Introduction to Reliability Engineering

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Basic Reliability Concepts

- Background
- Reliability Function
- Relationship to CDF & PDF
- Mean/Residual Time-To-Failure
- Failure &Hazard Rate
- Bathtub Curve
  - Decreasing Failure Rate
  - Constant Failure Rate
  - Increasing Failure Rate
- Comprehensive Example

Why is Reliability Important?

Reliability is important because things fail.
Failures occur due to many reasons.

- Causes of failure include poor design, faulty manufacturing or construction, poor maintenance, inadequate inspection, environmental stresses, and human error.
- The impact of a failure can range from minor inconveniences and costs to personal injury or death, and significant economic losses.

Why is Reliability Important

- Generally, failure can be viewed as a culmination of a progressive degradation process.
- In theory, if we understood the physics and chemistry associated with failure processes, then failures could be predicted with certainty.
- In practice, we have limited understanding of the physical and chemical processes that cause failure, and the randomness of external events. Thus, failures appear to be random.
- Although appear to be random , they do exhibit some pattern that can be modeled using probability, i.e., we can predict failures statistically.

Definition of Reliability

- Reliability is the **probability** that a product will operate or a service will be provided properly for a specified period of time (known as the design life) under the intended operating conditions (designated temperature, load, speed, etc.) without failure.
- Reliability Engineering attempts to characterize, measure, and analyze system failures in order to improve their operational use to reduce the likelihood of unexpected failures, and downtime, thereby increasing availability.

Definition of Reliability

- Reliability requires a clear definition of the following aspects:
  - Meaning of failure, in terms of an observable description of functionality or performance.
  - Units of time e.g., hours, days, cycles (aircraft take-off, cruise, and landing).
  - Normal operating conditions, this includes factors such as load environment (temperature, vibration, altitude).

Improving Reliability

- One can often improve reliability in several ways:
  - Increasing the redundancy or duplication.
  - Designing excess strength into components.
  - Derating; operating a system below its rated stress/loading level.
  - Reducing the complexity and number of components in a system.

Reliability Function

- Suppose n_o identical components are tested under their designed operating conditions. Let us assume that by some time t, $n_f(t)$ failed components, and $n_s(t)$ surviving components such that $n_f(t) + n_s(t) = n_o$
- Reliability at time t, $R(t)$, is defined as follows;

$$R(t) = \frac{n_s(t)}{n_s(t) + n_f(t)}$$

Reliability Function

- If *T* is a random variable denoting the time to failure, then the reliability function at time *t* can be expressed as;

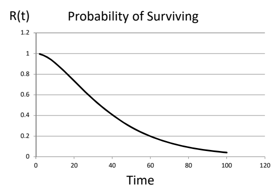
$$R(t) = \mathbb{P}(T > t)$$

- Reliability is related to the cumulative probability function

$$\mathrm{F(t) = 1 - R(t) = \mathbb{P}(T < t)}$$

- In fact, R(0)=1 and $\lim_{t\to\infty} \mathrm{R(t)} = 0$

$F(0) = 0$
$F(\infty) = 1$



Reliability, CDF, & PDF

- If *T* has a probability density function $f(t)$, then

$$R(t) = 1 - F(t) = 1 - \int_0^{t'} f(t)\,\mathrm{d}t$$

$$f(t) = -\frac{\mathrm{d}R(t)}{\mathrm{d}t}$$

$f(t) = \frac{dF(t)}{dt}$

- Also since, $R(t) = 1 - F(t)$

$$R(t) = \int_{t'}^{\infty} f(t)\,\mathrm{d}t$$

$$F(t) = \int_0^{t'} f(t)\,\mathrm{d}t$$

Example: Reliability Function

- Given the following PDF of the time-to-failure of a compressor, which we will denote as *T*, what is the reliability for 100-hr of operating life.

$$\mathrm{f(t)} = \begin{cases} \frac{0.001}{(0.001\,t + 1)^2} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$R(t) = \int_{t'}^{\infty} f(t)\,\mathrm{d}t \;\; = \int_{100}^{\infty} \frac{0.001}{(0.001\,t + 1)^2}\mathrm{d}t$$

$$R(100) = \frac{-1}{(0.001\,t + 1)}\Big|_{100}^{\infty} = \frac{-1}{\infty} - \frac{-1}{(0.001)(100) + 1} = 0.909$$

Example: Reliability Function

- Probability that a failure occurs within the interval of time $[10, 100]$ can be found using the following expression.

$$\mathbb{P}\{10 \leq 100\} = R(10) - R(100)$$

$$= \frac{1}{(0.01) + 1} - \frac{1}{(0.1) + 1} = 0.081$$

Mean Time to Failure

- One of the key measures of a system's reliability is the MTTF.
  - MTTF is usually used when the system is nonrepairable.
  - For repairable systems, the failure time between two successive failures is usually referred to as MTBF (B→Between)
- Consider n identical nonrepairable systems and their time to failure are given by $t_1, t_2, \ldots, t_n$. Then the mean time to failure is given as,

$$\widehat{MTTF} = \frac{1}{n}\sum_{i=1}^{n} t_i$$

Mean Time to Failure

- If t is a random variable representing time to failure, then the Mean-Time-To-Failure, MTTF can be defined as follows:

$$MTTF = \mathbb{E}[T] = \int_0^\infty t\, f(t)dt$$

$$\mu = \int_0^\infty x\, f(x)\,\mathrm{d}x \qquad \mu = \mathbb{E}[X]$$

- Another measure that is often used to describe the distribution of the time to failure is its variance σ^2

$$\sigma^2 = \int_0^\infty (t - MTTF)^2 f(t)dt$$

$$\sigma^2 = \int_0^\infty (x-\mu)^2 f(x)\,\mathrm{d}x$$

$$\sigma^2 = \int_0^\infty t^2 f(t)dt - (MTTF)^2$$

Mean Time to Failure

- MTTF can also be express in terms of the integral of reliability
- Recall that $f(t) = dF(t)/dt = -dR(t)/dt$, thus,

$$MTTF = \int_0^\infty t \times \left(-\frac{dR(t)}{dt}\right) dt$$

  - Using integration by parts, we see that

$$MTTF = -t\,R(t)\Big|_0^\infty + \int_0^\infty R(t)\,dt$$

  - Since $R(\infty) = 0$ and $R(0) = 1$, we have,

$$MTTF = \int_0^\infty R(t)\,dt$$

Residual MTTF

- If an equipment has been operating for some time ,T_0, we can still calculate its **residual** MTTF using the condition reliability function $R(t|T_0)$.

$$MTTF(T_0) = \int_{T_0}^{\infty} R(t|T_0)\,\mathrm{d}(t)$$

Conditional Reliability

- We define conditional reliability as the reliability of a component given that it has operated for time T_0.

$$\begin{aligned} R(t|T_0) &= \Pr\{T > t + T_0 | T > T_0\} \\ &= \frac{\Pr\{(T > T_0 + t) \cap (T > T_0)\}}{\Pr\{T > T_0\}} \\ &= \frac{\Pr\{(T > T_0 + t)\}}{\Pr\{T > T_0\}} \\ &= \frac{R(T_0 + t)}{R(T_0)} \end{aligned}$$

Residual MTTF

- If an equipment has been operating for some time ,T_0, we can still calculate its **residual** MTTF using the condition reliability function $R(t|T_0)$.

$$\begin{aligned} MTTF(T_0) &= \int_{T_0}^{\infty} R(t|T_0)\,\mathrm{d}(t) \\ &= \int_{T_0}^{\infty} \frac{R(t+T_0)}{R(T_0)}\mathrm{d}(t+T_0) \\ &= \frac{1}{R(T_0)}\int_{T_0}^{\infty} R(t')\,\mathrm{d}t' \end{aligned}$$

where $t' = t + T_0$

Example : Residual MTTF

- Consider the following reliability function

  $R(t) = e^{-0.002\,t}$

  - Calculate the MTTF

$$MTTF = \int_0^{\infty} R(t)\,\mathrm{d}t$$

$$MTTF = \int_0^{\infty} e^{-0.002\,t} dt = \frac{e^{-0.002\,t}}{-0.002}\Big|_0^{\infty}$$

$$= \frac{-e^{-\infty}}{0.002} - \frac{-1}{0.002} = \frac{1}{0.002} = 500\ hr$$

Failure/Hazard Rate

- Probability of failure of a component in a given time interval $[t_1, t_2]$ can be expressed as follows;

$$\int_{t_1}^{t_2} f(t)\,\mathrm{d}t = R(t_1) - R(t_2)$$

- To see this

$$\int_{t_1}^{t_2} f(t)\,\mathrm{d}t = F(t_2) - F(t_1)$$

$$= [1 - R(t_2)] - [1 - R(t_1)]$$

$$= R(t_1) - R(t_2)$$

Failure/Hazard Rate

- Probability of failure of a component in an time $[t_1, t_2]$ can be expressed as;

$$\int_{t_1}^{t_2} f(t)\,\mathrm{d}t = R(t_1) - R(t_2)$$

- **Failure Rate** is defined as the probability that a failure occurs within the interval $[t_1, t_2]$, given that no failure occurred prior to t_1,

$$\frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)}$$

- By replacing t_1 and t_2 with t and $t + \Delta t$,

$$\frac{R(t) - R(t + \Delta t)}{\Delta t R(t)}$$

Failure/Hazard Rate

- The **Hazard Rate Function** is defined as the limit of the failure rate as Δt approaches zero, i.e., it is the instantaneous Failure Rate.

$$\lambda(t) = h(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t R(t)} = \frac{f(t)}{R(t)}$$

Important remark:

$$f(t) = \frac{-d\,R(t)}{dt}$$

$$\lambda(t) = -\frac{1}{R(t)} \frac{d\,R(t)}{dt}$$

$$\int \frac{1}{x}\,\mathrm{d}x = \ln(x)$$

$$\ln\big(R(t)\big) = -\int \lambda(t)\,\mathrm{d}t$$

$$R(t) = e^{-\int \lambda(t)\,\mathrm{d}t}$$

$$R(t) = \exp\left[-\int_0^{t'} \lambda(t)\,dt\right]$$

Failure/Hazard Rate

- Another useful function is the **Average Hazard Rate**, denoted as $\mathrm{AFR}(t_1, t_2)$, and defined as,

$$\mathrm{AFR}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \lambda(t')\,dt'$$

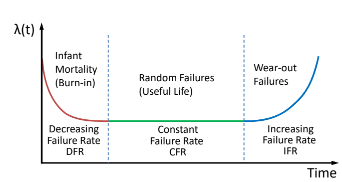
$$= \frac{\ln R(t_1) - \ln R(t_2)}{t_2 - t_1}$$

- If $t_1 = 0$ and $t_2 = t$, then AFR can be written as follows:

$$\mathrm{AFR}(t) = \frac{-\ln R(t)}{t}$$

Bathtub Curve

The bathtub curve provides a general description of the hazard function across the life cycle of a product. It is comprised of 3 main regions.



Bathtub Curve - DFR

- New units experience a high failure rates at the beginning of their use which then decreases over time, hence the term decreasing failure rate, **DFR**. This phase is known as **infant mortality**.
  - Typically results from manufacturing defects, cracks, poor workmanship, quality control, defective parts, contamination.
  - Can be reduced through **burn-in** testing where units are subjected to slightly more severe conditions than those encountered under normal operation.

Bathtub Curve - CFR

- The failure rate begins to level for a period of time which is characterized by a constant failure rate **(CFR)**. In this region, failures are random and do not follow a predictable pattern.
  - This phase is typically referred to as the "useful life".
  - Events are often "Act of God".
  - Failure may be caused by random loads, human error, or chance.
  - This phase can be reduced by redundancy or excess strength.

Bathtub Curve - IFR

- The third region also known as the wear-out phase is characterized by an increasing failure rate **(IFR)**. Failures in this phase are no longer characterized by being random and are mostly due to aging and wear
  - Typical causes of failure in this phase are fatigue due to cyclic loading, wear, corrosion.
  - Can be reduced through derating, preventive maintenance, parts replacement, condition monitoring using sensor technology

Bathtub Curve (Summary)

	Characterized By	Caused By	Reduced By
Burn-in	DFR	Manufacturing defects: Welding flaws, defective parts, poor quality/workmanship, contamination.	Burn-in testing, screening, acceptance testing, quality control
Useful Life	CFR	Environment, random loads, human error, chance events	Redundancy, excess strength
Wear-out	IFR	Fatigue, corrosion, aging, cyclic loading	Derating, part replacement, preventive maintenance

Section Summary

Reliability and its Important
Mathematical Definition of Reliability function
Relation to CDF and PDF
Defined MTTF (Mean time to failure)
Residual MTTF & Conditional Reliability
Defined failure/hazard rate
Hazard rate function of reliability
Bathtub curve

Knowledge Check

- A company manufactures widgets. The time to failure in years of these widgets has the following PDF.

$$f(t) = \frac{200}{(t+\mathbf{10})^3} \qquad \text{for } t \geq 0$$

a) Derive the reliability function and determine the reliability for the first year of operation.

$$R(t) = \int_{t'}^{\infty} f(t)\,\mathrm{d}t = 200\int_{t'}^{\infty} \frac{1}{(t+10)^3}\mathrm{d}t$$

$$= \frac{-200}{2\times(t+10)^2}\Big|_{t'}^{\infty} = 0 - \frac{-100}{(t'+10)^2} = \frac{100}{(t'+10)^2}$$

$$R(1) = \frac{100}{(1+10)^2} = 0.826$$

Knowledge Check

➢ The time to failure in years of these widgets has the following PDF.

$$f(t) = \frac{200}{(t+\mathbf{10})^3} \qquad \text{for } t \geq 0$$

b) Computer the MTTF.

$$MTTF = \int_0^\infty R(t)\,\mathrm{dt} = \int_0^\infty \frac{100}{(t+10)^2}\,\mathrm{dt}$$

$$= \frac{-100}{(t+10)^1}\Big|_0^\infty = 10 \text{ years}$$

Knowledge Check

➢ The time to failure in years of these widgets has the following PDF.

$$f(t) = \frac{200}{(t+\mathbf{10})^3} \qquad \text{for } t \geq 0$$

c) What is the design life for a reliability of 0.95?

$R(t) = \frac{100}{(t+100)^2} = 0.95$ → Solve for t

$$t = \sqrt{\frac{100}{0.95}} - 10 = 0.26 \text{ years}$$

Knowledge Check

➢ The time to failure in years of these widgets has the following PDF.

$$f(t) = \frac{200}{(t+\mathbf{10})^3} \qquad \text{for } t \geq 0$$

d) Is the failure rate DFR, CFR, IFR?

Failure rate is decreasing, DFR, because $\lambda(0) = 0.2$ and $\lambda(t \to \infty) = 0$

$$\lambda(t) = \frac{f(t)}{R(T)} = \frac{200}{(t+10)^3} \Big/ \frac{100}{(t+10)^2} = \frac{2}{t+10}$$

Knowledge Check

➢ The time to failure in years of these widgets has the following PDF.

$$f(t) = \frac{200}{(t+\mathbf{10})^3} \qquad \text{for } t \geq 0$$

e) Will a one year burn-in period improve the reliability in part (a)? Calculate the new reliability.

$$R(1|1) = \frac{R(1+1)}{R(1)} = \frac{100}{(2+10)^2} \Big/ \frac{100}{(1+10)^2}$$

$$= \frac{11^2}{12^2} = 0.84$$