ISyE 6810 Systems Monitoring and Prognostics
Spring 2021 Exam 2

Due Sunday, April 25 by 11:59 p.m.

Question 1: (20 pts.)
Suppose we are using an Exponential(λ) distribution to model the lifetimes of n items.

(a) Find the maximum likelihood estimator of λ.

(b) Suppose that we observed $n = 50$ items and that $\sum_{i=1}^{50} t_i = 25$, find a 90% confidence interval for λ. (Hint: Standard error of $\hat{\lambda}$ is given by the following expression $se\hat{\lambda} = \frac{\sqrt{n}}{\sum_{i=1}^{n} t_i}$)

(c) Suppose that $\lambda \sim Gamma(1, 2)$, find the posterior distribution for λ

(d) Suppose we observed $n = 50$ items and that $\sum_{i=1}^{50} t_i = 25$, what is the posterior probability that λ falls in the 90% confidence interval found calculate in second bullet?
Hint: $Gamma(\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ for $x \geq 0$.

Question 2: (40 pts.)
Suppose the actual degradation path of a particular family of components is given by the following degradation model

$$S(t) = \phi + \theta t + \epsilon(t)$$

where $\phi = S(0)$ and is a constant, $\theta \sim N(\mu_0; \sigma_0^2)$, and $\epsilon(t)$ is a Brownian motion with diffusion parameter σ, i.e., $\epsilon(t) \sim N(0; \sigma^2 t)$

(a) Derive an expression for the posterior mean μ_p and variance σ_p^2 of θ for a unit operating in the field given that a partial degradation signal $S(t_1), \ldots, S(t_k)$ has been observed.

(b) Using the posterior distribution of θ, derive an expression for the remaining lifetime distribution of the unit, denoted by the random variable T.

(c) Using the historical degradation signals provided in the "Estimation sheet" (20 component signals) of the attached excel file, estimate the parameters of the degradation model.

(d) Choose an appropriate failure threshold and use the model to predict the expected remaining lifetime of the partially observed data in the "Prediction sheet" (10 component signals).

(e) Propose an approach that can help improve the prediction accuracy and apply it to the same "prediction" dataset. Record your predicted failure times for the 10 component signals in the "Prediction sheet".

Question 3: (20 pts.)
Using the degradation model of Question 2, calculate the optimal replacement time and the optimal spare parts ordering time for a component with the partially observed degradation signals provided "Question 2 Repair & Inventory" sheet of the attached excel file (provide details of your procedure to help with the partial credit).

- Assume that the lead time is 7 days.
- Assume that the cost of unexpected failure of a component from the previous question is \$100 and the cost of preventive maintenance is \$50.
- Assume that the cost of holding the component in stock costs \$0.5 per day and the cost of a stockout is \$200.

Question 4: (20 pts.)
Let $A[t_i, t_j]$ denote the event that the degradation signal does not cross the failure threshold, $\mathcal{D}$, within the interval $[t_i, t_j]$. Using the linear degradation model provided in question 2, show that:

$$f(S_1, \ldots, S_k | A_{[0,t_k]}, \theta) = \prod_{j=1}^{k} f(S_j, A_{[t_{j-1},t_j]} | s_{j-1}, \theta) \frac{1}{\mathbb{P}(A_{[0,t_k]} | \theta)}$$

where $S_j = s_j - s_{j-1}$

Good Luck