A Belief-Driven Taylor Rule: Expectations As a Policy Tool *

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Abstract

This paper explores the role of the private sector's information in the Taylor-rule-based monetary policy, which is referred to as the belief-driven Taylor rule. We employ a New Keynesian dynamic stochastic general equilibrium model that includes non-nested information among all sectors and investigate the effects of incorporating the information sets of households, firms, and the central bank into the belief-driven Taylor rule. The results suggest that incorporating the private sector's information into monetary policy can effectively anchor the private sector's inflation forecasts. Moreover, when the household has an asymmetric information set compared to firms', incorporating the household's information in the belief-driven Taylor rule leads to better anchoring results and results in the lowest welfare loss. This finding is further reinforced by our examination of a weighted strategy in the belief-driven Taylor rule. This is a pioneering study as it introduces a new approach in which a DSGE model with a belief-driven Taylor rule is employed to investigate the impact of private sector information on monetary policy.

Keywords: Incomplete Information, Monetary Policy, Asymmetric Information, Multiple Endogenous Signals

JEL Codes: D82, E52

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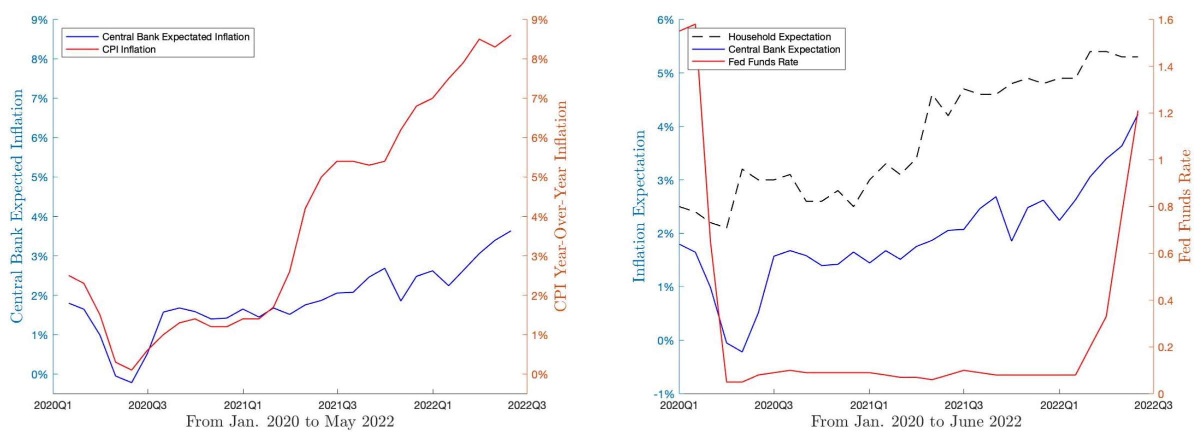
1 Introduction

Contemporary monetary policy is centered on achieving the inflation target, with the central bank serving as the monetary authority and setting the monetary rules. Following the 2008 financial crisis, the Federal Reserve initiated discussions on the implementation of an Average-Inflation-Targeting (AIT) policy as a means to stimulate economic recovery. In response to the COVID-19 pandemic in 2020, the Federal Reserve announced the adoption of AIT as a component of its monetary strategy. This policy grants the central bank a certain degree of flexibility in tolerating temporary inflation. Given the central bank's expectation of low inflation, the AIT policy aims to support economic recovery by allowing for a more accommodative monetary stance.[1], the federal funds rate was maintained at a low level of around 0.08% throughout the year 2021. Meanwhile, the US Consumer Price Index (CPI) continued to rise, with a year-over-year increase of 8.6% by May 2022, reaching a historical high not seen since the late 1980s. [See Figure 1:a].



(a) Central Bank Expected Inflation and CPI Inflation

(b) Inflation Expectations and Federal Funds Rate

Figure 1: CPI Inflation, Expected Inflation, and Fed Funds Rate

Notes: The data range spans from January 2020, the beginning of the COVID-19 pandemic, to June 2022. The households' inflation expectations are obtained from Surveys of Consumers by the University of Michigan.

As the inflation data became apparent, the Federal Reserve adjusted its beliefs and took successive measures to bring inflation back to the target level of 2%. Since March 2022, the Federal Reserve has raised the federal funds rate three times and, in July 2022, announced a 0.75-percentage-point increase, which is also a historical high within the past 28 years. The

[1]In simpler terms, the central bank viewed inflation in 2021 as a transitory phenomenon.

right axis in Figure 1:b documents a sharp increase in the federal funds rate. It is evident from the previous evidence that the central bank makes its monetary decisions based on its own information set. However, as pointed out in Coibion and Gorodnichenko (2012), relying solely on its information set can result in higher inflation volatility, which can pose a problem. This is particularly important to consider in light of Chair Jerome H. Powell's emphasis on the central bank's duty to deliver low and stable inflation during his speech at the Jackson Hole conference on August 26, 2022. Motivated by this, we investigate a New Keynesian model in which the private sector's beliefs are integrated into monetary policy through the Taylor rule. The conventional Taylor rule determines the interest rate based on the inflation target and the gap between actual and potential output. In this study, we consider the option for the central bank to use either its own information set or the private sector's information to form its now-cast of inflation and the output gap. This modified version of the Taylor rule is referred to as the belief-driven Taylor rule. Additionally, we explore the impact of the central bank's choice of the information set in the belief-driven Taylor rule on the private sector's inflation expectations. This is relevant given the emphasis placed by Powell on public inflation expectations and his argument that the central bank should utilize its tools to avoid the establishment of high inflation expectations among the public.

This paper is structured into two main parts: analytical analysis and numerical analysis. Both sections are built on a baseline New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model, in which households and firms hold asymmetric information sets. This is supported by empirical evidence from Candia, Coibion and Gorodnichenko (2020) and Gorodnichenko, Coibion and Candia (forthcoming), which shows that firms hold different expectations than households, indicating they possess distinct information sets.

In the analytical analysis section, we exclude the central bank's information set and examine the scenario in which the central bank uses either the households' or firms' information set in the belief-driven Taylor rule. The objective is to analyze the impact of different information choices in the belief-driven Taylor rule on the transmission of information from the public signal, interest rate, to households or firms. After model simplification, we obtain the closed-form solution using a Gaussian projection method by making an initial guess and then verifying it. We show that if the information set used in the belief-driven Taylor rule does not encompass an exogenous shock, the public signal, interest rate, will not be able to transmit any information regarding that shock. In other words, if the central bank relies on the firms' information set, the firms will not be able to obtain any relevant information from the interest rate about a specific shock that only enters the households' information set. This conclusion is further reinforced by the results of the numerical analysis that follows.

We then calculate the welfare loss associated with different choices of information sets in the belief-driven Taylor rule. We find that the welfare loss varies depending on the information set used and is determined by the relative size of the variance of exogenous shocks.

In the numerical analysis section, to begin, we study the case where the private sector, encompassing both households and firms, holds an asymmetric information set relative to the central bank. This asymmetric information structure is backed by the well-documented disagreement in inflation expectations, providing supporting evidence. (see Candia, Coibion and Gorodnichenko (2020),Andrade et al. (2016), Dovern, Fritsche and Slacalek (2012)) The left axis in Figure 1:b also illustrates the consistent disagreement between households' inflation expectations and those of the central bank, with the former consistently higher.[2] We adopt the central bank communications error between the private sector and the central bank as in Kohlhas (2021). As a benchmark, we consider the scenario where the central bank has a complete information set. From there, we examine three alternative belief-driven Taylor rules. The central bank can determine the interest rate based on its own information set, the private sector's information set, or by filtering the private sector's information set with its own information set and using the resulting beliefs to set the interest rate. In our study, we mainly examine the information transmission of the interest rate, the anchoring of inflation forecasts, and the impact on social welfare. Our main result in this section is that when the belief-driven Taylor rule incorporates the private sector's information, the private sector's inflation forecast is more anchored than other information sets. With regard to social welfare, we find that the central bank's information set is superior to the others, 34 percent higher, because both the private sector and the central bank observe similar signals, with the only difference being the level of noise associated with each signal. In this scenario, the central bank communication error prevails over the others and results in the lowest welfare loss when the central bank's information set is used. The differences in interest rate information transmission across the different Taylor rules are not as pronounced for the same reason.

Subsequently, we delve deeper into the information structure of the private sector by separating households and firms, creating non-nested information sets for households, firms, and the central bank. We differentiate households from firms based on demand and supply side signals. The central bank, households, and firms can observe public signals, such as the interest rate and inflation. Households have access to private demand-side signals, such as

[2] From the third quarter of 2020 to the first quarter of 2022, the central bank's inflation expectations remained around 2 percent, leading to no need to raise the Federal Funds Rate. A study by Binder, Janson and Verbrugge (2022) shows that compared to the Survey of Professional Forecasters (SPF), which displays stabilized expectations around the target, most individual forecasters' long-run inflation expectations are subject to significant fluctuations, with significant deviations from the target.

a preference shock, while supply-side signals enter both the central bank's and firms' information sets, albeit with different levels of noise. We assume that general consumers do not pay much attention to central bank announcements; hence the central bank communication error only exists between the central bank and firms. We conduct similar studies as in the previous scenario. As a result, the belief-driven Taylor rule can be operated based on the information set of households, firms, or the central bank. The results of the analysis of interest rate information transmission are consistent with our analytical findings. Our key result is that, by incorporating the household's information set in the belief-driven Taylor rule, the inflation expectations of both households and firms (i.e., the private sector) become the most anchored compared to the other information sets, also resulting in the lowest welfare loss. Rather than trying attempting to alter the private sector's perceptions of the economy, our results show that incorporating private sector now-casts into monetary policy helps anchor their forecasts. The finding of the effectiveness of the household's information set provides a potential solution to the concerns raised in Coibion et al. (2020), where it is argued that inflation expectations should not be used as a policy tool due to the limitations of firm-level survey data. Moreover, The average inflation targeting policy, on the one hand, allows for some tolerance of inflation, but on the other hand, it may pose challenges due to the absence of real-time data and slow policy actions, leading to current severe inflation. Within the average inflation targeting policy framework, where the Taylor rule remains the standard tool, incorporating public expectations offers valuable insights. Incorporating public expectations into the Taylor rule would help stabilize the public's expectations and may also serve as a source of unobservable real-time data for the monetary authority.[3] Finally, we investigate a weighted strategy in the belief-driven Taylor rule. The idea is that, in reality, the central bank's decisions are not solely based on one particular type of information set. As a result, we allow the central bank to assign varying weights to inflation and output gap now-casts formed by households or itself. Our results indicate that incorporating households' expectations into monetary decisions effectively anchors the private sector's inflation forecasts, even with a weight as low as 0.1.

The key contributions of this paper are threefold. First, we present various belief-driven Taylor rules in an environment of incomplete information, where the monetary authority sets interest rates based on different inflation and output gap expectations, demonstrating our concept of incorporating public expectations into policy decisions. Second, we examine a scenario of dispersed information where all three sectors have non-nested information

[3]Since the model structure is assumed to be publicly known among economic participants, the public would be aware of the information set used in monetary policy. As a result, their high inflation expectations would inevitably result in an increase in the Federal Funds Rate, avoiding any unexpected shocks to the nominal interest rate.

sets. Finally, we theoretically derive analytical solutions to analyze the effects of various information implementations in monetary policy on interest rate information transmission and total volatility. Additionally, we perform numerical simulations to examine the utilization of private sector information in monetary policy and its impact on stabilizing private sector inflation forecasts. To the best of our knowledge, these three contributions are novel advancements in the field.

Related Literature: This paper contributes to the existing literature on the implications of information frictions at a macro level [see Nimark (2008), Gorodnichenko and Coibion (2012), and Angeletos and Huo (2021)]. Previous studies in Melosi (2017), Han, Tan and Wu (2021b) explore the signaling effects, primarily focusing on incomplete information within firms. Empirical evidence in Candia, Coibion and Gorodnichenko (2020) and Gorodnichenko, Coibion and Candia (forthcoming) shows that firms hold different expectations compared to those held by the SPF or households. Thus, we examine scenarios in which the information sets among households, firms, and central banks are incomplete and asymmetric.

The papers Carboni and Ellison (2011) and Kohlhas (2021) examine scenarios in which the central bank holds incomplete information sets that differ from those of the private sector. In the first paper, monetary policy does not incorporate an output gap, and there is no process of signal extraction between the monetary authority and the private sector. In the second paper, the monetary policy lacks expected inflation. Our benchmark theory model incorporates both elements. Our numerical analysis incorporates the central bank communication errors between the central bank and firms, as studied in Kohlhas (2021). Nakamura and Steinsson (2018) highlights the significance of the central bank's communication channel and examines the effects of central bank announcements.

Additionally, Falck, Hoffmann and Hürtgen (2021) examines the monetary policy signaling channel with varying levels of disagreement in monetary policy transmission. Our paper introduces a way of incorporating disagreed expectations into policy decisions and investigates the associated transmission channels. Both theoretically and numerically, we discuss the results in the context of social welfare, as in Morris and Shin (2002). This paper also provides potential answers to the concerns raised in Coibion et al. (2020) regarding the use of expectations as a policy tool. Moreover, this paper reaches the same conclusion as Gorodnichenko, Coibion and Weber (forthcoming) that a central bank should prioritize effective communication with the private sector, particularly households. Finally, the solution method follows Han, Tan and Wu (2021a), which provides a user-friendly toolkit for solving models with dispersed information.

The paper is organized as follows: Section 2 presents the benchmark model. Section 3 derives analytical solutions for the benchmark model. Section 4 provides numerical solutions for the benchmark model, incorporating central bank communication errors. Finally, Section 5 concludes the paper.

2 Model Environment

We consider a prototypical New Keynesian model, which consists of a representative household, a monetary authority, and a large number of intermediate firms with nominal rigidities characterized by quadratic price adjustment costs. Each sector forms conditional expectations based on its own information set. If the household and firms share the same information set, we refer to it as a symmetrical information structure within the private sector. The economy's fluctuations are primarily driven by a preference shock affecting the household's discount factor, an aggregate technology shock affecting productivity levels, and an aggregate markup shock affecting the price level.

Timing: There are three stages within a time period. After all the innovations arrived, firms pre-set prices with partial information. Then, the central bank reacts to its information and delivers a public interest rate. Last, firms adjust the prices, and households respond to clear the goods and labor market.

2.1 Household

A representative household maximizes the utility function

$$\mathbb{E}_0^{HH} \sum_{t=0}^{\infty} \beta^t \exp\left(\varepsilon_t^{\beta}\right) \left[\log\left(C_t\right) - \frac{N_t^{1+\eta}}{1+\eta}\right],$$

where $\beta \in (0,1)$ is the discount factor and $1/\eta > 0$ is the Frisch elasticity of labor supply. $\mathbb{E}_t^{HH}$ is the household expectation operator that operates on a private information set, Ω_t^{HH}, which will be clarified later. ε_t^{β} is a demand shock affecting the household's intertemporal substitution. This shock is assumed to follow *i.i.d.* Gaussian distribution

$$\varepsilon_t^{\beta} \sim \mathbb{N}\left(0, \sigma_{\beta}^2\right).$$

The household's budget constraint is given as

$$\int_0^1 P_{it} C_{it} di + (1 + i_t) B_t \leq \int_0^1 \Pi_{it} di + W_t N_t + B_{t-1},$$

where P_{it} and C_{it} are price and consumption for the intermediate good i. The price index is aggregated by $P_t = \left(\int_0^1 P_{it}^{1-\rho} di\right)^{\frac{1}{1-\rho}}$. The final good consumption is aggregated by $C_t = \left(\int_0^1 C_{it}^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}}$. ρ is the elasticity of substitution. B_t is the one-period risk-free bond, Π_{it} is the profit for intermediate good firm i, W_t is the nominal wage and N_t is the labor supply.

Let c_t and π_t denote log deviations from steady states of consumption and inflation. We have the Euler Equation, which governs the intertemporal behavior of the household

$$c_t = \mathbb{E}_t^{HH} [c_{t+1}] - \left(i_t - \varepsilon_t^{\beta} - \mathbb{E}_t^{HH} [\pi_{t+1}]\right),$$

where i_t is the nominal interest rate and $\mathbb{E}_t^{HH}$ is the expectation operator for the household. Let y_t denote the final output's log deviation from the steady state. In equilibrium, we have the IS curve

$$y_t = \mathbb{E}_t^{HH} [y_{t+1}] - \left(i_t - \varepsilon_t^{\beta} - \mathbb{E}_t^{HH} [\pi_{t+1}]\right). \tag{2.1}$$

2.2 Firms

A continuum of intermediate firms is indexed by $i \in [0, 1]$. Firms are endowed with the same linear production function $Y_{i,t} = A_t N_{i,t}$, where $Y_{i,t}$ is the production of good i and $N_{i,t}$ is the labor input of firm i. A_t is the aggregate technology level. It is modeled as a persistent AR(1) process with a *i.i.d.* Gaussian noise θ_t

$$A_t = A_{t-1}^{\rho_a} \exp(\theta_t): \quad \rho_a \in (0, 1), \theta_t \sim \mathbb{N}\left(0, \sigma_\theta^2\right)$$

Aggregate technology signal is assumed to be non-observable to intermediate firms. Instead, the signal of aggregate technology contains two noises: (i) an industry level *i.i.d.* Gaussian noise, ε_{xt}^a, and (ii) an idiosyncratic *i.i.d.* Gaussian noise, ε_{it}^a

$$x_{it}^a = a_t + \varepsilon_{xt}^a + \varepsilon_{ixt}^a: \quad \varepsilon_{xt}^a \sim \mathcal{N}\left(0, \sigma_{a,x}^2\right), \varepsilon_{ixt}^a \sim \mathcal{N}\left(0, \sigma_{a,i}^2\right), a_t \equiv ln(A_t).$$

Each firm faces a quadratic price adjustment cost, as in Rotemberg (1982), and maximizes its profit

$$\Pi_{it} = (1 + T_t^s) P_{it} Y_{it} - W_t N_{it} - \frac{\psi}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2 P_t Y_t,$$

where $1 + T_t^s$ is stochastic with mean $\frac{\rho}{1-\rho}$. A mark-up shock is defined as $\mathcal{M}_t \equiv \frac{P_t}{W_t/A_t}$. In a symmetric equilibrium, all firms set the same price, and we have $\mathcal{M}_t = \frac{\rho}{\rho-1}\frac{1}{1+T_t^s}$. This mark-up shock also follows AR(1) process with a *i.i.d.* Gaussian noise ξ_t

$$\mathcal{M}_t = \mathcal{M}_{t-1}^{\rho_\mu} \exp(\xi_t) : \quad \rho_\mu \in (0,1), \xi_t \sim \mathbb{N}\left(0, \sigma_\xi^2\right).$$

Similar to the aggregate technology signal, firms cannot observe the exact signal of markup shock. The markup shock signal for each intermediate firm also contains two noises: (i)an industry level *i.i.d.* Gaussian noise, ε_{xt}^μ, and (ii) an idiosyncratic *i.i.d.* Gaussian noise, ε_{it}^μ

$$x_{it}^\mu = \mu_t + \varepsilon_{xt}^\mu + \varepsilon_{ixt}^\mu : \quad \varepsilon_{xt}^\mu \sim \mathcal{N}\left(0, \sigma_{\mu,x}^2\right), \varepsilon_{ixt}^\mu \sim \mathcal{N}\left(0, \sigma_{\mu,i}^2\right), \mu_t \equiv ln(\mathcal{M}_t).$$

The firms' optimization problem gives a New Keynesian Phillips Curve that explains the relationship among current inflation with firms' nowcast of the output gap, nowcast of markup shock, and one-period forecast of inflation

$$\pi_t = \beta \overline{\mathbb{E}}_t^F [\pi_{t+1}] + \lambda \overline{\mathbb{E}}_t^F [y_t - a_t] + \overline{\mathbb{E}}_t^F [\mu_t], \tag{2.2}$$

where $\lambda \equiv \frac{1+\eta}{\psi}\rho$. Inflation is defined as $\pi_t = p_t - p_{t-1}$. y_t is the log deviation of output. a_t represents the natural output rate. Therefore output gap is defined as $y_t - a_t$. The realization of markup shock μ_t is not observable to each firm. Each intermediate firm i forms conditional expectations, $\mathbb{E}_{i,t}^F$, based on its information set $\Omega_{i,t}^F$, and $\overline{\mathbb{E}}_t^F \equiv \int \mathbb{E}_{it}^F di$ denotes the average expectation across firms.

2.3 Monetary Authority

The central bank employs a simple Taylor-type rule which aims at hitting the output gap and inflation target

$$i_t = \phi_y \mathbb{E}_t^* [y_t - a_t] + \phi_\pi \mathbb{E}_t^* \pi_t + \varepsilon_t^m \tag{2.3}$$

where ϕ_y and ϕ_π measure monetary rule responses to the output gap and inflation, and there is an *i.i.d.* Gaussian monetary policy noise $\varepsilon_t^m \sim \mathbb{N}(0, \sigma_m^2)$. We assume the central bank is also equipped with incomplete information and cannot observe a complete set of endogenous variables and fundamental realizations.[4] The main contribution of this paper lies in the expectation operator $\mathbb{E}_t^*$. Instead of sticking with its information, different information sets could be applied by the central bank to form expectations of current economic variables, i.e., total output, technology level, and inflation. Private sectors' expectations are policy-wise internalized when the nominal interest rate is set correspondingly to their expectations/beliefs. We thereby call this Taylor-type rule belief-driven.

2.4 Welfare Loss Function

We follow Nisticò (2007) and derive the welfare loss as a second-order approximation of the household utility function with quadratic adjustment costs[5] :

$$\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1+\eta)\, \tilde{y}_t^2 \right] + t.i.p.$$

t.i.p. collects the terms that are independent of policies. η is the inverse of Frisch elasticity. $\tilde{y}_t$ is the log deviation of the output gap. We present the total welfare loss as

$$\mathbb{L} = \frac{1}{1-\beta} \frac{1+\eta}{2} var(\tilde{y}_t). \tag{2.4}$$

3 Analytical Analysis

In this section, we derive a closed-form solution to a simplified model. The household and firms have different information sets, both containing an endogenous signal and an exogenous signal. The central bank can operate on either of the information sets, and information choices lead to different welfare results. The detailed proof is provided in the appendix.

3.1 Closed-Form Solution

We simplify the model with the following assumption. The simplified model provides an analytical characterization of the economy where information dispersion exists among all sectors.

[4] Orphanides (2001) has emphasized the difference between the real-time policy recommendations and those obtained with the ex-post revised data, demonstrating the incompleteness of the central bank's information set.

[5] Measured by ψ.

It also highlights a crucial argument that different policy-wise information implementation results in different signal extraction processes and total economic volatility.

Assumption 1. *There is no mark-up shock and its associated industry level and idiosyncratic noises ($\mu_t = 0, \sigma_{\mu,x} = \sigma_{\mu,i} = 0$). There is neither industry-level noise nor idiosyncratic noise of technology for intermediate firms ($\sigma_{a,x} = \sigma_{a,i} = 0$). Aggregate technology shock is i.i.d ($\rho_a = 0$).*

Based on Assumption 1, all remaining shocks $(\varepsilon_t^a, \varepsilon_t^\beta, \varepsilon_t^m)$ are *i.i.d.*. Therefore, all one-period forward expectations are essentially zeros. The household's Euler equation is degenerated to

$$y_t = i_t + \varepsilon_t^\beta : \text{with the household's information set } \Omega^{HH} = \{i_t, \varepsilon_t^\beta\}. \tag{3.1}$$

i_t is the interest rate, and it is a public endogenous signal for both households and firms. ε_t^β is the household's private information about exogenous preference shock. Because there are no idiosyncratic noises, we can remove intermediate firms' index i and average expectations in New Keynesian Phillips Curve

$$\pi_t = \lambda \mathbb{E}_t^F \left[y_t - \varepsilon_t^a\right] : \text{ with firms' information set } \Omega^F = \{i_t, \varepsilon_t^a\} \tag{3.2}$$

i_t is a public signal, and ε_t^a is firms' private information about exogenous technology shock. We consider two monetary policies and solve them accordingly

$$\textbf{Policy I: } i_t = \phi_y \mathbb{E}_t^F \left[y_t - \varepsilon_t^a\right] + \phi_\pi \mathbb{E}_t^F \pi_t + \varepsilon_t^m$$

$$\textbf{Policy II: } i_t = \phi_y \mathbb{E}_t^{HH} \left[y_t - \varepsilon_t^a\right] + \phi_\pi \mathbb{E}_t^{HH} \pi_t + \varepsilon_t^m.$$

In Policy I, the Taylor rule operates on firms' information set, $\Omega^F = \{i_t, \varepsilon_t^a\}$, to form expectations about current output, technology level, and inflation; Policy II adopts household's information set, $\Omega^{HH} = \{i_t, \varepsilon_t^\beta\}$. The following propositions characterize unique incomplete information equilibrium for both policies in closed form.

Proposition 1. *For Policy I, given Assumption 1 and information structures (3.1) and (3.2), the model features a unique equilibrium where output, inflation, and interest rate follow*

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{3.3}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{3.4}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{3.5}$$

where coefficients $(S_{i,h}, S_\beta, S_{i,f}, S_a, C_\beta, C_m, C_a)$ *are given by*

$$S_{i,h} = -1, S_\beta = 1, S_{i,f} = -\lambda, S_a = -\lambda,$$
$$C_\beta = 0, C_m = \frac{1}{1+\phi_y+\lambda\phi_\pi}, C_a = \frac{\phi_y+\lambda\phi_\pi}{\lambda+\lambda\phi_y+\lambda\phi_\pi}.$$

Proposition 2. *For Policy II, given Assumption 1 and information structures (3.1) and (3.2), the model features a unique equilibrium where output, inflation and interest rate follow*

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{3.6}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{3.7}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{3.8}$$

where coefficients $(S_{i,h}, S_\beta, S_{i,f}, S_a, C_\beta, C_m, C_a)$ *are given by*

$$S_{i,h} = -1, S_\beta = 1, S_{i,f} = \Delta, S_a = -\lambda,$$
$$C_\beta = \frac{\phi_y}{1+\phi_y+\Delta\phi_\pi}, C_m = \frac{1}{1+\phi_y+\Delta\phi_\pi}, C_a = 0,$$

where $\Delta = \frac{\lambda\phi_y(1+\phi_y)\sigma_\beta^2 - \lambda(\phi_y^2\sigma_\beta^2+\sigma_m^2)}{\phi_y^2\sigma_\beta^2+\sigma_m^2-\lambda\phi_y\phi_\pi\sigma_\beta^2}$.

The closed-form solution straightforwardly helps understand the consequences of the central bank's different choices of information sets when conducting policy decisions. We first focus on the signal extraction process under Policy I. When the central bank operates on firms' information set, they lack information about preference shock. Hence interest rate does not respond to preference shock, i.e., $C_\beta = 0$. Firms can neither extract any useful information of preference shock, i.e., $\mathbb{E}^F[\varepsilon^\beta] = 0$, because both of their signals (i_t, ε_t^a) contain no information about this demand shock [6]. The situation is the same for Policy II.Households cannot extract supply shock information when the central bank adopts the household's information and ignores firms' information. Then, different signal extraction processes no doubt lead to different dynamics for endogenous economic variables. We see coefficients $S_\beta \vee S_a$ as **direct** signaling effects since they are associated with observable exogenous shocks $\varepsilon_t^\beta \vee \varepsilon_t^a$, and $S_{i,h} \vee S_{i,f}$ as **indirect** signaling effects, which are associated with endogenous monetary policy. With Policy I, a positive preference shock **directly** increases output and dose not influence inflation and interest rate. A positive technology shock decreases inflation both **directly** and **indirectly**. Because firms can observe technology shock, a positive technology shock results in a higher output gap, therefore, lower inflation, $S_a < 0$. Firms can also observe a higher interest rate[7], and inflation will also decrease because of $S_{i,f} < 0$. This shock surprisingly decreases output **indirectly** because households observe an increase in the interest rate and $S_{i,h} < 0$. With Policy II, a positive technology shock will directly decrease inflation and not influence output and the interest rate. A positive preference shock influences inflation indirectly, influencing output both directly and indirectly. This influence depends on parameterization since Δ is undetermined.

3.2 Welfare Analysis

Social welfare here is measured by output gap volatility which is determined by its dynamics hence choices of information sets for policy makers. Different welfare loss of both policies are calculated below

$$\textbf{Policy I: } var(y_t - \varepsilon_t^a) = \sigma_\beta^2 + C_m^2\sigma_m^2 + (C_a + 1)^2\sigma_a^2 \tag{3.9}$$

$$C_m = \frac{1}{1 + \phi_y + \lambda\phi_\pi}, C_a = \frac{\phi_y + \lambda\phi_\pi}{\lambda + \lambda\phi_y + \lambda\phi_\pi}$$

and

$$\textbf{Policy II: } var(y_t - \varepsilon_t^a) = (1 - C_\beta)^2\sigma_\beta^2 + C_m^2\sigma_m^2 + \sigma_a^2 \tag{3.10}$$

[6] Mathematically, the projection of ε_t^β on the information space of $\Omega^F = \{i_t, \varepsilon_t^a\}$ is technically zero.

[7] In Equation (3.5), $C_a > 0$.

$$C_\beta = \frac{\phi_y}{1+\phi_y+\Delta\phi_\pi}, C_m = \frac{1}{1+\phi_y+\Delta\phi_\pi}, \Delta = \frac{\lambda\phi_y(1+\phi_y)\sigma_\beta^2 - \lambda(\phi_y^2\sigma_\beta^2+\sigma_m^2)}{\phi_y^2\sigma_\beta^2+\sigma_m^2-\lambda\phi_y\phi_\pi\sigma_\beta^2}.$$

Equations (3.9) and (3.10) document output gap volatility under two policies. It is straightforward to see that different policy information sets deliver different macroeconomic volatility. In traditional New Keynesian literature, the Taylor rule focuses on the output gap and inflation, which mainly involves supply shocks besides the monetary shock. Therefore, variable dynamics and volatility change when we apply the household's information set. When the monetary authority considers household information as their policy regime, they need to realize the possibility of asymmetric information structure between producers and consumers. Consumers may form their expectations of supply shocks using private and public signals instead of direct observation. Different signal extraction process ends up with different welfare loss. Even if the Taylor rule is not directly linked with demand shocks, welfare loss does. Comparing equations (3.9) and (3.10), we can tell that welfare loss bringing from the demand side, σ_β^2, decreases[8] when applying household's information in Taylor rule. Although total volatility is associated with all shocks and parameterization, this explains why we observe lower welfare loss when the Taylor rule operates on the household's information set in the later section.

4 Quantitative Analysis

This section introduces central bank communication errors and numerically solves the model. We first study symmetric information between consumers and producers and then move on to asymmetric information structure.

4.1 Symmetric Information

We introduce the communication error between the private sectors and a central bank from Kohlhas (2021). This section eliminates preference shock, ε_t^β, from the model. The central bank observes noised technology shock and markup shock. Instead, they receive signals (z_t^a and z_t^μ) accordingly.

$$z_t^a = a_t + \varepsilon_{zt}^a : \varepsilon_{zt}^a \sim \mathcal{N}\left(0, \sigma_{z,a}^2\right), \quad z_t^\mu = \mu_t + \varepsilon_{zt}^\mu : \varepsilon_{zt}^\mu \sim \mathcal{N}\left(0, \sigma_{z,\mu}^2\right),$$

where ε_{zt}^a and ε_{zt}^μ are assumed to be *i.i.d.* Gaussian distributed. They are the central bank's observational noises about aggregate technology level and markup shock. Signals z_t^a and z_t^μ

[8] From the coefficient that $(1-C_\beta)^2 < 1$.

would be delivered from the central bank to private sectors, but there exists communication error (i.e., $\varepsilon^a_{\omega t}$ and $\varepsilon^\mu_{\omega t}$). Instead, private sectors receive signals, ω^a_t and ω^μ_t regarding those economic indicators published by the central bank,

$$\omega^a_t = z^a_t + \varepsilon^a_{\omega t} : \varepsilon^a_{\omega t} \sim \mathcal{N}\left(0, \sigma^2_{\omega,a}\right), \quad \omega^\mu_t = z^\mu_t + \varepsilon^\mu_{\omega t} : \varepsilon^\mu_{\omega t} \sim \mathcal{N}\left(0, \sigma^2_{\omega,\mu}\right),$$

where noises $\varepsilon^a_{\omega t}$ and $\varepsilon^\mu_{\omega t}$ are *i.i.d.* Gaussian distributed and signals ω^a_t and ω^μ_t would enter information sets. Last, we include inflation measurement error. We assume that neither the central bank nor the private sectors can observe true inflation. The inflation signal they receive ($\bar{\pi}_t$) contains an *i.i.d.* Gaussian noise ε^p_t

$$\bar{\pi}_t = \pi_t + \varepsilon^p_t : \varepsilon^p_t \sim \mathcal{N}(0, \sigma^2_p).$$

We conclude information sets for the household, firms, and the central bank as follow

$$\Omega^{HH}_t = \Omega^F_{it} = \{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}^\infty_{j=0}, \quad \Omega^{cb}_t = \{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}^\infty_{j=0}. \tag{4.1}$$

The representative household shares the same information set with firms. They receive their private signals about exogenous technology and markup shocks (x_{it}), public signals (ω_t) of exogenous shocks sent out by the central bank, and public signals of endogenous variables ($\bar{\pi}_t$ and i_t). Public signals are common knowledge and included in central bank's information set. Besides, the central bank receives private signals about exogenous technology and markup shocks (z_t).

For monetary policies, we consider four following cases:

$$\textbf{Benchmark: } i_t = \phi_y [y_t - a_t] + \phi_\pi \pi_t + \varepsilon^m_t,$$

$$\textbf{Policy I: } i_t = \phi_y \mathbb{E}^{cb}_t [y_t - a_t] + \phi_\pi \mathbb{E}^{cb}_t \pi_t + \varepsilon^m_t,$$

$$\textbf{Policy II: } i_t = \phi_y \mathbb{E}^{Private}_t [y_t - a_t] + \phi_\pi \mathbb{E}^{Private}_t \pi_t + \varepsilon^m_t,$$

$$\textbf{Policy III: } i_t = \phi_y \mathbb{E}^{cb}_t [\mathbb{E}^{Private}_t (y_t - a_t)] + \phi_\pi \mathbb{E}^{cb}_t [\mathbb{E}^{Private}_t \pi_t] + \varepsilon^m_t,$$

where $\varepsilon^m_t \sim \mathbb{N}(0, \sigma^2_m)$ is the exogenous monetary shock.

Benchmark monetary policy follows the traditional framework of complete information. It assumes that a central bank can observe all the realizations of economic variables. A nominal interest rate is set accordingly. However, a central bank is endowed with much less information. In Policy I, we assume the central bank conducts policy with its own information set, i.e., Ω^{cb}_t in (4.1). We denote $\Omega^{Private}_t$ as the private sectors' information set since the household and firms share the same information set. Policy II allows the central bank to internalize private sectors' information by operating on the information set $\Omega_t^{Private}$. Policy III serves as another possibility for information implementation. Higher-order belief $\mathbb{E}_t^{cb}[\mathbb{E}_t^{Private}]$ contains another information filtering process. While obtaining the private sectors' expectations, the central bank filters them with its own information set.[9] It is worth noting that this is different from a measurement error of survey data; a measurement error case is equivalent to Policy II with another exogenous shock.

4.1.1 Solution Method and Parameterization

This incomplete information model involves multiple endogenous signals, i.e., interest rate and inflation, and exogenous signals, i.e., technology signal and markup signal. The symmetric private information structure model contains two conditional expectations, i.e., $\mathbb{E}_t^{private}$ and $\mathbb{E}_t^{cb}$. In the next section, with the introduction of asymmetric information structure between households and firms, the model contains three types of conditional expectations, i.e., $\mathbb{E}_t^{HH}$, $\mathbb{E}^{Firms}$, and $\mathbb{E}_t^{cb}$. Instead of applying time-domain methods in Nimark (2008), we apply the solution method from Han, Tan and Wu (2021a) with the toolbox zTran. Their approach is based on policy function iterations in the frequency domain. The canonical representation can be found in Appendix A

Parameterization is performed before simulations. Table 1 documents conventional parameter values for quarterly models and parameter values for exogenous processes. The discount factor, β, is set as 0.99 and can be interpreted as a quarterly value. The inverse of Frisch elasticity, η, is set to 1. The elasticity of substitution, ρ, among intermediate goods is set to 6. The price adjustment cost, ψ, is established as the slope of NKPC, λ, equal to 0.25. These are commonly used in the literature. For the Taylor rule coefficients, we follow Kohlhas (2021) and set $\phi_y = 1.81$, and we follow Taylor (1999) to set $\phi_\pi = 0.5$; as Yellen (2012) suggests, monetary policy should respond more to the output gap than inflation. We follow Rudebusch (2002) and set $\sigma_m = 0.4$. The rest parameters values of the technology and markup process are from Kohlhas (2021). We focus on the analysis of information implementation within the monetary policy. We thereby will not calibrate the precision of central bank signals with different values, and we set both central bank's communication noises ($\sigma_{\omega,a}$ and $\sigma_{\omega,\mu}$) to 1.

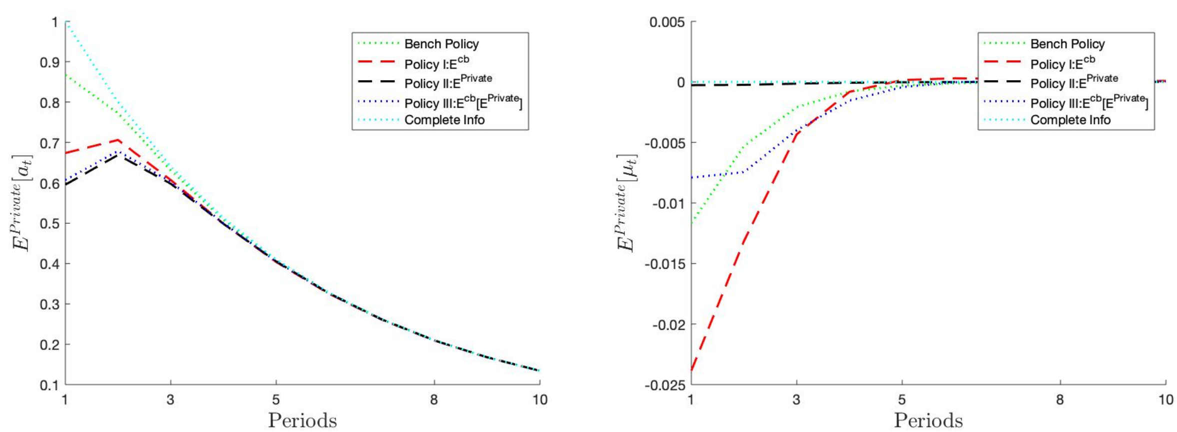
[9] In other words, the central bank has less confidence in directly adopting results from survey data and attempts to interpret those results in its own way.

Table 1: Symmetric Model Parameterization

Parameter	Value	Description
Households		
β	0.99	Discount factor
η	1	Inverse of Frisch elasticity
Firms		
ρ	6	Elasticity of substitution
ψ	48	Price adjustment cost
ρ_a	0.8	Persistence of technology shock
ρ_μ	0.7	Persistence of markup shock
σ_θ	0.6	Tech. process white noisy
σ_ξ	0.16	Markup process white noisy
$\sigma_{x,a}$	0.65	Industry tech. noisy
$\sigma_{x,a,i}$	0.2	Firm idiosyncratic tech. noisy
$\sigma_{x,\mu}$	0.2	Industry markup noisy
$\sigma_{x,\mu,i}$	0.11	Firm idiosyncratic markup noisy
$\sigma_{\omega,a}$	1	Industry's policy-signal tech. noisy
$\sigma_{\omega,\mu}$	1	Industry's policy-signal markup noisy
Central Bank		
ϕ_y	1.81	MP response to output gap
ϕ_π	0.5	MP response to inflation
$\sigma_{z,a}$	0.4	Central bank tech. noisy
$\sigma_{z,\mu}$	0.1	Central bank markup noisy
σ_p	0.8	Inflation measurement error
σ_m	0.4	Monetary policy shock

4.1.2 Now-cast and Forecast Impulse Responses

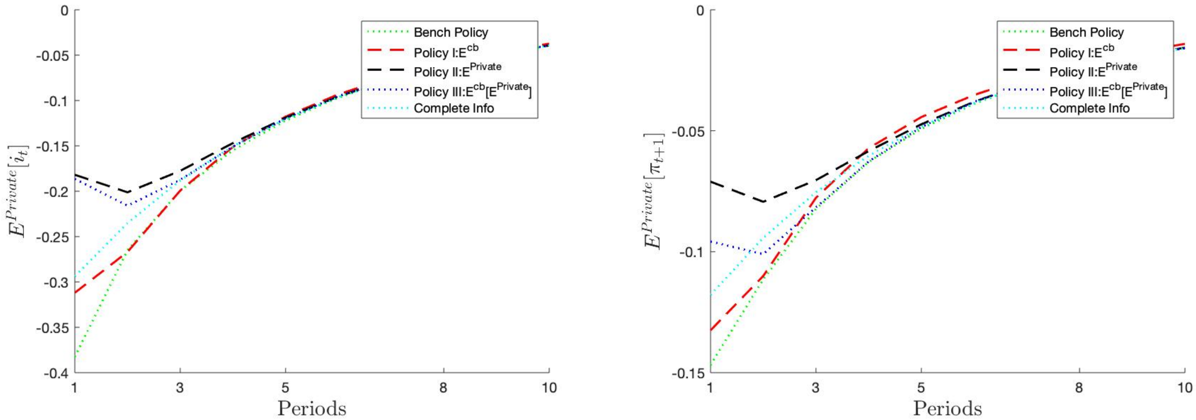
An Aggregate Technology Shock: We study private sectors' now-cast of exogenous shocks and forecast of endogenous variables with different policies. Figure 2 documents the private sectors' now-cast of technology level, $\mathbb{E}^{Private}[a_t]$, and markup shock, $\mathbb{E}^{Private}[\mu_t]$, to a positive technological innovation. The cyan dotted line corresponds to the complete information setting, where no information friction exists. The green dotted line corresponds to benchmark policy, where central bank is endowed with full information while households and firms receive noised signals. The red dashed line corresponds to Policy I, where nominal interest rate is determined by central bank's information. The black dashed line corresponds to Policy II, where nominal interest rate is determined by private sectors' information. The blue dotted line corresponds to Policy III, where central bank filters private information when deciding nominal interest rate.



(a) $\mathbb{E}^{Private}[a_t|\varepsilon_t^a]$ (b) $\mathbb{E}^{Private}[\mu_t|\varepsilon_t^a]$

Figure 2: Responses of Private Sectors' Now-cast of Technology Level and Markup Level to a Positive Innovation in Technology

With complete information, a positive technology shock shifts the aggregate supply curve to the right, decreasing the price level. With incomplete information, households and firms receive imperfect observation of shocks. White noises will dampen their beliefs about realized shocks in the first period. Because all white noises are *i.i.d.*, they revise their understanding of fundamentals. Therefore, in Figure 2:a, we observe hump-shape beliefs of aggregate technology shock. When the nominal interest rate responds to the private sectors' information, communication errors serve as additional noises compared with responding to the central bank's information. The private sectors' perception of the realized shock would be further dampened. This explains why black and blue lines are below the red line in Figure 2:a. When central bank implements private sectors' information within monetary policy, all



(a) $\mathbb{E}^{Private}[i_t|\varepsilon_t^a]$ (b)$\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^a]$

Figure 3: Response of Private Sectors' Now-cast Nominal Interest Rate and One-period Inflation Forecast to a Positive Innovation in Technology

model information sets become nested, and shocks are anticipated for households and firms. They can distinguish aggregate technology shock, a_t, from markup shock, μ_t, and therefore

$$\mathbb{E}^{Private}[\mu_t|\varepsilon_t^a] = 0.$$

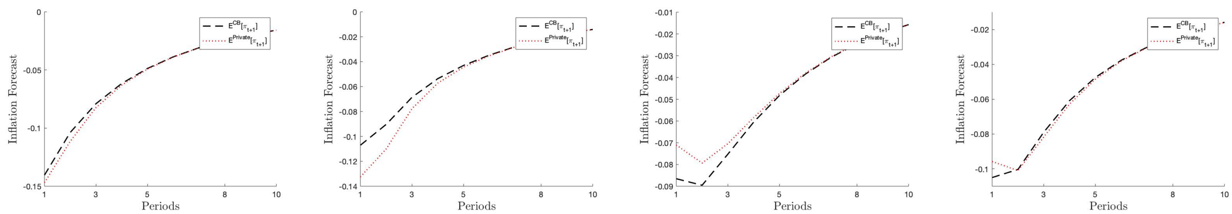
In Figure 2:b, the black dashed line does not respond to the aggregate technology innovation. Therefore, for the rest policy rules in Figure 2:b, households and firms will place probability on a negative markup innovation when observing a price decrease.

Figure 3 documents the private sectors' now-cast of nominal interest rate and one-period forecast of inflation with one standard deviation positive technology shock under different policy regimes.

With complete information, a positive aggregate technology innovation increases the output gap and decreases the price level, hence decreasing the nominal interest rate. Now-cast of nominal interest rate depends on how households and firms understand signals. With Policy II, the Taylor rule is driven by private sectors' information:

$$i_t = \phi_y \mathbb{E}_t^{Private}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{Private}\pi_t + \varepsilon_t^m.$$

Consistent with the above, households and firms underestimate realized technology shock size because of white noise and communication errors. Therefore, they expect a smaller output gap and a smaller nominal interest rate decrease. With *i.i.d.* noises vanishing in the next period, they will update their beliefs. In Figure 3:a, the black dashed line is hump-shape and responds the least, and it is worth noting that households and firms tend to



(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

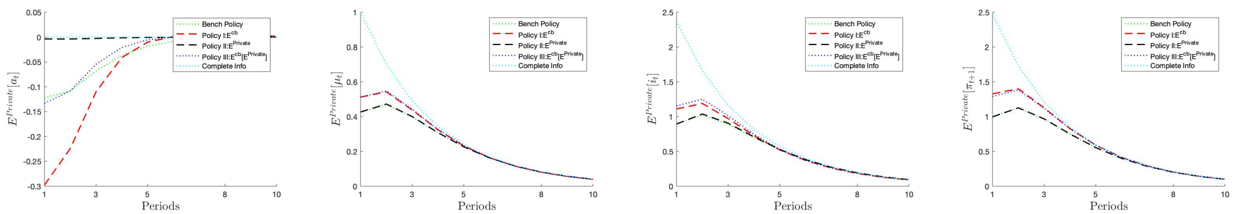
Figure 4: Comparison between Central Bank and Private Sectors' One-period Inflation Forecast to a Positive Aggregate Technology Innovation, i.e., $\mathbb{E}^{CB}[\pi_{t+1}|\varepsilon_t^a]$ and $\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^a]$.

overestimate when monetary policy works on other information sets. The cyan dotted line represents the private sectors' expectation of nominal interest rate change when we shut down information frictions at every layer. Whether the central bank operates on a full or its own noised information set, households and firms would expect a more significant decrease in nominal interest rate. Green dotted, and red dashed lines are below the cyan dotted line.

Characteristics in their now-cast are also reflected in their forecast. In Figure 3:b, inflation forecast also responds the least when the private sectors' information is implemented in monetary policy. They would also overestimate inflation responses when their information is not involved in monetary policy. This implies different policy-wise information implementations may affect the private sectors' inflation forecast.

Our model also generates disagreement in inflation expectations between the private sectors and the central bank. As shown in Figure 4, the black dashed line corresponds to the central bank's inflation forecast, and the red dotted line corresponds to private sectors' inflation forecast. Results show that the private sectors' inflation forecasts are less volatile than the central bank's inflation forecast when the private sectors' information set is involved in policy. This implies that policy-wise informational choices may affect, even rotate, inflation expectation disagreement.

A Markup Shock: Figure 5 documents private sectors' now-cast of technology level, $\mathbb{E}^{Private}[a_t]$, markup level, $\mathbb{E}^{Private}[\mu_t]$, nominal interest rate, $\mathbb{E}^{Private}[i_t]$, and forecast of inflation, $\mathbb{E}^{Private}[\pi_{t+1}]$ to a positive innovation in markup.



(a) $\mathbb{E}^{Private}[a_t|\varepsilon_t^{\mu}]$ (b) $\mathbb{E}^{Private}[\mu_t|\varepsilon_t^{\mu}]$ (c) $\mathbb{E}^{Private}[i_t|\varepsilon_t^{\mu}]$ (d) $\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^{\mu}]$

Figure 5: Private Sectors' Now-cast and Forecast Responses to a Positive Innovation in Markup.

With complete information, a markup shock increases the price level and the nominal interest rate. Consistent with previous findings, households and firms underestimate the realized shock size, Figure 5:b, and can distinguish shocks with Policy II, Figure 5:a. For Figure 5:c, recall monetary policies

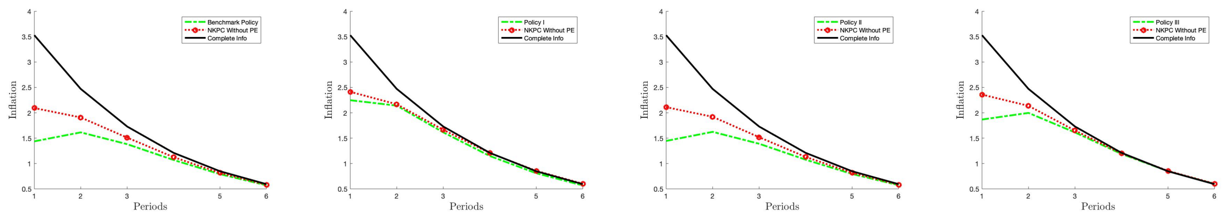
$$\text{Policy I} : i_t = \phi_y \mathbb{E}_t^{CB} [y_t - a_t] + \phi_\pi \mathbb{E}_t^{CB} \pi_t + \varepsilon_t^m$$

$$\text{Policy II} : i_t = \phi_y \mathbb{E}_t^{Private} [y_t - a_t] + \phi_\pi \mathbb{E}_t^{Private} \pi_t + \varepsilon_t^m$$

Their underestimation of actual markup innovation leads to an underestimation of current inflation compared with the complete information case. With Policy I, the non-nested information structure induces private sectors to assign probability on a negative productivity shock after observing an increase in nominal interest rate. Therefore, they form a higher now-cast of nominal interest rate response with Policy I because ϕ_y is greater than ϕ_π. However, their inflation forecast contradicts their now-cast of nominal interest rate. We compare Policy I with Policy II. Private sectors expect a larger increase in the nominal interest rate today with Policy I. They are supposed to expect lower inflation tomorrow. However, in 5:d, private sectors associate higher interest rates today with higher inflation tomorrow. This phenomenon also implies that the private sectors' inflation forecast is anchored when their information is involved within monetary policy.

4.1.3 Higher-order Decomposition

We follow Angeletos and Huo (2021), or AH for short, to study a higher-order decomposition in our NKPC's impulse responses. We explore how we can explain the quantitative difference in responses between complete and incomplete information. We also investigate how much is due to a lack of exogenous information (the partial equilibrium (PE) component) or endogenous uncertainty of inflation (the general equilibrium (GE) component). While AH focuses on moment estimation and calibration, our analysis focuses on the effects of different policy choices.



(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

Figure 6: Responses of Inflation to a Positive Innovation in Markup: Isolate Partial Equilibrium Effects in NKPC

To formalize the idea, we first take a look at equation 2.2:

$$\pi_t = \underbrace{\beta\overline{\mathbb{E}}_t^F\left[\pi_{t+1}\right]}_{\text{GE Component}} + \underbrace{\lambda\overline{\mathbb{E}}_t^F\left[y_t - a_t\right] + \overline{\mathbb{E}}_t^F\left[\mu_t\right]}_{\text{PE component}}. \tag{4.2}$$

We decompose NKPC into two parts: the GE component and the PE component. As in AH, the first part captures GE effects, also see Angeletos and Lian (2018), and the second part captures PE effects. Unlike AH, we shut down the PE component and numerically solve the structure model. It is worth noting that AH assumes the PE component following an AR(1) process to obtain analytical solutions. We drop this assumption when we solve the structural model numerically using the toolbox from Han, Tan and Wu (2021a). Figure 6 documents impulse responses of inflation to a positive innovation in markup with the decomposition above in different policy regimes. The solid black line represents the inflation responses in the complete information scenario. The red dotted circled line represents the inflation responses without exogenous uncertainty in NKPC with different policies. The green dashed line represents the inflation responses with different policies.

Quantitative bites of information friction are significant.[10] We first notice that the GE channel dominates the PE channel only with Policy I, i.e. when the central bank's information drives the Taylor rule. This is because the effects of firms' uncertainty of the next period's inflation override the effects of their now-cast of exogenous shocks. This is in line with Angeletos and Huo (2021). To understand this, we plug NKPC into different policies and compare the benchmark policy with Policy II:

$$i_t = \phi_y\mathbb{E}_t^{CB}\left[y_t - a_t\right] + \phi_\pi\mathbb{E}_t^{CB}\{\beta\overline{\mathbb{E}}_t^{Private}\left[\pi_{t+1}\right] + \lambda(y_t - a_t) + \mu_t\} + \varepsilon_t^m, \tag{4.3}$$

$$i_t = \phi_y\left[y_t - a_t\right] + \phi_\pi\{\beta\overline{\mathbb{E}}_t^{Private}\left[\pi_{t+1}\right] + \lambda(y_t - a_t) + \mu_t\} + \varepsilon_t^m. \tag{4.4}$$

[10]See the difference between the green dashed line and the black solid line in Figure 6.

From Equation (4.3), since firms know the model structure, they know the central bank will filter μ_t with its information set even if firms can observe the actual size of it.[11] Therefore, their information about the markup shock is not as crucial as in Equation (4.4) because μ_t enters the model directly. Also, Figure 6:a and Figure 6:c deliver the same results because if we plug NKPC into monetary policy, μ_t shares the same characteristics as in Equation (4.4).

4.1.4 Social Welfare

Table 2 documents the average period welfare loss associated with each policy. The first row is the benchmark policy. Rows $3-5$ correspond to average period welfare loss with Policy I-III.

Table 2: Welfare Analysis of Polices

Policy Rule	Welfare Loss
Benchmark Policy	5.2986
$\mathbb{E}^{cb}$	15.3219
$\mathbb{E}^{private}$	23.2522
$\mathbb{E}^{cb}[\mathbb{E}^{private}]$	30.0245

Not surprisingly, the benchmark policy achieves the least welfare loss, where the central bank has full economic information. The interest rate is generated by the most accurate information. The table shows that the central bank should set the monetary rule according to their expectation rather than adopting private sector expectations or further guessing the private sector expectations when private sectors observe the same noisy signals. This is intuitive when we re-examine the formation sets.

$$\Omega_t^{HH} = \Omega_{it}^{F} = \{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}, \text{ and } ,\Omega_t^{cb} = \{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}.$$

While inflation, $\bar{\pi}_{t-j}$, and the interest rate, i_{t-j}, are common knowledge, private sectors receive private noises, contained in x_{it-j}, and bank disclosure noise, contained in ω_{t-j}. On the other hand, the central bank only receives its private noise, contained in z_{t-j}, because z_{t-j} and ω_{t-j} together will cancel out the bank disclosure noises. Therefore, the central bank's information set is determined to be more "valuable" than the private sectors' information set, both from our analysis and welfare results.

[11]We shut down the PE component, so a_t and μ_t become common knowledge for firms.

4.2 Asymmetric Information

While keeping communication frictions between firms and the central bank, we take the preference shock, ε_t^{β}, back. We assume households cannot observe technology and markup signals, but the demand side shock can only be observed by households. It is natural to assume heterogeneity in information sets for different sectors. Each sector—households, firms and the central bank—has its information structure. However, the results still hold that the monetary authority should rely on private expectations. Thus, we study an optimal weighting strategy for monetary policy given heterogeneous expectations from the private sector.

Household: The corresponding Euler Equation is given as

$$y_t = \mathbb{E}_t^{HH}\left[y_{t+1}\right] - \left(i_t - \mathbb{E}_t^{HH}\left[\pi_{t+1}\right] - \varepsilon_t^{\beta}\right). \tag{4.5}$$

Firms: Firms share the same properties and information structure as in the previous model.

$$\pi_t = \beta\overline{\mathbb{E}}_t^F\left[\pi_{t+1}\right] + \lambda\overline{\mathbb{E}}_t^F\left[y_t - a_t\right] + \overline{\mathbb{E}}_t^F\left[\mu_t\right], \tag{4.6}$$

Monetary Authority: With heterogeneous information sets, the central bank employs a typical Taylor rule in responding to the output gap and inflation. We consider four cases:

$$\textbf{Benchmark: } i_t = \phi_y\left[y_t - a_t\right] + \phi_\pi\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy I: } i_t = \phi_y\mathbb{E}_t^{cb}\left[y_t - a_t\right] + \phi_\pi\mathbb{E}_t^{cb}\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy II: } i_t = \phi_y\mathbb{E}_t^{HH}\left[y_t - a_t\right] + \phi_\pi\mathbb{E}_t^{HH}\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy III: } i_t = \phi_y\mathbb{E}_t^{Firm}\left[y_t - a_t\right] + \phi_\pi\mathbb{E}_t^{Firm}\pi_t + \varepsilon_t^m,$$

where a_t is the log-deviation of potential output, ε_t^m is the monetary shock, and $\varepsilon_t^m \sim \mathbb{N}\left(0, \sigma_m^2\right)$.

Information structure: Different from the previous section, we assume each sector observes public signals and sector-related signals. Public signals are inflation with measurement error, $\bar{\pi}_t$, and the nominal interest rate i_t, which are included in all information sets. Preference shock signal exclusively enters the household's information set. Signals about technology and markup enter firms' and the central bank's information sets. Central bank communication error exists between firms and the central bank. We conclude information sets as follow

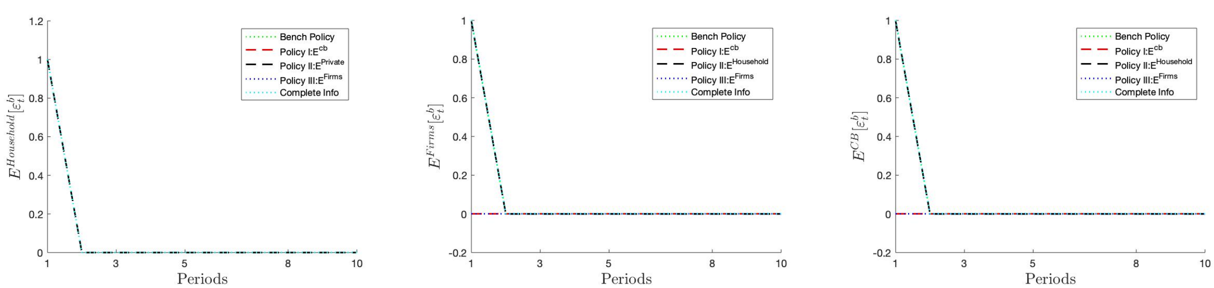
$$\Omega_t^{HH} = \left\{\varepsilon_{t-j}^{\beta}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}, \Omega_{it}^{F} = \{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty} \text{ and } \Omega_t^{cb} = \{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$$

The solution method adopts the frequency-domain policy function iteration approach in Han, Tan and Wu (2021a) with information sets defined above.

4.2.1 Now-cast and Forecast Impulse Responses

In this section, we study households, firms, and the central bank's now-cast and forecast of endogenous and exogenous variables. We begin with demand shock, namely ε_t^{β}, and then move to supply shocks, namely ε_t^{a} and ε_t^{μ}.

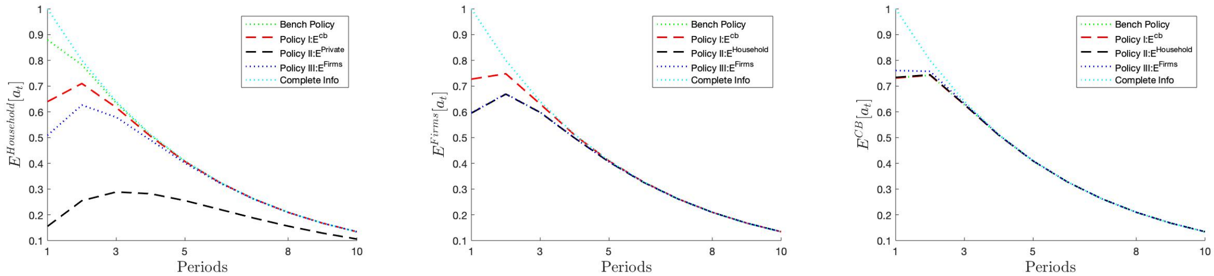
A Preference Shock: Figure 7 documents all sectors' now-cast of preference shock to a positive innovation in preference with different policies. This preference innovation signal only enters households' information set. The cyan dotted line corresponds to the complete information setting, where no information friction exists. The green dotted line corresponds to benchmark policy, where the central bank is endowed with full information while households and firms receive noised signals. The red dashed line corresponds to Policy I, where the central bank's information determines the nominal interest rate. The black dashed line corresponds to Policy II, where households' information determines the nominal interest rate. The blue dotted line corresponds to Policy III, where firms' information determines the nominal interest rate.



(a) $\mathbb{E}^{Households}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$ (b) $\mathbb{E}^{Firms}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$ (c) $\mathbb{E}^{CB}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$

Figure 7: Now-cast of Preference Shock to a Positive Innovation in Preference

Responses in Figure 7 accord with our theoretical analysis in Section 3. With this non-nested incomplete information environment, firms and the central bank are not endowed with the preference innovation signal. When implementing their information sets in monetary policy, the nominal interest rate would not respond to a preference shock, as shown in



(a) $\mathbb{E}^{Households}[a_t|\varepsilon_t^a]$ (b) $\mathbb{E}^{Firms}[a_t|\varepsilon_t^a]$ (c) $\mathbb{E}^{CB}[a_t|\varepsilon_t^a]$

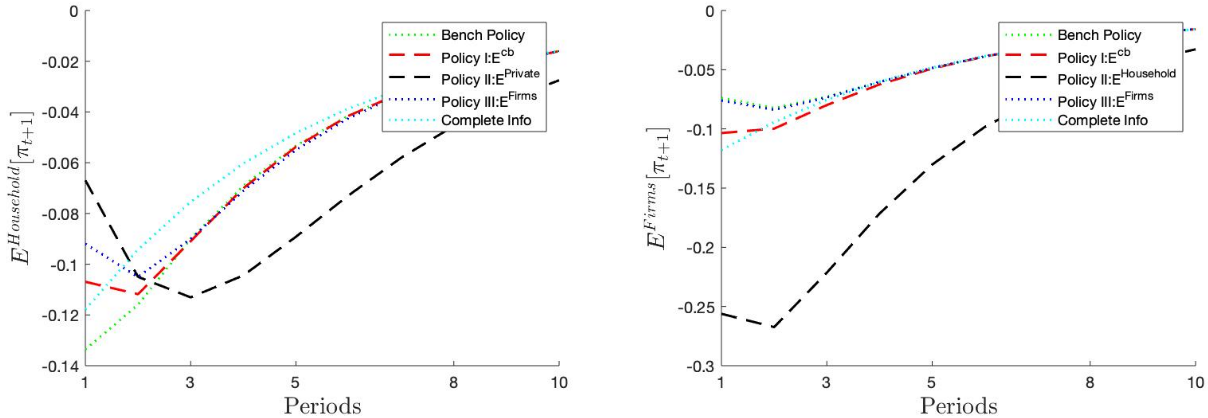
Figure 8: Now-cast of Technology Shock to a Positive Innovation in Technology

Proposition 1 where $C_\beta = 0$. They would not learn about the realized preference shock unless the nominal interest rate responds to that shock. When we adopt households' information set in the policy, the movement of nominal interest rate informs firms and central bank about *some* fundamental realization. Combined with their own information sets, they can infer the realization of a preference innovation.

An Aggregate Technology Shock: Figure 8 documents all sectors' now-cast of technology level to a positive innovation in aggregate technology with different policies.

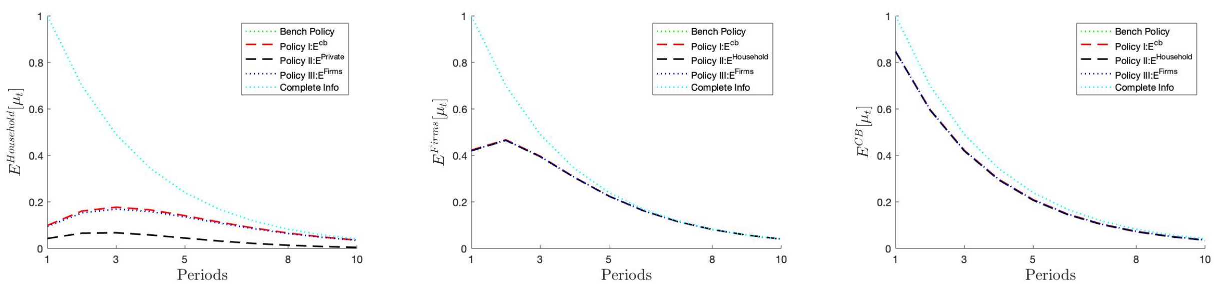
In Figure 8:b and Figure 8:c, the central bank forms a more precise now-cast of technology innovation than firms', which implies a dominance of the central bank's information over firms' information, because the central bank only receives noises at the aggregate level. In contrast, firms receive noises at the aggregate level, idiosyncratic level and communication error. Figure 8:a documents the learning process of households. Firms and the central bank are both endowed with technology signals. When implementing their information sets, the nominal interest rate movement contains information about technology innovation. Households are thereby able to infer the realized innovation. Figure 8:a shows households can better infer the realization with the implementation of the central bank's information set than firms', which is consistent with our argument. Households may not update their beliefs about the realized technology innovation if the monetary policy contains no information about it, i.e., implementing households' own information set.

Figure 9 documents households' and firms' inflation forecast to a positive technology innovation. Inflation forecast disagreement also exists between households and firms. Households' inflation forecast is more stable than firms'. Significantly, firms expect a significant inflation decrease when the policy information set contains no technology signal, i.e., Policy II. In such a scenario, firms expect a mild response of nominal interest rate, which is not enough to curb deflation. The lack of technology signal within policy rule also prolongs economic recovery.



(a) $\mathbb{E}^{Household}[\pi_{t+1}|\varepsilon_t^a]$ (b)$\mathbb{E}^{Firms}[\pi_{t+1}|\varepsilon_t^a]$

Figure 9: Households and Firms' Inflation Forecast to a Positive Innovation in Technology



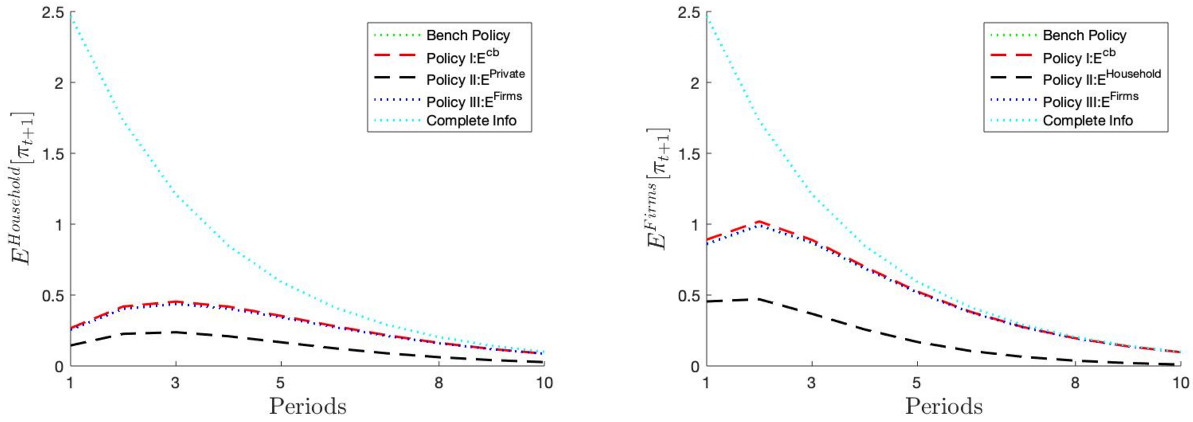
(a) $\mathbb{E}^{Households}[\mu_t|\varepsilon_t^\mu]$ (b) $\mathbb{E}^{Firms}[\mu_t|\varepsilon_t^\mu]$ (c) $\mathbb{E}^{CB}[\mu_t|\varepsilon_t^\mu]$

Figure 10: Now-cast of Markup Shock to a Positive Innovation in Markup

A Markup Shock: Figure 10 documents all sectors' now-cast of markup level to a positive innovation in markup with different policies. Firms and the central bank can better infer realized markup innovation than households. However, households are worse at recovering markup innovation than technology innovation with all information implementation. Because markup innovation is not directly involved in policy rules. The nominal interest rate movement contains less information on markup innovation than technology for households to update their beliefs.

Figure 11 documents households' and firms' inflation forecast to a positive markup innovation. Information frictions dampen households and firms' inflation forecast. Their forecast is further anchored when the nominal interest rate responds to households' information set. This is an exciting finding. Recall Policy II:

$$i_t = \phi_y \mathbb{E}_t^{HH}[y_t - a_t] + \phi_\pi \mathbb{E}_t^{HH}\pi_t + \varepsilon_t^m \text{ with: } \Omega_t^{HH} = \left\{\varepsilon_{t-j}^\beta, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^\infty.$$



(a) $\mathbb{E}^{Household}[\pi_{t+1}|\varepsilon_t^{\mu}]$ (b)$\mathbb{E}^{Firms}[\pi_{t+1}|\varepsilon_t^{\mu}]$

Figure 11: Households and Firms' Inflation Forecast to a Positive Innovation in Markup

When households do not receive the markup signal, firms would expect a mild nominal interest rate response to a positive markup innovation. In principle, this response would not be enough to curb inflation in the next period. Therefore, firms should expect higher inflation with Policy II than the other, which is different from their forecast in Figure 11:b. For anchoring private sectors' inflation forecast, this result demonstrates the importance of households'/demand side information.

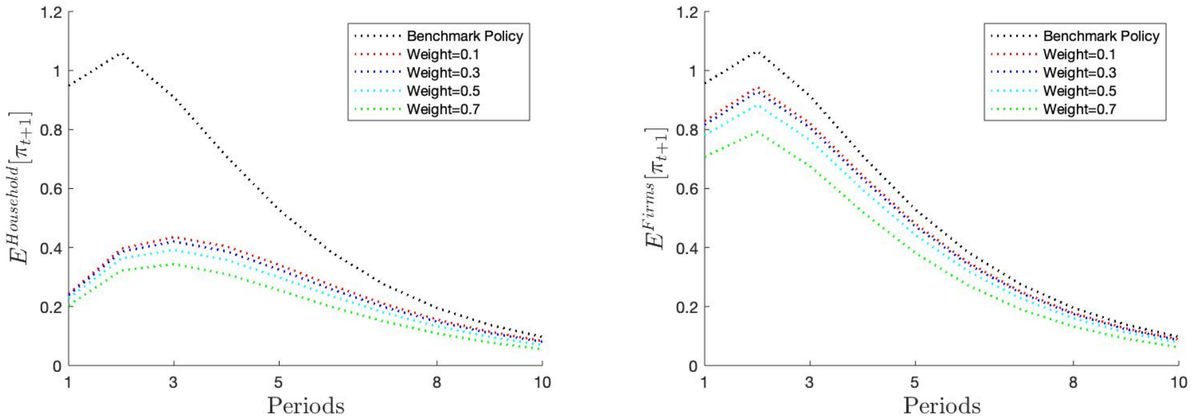
4.2.2 Weighted Strategy in Taylor Rule

Households' expectation matters in anchoring the private sectors' expectations with previous policy rules. Is this "anchoring effect" robust? Because, practically, monetary decisions would not be made solely corresponding to specific expectations. To answer such a question, we study another policy rule that includes both the central bank and households' expectations. The relative importance of households' expectations is variant, which is captured by coefficient $\omega_{HH} \in [0, 1]$. This coefficient ω_{HH} can be understood as the attention the central bank pays to households' now-cast.

$$i_t = \omega_{HH}\left[\phi_y \mathbb{E}_t^{HH}[y_t - a_t] + \phi_\pi \mathbb{E}_t^{HH}\pi_t\right] + (1-\omega_{HH})\left[\phi_y \mathbb{E}_t^{CB}[y_t - a_t] + \phi_\pi \mathbb{E}_t^{CB}\pi_t\right] + \varepsilon_t^m.$$

The central bank is stuck with its now-cast while considering households' expectations.[12] We provide a potential answer to concerns in Coibion et al. (2020), where they argue that the limitation of firm-level surveys is the main reason why expectations should not be considered

[12] Central bank would not consider firms' information because, in the previous section, we observe a dominance of central bank's information over firms'.



(a) $\mathbb{E}^{Household}[\pi_{t+1}|\varepsilon_t^{\mu}]$ (b)$\mathbb{E}^{Firms}[\pi_{t+1}|\varepsilon_t^{\mu}]$

Figure 12: Households and Firms' Inflation Forecast to a Positive Innovation in Markup with Various ω_{HH}.

as a policy tool. We calibrate ω_{HH}, and Figure 12 documents households and firms' inflation forecast with respect to a positive markup shock innovation.

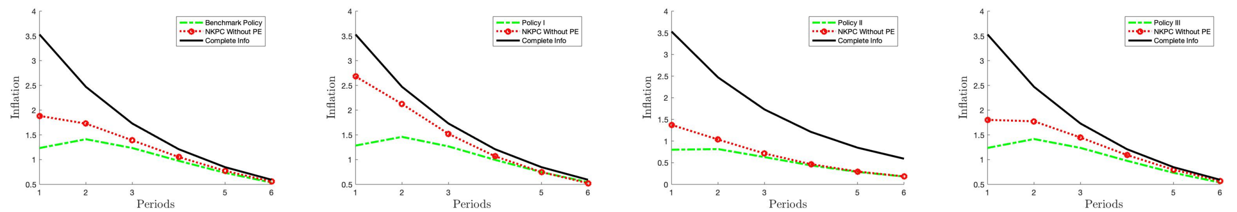
The black dotted line represents $\omega_{HH} = 0$, meaning the central bank does not consider households' expectations. When ω_{HH} increases, the results in Figure 12 approach what is in Figure 11, demonstrating consistency in the solution method. An exciting and crucial finding is that the private sectors' inflation forecast drops significantly even with $\omega_{HH} = 0.1$. This validates the distinction between the exclusion and inclusion of private information when conducting monetary policy. The responsiveness of the nominal interest rate to the private sectors' expectations helps anchor the private sectors' inflation forecast.

4.2.3 Higher-order Decomposition

We study the information acquisition problem for firms between exogenous information (the PE component) and endogenous uncertainty (the GE component) under the framework of asymmetric information:

$$\pi_t = \underbrace{\beta\overline{\mathbb{E}}_t^F[\pi_{t+1}]}_{\text{GE Component}} + \underbrace{\lambda\overline{\mathbb{E}}_t^F[y_t - a_t] + \overline{\mathbb{E}}_t^F[\mu_t]}_{\text{PE component}}.$$

Figure 13 documents inflation impulse responses to a positive innovation in markup with different policies. The black solid lines represent complete information cases. The red circle-dotted lines represent cases in which we eliminate the uncertainty of partial equilibrium effects (PE component), meaning that the quantitative difference between the black solid line and the red circled-dotted line is due to uncertainty in $\mathbb{E}_t[\pi_{t+1}]$ (GE component). Finally, the green dashed lines are the responses where both the PE and GE components exist. We shall now compare each policy in Figure 13 with Figure 6 to understand how different information structures affect firms' myopia and anchoring.



(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

Figure 13: Responses of Inflation to a Positive Innovation in Markup: Isolate Partial Equilibrium Effects in NKPC

The benchmark policy delivers similar results in both symmetric and asymmetric information settings. This means that the asymmetric information setting makes an insignificant difference to firms' informational acquisition between exogenous signals and endogenous beliefs when monetary authority is endowed with perfect information. With Policy I, where the central bank's information drives the Taylor rule, the PE component generates a significant quantitative bite compared to Figure: 6:b. This means that firms' uncertainty about the current economy largely influences their behavior when they know that the central bank operates its information. While with Policy II and Policy III, the overall responses are milder because of the increase in signal structure complexity, the ratio between the GE effects and PE effects is similar to those in Figure 6:c and Figure 6:d.

4.2.4 Social Welfare

Table 3 documents the average period welfare loss associated with each policy concerning different preference shock variances. The complete information case represents no information friction, and the benchmark policy is where the Taylor rule is driven by full information.

In this section, the information structure is asymmetric between the private sector, leading to an increase in total volatility and, as a result, a higher welfare loss in all scenarios. Similar to the symmetric information setting, the central bank's information is more valuable than the firms' due to the noisy nature of the signals observed by firms. Hence, executing the Taylor rule using the central bank's information results in a lower welfare loss compared to using the firms' information. However, the significance of household information becomes apparent. When the variance of the preference shock is low, using the household's expectations in the Taylor rule may not be advantageous because it disregards other supply signals.

Table 3: Welfare Analysis of Polices

Policy Rule	Welfare Loss
$\sigma_\beta^2 = 0.5^2$	
Complete Info	6.80
Benchmark Policy	9.15
$\mathbb{E}^{cb}$	42.32
$\mathbb{E}^{HH}$	69.46
$\mathbb{E}^{Firm}$	50.61
$\sigma_\beta^2 = 1^2$	
Complete Info	15.51
Benchmark Policy	17.85
$\mathbb{E}^{cb}$	117.32
$\mathbb{E}^{HH}$	78.23
$\mathbb{E}^{Firm}$	125.61
$\sigma_\beta^2 = 2^2$	
Complete Info	50.33
Benchmark Policy	52.68
$\mathbb{E}^{cb}$	417.31
$\mathbb{E}^{HH}$	113.07
$\mathbb{E}^{Firm}$	425.61
$\sigma_\beta^2 = 3^2$	
Complete Info	108.38
Benchmark Policy	110.72
$\mathbb{E}^{cb}$	917.32
$\mathbb{E}^{HH}$	171.12
$\mathbb{E}^{Firm}$	925.61
$\sigma_\beta^2 = 4^2$	
Complete Info	189.64
Benchmark Policy	191.98
$\mathbb{E}^{cb}$	1617.30
$\mathbb{E}^{HH}$	252.38
$\mathbb{E}^{Firm}$	1625.60

However, as the variance of the preference shock approaches or surpasses the variance of other shocks, it becomes influential and dominates either the firms' or the central bank's information. By incorporating household information within the Taylor rule, the average period welfare loss decreases significantly. The contrast between Policy II and other policies becomes more pronounced as the variance of the preference shock increases. This suggests that when there are heterogeneous shocks and information signals across different sectors, the most effective way to stabilize the economy is to adopt the household's information. This highlights the importance of the central bank in understanding household expectations more deeply.

The importance of communication between the monetary authority and the public has been emphasized by Candia, Coibion and Gorodnichenko (2020). Our model highlights that stabilizing the economy requires convincing the public that the nominal interest rate is based on households' understanding of current economic conditions rather than the central bank's role. This is because the private sector drives the value-added process and the central bank serves neither the supply nor demand sides. To reduce aggregate volatility and anchor expectations, the central bank needs to understand the private sector's information, with a particular focus on households, which surprisingly dominate firms.

The use of inflation expectations as a policy tool has been questioned by Coibion et al. (2020) due to two reasons: the private sector's reduced response to policy announcements in a low-inflation environment and the need for more comprehensive firm-level survey data. This paper provides potential answers to these problems. By incorporating the private sector's expectations into monetary policy, it becomes endogenous to the private sector rather than just being a tool to influence expectations through announcements. Additionally, the dominance of household information in our results means that we do not need comprehensive firm-level survey data and consumer survey data is sufficient.

We evaluate the impact of alternative information sets on welfare by using Policy I as a benchmark. All variations in information sets are made in parallel. First, we remove the communication errors between the central bank and the private sector. In this scenario, both households and firms receive the central bank's signals of the technology and markup shocks, z_t^a and z_t^μ, as well as the public signals sent out by the central bank, ω_t^a and ω_t^μ. This results in the following information structure for Case I:

Case I	Information Set
Households	$\left\{z_{t-j}, \omega_{t-j}, \varepsilon^{\beta}_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$
Firms	$\{z_{t-j}, \omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$
Central Bank	$\{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$

In Case II, we keep communication errors between the central bank and firms but allow the central bank to observe actual realizations about technology and markup shocks (a_t and μ_t). Information structure is

Case II	Information Set
Households	$\left\{\varepsilon^{\beta}_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$
Firms	$\{\omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$
Central Bank	$\{a_t, \mu_t, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$

In Case III, we introduce a noised signal of preference shock to the central bank. The central bank now receives a noised signal of the preference shock ε^{β}_t as $\tilde{\varepsilon}^{\beta}_t$. Information structure is

Case III	Information Set
Households	$\left\{\varepsilon^{\beta}_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$
Firms	$\{\omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$
Central Bank	$\left\{\tilde{\varepsilon}^{\beta}_t, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$

In Case IV, we combine Case II and Case III to verify that more information generates less welfare loss

Case IV	Information Set
Households	$\left\{\varepsilon^{\beta}_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$
Firms	$\{\omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$
Central Bank	$\left\{\tilde{\varepsilon}^{\beta}_t, a_t, \mu_t, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}$

Table 4 summarizes the impact of different signal structures on welfare.[13] The results in Case I show that the benefits of eliminating communication frictions are relatively small

[13]The focus is on $\sigma^2_{\beta} = 4^2$.

when there is asymmetric information. Even when both private sectors have full access to the central bank's "announcement noises", the welfare increase is only about 0.130%. In contrast, when the central bank has full access to supply-side signals in Case II, the welfare increase is around 0.748%, which is almost six times the increase from eliminating communication noises. This highlights the significance of actual information frictions related to real economic fundamentals, compared to "artificial" frictions such as communication frictions. The results in Case III show that information from the demand-side shock (preference shock) has a significant impact on welfare.[14] This reinforces our previous findings. When the household's information drives the Taylor rule, the welfare loss reaches its minimum at 252.38. If the central bank considers even the noisy household information, the corresponding public signal i_t would have a significant positive impact on the economy. Finally, the results in Case IV further support our argument that the more information the central bank considers, the better the public signal it can send out.

Table 4: Changes in Information Sets and Welfare Loss

Signal Structures	Welfare Loss
Policy I: $\mathbb{E}^{cb}$	1617.30
Case I	1615.2
Case II	1605.2
Case III	238.12
Case IV	229.23

5 Conclusion

This paper extends the standard Taylor rule by incorporating different information sets, referred to as the belief-driven Taylor rule, within a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with non-nested information. The model also includes an extension that considers information dispersion across all sectors.

Initially, we simplify the benchmark model and employ Gaussian projection to obtain a closed-form solution. This allows us to analyze the transmission of interest rate information from the central bank to households and firms by incorporating different information sets into the belief-driven Taylor rule. The results indicate that if the information set used in the belief-driven Taylor rule does not encompass an exogenous shock, the public signal, interest rate, will not be able to transmit any information regarding that shock.

[14] It is worth noting that the central bank receives a "noisy" signal.

Next, we incorporate central bank communication errors and perform numerical analysis. We study the case where the private sector, encompassing both households and firms, holds an asymmetric information set relative to the central bank. we examine three different belief-driven Taylor rules. The central bank can determine the interest rate based on its own information set, the private sector's information set, or by filtering the private sector's information set with its own information set and using the resulting beliefs to set the interest rate. Our main result in this section is that when the belief-driven Taylor rule incorporates the private sector's information, the private sector's inflation forecast is more anchored than other information sets and the central bank's information set is superior to the others in terms of social welfare.

Subsequently, we delve deeper into the information structure of the private sector by separating households and firms, creating non-nested information sets for households, firms, and the central bank. The results of the analysis of interest rate information transmission are consistent with our analytical findings. More importantly, we find that, by incorporating the household's information set in the belief-driven Taylor rule, the inflation expectations of both households and firms (i.e., the private sector) become the most anchored compared to the other information sets, also resulting in the lowest welfare loss. Finally, we investigate a weighted strategy in the belief-driven Taylor rule and find that incorporating households' expectations into monetary decisions effectively anchors the private sector's inflation forecasts, even with a weight as low as 0.1.

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Appendices

A Proof of Propositions

This section provides solutions to both propositions.

A.1 Policy I

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{A.1}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{A.2}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{A.3}$$

where $s_t^i = i_t = -y_t + \varepsilon_t^\beta$.

From firms' NKPC and the Taylor rule, we have

$$i_t = \frac{\phi_y + \lambda \phi_\pi}{\lambda} \pi_t + \varepsilon_t^m. \tag{A.4}$$

Denote $\Phi \equiv \frac{\phi_y + \lambda \phi_u}{\lambda}$, we have

$$i_t = \Phi(S_{i,f} i_t + S_a \varepsilon_t^a) + \varepsilon_t^m \implies i_t = \frac{1}{1 - \Phi S_{i,f}} (\Phi S_a \varepsilon_t^a + \varepsilon_t^m), \tag{A.5}$$

therefore we have

$$C_\beta = 0, C_m = \frac{1}{1 - \Phi S_{i,f}}, C_a = \frac{\Phi}{1 - \Phi S_{i,f}}. \tag{A.6}$$

Notice that

$$s_t = \begin{bmatrix} s_t^i \\ s_t^a \end{bmatrix} = \begin{bmatrix} i_t \\ \varepsilon_t^a \end{bmatrix} = \underbrace{\begin{bmatrix} C_\beta & C_m & C_a \\ 0 & 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \varepsilon_t^\beta \\ \varepsilon_t^m \\ \varepsilon_t^a \end{bmatrix}}_{\varepsilon_t}, \tag{A.7}$$

and

$$\pi_t = \lambda \mathbb{E}_t^F y_t - \lambda \varepsilon_t^a = -\lambda i_t - \lambda \varepsilon_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta = -\lambda s_t^i - \lambda s_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta \tag{A.8}$$

By Gaussian projection, we have

$$\pi_t = S_{i,f} s_t^i + S_a s_t^a = -\lambda s_t^i - \lambda s_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta, \tag{A.9}$$

$$(S_{i,f} \quad S_a) = [-\lambda \quad -\lambda] + \lambda \Sigma_{\beta s} \Sigma_s^{-1} \tag{A.10}$$

where

$$\Sigma_{\beta s} = [1 \quad 0 \quad 0] \begin{bmatrix} \sigma_\beta^2 & 0 & 0 \\ 0 & \sigma_m^2 & 0 \\ 0 & 0 & \sigma_a^2 \end{bmatrix} \begin{bmatrix} C_\beta & 0 \\ C_m & 0 \\ C_a & 1 \end{bmatrix} = \begin{bmatrix} C_\beta \sigma_\beta^2 & 0 \end{bmatrix}, \tag{A.11}$$

and

$$\Sigma_s = B\Sigma B' = \begin{bmatrix} C_\beta^2\sigma_\beta^2 + C_m^2\sigma_m^2 + C_a^2\sigma_a^2 & C_a\sigma_a^2 \\ C_a\sigma_a^2 & \sigma_a^2 \end{bmatrix}. \tag{A.12}$$

Notice that $C_\beta = 0$, then

$$\Sigma_{\beta s} = [0 \quad 0], \quad [S_{i,f} \quad S_a] = [-\lambda \quad -\lambda]. \tag{A.13}$$

Also we have $\mathbb{E}_t^F \varepsilon_t^\beta = \frac{\lambda + S_{i,f}}{\lambda} i_t = 0$.

Therefore

$$\pi_t = \lambda s_t^i - \lambda s_t^a = \lambda i_t - \lambda \varepsilon_t^a, \tag{A.14}$$

Then we move on to output

$$y_t = S_{i,h} s_t^i + S_\beta s_t^\beta = S_{i,h} i_t + S_\beta \varepsilon_t^\beta \implies y_t = -i_t + \varepsilon_t^\beta, \tag{A.15}$$

which gives

$$\begin{bmatrix} C_\beta & C_m & C_a \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{1-\Phi S_{i,f}} & \frac{\Phi}{1-\Phi S_{i,f}} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{1+\phi_y+\lambda\phi_\pi} & \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\phi_\pi)} \end{bmatrix} \tag{A.16}$$

We reach the solution that

$$y_t = -i_t + \varepsilon_t^\beta = \varepsilon^\beta - \frac{1}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m - \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\lambda\phi_\pi)}\varepsilon_t^a \tag{A.17}$$

$$\pi_t = -\lambda i_t - \lambda\varepsilon_t^a = -\frac{\lambda}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m - \lambda(1+\frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\lambda\phi_\pi)})\varepsilon_t^a \tag{A.18}$$

$$i_t = \frac{1}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m + \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\phi_\pi)}\varepsilon_t^a. \tag{A.19}$$

A.2 Policy II

$$y_t = S_{i,h}i_t + S_\beta\varepsilon_t^\beta, \tag{A.20}$$

$$\pi_t = S_{i,f}i_t + S_a\varepsilon_t^a, \tag{A.21}$$

$$i_t = C_\beta\varepsilon_t^\beta + C_m\varepsilon_t^m + C_a\varepsilon_t^a, \tag{A.22}$$

where $s_t^i = i_t = -y_t + \varepsilon_t^\beta$.

Combine y_t and π_t, we have

$$\pi_t = \lambda\mathbb{E}_t^F(-i_t + \varepsilon_t^\beta) - \lambda\varepsilon_t^a = -\lambda i_t - \lambda\varepsilon_t^a + \lambda\mathbb{E}_t^F\varepsilon_t^\beta. \tag{A.23}$$

Then

$$-\lambda i_t - \lambda\varepsilon_t^a + \lambda\mathbb{E}_t^F\varepsilon_t^\beta = S_{i,f}i_t + S_a\varepsilon_t^a \implies \mathbb{E}_t^F\varepsilon_t^\beta = \frac{\lambda+S_{i,f}}{\lambda}i_t. \tag{A.24}$$

Using Gaussian projection

$$[S_{i,f} \quad S_a] = [-\lambda \quad -\lambda] + \lambda\Sigma_{\beta s} = \begin{bmatrix} -\lambda & -\lambda \end{bmatrix} + \lambda\begin{bmatrix} c_\beta\sigma_\beta^2 & 0 \end{bmatrix}\Sigma_s^{-1}, \tag{A.25}$$

where

$$\Sigma_s = \begin{bmatrix} C_\beta^2\sigma_\beta^2 + C_m^2\sigma_m^2 + C_a^2\sigma_a^2 & C_a\sigma_a^2 \\ C_a\sigma a^2 & \sigma_a^2 \end{bmatrix}. \tag{A.26}$$

Therefore, we have

$$S_a = -\lambda. \tag{A.27}$$

Combine (A.20), (A.21), (A.22) and (A.24),

$$\begin{aligned} i_t &= \phi_y \mathbb{E}_t^{HH}[y_t - \varepsilon_t^a] + \phi_\pi \mathbb{E}_t^{HH}[\lambda \mathbb{E}_t^F y_t - \lambda\varepsilon_t^a] + \varepsilon_t^m \\ &= -\phi_y i_t + \phi_y \varepsilon_t^\beta - \phi_y \mathbb{E}_t^{HH}\varepsilon_t^a + \phi_\pi \mathbb{E}_t^{HH}[\lambda(-i_t + \mathbb{E}_t^F \varepsilon_t^\beta - \lambda\varepsilon_t^a] + \varepsilon_t^m \\ &= -\left(\phi_y + S_{i,f}\phi_\pi\right) i_t + \phi_y\varepsilon_t^\beta - \left(\phi_y + \lambda\phi_\pi\right)\mathbb{E}_t^{HH}\varepsilon_t^a + \varepsilon_t^m. \end{aligned}$$

which implies

$$i_t = \frac{1}{\Phi_2}(\phi_y\varepsilon_t^\beta - (\phi_y + \lambda\phi_\pi)\mathbb{E}_t^{HH}\varepsilon_t^a + \varepsilon_t^m). \tag{A.28}$$

Notice that $\mathbb{E}_t^F \varepsilon_t^\beta = \frac{\lambda + S_{i,f}}{\lambda} i_t$, and then the projection of ε_t^β on ε_t^a should be zero. We set that $\mathbb{E}_t^{HH}\varepsilon_t^a = \Phi_a i_t$.

From (A.28), we have

$$i_t = \frac{1}{\Phi_2\Phi_3}\phi_y\varepsilon_t^\beta + \frac{1}{\Phi_2\Phi_3}\varepsilon_t^m \tag{A.29}$$

where

$$\Phi_3 = 1 + \frac{\phi_y + \lambda\phi_\pi}{\Phi_2}\Phi_a. \tag{A.30}$$

We can tell that $i_t \perp \varepsilon_t^a$, then $\mathbb{E}_t^{HH}\varepsilon_t^a = 0$ and $\Phi_a = 0$.

Hence we have

$$i_f = \frac{\phi_y}{\Phi_2}\varepsilon_t^\beta + \frac{1}{\Phi_2}\varepsilon_t^m \tag{A.31}$$

where

$$C_\beta = \frac{\phi_y}{\Phi_2}, \quad C_m = \frac{1}{\Phi_2}, \quad C_a = 0. \tag{A.32}$$

Also,

$$\Sigma_S = \begin{bmatrix} C_c^2\sigma_\beta^2 + C_m^2\sigma_m^2 & 0 \\ 0 & \sigma_a^2 \end{bmatrix}. \tag{A.33}$$

From (A.25),

$$
\begin{aligned}
S_{i,f} &= -\lambda + \lambda \frac{C_\beta \sigma_\beta^2}{C_\beta^2 \sigma_\beta^2 + C_m^2 \sigma_m^2} \\
&= -\lambda + \lambda \frac{\frac{\phi_y}{\Phi_2} \sigma_\beta^2}{\frac{\phi_y^2}{\Phi_2^2} \sigma_\beta^2 + \frac{1}{\Phi_2^2 \sigma_m^2}} \\
&= -\lambda + \lambda \frac{\phi_y \Phi_2 \sigma_\beta^2}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2} \\
&= -\lambda + \frac{\lambda \phi_y \phi_\pi \sigma_\beta^2 S_{i,f} + \lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2},
\end{aligned}
$$

which implies that

$$
S_{i,f} = \frac{\lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2 - \lambda \left(\phi_y^2 \sigma_\beta^2 + \sigma_m^2\right)}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2 - \lambda \phi_y \phi_\pi \sigma_\beta^2} \tag{A.34}
$$

We reach the solution that

$$
y_t = -i_t + \varepsilon_t^\beta = (1 - C_\beta) \varepsilon_t^\beta - C_m \varepsilon_t^m \tag{A.35}
$$

$$
\pi_t = -\Delta i_t - \lambda \varepsilon_t^a = \Delta C_\beta \varepsilon_t^\beta + \Delta C_m \varepsilon_t^m - \lambda \varepsilon_t^a \tag{A.36}
$$

$$
i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m, \tag{A.37}
$$

where

$$
\Delta = \frac{\lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2 - \lambda \left(\phi_y^2 \sigma_\beta^2 + \sigma_m^2\right)}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2 - \lambda \phi_y \phi_\pi \sigma_\beta^2} \tag{A.38}
$$

$$
C_\beta = \frac{\phi_y}{1 + \phi_y + \Delta \phi_\pi} \tag{A.39}
$$

$$
C_m = \frac{1}{1 + \phi_y + \Delta \phi_\pi}. \tag{A.40}
$$

B Numerical zTran Solution for Symmetric Model

The canonical representation is given as

$$\sum_{k=0}^{l} A_k \Psi_{t-k} + \sum_{k=0}^{h} B_k \mathbb{E}_t \Psi_{t+k} = \mathbf{0}_{n_x \times 1},$$

and coefficients are grouped by

$$\Psi_t \equiv \begin{bmatrix} \chi_t \\ \upsilon_t \\ s_t \end{bmatrix}, \quad A_k \equiv \begin{bmatrix} A_k^{\chi} & A_k^{\upsilon} & A_k^{s} \end{bmatrix}, \quad B_k \equiv \begin{bmatrix} B_k^{\chi} & B_k^{\upsilon} & B_k^{s} \end{bmatrix},$$

where χ_t is endogenous variable, s_t is exogenous signals and υ_t is idiosyncratic shock aggregator. Define $s_t^m \equiv \epsilon_{mt}$ and $s_t^{\pi} \equiv \epsilon_t^p$, we have

$$\chi_t = \begin{bmatrix} i_t \\ y_t \\ \pi_t \\ \bar{\pi}_t \end{bmatrix}, s_t = \begin{bmatrix} a_t \\ \mu_t \\ s_t^m \\ \omega_t^a \\ \omega_t^{\mu} \\ s_t^{\pi} \\ z_t^a \\ z_t^{\mu} \\ x_{it}^a \\ x_{it}^{\mu} \end{bmatrix}, \epsilon_t = \begin{bmatrix} \epsilon_t^{\theta} \\ \epsilon_t^{\xi} \\ \epsilon_t^{xa} \\ \epsilon_t^{x\mu} \\ \epsilon_t^{\omega a} \\ \epsilon_t^{\omega\mu} \\ \epsilon_t^{za} \\ \epsilon_t^{z\mu} \\ \epsilon_t^{p} \\ \epsilon_t^{m} \\ \epsilon_t^{xa,i} \\ \epsilon_t^{x\mu,i} \end{bmatrix}, \tag{B.1}$$

where χ_t collects the endogenous variables but π_t cannot be observed, s_t collects the signals, but a_t and μ_t cannot be observed, and ϵ_t collects all the innovations that hit the economy.

C Impulse response

C.1 Symmetric Information

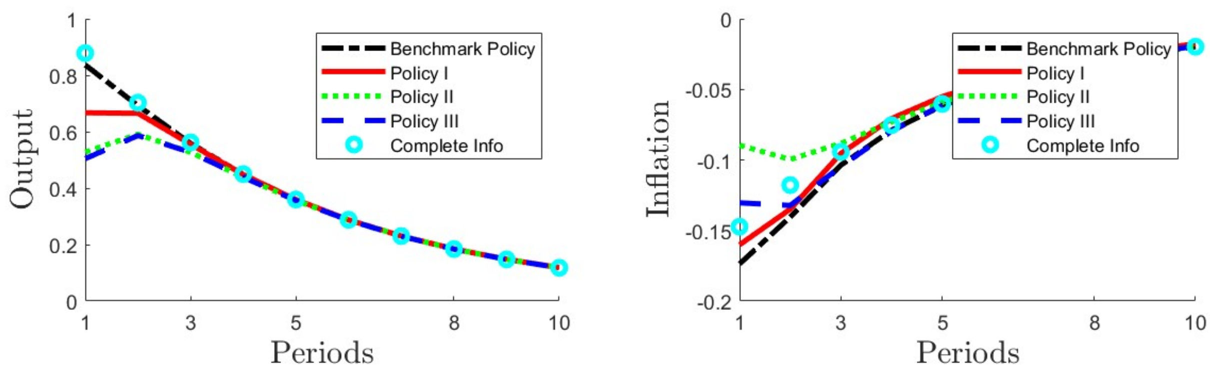

Figure 14: Technology shock

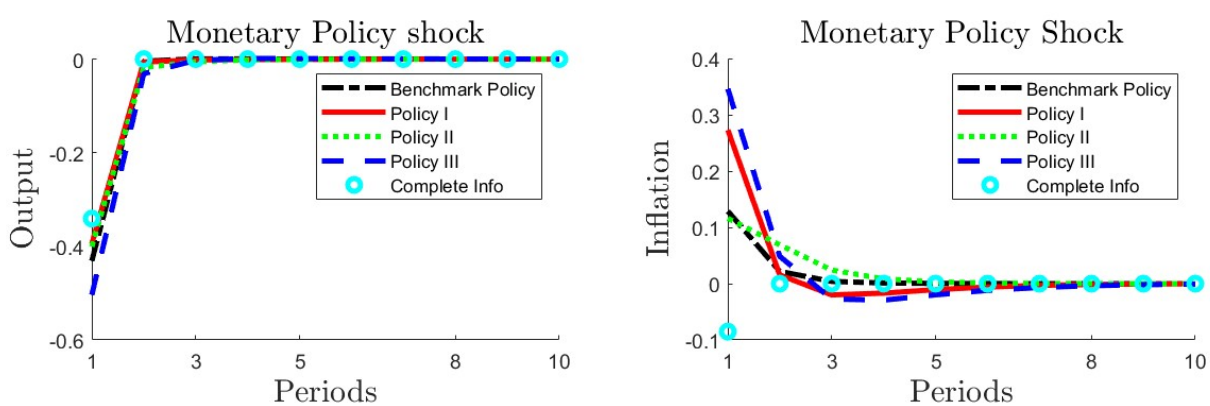

Figure 15: Monetary Policy shock

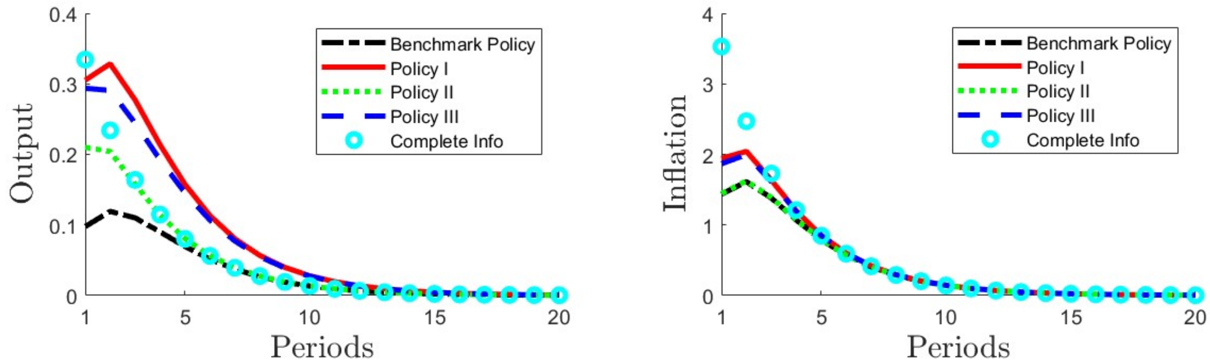

Figure 16: Markup shock

C.2 Asymmetric Information

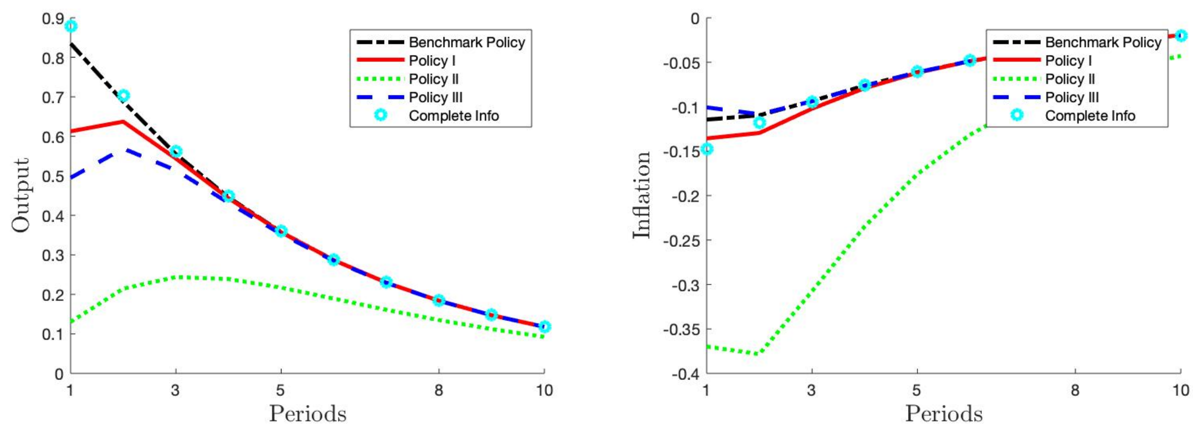

Figure 17: Technology shock

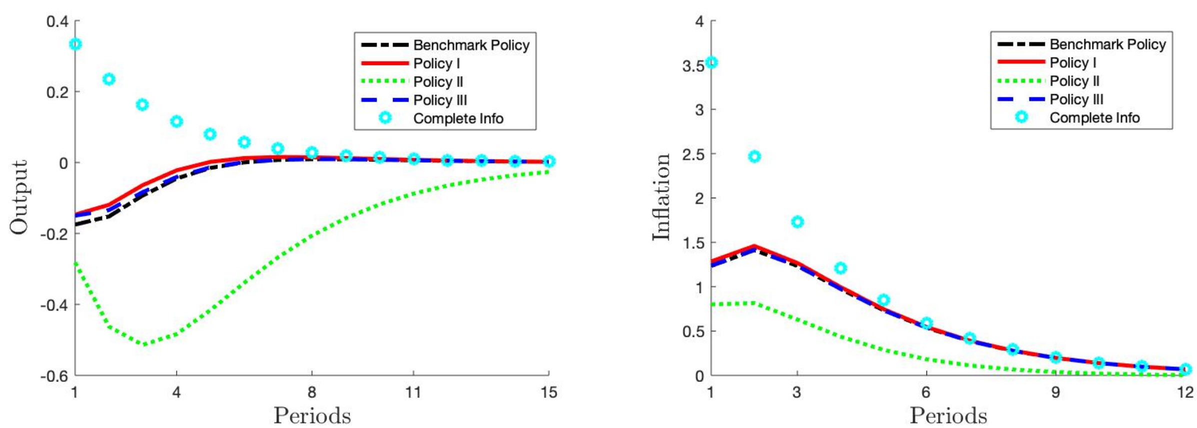

Figure 18: Markup shock

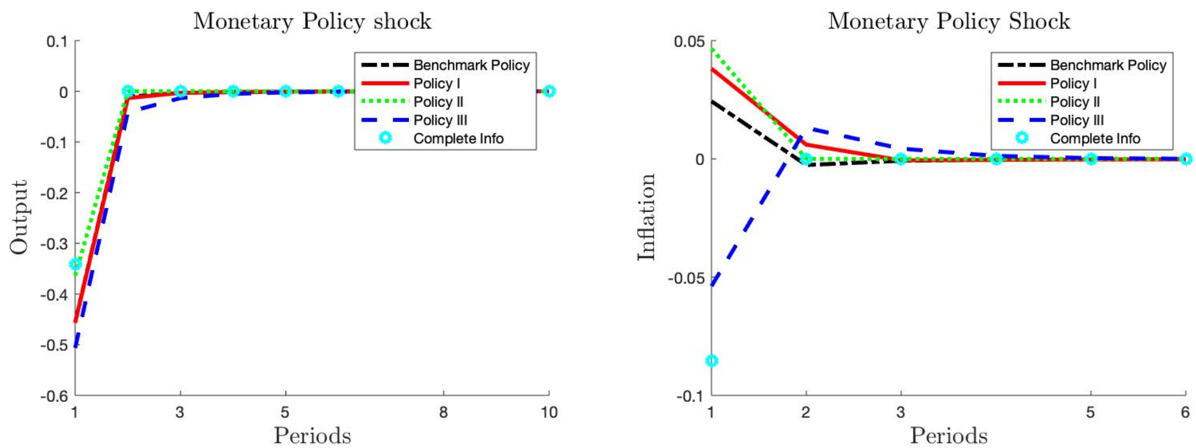

Figure 19: Monetary Policy shock

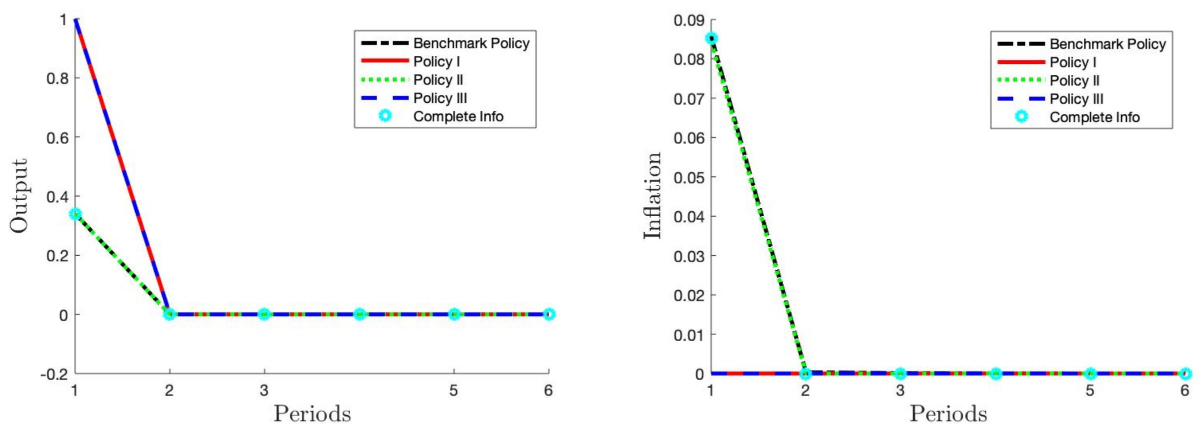

Figure 20: Preference shock