A Belief-Driven Taylor Rule: Expectation As a Policy Tool *

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Abstract

In reality, a central bank does not observe fundamentals within an economy and information dispersion exists among all the sectors. This paper employs a New Keynesian dynamic stochastic general equilibrium (DSGE) model with non-nested information and extends the information dispersion to all sectors. We study monetary polices with internalization of public expectations by means of policy-wise information implementation. Theoretically, we show that this implementation determines signal extraction processes, dynamics of macro-variables and hence macro-volatility. Numerically, we find an increasing importance in private sectors' information in the context of anchoring public expectations. This paper aids in understanding impacts of policy-rule information choices .

Keywords: Incomplete Information, Monetary Policy, Asymmetric Information, Multiple Endogenous Signals

JEL Codes: D82, E52

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1 Introduction

Modern monetary policy focuses on hitting the target inflation under the circumstance of rational expectation, and the monetary authority, i.e., the central bank, dictates the monetary rules. After the financial crisis in 2008, the US Federal Reserve began discussing an average inflation targeting policy aimed at enhancing economic recovery. In 2020, during the COVID-19 pandemic, the Federal Reserve announced the adoption of this policy as a part of its monetary strategy. This policy enlarges policy space with a temporary tolerance of inflation. Therefore, the federal funds effective rate was kept at approximately 0.08% during 2021. Meanwhile, the consumer price index (CPI) has increased 6.6%[1] because of the global crisis. There are consequences of this temporary tolerance of inflation. From Figure 1, the CPI of the United States had reached 292.29[2] and its year-over-year increase had reached 8.6% by May 2022, which is a historical high since the late 80s.

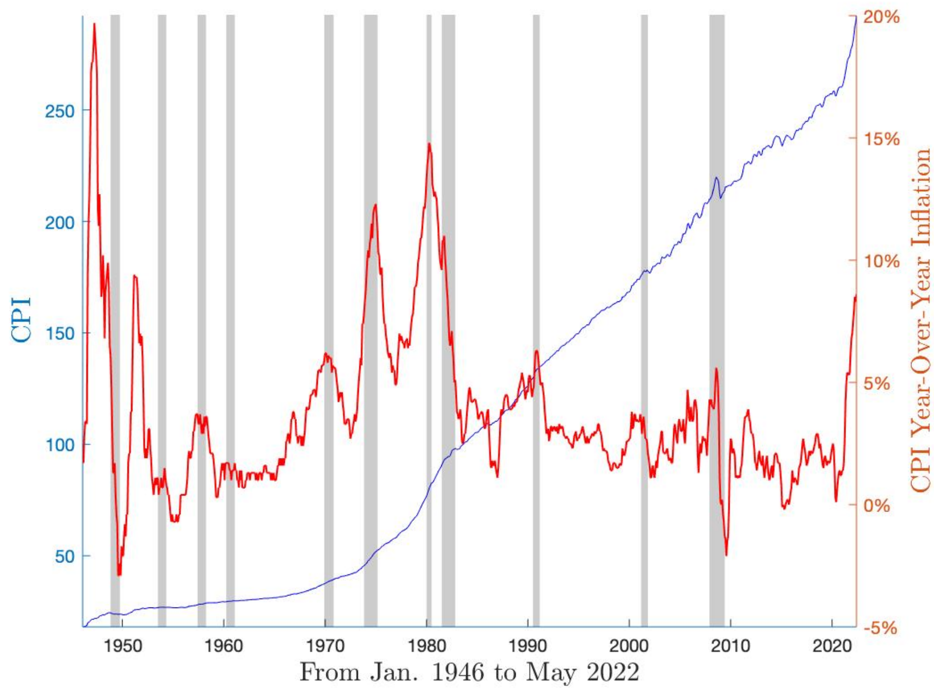


Figure 1: Inflation Measured by CPI

Facing such high inflation, the Federal Reserve takes consequential actions aiming to pull back inflation to its targeted value, 2%. Since March 2022, the Federal Reserve has raised Federal funds rate three times and, in July 2022, announced a 0.75-percentage-point

[1]From 261.58 in January 2021 to 278.80 in December 2021.
[2]Based on 1982–1984.

increase, which is also a historical high within the past 28 years. The right-axis in Figure 2 documents a steep increase in Federal funds rate. A restrictive monetary policy tends to continue according to Chair Jerome H. Powell's Jackson Hole speech on August 26th, 2022. He has emphasized central banks' duty and determination of delivering low and stable inflation. Furthermore, he has brought out the importance of the public's expectations about inflation, arguing that a central bank should use its tools to prevent the public's high expectations about future inflation from being entrenched. Because if those expectations realized, it would reinforce the public's beliefs about an economy and induce higher wage and pricing decisions due to rational inattention. Instead of acting to shape private sectors' beliefs about the economy, we study the model where private sectors' beliefs are implemented as policy tools and internalized within monetary policy.

Typically, a central bank makes decisions based on its own information set. This information set is generally assumed to be complete. However, as shown by various research in the literature, both private and public sectors have been found to be partially informed and there exists pronounced expected inflation disagreement across economic agents, meaning no sector has a complete set of information(Candia, Coibion and Gorodnichenko (2020),Andrade et al. (2016), Dovern, Fritsche and Slacalek (2012)). The left-axis in Figure 2 includes households' inflation expectation[3] and the central bank's inflation expectation starting from Jan. 2020, which is the beginning of COVID-19 pandemic, to Jun. 2022. We firstly notice that the households' inflation expectation is consistently above that of central bank. This phenomena can be interpreted as different economic agents form expectations based on different information sets. Secondly, the Federal Funds Rate sharply increases not until recently. This is potentially because of central bank's relatively stable inflation expectation during past two years [4]. It is important to study monetary policies in the context of incomplete information setting, especially with the shattering of the complete information rational expectation assumption for central banks. The average inflation targeting policy on the one hand provides certain tolerance of inflation, on the other hand it may be problematic due to sluggish policy actions which would cause severe inflation as now.

Under the framework of the average inflation targeting policy with the fact that the Taylor rule is still treated as the benchmark tool, an internalization of public expectations provides us with useful insights. With internalizing public expectations into the Taylor rule, it would keep public's expectations from being entrenched.[5] It may also deliver a smooth

[3]Based on Surveys of Consumers by the University of Michigan.

[4]From 2020-Q3 to 2022-Q1, central bank's inflation expectation is around 2 percent. Therefore they have no incentive to raise the Federal Funds Rate. Binder, Janson and Verbrugge (2022) show that "most individual forecasters' long-run inflation expectations fluctuate substantially, with sizeable departures from target" comparing with SPF's stabilized expectations around target.

[5]Since the model structure is assumed to be common knowledge for economic participants, the public

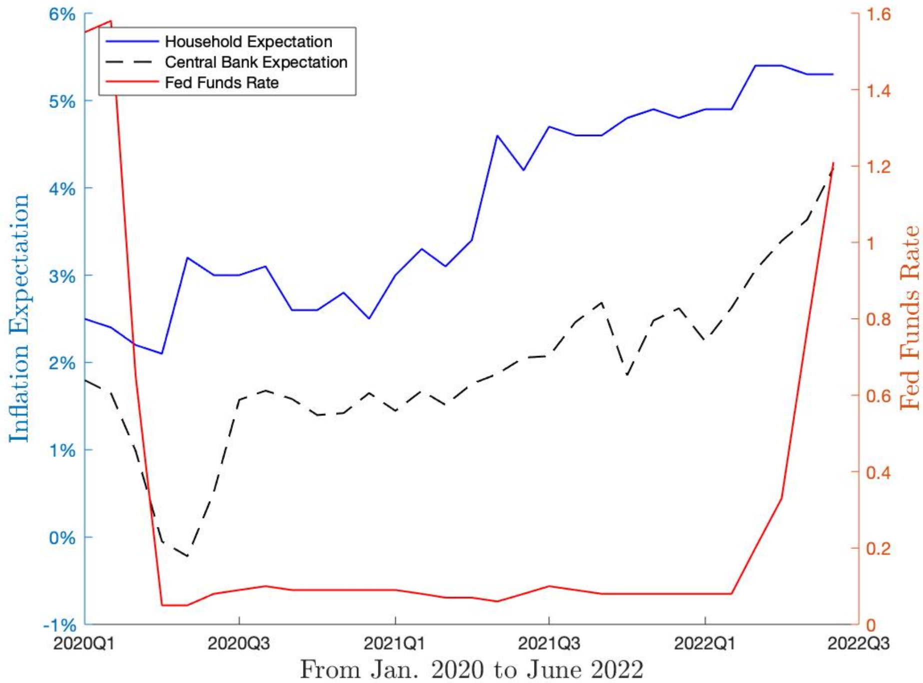


Figure 2: Inflation Expectations and Federal Funds Rate

transition of the Federal Funds Rate. Moreover, this internalization may reduce the total economic volatility and result in better social welfare. We then study a more realistic case where heterogeneous information exists between private sectors .

This paper employs a New Keynesian DSGE model with an extension and argues that a central bank should learn more about the private sector's information with the increase of signal complexity, especially demand shocks. First, we consider a DSGE model with heterogeneous information between private sectors and the central bank. We then introduce an asymmetric information structure within private sectors. In the latter setting, a household has its own private information, a signal of its own preference shock, and technology shock and markup shock become private for firms and the central bank.

The contribution of this paper is threefold. First, we propose different belief-driven Taylor rules in the context of incomplete information environment, in the sense that monetary authority sets interest rates corresponding to different information. This illustrates our idea of internalizing public expectations within policy decisions. Second, we study a dispersed information case where all three sectors are endowed with non-nested information sets. Last,

would know which information set is applied within the monetary policy. Therefore, their high expectations about inflation would certainly lead to an increase in the Federal Funds Rate. There would be no unanticipated shock to affect the nominal interest rate.

we both theoretically derive analytical solutions to describe impacts of various policy-wise information implementation on macro-dynamics and macro-volatility, and numerically study utilization of private sector's information with an increase with economy's information structure complexity. To our knowledge, these three points are novel enrichment to the literature.

The main result of this paper shows when there is no information dispersion within private sectors, a central bank should believe in its own information; as the signal complexity [6] increases, depending on the volatility of household shocks, if it is not negligible, the central bank should consider household expectations other than its own in the belief-driven Taylor rule. This sheds light on the importance of newly emerging survey data. This result can be understood through thinking about the original goal of monetary policy and micro structure of the economy. The goal of a monetary policy is to stabilize an economy. With full information in rational expectation literature, a central bank can observe all the fundamentals then set the interest rate to hit the target inflation rate. However, with incomplete information, a central bank cannot observe all shocks and private signals of private sectors but has to form its expectations about both endogenous and exogenous variables. Furthermore, a central bank does not serve in the economy as a producer or a consumer. With the simple signal structure[7], a central bank's information dominates the private sector's information because private signals are "noisy" central bank signals due to noises from bank disclosure. In this scenario, intuitively, a central bank's information is more precise and valuable. This is consistent with welfare analysis results. Adoption of a central bank's information within the belief-driven Taylor rule increases total welfare by 34 percent more than adopting private information.

However, the story becomes different when we introduce a simple asymmetric information structure within private sectors and differentiate the household from firms. A demand side shock [8] can only be observed by the household and supply shocks [9] will only enter firms' and the central bank's information sets. The bank disclosure problem still exists but only between firms and the central bank. The difference is that when the representative household is endowed with demand side information, supply shocks will not be misinterpreted as they were in the symmetric information setting. For example, when a positive markup shock hits, the representative household will only see an increase in price level and lower their aggregate demand instead of confusing it with a decrease in marginal cost. With different information structures, signal extraction processes also differ. Not surprisingly, the total volatility of an economy increases with an increase in signal structure complexity. Compared

[6] The existence of information dispersion within private sectors.

[7] Households and firms hold the same information set

[8] A preference shock, ε^{β}.

[9] Aggregate and idiosyncratic technology and markup shocks.

with private sectors sharing a symmetric information set, we observe larger welfare loss (i.e macro-volatility) for each kind of policy-wise information implementation when non-nested information structure extends to all sectors. Because no matter which information set is applied[10], there will be a higher-order belief formation process[11], and the choice of any particular information set will not increase signal extraction efficiency. The crucial finding here is that a simple demand shock now dominates supply shocks, and this result is seconded by a study of weighted strategy in monetary policy. We suggest central banks should consider implementing household expectations when conduct monetary policy decisions. Because of central bank's communication error, firms still receive noised central bank signals. Together with firms' own idiosyncratic noises, their information set (Ω_t^{firm}) is dominated by central bank's information. Numerical analysis shows that with implementation of household information, economy's average period welfare loss approaches benchmark results where central bank is endowed with full information.

Related Literature: This paper relates to the ongoing literature of information frictions and their macro-size implications. [see Nimark (2008), Gorodnichenko and Coibion (2012) and Angeletos and Huo (2021)] Melosi (2017), Han, Tan and Wu (2021b) study the signaling effects, where they mainly focus on firm-side incomplete information. Empirical evidence in Candia, Coibion and Gorodnichenko (2020) and Gorodnichenko, Coibion and Candia (forthcoming) find that firms hold different expectations with respect to SPF or households. Therefore, we examine scenarios in which information sets are incomplete and asymmetric among households, firms, and central banks.

Carboni and Ellison (2011) and Kohlhas (2021) both study the scenario that central bank shares incomplete but different information sets against private sectors, whereas in the first paper, there is no output gap in monetary policy and no signal extraction process between monetary authority and private sectors; in the second paper, there is no expected inflation in monetary policy. Our benchmark theory model combines. In our numerical analysis, we introduce central bank communication between central bank and firms as in Kohlhas (2021). The importance of central communication channel is seconded by the study of Nakamura and Steinsson (2018) where they study impacts of central bank announcements.

For monetary policy transmission, Falck, Hoffmann and Hürtgen (2021) study the monetary policy signaling channel with heterogeneous degree of disagreement. Our paper introduces a way of policy-wise internalizing disagreed expectations and studies associated transmission channels. We both theoretically and numerically discuss results in the context

[10] In sum, there are three different information sets that a central bank may apply to set the interest rate: the representative household's information (Ω_t^{HH}), firms' information (Ω_t^{firms}) and its own information (Ω_t^{cb})

[11] $\mathbb{E}^{Household}[\mathbb{E}^{CentralBank}]$, $\mathbb{E}^{Firms}[\mathbb{E}^{CentralBank}]$, $\mathbb{E}^{Firms}[\mathbb{E}^{Household}]$, etc.

of social welfare as in Morris and Shin (2002). This paper also provides possible answers to the concerns in Coibion et al. (2020) regarding the use of expectation as a policy tool. Furthermore, this paper draws the same conclusion as in Gorodnichenko, Coibion and Weber (forthcoming) that a central bank should emphasize the efficiency of communication with private sectors, especially households. Finally, the solution method follows Han, Tan and Wu (2021a), where they provide a user-friendly tool box to solve a disperse information model.

The paper is structured as follow: Section 2 illustrates the benchmark model. Section 3 derives analytical solutions of the benchmark model. Section 4 provides numerical solutions to the benchmark model adding central bank communication error. Section 5 concludes.

2 Model Environment

We consider a prototypical new Keynesian model. The model consists of a representative household, a monetary authority and a continuum of intermediate firms with nominal rigidities in terms of quadratic price adjustment costs. Each sector operates on its own information set when forming conditional expectations. We call it symmetric information structure within private sectors if the household and firms share a same information set. The economy's fluctuation is primarily driven by a preference shock, which affects the household's discount factor, an aggregate technology shock, which affects productivity level, and an aggregate markup shock, which affects price level.

Timing: There are three stages within a time period. After all the innovations arriving, firms pre-set prices with partial information. Then, the central bank reacts to their information and delivers a public interest rate. Last, firms adjust the prices and households respond to clear the good and labor market.

2.1 Household

A representative household maximizes the utility function

$$\mathbb{E}_0^{HH} \sum_{t=0}^{\infty} \beta^t \exp\left(\varepsilon_t^{\beta}\right) \left[\log\left(C_t\right) - \frac{N_t^{1+\eta}}{1+\eta}\right],$$

where $\beta \in (0,1)$ is the discount factor and $1/\eta > 0$ is the Frisch elasticity of labor supply. $\mathbb{E}_t^{HH}$ is the household expectation operator that operates on a private information set,

Ω_t^{HH}, which will be clarified later. ε_t^β is a demand shock, which affects the household's intertemporal substitution. This shock is assumed to follow *i.i.d.* Gaussian distribution

$$\varepsilon_t^\beta \sim \mathbb{N}\left(0, \sigma_\beta^2\right).$$

The household's budget constraint is given as

$$\int_0^1 P_{it} C_{it} di + (1 + i_t) B_t \leq \int_0^1 \Pi_{it} di + W_t N_t + B_{t-1},$$

where P_{it} and C_{it} are price and consumption for intermediate good i and final good. The price index is aggregated by $P_t = \left(\int_0^1 P_{it}^{1-\rho} di\right)^{\frac{1}{1-\rho}}$. The final good consumption is aggregated by $C_t = \left(\int_0^1 C_{it}^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}}$. ρ is the elasticity of substitution. B_t is the one-period risk-free bond, Π_{it} is the profit for intermediate good firm i, W_t is the nominal wage and N_t is the labor supply.

Let c_t and π_t denote log deviations from steady states of consumption and inflation. We have the Euler Equation which governs intertemporal behavior of the household

$$c_t = \mathbb{E}_t^{HH}\left[c_{t+1}\right] - \left(i_t - \varepsilon_t^\beta - \mathbb{E}_t^{HH}\left[\pi_{t+1}\right]\right),$$

where i_t is the nominal interest rate and $\mathbb{E}_t^{HH}$ is the expectation operator for the household. Let y_t denote final output's log deviation from steady state. In equilibrium, we have the IS curve

$$y_t = \mathbb{E}_t^{HH}\left[y_{t+1}\right] - \left(i_t - \varepsilon_t^\beta - \mathbb{E}_t^{HH}\left[\pi_{t+1}\right]\right). \tag{2.1}$$

2.2 Firms

There is a continuum of intermediate firms indexed by $i \in [0, 1]$. Firms are endowed with the same linear production function $Y_{i,t} = A_t N_{i,t}$, where $Y_{i,t}$ is production of good i and $N_{i,t}$ is the labor input of firm i. A_t is the aggregate technology level. It is modeled as a persistent AR(1) process with a *i.i.d.* Gaussian noise θ_t

$$A_t = A_{t-1}^{\rho_a} \exp\left(\theta_t\right): \quad \rho_a \in (0, 1), \theta_t \sim \mathbb{N}\left(0, \sigma_\theta^2\right)$$

Aggregate technology signal is assumed to be non-observable to intermediate firms. Instead, the signal of aggregate technology contains two noises: (i) an industry level *i.i.d.* Gaussian noise, ε^a_{xt}, and (ii) an idiosyncratic *i.i.d.* Gaussian noise, ε^a_{it}

$$x^a_{it} = a_t + \varepsilon^a_{xt} + \varepsilon^a_{ixt} : \quad \varepsilon^a_{xt} \sim \mathcal{N}\left(0, \sigma^2_{a,x}\right), \varepsilon^a_{ixt} \sim \mathcal{N}\left(0, \sigma^2_{a,i}\right), a_t \equiv ln(A_t).$$

Each firm faces a quadratic price adjustment cost as in Rotemberg (1982) and maximizes its profit

$$\Pi_{it} = (1 + T^s_t) P_{it}Y_{it} - W_t N_{it} - \frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 P_t Y_t,$$

where $1 + T^s_t$ is stochastic with mean $\frac{\rho}{1-\rho}$. A mark-up shock is defined as $\mathcal{M}_t \equiv \frac{P_t}{W_t/A_t}$. In a symmetric equilibrium, all firms set a same price and we have $\mathcal{M}_t = \frac{\rho}{\rho-1}\frac{1}{1+T^s_t}$. This mark-up shock also follows AR(1) process with a *i.i.d.* Gaussian noise ξ_t

$$\mathcal{M}_t = \mathcal{M}^{\rho_\mu}_{t-1} \exp(\xi_t) : \quad \rho_\mu \in (0, 1), \xi_t \sim \mathbb{N}\left(0, \sigma^2_\xi\right).$$

Same as aggregate technology signal, firms are not able to observe the exact signal of mark-up shock. The mark-up shock signal for each intermediate firm also contains two noises: (i)an industry level *i.i.d.* Gaussian noise, ε^μ_{xt}, and (ii) an idiosyncratic *i.i.d.* Gaussian noise, ε^μ_{it}

$$x^\mu_{it} = \mu_t + \varepsilon^\mu_{xt} + \varepsilon^\mu_{ixt} : \quad \varepsilon^\mu_{xt} \sim \mathcal{N}\left(0, \sigma^2_{\mu,x}\right), \varepsilon^\mu_{ixt} \sim \mathcal{N}\left(0, \sigma^2_{\mu,i}\right), \mu_t \equiv ln(\mathcal{M}_t).$$

Firm's optimization problem gives a New Keynesian Phillips Curve that explains the relationship among current inflation with firms' nowcast of output gap, nowcast of markup shock and one-period forecast of inflation

$$\pi_t = \beta\overline{\mathbb{E}}^F_t[\pi_{t+1}] + \lambda\overline{\mathbb{E}}^F_t[y_t - a_t] + \overline{\mathbb{E}}^F_t[\mu_t], \tag{2.2}$$

where $\lambda \equiv \frac{1+\eta}{\psi}\rho$. Inflation is defined as $\pi_t = p_t - p_{t-1}$. y_t is the log-deviation of output. a_t represents the natural output rate. Therefore output gap is defined as $y_t - a_t$. Realization of markup shock μ_t is not observable to each firm. Each intermediate firm i forms conditional expectations, $\mathbb{E}^F_{i,t}$, based on its own information set $\Omega^F_{i,t}$ and $\overline{\mathbb{E}}^F_t \equiv \int \mathbb{E}^F_{it} di$ denotes the average expectation across firms.

2.3 Monetary Authority

The central bank employs a simple Taylor-type rule which aims at hitting output gap and inflation target

$$i_t = \phi_y \mathbb{E}_t^* [y_t - a_t] + \phi_\pi \mathbb{E}_t^* \pi_t + \varepsilon_t^m \tag{2.3}$$

where ϕ_y and ϕ_π measure monetary rule responses to output gap and inflation and there is a *i.i.d.* Gaussian monetary policy noise $\varepsilon_t^m \sim \mathbb{N}(0, \sigma_m^2)$. We assume central is also equipped with incomplete information and cannot observe a full set of endogenous variables and fundamental realizations.[12] The main contribution of this paper lies in the expectation operator $\mathbb{E}_t^*$. Instead of sticking with its own information, different information sets could be applied by the central bank to form expectations of current economic variables, i.e. total output, technology level and inflation. Private sectors' expectations are policy-wise internalized when the nominal interest rate is set correspondingly to their expectations/beliefs. We thereby call this Taylor-type rule as belief-driven.

2.4 Welfare Loss Function

We follow Nisticò (2007) and derive the welfare loss as a second-order approximation of the household utility function with quadratic adjustment costs[13] :

$$\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1+\eta) \tilde{y}_t^2 \right] + t.i.p.$$

t.i.p. collects the terms that are independent of policies. η is the inverse of Frisch elasticity. $\tilde{y}_t$ is the log-deviation of output gap. We present the total welfare loss as

$$\mathbb{L} = \frac{1}{1-\beta} \frac{1+\eta}{2} var(\tilde{y}_t). \tag{2.4}$$

[12] Orphanides (2001) has emphasized the difference between the real-time policy recommendations and those obtained with the ex post revised data, demonstrating the incompleteness of central bank's information set.

[13] Measured by ψ.

3 Analytical Analysis

In this section, we derive a closed-form solution to a simplified model. The household and firms have different information sets, both of which contain an endogenous signal and an exogenous signal. The central bank can operate on either of information sets, and different choices of information lead to different welfare results. A detailed proof is provided in the appendix.

3.1 Closed-Form Solution

We simplify the model with the following assumption. The simplified model, on the one hand, provides us with an analytical characterization of the economy where information dispersion exists among all sectors, on the other hand, highlights a crucial argument that different policy-wise information implementation results in different signal extraction processes and then different total economic volatility.

Assumption 1. *There is no mark-up shock and its associated industry level and idiosyncratic noises ($\mu_t = 0, \sigma_{\mu,x} = \sigma_{\mu,i} = 0$). There are neither industry level noise nor idiosyncratic noise of technology for intermediate firms ($\sigma_{a,x} = \sigma_{a,i} = 0$). Aggregate technology shock is i.i.d ($\rho_a = 0$).*

Based on Assumption 1, all remaining shocks $(\varepsilon_t^a, \varepsilon_t^\beta, \varepsilon_t^m)$ are *i.i.d.*. Therefore, all one-period forward expectations are essentially zeros. The household's Euler equation is degenerated to

$$y_t = i_t + \varepsilon_t^\beta : \text{with household's information set } \Omega^{HH} = \{i_t, \varepsilon_t^\beta\}. \tag{3.1}$$

i_t is the interest rate and it is a public endogenous signal for both household and firms. ε_t^β is household's private information about exogenous preference shock. Because there are no idiosyncratic noises, we can remove intermediate firms' index i and average expectations in New Keynesian Phillips Curve

$$\pi_t = \lambda \mathbb{E}_t^F [y_t - \varepsilon_t^a] : \text{ with firms' information set } \Omega^F = \{i_t, \varepsilon_t^a\} \tag{3.2}$$

i_t is a public signal and ε_t^a is firm's private information about exogenous technology shock. We consider two monetary policies and solve them accordingly

$$\textbf{Policy I: } i_t = \phi_y \mathbb{E}_t^F [y_t - \varepsilon_t^a] + \phi_\pi \mathbb{E}_t^F \pi_t + \varepsilon_t^m$$

$$\textbf{Policy II: } i_t = \phi_y \mathbb{E}_t^{HH}\left[y_t - \varepsilon_t^a\right] + \phi_\pi \mathbb{E}_t^{HH} \pi_t + \varepsilon_t^m.$$

In Policy I, Taylor rule operates on firms' information set, $\Omega^F = \{i_t, \varepsilon_t^a\}$, to form expectations about current output, technology level and inflation; Policy II adopts household's information set, $\Omega^{HH} = \{i_t, \varepsilon_t^\beta\}$. Following propositions characterize unique incomplete information equilibrium for two policies in closed-form.

Proposition 1. *For Policy I, given Assumption 1 and information structures (3.1) and (3.2), the model features a unique equilibrium where output, inflation and interest rate follow*

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{3.3}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{3.4}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{3.5}$$

where coefficients $(S_{i,h}, S_\beta, S_{i,f}, S_a, C_\beta, C_m, C_a)$ are given by

$$S_{i,h} = -1, S_\beta = 1, S_{i,f} = -\lambda, S_a = -\lambda,$$
$$C_\beta = 0, C_m = \frac{1}{1+\phi_y+\lambda\phi_\pi}, C_a = \frac{\phi_y+\lambda\phi_\pi}{\lambda+\lambda\phi_y+\lambda\phi_\pi}.$$

Proposition 2. *For Policy II, given Assumption 1 and information structures (3.1) and (3.2), the model features a unique equilibrium where output, inflation and interest rate follow*

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{3.6}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{3.7}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{3.8}$$

where coefficients $(S_{i,h}, S_\beta, S_{i,f}, S_a, C_\beta, C_m, C_a)$ are given by

$$S_{i,h} = -1, S_\beta = 1, S_{i,f} = \Delta, S_a = -\lambda,$$
$$C_\beta = \frac{\phi_y}{1+\phi_y+\Delta\phi_\pi}, C_m = \frac{1}{1+\phi_y+\Delta\phi_\pi}, C_a = 0,$$

where $\Delta = \frac{\lambda\phi_y(1+\phi_y)\sigma_\beta^2 - \lambda(\phi_y^2\sigma_\beta^2+\sigma_m^2)}{\phi_y^2\sigma_\beta^2+\sigma_m^2-\lambda\phi_y\phi_\pi\sigma_\beta^2}$.

The closed-form solution straightforwardly helps understand consequences of central bank's different choices of information sets when conducting policy decisions. We firstly focus on signal extraction process under Policy I. When central bank operates on firms' information set, they are lack of information about preference shock. Hence interest rate does not respond to preference shock, i.e. $C_\beta = 0$. Firms can neither extract any useful information of preference shock, i.e. $\mathbb{E}^F[\varepsilon^\beta] = 0$, because both of their signals (i_t, ε_t^a) contain no information about this demand shock [14]. The situation is the same for Policy II. Households will not be able to extract information about supply shocks when central bank adopts household's information and ignores firms' information. Then, different signal extraction processes with no doubt lead to different dynamics for endogenous economic variables. We see coefficients $S_\beta \vee S_a$ as **direct** signalling effects, since they are associated with observable exogenous shocks $\varepsilon_t^\beta \vee \varepsilon_t^a$, and $S_{i,h} \vee S_{i,f}$ as **indirect** signalling effects, which are associated with endogenous monetary policy. With Policy I, a positive preference shock **directly** increases output, and has no influence in both inflation and interest rate. A positive technology shock decreases inflation both **directly** and **indirectly**. Because firms can observe technology shock, a positive technology shock results in a higher output gap therefore a lower inflation, $S_a < 0$. Firms can also observe a higher interest rate [15] and because $S_{i,f} < 0$, inflation will also decrease. This shock surprisingly decreases output **indirectly**, because households observe an increase in interest rate and $S_{i,h} < 0$. With Policy II, a positive technology shock will only directly decrease inflation and have no influence in both output and interest rate. A positive preference shock influences inflation indirectly and output both directly and indirectly. This influence depends on parameterization since Δ is undetermined.

3.2 Welfare Analysis

Social welfare here is measured by output gap volatility which is determined by its dynamics hence choices of information sets for policy makers. Different welfare loss of both policies are calculated below

$$\textbf{Policy I: } var(y_t - \varepsilon_t^a) = \sigma_\beta^2 + C_m^2\sigma_m^2 + (C_a+1)^2\sigma_a^2 \tag{3.9}$$

[14] Mathematically, the projection of ε_t^β on the information space of $\Omega^F = \{i_t, \varepsilon_t^a\}$ is technically zero.

[15] In equation (3.5), $C_a > 0$.

$$C_m = \frac{1}{1+\phi_y+\lambda\phi_\pi}, C_a = \frac{\phi_y+\lambda\phi_\pi}{\lambda+\lambda\phi_y+\lambda\phi_\pi}$$

and

$$\textbf{Policy II: } var(y_t - \varepsilon_t^a) = (1-C_\beta)^2\sigma_\beta^2 + C_m^2\sigma_m^2 + \sigma_a^2 \tag{3.10}$$

$$C_\beta = \frac{\phi_y}{1+\phi_y+\Delta\phi_\pi}, C_m = \frac{1}{1+\phi_y+\Delta\phi_\pi}, \Delta = \frac{\lambda\phi_y(1+\phi_y)\sigma_\beta^2 - \lambda(\phi_y^2\sigma_\beta^2+\sigma_m^2)}{\phi_y^2\sigma_\beta^2+\sigma_m^2-\lambda\phi_y\phi_\pi\sigma_\beta^2}.$$

Equations (3.9) and (3.10) document output gap volatility under two policies. It is straightforward to see that different choices of policy information sets deliver different macroeconomic volatility. In traditional New Keynesian literature, the Taylor rule focuses on output gap and inflation which mainly involves supply shocks besides the monetary shock. Therefore, variable dynamics and volatility change dramatically when we apply household's information set. When the monetary authority considers household's information as their policy regime, they need realize a possibility of asymmetric information structure between producers and consumers. Consumers may form their expectations of supply shocks using both private signals and public signals instead of a direct observation. Different signal extraction process ends up with different welfare loss. Even if the Taylor rule is not directly linked with demand shocks, welfare loss does. Comparing equations (3.9) and (3.10), we can tell that welfare loss bringing from demand side, σ_β^2, decreases[16] when applying household's information in Taylor rule. Although total volatility is associated with all shocks and parameterization, this explains why we observe lower welfare loss when the Taylor rule operates on household's information set in the later section where we introduce a complex information structure and solve the model numerically.

4 Quantitative Analysis

In this section, we introduce central bank communication error and solve the model numerically. We firstly study symmetric information between consumers and producers and then move on to asymmetric information structure.

[16]From the coefficient that $(1-C_\beta)^2 < 1$.

4.1 Symmetric Information

We introduce the communication error between private sectors and a central bank from Kohlhas (2021). In this section, preference shock, ε_t^β, is eliminated from the model. The central bank observes noised technology shock and markup shock. Instead, they receive signals (z_t^a and z_t^μ) accordingly.

$$z_t^a = a_t + \varepsilon_{zt}^a : \varepsilon_{zt}^a \sim \mathcal{N}\left(0, \sigma_{z,a}^2\right), \quad z_t^\mu = \mu_t + \varepsilon_{zt}^\mu : \varepsilon_{zt}^\mu \sim \mathcal{N}\left(0, \sigma_{z,\mu}^2\right),$$

where ε_{zt}^a and ε_{zt}^μ are assumed to be *i.i.d.* Gaussian distributed and they are central bank's observational noises about aggregate technology level and markup shock. Signals z_t^a and z_t^μ would be delivered from the central bank to private sectors, but there exists communication error (i.e. $\varepsilon_{\omega t}^a$ and $\varepsilon_{\omega t}^\mu$). Instead, private sectors receive signals, ω_t^a and ω_t^μ regarding to those economic indicators published by central bank,

$$\omega_t^a = z_t^a + \varepsilon_{\omega t}^a : \varepsilon_{\omega t}^a \sim \mathcal{N}\left(0, \sigma_{\omega,a}^2\right), \quad \omega_t^\mu = z_t^\mu + \varepsilon_{\omega t}^\mu : \varepsilon_{\omega t}^\mu \sim \mathcal{N}\left(0, \sigma_{\omega,\mu}^2\right),$$

where noises $\varepsilon_{\omega t}^a$ and $\varepsilon_{\omega t}^\mu$ are *i.i.d.* Gaussian distributed and signals ω_t^a and ω_t^μ would enter information sets. Last, we include inflation measurement error. We assume that neither the central bank nor private sectors can observe the true inflation. The inflation signal they receive ($\bar{\pi}_t$) contains an *i.i.d.* Gaussian noise ε_t^p

$$\bar{\pi}_t = \pi_t + \varepsilon_t^p : \varepsilon_t^p \sim \mathcal{N}(0, \sigma_p^2).$$

We conclude information sets for the household, firms and the central bank as follow

$$\Omega_t^{HH} = \Omega_{it}^F = \left\{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^\infty, \quad \Omega_t^{cb} = \left\{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^\infty. \tag{4.1}$$

The representative household shares the same information set with firms. They receive their private signals about exogenous technology and markup shocks (x_{it}), public signals (ω_t) of exogenous shocks sent out by the central bank and public signals of endogenous variables ($\bar{\pi}_t$ and i_t). Public signals serve as common knowledge and are included in central bank's information set. Besides, the central bank receives their private signals about exogenous technology and markup shocks (z_t).

For monetary policies, we consider four following cases:

Benchmark: $i_t = \phi_y \left[y_t - a_t\right] + \phi_\pi \pi_t + \varepsilon_t^m,$

Policy I: $i_t = \phi_y \mathbb{E}_t^{cb} \left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{cb} \pi_t + \varepsilon_t^m,$

$$\textbf{Policy II: } i_t = \phi_y \mathbb{E}_t^{Private}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{Private} \pi_t + \varepsilon_t^m,$$

$$\textbf{Policy III: } i_t = \phi_y \mathbb{E}_t^{cb}[\mathbb{E}_t^{Private}(y_t - a_t)] + \phi_\pi \mathbb{E}_t^{cb}[\mathbb{E}_t^{Private} \pi_t] + \varepsilon_t^m,$$

where $\varepsilon_t^m \sim \mathbb{N}\left(0, \sigma_m^2\right)$ is the exogenous monetary shock.

Benchmark monetary policy follows the traditional framework of complete information. It assumes that a central bank can observe all the realizations of economic variables. A nominal interest rate is set accordingly. However, in reality a central bank is endowed with much less information. In Policy I, we assume the central bank conducts policy with its own information set, i.e. Ω_t^{cb} in (4.1). We denote $\Omega_t^{Private}$ as private sectors' information set since the household and firms share a same information set. In Policy II, we allow the central bank to internalize private sectors' information by operating on information set $\Omega_t^{Private}$. Policy III serves as another possibility of information implementation. Higher-order belief $\mathbb{E}_t^{cb}[\mathbb{E}_t^{Private}]$ contains another information filtering process. While obtaining private sectors' expectations, the central bank filters these expectations with its own information set.[17] It is worth noting that this is different from a measurement error of survey data; a measurement error case is equivalent to Policy II with another exogenous shock.

4.1.1 Solution Method and Parameterization

This incomplete information model involves multiple endogenous signals, i.e. interest rate and inflation, and exogenous signals, i.e. technology signal and markup signal. The model with symmetric private information structure contains two types of conditional expectations, i.e. $\mathbb{E}_t^{private}$ and $\mathbb{E}_t^{cb}$; in the next section, with introduction of asymmetric information structure between households and firms, the model contains three types of conditional expectations, i.e. $\mathbb{E}_t^{HH}$, $\mathbb{E}^{Firms}$ and $\mathbb{E}_t^{cb}$. Instead of applying time-domain methods in Nimark (2008), we apply the solution method from Han, Tan and Wu (2021a) with the toolbox, zTran. Their approach is based on policy function iterations in the frequency domain. The canonical representation can be found in Appendix A

Parameterization is performed before simulations. Table 1 documents conventional parameters values for quarterly models and parameters values for exogenous processes. Discount factor, β, is set as 0.99 and can be interpreted as quarterly value. The inverse of Frisch elasticity, η, is set to 1. The elasticity of substitution, ρ, among intermediate goods is set to 6. The price adjustment cost, ψ, is established as the slope of NKPC, λ, equal 0.25. These are commonly used in the literature. For the Taylor rule coefficients, we follow Kohlhas (2021) and set $\phi_y = 1.81$, and we follow Taylor (1999) to set $\phi_\pi = 0.5$; as Yellen

[17] In another word, the central bank has less confidence in adopting results from survey data directly and attempts to interpret those results in its own way.

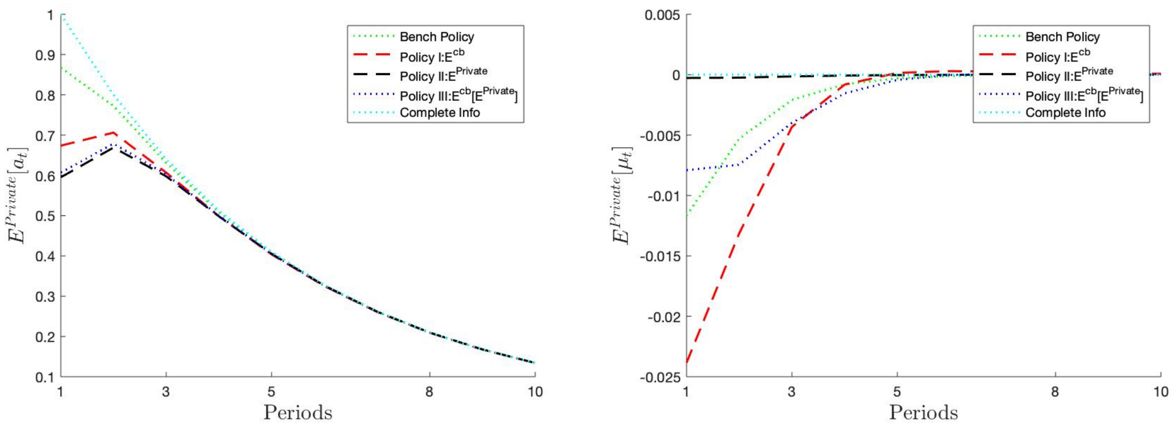
Table 1: Symmetric Model Parameterization

Parameter	Value	Description
Households		
β	0.99	Discount factor
η	1	Inverse of Frisch elasticity
Firms		
ρ	6	Elasticity of substitution
ψ	48	Price adjustment cost
ρ_a	0.8	Persistence of technology shock
ρ_μ	0.7	Persistence of markup shock
σ_θ	0.6	Tech. process white noisy
σ_ξ	0.16	Markup process white noisy
$\sigma_{x,a}$	0.65	Industry tech. noisy
$\sigma_{x,a,i}$	0.2	Firm idiosyncratic tech. noisy
$\sigma_{x,\mu}$	0.2	Industry markup noisy
$\sigma_{x,\mu,i}$	0.11	Firm idiosyncratic markup noisy
$\sigma_{\omega,a}$	1	Industry's policy-signal tech. noisy
$\sigma_{\omega,\mu}$	1	Industry's policy-signal markup noisy
Central Bank		
ϕ_y	1.81	MP response to output gap
ϕ_π	0.5	MP response to inflation
$\sigma_{z,a}$	0.4	Central bank tech. noisy
$\sigma_{z,\mu}$	0.1	Central bank markup noisy
σ_p	0.8	Inflation measurement error
σ_m	0.4	Monetary policy shock

(2012) suggests, monetary policy should respond more to the output gap than inflation. We follow Rudebusch (2002) and set $\sigma_m = 0.4$. The rest parameters values of technology and markup process are from Kohlhas (2021). We focus on the analysis of information implementation within the monetary policy. We thereby will not calibrate the precision of central bank signals with different values and we set both central bank's communication noisy ($\sigma_{\omega,a}$ and $\sigma_{\omega,\mu}$) to 1.

4.1.2 Now-cast and Forecast Impulse Responses

An Aggregate Technology Shock : We study private sectors' now-cast of exogenous shocks and forecast of endogenous variables with different policies. Figure 3 documents private sectors' now-cast of technology level, $\mathbb{E}^{Private}[a_t]$, and markup shock, $\mathbb{E}^{Private}[\mu_t]$, to a positive innovation in technology. The cyan dotted line corresponds to complete information setting, where no information friction exists. The green dotted line corresponds to benchmark policy, where central bank is endowed with full information while households and firms receive noised signals. The red dashed line corresponds to Policy I, where nominal interest rate is determined by central bank's information. The black dashed line corresponds to Policy II, where nominal interest rate is determined by private sectors' information. The blue dotted line corresponds to Policy III, where central bank filters private information when deciding nominal interest rate.



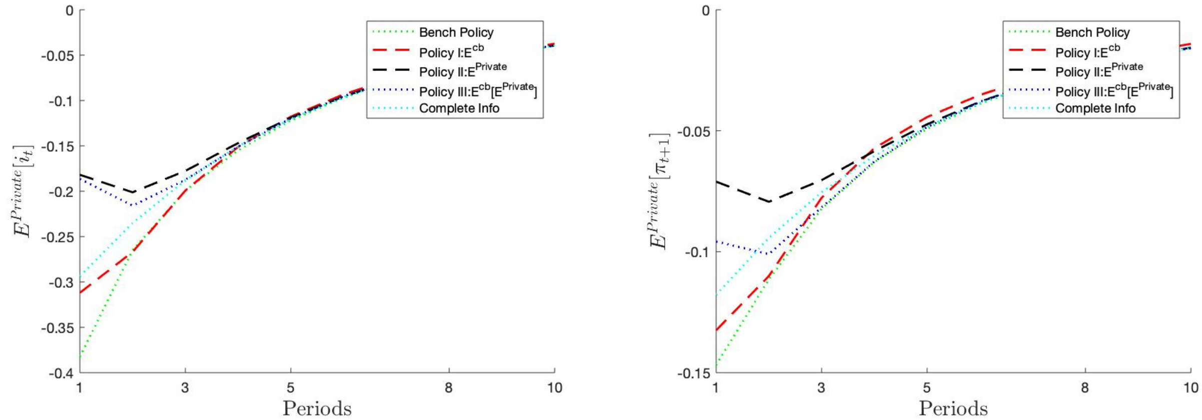
(a) $\mathbb{E}^{Private}[a_t|\varepsilon_t^a]$ (b) $\mathbb{E}^{Private}[\mu_t|\varepsilon_t^a]$

Figure 3: Responses of Private Sectors' Now-cast of Technology Level and Markup Level to a Positive Innovation in Technology

With complete information, a positive technology shock shifts aggregate supply curve to the right and hence price level decreases. With incomplete information, households and firms receive imperfect observation of shocks. White noises will dampen their beliefs about realized shocks in the first time period. Because all white noises are *i.i.d.*, they revise their understanding about fundamentals. Therefore, in Figure 3:a, we observe a hump-shape beliefs of aggregate technology shock. When nominal interest rate responds to private sectors' information, communication error serves as an additional noise comparing with responding to central bank's information. Private sectors' perception of the realized shock would be further dampened. This explains why black and blue lines are below the red line in Figure 3:a. When central bank implements private sectors' information within monetary policy, all model information sets become nested and shock are anticipated for households and firms. They can distinguish aggregate technology shock, a_t, from markup shock, μ_t, and therefore

$$\mathbb{E}^{Private}[\mu_t|\varepsilon_t^a] = 0.$$

In Figure 3:b, the black dashed line does not respond to the aggregate technology innovation. For the rest policy rules in Figure 3:b, when observe a decrease in price, households and firms will place probability on a negative markup innovation.



(a) $\mathbb{E}^{Private}[i_t|\varepsilon_t^a]$ (b)$\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^a]$

Figure 4: Response of Private Sectors' Now-cast Nominal Interest Rate and One-period Inflation Forecast to a Positive Innovation in Technology

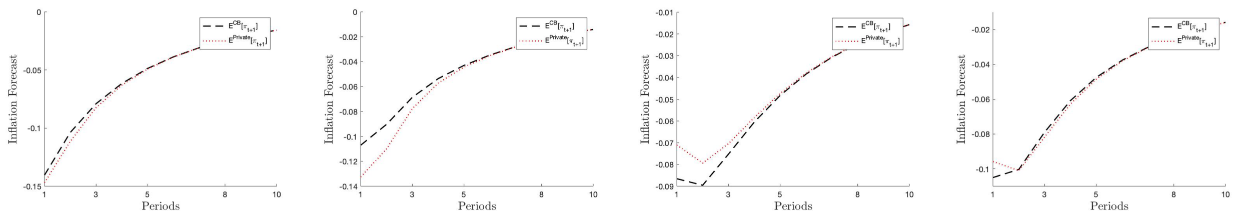
Figure 4 documents private sectors' now-cast of nominal interest rate and one-period forecast of inflation with one standard deviation positive technology shock under different policy regimes.

With complete information, a positive aggregate technology innovation increases the output gap, together with the decrease in price level, and hence decreases nominal interest rate. Now-cast of nominal interest rate depends on how households and firms understand signals. With Policy II, Taylor rule is driven by private sectors' information:

$$i_t = \phi_y \mathbb{E}_t^{Private}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{Private} \pi_t + \varepsilon_t^m.$$

Consistent with above, households and firms underestimate realized technology shock size because of white noises and communication error. They expect a smaller output gap as well as a smaller decrease in nominal interest rate. With $i.i.d.$ noises vanished in the next period, they will update their beliefs. In Figure 4:a, the black dashed line is hump-shape and responds the least, and it is worthy noting that households and firms tend to over-estimate when monetary policy works on other information sets. The cyan dotted line represents private sectors' expectation of nominal interest rate change when we shut down information frictions at every layer. No matter central bank operates on full information set or its own noised information set, households and firms would expect a larger decrease in nominal interest rate, i.e. green dotted and red dashed lines are below the cyan dotted line.

Characteristics in their now-cast are also reflected in their forecast. In Figure 4:b, inflation forecast also responds the least when private sectors' information is implemented in monetary policy. They would also over-estimate inflation responses when their information is not



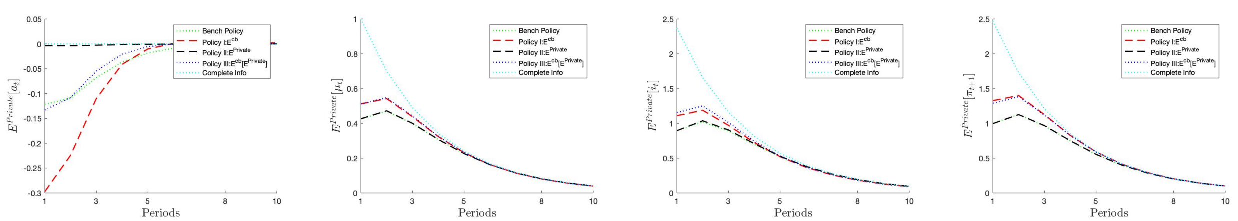
(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

Figure 5: Comparison between Central Bank and Private Sectors' One-period Inflation Forecast to a Positive Aggregate Technology Innovation, i.e. $\mathbb{E}^{CB}[\pi_{t+1}|\varepsilon_t^a]$ and $\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^a]$.

involved in monetary policy. This implies different policy-wise information implementation may affect private sectors' inflation forecast.

Our model also generates disagreement in inflation expectation between private sectors and central bank. As shown in Figure 5, the black dashed line corresponds to central bank's inflation forecast and the red dotted line corresponds to private sectors' inflation forecast. Results show that private sectors' inflation forecast are less volatile than central bank's inflation forecast when private sectors' information set is involved in policy. This implies policy-wise informational choices may affect, even rotate, inflation expectation disagreement.

A Markup Shock : Figure 6 documents private sectors' now-cast of technology level, $\mathbb{E}^{Private}[a_t]$, markup level, $\mathbb{E}^{Private}[\mu_t]$, nominal interest rate, $\mathbb{E}^{Private}[i_t]$, and forecast of inflation, $\mathbb{E}^{Private}[\pi_{t+1}]$ to a positive innovation in markup.



(a) $\mathbb{E}^{Private}[a_t|\varepsilon_t^\mu]$ (b) $\mathbb{E}^{Private}[\mu_t|\varepsilon_t^\mu]$ (c) $\mathbb{E}^{Private}[i_t|\varepsilon_t^\mu]$ (d) $\mathbb{E}^{Private}[\pi_{t+1}|\varepsilon_t^\mu]$

Figure 6: Private Sectors' Now-cast and Forecast Responses to a Positive Innovation in Markup.

With complete information, a markup shock increases price level and hence increases nominal interest rate. Consistent with previous findings, households and firms underestimate the realized shock size, Figure 6:b, and are able to distinguish shocks with Policy II, Figure 6:a. For Figure 6:c, recall monetary policies

$$\text{Policy I} : i_t = \phi_y \mathbb{E}_t^{CB}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{CB}\pi_t + \varepsilon_t^m$$

$$\text{Policy II} : i_t = \phi_y \mathbb{E}_t^{Private}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{Private}\pi_t + \varepsilon_t^m$$

Their underestimation of actual markup innovation leads to an underestimation of current inflation comparing with complete information case. With Policy I, the non-nested information structure induces private sectors to assign probability on a negative productivity shock after observing an increase in nominal interest rate. Therefore, they form higher now-cast of nominal interest rate response with Policy I because ϕ_y is greater than ϕ_π. However, their inflation forecast contradicts their now-cast of nominal interest rate. While they expect larger increase in nominal interest rate today with Policy I than Policy II, they also expect a larger increase in inflation tomorrow. This phenomena implies the same as above: private sectors' inflation forecast is anchored when their information is involved within monetary policy.

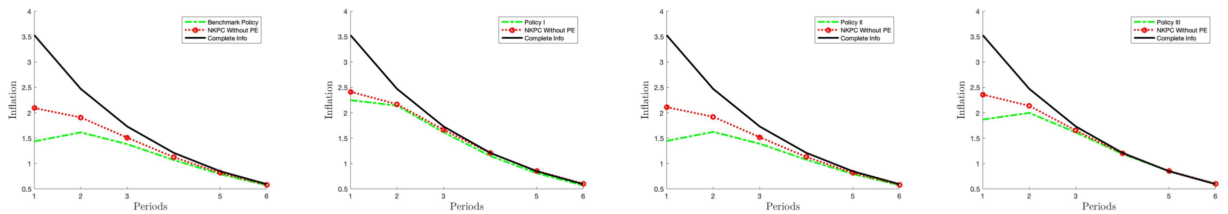
4.1.3 Higher-order Decomposition

We follow Angeletos and Huo (2021), or AH for short, to study a higher-order decomposition in our NKPC's impulse responses. We explore how can we explain the quantitative difference in responses between complete and incomplete information. We also investigate how much is due to lack of exogenous information (the partial equilibrium (PE) component) or endogenous uncertainty of inflation (the general equilibrium (GE) component). While AH focuses on moment estimation and calibration, our analysis focuses on the effects resulting from different policy choices.

To formalize the idea, we first take a look at equation 2.2:

$$\pi_t = \underbrace{\beta \overline{\mathbb{E}}_t^F \left[\pi_{t+1}\right]}_{\text{GE Component}} + \underbrace{\lambda \overline{\mathbb{E}}_t^F \left[y_t - a_t\right] + \overline{\mathbb{E}}_t^F \left[\mu_t\right]}_{\text{PE component}}. \tag{4.2}$$

We decompose NKPC into two parts: the GE component and the PE component. As in AH, the first part captures GE effects, also see Angeletos and Lian (2018), and the second part captures PE effects. Different from AH, we shut down the PE component and solve the structure model numerically. It is worth noting that AH assumes the PE component following an AR(1) process to obtain analytical solutions. We drop this assumption when we solve the structural model numerically using the tool box from Han, Tan and Wu (2021a). Figure 7 documents impulse responses of inflation to a positive innovation in markup with the decomposition above in different policy regimes. The black solid line represents the inflation responses in the complete information scenario. The red dotted, circled line represents the inflation responses without exogenous uncertainty in NKPC with different policies. The green dashed line represents the inflation responses with different policies.



(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

Figure 7: Responses of Inflation to a Positive Innovation in Markup: Isolate Partial Equilibrium Effects in NKPC

Quantitative bites of information friction are significant.[18] We first notice that the GE channel dominates the PE channel only with Policy I, i.e., when the Taylor rule is driven by the central bank's information, effects of firms' uncertainty of next period's inflation overrides effects of their nowcast of exogenous shocks and this is in line with Angeletos and Huo (2021). To understand this, we plug NKPC into different policies and compare benchmark policy with Policy II:

$$i_t = \phi_y \mathbb{E}_t^{CB}\left[y_t - a_t\right] + \phi_\pi \mathbb{E}_t^{CB}\{\beta \overline{\mathbb{E}}_t^{Private}\left[\pi_{t+1}\right] + \lambda(y_t - a_t) + \mu_t\} + \varepsilon_t^m, \tag{4.3}$$

$$i_t = \phi_y \left[y_t - a_t\right] + \phi_\pi \{\beta \overline{\mathbb{E}}_t^{Private}\left[\pi_{t+1}\right] + \lambda(y_t - a_t) + \mu_t\} + \varepsilon_t^m. \tag{4.4}$$

From Equation (4.3), since firms know the model structure, they know the central bank will filter μ_t with its information set even if firms can observe the actual size of it.[19] Therefore, their information about the markup shock is not as important as in Equation (4.4), because μ_t enters the model directly. Also, Figure 7:a and Figure 7:c deliver the same results because if we plug in NKPC into monetary policy, μ_t shares the same characteristics as in Equation (4.4).

4.1.4 Social Welfare

Table 2 documents the average period welfare loss associated with each policy. The first row is the benchmark policy. Rows $3-5$ correspond to average period welfare loss with Policy I-III.

[18]See the difference between the green dashed line and black solid line in Figure 7.

[19]We shut down PE component, so a_t and μ_t become common knowledge for firms.

Table 2: Welfare Analysis of Polices

Policy Rule	Welfare Loss
Benchmark Policy	5.2986
$\mathbb{E}^{cb}$	15.3219
$\mathbb{E}^{private}$	23.2522
$\mathbb{E}^{cb}[\mathbb{E}^{private}]$	30.0245

Not surprisingly, the benchmark policy, where the central bank has full information on the economy, achieves the least welfare loss. The interest rate is generated by the most accurate information. The table shows that it is better for the central bank to set the monetary rule according to their own expectation, rather than adopting private sector expectations or further guessing the private sector expectations, when private sectors observe same noisy signals. This is intuitive when we re-examine the formation sets.

$$\Omega_t^{HH} = \Omega_{it}^F = \{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}, \text{ and } ,\Omega_t^{cb} = \{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}.$$

While inflation, $\bar{\pi}_{t-j}$, and the interest rate, i_{t-j}, are common knowledge among all, private sectors receive private noises, contained in x_{it-j}, and bank disclosure noise, contained in ω_{t-j}. The central bank only receives its own private noise, contained in z_{t-j}, because z_{t-j} and ω_{t-j} together will cancel out the bank disclosure noises. Therefore, the central bank's information set is determined to be more "valuable" than the private sectors' information set, both from our analysis and welfare results.

4.2 Asymmetric Information

While keeping communication frictions between firms and central bank, we take the preference shock, ε_t^{β}, back. We assume households cannot observe technology and mark-up signals, and demand side shock can only be observed by households. It is natural to assume heterogeneity in information sets for different sectors. Now, each of the sectors—households, firms and the central bank—has its own information structure. However, the results still hold that the monetary authority should still rely on private expectations. Thus, we study an optimal weighting strategy for monetary policy given heterogeneous expectations from the private sector.

Household: The corresponding Euler Equation is given as

$$y_t = \mathbb{E}_t^{HH}[y_{t+1}] - \left(i_t - \mathbb{E}_t^{HH}[\pi_{t+1}] - \varepsilon_t^{\beta}\right). \tag{4.5}$$

Firms: Firms share the same properties and information structure as in the previous model.

$$\pi_t = \beta\overline{\mathbb{E}}_t^F[\pi_{t+1}] + \lambda\overline{\mathbb{E}}_t^F[y_t - a_t] + \overline{\mathbb{E}}_t^F[\mu_t], \tag{4.6}$$

Monetary Authority: With heterogeneous information sets, the central bank employs a typical Taylor rule responding to the output gap and inflation. We consider four cases:

$$\textbf{Benchmark:}\ i_t = \phi_y[y_t - a_t] + \phi_\pi\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy I:}\ i_t = \phi_y\mathbb{E}_t^{cb}[y_t - a_t] + \phi_\pi\mathbb{E}_t^{cb}\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy II:}\ i_t = \phi_y\mathbb{E}_t^{HH}[y_t - a_t] + \phi_\pi\mathbb{E}_t^{HH}\pi_t + \varepsilon_t^m,$$

$$\textbf{Policy III:}\ i_t = \phi_y\mathbb{E}_t^{Firm}[y_t - a_t] + \phi_\pi\mathbb{E}_t^{Firm}\pi_t + \varepsilon_t^m,$$

where a_t is the log-deviation of potential output, ε_t^m is the monetary shock and $\varepsilon_t^m \sim \mathbb{N}(0, \sigma_m^2)$
.

Information structure: Different from previous section, we assume each sector observes public signals and sector-related signals. Public signals are inflation with measurement error, $\bar{\pi}_t$, and nominal interest rate i_t, both of which are included in all information sets. Preference shock signal exclusively enters the household's information set. Signals about technology and markup enter firms' and central bank's information sets. Central bank communication error exists between firms and the central bank. We conclude information sets as follow

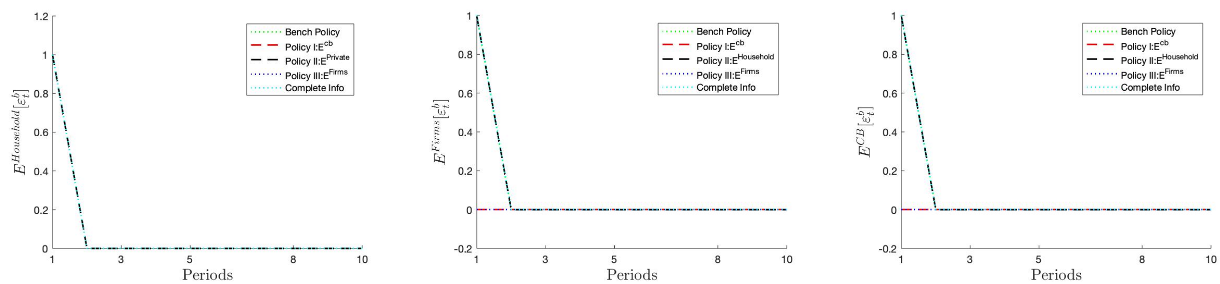
$$\Omega_t^{HH} = \left\{\varepsilon_{t-j}^{\beta}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}, \Omega_{it}^F = \{x_{it-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty} \text{ and } \Omega_t^{cb} = \{z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\}_{j=0}^{\infty}$$

The solution method adopts the frequency-domain policy function iteration approach in Han, Tan and Wu (2021a) with information sets as defined above.

4.2.1 Now-cast and Forecast Impulse Responses

In this section, we study households, firms and central bank's now-cast and forecast of endogenous and exogenous variables. We begin with demand shock, namely ε_t^{β}, and then move to supply shocks, namely ε_t^a and ε_t^{μ}.

A Preference Shock : Figure 8 documents all sectors' now-cast of preference shock to a positive innovation in preference with different policies. This preference innovation signal only enters households' information set. The cyan dotted line corresponds to complete information setting, where no information friction exists. The green dotted line corresponds to benchmark policy, where central bank is endowed with full information while households and firms receive noised signals. The red dashed line corresponds to Policy I, where nominal interest rate is determined by central bank's information. The black dashed line corresponds to Policy II, where nominal interest rate is determined by households' information. The blue dotted line corresponds to Policy III, where nominal interest rate is determined by firms' information.



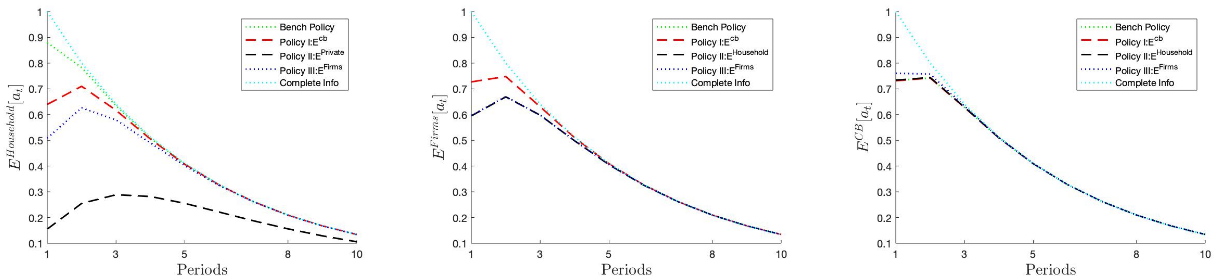
(a) $\mathbb{E}^{Households}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$ (b) $\mathbb{E}^{Firms}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$ (c) $\mathbb{E}^{CB}[\varepsilon_t^{\beta}|\varepsilon_t^{\beta}]$

Figure 8: Now-cast of Preference Shock to a Positive Innovation in Preference

Responses in Figure 8 accord with our theoretical analysis in Section 3. With this non-nested incomplete information environment, firms and central bank are not endowed with preference innovation signal. When implementing their information sets in monetary policy, nominal interest rate would not respond to a preference shock, as shown in Proposition 1 where $C_{\beta} = 0$. They would not learn about the realized preference shock unless nominal interest rate responds to that shock. When we adopt households' information set in the policy, movement of nominal interest rate informs firms and central bank about *some* fundamental realization. Combined with their own information sets, they are able to infer the realization of a preference innovation.

An Aggregate Technology Shock : Figure 9 documents all sectors' now-cast of technology level to a positive innovation in aggregate technology with different policies.

In Figure 9:b and Figure 9:c, central bank forms a more precise now-cast of technology innovation than firms', which implies a dominance of central bank's information over firms' information, because central bank only receives noises at aggregate level while firms receive noises at aggregate level, idiosyncratic level and communication error. Figure 9:a documents the learning process of households. Firms and central bank are both endowed with technology signals. When we implement their information sets, nominal interest rate movement contains information about technology innovation. Households are thereby able to infer the

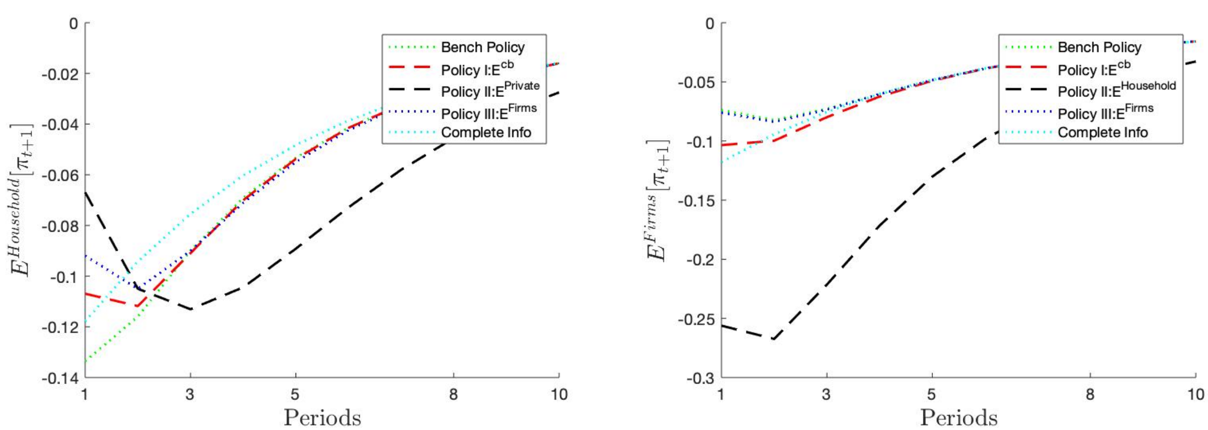


(a) $\mathbb{E}^{Households}[a_t|\varepsilon_t^a]$ (b) $\mathbb{E}^{Firms}[a_t|\varepsilon_t^a]$ (c) $\mathbb{E}^{CB}[a_t|\varepsilon_t^a]$

Figure 9: Now-cast of Technology Shock to a Positive Innovation in Technology

realized innovation, and Figure 9:a shows households can better infer the realization with implementation of central bank's information set than firms', which is consistent with our argument. Households may not update their beliefs about the realized technology innovation if monetary policy contains no information about it, i.e. implementing households' own information set.

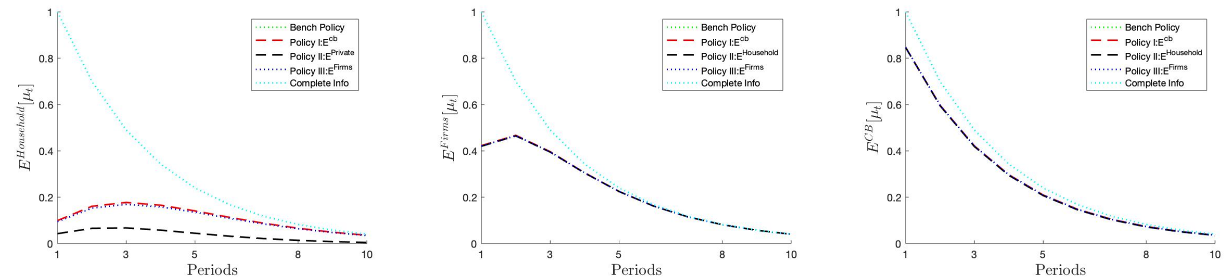
Figure 10 documents households and firms' inflation forecast to a positive technology innovation. Inflation forecast disagreement also exists between households and firms. Households' inflation forecast is more stable than firms'. Especially, firms expect a significant inflation decrease when policy information set contains no technology signal, i.e. Policy II. In such scenario, firms expect a mild response of nominal interest rate that is not enough to curb deflation. The lack of technology signal within policy rule also prolongs the economic recovery.



(a) $\mathbb{E}^{Household}[\pi_{t+1}|\varepsilon_t^a]$ (b)$\mathbb{E}^{Firms}[\pi_{t+1}|\varepsilon_t^a]$

Figure 10: Households and Firms' Inflation Forecast to a Positive Innovation in Technology

A Markup Shock : Figure 11 documents all sectors' now-cast of markup level to a positive



(a) $\mathbb{E}^{Households}[\mu_t|\varepsilon_t^{\mu}]$ (b) $\mathbb{E}^{Firms}[\mu_t|\varepsilon_t^{\mu}]$ (c) $\mathbb{E}^{CB}[\mu_t|\varepsilon_t^{\mu}]$

Figure 11: Now-cast of Markup Shock to a Positive Innovation in Markup

innovation in markup with different policies. Firms and central bank can better infer realized markup innovation than households. However, households are worse at recovering markup innovation than technology innovation with all information implementation. Because markup innovation is not directly involved in policy rules. Nominal interest rate movement contains less information of markup innovation than technology for households to update their beliefs.

Figure 12 documents households and firms' inflation forecast to a positive markup innovation. Households and firms' inflation forecast is dampened by information frictions. Their forecast is further anchored when nominal interest rate responds to households' information set. This is an interesting finding. Recall Policy II:

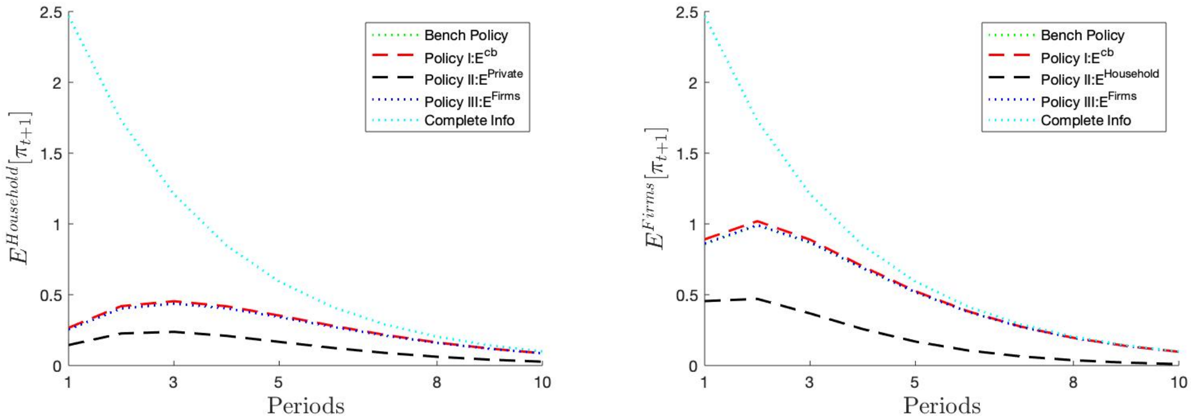
$$i_t = \phi_y \mathbb{E}_t^{HH}[y_t - a_t] + \phi_\pi \mathbb{E}_t^{HH}\pi_t + \varepsilon_t^m \text{ with: } \Omega_t^{HH} = \left\{\varepsilon_{t-j}^{\beta}, \bar{\pi}_{t-j}, i_{t-j}\right\}_{j=0}^{\infty}.$$

When households do not receive markup signal, firms would expect a mild nominal interest rate response to a positive markup innovation. In principle, this response would not be enough to curb next period inflation. Firms should expect a higher inflation with Policy II than the other, which is different from their forecast in Figure 12:b. For anchoring private sectors' inflation forecast, this result demonstrates an importance in households'/demand side information.

4.2.2 Higher-order Decomposition

We study the information acquisition problem for firms between exogenous information (the PE component) and endogenous uncertainty (the GE component) under the framework of asymmetric information:

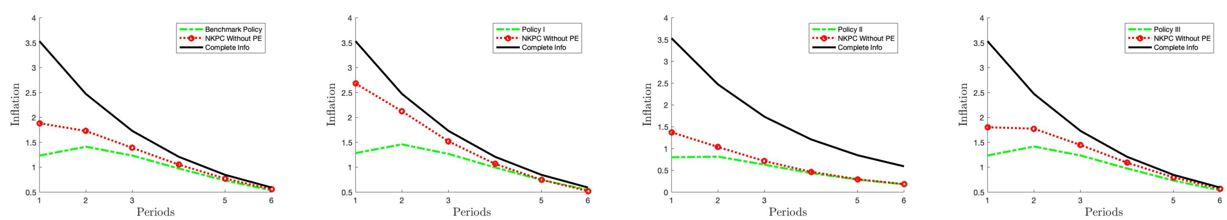
$$\pi_t = \underbrace{\beta \overline{\mathbb{E}}_t^F[\pi_{t+1}]}_{\text{GE Component}} + \underbrace{\lambda \overline{\mathbb{E}}_t^F[y_t - a_t] + \overline{\mathbb{E}}_t^F[\mu_t]}_{\text{PE component}}.$$



(a) $\mathbb{E}^{Household}[\pi_{t+1}|\varepsilon_t^{\mu}]$ (b)$\mathbb{E}^{Firms}[\pi_{t+1}|\varepsilon_t^{\mu}]$

Figure 12: Households and Firms' Inflation Forecast to a Positive Innovation in Markup

Figure 13 documents inflation impulse responses to a positive innovation in markup with different policies. The black solid lines represent complete information cases. The red circle-dotted lines represent cases in which we eliminate the uncertainty of partial equilibrium effects (PE component), meaning that the quantitative difference between the black solid line and the red circled-dotted line is due to uncertainty in $\mathbb{E}_t[\pi_{t+1}]$ (GE component). The green dashed lines are the responses where both the PE and GE components exist. We shall now compare each policy in Figure 13 with those in Figure 7 to understand how different information structures effect firms' myopia and anchoring.



(a) Benchmark Policy (b) Policy I (c) Policy II (d) Policy III

Figure 13: Responses of Inflation to a Positive Innovation in Markup: Isolate Partial Equilibrium Effects in NKPC

The benchmark policy delivers similar results in both symmetric and asymmetric information settings. This means that the asymmetric information setting makes insignificant difference to firms' informational acquisition between exogenous signals and endogenous beliefs when monetary authority is endowed with perfect information. With Policy I, where the Taylor rule is driven by the central bank's information, the PE component generates a significant quantitative bite when compared with Figure 7:b. This means that firms' uncertainty of current economy largely influences their behavior when they know that the central bank is operating based on its own information. While with Policy II and Policy III, the overall responses are milder because of the increase in signal structure complexity, the ratio between the GE effects and PE effects is similar to those in Figure 7:c and Figure 7:d.

4.2.3 Social Welfare

Table 3 documents the period welfare loss associated with each policy with respect to different preference shock variances. The complete information case represents no information friction among all parties and the benchmark policy is where the Taylor rule is driven by full information.

Different from the previous section, we now have asymmetric information between private sectors. It is clear to see that the total volatility increases as the information structure becomes more complicated; therefore, the welfare loss, or the total volatility, increases in all scenarios. Same as the symmetric information setting, the central bank's information is more valuable than firms', since firms still observe noisy bank signals. Thus, we have a lower welfare loss when we execute the Taylor rule with the central bank's information rather than firms' information. However, the salient feature here is the importance of household information. When the preference shock variance is small, the action of adopting the household expectation is not very beneficial because if we apply their expectation in Taylor rule, we are ignoring other supply signals. However, once this preference shock variance approaches or surpasses other shock variance, it becomes influential and dominates either firms' or the central bank's information. When we consider the household information within the Taylor rule, the average period welfare loss drops significantly. The polarization between Policy II and the rest increases as the preference shock variance increases. This result suggests that when there are heterogeneous shocks and information signals for different sectors, the relatively efficient way to stabilize the economy is to adopt the household's information. This implies that the central bank should learn more about the household's expectation.

Candia, Coibion and Gorodnichenko (2020) argues the importance of communication between the monetary authority and the public. Our model suggests that, to stabilize the economy, it is more crucial to convince private sectors that the nominal interest rate is determined by the household's understanding of current economic conditions and a weakened the role of the monetary authority. This is understandable since, in reality, the central bank serves neither on the supply side, nor the demand side. It is the private sector, that drives the value-added process. To lower the aggregate volatility of the economy, by a means of anchoring expectation, the central bank needs to study private sectors' information. Moreover, household information surprisingly dominates firms'.

Table 3: Welfare Analysis of Polices

Policy Rule	Welfare Loss
$\sigma_\beta^2 = 0.5^2$	
Complete Info	6.80
Benchmark Policy	9.15
$\mathbb{E}^{cb}$	42.32
$\mathbb{E}^{HH}$	69.46
$\mathbb{E}^{Firm}$	50.61
$\sigma_\beta^2 = 1^2$	
Complete Info	15.51
Benchmark Policy	17.85
$\mathbb{E}^{cb}$	117.32
$\mathbb{E}^{HH}$	78.23
$\mathbb{E}^{Firm}$	125.61
$\sigma_\beta^2 = 2^2$	
Complete Info	50.33
Benchmark Policy	52.68
$\mathbb{E}^{cb}$	417.31
$\mathbb{E}^{HH}$	113.07
$\mathbb{E}^{Firm}$	425.61
$\sigma_\beta^2 = 3^2$	
Complete Info	108.38
Benchmark Policy	110.72
$\mathbb{E}^{cb}$	917.32
$\mathbb{E}^{HH}$	171.12
$\mathbb{E}^{Firm}$	925.61
$\sigma_\beta^2 = 4^2$	
Complete Info	189.64
Benchmark Policy	191.98
$\mathbb{E}^{cb}$	1617.30
$\mathbb{E}^{HH}$	252.38
$\mathbb{E}^{Firm}$	1625.60

Coibion et al. (2020) argues that inflation expectation should not be used as a policy tool mainly based on two reasons. First, neither private sector will respond too much to the policy announcements in a low-inflation environment. Second, firms' survey data are insufficient. This paper provides possible answers to both of these problems. First, other than "announcements," the monetary policy associated with private sectors' expectation becomes "endogenous" for private sectors. This is fundamentally different from "announcing" an inflation target to the market, which was viewed as a "tool" to influence their expectation. Our previously discussed results solve the second concern in that we do not need comprehensive firm level survey data through the domination of household information; thus, consumer survey data is satisfactory.

To further understand how information affects welfare, we exercise alternative information sets based on Policy I. All changes about information sets are parallel. We first eliminate the communication noises between the central bank and private sectors. We allow both private parties to receive signals of the central bank's own signals of technology and markup, z_t^a and z_t^μ, as well as public signals about these two shocks that the central bank sends out, ω_t^a and ω_t^μ. Therefore, in Case I, we have the information structure as:

Case I	Information Set
Households	$\left\{ z_{t-j}, \omega_{t-j}, \varepsilon_{t-j}^\beta, \bar{\pi}_{t-j}, i_{t-j} \right\}_{j=0}^\infty$
Firms	$\{ z_{t-j}, \omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j} \}_{j=0}^\infty$
Central Bank	$\{ z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j} \}_{j=0}^\infty$

In Case II, we keep communication error between central bank and firms but allow the central bank to observe actual realizations about technology and markup shocks (a_t and μ_t). Information structure is stated as follow

Case II	Information Set
Households	$\left\{ \varepsilon_{t-j}^\beta, \bar{\pi}_{t-j}, i_{t-j} \right\}_{j=0}^\infty$
Firms	$\{ \omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j} \}_{j=0}^\infty$
Central Bank	$\{ a_t, \mu_t, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j} \}_{j=0}^\infty$

In Case III, we introduce an noised signal of preference shock to the central bank. The central bank now receives a noised signal of the preference shock ε_t^β as $\tilde{\varepsilon}_t^\beta$. Information structure is stated as follow

Case III	Information Set
Households	$\left\{\varepsilon^{\beta}_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}^{\infty}_{j=0}$
Firms	$\{\omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}^{\infty}_{j=0}$
Central Bank	$\left\{\tilde{\varepsilon}^{\beta}_{t}, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}^{\infty}_{j=0}$

In Case IV, we combine Case II and Case III to verify that more information generates less welfare loss

Case IV	Information Set
Households	$\left\{\varepsilon^{\beta}_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}^{\infty}_{j=0}$
Firms	$\{\omega_{t-j}, x_{it-j}, \bar{\pi}_{t-j}, i_{t-j}\}^{\infty}_{j=0}$
Central Bank	$\left\{\tilde{\varepsilon}^{\beta}_{t}, a_t, \mu_t, z_{t-j}, \omega_{t-j}, \bar{\pi}_{t-j}, i_{t-j}\right\}^{\infty}_{j=0}$

Table 4 documents different welfare changes with respect to different signal structures.[20] We first notice that, within Case I, welfare gains from eliminating communication frictions are relatively small under asymmetric information. Even when both private sectors can fully observe "announcement noises" from the central bank, the welfare increase is around 0.130%. We also find that, within Case II, when the central bank is fully endowed with supply-side signals, the welfare increase will be approximately 0.748%, which is almost six times the welfare increase from eliminating communication noises. This shows that the actual information friction regarding real economic fundamentals is more important than "artificial" friction, such as communication friction. With Case III, we find that information from demand side shock, i.e., the preference shock, causes a significant increase in welfare. [21] This supports our previous findings. As discussed earlier, when the Taylor rule is driven by the household information, the representative household does not observe supply shock signals but only its own preference shock, and welfare loss falls to its lowest at 252.38. Now, if the central bank ever considers this household's private information, even with noise, the corresponding public signal i_t they sent out would largely benefit the whole economy. In the end, Case IV supports our arguments that the more information the central bank considers, the better public signal they can send out.

[20] We focus on $\sigma^2_\beta = 4^2$.

[21] It is worth noting that the central bank observes the "noisy" signal.

Table 4: Changes in Information Sets and Welfare Loss

Signal Structures	Welfare Loss
Policy I: $\mathbb{E}^{cb}$	1617.30
Case I	1615.2
Case II	1605.2
Case III	238.12
Case IV	229.23

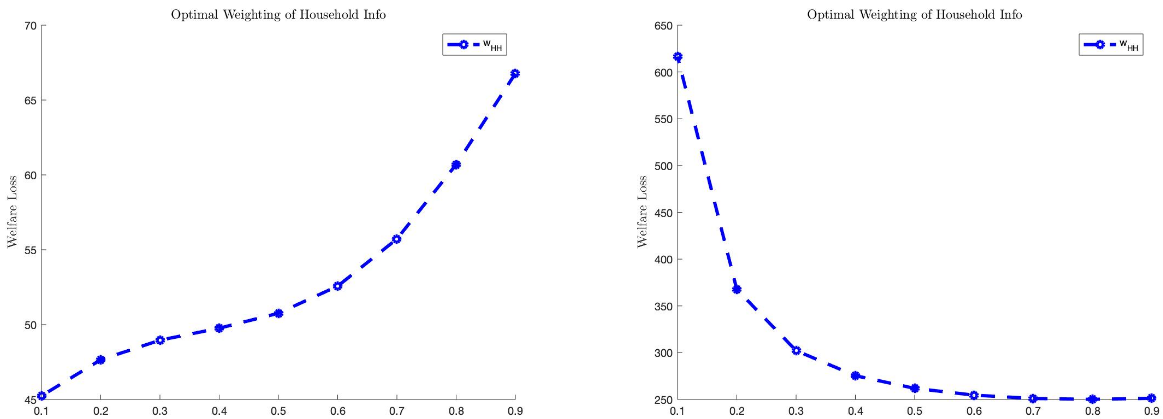
4.2.4 Weighted Strategy in Taylor Rule

We realize the importance of household information and suggest the central bank should pay more attention to households' private signals. This can be learned through survey data. We next address the question of how the central bank can reconcile the household expectation with its own expectation through the proposal of the following policy:

$$i_t = \omega_{HH}\left[\phi_y \mathbb{E}_t^{HH}\left[y_t - y_t^n\right] + \phi_\pi \mathbb{E}_t^{HH}\pi_t\right] + (1-\omega_{HH})\left[\phi_y \mathbb{E}_t^{cb}\left[y_t - y_t^n\right] + \phi_\pi \mathbb{E}_t^{cb}\pi_t\right] + \epsilon_{mt},$$

where ω_{HH} is the assigned weight for the household expectation. This policy rule helps determine how much attention the central bank should pay to the demand side data of households when it decides to apply survey expectations. Starting from $\omega_{HH} = 0.1$, with the increment size 0.1, to $\omega_{HH} = 0.9$.

We use the central bank's expectation instead of firms' because Table 3 shows that the central bank's information clearly dominates firms' information. This solves the concerns in Coibion et al. (2020), where they state the limitation of firm level survey is the main cause of why expectation should not be considered as a policy tool. We also notice that the volatility of demand shocks is important in terms of welfare, and if it is relatively small comparing with supply shocks, its welfare contribution will be diminished; therefore, we conduct the welfare analysis corresponding to two different variances, and Figure 14 shows the results,



(a) $\sigma_\beta^2 = 0.5^2$ (b) $\sigma_\beta^2 = 4^2$

Figure 14: Changes of Welfare Loss to the Weight of Household's Expectation

The welfare loss changes monotonically in both cases. When the demand side variance is small, welfare loss increases as the central bank increases its weight. Because the household cannot observe supply shocks, there is a trade-off between demand shock volatility and supply shock volatility while we change information sets to form expectations of endogenous variables. If the central bank realizes there is a fair amount of volatility from the demand side and it is not viewed as public information, the central bank should consider survey data and adopt the household expectation when applying the Taylor rule.

5 Conclusion

In this paper, we extend information dispersion to all sectors and each sector is endowed with non-nested information set. Households receive private signals about individual preference shocks. Firms receive idiosyncratic signals about technology shock and markup shocks and central bank announcement signals. A central bank receives signals about aggregate technology shock and markup shock, and delivers associated signals to firms with communication error. We introduce a belief-driven Taylor rule as an approach to policy-wise internalize private sectors' expectations. Theoretically, we show that policy-wise information choices determine signal extraction processes about exogenous shocks and thereby determine dynamics of endogenous variables and macro-volatility. Numerically, we study utilization of each sector's information set as well as specific signals. We find that an increasing importance in demand-side information set when economy's information structure becomes more complicated.

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Appendices

A Proof of Propositions

This section provides solutions of both propositions.

A.1 Policy I

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{A.1}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{A.2}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{A.3}$$

where $s_t^i = i_t = -y_t + \varepsilon_t^\beta$.

From firms' NKPC and Taylor rule, we have

$$i_t = \frac{\phi_y + \lambda \phi_\pi}{\lambda} \pi_t + \varepsilon_t^m. \tag{A.4}$$

Denote $\Phi \equiv \frac{\phi_y + \lambda \phi_u}{\lambda}$, we have

$$i_t = \Phi(S_{i,f} i_t + S_a \varepsilon_t^a) + \varepsilon_t^m \implies i_t = \frac{1}{1 - \Phi S_{i,f}} (\Phi S_a \varepsilon_t^a + \varepsilon_t^m), \tag{A.5}$$

therefore we have

$$C_\beta = 0, C_m = \frac{1}{1 - \Phi S_{i,f}}, C_a = \frac{\Phi}{1 - \Phi S_{i,f}}. \tag{A.6}$$

Notice that

$$s_t = \begin{bmatrix} s_t^i \\ s_t^a \end{bmatrix} = \begin{bmatrix} i_t \\ \varepsilon_t^a \end{bmatrix} = \underbrace{\begin{bmatrix} C_\beta & C_m & C_a \\ 0 & 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \varepsilon_t^\beta \\ \varepsilon_t^m \\ \varepsilon_t^a \end{bmatrix}}_{\varepsilon_t}, \tag{A.7}$$

and

$$\pi_t = \lambda \mathbb{E}_t^F y_t - \lambda \varepsilon_t^a = -\lambda i_t - \lambda \varepsilon_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta = -\lambda s_t^i - \lambda s_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta \tag{A.8}$$

By Gaussian projection, we have

$$\pi_t = S_{i,f} s_t^i + S_a s_t^a = -\lambda s_t^i - \lambda s_t^a + \lambda \mathbb{E}_t^F \varepsilon_t^\beta, \tag{A.9}$$

$$(S_{i,f} \quad S_a) = [-\lambda \quad -\lambda] + \lambda \Sigma_{\beta s} \Sigma_s^{-1} \tag{A.10}$$

where

$$\Sigma_{\beta s} = [1 \quad 0 \quad 0] \begin{bmatrix} \sigma_\beta^2 & 0 & 0 \\ 0 & \sigma_m^2 & 0 \\ 0 & 0 & \sigma_a^2 \end{bmatrix} \begin{bmatrix} C_\beta & 0 \\ C_m & 0 \\ C_a & 1 \end{bmatrix} = \begin{bmatrix} C_\beta \sigma_\beta^2 & 0 \end{bmatrix}, \tag{A.11}$$

and

$$\Sigma_s = B\Sigma B' = \begin{bmatrix} C_\beta^2\sigma_\beta^2 + C_m^2\sigma_m^2 + C_a^2\sigma_a^2 & C_a\sigma_a^2 \\ C_a\sigma_a^2 & \sigma_a^2 \end{bmatrix}. \tag{A.12}$$

Notice that $C_\beta = 0$, then

$$\Sigma_{\beta s} = [0 \quad 0], \quad [S_{i,f} \quad S_a] = [-\lambda \quad -\lambda]. \tag{A.13}$$

Also we have $\mathbb{E}_t^F \varepsilon_t^\beta = \frac{\lambda + S_{i,f}}{\lambda} i_t = 0$.

Therefore

$$\pi_t = \lambda s_t^i - \lambda s_t^a = \lambda i_t - \lambda \varepsilon_t^a, \tag{A.14}$$

Then we move on to output

$$y_t = S_{i,h} s_t^i + S_\beta s_t^\beta = S_{i,h} i_t + S_\beta \varepsilon_t^\beta \implies y_t = -i_t + \varepsilon_t^\beta, \tag{A.15}$$

which gives

$$\begin{bmatrix} C_\beta & C_m & C_a \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{1-\Phi S_{i,f}} & \frac{\Phi}{1-\Phi S_{i,f}} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{1+\phi_y+\lambda\phi_\pi} & \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\phi_\pi)} \end{bmatrix} \tag{A.16}$$

We reach the solution that

$$y_t = -i_t + \varepsilon_t^\beta = \varepsilon^\beta - \frac{1}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m - \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\lambda\phi_\pi)}\varepsilon_t^a \tag{A.17}$$

$$\pi_t = -\lambda i_t - \lambda\varepsilon_t^a = -\frac{\lambda}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m - \lambda(1+\frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\lambda\phi_\pi)})\varepsilon_t^a \tag{A.18}$$

$$i_t = \frac{1}{1+\phi_y+\lambda\phi_\pi}\varepsilon_t^m + \frac{\phi_y+\lambda\phi_\pi}{\lambda(1+\phi_y+\phi_\pi)}\varepsilon_t^a. \tag{A.19}$$

A.2 Policy II

$$y_t = S_{i,h} i_t + S_\beta \varepsilon_t^\beta, \tag{A.20}$$

$$\pi_t = S_{i,f} i_t + S_a \varepsilon_t^a, \tag{A.21}$$

$$i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m + C_a \varepsilon_t^a, \tag{A.22}$$

where $s_t^i = i_t = -y_t + \varepsilon_t^\beta$.

Combine y_t and π_t, we have

$$\pi_t = \lambda \mathbb{E}_t^F(-i_t + \varepsilon_t^\beta) - \lambda\varepsilon_t^a = -\lambda i_t - \lambda\varepsilon_t^a + \lambda\mathbb{E}_t^F\varepsilon_t^\beta. \tag{A.23}$$

Then

$$-\lambda i_t - \lambda\varepsilon_t^a + \lambda\mathbb{E}_t^F\varepsilon_t^\beta = S_{i,f} i_t + S_a\varepsilon_t^a \implies \mathbb{E}_t^F\varepsilon_t^\beta = \frac{\lambda + S_{i,f}}{\lambda} i_t. \tag{A.24}$$

Using Gaussian projection

$$[S_{i,f} \quad S_a] = [-\lambda \quad -\lambda] + \lambda\Sigma_{\beta s} = \begin{bmatrix} -\lambda & -\lambda \end{bmatrix} + \lambda \begin{bmatrix} c_\beta\sigma_\beta^2 & 0 \end{bmatrix} \Sigma_s^{-1}, \tag{A.25}$$

where

$$\Sigma_s = \begin{bmatrix} C_\beta^2\sigma_\beta^2 + C_m^2\sigma_m^2 + C_a^2\sigma_a^2 & C_a\sigma_a^2 \\ C_a\sigma a^2 & \sigma_a^2 \end{bmatrix}. \tag{A.26}$$

Therefore, we have

$$S_a = -\lambda. \tag{A.27}$$

Combine (A.20), (A.21), (A.22) and (A.24),

$$\begin{aligned} i_t &= \phi_y \mathbb{E}_t^{HH}[y_t - \varepsilon_t^a] + \phi_\pi \mathbb{E}_t^{HH}[\lambda \mathbb{E}_t^F y_t - \lambda\varepsilon_t^a] + \varepsilon_t^m \\ &= -\phi_y i_t + \phi_y \varepsilon_t^\beta - \phi_y \mathbb{E}_t^{HH}\varepsilon_t^a + \phi_\pi \mathbb{E}_t^{HH}[\lambda(-i_t + \mathbb{E}_t^F \varepsilon_t^\beta - \lambda\varepsilon_t^a] + \varepsilon_t^m \\ &= -\left(\phi_y + S_{i,f}\phi_\pi\right) i_t + \phi_y\varepsilon_t^\beta - \left(\phi_y + \lambda\phi_\pi\right)\mathbb{E}_t^{HH}\varepsilon_t^a + \varepsilon_t^m. \end{aligned}$$

which implies

$$i_t = \frac{1}{\Phi_2}(\phi_y\varepsilon_t^\beta - (\phi_y + \lambda\phi_\pi)\mathbb{E}_t^{HH}\varepsilon_t^a + \varepsilon_t^m). \tag{A.28}$$

Notice that $\mathbb{E}_t^F \varepsilon_t^\beta = \frac{\lambda + S_{i,f}}{\lambda} i_t$, and then the projection of ε_t^β on ε_t^a should be zero. We set that $\mathbb{E}_t^{HH}\varepsilon_t^a = \Phi_a i_t$.

From (A.28), we have

$$i_t = \frac{1}{\Phi_2 \Phi_3} \phi_y \varepsilon_t^{\beta} + \frac{1}{\Phi_2 \Phi_3} \varepsilon_t^m \tag{A.29}$$

where

$$\Phi_3 = 1 + \frac{\phi_y + \lambda \phi_\pi}{\Phi_2} \Phi_a. \tag{A.30}$$

We can tell that $i_t \perp \varepsilon_t^a$, then $\mathbb{E}_t^{HH} \varepsilon_t^a = 0$ and $\Phi_a = 0$.

Hence we have

$$i_f = \frac{\phi_y}{\Phi_2} \varepsilon_t^{\beta} + \frac{1}{\Phi_2} \varepsilon_t^m \tag{A.31}$$

where

$$C_\beta = \frac{\phi_y}{\Phi_2}, \quad C_m = \frac{1}{\Phi_2}, \quad C_a = 0. \tag{A.32}$$

Also,

$$\Sigma_S = \begin{bmatrix} C_c^2 \sigma_\beta^2 + C_m^2 \sigma_m^2 & 0 \\ 0 & \sigma_a^2 \end{bmatrix}. \tag{A.33}$$

From (A.25),

$$
\begin{aligned}
S_{i,f} &= -\lambda + \lambda \frac{C_\beta \sigma_\beta^2}{C_\beta^2 \sigma_\beta^2 + C_m^2 \sigma_m^2} \\
&= -\lambda + \lambda \frac{\frac{\phi_y}{\Phi_2} \sigma_\beta^2}{\frac{\phi_y^2}{\Phi_2^2} \sigma_\beta^2 + \frac{1}{\Phi_2^2 \sigma_m^2}} \\
&= -\lambda + \lambda \frac{\phi_y \Phi_2 \sigma_\beta^2}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2} \\
&= -\lambda + \frac{\lambda \phi_y \phi_\pi \sigma_\beta^2 S_{i,f} + \lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2},
\end{aligned}
$$

which implies that

$$
S_{i,f} = \frac{\lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2 - \lambda \left(\phi_y^2 \sigma_\beta^2 + \sigma_m^2\right)}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2 - \lambda \phi_y \phi_\pi \sigma_\beta^2} \tag{A.34}
$$

We reach the solution that

$$
y_t = -i_t + \varepsilon_t^\beta = (1 - C_\beta)\varepsilon_t^\beta - C_m \varepsilon_t^m \tag{A.35}
$$

$$
\pi_t = -\Delta i_t - \lambda \varepsilon_t^a = \Delta C_\beta \varepsilon_t^\beta + \Delta C_m \varepsilon_t^m - \lambda \varepsilon_t^a \tag{A.36}
$$

$$
i_t = C_\beta \varepsilon_t^\beta + C_m \varepsilon_t^m, \tag{A.37}
$$

where

$$
\Delta = \frac{\lambda \phi_y \left(1 + \phi_y\right) \sigma_\beta^2 - \lambda \left(\phi_y^2 \sigma_\beta^2 + \sigma_m^2\right)}{\phi_y^2 \sigma_\beta^2 + \sigma_m^2 - \lambda \phi_y \phi_\pi \sigma_\beta^2} \tag{A.38}
$$

$$
C_\beta = \frac{\phi_y}{1 + \phi_y + \Delta \phi_\pi} \tag{A.39}
$$

$$
C_m = \frac{1}{1 + \phi_y + \Delta \phi_\pi}. \tag{A.40}
$$

B Numerical zTran Solution for Symmetric Model

The canonical representation is given as

$$\sum_{k=0}^{l} A_k \Psi_{t-k} + \sum_{k=0}^{h} B_k \mathbb{E}_t \Psi_{t+k} = \mathbf{0}_{n_x \times 1},$$

and coefficients are grouped by

$$\Psi_t \equiv \begin{bmatrix} \chi_t \\ \upsilon_t \\ s_t \end{bmatrix}, \quad A_k \equiv \begin{bmatrix} A_k^{\chi} & A_k^{\upsilon} & A_k^{s} \end{bmatrix}, \quad B_k \equiv \begin{bmatrix} B_k^{\chi} & B_k^{\upsilon} & B_k^{s} \end{bmatrix},$$

where χ_t is endogenous variable, s_t is exogenous signals and υ_t is idiosyncratic shock aggregator. Define $s_t^m \equiv \epsilon_{mt}$ and $s_t^{\pi} \equiv \epsilon_t^p$, we have

$$\chi_t = \begin{bmatrix} i_t \\ y_t \\ \pi_t \\ \bar{\pi}_t \end{bmatrix}, s_t = \begin{bmatrix} a_t \\ \mu_t \\ s_t^m \\ \omega_t^a \\ \omega_t^{\mu} \\ s_t^{\pi} \\ z_t^a \\ z_t^{\mu} \\ x_{it}^a \\ x_{it}^{\mu} \end{bmatrix}, \epsilon_t = \begin{bmatrix} \epsilon_t^{\theta} \\ \epsilon_t^{\xi} \\ \epsilon_t^{xa} \\ \epsilon_t^{x\mu} \\ \epsilon_t^{\omega a} \\ \epsilon_t^{\omega \mu} \\ \epsilon_t^{za} \\ \epsilon_t^{z\mu} \\ \epsilon_t^{p} \\ \epsilon_t^{m} \\ \epsilon_t^{xa,i} \\ \epsilon_t^{x\mu,i} \end{bmatrix}, \tag{B.1}$$

where χ_t collects the endogenous variables but π_t cannot be observed, s_t collects the signals but a_t and μ_t cannot be observed, and ϵ_t collects all the innovations that hit the economy.

C Impulse response

C.1 Symmetric Information

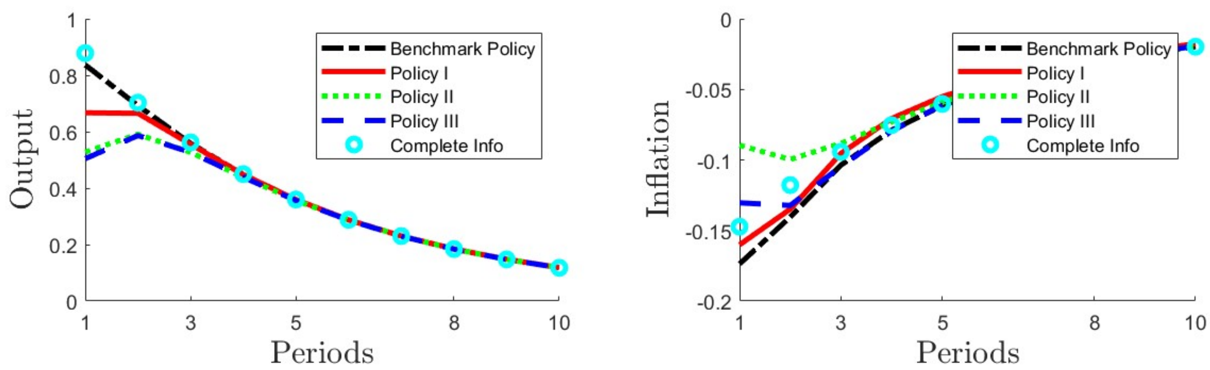

Figure 15: Technology shock

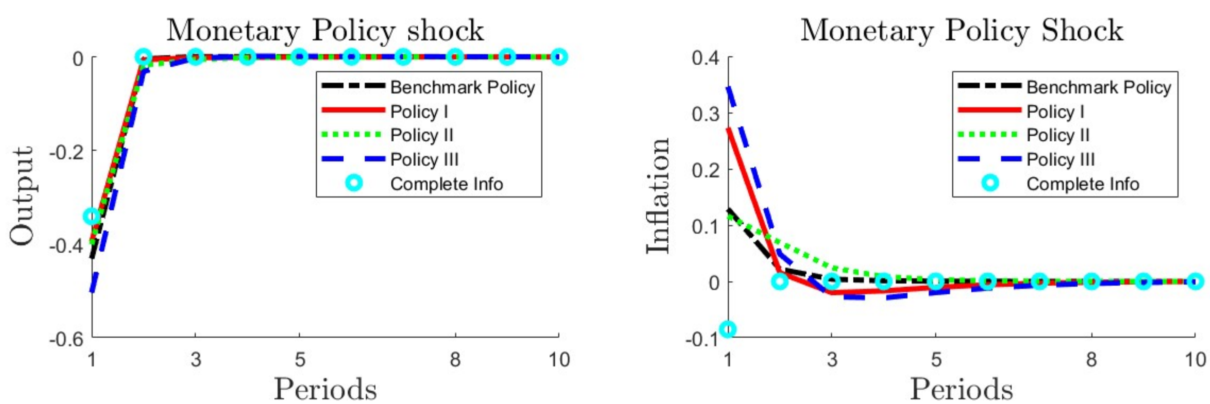

Figure 16: Monetary Policy shock

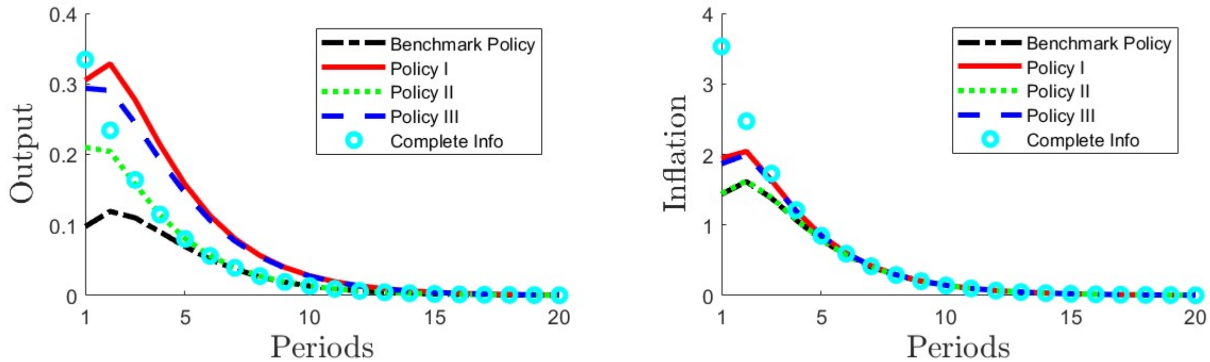

Figure 17: Markup shock

C.2 Asymmetric Information

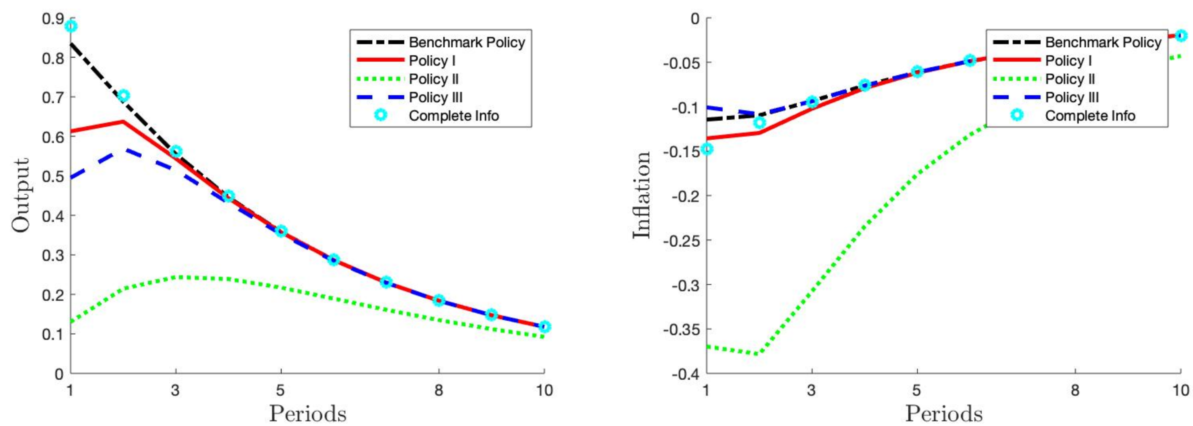

Figure 18: Technology shock

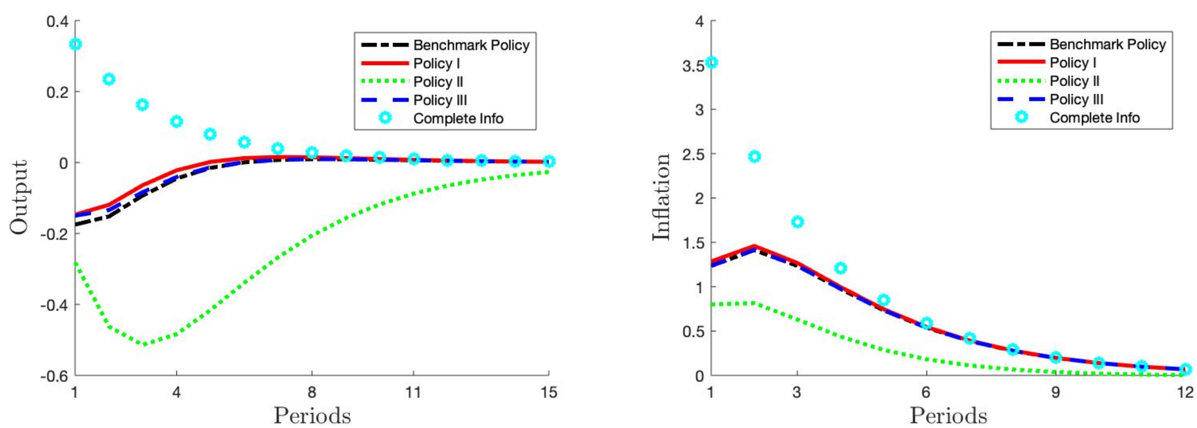

Figure 19: Markup shock

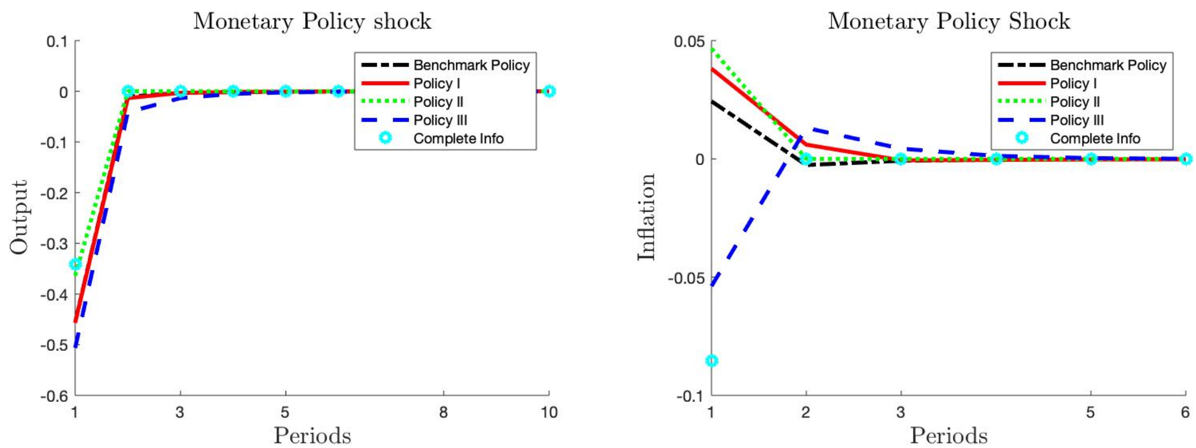

Figure 20: Monetary Policy shock

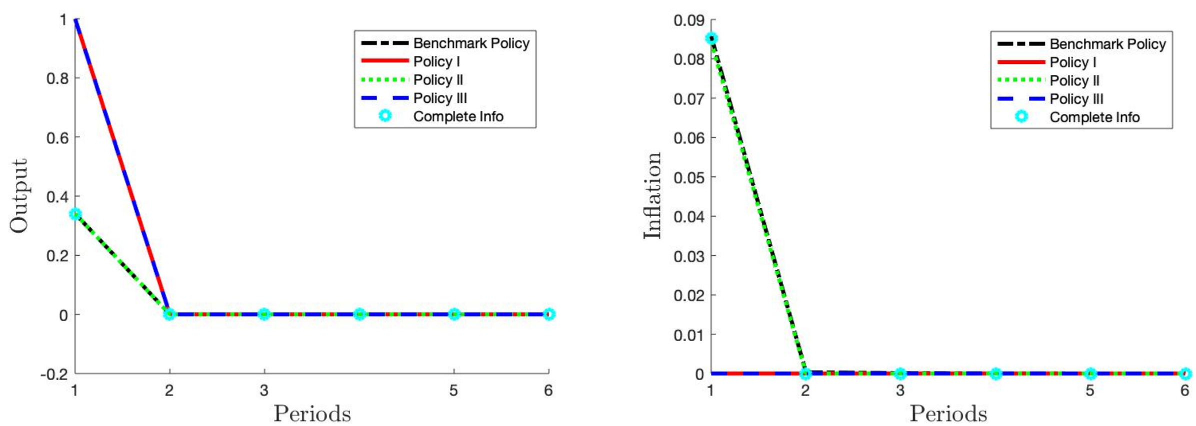

Figure 21: Preference shock