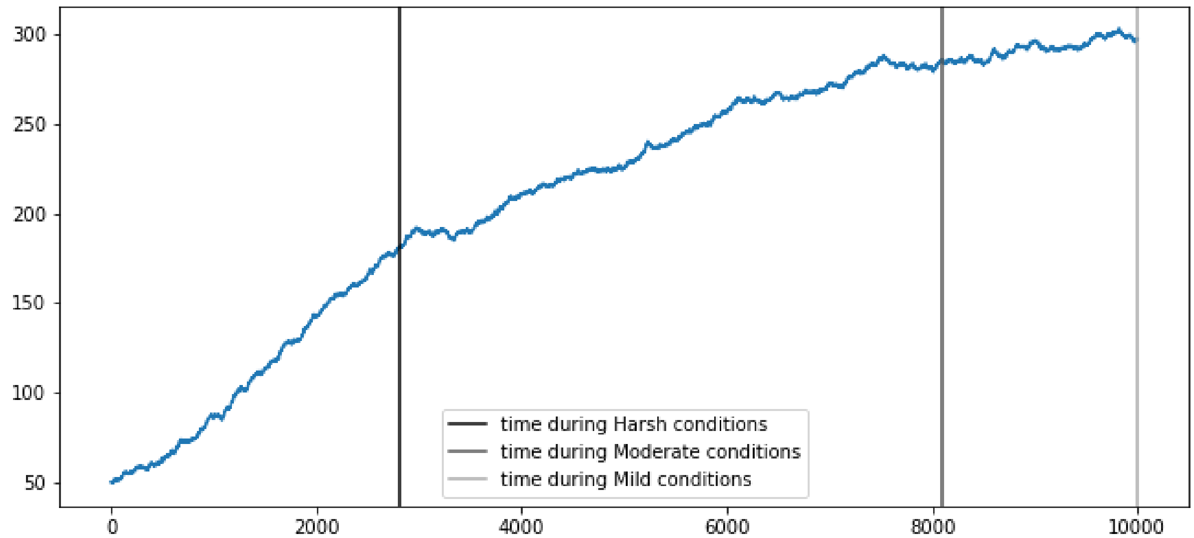
In [150]:
```python
import pandas as pd
import numpy as np
from sklearn.linear_model import LinearRegression
from scipy.stats import norm
pd.set_option('display.max_colwidth', None)
from scipy.optimize import minimize
from scipy.optimize import fsolve
from scipy.integrate import quad
import scipy.integrate as integrate
import scipy.special as special
import math
import random
%matplotlib inline
import matplotlib.pyplot as plt
plt.rcParams["figure.figsize"] = (11, 5)  #set default figure size
from mpl_toolkits.mplot3d import Axes3D
from math import sqrt
from pylab import plot, show, grid, xlabel, ylabel
from collections import defaultdict
import copy
```

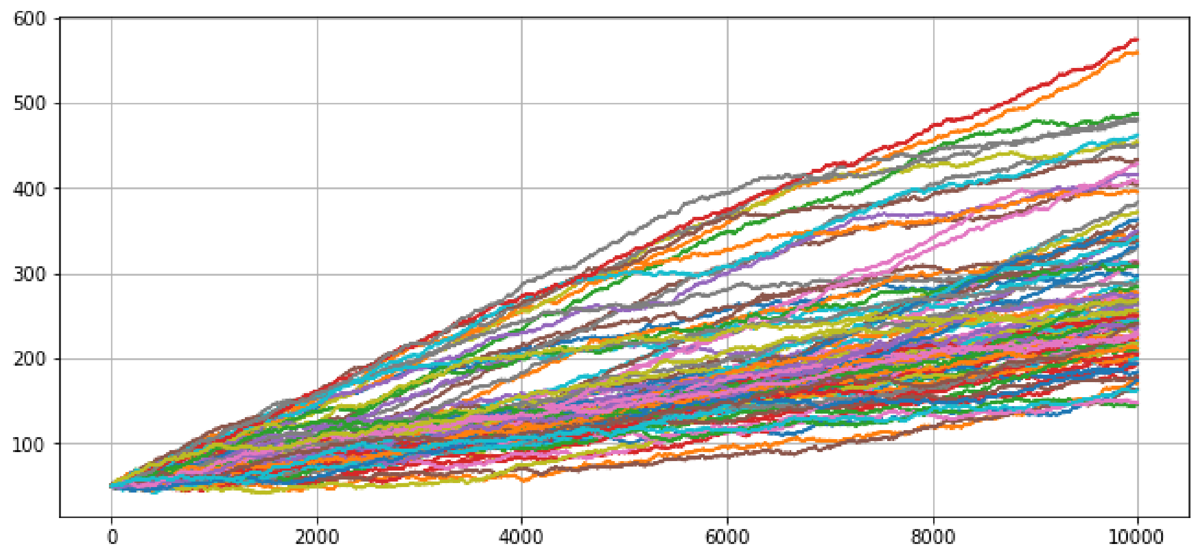
In [2]:
```python
import pickle

def save_in_pickle(save_object, save_file):
    with open(save_file, "wb") as pickle_out:
        pickle.dump(save_object, pickle_out)

def load_from_pickle(pickle_file):
    with open(pickle_file, "rb") as pickle_in:
        return pickle.load(pickle_in)
```

In [3]:
```python
# The statespace
states = ["Mild","Moderate","Harsh"]
paths = load_from_pickle("paths.pkl")
X_t = load_from_pickle("X_t.pkl")
```

In [4]:
```
plot(X_t[0])
for env, time in paths[0]:
    plt.axvline(time, 0, 1, label=f'time during {states[env]} condition
s',color = str(1-(env+1)/3))
plt.legend()

plt.show()
```

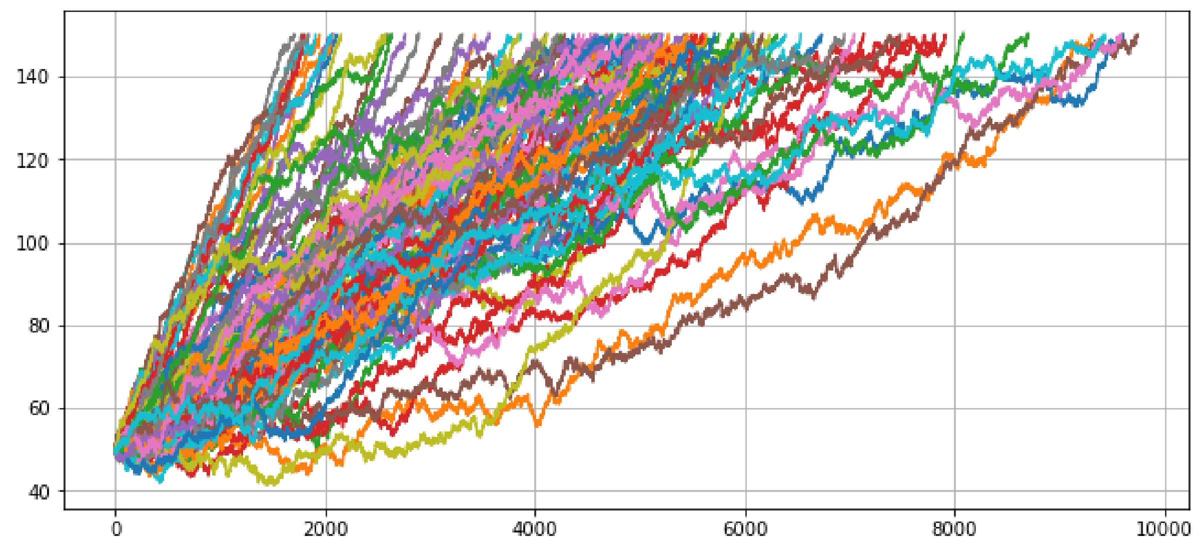


In [5]:
```
for k in range(100):
    plot(X_t[k])
grid(True)
```



In [16]:
```
clipped_paths = []
clipped_X_t = []
for i in range(100):
    limit = np.argmax(X_t[i]>150)
    new_path = []
    for env,time in paths[i]:
        if time < limit:
            new_path.append((env,time))
        else:
            new_path.append((env,limit))
            break
    clipped_X_t.append(X_t[i][:limit+1])
    clipped_paths.append(new_path)
```

In [17]:
```
for k in range(100):
    plot(clipped_X_t[k])
grid(True)
```



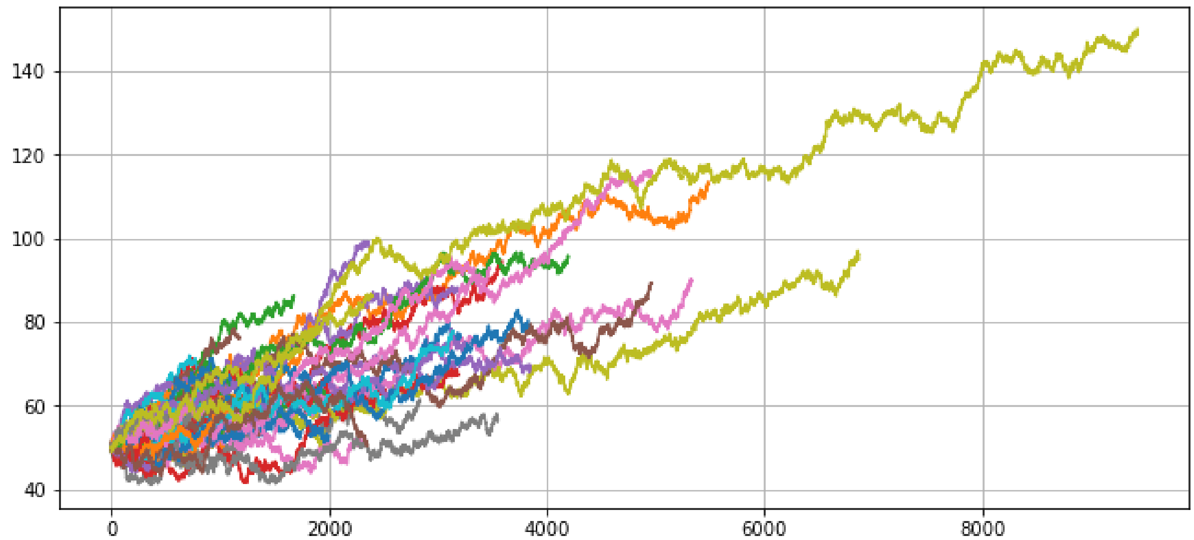
In [18]:
```
save_in_pickle(clipped_paths,'clipped_paths.pkl')
save_in_pickle(clipped_X_t,'clipped_X_t.pkl')
```

Estimating parameters for degradation model

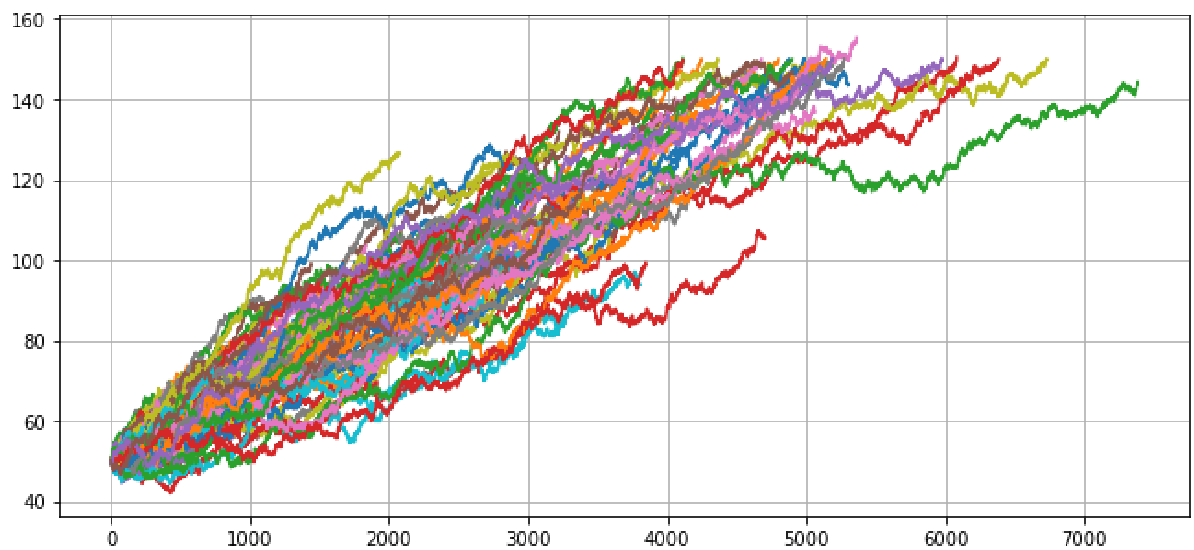
In [ ]:
```
phi = np.mean([X[0] for X in clipped_X_t])
```

In [84]:
```
X_t_env = [[],[],[]]
for i in range(100):
    start = 0
    for env,end in clipped_paths[i]:
        diff = clipped_X_t[i][start] - phi
        X_t_env[env].append(np.array(clipped_X_t[i][start:end+1]) - diff
)
        start = end+1
```

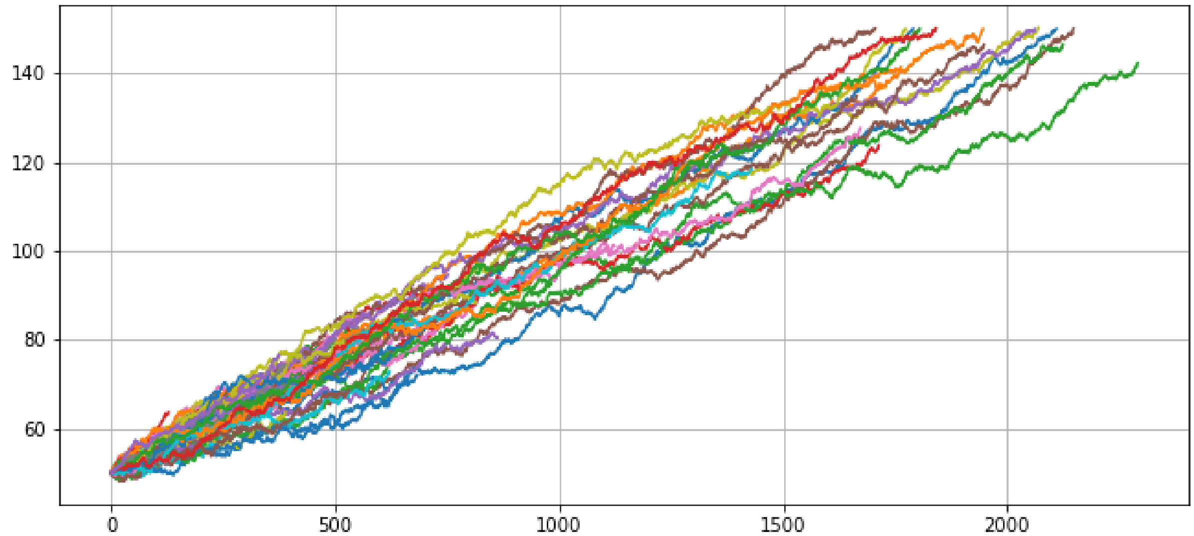
In [85]:
```python
for k in range(len(X_t_env[0])):
    plot(X_t_env[0][k])
grid(True)
```



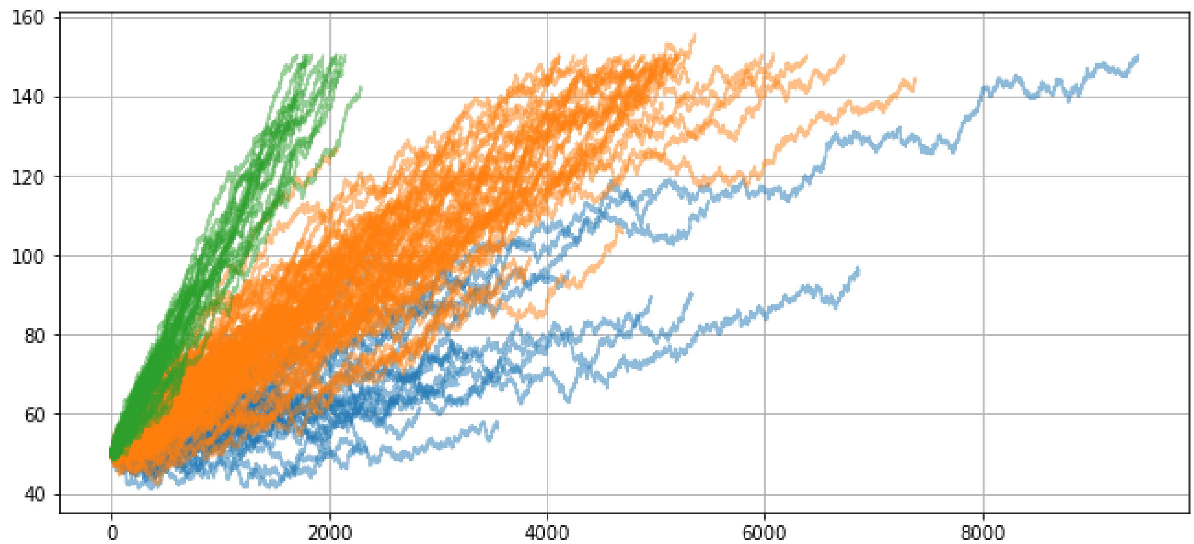
In [86]:
```python
for k in range(len(X_t_env[1])):
    plot(X_t_env[1][k])
grid(True)
```



In [87]:
```
for k in range(len(X_t_env[2])):
    plot(X_t_env[2][k])
grid(True)
```



In [88]:
```
colors = ['tab:blue','tab:orange','tab:green']
for i in range(3):
    for k in range(len(X_t_env[i])):
        fig = plot(X_t_env[i][k],color=colors[i], alpha=0.5)
    grid(True)
```



Estimating rate of increase μ

In [95]:
```
net_increases = [80.0,200.0,500.0]
T = 10000
true_mus = [i/T for i in net_increases]
true_mus
```

Out[95]: [0.008, 0.02, 0.05]

In [99]:
```
mus_env = []
sds_env = []
for i in range(3):
    mus = []
    for k in range(len(X_t_env[i])):
        y = np.array(X_t_env[i][k]).reshape((-1,1))
        X = np.arange(len(y)).reshape((-1,1))
        reg = LinearRegression().fit(X, y)
        mus.append(reg.coef_[0][0])
#         print(reg.coef_)
#         print(reg.intercept_)
    mus_env.append(np.mean(mus))
    sds_env.append(np.std(mus))
```

In [98]:
```
mus_env
```

Out[98]:
```
[0.008464419403965676, 0.020446425724897195, 0.04902512128315763]
```

In [100]:
```
sds_env
```

Out[100]:
```
[0.011343442692742825, 0.007892769134258489, 0.013995468987184349]
```

Starting distribution

In [103]:
```
np.bincount([path[0][0] for path in clipped_paths])
```

Out[103]:
```
array([32, 48, 20])
```

In [104]:
```
start_p = [0.3,.5,.2]
```

CTMC

In [ ]:
```
transition_matrix = [[0  ,0.7,0.3],
                     [0.6,0  ,0.4],
                     [0.3,0.7,0  ]]
```

In [132]:
```
trans_count = np.zeros((3,3),dtype=int)
for i in range(100):
    for j in range(len(paths[i])-1):
        trans_count[paths[i][j][0],paths[i][j+1][0]] += 1
```

In [135]:
```
trans_matrix = trans_count/trans_count.sum(axis=1).reshape((-1,1))
trans_matrix
```

Out[135]:
```
array([[0.        , 0.69026549, 0.30973451],
       [0.70338983, 0.        , 0.29661017],
       [0.26470588, 0.73529412, 0.        ]])
```

In [142]:
```
failure_times = [[],[],[]]
censored_failure_times = [[],[],[]]
for i in range(100):
    start = 0
    for env,end in paths[i]:
        if end == 10000:
            censored_failure_times[env].append(end-start)
        else:
            failure_times[env].append(end-start)
        start = end
```

In [145]:
```
lambdas = []
for i in range(3):
    r = len(failure_times[i])
    lambdas.append(r/(np.sum(failure_times[i])+np.sum(censored_failure_t
imes[i])))
```

In [146]:
```
lambdas
```

Out[146]:
```
[0.0005309676297698984, 0.00019811688224469784, 0.0003549560741858195]
```

In [147]:
```
[1/i for i in lambdas]
```

Out[147]:
```
[1883.353982300885, 5047.525423728814, 2817.25]
```

Delta/Variance

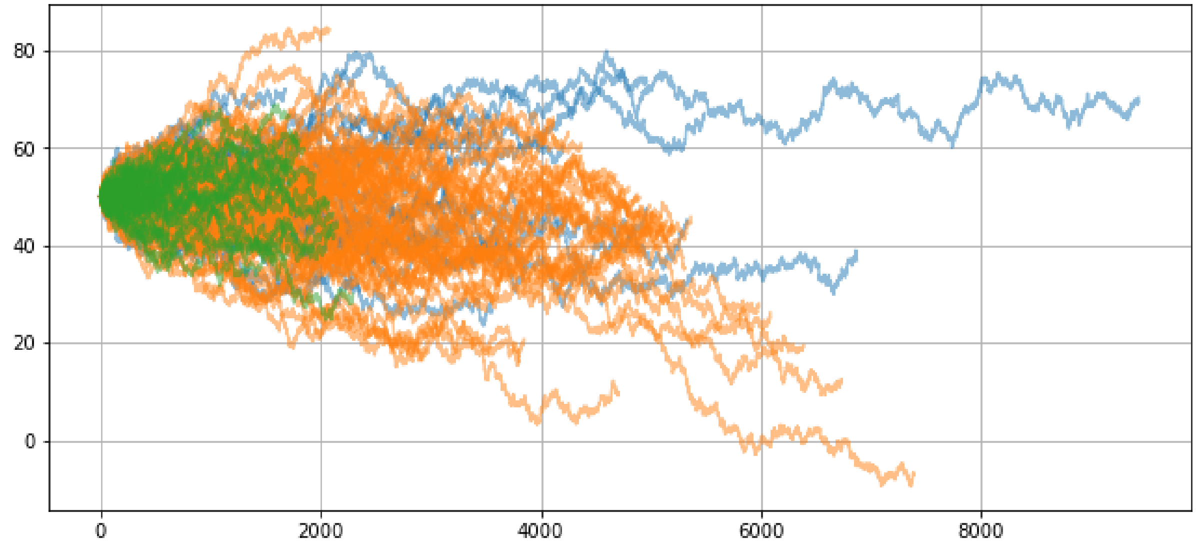
In [167]:
```
norm_X_t_env =  copy.deepcopy(X_t_env)
```

In [169]:
```
for env in range(3):
    for i in range(len(norm_X_t_env[env])):
        for j in range(len(norm_X_t_env[env][i])):
            norm_X_t_env[env][i][j] -= mus_env[env]*j
```

In [171]:
```
mus_env
```

Out[171]:
```
[0.008464419403965676, 0.020446425724897195, 0.04902512128315763]
```

In [172]:
```
colors = ['tab:blue','tab:orange','tab:green']
for i in range(3):
    for k in range(len(norm_X_t_env[i])):
        fig = plot(norm_X_t_env[i][k],color=colors[i], alpha=0.5)
    grid(True)
```



In [174]:
```
norm_diff = []
for env in range(3):
    for i in range(len(norm_X_t_env[env])):
        for j in range(len(norm_X_t_env[env][i])-1):
            norm_diff.append(norm_X_t_env[env][i][j+1]-norm_X_t_env[env]
[i][j])
```

In [176]:
```
norm.fit(norm_diff) # expecting 0, 0.25
```

Out[176]:
```
(-0.00029288222398599415, 0.25035960558049875)
```

Deriving RULs

- Simulation based on new CTMC and degradation model parameters is the most straightforward (can use Monte Carlo or Bootstrap)
- Closed form solution for CTMC and linear degradation model (haven't found a way to prove it)

In [ ]: