ISyE 6810 Systems Monitoring and Prognostics
Exam 1 (Spring 2021)

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Question 1: Find the general solution for a degraded system that fails completely from the degraded state only. In other words, the system must degrade before it can fail. Let λ_1 be the rate at which a system degrades and λ_2 the rate at which the system fails from the degraded state. Also derive a compact form of the MTTF of the system.

Question 2: Suppose the actual degradation path of a particular unit is given by $\mathcal{D}(t) = \beta_1 t$ where β_1 varies from unit-to-unit according to a $LogNorm(\mu_1, \sigma_1)$ lognormal distribution. Also suppose that failure occurs when $\mathcal{D}(t) > \mathcal{D}_f$ and $\mathcal{D}_f$ has a $LogNorm(\mu_2, \sigma_2)$ distribution. Derive an expression for the cdf of the failure time, $F(t)$.

Question 3: The TF99 turbofan jet engine is to be manufactured by Major Electric (ME) Aircraft Engines and will be used in the new Arialbus A555 passenger plane. The engine consists of five modules identified by subsystem codes of 23A, 23B, 23C, 23D, and 23E. There is concern about the overall reliability of the engine. The average round-trip flight time is 5 hours. Each A555 is scheduled to fly once a day. Each module has independently undergone considerable reliability testing. Repair times are based upon data collected from repairs of the TF85 engine that is used on similar types of aircraft. It is assumed that these times are representative of the repair times of the TF99 engine. Test data for each module is provided in the accompanying excel sheet. Note that some tests have been censored.

- From among the exponential, Weibull, normal, and lognormal distributions, find a best-fit failure distribution.
- Compute the following engine (subsystem) performance measurements (all modules are critical)
  - Reliability(1^{st} mission)
  - Reliability(25^{th} mission)
  - Median number of missions to failure
  - Mean time to repair
  - 90^{th} percentile of the repair time
- Which subsystem displays the worst reliability? What should the reliability of that subsystem be to achieve a system reliability specification of $R(10 \text{ missions}) = 0.9$?

Question 4: The attached excel sheet provides crack data for 15 specimens of Aluminum alloy. Each of the 15 specimens was placed on a test rig where it underwent a fatigue test (bending back and forth). Number of cycles were noted for various crack lengths as shown in the data. Each test was stopped (censored) at $120,000$ cycles. Assume soft failure occurs once a crack length of 15 mm is crossed. Use the data to compute the cdf of the time-to-failure using the approaches outlined below. Then compare your results for the $10^{th}, 25^{th}, 50^{th}, 75^{th}, 90^{th}$ percentiles of the cdf.

- Use the TTF information (only) to estimate the cdf at the required percentiles assuming that the TTF distribution follows a Weibull.
- Assume that the degradation signals follow a linear degradation path $\mathcal{D}(t) = \beta_1 + \beta_2 t$, where $\beta_1 isconstant$ and β_2 is normally distributed. Derive a closed-form expression for the cdf of the TTF and use it to calculate the cdf at the designated percentiles.
- Assume the same linear degradation path, implement the approximate degradation path approach to estimate psuedo failure times. Assuming a Weibull distribution, estimate the designated percentiles.
- Assume the same linear degradation path, implement the bootstrapping algorithms discussed int the lecture and calculate the cdf as well as the upper and lower bounds of the pointwise approximate percentiles.

Good Luck.