$X_i \sim \text{Poisson}(\lambda)$, $\lambda \sim \text{Gamma}(\alpha, \beta)$, let there be n X_i's

$$\pi(\lambda|x) = \frac{f(x|\lambda)\,\pi(\theta)}{\int_\lambda f(x|\theta)\,\pi(\theta)\,d\theta} = \frac{h(x,\lambda)}{m(x)} \propto h(x,\lambda)$$

$$h(x,\lambda) = \prod_i \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$= \frac{\lambda^{\Sigma x_i} e^{-n\lambda}}{\prod_i (x_i!)} \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$= \frac{\beta^\alpha}{\prod_i (x_i!) \cdot \Gamma(\alpha)} \lambda^{\Sigma x_i + \alpha - 1} e^{-(n\lambda + \beta\lambda)}$$

$$\propto \lambda^{\Sigma x_i + \alpha - 1} e^{-\lambda(n+\beta)}$$

$$\therefore \pi(\lambda|x) \propto \lambda^{\Sigma x_i + \alpha - 1} e^{-\lambda(n+\beta)}$$

$$\therefore \lambda | X_i \,\forall i \sim \text{Gamma}\left(\sum_i x_i + \alpha - 1, n + \beta\right)$$