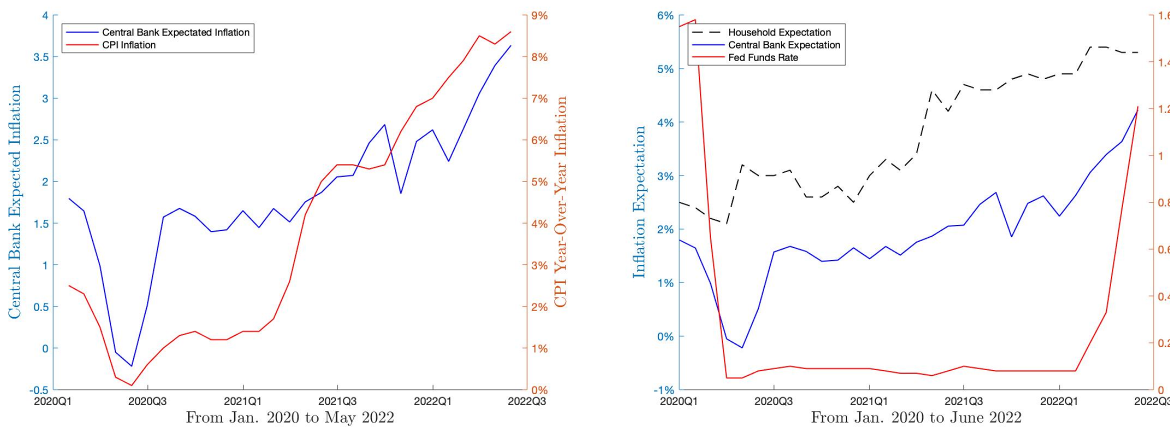
1 Introduction

Modern monetary policy focuses on hitting the inflation target, and the monetary authority, i.e., the central bank, dictates the monetary rules. After the financial crisis in 2008, the Federal Reserve began the discussion of the Average-Inflation-Targeting (AIT) policy to enhance economic recovery. In 2020, during the COVID-19 pandemic, the Federal Reserve announced the adoption of AIT as a part of its monetary strategy, which provided the central bank with *some* tolerance for temporary inflation. With the central bank's low inflation expectation[1], the federal funds rate was kept at approximately 0.08% during 2021. Meanwhile, US Consumer Price Index (CPI) kept climbing and its year-over-year increase reached 8.6% by May 2022, which is a historical high since the late 80s [See Figure 1:a].



(a) Central Bank Expected Inflation and CPI Inflation

(b) Inflation Expectations and Federal Funds Rate

Figure 1: CPI Inflation, Expected Inflation and Fed Funds Rate

Only when accumulated inflation data was realized, the Federal Reserve updates its beliefs and takes sequential actions to pull back inflation to the targeted value of 2%. Since March 2022, the Federal Reserve has raised the federal funds rate three times and, in July 2022, announced a 0.75-percentage-point increase, which is also a historical high within the past 28 years. The right axis in Figure 1:b documents a steep increase in the federal funds rate. A restrictive monetary policy tends to continue, according to Chair Jerome H. Powell's Jackson Hole speech on August 26, 2022. He has emphasized central banks' duty and determination to deliver low and stable inflation. Furthermore, he has brought out the importance of the public's expectations about inflation, arguing that a central bank should use its tools to prevent the public's high inflation expectations from getting entrenched. Because if those

[1] In other words, the central bank interpreted inflation in 2021 as temporary.

expectations were realized, it would reinforce the public's beliefs about an economy and induce higher wage and pricing decisions due to rational inattention. Instead of reshaping the private sectors' beliefs about the economy, we study the model where the private sectors' beliefs are implemented as policy tools and internalized within monetary policy. We find that the responsiveness of nominal interest rate to private sectors' nowcasts helps anchor private sectors' forecasts.

Typically, a central bank makes decisions based on its own information set, which may increase inflation volatility (Coibion and Gorodnichenko (2012)). This information set is generally assumed to be complete. However, as shown by various research in the literature, both private and public sectors are partially informed. There exists pronounced expected inflation disagreement across economic agents, meaning no sector has a complete set of information(Candia, Coibion and Gorodnichenko (2020),Andrade et al. (2016), Dovern, Fritsche and Slacalek (2012)). The left axis in Figure 1:b includes households' inflation expectation[2] and the central bank's inflation expectation starting from Jan. 2020, the beginning of the COVID-19 pandemic, to Jun. 2022. We first notice that households' inflation expectation is consistently above that of the central bank. We interpret this phenomenon as different economic agents forming expectations based on different information sets. Secondly, the federal funds rate sharply increased only recently because of the central bank's relatively stable inflation expectation during the past two years[3]. Therefore, studying monetary policies in an incomplete information setting is essential, especially with a shattering of the complete information assumption for central banks. The average inflation targeting policy, on the one hand, provides certain inflation tolerance; on the other hand, it may be problematic due to the lack of real-time data and sluggish policy actions, which would cause severe inflation as of now. Under the framework of the average inflation targeting policy, with the Taylor rule still treated as the benchmark tool, an internalization of public expectations provides valuable insights. Internalizing public expectations into the Taylor rule would help anchor the public's expectations,[4] and may also serve as unobservable real-time data for a monetary authority. Moreover, this internalization may reduce the total economic volatility and result in better social welfare. We further introduce heterogeneous information between households

[2]Based on Surveys of Consumers by the University of Michigan.

[3]From 2020-Q3 to 2022-Q1, the central bank's inflation expectation is around 2 percent. Therefore they have no incentive to raise the Federal Funds Rate. Binder, Janson and Verbrugge (2022) show that "most individual forecasters' long-run inflation expectations fluctuate substantially, with sizeable departures from target" compared with SPF's stabilized expectations around target.

[4]Since the model structure is assumed to be common knowledge for economic participants, the public would know which information set is applied within the monetary policy. Therefore, their high expectations about inflation would undoubtedly lead to an increase in the Federal Funds Rate. There would be no unanticipated shock to affect the nominal interest rate.

and firms.

This paper employs a New Keynesian DSGE model with an extension and argues that a central bank should learn and respond to the private sectors' information, especially the demand side. First, we consider a DSGE model with heterogeneous information between the central bank and the private sectors, i.e., households and firms. We then introduce an asymmetric information structure within the private sectors. In the latter setting, households have their private information of a preference shock; meanwhile, technology and markup shocks become private information for firms and the central bank.

The contribution of this paper is threefold. First, we propose different belief-driven Taylor rules in an incomplete information environment, where monetary authority sets interest rates corresponding to different inflation and output gap expectations. This illustrates our idea of internalizing public expectations within policy decisions. Second, we study a dispersed information case where all three sectors are endowed with non-nested information sets. Last, we theoretically derive analytical solutions to describe the impacts of various policy-wise information implementations on output, inflation, interest rate dynamics, and total volatility. Moreover, we numerically study monetary policy utilization of private sectors' information and their impacts on anchoring private sectors' expectations. To our knowledge, these three are novel enrichment to the literature.

The main result of this paper shows that when there is no information dispersion within the private sectors, the central bank should believe in its information. As the private sectors' asymmetric information arises, the policy rule should respond to households' expectations if the volatility of households' shocks is not negligible. This sheds light on the importance of newly emerging survey data. We explain this result by thinking about the original goal of monetary policy and the microstructure of the economy. The goal of a monetary policy is to stabilize an economy. With complete information, a central bank can observe all the fundamentals and set the interest rate to hit the target inflation rate. However, with incomplete information, a central bank can only observe some shocks and signals and has to form its expectations about both endogenous and exogenous variables. Furthermore, a central bank does not serve the economy as a producer or a consumer. With the simple signal structure[5], a central bank's information dominates the private sector's information because private signals are "noised" central bank signals due to bank disclosure noises. In this scenario, a central bank's information is intuitively more precise and valuable. This is consistent with welfare analysis results. Adoption of a central bank's information within the belief-driven Taylor rule increases total welfare by 34 percent more than adopting private information.

[5]Households and firms hold the same information set.

The story becomes different when we introduce a simple asymmetric information structure within private sectors and differentiate households from firms. A demand-side shock [6] can only be observed by households; supply shocks [7] will only enter firms' and the central bank's information sets. The bank disclosure problem exists but only between firms and the central bank. The difference is that when households are endowed with demand-side information, supply shocks will not be misinterpreted as they were in the symmetric information setting. For example, when a positive markup shock hits, the representative household will only see an increase in price level and lower their aggregate demand instead of confusing it with a decrease in marginal cost. With different information structures, signal extraction processes also differ. Not surprisingly, the total volatility of an economy increases with an increase in signal structure complexity. Compared with private sectors sharing a symmetric information set, we observe more considerable welfare loss (i.e., macro-volatility) for each kind of policy-wise information implementation when a non-nested information structure extends to all sectors. Because no matter which information set is applied[8], there will be a higher-order belief formation process[9], and the choice of any particular information set will not increase signal extraction efficiency. The crucial finding here is that a simple demand shock now dominates supply shocks, and a study of weighted strategy in monetary policy seconds this result. The central bank should consider responding to household expectations when conducting monetary policy decisions. Because of the central bank's communication error, firms still receive noised central bank signals. Together with firms' idiosyncratic noises, their information set (Ω_t^{firm}) is dominated by central bank's information. Numerical analysis shows that with the implementation of household information, the economy's average period welfare loss approaches benchmark results where the central bank is endowed with full information.

Related Literature: This paper relates to the ongoing literature on information frictions and their macro-size implications. [see Nimark (2008), Gorodnichenko and Coibion (2012) and Angeletos and Huo (2021)] Melosi (2017), Han, Tan and Wu (2021b) study the signaling effects, where they mainly focus on firm-side incomplete information. Empirical evidence in Candia, Coibion and Gorodnichenko (2020) and Gorodnichenko, Coibion and Candia (forthcoming) find that firms hold different expectations to SPF or households. Therefore, we examine scenarios in which information sets are incomplete and asymmetric among households, firms, and central banks.

[6] A preference shock, ε^{β}.

[7] Aggregate and idiosyncratic technology and markup shocks.

[8] In sum, there are three different information sets that a central bank may apply to set the interest rate: the representative household's information (Ω_t^{HH}), firms' information (Ω_t^{firms}) and its own information (Ω_t^{cb})

[9] $\mathbb{E}^{Household}[\mathbb{E}^{CentralBank}]$, $\mathbb{E}^{Firms}[\mathbb{E}^{CentralBank}]$, $\mathbb{E}^{Firms}[\mathbb{E}^{Household}]$, etc.

Carboni and Ellison (2011) and Kohlhas (2021) study the scenario where the central bank shares incomplete but different information sets against private sectors. In the first paper, there is no output gap in monetary policy and no signal extraction process between the monetary authority and private sectors; in the second paper, there is no expected inflation in monetary policy. Our benchmark theory model combines both. Our numerical analysis includes communication errors between the central bank and firms from Kohlhas (2021). Nakamura and Steinsson (2018) voices the importance of the central bank's communication channel. They study the impacts of central bank announcements.

Falck, Hoffmann and Hürtgen (2021) study the monetary policy signaling channel with heterogeneous degrees of disagreement for monetary policy transmission. Our paper introduces a way of policy-wise internalizing disagreed expectations and studies associated transmission channels. We both theoretically and numerically discuss results in the context of social welfare, as in Morris and Shin (2002). This paper also provides possible answers to the concerns in Coibion et al. (2020) regarding the use of expectation as a policy tool. Furthermore, this paper draws the same conclusion as Gorodnichenko, Coibion and Weber (forthcoming) that a central bank should emphasize communication efficiency with private sectors, especially households. Finally, the solution method follows Han, Tan and Wu (2021a), where they provide a user-friendly toolbox to solve dispersed information models.

The paper is structured as follow: Section 2 illustrates the benchmark model. Section 3 derives analytical solutions of the benchmark model. Section 4 provides numerical solutions to the benchmark model adding central bank communication errors. Section 5 concludes.

2 Model Environment

We consider a prototypical new Keynesian model. The model consists of a representative household, a monetary authority, and a continuum of intermediate firms with nominal rigidities in terms of quadratic price adjustment costs. Each sector operates on its own information set when forming conditional expectations. We call it a symmetric information structure within private sectors if the household and firms share the same information set. The economy's fluctuation is primarily driven by a preference shock, which affects the household's discount factor, an aggregate technology shock, which affects the productivity level, and an aggregate markup shock, which affects the price level.

Timing: There are three stages within a time period. After all the innovations arrived, firms pre-set prices with partial information. Then, the central bank reacts to its information and delivers a public interest rate. Last, firms adjust the prices, and households respond to clear the goods and labor market.

2.1 Household

A representative household maximizes the utility function

$$\mathbb{E}_0^{HH} \sum_{t=0}^{\infty} \beta^t \exp\left(\varepsilon_t^{\beta}\right) \left[\log\left(C_t\right) - \frac{N_t^{1+\eta}}{1+\eta}\right],$$

where $\beta \in (0,1)$ is the discount factor and $1/\eta > 0$ is the Frisch elasticity of labor supply. $\mathbb{E}_t^{HH}$ is the household expectation operator that operates on a private information set, Ω_t^{HH}, which will be clarified later. ε_t^{β} is a demand shock, which affects the household's intertemporal substitution. This shock is assumed to follow *i.i.d.* Gaussian distribution

$$\varepsilon_t^{\beta} \sim \mathbb{N}\left(0, \sigma_{\beta}^2\right).$$

The household's budget constraint is given as

$$\int_0^1 P_{it} C_{it} di + \left(1 + i_t\right) B_t \leq \int_0^1 \Pi_{it} di + W_t N_t + B_{t-1},$$

where P_{it} and C_{it} are price and consumption for the intermediate good i. The price index is aggregated by $P_t = \left(\int_0^1 P_{it}^{1-\rho} di\right)^{\frac{1}{1-\rho}}$. The final good consumption is aggregated by $C_t = \left(\int_0^1 C_{it}^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}}$. ρ is the elasticity of substitution. B_t is the one-period risk-free bond, Π_{it} is the profit for intermediate good firm i, W_t is the nominal wage and N_t is the labor supply.

Let c_t and π_t denote log deviations from steady states of consumption and inflation. We have the Euler Equation, which governs the intertemporal behavior of the household

$$c_t = \mathbb{E}_t^{HH}\left[c_{t+1}\right] - \left(i_t - \varepsilon_t^{\beta} - \mathbb{E}_t^{HH}\left[\pi_{t+1}\right]\right),$$

where i_t is the nominal interest rate and $\mathbb{E}_t^{HH}$ is the expectation operator for the household. Let y_t denote the final output's log deviation from the steady state. In equilibrium, we have the IS curve

$$y_t = \mathbb{E}_t^{HH}\left[y_{t+1}\right] - \left(i_t - \varepsilon_t^{\beta} - \mathbb{E}_t^{HH}\left[\pi_{t+1}\right]\right). \tag{2.1}$$

2.2 Firms

There is a continuum of intermediate firms indexed by $i \in [0,1]$. Firms are endowed with the same linear production function $Y_{i,t} = A_t N_{i,t}$, where $Y_{i,t}$ is the production of good i and $N_{i,t}$ is the labor input of firm i. A_t is the aggregate technology level. It is modeled as a persistent AR(1) process with a $i.i.d.$ Gaussian noise θ_t

$$A_t = A_{t-1}^{\rho_a} \exp(\theta_t): \quad \rho_a \in (0,1), \theta_t \sim \mathbb{N}\left(0, \sigma_\theta^2\right)$$

Aggregate technology signal is assumed to be non-observable to intermediate firms. Instead, the signal of aggregate technology contains two noises: (i) an industry level $i.i.d.$ Gaussian noise, ε_{xt}^a, and (ii) an idiosyncratic $i.i.d.$ Gaussian noise, ε_{it}^a

$$x_{it}^a = a_t + \varepsilon_{xt}^a + \varepsilon_{ixt}^a: \quad \varepsilon_{xt}^a \sim \mathcal{N}\left(0, \sigma_{a,x}^2\right), \varepsilon_{ixt}^a \sim \mathcal{N}\left(0, \sigma_{a,i}^2\right), a_t \equiv ln(A_t).$$

Each firm faces a quadratic price adjustment cost, as in Rotemberg (1982), and maximizes its profit

$$\Pi_{it} = (1 + T_t^s) P_{it} Y_{it} - W_t N_{it} - \frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 P_t Y_t,$$

where $1 + T_t^s$ is stochastic with mean $\frac{\rho}{1-\rho}$. A mark-up shock is defined as $\mathcal{M}_t \equiv \frac{P_t}{W_t/A_t}$. In a symmetric equilibrium, all firms set the same price, and we have $\mathcal{M}_t = \frac{\rho}{\rho-1}\frac{1}{1+T_t^s}$. This mark-up shock also follows AR(1) process with a $i.i.d.$ Gaussian noise ξ_t

$$\mathcal{M}_t = \mathcal{M}_{t-1}^{\rho_\mu} \exp(\xi_t): \quad \rho_\mu \in (0,1), \xi_t \sim \mathbb{N}\left(0, \sigma_\xi^2\right).$$

Similar to the aggregate technology signal, firms cannot observe the exact signal of markup shock. The markup shock signal for each intermediate firm also contains two noises: (i)an industry level $i.i.d.$ Gaussian noise, ε_{xt}^μ, and (ii) an idiosyncratic $i.i.d.$ Gaussian noise, ε_{it}^μ

$$x_{it}^\mu = \mu_t + \varepsilon_{xt}^\mu + \varepsilon_{ixt}^\mu: \quad \varepsilon_{xt}^\mu \sim \mathcal{N}\left(0, \sigma_{\mu,x}^2\right), \varepsilon_{ixt}^\mu \sim \mathcal{N}\left(0, \sigma_{\mu,i}^2\right), \mu_t \equiv ln(\mathcal{M}_t).$$

The firms' optimization problem gives a New Keynesian Phillips Curve that explains the relationship among current inflation with firms' nowcast of the output gap, nowcast of markup shock, and one-period forecast of inflation

$$\pi_t = \beta \overline{\mathbb{E}}_t^F [\pi_{t+1}] + \lambda \overline{\mathbb{E}}_t^F [y_t - a_t] + \overline{\mathbb{E}}_t^F [\mu_t], \tag{2.2}$$

where $\lambda \equiv \frac{1+\eta}{\psi}\rho$. Inflation is defined as $\pi_t = p_t - p_{t-1}$. y_t is the log deviation of output. a_t represents the natural output rate. Therefore output gap is defined as $y_t - a_t$. The realization of markup shock μ_t is not observable to each firm. Each intermediate firm i forms conditional expectations, $\mathbb{E}^F_{i,t}$, based on its information set $\Omega^F_{i,t}$, and $\overline{\mathbb{E}}^F_t \equiv \int \mathbb{E}^F_{it} di$ denotes the average expectation across firms.

2.3 Monetary Authority

The central bank employs a simple Taylor-type rule which aims at hitting the output gap and inflation target

$$i_t = \phi_y \mathbb{E}^*_t \left[y_t - a_t\right] + \phi_\pi \mathbb{E}^*_t \pi_t + \varepsilon^m_t \tag{2.3}$$

where ϕ_y and ϕ_π measure monetary rule responses to the output gap and inflation, and there is an *i.i.d.* Gaussian monetary policy noise $\varepsilon^m_t \sim \mathbb{N}\left(0, \sigma^2_m\right)$. We assume the central bank is also equipped with incomplete information and cannot observe a complete set of endogenous variables and fundamental realizations.[10] The main contribution of this paper lies in the expectation operator $\mathbb{E}^*_t$. Instead of sticking with its information, different information sets could be applied by the central bank to form expectations of current economic variables, i.e., total output, technology level, and inflation. Private sectors' expectations are policy-wise internalized when the nominal interest rate is set correspondingly to their expectations/beliefs. We thereby call this Taylor-type rule belief-driven.

2.4 Welfare Loss Function

We follow Nisticò (2007) and derive the welfare loss as a second-order approximation of the household utility function with quadratic adjustment costs[11] :

$$\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(1+\eta\right) \tilde{y}^2_t\right] + t.i.p.$$

t.i.p. collects the terms that are independent of policies. η is the inverse of Frisch elasticity. $\tilde{y}_t$ is the log deviation of the output gap. We present the total welfare loss as

[10] Orphanides (2001) has emphasized the difference between the real-time policy recommendations and those obtained with the ex-post revised data, demonstrating the incompleteness of the central bank's information set.

[11] Measured by ψ.

$$\mathbb{L} = \frac{1}{1-\beta}\frac{1+\eta}{2}var(\tilde{y}_t). \tag{2.4}$$

3 Analytical Analysis

In this section, we derive a closed-form solution to a simplified model. The household and firms have different information sets, both containing an endogenous signal and an exogenous signal. The central bank can operate on either of the information sets, and information choices lead to different welfare results. The detailed proof is provided in the appendix.

3.1 Closed-Form Solution

We simplify the model with the following assumption. The simplified model provides an analytical characterization of the economy where information dispersion exists among all sectors. It also highlights a crucial argument that different policy-wise information implementation results in different signal extraction processes and total economic volatility.

Assumption 1. *There is no mark-up shock and its associated industry level and idiosyncratic noises ($\mu_t = 0, \sigma_{\mu,x} = \sigma_{\mu,i} = 0$). There is neither industry-level noise nor idiosyncratic noise of technology for intermediate firms ($\sigma_{a,x} = \sigma_{a,i} = 0$). Aggregate technology shock is i.i.d ($\rho_a = 0$).*

Based on Assumption 1, all remaining shocks ($\varepsilon_t^a, \varepsilon_t^\beta, \varepsilon_t^m$) are *i.i.d.*. Therefore, all one-period forward expectations are essentially zeros. The household's Euler equation is degenerated to

$$y_t = i_t + \varepsilon_t^\beta : \text{with the household's information set } \Omega^{HH} = \{i_t, \varepsilon_t^\beta\}. \tag{3.1}$$

i_t is the interest rate, and it is a public endogenous signal for both households and firms. ε_t^β is the household's private information about exogenous preference shock. Because there are no idiosyncratic noises, we can remove intermediate firms' index i and average expectations in New Keynesian Phillips Curve

$$\pi_t = \lambda \mathbb{E}_t^F [y_t - \varepsilon_t^a] : \text{ with firms' information set } \Omega^F = \{i_t, \varepsilon_t^a\} \tag{3.2}$$

i_t is a public signal, and ε_t^a is firms' private information about exogenous technology shock. We consider two monetary policies and solve them accordingly

$$\textbf{Policy I: } i_t = \phi_y \mathbb{E}_t^F \left[y_t - \varepsilon_t^a \right] + \phi_\pi \mathbb{E}_t^F \pi_t + \varepsilon_t^m$$

$$\textbf{Policy II: } i_t = \phi_y \mathbb{E}_t^{HH} \left[y_t - \varepsilon_t^a \right] + \phi_\pi \mathbb{E}_t^{HH} \pi_t + \varepsilon_t^m.$$

In Policy I, the Taylor rule operates on firms' information set, $\Omega^F = \{i_t, \varepsilon_t^a\}$, to form expectations about current output, technology level, and inflation; Policy II adopts household's information set, $\Omega^{HH} = \{i_t, \varepsilon_t^\beta\}$. The following propositions characterize unique incomplete information equilibrium for both policies in closed form.

Proposition 1. *For Policy I, given Assumption 1 and information structures (3.1) and (3.2), the model features a unique equilibrium where output, inflation, and interest rate follow*

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{3.3}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{3.4}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{3.5}$$

where coefficients $(S_{i,h}, S_\beta, S_{i,f}, S_a, C_\beta, C_m, C_a)$ *are given by*

$$\begin{aligned} & S_{i,h} = -1, S_\beta = 1, S_{i,f} = -\lambda, S_a = -\lambda, \\ & C_\beta = 0, C_m = \frac{1}{1 + \phi_y + \lambda\phi_\pi}, C_a = \frac{\phi_y + \lambda\phi_\pi}{\lambda + \lambda\phi_y + \lambda\phi_\pi}. \end{aligned}$$

Proposition 2. *For Policy II, given Assumption 1 and information structures (3.1) and (3.2), the model features a unique equilibrium where output, inflation and interest rate follow*

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{3.6}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{3.7}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{3.8}$$

where coefficients $(S_{i,h}, S_\beta, S_{i,f}, S_a, C_\beta, C_m, C_a)$ *are given by*

$$S_{i,h} = -1, S_\beta = 1, S_{i,f} = \Delta, S_a = -\lambda,$$
$$C_\beta = \frac{\phi_y}{1+\phi_y+\Delta\phi_\pi}, C_m = \frac{1}{1+\phi_y+\Delta\phi_\pi}, C_a = 0,$$

where $\Delta = \frac{\lambda\phi_y(1+\phi_y)\sigma_\beta^2 - \lambda(\phi_y^2\sigma_\beta^2+\sigma_m^2)}{\phi_y^2\sigma_\beta^2+\sigma_m^2-\lambda\phi_y\phi_\pi\sigma_\beta^2}$.

The closed-form solution straightforwardly helps understand the consequences of the central bank's different choices of information sets when conducting policy decisions. We first focus on the signal extraction process under Policy I. When the central bank operates on firms' information set, they lack information about preference shock. Hence interest rate does not respond to preference shock, i.e., $C_\beta = 0$. Firms can neither extract any useful information of preference shock, i.e., $\mathbb{E}^F[\varepsilon^\beta] = 0$, because both of their signals (i_t, ε_t^a) contain no information about this demand shock [12]. The situation is the same for Policy II.Households cannot extract supply shock information when the central bank adopts the household's information and ignores firms' information. Then, different signal extraction processes no doubt lead to different dynamics for endogenous economic variables. We see coefficients $S_\beta \vee S_a$ as **direct** signaling effects since they are associated with observable exogenous shocks $\varepsilon_t^\beta \vee \varepsilon_t^a$, and $S_{i,h} \vee S_{i,f}$ as **indirect** signaling effects, which are associated with endogenous monetary policy. With Policy I, a positive preference shock **directly** increases output and dose not influence inflation and interest rate. A positive technology shock decreases inflation both **directly** and **indirectly**. Because firms can observe technology shock, a positive technology shock results in a higher output gap, therefore, lower inflation, $S_a < 0$. Firms can also observe a higher interest rate[13], and inflation will also decrease because of $S_{i,f} < 0$. This shock surprisingly decreases output **indirectly** because households observe an increase in the interest rate and $S_{i,h} < 0$. With Policy II, a positive technology shock will directly decrease inflation and not influence output and the interest rate. A positive preference shock influences inflation indirectly, influencing output both directly and indirectly. This influence depends on parameterization since Δ is undetermined.

3.2 Welfare Analysis

Social welfare here is measured by output gap volatility which is determined by its dynamics hence choices of information sets for policy makers. Different welfare loss of both policies are calculated below

[12] Mathematically, the projection of ε_t^β on the information space of $\Omega^F = \{i_t, \varepsilon_t^a\}$ is technically zero.

[13] In Equation (3.5), $C_a > 0$.

$$\textbf{Policy I: } var(y_t - \varepsilon_t^a) = \sigma_\beta^2 + C_m^2\sigma_m^2 + (C_a+1)^2\sigma_a^2 \tag{3.9}$$

$$C_m = \frac{1}{1+\phi_y+\lambda\phi_\pi}, C_a = \frac{\phi_y+\lambda\phi_\pi}{\lambda+\lambda\phi_y+\lambda\phi_\pi}$$

and

$$\textbf{Policy II: } var(y_t - \varepsilon_t^a) = (1-C_\beta)^2\sigma_\beta^2 + C_m^2\sigma_m^2 + \sigma_a^2 \tag{3.10}$$

$$C_\beta = \frac{\phi_y}{1+\phi_y+\Delta\phi_\pi}, C_m = \frac{1}{1+\phi_y+\Delta\phi_\pi}, \Delta = \frac{\lambda\phi_y(1+\phi_y)\sigma_\beta^2 - \lambda(\phi_y^2\sigma_\beta^2+\sigma_m^2)}{\phi_y^2\sigma_\beta^2+\sigma_m^2-\lambda\phi_y\phi_\pi\sigma_\beta^2}.$$

Equations (3.9) and (3.10) document output gap volatility under two policies. It is straightforward to see that different policy information sets deliver different macroeconomic volatility. In traditional New Keynesian literature, the Taylor rule focuses on the output gap and inflation, which mainly involves supply shocks besides the monetary shock. Therefore, variable dynamics and volatility change when we apply the household's information set. When the monetary authority considers household information as their policy regime, they need to realize the possibility of asymmetric information structure between producers and consumers. Consumers may form their expectations of supply shocks using private and public signals instead of direct observation. Different signal extraction process ends up with different welfare loss. Even if the Taylor rule is not directly linked with demand shocks, welfare loss does. Comparing equations (3.9) and (3.10), we can tell that welfare loss bringing from the demand side, σ_β^2, decreases[14] when applying household's information in Taylor rule. Although total volatility is associated with all shocks and parameterization, this explains why we observe lower welfare loss when the Taylor rule operates on the household's information set in the later section.

4 Quantitative Analysis

This section introduces central bank communication errors and numerically solves the model. We first study symmetric information between consumers and producers and then move on

[14]From the coefficient that $(1-C_\beta)^2 < 1$.

to asymmetric information structure.

4.1 Symmetric Information

We introduce the communication error between the private sectors and a central bank from Kohlhas (2021). This section eliminates preference shock, ε_t^β, from the model. The central bank observes noised technology shock and markup shock. Instead, they receive signals (z_t^a and z_t^μ) accordingly.

$$z_t^a = a_t + \varepsilon_{zt}^a : \varepsilon_{zt}^a \sim \mathcal{N}\left(0, \sigma_{z,a}^2\right), \quad z_t^\mu = \mu_t + \varepsilon_{zt}^\mu : \varepsilon_{zt}^\mu \sim \mathcal{N}\left(0, \sigma_{z,\mu}^2\right),$$

where ε_{zt}^a and ε_{zt}^μ are assumed to be *i.i.d.* Gaussian distributed. They are the central bank's observational noises about aggregate technology level and markup shock. Signals z_t^a and z_t^μ would be delivered from the central bank to private sectors, but there exists communication error (i.e., $\varepsilon_{\omega t}^a$ and $\varepsilon_{\omega t}^\mu$). Instead, private sectors receive signals, ω_t^a and ω_t^μ regarding those economic indicators published by the central bank,

$$\omega_t^a = z_t^a + \varepsilon_{\omega t}^a : \varepsilon_{\omega t}^a \sim \mathcal{N}\left(0, \sigma_{\omega,a}^2\right), \quad \omega_t^\mu = z_t^\mu + \varepsilon_{\omega t}^\mu : \varepsilon_{\omega t}^\mu \sim \mathcal{N}\left(0, \sigma_{\omega,\mu}^2\right),$$

where noises $\varepsilon_{\omega t}^a$ and $\varepsilon_{\omega t}^\mu$ are *i.i.d.* Gaussian distributed and signals ω_t^a and ω_t^μ would enter information sets. Last, we include inflation measurement error. We assume that neither the central bank nor the private sectors can observe true inflation. The inflation signal they receive ($\bar{\pi}_t$) contains an *i.i.d.* Gaussian noise ε_t^p

$$\bar{\pi}_t = \pi_t + \varepsilon_t^p : \varepsilon_t^p \sim \mathcal{N}(0, \sigma_p^2).$$

We conclude information sets for the household, firms, and the central bank as follow

$$\Omega_t^{HH} = \Omega_{it}^F = \{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^\infty, \quad \Omega_t^{cb} = \{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^\infty. \tag{4.1}$$

The representative household shares the same information set with firms. They receive their private signals about exogenous technology and markup shocks (x_{it}), public signals (ω_t) of exogenous shocks sent out by the central bank, and public signals of endogenous variables ($\bar{\pi}_t$ and i_t). Public signals are common knowledge and included in central bank's information set. Besides, the central bank receives private signals about exogenous technology and markup shocks (z_t).

For monetary policies, we consider four following cases:

$$\textbf{Benchmark:}\; i_t = \phi_y\left[y_t - a_t\right] + \phi_\pi \pi_t + \varepsilon_t^m,$$

$$\textbf{Policy I: } i_t = \phi_y \mathbb{E}_t^{cb}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{cb}\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy II: } i_t = \phi_y \mathbb{E}_t^{Private}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{Private}\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy III: } i_t = \phi_y \mathbb{E}_t^{cb}[\mathbb{E}_t^{Private}(y_t - a_t)] + \phi_\pi \mathbb{E}_t^{cb}[\mathbb{E}_t^{Private}\pi_t] + \varepsilon_t^m,$$

where $\varepsilon_t^m \sim \mathbb{N}\left(0, \sigma_m^2\right)$ is the exogenous monetary shock.

Benchmark monetary policy follows the traditional framework of complete information. It assumes that a central bank can observe all the realizations of economic variables. A nominal interest rate is set accordingly. However, a central bank is endowed with much less information. In Policy I, we assume the central bank conducts policy with its own information set, i.e., Ω_t^{cb} in (4.1). We denote $\Omega_t^{Private}$ as the private sectors' information set since the household and firms share the same information set. Policy II allows the central bank to internalize private sectors' information by operating on the information set $\Omega_t^{Private}$. Policy III serves as another possibility for information implementation. Higher-order belief $\mathbb{E}_t^{cb}[\mathbb{E}_t^{Private}]$ contains another information filtering process. While obtaining the private sectors' expectations, the central bank filters them with its own information set.[15] It is worth noting that this is different from a measurement error of survey data; a measurement error case is equivalent to Policy II with another exogenous shock.

4.1.1 Solution Method and Parameterization

This incomplete information model involves multiple endogenous signals, i.e., interest rate and inflation, and exogenous signals, i.e., technology signal and markup signal. The symmetric private information structure model contains two conditional expectations, i.e., $\mathbb{E}_t^{private}$ and $\mathbb{E}_t^{cb}$. In the next section, with the introduction of asymmetric information structure between households and firms, the model contains three types of conditional expectations, i.e., $\mathbb{E}_t^{HH}$, $\mathbb{E}^{Firms}$, and $\mathbb{E}_t^{cb}$. Instead of applying time-domain methods in Nimark (2008), we apply the solution method from Han, Tan and Wu (2021a) with the toolbox zTran. Their approach is based on policy function iterations in the frequency domain. The canonical representation can be found in Appendix A

Parameterization is performed before simulations. Table 1 documents conventional parameter values for quarterly models and parameter values for exogenous processes. The discount factor, β, is set as 0.99 and can be interpreted as a quarterly value. The inverse of Frisch elasticity, η, is set to 1. The elasticity of substitution, ρ, among intermediate goods is set to 6. The price adjustment cost, ψ, is established as the slope of NKPC, λ, equal to

[15] In other words, the central bank has less confidence in directly adopting results from survey data and attempts to interpret those results in its own way.

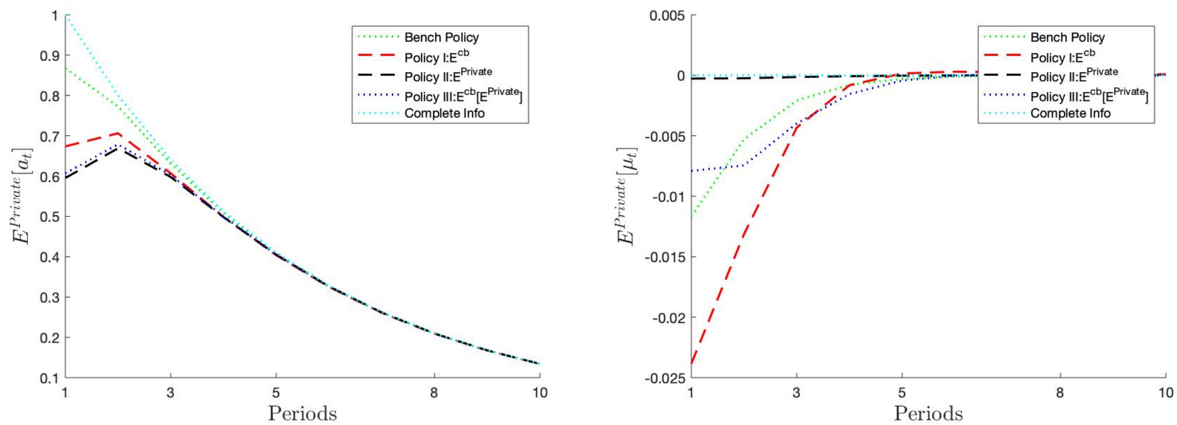
Table 1: Symmetric Model Parameterization

Parameter	Value	Description
Households		
β	0.99	Discount factor
η	1	Inverse of Frisch elasticity
Firms		
ρ	6	Elasticity of substitution
ψ	48	Price adjustment cost
ρ_a	0.8	Persistence of technology shock
ρ_μ	0.7	Persistence of markup shock
σ_θ	0.6	Tech. process white noisy
σ_ξ	0.16	Markup process white noisy
$\sigma_{x,a}$	0.65	Industry tech. noisy
$\sigma_{x,a,i}$	0.2	Firm idiosyncratic tech. noisy
$\sigma_{x,\mu}$	0.2	Industry markup noisy
$\sigma_{x,\mu,i}$	0.11	Firm idiosyncratic markup noisy
$\sigma_{\omega,a}$	1	Industry's policy-signal tech. noisy
$\sigma_{\omega,\mu}$	1	Industry's policy-signal markup noisy
Central Bank		
ϕ_y	1.81	MP response to output gap
ϕ_π	0.5	MP response to inflation
$\sigma_{z,a}$	0.4	Central bank tech. noisy
$\sigma_{z,\mu}$	0.1	Central bank markup noisy
σ_p	0.8	Inflation measurement error
σ_m	0.4	Monetary policy shock

0.25. These are commonly used in the literature. For the Taylor rule coefficients, we follow Kohlhas (2021) and set $\phi_y = 1.81$, and we follow Taylor (1999) to set $\phi_\pi = 0.5$; as Yellen (2012) suggests, monetary policy should respond more to the output gap than inflation. We follow Rudebusch (2002) and set $\sigma_m = 0.4$. The rest parameters values of the technology and markup process are from Kohlhas (2021). We focus on the analysis of information implementation within the monetary policy. We thereby will not calibrate the precision of central bank signals with different values, and we set both central bank's communication noises ($\sigma_{\omega,a}$ and $\sigma_{\omega,\mu}$) to 1.

4.1.2 Now-cast and Forecast Impulse Responses

An Aggregate Technology Shock: We study private sectors' now-cast of exogenous shocks and forecast of endogenous variables with different policies. Figure 2 documents the private sectors' now-cast of technology level, $\mathbb{E}^{Private}[a_t]$, and markup shock, $\mathbb{E}^{Private}[\mu_t]$, to a positive technological innovation. The cyan dotted line corresponds to the complete information setting, where no information friction exists. The green dotted line corresponds to benchmark policy, where central bank is endowed with full information while households and firms receive noised signals. The red dashed line corresponds to Policy I, where nominal interest rate is determined by central bank's information. The black dashed line corresponds to Policy II, where nominal interest rate is determined by private sectors' information. The blue dotted line corresponds to Policy III, where central bank filters private information when deciding nominal interest rate.



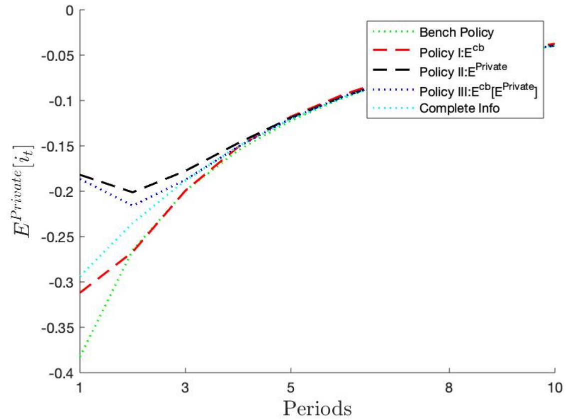
(a) $\mathbb{E}^{Private}[a_t|\varepsilon_t^a]$ (b) $\mathbb{E}^{Private}[\mu_t|\varepsilon_t^a]$

Figure 2: Responses of Private Sectors' Now-cast of Technology Level and Markup Level to a Positive Innovation in Technology

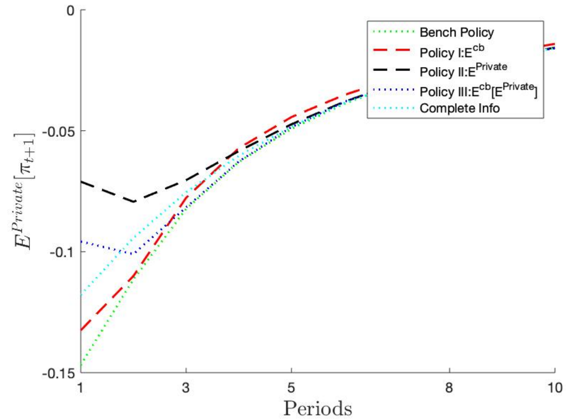
With complete information, a positive technology shock shifts the aggregate supply curve to the right, decreasing the price level. With incomplete information, households and firms receive imperfect observation of shocks. White noises will dampen their beliefs about realized shocks in the first period. Because all white noises are *i.i.d.*, they revise their understanding of fundamentals. Therefore, in Figure 2:a, we observe hump-shape beliefs of aggregate technology shock. When the nominal interest rate responds to the private sectors' information, communication errors serve as additional noises compared with responding to the central bank's information. The private sectors' perception of the realized shock would be further dampened. This explains why black and blue lines are below the red line in Figure 2:a. When central bank implements private sectors' information within monetary policy, all model information sets become nested, and shocks are anticipated for households and firms. They can distinguish aggregate technology shock, a_t, from markup shock, μ_t, and therefore

$$\mathbb{E}^{Private}[\mu_t|\varepsilon_t^a] = 0.$$

In Figure 2:b, the black dashed line does not respond to the aggregate technology innovation.



(a) $\mathbb{E}^{Private}[i_t|\varepsilon_t^a]$



(b)$\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^a]$

Figure 3: Response of Private Sectors' Now-cast Nominal Interest Rate and One-period Inflation Forecast to a Positive Innovation in Technology

Therefore, for the rest policy rules in Figure 2:b, households and firms will place probability on a negative markup innovation when observing a price decrease.

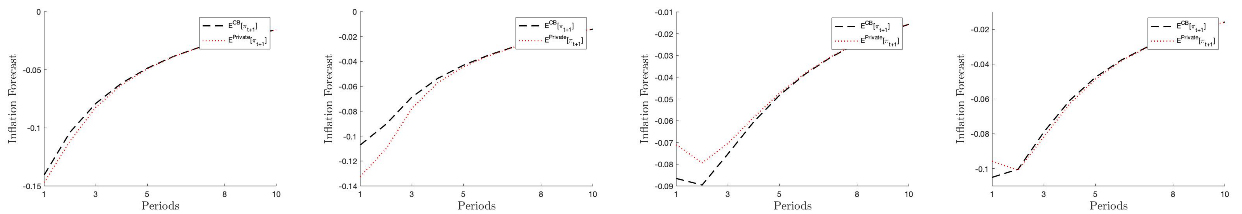
Figure 3 documents the private sectors' now-cast of nominal interest rate and one-period forecast of inflation with one standard deviation positive technology shock under different policy regimes.

With complete information, a positive aggregate technology innovation increases the output gap and decreases the price level, hence decreasing the nominal interest rate. Now-cast of nominal interest rate depends on how households and firms understand signals. With Policy II, the Taylor rule is driven by private sectors' information:

$$i_t = \phi_y \mathbb{E}_t^{Private}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{Private} \pi_t + \varepsilon_t^m.$$

Consistent with the above, households and firms underestimate realized technology shock size because of white noise and communication errors. Therefore, they expect a smaller output gap and a smaller nominal interest rate decrease. With *i.i.d.* noises vanishing in the next period, they will update their beliefs. In Figure 3:a, the black dashed line is hump-shape and responds the least, and it is worth noting that households and firms tend to overestimate when monetary policy works on other information sets. The cyan dotted line represents the private sectors' expectation of nominal interest rate change when we shut down information frictions at every layer. Whether the central bank operates on a full or its own noised information set, households and firms would expect a more significant decrease in nominal interest rate. Green dotted, and red dashed lines are below the cyan dotted line.

Characteristics in their now-cast are also reflected in their forecast. In Figure 3:b, infla-



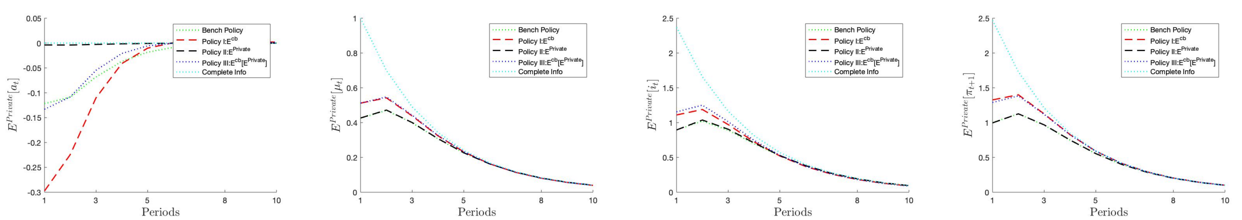
(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

Figure 4: Comparison between Central Bank and Private Sectors' One-period Inflation Forecast to a Positive Aggregate Technology Innovation, i.e., $\mathbb{E}^{CB}[\pi_{t+1}|\varepsilon_t^a]$ and $\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^a]$.

tion forecast also responds the least when the private sectors' information is implemented in monetary policy. They would also overestimate inflation responses when their information is not involved in monetary policy. This implies different policy-wise information implementations may affect the private sectors' inflation forecast.

Our model also generates disagreement in inflation expectations between the private sectors and the central bank. As shown in Figure 4, the black dashed line corresponds to the central bank's inflation forecast, and the red dotted line corresponds to private sectors' inflation forecast. Results show that the private sectors' inflation forecasts are less volatile than the central bank's inflation forecast when the private sectors' information set is involved in policy. This implies that policy-wise informational choices may affect, even rotate, inflation expectation disagreement.

A Markup Shock: Figure 5 documents private sectors' now-cast of technology level, $\mathbb{E}^{Private}[a_t]$, markup level, $\mathbb{E}^{Private}[\mu_t]$, nominal interest rate, $\mathbb{E}^{Private}[i_t]$, and forecast of inflation, $\mathbb{E}^{Private}[\pi_{t+1}]$ to a positive innovation in markup.



(a) $\mathbb{E}^{Private}[a_t|\varepsilon_t^\mu]$ (b) $\mathbb{E}^{Private}[\mu_t|\varepsilon_t^\mu]$ (c) $\mathbb{E}^{Private}[i_t|\varepsilon_t^\mu]$ (d) $\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^\mu]$

Figure 5: Private Sectors' Now-cast and Forecast Responses to a Positive Innovation in Markup.

With complete information, a markup shock increases the price level and the nominal interest rate. Consistent with previous findings, households and firms underestimate the realized shock size, Figure 5:b, and can distinguish shocks with Policy II, Figure 5:a. For Figure 5:c, recall monetary policies

$$\text{Policy I} : i_t = \phi_y \mathbb{E}_t^{CB}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{CB}\pi_t + \varepsilon_t^m$$

$$\text{Policy II} : i_t = \phi_y \mathbb{E}_t^{Private}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{Private}\pi_t + \varepsilon_t^m$$

Their underestimation of actual markup innovation leads to an underestimation of current inflation compared with the complete information case. With Policy I, the non-nested information structure induces private sectors to assign probability on a negative productivity shock after observing an increase in nominal interest rate. Therefore, they form a higher now-cast of nominal interest rate response with Policy I because ϕ_y is greater than ϕ_π. However, their inflation forecast contradicts their now-cast of nominal interest rate. We compare Policy I with Policy II. Private sectors expect a larger increase in the nominal interest rate today with Policy I. They are supposed to expect lower inflation tomorrow. However, in 5:d, private sectors associate higher interest rates today with higher inflation tomorrow. This phenomenon also implies that the private sectors' inflation forecast is anchored when their information is involved within monetary policy.

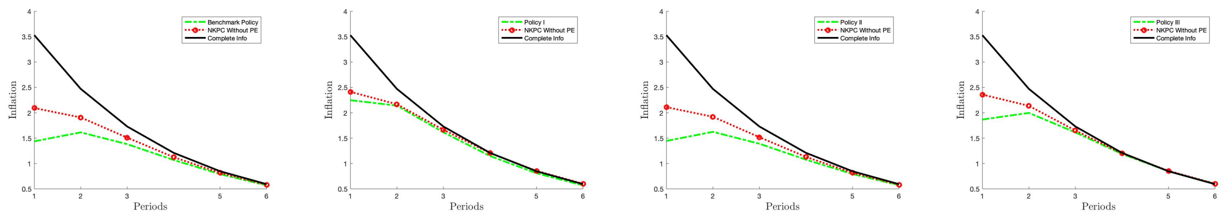
4.1.3 Higher-order Decomposition

We follow Angeletos and Huo (2021), or AH for short, to study a higher-order decomposition in our NKPC's impulse responses. We explore how we can explain the quantitative difference in responses between complete and incomplete information. We also investigate how much is due to a lack of exogenous information (the partial equilibrium (PE) component) or endogenous uncertainty of inflation (the general equilibrium (GE) component). While AH focuses on moment estimation and calibration, our analysis focuses on the effects of different policy choices.

To formalize the idea, we first take a look at equation 2.2:

$$\pi_t = \underbrace{\beta \overline{\mathbb{E}}_t^F\left[\pi_{t+1}\right]}_{\text{GE Component}} + \underbrace{\lambda \overline{\mathbb{E}}_t^F\left[y_t - a_t\right] + \overline{\mathbb{E}}_t^F\left[\mu_t\right]}_{\text{PE component}}. \tag{4.2}$$

We decompose NKPC into two parts: the GE component and the PE component. As in AH, the first part captures GE effects, also see Angeletos and Lian (2018), and the second part captures PE effects. Unlike AH, we shut down the PE component and numerically solve the structure model. It is worth noting that AH assumes the PE component following an AR(1) process to obtain analytical solutions. We drop this assumption when we solve the structural model numerically using the toolbox from Han, Tan and Wu (2021a). Figure



(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

Figure 6: Responses of Inflation to a Positive Innovation in Markup: Isolate Partial Equilibrium Effects in NKPC

6 documents impulse responses of inflation to a positive innovation in markup with the decomposition above in different policy regimes. The solid black line represents the inflation responses in the complete information scenario. The red dotted circled line represents the inflation responses without exogenous uncertainty in NKPC with different policies. The green dashed line represents the inflation responses with different policies.

Quantitative bites of information friction are significant.[16] We first notice that the GE channel dominates the PE channel only with Policy I, i.e. when the central bank's information drives the Taylor rule. This is because the effects of firms' uncertainty of the next period's inflation override the effects of their now-cast of exogenous shocks. This is in line with Angeletos and Huo (2021). To understand this, we plug NKPC into different policies and compare the benchmark policy with Policy II:

$$i_t = \phi_y \mathbb{E}_t^{CB}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{CB}\{\beta \overline{\mathbb{E}}_t^{Private}\left[\pi_{t+1}\right] + \lambda(y_t - a_t) + \mu_t\} + \varepsilon_t^m, \tag{4.3}$$

$$i_t = \phi_y \left[y_t - a_t\right] + \phi_\pi \{\beta \overline{\mathbb{E}}_t^{Private}\left[\pi_{t+1}\right] + \lambda(y_t - a_t) + \mu_t\} + \varepsilon_t^m. \tag{4.4}$$

From Equation (4.3), since firms know the model structure, they know the central bank will filter μ_t with its information set even if firms can observe the actual size of it.[17] Therefore, their information about the markup shock is not as crucial as in Equation (4.4) because μ_t enters the model directly. Also, Figure 6:a and Figure 6:c deliver the same results because if we plug NKPC into monetary policy, μ_t shares the same characteristics as in Equation (4.4).

4.1.4 Social Welfare

Table 2 documents the average period welfare loss associated with each policy. The first row is the benchmark policy. Rows $3-5$ correspond to average period welfare loss with Policy

[16] See the difference between the green dashed line and the black solid line in Figure 6.

[17] We shut down the PE component, so a_t and μ_t become common knowledge for firms.

I-III.

Table 2: Welfare Analysis of Polices

Policy Rule	Welfare Loss
Benchmark Policy	5.2986
$\mathbb{E}^{cb}$	15.3219
$\mathbb{E}^{private}$	23.2522
$\mathbb{E}^{cb}[\mathbb{E}^{private}]$	30.0245

Not surprisingly, the benchmark policy achieves the least welfare loss, where the central bank has full economic information. The interest rate is generated by the most accurate information. The table shows that the central bank should set the monetary rule according to their expectation rather than adopting private sector expectations or further guessing the private sector expectations when private sectors observe the same noisy signals. This is intuitive when we re-examine the formation sets.

$$\Omega_t^{HH} = \Omega_{it}^F = \{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}, \text{ and } ,\Omega_t^{cb} = \{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}.$$

While inflation, $\bar{\pi}_{t-j}$, and the interest rate, i_{t-j}, are common knowledge, private sectors receive private noises, contained in x_{it-j}, and bank disclosure noise, contained in ω_{t-j}. On the other hand, the central bank only receives its private noise, contained in z_{t-j}, because z_{t-j} and ω_{t-j} together will cancel out the bank disclosure noises. Therefore, the central bank's information set is determined to be more "valuable" than the private sectors' information set, both from our analysis and welfare results.

4.2 Asymmetric Information

While keeping communication frictions between firms and the central bank, we take the preference shock, ε_t^{β}, back. We assume households cannot observe technology and mark-up signals, but the demand side shock can only be observed by households. It is natural to assume heterogeneity in information sets for different sectors. Each sector—households, firms and the central bank—has its information structure. However, the results still hold that the monetary authority should rely on private expectations. Thus, we study an optimal weighting strategy for monetary policy given heterogeneous expectations from the private sector.

Household: The corresponding Euler Equation is given as

$$y_t = \mathbb{E}_t^{HH}[y_{t+1}] - \left(i_t - \mathbb{E}_t^{HH}[\pi_{t+1}] - \varepsilon_t^{\beta}\right). \tag{4.5}$$

Firms: Firms share the same properties and information structure as in the previous model.

$$\pi_t = \beta\overline{\mathbb{E}}_t^F[\pi_{t+1}] + \lambda\overline{\mathbb{E}}_t^F[y_t - a_t] + \overline{\mathbb{E}}_t^F[\mu_t], \tag{4.6}$$

Monetary Authority: With heterogeneous information sets, the central bank employs a typical Taylor rule in responding to the output gap and inflation. We consider four cases:

$$\textbf{Benchmark: } i_t = \phi_y[y_t - a_t] + \phi_\pi\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy I: } i_t = \phi_y\mathbb{E}_t^{cb}[y_t - a_t] + \phi_\pi\mathbb{E}_t^{cb}\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy II: } i_t = \phi_y\mathbb{E}_t^{HH}[y_t - a_t] + \phi_\pi\mathbb{E}_t^{HH}\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy III: } i_t = \phi_y\mathbb{E}_t^{Firm}[y_t - a_t] + \phi_\pi\mathbb{E}_t^{Firm}\pi_t + \varepsilon_t^m,$$

where a_t is the log-deviation of potential output, ε_t^m is the monetary shock, and $\varepsilon_t^m \sim \mathbb{N}(0, \sigma_m^2)$.

Information structure: Different from the previous section, we assume each sector observes public signals and sector-related signals. Public signals are inflation with measurement error, $\bar{\pi}_t$, and the nominal interest rate i_t, which are included in all information sets. Preference shock signal exclusively enters the household's information set. Signals about technology and markup enter firms' and the central bank's information sets. Central bank communication error exists between firms and the central bank. We conclude information sets as follow

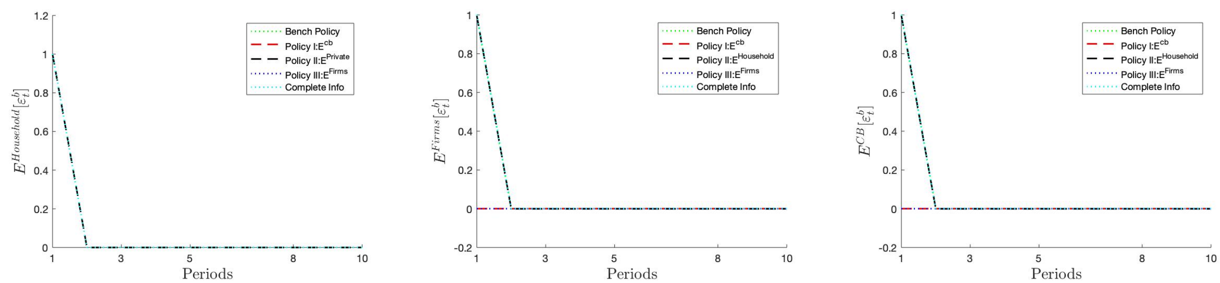
$$\Omega_t^{HH} = \left\{\varepsilon_{t-j}^{\beta}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}, \Omega_{it}^{F} = \{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty} \text{ and } \Omega_t^{cb} = \{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$$

The solution method adopts the frequency-domain policy function iteration approach in Han, Tan and Wu (2021a) with information sets defined above.

4.2.1 Now-cast and Forecast Impulse Responses

In this section, we study households, firms, and the central bank's now-cast and forecast of endogenous and exogenous variables. We begin with demand shock, namely ε_t^{β}, and then move to supply shocks, namely ε_t^a and ε_t^{μ}.

A Preference Shock: Figure 7 documents all sectors' now-cast of preference shock to a positive innovation in preference with different policies. This preference innovation signal only enters households' information set. The cyan dotted line corresponds to the complete information setting, where no information friction exists. The green dotted line corresponds to benchmark policy, where the central bank is endowed with full information while households and firms receive noised signals. The red dashed line corresponds to Policy I, where the central bank's information determines the nominal interest rate. The black dashed line corresponds to Policy II, where households' information determines the nominal interest rate. The blue dotted line corresponds to Policy III, where firms' information determines the nominal interest rate.



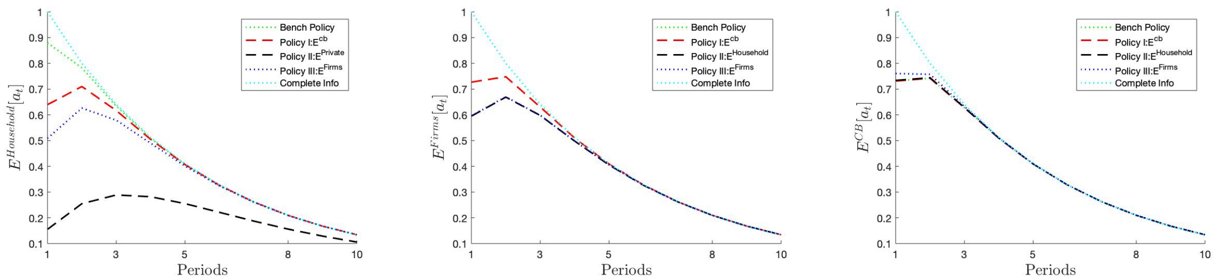
(a) $\mathbb{E}^{Households}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$ (b) $\mathbb{E}^{Firms}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$ (c) $\mathbb{E}^{CB}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$

Figure 7: Now-cast of Preference Shock to a Positive Innovation in Preference

Responses in Figure 7 accord with our theoretical analysis in Section 3. With this non-nested incomplete information environment, firms and the central bank are not endowed with the preference innovation signal. When implementing their information sets in monetary policy, the nominal interest rate would not respond to a preference shock, as shown in Proposition 1 where $C_{\beta} = 0$. They would not learn about the realized preference shock unless the nominal interest rate responds to that shock. When we adopt households' information set in the policy, the movement of nominal interest rate informs firms and central bank about *some* fundamental realization. Combined with their own information sets, they can infer the realization of a preference innovation.

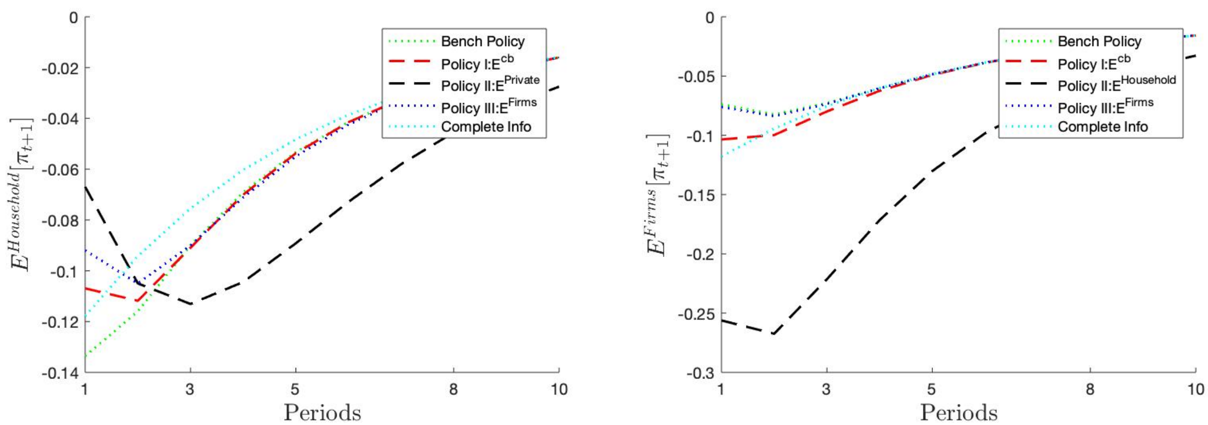
An Aggregate Technology Shock: Figure 8 documents all sectors' now-cast of technology level to a positive innovation in aggregate technology with different policies.

In Figure 8:b and Figure 8:c, the central bank forms a more precise now-cast of technology innovation than firms', which implies a dominance of the central bank's information over firms' information, because the central bank only receives noises at the aggregate level. In contrast, firms receive noises at the aggregate level, idiosyncratic level and communication error. Figure 8:a documents the learning process of households. Firms and the central



(a) $\mathbb{E}^{Households}[a_t|\varepsilon_t^a]$ (b) $\mathbb{E}^{Firms}[a_t|\varepsilon_t^a]$ (c) $\mathbb{E}^{CB}[a_t|\varepsilon_t^a]$

Figure 8: Now-cast of Technology Shock to a Positive Innovation in Technology

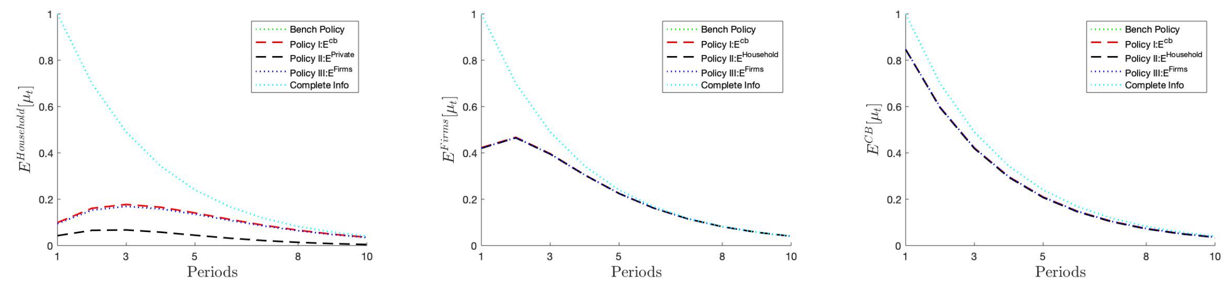


(a) $\mathbb{E}^{Household}[\pi_{t+1}|\varepsilon_t^a]$ (b)$\mathbb{E}^{Firms}[\pi_{t+1}|\varepsilon_t^a]$

Figure 9: Households and Firms' Inflation Forecast to a Positive Innovation in Technology

bank are both endowed with technology signals. When implementing their information sets, the nominal interest rate movement contains information about technology innovation. Households are thereby able to infer the realized innovation. Figure 8:a shows households can better infer the realization with the implementation of the central bank's information set than firms', which is consistent with our argument. Households may not update their beliefs about the realized technology innovation if the monetary policy contains no information about it, i.e., implementing households' own information set.

Figure 9 documents households' and firms' inflation forecast to a positive technology innovation. Inflation forecast disagreement also exists between households and firms. Households' inflation forecast is more stable than firms'. Significantly, firms expect a significant inflation decrease when the policy information set contains no technology signal, i.e., Policy II. In such a scenario, firms expect a mild response of nominal interest rate, which is not enough to curb deflation. The lack of technology signal within policy rule also prolongs economic recovery.



(a) $\mathbb{E}^{Households}[\mu_t|\varepsilon_t^{\mu}]$ (b) $\mathbb{E}^{Firms}[\mu_t|\varepsilon_t^{\mu}]$ (c) $\mathbb{E}^{CB}[\mu_t|\varepsilon_t^{\mu}]$

Figure 10: Now-cast of Markup Shock to a Positive Innovation in Markup

A Markup Shock: Figure 10 documents all sectors' now-cast of markup level to a positive innovation in markup with different policies. Firms and the central bank can better infer realized markup innovation than households. However, households are worse at recovering markup innovation than technology innovation with all information implementation. Because markup innovation is not directly involved in policy rules. The nominal interest rate movement contains less information on markup innovation than technology for households to update their beliefs.

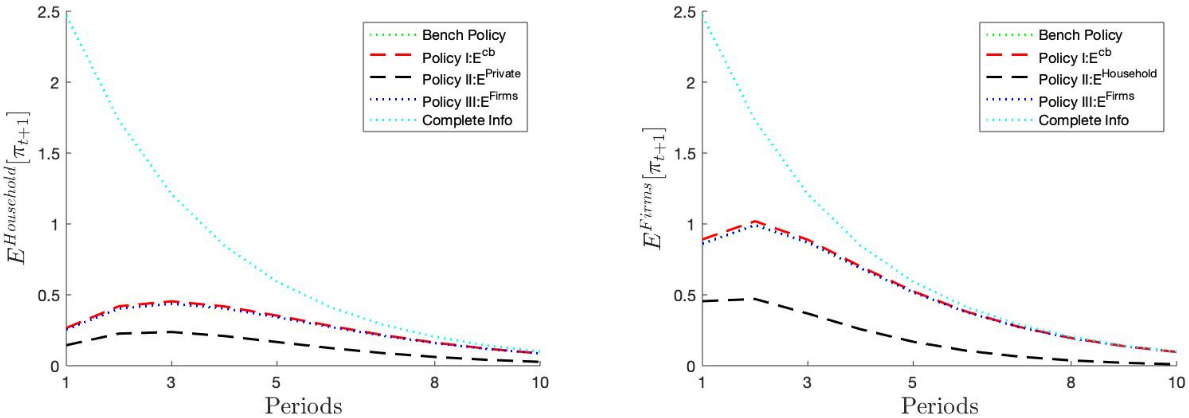
Figure 11 documents households' and firms' inflation forecast to a positive markup innovation. Information frictions dampen households and firms' inflation forecast. Their forecast is further anchored when the nominal interest rate responds to households' information set. This is an exciting finding. Recall Policy II:

$$i_t = \phi_y \mathbb{E}_t^{HH}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{HH}\pi_t + \varepsilon_t^m \text{ with: } \Omega_t^{HH} = \left\{\varepsilon_{t-j}^{\beta}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}.$$

When households do not receive the markup signal, firms would expect a mild nominal interest rate response to a positive markup innovation. In principle, this response would not be enough to curb inflation in the next period. Therefore, firms should expect higher inflation with Policy II than the other, which is different from their forecast in Figure 11:b. For anchoring private sectors' inflation forecast, this result demonstrates the importance of households'/demand side information.

4.2.2 Weighted Strategy in Taylor Rule

Households' expectation matters in anchoring the private sectors' expectations with previous policy rules. Is this "anchoring effect" robust? Because, practically, monetary decisions would not be made solely corresponding to specific expectations. To answer such a question, we study another policy rule that includes both the central bank and households' expecta-



(a) $\mathbb{E}^{Household}[\pi_{t+1}|\varepsilon_t^{\mu}]$ (b)$\mathbb{E}^{Firms}[\pi_{t+1}|\varepsilon_t^{\mu}]$

Figure 11: Households and Firms' Inflation Forecast to a Positive Innovation in Markup

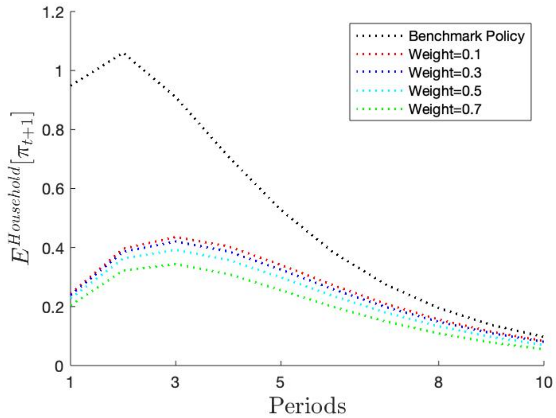
tions. The relative importance of households' expectations is variant, which is captured by coefficient $\omega_{HH} \in [0, 1]$. This coefficient ω_{HH} can be understood as the attention the central bank pays to households' now-cast.

$$i_t = \omega_{HH}\left[\phi_y \mathbb{E}_t^{HH}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{HH}\pi_t\right] + (1 - \omega_{HH})\left[\phi_y \mathbb{E}_t^{CB}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{CB}\pi_t\right] + \varepsilon_t^m.$$

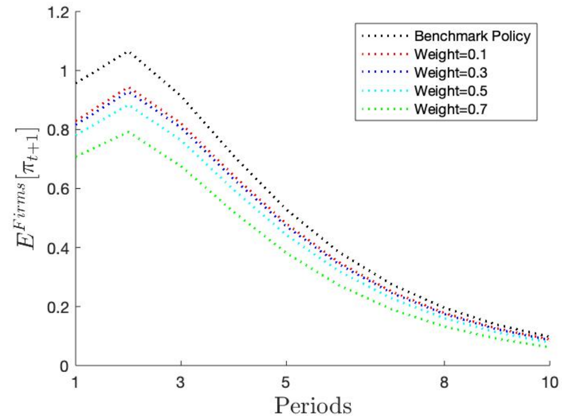
The central bank is stuck with its now-cast while considering households' expectations.[18] We provide a potential answer to concerns in Coibion et al. (2020), where they argue that the limitation of firm-level surveys is the main reason why expectations should not be considered as a policy tool. We calibrate ω_{HH}, and Figure 12 documents households and firms' inflation forecast with respect to a positive markup shock innovation.

The black dotted line represents $\omega_{HH} = 0$, meaning the central bank does not consider households' expectations. When ω_{HH} increases, the results in Figure 12 approach what is in Figure 11, demonstrating consistency in the solution method. An exciting and crucial finding is that the private sectors' inflation forecast drops significantly even with $\omega_{HH} = 0.1$. This validates the distinction between the exclusion and inclusion of private information when conducting monetary policy. The responsiveness of the nominal interest rate to the private sectors' expectations helps anchor the private sectors' inflation forecast.

[18]Central bank would not consider firms' information because, in the previous section, we observe a dominance of central bank's information over firms'.



(a) $\mathbb{E}^{Household}[\pi_{t+1}|\varepsilon_t^{\mu}]$



(b)$\mathbb{E}^{Firms}[\pi_{t+1}|\varepsilon_t^{\mu}]$

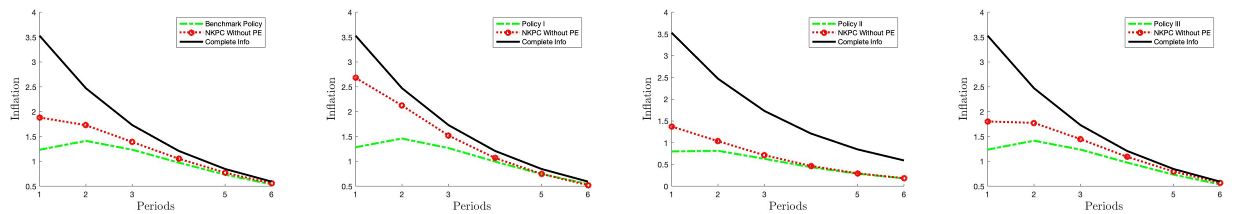
Figure 12: Households and Firms' Inflation Forecast to a Positive Innovation in Markup with Various ω_{HH}.

4.2.3 Higher-order Decomposition

We study the information acquisition problem for firms between exogenous information (the PE component) and endogenous uncertainty (the GE component) under the framework of asymmetric information:

$$\pi_t = \underbrace{\beta\overline{\mathbb{E}}_t^F[\pi_{t+1}]}_{\text{GE Component}} + \underbrace{\lambda\overline{\mathbb{E}}_t^F[y_t - a_t] + \overline{\mathbb{E}}_t^F[\mu_t]}_{\text{PE component}}.$$

Figure 13 documents inflation impulse responses to a positive innovation in markup with different policies. The black solid lines represent complete information cases. The red circle-dotted lines represent cases in which we eliminate the uncertainty of partial equilibrium effects (PE component), meaning that the quantitative difference between the black solid line and the red circled-dotted line is due to uncertainty in $\mathbb{E}_t[\pi_{t+1}]$ (GE component). Finally, the green dashed lines are the responses where both the PE and GE components exist. We shall now compare each policy in Figure 13 with Figure 6 to understand how different information structures affect firms' myopia and anchoring.



(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

Figure 13: Responses of Inflation to a Positive Innovation in Markup: Isolate Partial Equilibrium Effects in NKPC

The benchmark policy delivers similar results in both symmetric and asymmetric information settings. This means that the asymmetric information setting makes an insignificant difference to firms' informational acquisition between exogenous signals and endogenous beliefs when monetary authority is endowed with perfect information. With Policy I, where the central bank's information drives the Taylor rule, the PE component generates a significant quantitative bite compared to Figure: 6:b. This means that firms' uncertainty about the current economy largely influences their behavior when they know that the central bank operates its information. While with Policy II and Policy III, the overall responses are milder because of the increase in signal structure complexity, the ratio between the GE effects and PE effects is similar to those in Figure 6:c and Figure 6:d.

4.2.4 Social Welfare

Table 3 documents the average period welfare loss associated with each policy concerning different preference shock variances. The complete information case represents no information friction, and the benchmark policy is where the Taylor rule is driven by full information.

Unlike the previous section, we now have asymmetric information between private sectors. It is clear that the total volatility increases when the information structure becomes more complicated; therefore, the welfare loss, or the total volatility, increases in all scenarios. Similar to the symmetric information setting, the central bank's information is more valuable than the firms' since firms still observe noisy bank signals. Thus, we have a lower welfare loss when we execute the Taylor rule with the central bank's information rather than the firms' information. However, the salient feature here is the importance of household information. When the preference shock variance is slight, adopting the household expectation is not very beneficial because if we apply their expectation to the Taylor rule, we ignore other supply signals. However, once this preference shock variance approaches or surpasses other shock variance, it becomes influential and dominates either firms' or the central bank's information. When considering the household information within the Taylor rule, the average

Table 3: Welfare Analysis of Polices

Policy Rule	Welfare Loss
$\sigma^2_\beta = 0.5^2$	
Complete Info	6.80
Benchmark Policy	9.15
$\mathbb{E}^{cb}$	42.32
$\mathbb{E}^{HH}$	69.46
$\mathbb{E}^{Firm}$	50.61
$\sigma^2_\beta = 1^2$	
Complete Info	15.51
Benchmark Policy	17.85
$\mathbb{E}^{cb}$	117.32
$\mathbb{E}^{HH}$	78.23
$\mathbb{E}^{Firm}$	125.61
$\sigma^2_\beta = 2^2$	
Complete Info	50.33
Benchmark Policy	52.68
$\mathbb{E}^{cb}$	417.31
$\mathbb{E}^{HH}$	113.07
$\mathbb{E}^{Firm}$	425.61
$\sigma^2_\beta = 3^2$	
Complete Info	108.38
Benchmark Policy	110.72
$\mathbb{E}^{cb}$	917.32
$\mathbb{E}^{HH}$	171.12
$\mathbb{E}^{Firm}$	925.61
$\sigma^2_\beta = 4^2$	
Complete Info	189.64
Benchmark Policy	191.98
$\mathbb{E}^{cb}$	1617.30
$\mathbb{E}^{HH}$	252.38
$\mathbb{E}^{Firm}$	1625.60

period welfare loss drops significantly. The polarization between Policy II and the rest increases as the preference shock variance increases. This result suggests that when there are heterogeneous shocks and information signals for different sectors, the relatively efficient way to stabilize the economy is to adopt the household's information. This implies that the central bank should learn more about the household's expectations.

Candia, Coibion and Gorodnichenko (2020) argue the importance of communication between the monetary authority and the public. Our model suggests that: in order to stabilize the economy, it is crucial to convince the public that the nominal interest rate is determined by the households' understanding of current economic conditions and the weakened role of the monetary authority. This is understandable since the central bank serves neither the supply side nor the demand side. It is the private sector that drives the value-added process. To lower the economy's aggregate volatility by anchoring expectations, the central bank needs to study the private sectors' information. Moreover, household information surprisingly dominates firms'.

Coibion et al. (2020) argue that inflation expectation should not be used as a policy tool mainly based on two reasons. First, the private sectors will respond less to policy announcements in a low-inflation environment. Second, firms' survey data need to be more comprehensive. This paper provides possible answers to both of these problems. First, other than "announcements," the monetary policy associated with private sectors' expectations becomes "endogenous" for the private sectors. This is fundamentally different from "announcing" an inflation target to the market, which was viewed as a "tool" to influence their expectation. Our previously discussed results solve the second concern: we do not need comprehensive firm-level survey data through the domination of household information; thus, consumer survey data is satisfactory.

We exercise alternative information sets based on Policy I to understand further how information affects welfare. All changes in information sets are parallel. We first eliminate the communication noises between the central bank and private sectors. We allow both private parties to receive the central bank's signals of technology and markup, z_t^a and z_t^μ, and public signals the central bank sends out, ω_t^a and ω_t^μ. Therefore, in Case I, we have the information structure as:

Case I	Information Set
Households	$\left\{z_{t-j}, \omega_{t-j}, \varepsilon^{\beta}_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$
Firms	$\{z_{t-j}, \omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$
Central Bank	$\{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$

In Case II, we keep communication errors between the central bank and firms but allow the central bank to observe actual realizations about technology and markup shocks (a_t and μ_t). Information structure is

Case II	Information Set
Households	$\left\{\varepsilon_{t-j}^{\beta}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$
Firms	$\{\omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$
Central Bank	$\{a_t, \mu_t, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$

In Case III, we introduce a noised signal of preference shock to the central bank. The central bank now receives a noised signal of the preference shock ε_t^{β} as $\tilde{\varepsilon}_t^{\beta}$. Information structure is

Case III	Information Set
Households	$\left\{\varepsilon_{t-j}^{\beta}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$
Firms	$\{\omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$
Central Bank	$\left\{\tilde{\varepsilon}_t^{\beta}, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$

In Case IV, we combine Case II and Case III to verify that more information generates less welfare loss

Case IV	Information Set
Households	$\left\{\varepsilon_{t-j}^{\beta}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$
Firms	$\{\omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$
Central Bank	$\left\{\tilde{\varepsilon}_t^{\beta}, a_t, \mu_t, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$

Table 4 documents different welfare changes concerning different signal structures.[19] We first notice that, within Case I, welfare gains from eliminating communication frictions are relatively small under asymmetric information. Even when both private sectors can fully observe "announcement noises" from the central bank, the welfare increase is around 0.130%. We also find that, within Case II, when the central bank is fully endowed with supply-side signals, the welfare increase will be approximately 0.748%, which is almost six times the welfare increase from eliminating communication noises. This shows that the actual information friction regarding real economic fundamentals is more important than "artificial"

[19]We focus on $\sigma_{\beta}^2 = 4^2$.

friction, such as communication friction. With Case III, we find that information from demand-side shock (the preference shock) causes a significant increase in welfare. [20] This supports our previous findings. As discussed earlier, when the households' information drives the Taylor rule, the welfare loss falls to the lowest at 252.38. Now, if the central bank ever considers this household's private information, even with noises, the corresponding public signal i_t they sent out would primarily benefit the economy. In the end, Case IV supports our arguments that the more information the central bank considers, the better public signal it can send out.

Table 4: Changes in Information Sets and Welfare Loss

Signal Structures	Welfare Loss
Policy I: $\mathbb{E}^{cb}$	1617.30
Case I	1615.2
Case II	1605.2
Case III	238.12
Case IV	229.23

5 Conclusion

This paper employs a New Keynesian dynamic stochastic general equilibrium (DSGE) model with non-nested information to extend information dispersion to all sectors. Households receive private signals about individual preference shocks. Firms receive idiosyncratic signals about technology shock and markup shocks, and central bank announcement signals. A central bank receives signals about aggregate technology shock and markup shock and delivers associated signals to firms with communication errors. We introduce a belief-driven Taylor rule to internalize private sectors' expectations policy-wise. Theoretically, we show that policy-wise information choices determine signal extraction processes about exogenous shocks and thereby determine dynamics of endogenous variables and macro-volatility. Numerically, we study the utilization of each sector's information set and specific signals. We find increasing importance in demand-side information sets when the economy's information structure becomes more complicated.

[20] It is worth noting that the central bank observes the "noisy" signal.

References

Andrade, P., R. K. Crump, S. Eusepi, and E. Moench (2016): "Fundamental disagreement," *Journal of Monetary Economics*, 83, 106–128.

Angeletos, G., and Z. Huo (2021): "Myopia and Anchoring," *American Economic Review*, 111(4), 1166–1200.

Angeletos, G.-M., and C. Lian (2018): "Forward Guidance without Common Knowledge," *American Economic Review*, 108(9), 2477–2512.

Binder, C., W. Janson, and R. Verbrugge (2022): "Out of Bounds: Do SPF Respondents Have Anchored Inflation Expectations?," *Journal of Money, Credit and Banking*.

Candia, B., O. Coibion, and Y. Gorodnichenko (2020): "Communication and the Beliefs of Economic Agents," Economic policy symposium, Federal Reserve Bank of Kansas City.

Carboni, G., and M. Ellison (2011): "Inflation and output volatility under asymmetric incomplete information," *Journal of Economic Dynamics and Control*, 35, 40–51.

Coibion, O., and Y. Gorodnichenko (2012): "Why are target interest rate changes so persistent?," *American Economic Journal: Macroeconomics*.

Coibion, O., S. Kumar, M. Pedemonte, and Y. Gorodnichenko (2020): "Inflation Expectations as a Policy Tool?," *Journal of International Economics*, 124.

Dovern, J., U. Fritsche, and J. Slacalek (2012): "Disagreement among forecasters in G7 countries," *Review of Economics and Statistics*, 94(4), 1081–1096.

Falck, E., M. Hoffmann, and P. Hürtgen (2021): "Disagreement about inflation expectations and monetary policy transmission," *Journal of Monetary Economics*, 118, 15–31.

Gorodnichenko, Y., and O. Coibion (2012): "What Can Survey Forecasts Tell Us about Information Rigidities?," *Journal of Political Economy*, 120(1), 116–159.

Gorodnichenko, Y., O. Coibion, and B. Candia (forthcoming): "The Macroeconomic Expectations of Firms," *Handbook of Economic Expectations*.

GORODNICHENKO, Y., O. COIBION, AND M. WEBER (forthcoming): "Monetary policy communications and their effects on household inflation expectations," *Journal of Political Economy*.

HAN, Z., F. TAN, AND J. WU (2021a): "Analytic Policy Function Iteration," *Journal of Economic Theory*.

——— (2021b): "Learning from Monetary and Fiscal Policy," Working Paper.

KOHLHAS, A. N. (2021): "Learning by Sharing: Monetary Policy and Common Knowledge," *American Economic Journal: Macroeconomics*.

MELOSI, L. (2017): "Signalling Effects of Monetary Policy," *The Review of Economic Studies*, 84(2), 853–884.

MORRIS, S., AND H. S. SHIN (2002): "The Social Value of Public Information," *American Economic Review*, 92, 1521–1534.

NAKAMURA, E., AND J. STEINSSON (2018): "High-frequency identification of monetary non-neutrality: the information effect," *The Quarterly Journal of Economics*, 133(3), 1283–1330.

NIMARK, K. (2008): "Dynamic pricing and imperfect common knowledge," *Journal of Monetary Economics*, 55(2), 365 – 382.

NISTICÒ, S. (2007): "The welfare loss from unstable inflation," *Economics Letters*, 96, 51–57.

ORPHANIDES, A. (2001): "Monetary Policy Rules Based on Real-Time Data," *American Economic Review*, 91(September), 964–985.

ROTEMBERG, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90(December), 1187–1211.

RUDEBUSCH, G. D. (2002): "Term structure evidence on interest rate smoothing and monetary policy inertia," *Journal of Monetary Economics*, 49, 1161–1187.

TAYLOR, J. B. (ed.) (1999): *Monetary Policy Rules*. University of Chicago Press, Chicago.

YELLEN, J. L. (2012): "The Economic Outlook and Monetary Policy," Remarks by Vice

Chair Janet L. Yellen At the Money Marketeers of New York University, New York, New York.

Appendices

A Proof of Propositions

This section provides solutions to both propositions.

A.1 Policy I

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{A.1}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{A.2}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{A.3}$$

where $s_t^i = i_t = -y_t + \varepsilon_t^\beta$.

From firms' NKPC and the Taylor rule, we have

$$i_t = \frac{\phi_y + \lambda \phi_\pi}{\lambda} \pi_t + \varepsilon_t^m. \tag{A.4}$$

Denote $\Phi \equiv \frac{\phi_y + \lambda \phi_u}{\lambda}$, we have

$$i_t = \Phi(S_{i,f} i_t + S_a \varepsilon_t^a) + \varepsilon_t^m \implies i_t = \frac{1}{1 - \Phi S_{i,f}} (\Phi S_a \varepsilon_t^a + \varepsilon_t^m), \tag{A.5}$$

therefore we have

$$C_\beta = 0, C_m = \frac{1}{1 - \Phi S_{i,f}}, C_a = \frac{\Phi}{1 - \Phi S_{i,f}}. \tag{A.6}$$

Notice that

$$s_t = \begin{bmatrix} s_t^i \\ s_t^a \end{bmatrix} = \begin{bmatrix} i_t \\ \varepsilon_t^a \end{bmatrix} = \underbrace{\begin{bmatrix} C_\beta & C_m & C_a \\ 0 & 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \varepsilon_t^\beta \\ \varepsilon_t^m \\ \varepsilon_t^a \end{bmatrix}}_{\varepsilon_t}, \tag{A.7}$$

and

$$\pi_t = \lambda \mathbb{E}_t^F y_t - \lambda \varepsilon_t^a = -\lambda i_t - \lambda \varepsilon_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta = -\lambda s_t^i - \lambda s_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta \tag{A.8}$$

By Gaussian projection, we have

$$\pi_t = S_{i,f} s_t^i + S_a s_t^a = -\lambda s_t^i - \lambda s_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta, \tag{A.9}$$

$$(S_{i,f} \quad S_a) = [-\lambda \quad -\lambda] + \lambda \Sigma_{\beta s} \Sigma_s^{-1} \tag{A.10}$$

where

$$\Sigma_{\beta s} = [1 \quad 0 \quad 0] \begin{bmatrix} \sigma_\beta^2 & 0 & 0 \\ 0 & \sigma_m^2 & 0 \\ 0 & 0 & \sigma_a^2 \end{bmatrix} \begin{bmatrix} C_\beta & 0 \\ C_m & 0 \\ C_a & 1 \end{bmatrix} = \begin{bmatrix} C_\beta \sigma_\beta^2 & 0 \end{bmatrix}, \tag{A.11}$$

and

$$\Sigma_s = B\Sigma B' = \begin{bmatrix} C_\beta^2\sigma_\beta^2 + C_m^2\sigma_m^2 + C_a^2\sigma_a^2 & C_a\sigma_a^2 \\ C_a\sigma_a^2 & \sigma_a^2 \end{bmatrix}. \tag{A.12}$$

Notice that $C_\beta = 0$, then

$$\Sigma_{\beta s} = [0 \quad 0], \quad [S_{i,f} \quad S_a] = [-\lambda \quad -\lambda]. \tag{A.13}$$

Also we have $\mathbb{E}_t^F \varepsilon_t^\beta = \frac{\lambda + S_{i,f}}{\lambda} i_t = 0$.

Therefore

$$\pi_t = \lambda s_t^i - \lambda s_t^a = \lambda i_t - \lambda \varepsilon_t^a, \tag{A.14}$$

Then we move on to output

$$y_t = S_{i,h} s_t^i + S_\beta s_t^\beta = S_{i,h} i_t + S_\beta \varepsilon_t^\beta \implies y_t = -i_t + \varepsilon_t^\beta, \tag{A.15}$$

which gives

$$\begin{bmatrix} C_\beta & C_m & C_a \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{1-\Phi S_{i,f}} & \frac{\Phi}{1-\Phi S_{i,f}} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{1+\phi_y+\lambda\phi_\pi} & \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\phi_\pi)} \end{bmatrix} \tag{A.16}$$

We reach the solution that

$$y_t = -i_t + \varepsilon_t^{\beta} = \varepsilon^{\beta} - \frac{1}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m - \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\lambda\phi_\pi)}\varepsilon_t^a \tag{A.17}$$

$$\pi_t = -\lambda i_t - \lambda\varepsilon_t^a = -\frac{\lambda}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m - \lambda(1+\frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\lambda\phi_\pi)})\varepsilon_t^a \tag{A.18}$$

$$i_t = \frac{1}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m + \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\phi_\pi)}\varepsilon_t^a. \tag{A.19}$$

A.2 Policy II

$$y_t = S_{i,h}i_t + S_\beta\varepsilon_t^\beta, \tag{A.20}$$

$$\pi_t = S_{i,f}i_t + S_a\varepsilon_t^a, \tag{A.21}$$

$$i_t = C_\beta\varepsilon_t^\beta + C_m\varepsilon_t^m + C_a\varepsilon_t^a, \tag{A.22}$$

where $s_t^i = i_t = -y_t + \varepsilon_t^\beta$.

Combine y_t and π_t, we have

$$\pi_t = \lambda\mathbb{E}_t^F(-i_t + \varepsilon_t^\beta) - \lambda\varepsilon_t^a = -\lambda i_t - \lambda\varepsilon_t^a + \lambda\mathbb{E}_t^F\varepsilon_t^\beta. \tag{A.23}$$

Then

$$-\lambda i_t - \lambda\varepsilon_t^a + \lambda\mathbb{E}_t^F\varepsilon_t^\beta = S_{i,f}i_t + S_a\varepsilon_t^a \implies \mathbb{E}_t^F\varepsilon_t^\beta = \frac{\lambda+S_{i,f}}{\lambda}i_t. \tag{A.24}$$

Using Gaussian projection

$$[S_{i,f} \quad S_a] = [-\lambda \quad -\lambda] + \lambda\Sigma_{\beta s} = \begin{bmatrix} -\lambda & -\lambda \end{bmatrix} + \lambda\begin{bmatrix} c_\beta\sigma_\beta^2 & 0 \end{bmatrix}\Sigma_s^{-1}, \tag{A.25}$$

where

$$
\Sigma_s = \begin{bmatrix} C_\beta^2 \sigma_\beta^2 + C_m^2 \sigma_m^2 + C_a^2 \sigma_a^2 & C_a \sigma_a^2 \\ C_a \sigma a^2 & \sigma_a^2 \end{bmatrix}. \tag{A.26}
$$

Therefore, we have

$$
S_a = -\lambda. \tag{A.27}
$$

Combine (A.20), (A.21), (A.22) and (A.24),

$$
\begin{aligned}
i_t &= \phi_y \mathbb{E}_t^{HH}[y_t - \varepsilon_t^a] + \phi_\pi \mathbb{E}_t^{HH}[\lambda \mathbb{E}_t^F y_t - \lambda \varepsilon_t^a] + \varepsilon_t^m \\
&= -\phi_y i_t + \phi_y \varepsilon_t^\beta - \phi_y \mathbb{E}_t^{HH} \varepsilon_t^a + \phi_\pi \mathbb{E}_t^{HH}[\lambda(-i_t + \mathbb{E}_t^F \varepsilon_t^\beta - \lambda \varepsilon_t^a] + \varepsilon_t^m \\
&= -\left(\phi_y + S_{i,f}\phi_\pi\right) i_t + \phi_y \varepsilon_t^\beta - \left(\phi_y + \lambda\phi_\pi\right) \mathbb{E}_t^{HH} \varepsilon_t^a + \varepsilon_t^m.
\end{aligned}
$$

which implies

$$
i_t = \frac{1}{\Phi_2}(\phi_y \varepsilon_t^\beta - (\phi_y + \lambda\phi_\pi)\mathbb{E}_t^{HH} \varepsilon_t^a + \varepsilon_t^m). \tag{A.28}
$$

Notice that $\mathbb{E}_t^F \varepsilon_t^\beta = \frac{\lambda + S_{i,f}}{\lambda} i_t$, and then the projection of ε_t^β on ε_t^a should be zero. We set that $\mathbb{E}_t^{HH} \varepsilon_t^a = \Phi_a i_t$.

From (A.28), we have

$$i_t = \frac{1}{\Phi_2 \Phi_3} \phi_y \varepsilon_t^\beta + \frac{1}{\Phi_2 \Phi_3} \varepsilon_t^m \tag{A.29}$$

where

$$\Phi_3 = 1 + \frac{\phi_y + \lambda \phi_\pi}{\Phi_2} \Phi_a. \tag{A.30}$$

We can tell that $i_t \perp \varepsilon_t^a$, then $\mathbb{E}_t^{HH} \varepsilon_t^a = 0$ and $\Phi_a = 0$.

Hence we have

$$i_f = \frac{\phi_y}{\Phi_2} \varepsilon_t^\beta + \frac{1}{\Phi_2} \varepsilon_t^m \tag{A.31}$$

where

$$C_\beta = \frac{\phi_y}{\Phi_2}, \quad C_m = \frac{1}{\Phi_2}, \quad C_a = 0. \tag{A.32}$$

Also,

$$\Sigma_S = \begin{bmatrix} C_c^2 \sigma_\beta^2 + C_m^2 \sigma_m^2 & 0 \\ 0 & \sigma_a^2 \end{bmatrix}. \tag{A.33}$$

From (A.25),

$$
\begin{aligned}
S_{i,f} &= -\lambda + \lambda \frac{C_\beta \sigma_\beta^2}{C_\beta^2 \sigma_\beta^2 + C_m^2 \sigma_m^2} \\
&= -\lambda + \lambda \frac{\frac{\phi_y}{\Phi_2} \sigma_\beta^2}{\frac{\phi_y^2}{\Phi_2^2} \sigma_\beta^2 + \frac{1}{\Phi_2^2 \sigma_m^2}} \\
&= -\lambda + \lambda \frac{\phi_y \Phi_2 \sigma_\beta^2}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2} \\
&= -\lambda + \frac{\lambda \phi_y \phi_\pi \sigma_\beta^2 S_{i,f} + \lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2},
\end{aligned}
$$

which implies that

$$
S_{i,f} = \frac{\lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2 - \lambda \left(\phi_y^2 \sigma_\beta^2 + \sigma_m^2\right)}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2 - \lambda \phi_y \phi_\pi \sigma_\beta^2} \tag{A.34}
$$

We reach the solution that

$$
y_t = -i_t + \varepsilon_t^\beta = (1 - C_\beta) \varepsilon_t^\beta - C_m \varepsilon_t^m \tag{A.35}
$$

$$
\pi_t = -\Delta i_t - \lambda \varepsilon_t^a = \Delta C_\beta \varepsilon_t^\beta + \Delta C_m \varepsilon_t^m - \lambda \varepsilon_t^a \tag{A.36}
$$

$$
i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m, \tag{A.37}
$$

where

$$
\Delta = \frac{\lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2 - \lambda \left(\phi_y^2 \sigma_\beta^2 + \sigma_m^2\right)}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2 - \lambda \phi_y \phi_\pi \sigma_\beta^2} \tag{A.38}
$$

$$
C_\beta = \frac{\phi_y}{1 + \phi_y + \Delta \phi_\pi} \tag{A.39}
$$

$$
C_m = \frac{1}{1 + \phi_y + \Delta \phi_\pi}. \tag{A.40}
$$

B Numerical zTran Solution for Symmetric Model

The canonical representation is given as

$$\sum_{k=0}^{l} A_k \Psi_{t-k} + \sum_{k=0}^{h} B_k \mathbb{E}_t \Psi_{t+k} = \mathbf{0}_{n_x \times 1},$$

and coefficients are grouped by

$$\Psi_t \equiv \begin{bmatrix} \chi_t \\ \upsilon_t \\ s_t \end{bmatrix}, \quad A_k \equiv \begin{bmatrix} A_k^\chi & A_k^\upsilon & A_k^s \end{bmatrix}, \quad B_k \equiv \begin{bmatrix} B_k^\chi & B_k^\upsilon & B_k^s \end{bmatrix},$$

where χ_t is endogenous variable, s_t is exogenous signals and υ_t is idiosyncratic shock aggregator. Define $s_t^m \equiv \epsilon_{mt}$ and $s_t^\pi \equiv \epsilon_t^p$, we have

$$\chi_t = \begin{bmatrix} i_t \\ y_t \\ \pi_t \\ \bar{\pi}_t \end{bmatrix}, s_t = \begin{bmatrix} a_t \\ \mu_t \\ s_t^m \\ \omega_t^a \\ \omega_t^\mu \\ s_t^\pi \\ z_t^a \\ z_t^\mu \\ x_{it}^a \\ x_{it}^\mu \end{bmatrix}, \epsilon_t = \begin{bmatrix} \epsilon_t^\theta \\ \epsilon_t^\xi \\ \epsilon_t^{xa} \\ \epsilon_t^{x\mu} \\ \epsilon_t^{\omega a} \\ \epsilon_t^{\omega\mu} \\ \epsilon_t^{za} \\ \epsilon_t^{z\mu} \\ \epsilon_t^p \\ \epsilon_t^m \\ \epsilon_t^{xa,i} \\ \epsilon_t^{x\mu,i} \end{bmatrix}, \tag{B.1}$$

where χ_t collects the endogenous variables but π_t cannot be observed, s_t collects the signals, but a_t and μ_t cannot be observed, and ϵ_t collects all the innovations that hit the economy.

C Impulse response

C.1 Symmetric Information

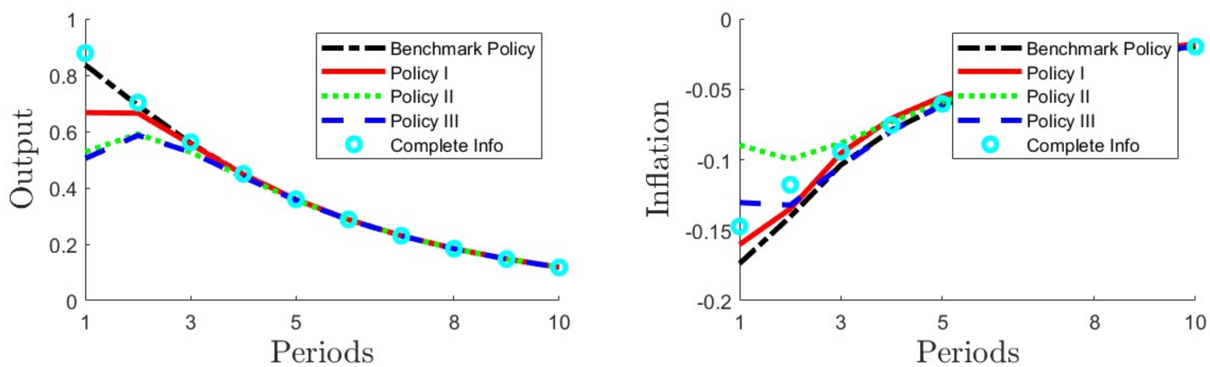


Figure 14: Technology shock

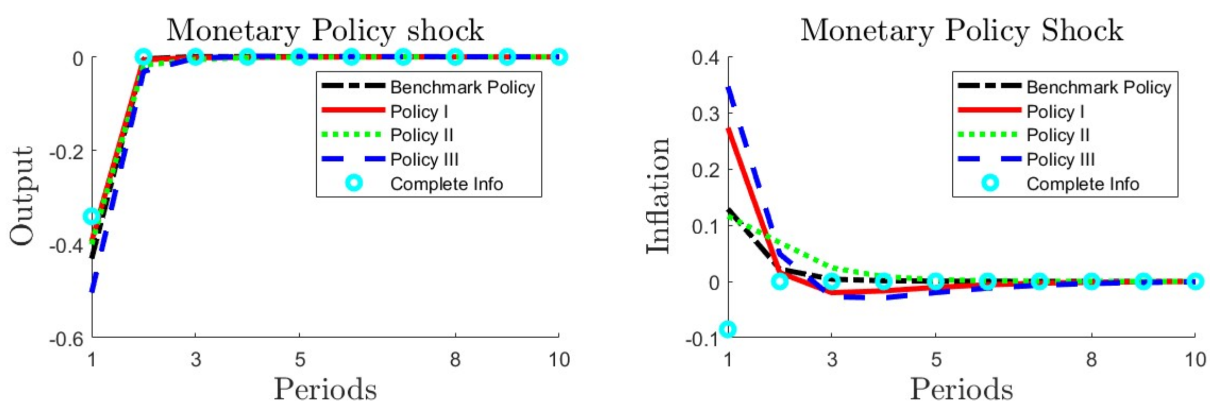


Figure 15: Monetary Policy shock

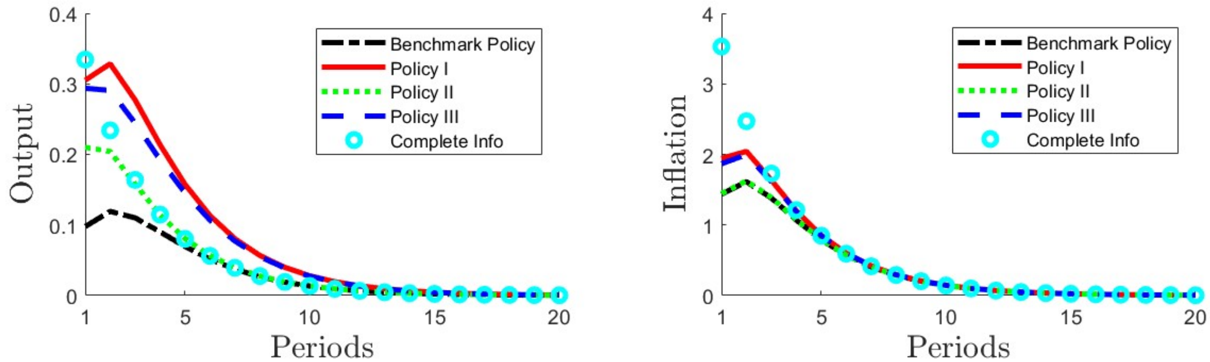


Figure 16: Markup shock

C.2 Asymmetric Information

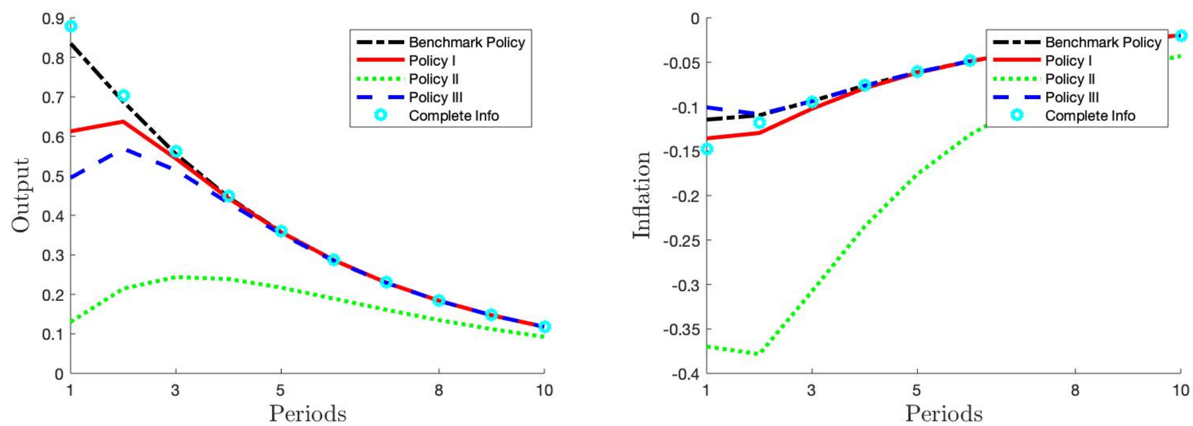


Figure 17: Technology shock

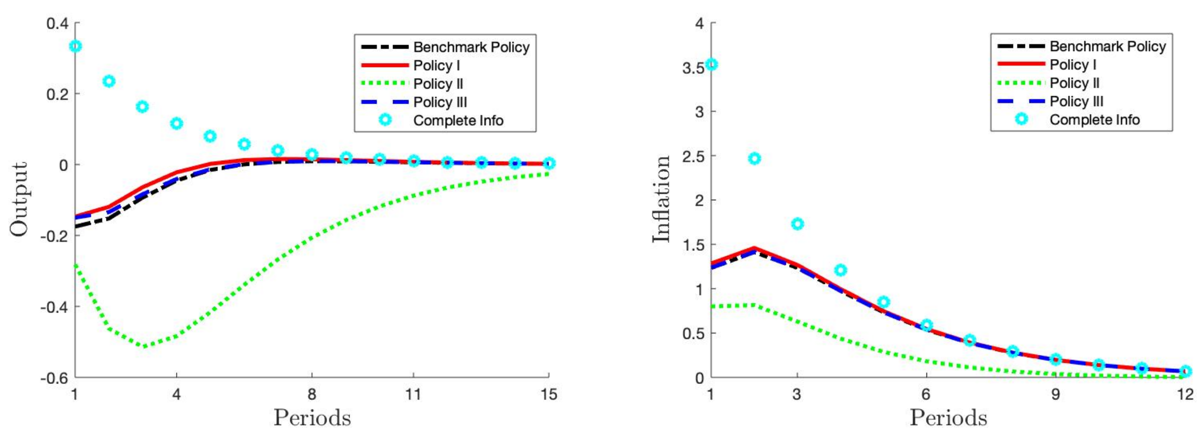


Figure 18: Markup shock

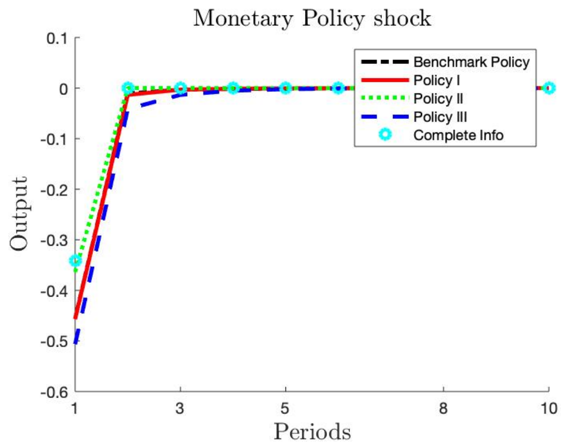


Figure 19: Monetary Policy shock

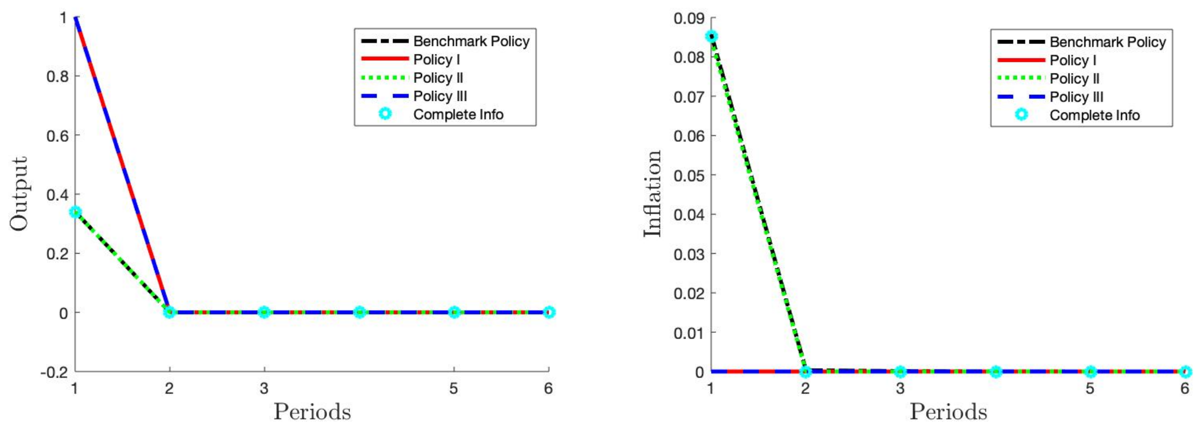


Figure 20: Preference shock