Statistical Degradation Models of Degradation Data

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Degradation Modeling Using Degradation Data

- Degradation data often provides considerably more reliability information than censored failure-time data (especially with few or no failures).
- Direct observation of the degradation process allows direct modeling of the failure-causing mechanism.
- Most failures can be traced to an underlying degradation process.
- Failure occurs when degradation crosses a threshold.
- Degradation curves can have different shapes.

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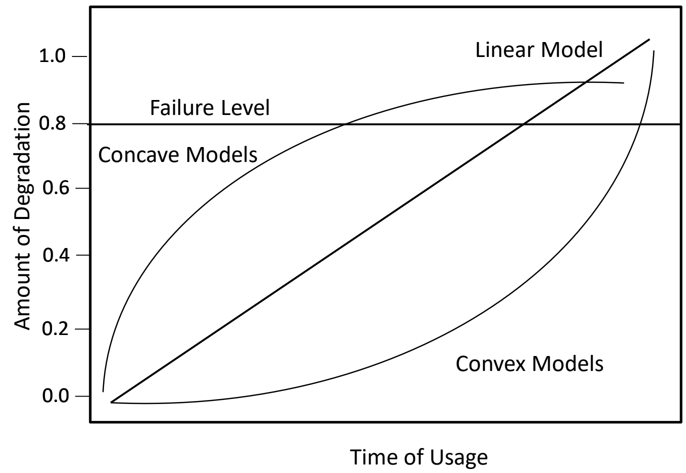
Continuous-State Degradation Models

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- Degradation curves can have different shapes.
- Some applications have more than one degradation variable or more than one underlying degradation process.
- Examples here have only one degradation variable and underlying degradation process.



Continuous-State Degradation Models

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- **Linear degradation:** Degradation rate

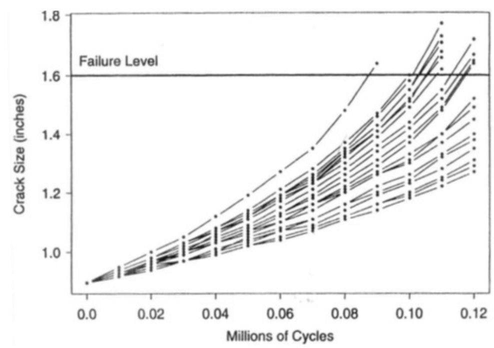
$$\frac{d\mathcal{D}(t)}{dt} = C$$

- is constant over time., i.e., degradation level at time t, $\mathcal{D}(t) = \mathcal{D}(0) + C \times t$ is linear in t. This can be often seen when partial degradation does not accelerate/decelerate subsequent degradation.
  - Examples include the amount of automobile tire tread wear.
- **Concave degradation:** Degradation rate decreasing in time. Degradation level increasing at a decreasing rate.
  - Examples include chemical processes with a limited amount of material to react.
- **Convex degradation:** Degradation rate increasing in time. Degradation level increasing at an increasing rate.
  - Examples include crack growth.

Continuous-State Degradation Models

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

- Example→ Fatigue Crack-Size Data
  - Aluminum alloy fatigue size data.
  - The initial crack size at time 0 for each path was 0.9 inches (the size of a notch cut into each specimen).
  - The experiments are terminated at the first inspection after a unit's crack reaches 1.6 inches or censored at 1.2 million cycles.
  - The horizontal line at 1.6 inches represents the approximate level at which failure would occur.
  - In some applications, there may be more than one variable, or more than one underlying degradation process.



Meeker & Escobar, *Statistical Methods for Reliability Data,* 1998.

Introduction to Degradation Modeling

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

- In degradation modeling:
  - It is necessary to define models that characterize the evolution of degradation variables.
  - These models may be defined based on the data or derived from basic principles related to the underlying degradation process.
  - Most models start with a deterministic description of the degradation process, often in the form of a differential equation or a system of differential equations.
  - Randomness can be introduced using probability distributions to describe variability in the initial conditions and model parameters like rate constant or material properties.
  - Models that cannot be defined based on physics are derived using data-driven approaches (statistical and stochastic models).

Example using Paris Model

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- Paris Model is used to model crack propagation during fatigue processes.

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = C \times [\Delta K(a)]^m$$

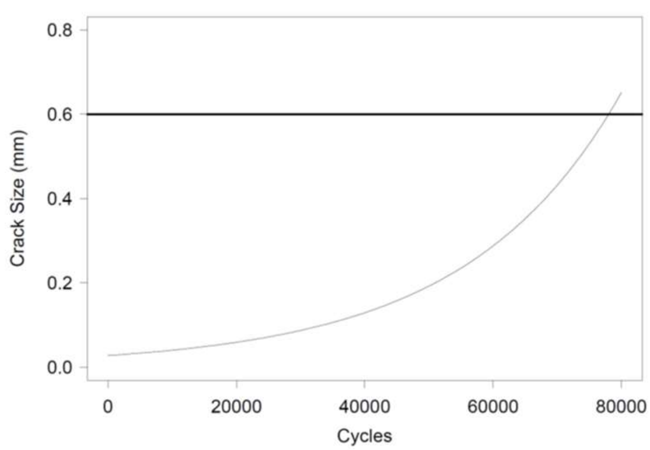
  - where $C > 0$ and $m > 0$ are material properties.
  - $\Delta K(a)$ is the stress intensity function of a (crack length), where the form of $K(a)$ depends on the applied stress, part dimension and geometry
- To model a 2-dimensional edge-crack in a plate with a crack that is small relative to the width of the plate (say less than 3%), $K(a) = \text{Stress} \times \sqrt{\pi a}$ and the solution to the resulting differential equation is

$$a(t) = \begin{cases} \left[\{a(0)\}^{1-m/2} + (1 - m/2) \times C \times (\text{Stress}\sqrt{\pi})^m \times t\right]^{\frac{2}{2-m}}, & m \neq 2 \\ a(0) \times \exp\left[C \times (\text{Stress}\sqrt{\pi})^2 \times t\right], & m = 2 \end{cases}$$

Paris Model

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

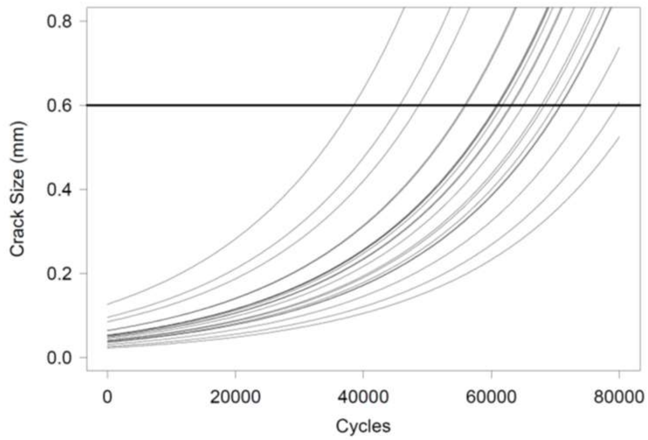
- Paris Model with No Variability
- $\frac{\mathrm{d}a(t)}{\mathrm{d}t} = C \times [\Delta K(a)]^m$



Paris Model

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

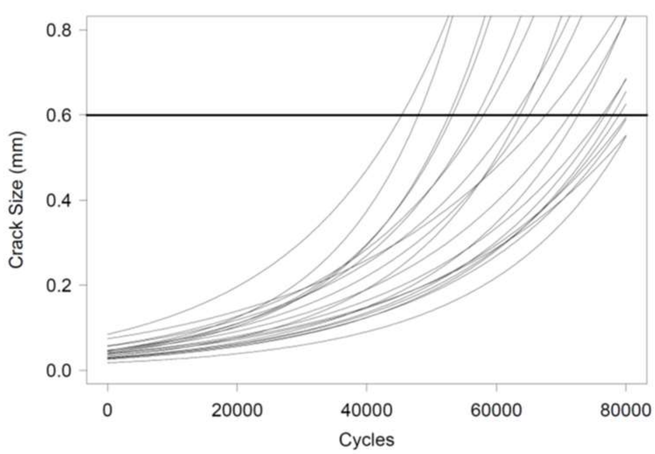
- Paris Model with **Unit-to-Unit Variability in Initial Crack Size** but with Fixed Materials Parameters and Constant Stress



Paris Model

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

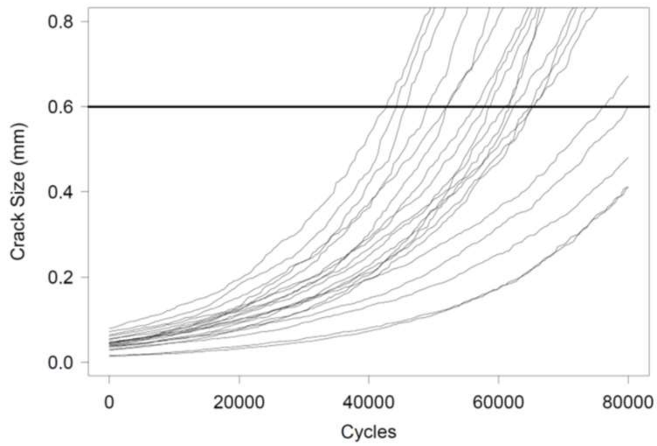
- Paris Model with **Unit-to-Unit Variability in the Initial Crack Size and Materials Parameters** but Constant Stress Cycles



Paris Model

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

- Paris Model with **Unit-to-Unit Variability in the Initial Crack Size and Materials Parameters and Stochastic Stress Cycles**



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General Degradation Path Model

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

- $\mathcal{D}_{ij} = \mathcal{D}(t_{ij}, ; \beta_{1i}, \beta_{2i}, \ldots, \beta_{ki})$ is the degradation path for unit i at time t which can be measured in units of time or usage.
- Observed sample degradation path of unit i at time t_j is given as

$$S_{ij} = \mathcal{D}_{ij} + \epsilon_{ij}, \qquad i = 1, \ldots n, \qquad j = 1, \ldots, m_i$$

- $\epsilon_{ij} \sim N(o, \sigma_\epsilon)$ describe a combination of measurement and modeling errors. It is the residual deviation for unit i at time t_j.
- The total number of observations on unit i is denoted by m_i.
- For unit i, $\beta_{1i}, \beta_{2i}, \ldots, \beta_{ki}$ is a vector k of unknown parameters. Some of the $\beta_{1i}, \beta_{2i}, \ldots, \beta_{ki}$ are random and differ from unit to unit while others are fixed across representing a phenomenon that does not change from one unit to another.
  - $\beta_{1i}, \beta_{2i}, \ldots, \beta_{ki}$ are often modeled using a multivariate normal distribution with parameters $\boldsymbol{\mu}_\beta$ and covariance $\boldsymbol{\Sigma}_\beta$

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General Degradation Path Model

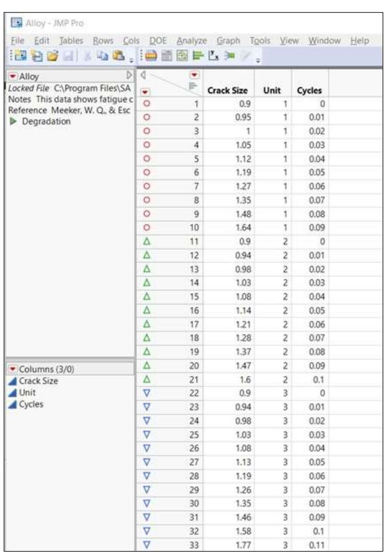
- The likelihood for the random-parameter degradation model is

$$\mathcal{L}(\boldsymbol{\mu_\beta}, \boldsymbol{\Sigma_\beta}, \sigma_\epsilon | DATA) = \prod_{i=1}^{n} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \left[\prod_{i=1}^{m_j} \frac{1}{\sigma_\epsilon} \phi_{nor}(\zeta_{ij}) \right]$$
$$\times f_\beta(\beta_{1i}, \beta_{2i}, \dots, \beta_{ki}; \boldsymbol{\mu}_\beta, \boldsymbol{\Sigma_\beta}) d\beta_{1i}, \dots, d\beta_{ki}$$

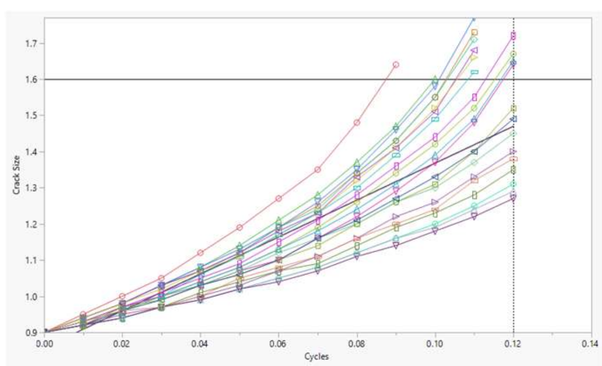
  - Where $\zeta_{ij} = [S_{ij} - \mathcal{D}(t_{ij},; \beta_{1i}, \beta_{2i}, \dots, \beta_{ki})]/\sigma_\epsilon$ and $f_\beta(\beta_{1i}, \beta_{2i}, \dots, \beta_{ki}; \boldsymbol{\mu}_\beta, \boldsymbol{\Sigma_\beta})$ is a multivariate normal distribution density function, where n is the number of sample paths and k is the number of random parameters in each path.

- Each evaluation of $\mathcal{L}(\boldsymbol{\mu_\beta}, \boldsymbol{\Sigma_\beta}, \sigma_\epsilon | DATA)$ will, in general, require numerical approximation of n integrals of dimension k.

Example: Alloy Fatigue Crack-Size Observations (JMP)



	Crack Size	Unit	Cycles
1	0.9	1	0
2	0.95	1	0.01
3	1	1	0.02
4	1.05	1	0.03
5	1.12	1	0.04
6	1.19	1	0.05
7	1.27	1	0.06
8	1.35	1	0.07
9	1.48	1	0.08
10	1.64	1	0.09
11	0.9	2	0
12	0.94	2	0.01
13	0.98	2	0.02
14	1.03	2	0.03
15	1.08	2	0.04
16	1.14	2	0.05
17	1.21	2	0.06
18	1.28	2	0.07
19	1.37	2	0.08
20	1.47	2	0.09
21	1.6	2	0.1
22	0.9	3	0
23	0.94	3	0.01
24	0.98	3	0.02
25	1.03	3	0.03
26	1.08	3	0.04
27	1.13	3	0.05
28	1.19	3	0.06
29	1.26	3	0.07
30	1.35	3	0.08
31	1.46	3	0.09
32	1.58	3	0.1
33	1.77	3	0.11



Estimate

Parameter	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.857791	0.0108	79.42207	<.0001*
Slope	5.099258	0.158561	32.15966	<.0001*

Failures in the Context of Degradation Path Models

- **Soft Failures are typically defined in terms of a specified degradation level**
  - In some engineering systems that exhibit a gradual loss of performance, failures tend to be defined sometimes arbitrarily yet in a purposeful manner as a specified level of degradation.
  - This is referred to as Soft Failure.
  - A fixed value of $\mathcal{D}_f$ is used to denote the critical level for the degradation path beyond which failure is assumed to have occurred.
  - The failure time T is defined as the time when the actual degradation path $\mathcal{D}_{ij}$ crosses the critical degradation level $\mathcal{D}_f$.

Failures in the Context of Degradation Path Models

- **Hard Failures are defined in terms of loss of functionality**
  - For some products, the definition of the failure event is clear, i.e., the product stops working. These are called hard failures.
  - Failure times will not usually correspond exactly with a particular level of degradation. Instead, the level of degradation at which failure occurs will be random from unit-to-unit.
  - This can be modeled by using a joint distribution to describe unit-to-unit variability in $\mathcal{D}_f$ as well as other random model parameters.

Distribution of the Time to Failure

- In general, a specified model for $\mathcal{D}_{ij}$ and $\mathcal{D}_f$ defines a failure-time distribution.
- The distribution can be written as a function of the degradation model parameters. Specifically, we assume that a unit fails when the degradation level first reaches $\mathcal{D}_f$,

$$\mathbb{P}(T \leq t) = \mathbb{P}\left(\mathcal{D}(t; \beta_1, \beta_2, \ldots, \beta_k) \geq \mathcal{D}_f\right)$$

- For a fixed $\mathcal{D}_f$ the distribution of T depends on the distribution of $\beta_{1i}, \beta_{2i}, \ldots, \beta_{ki}$.

Distribution of the Time to Failure

- In some cases it is possible to **directly evaluate $\boldsymbol{F(t)}$** where closed forms can be available for simple problems
- Some other cases require **numerical integration**:
  - Useful for a small number of random variables (e.g., 2 or 3).
- Complex models (e.g. nonlinear) often require **Monte Carlo simulation**, which is a general method for generating realization of the modeled data.

Example: Degradation Path Model

- Suppose that the actual degradation path of a particular unit is given by

$$\mathcal{D}(t) = \beta_1 + \beta_2 t$$

  - Where the parameter β_1 is fixed and β_2 varies from one unit to another according to $\beta_2 \sim N(\mu, \sigma^2)$ with $\sigma \ll \mu$ so that $\mathbb{P}(\beta_2 \leq 0)$ is negligible.

- We can derive the distribution $F_T(t)$ by acknowledging that the proportion failing by time t is equal to the proportion of degradation measure exceeding the critical threshold $\mathcal{D}_f$ at that time.

- Based on the assumptions, we $\mathcal{D}(t) \sim N\left(\beta_1 + \mu_{\beta_2} t, \sigma^2 t^2\right)$

Example: Degradation Path Model

- For the critical level $\mathcal{D}_f$, we know that $F_T(t) = \mathbb{P}(\beta_1 + \beta_2 t > \mathcal{D}_f)$, therefore;
- Recall that $\beta_2 \sim N(\mu, \sigma^2)$ and $\mathbb{P}(\beta_2 \leq x) = \Phi\left[\frac{x-\mu}{\sigma}\right]$, therefore;

$$\mathbb{P}\left(\beta_2 > \frac{\left(\mathcal{D}_f - \beta_1\right)}{t}\right) = 1 - \Phi\left[\frac{\frac{\left(\mathcal{D}_f - \beta_1\right)}{t} - \mu}{\sigma}\right]$$

$$F_T(t) = p\{T \leq t\} \approx \Phi\left[\frac{t - \frac{\left(\mathcal{D}_f - \beta_1\right)}{\mu}}{\frac{\sigma t}{\mu}}\right], \qquad t > 0$$

Knowledge Check

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- Suppose that the actual degradation path of a particular unit is given by

$$\mathcal{D}(t) = \phi + \theta t$$

  - Where the parameter ϕ is fixed and θ varies from one unit to another according toθ Weibull (α, β) distribution. Find the distribution function of the time-to-failure T.

- Let $G_\theta(\vartheta) = p\{\theta \le \vartheta\} = 1 - \exp\left[-\left(\frac{\vartheta}{\alpha}\right)^\beta\right]$
- For critical level $\mathcal{D}_f$, we can write $\mathcal{D}_f = \phi + \theta T$, then $T = \frac{(\mathcal{D}_f - \phi)}{\theta}$

$$F_T(t) = p\{T \le t\} = p\left\{\frac{(\mathcal{D}_f - \phi)}{\theta} \le t\right\}$$

$$= p\left\{\theta \ge \frac{(\mathcal{D}_f - \phi)}{t}\right\} = 1 - G_\theta\left(\frac{(\mathcal{D}_f - \phi)}{t}\right) = \exp\left[-\left(\frac{\mathcal{D}_f - \phi}{\alpha t}\right)^\beta\right]$$

Numerical Evaluation of $F_T(t)$

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- Suppose that the actual degradation path of a particular unit is given by

$$\mathcal{D}(t) = \beta_1 + \beta_2 t$$

  - Where the parameters (β_1, β_2) follow a bivariate normal distribution with parameters $\mu_{\beta_1}, \mu_{\beta_2}, \sigma^2_{\beta_1}, \sigma^2_{\beta_2}$, and ρ_{β_1,β_2}.

- The distribution of the time to failure $F_T(t)$ is given as:

$$F(t) = \int_{-\infty}^{\infty} \Phi_{nor}\left[\frac{g(\mathcal{D}_f, t, \beta_1) - \mu_{\beta_2|\beta_1}}{\sigma_{\beta_2|\beta_1}}\right]\frac{1}{\sigma_{\beta_1}}\phi_{nor}\left(\frac{\beta_1 - \mu_{\beta_1}}{\sigma_{\beta_1}}\right) \mathrm{d}\beta_1$$

  - Where $g(\mathcal{D}_f, t, \beta_1)$ is the value of β_2 that gives $\mathcal{D}(t) = \mathcal{D}_f$ for a specified β_1 and $\mu_{\beta_2|\beta_1} = \mu_{\beta_2} + \rho\sigma_{\beta_2}\left(\frac{\beta_1 - \mu_{\beta_1}}{\sigma_{\beta_1}}\right)$ and $\sigma^2_{\beta_2|\beta_1} = \sigma^2_{\beta_2}(1 - \rho^2)$

Monte Carlo Simulations for Evaluating $F_T(t)$

"Statistical Methods for Reliability Data"

- Monte Carlo Simulation can be used for evaluating $F_T(t)$
- Evaluation is performed by generating a large number of random sample paths from an assumed degradation path model, then using the proportion of paths crossing the threshold $\mathcal{D}_f$ to estimate $F_T(t)$
- **The Algorithm:**
  1. Generate N simulated realizations $\hat{\beta}_1, \hat{\beta}_2, \dots \hat{\beta}_k$ of $\beta_1, \beta_2, \dots, \beta_k$ from a multivariate normal distribution with mean $\widehat{\boldsymbol{\theta}}_{\boldsymbol{\beta}}$ and variance-covariance matrix $\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{\beta}}$ where N is a large number.
  2. Compute N simulated failure times corresponding to the N realizations of $\hat{\beta}_1, \hat{\beta}_2, \dots \hat{\beta}_k$ by substituting the simulated values into $\mathcal{D}(t; \beta_1, \beta_2, \dots, \beta_k)$ and finding the crossing time for each, given a predefined failure threshold.

Monte Carlo Simulations for Evaluating $F_T(t)$

"Statistical Methods for Reliability Data"

- **The Algorithm:**
  3. For any desired value to t use the following expression to evaluate $F(t)$

$$F(t) \approx \frac{\text{Numer of Simulated First Crossing Times} \leq t}{N}$$

- The resulting error in this procedure can be made arbitrarily small by making N large enough.
- In particular, the standard deviation of the Monte Carlo error in the evaluation of $F(t)$ at a given point is $\sqrt{F(t)(1 - F(t))/N}$.

Bootstrap Confidence Intervals

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- This method is useful for creating confidence intervals or standard errors for a distribution of a statistic when the (sampling) distribution is unknown.
- In bootstrapping, we use the distribution defined by the data to approximate the sampling distribution of the statistic.
- The procedure proceeds by simulating data from the observed data set (with replacement).
- Calculate the statistic of interest from each simulated data set.
- Repeat the resampling and the recalculation process
- Use the statistic estimated using the simulated data sets to construct a confidence interval.

Bootstrap Confidence Intervals

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- This method is used for computing confidence intervals for $F(t)$ from a sample of degradation data.
  1. Use the observed data from n sample paths to compute the estimates of $\widehat{\boldsymbol{\theta}}_{\boldsymbol{\beta}}$, i.e., and $\widehat{\boldsymbol{\sigma}}_{\boldsymbol{\epsilon}}^2$.
  2. Use Monte Carlo simulation (or in some cases numerical integration) to estimate $\widehat{F}(t)$ at desired values of t.
  3. Generate a large number $\boldsymbol{B}$ of bootstrap samples that mimic the original sample and compute the corresponding bootstrap estimates $\widehat{F}^*(t)$ using the following steps:
     a) Generate from $\widehat{\boldsymbol{\theta}}_{\boldsymbol{\beta}}$, n simulated realizations of the random path parameters $\beta_{1i}^*, \beta_{2i}^*, \dots, \beta_{ki}^*$, for $i = 1, \dots, n$

Bootstrap Confidence Intervals

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

a) Generate from $\widehat{\boldsymbol{\theta}}_{\boldsymbol{\beta}}$, n simulated realizations of the random path parameters $\beta_{1i}^*, \beta_{2i}^*, \ldots, \beta_{ki}^*$, for $i = 1, \ldots, n$ (with replacement)

b) Using the same sampling scheme, compute n simulated observed paths from the expression below up to the planned stopping time;

$$S_{ij}^* = \mathcal{D}(t_{ij}; \beta_{1i}^*, \beta_{2i}^*, \ldots, \beta_{ki}^*) + \epsilon_{ij}^*$$

where ϵ_{ij}^* values are independent simulated residual values generated from $N(0, \hat{\sigma}_\epsilon^2)$

c) Use the n simulated paths to estimate parameters of the path model, giving the bootstrap estimate $\widehat{\boldsymbol{\theta}}^*{}_{\boldsymbol{\beta}}$

d) Use Monte Carlo (or numerical integration) with $\widehat{\boldsymbol{\theta}}^*{}_{\boldsymbol{\beta}}$ as input to compute the bootstrap estimates $\hat{F}^*(t)$ at desired values of t.

Bootstrap Confidence Intervals

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

4. For each desired value of t, the bootstrap confidence interval for $F(t)$ is computed using the following steps:

a) Sort the B bootstrap estimates $\hat{F}^*(t)_1, \ldots, \hat{F}^*(t)_B$ in increasing order resulting in $\hat{F}^*(t)_{(b)}$ for $b = 1, \ldots, B$.

b) Following *Efron and Tibshirani* (1993), the lower and upper bounds of the point-wise approximate
$100(1-\alpha)\%$ confidence intervals for the distribution function $F(t)$ are:

$$[\underline{F}(t), \overline{F}(t)] = [\hat{F}^*(t)_{(l)}, \hat{F}^*(t)_{(u)}],$$

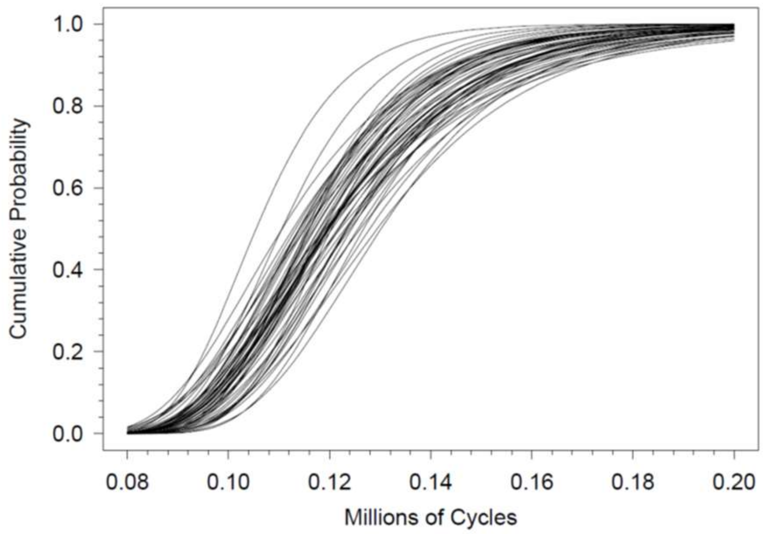
Where $l = B \times \Phi_{nor}\left[2\Phi_{nor}^{-1}(q) + 2\Phi_{nor}^{-1}\left(\frac{\alpha}{2}\right)\right]$

$$u = B \times \Phi_{nor}\left[2\Phi_{nor}^{-1}(q) + 2\Phi_{nor}^{-1}\left(1 - \frac{\alpha}{2}\right)\right]$$

And q is the proportion of the B values of $\hat{F}^*(t)$ that are less than $\hat{F}(t)$.
$q = 0.5$ is equivalent to the percentile bootstrap method.

Bootstrap Estimates of $F(t)$

Meeker & Escobar, "*Statistical Methods for Reliability Data*"



Approximate Degradation Models

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- An alternative (but only approximately correct) method of analyzing degradation data is as follows.
- Do a separate analysis for each unit to predict the time at which the unit will reach the critical degradation level corresponding to failure.
- These predicted failure times are called pseudo failure times.
- The n pseudo-failure times are analyzed as a complete sample of failure times to estimate $F(t)$.

Approximate Degradation Models

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

- Formally, the method is as follows
  - For the unit i use the path model $S_{ij} = \mathcal{D}_{ij} + \epsilon_{ij}$, $i = 1, \dots n$, $j = 1, \dots, m_i$ and the sample path data $(t_{i1}, y_{i1}), \dots, (t_{i\,m_i}, y_{i\,m_i})$ to estimate of $\boldsymbol{\beta}_i = (\beta_{1i}, \beta_{2i}, \dots, \beta_{ki})$, say $\widehat{\boldsymbol{\beta}}_i$.
  - Solve the equation $\mathcal{D}(t, \widehat{\boldsymbol{\beta}}_i) = \mathcal{D}_f$ for t and call the solution $\hat{t}_i$.
  - Repeat the procedure for each sample path to obtain the pseudo failure times $\hat{t}_1, \dots, \hat{t}_n$.
  - Do a single distribution analysis of the data $\hat{t}_1, \dots, \hat{t}_n$ to estimate $F(t)$.

Approximate Degradation Model Example

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

- For a simple linear degradation path $\mathcal{D}(t) = \beta_1 + \beta_2 t$, the pseudo failure times are obtained as follows:

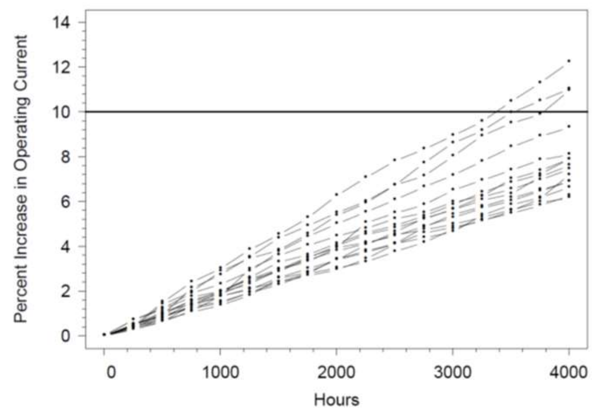
$$\widehat{t_i} = \frac{\mathcal{D}_f - \hat{\beta}_{1i}}{\hat{\beta}_{2i}}$$

  - where $\hat{\beta}_{1i} = \bar{y}_i - \hat{\beta}_{2i} \times \bar{t}_i$ and $\hat{\beta}_{2i} = \frac{\sum_{j=1}^{m_i}(t_{ij} - \bar{t}_i) \times y_{ij}}{\sum_{j=1}^{m_i}(t_{ij} - \bar{t}_i)^2}$
  - and $\bar{t}_i$ and $\bar{y}_i$ are the means of $t_{i1}, \dots, t_{i\,m_i}$ and $y_{i1}, \dots, y_{i\,m_i}$, respectively.

Example: Laser Life Data

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

- Percentage increase in operating current for GaAs lasers tested at 80°C.
- Fifteen (15) devices each measured every 250 hours up to 4000 hours of operation.
- For these devices and the corresponding application, a $\mathcal{D}_f = 10\%$ increase in current was the specified failure level.
- Plot of the Laser operating current as a function of time.



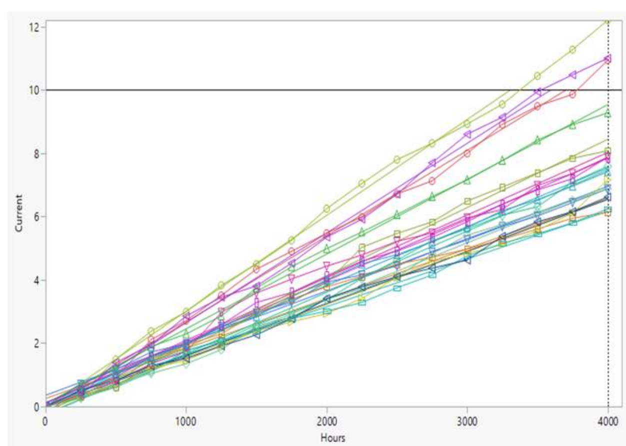
Example: Laser Life Data

Meeker & Escobar, *"Statistical Methods for Reliability Data"*

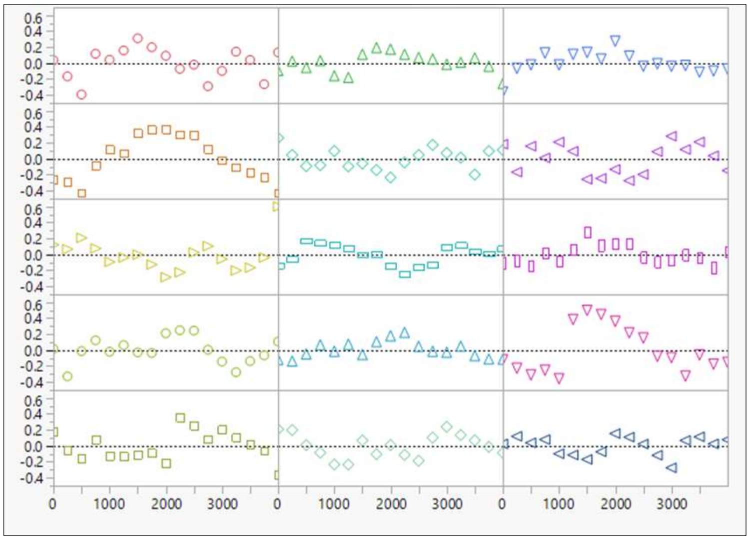
- For some degradation processes, all paths start at the origin so, $(t_{i1} = 0, y_{i1} = 0)$.
- The degradation path has the form $\mathcal{D}(t) = \beta_2 t$, and pseudo-failure times can be obtained as follows,

$$\hat{t_i} = \frac{\mathcal{D}_f}{\hat{\beta}_{2i}}$$

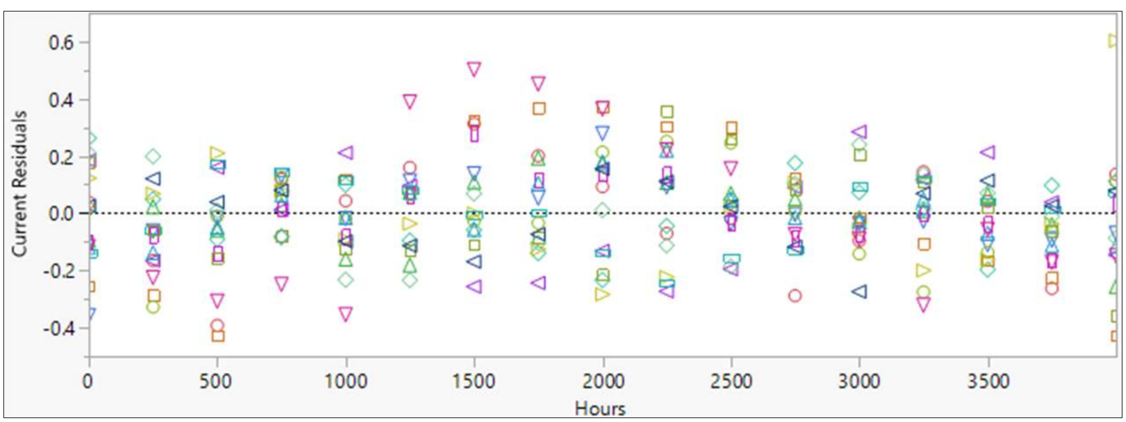
  - Where $\hat{\beta}_{2i} = \frac{\sum_{j=1}^{m_i} t_{ij} \times y_{ij}}{\sum_{j=1}^{m_i} {t_{ij}}^2}$



Example: Individual Residuals



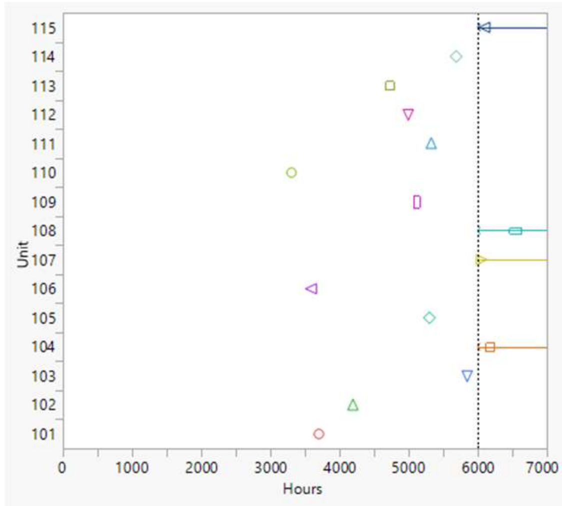
Example: Collective Residuals



Example: Laser Life Data

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

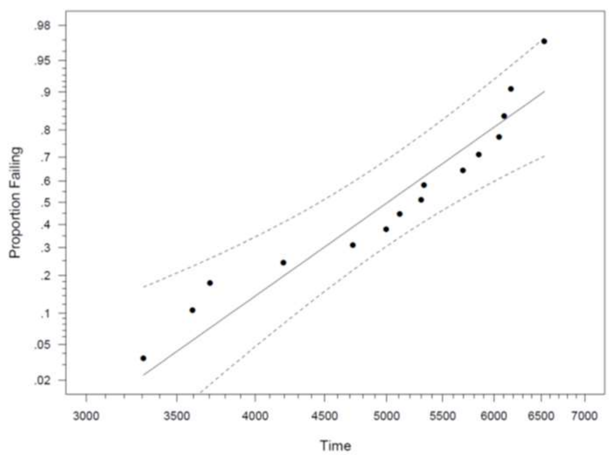
- The failure times (for paths exceeding $\mathcal{D}_f = 10\%$ increase in current before 4000 hours) and the **pseudo** failure times were obtained by fitting straight lines through the data for each path.
- These pseudo times-to-failure are: 3702, 4194, 5847, 6172, 5301, 3592, 6051, 6538, 5110, 3306, 5326, 4995, 4721, 5689, and 6102 hours.
- Inverse prediction



Example: Laser Life Data

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- Weibull Probability Plot of the Laser Pseudo Times to Failure Showing the ML Estimate of $F(t)$ and Approximate 95% Pointwise Confidence Intervals



Approximate Degradation Modeling

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- Approximate degradation modeling analysis may give adequate results if
  - The degradation paths are relatively simple.
  - The fitted path model is approximately correct.
  - There are enough data for precise estimation of the β_i parameters for each device.
  - The amount of measurement error is small.
  - There is not too much extrapolation in predicting the $\widehat{t_i}$ times to failure.

Approximate Degradation Modeling

Meeker & Escobar, "*Statistical Methods for Reliability Data*"

- Potential Problems With the Approximate Degradation Analysis
  - The method ignores the prediction error in $\hat{t}$ and does not account for measurement error in the observed sample paths.
  - The distributions fitted to the **pseudo** times to failure will not, in general, correspond to the distribution induced by the degradation model.
  - Some of the sample paths may not contain enough information to estimate all of the path parameters (e.g., when the path model has an asymptote but the sample path has not begun to level off).
  - This might necessitate fitting different models for different sample paths in order to predict the crossing time..