Residual Life Predictions in the Absence of Prior Degradation Knowledge

Nagi Gebraeel, Alaa Elwany, and Jing Pan

***Abstract*—Recent developments in degradation modeling have been targeted towards utilizing degradation-based sensory signals to predict residual life distributions. Typically, these models consist of stochastic parameters that are estimated with the aid of an historical database of degradation signals. In many applications, building a degradation database, where components are run-to-failure, may be very expensive and time consuming, as in the case of generators or jet engines. The degradation modeling framework presented herein addresses this challenge by utilizing failure time data, which are easier to obtain, and readily available (relative to sensor-based degradation signals) from historical maintenance/repair records. Failure time values are first fitted to a Bernstein distribution whose parameters are then used to estimate the prior distributions of the stochastic parameters of an initial degradation model. Once a complete realization of a degradation signal is observed, the assumptions of the initial degradation model are revised and improved for future predictions. This approach is validated using real world vibration-based degradation information from a rotating machinery application.**

***Index Terms*—Bernstein distribution, degradation modeling, prognostics, random coefficients models.**

ACRONYM[1]

BPFI	Ball Pass Frequency on Inner Race
BPFO	Ball Pass Frequency on Outer Race
BSF	Ball Spin Frequency
CDF	Cumulative Distribution Function
CI	Confidence Interval
DAQ	Data Acquisition
ELHR	Extended Linear Hazard Regression
FTF	Cage Rotating Frequency
PDF	Probability Density Function
RMS	Root Mean Square
RPM	Revolution Per Minute
SSR	Stress-Strength Interference

NOTATIONS

b	Slope of fitted regression line
D	Failure threshold
$F(\cdot)$	CDF of a random variable
$f(\cdot)$	PDF of a random variable
$S(t_i)$	Level of degradation signal at time t_i
L_i	Logarithm of the degradation signal at time t_i
L	Likelihood function
$W(t)$	Wear at time t
T_R	Random variable representing the residual life
t_i	time at epoch i
t_k	Current operating time at epoch k
t	time (Support of the remaining life distribution)
$\eta(\cdot)$	Path followed by the degradation signal
λ	Intercept of the Bernstein wear model
ω	Slope of the Bernstein wear model
c, α, γ	Parameters of the Bernstein distribution
β	Stochastic parameter of the degradation model
$\pi(\beta)$	Prior distribution of β
ϕ	Deterministic parameter of the degradation model
$\varepsilon(t_i)$	Error term at time t_i
σ^2	Variance of the error term
σ_β^2	Prior variance of β
$\widetilde{\sigma}_\beta^2$	Posterior variance of β
$\widetilde{\sigma}^2(t+t_k)$	Variance of the predictive distribution of the degradation signal
$\mu(t+t_k)$	Mean of the predictive distribution of the degradation signal
μ_β	Prior mean of β
$\widetilde{\mu}_\beta$	Posterior mean of β
$\Phi(\cdot)$	CDF of a standardized normal random variable
σ_c	Maximum contact stress between the ball and bearing raceway
P	Load per rolling element
K_D	Ball diameter
C_E	Contact modulus
E	Modulus of elasticity

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N. Gebraeel and A. Elwany are with the Milton H. Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30313 USA (e-mail: nagi@isye.gatech.edu; elwany@gatech.edu).

J. Pan is with the Department of Actuarial Sciences and Statistics, University of Iowa, Iowa City, IA 52242 USA (e-mail: jing-pan@uiowa.edu).

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[1]The singular and plural of an acronym are always spelled the same.

δ	Poisson's ratio
d_r	Diameter of rolling elements
d_c	Diameter of cage
d_{in}	Inner race diameter
d_{out}	Outer race diameter
z	Number of rolling elements
ν	Contact angle between the rolling elements and the rolling surfaces
R^2	Coefficient of determination
B_j	Bearing number j
D_j^i	Prediction error for bearing B_j at epoch i
t_j^F	Actual (observed) failure time of bearing B_j
p_j^i	Total service time of bearing B_j up to epoch i
$\widetilde{t}_j^i$	Updated median residual life of bearing B_j at epoch
d_r	Diameter of the rolling elements
d_c	Diameter of the cage
d_{out}	Diameter of the outer race
d_{in}	Diameter of the inner race
z	Number of rolling elements
ν	Contact angle between the rolling elements and the rolling surfaces
RPM	Rotational speed of the shaft.

I. Introduction

THE stochastic nature, and our limited understanding, of the physics-of-failure are major challenges in predicting the remaining lifetimes of complex engineering systems. A plethora of research efforts have devised approaches to make accurate predictions of a component's residual life. In conventional reliability approaches, this is achieved by evaluating conditional failure time distributions given that a system has survived up to a specific point in time. However, these failure time distributions are generally based on the failure characteristics of a population of identical devices. Thus, there is minimal or no consideration for the differences in degradation rates among the individual units, or their latest state of health.

Hence, it is useful to evaluate the condition-based residual life to account for these different degradation rates. In addition to the failure characteristics of the population of components, the condition-based residual life incorporates the level of degradation as a valuable indicator for the components' states of health. In many applications, condition-based information can be captured using sensor technology, and health monitoring systems. These sensory signals are often directly correlated with the underlying physical transitions that occur during degradation processes, and eventually lead to failure. The characteristic patterns associated with these signals are known as degradation signals [1]. Degradation models can be used to model these degradation signals, and make accurate condition-based remaining life predictions. These predictions provide increased accuracy of failure predictability over the conventional reliability approach [5], [16], [17].

Degradation modeling attempts to probabilistically characterize the evolution of physical degradation processes. Random coefficients models have been widely used to model the progression of degradation signals using both deterministic, and stochastic model parameters [2]–[5]. The objective of these models is to estimate the time it takes the signal to hit a predefined failure threshold. Due to the stochastic nature of the signal's evolution (captured by the stochastic model parameters), the resulting "hitting time" is a random variable. Its distribution is typically referred to as the residual life distribution.

There are several challenges associated with the development of random coefficients degradation models. One of the major challenges lies in estimating the random coefficients of these models. Typically, a database of degradation signals is acquired, and degradation signals are fitted to the desired functional form. The fitted model parameters are then used to estimate the distribution of the random coefficients. From the prediction accuracy standpoint, this approach provides the best estimates of the random coefficients. In many applications, building a degradation database, where components are run-to-failure, may be very expensive and time consuming, as in the case of jet engines for instance. One alternative is to define a critical degradation level (pre-failure), and use a partial degradation signal to estimate pseudo-failure times. Nevertheless, both cases involve the time consuming process of collecting degradation signals to build the degradation database.

Furthermore, despite the fact that significant advances in sensor technology have been achieved, some legacy systems still lack the capability of acquiring sensor-based information. This is not the case with failure/repair time data. Such data can be easily retrieved from historical maintenance records. Rather than using this information solely to predict the remaining life, this paper proposes a framework that couples existing failure time data with degradation models to predict the condition-based remaining life. We capitalize on prior engineering knowledge (or subjective hypotheses) about the functional form that a component's degradation signal will follow. For example, we expect the vibration level of a bearing to increase as it degrades. This increase may be linear, polynomial, or exponential.

In this paper, we assume that the distribution of the failure times is a Bernstein distribution. We hypothesize a preliminary functional form for the expected path of the degradation signal of the component being monitored (hereafter referred to as the degradation path). The functional properties of the hypothesized degradation model, along with the characteristics of the Bernstein distribution, are used to derive prior distributions of the stochastic parameters. The degradation model is then used to compute residual life distributions. Subsequent real-time sensory signals are used to update these distributions.

The following section presents a review of the relevant literature. Section III discusses the degradation modeling framework, and presents the recent advances related to sensor-based updating of residual life distributions. Section IV discusses the proposed methodology for estimating the prior distributions of the stochastic parameters using the Bernstein distribution. Section V presents a case study where vibration-based degradation signals and failure times of rolling element bearings are used to compare the two methodologies presented in Sections III and IV. Concluding remarks, and directions for future research are then outlined in Section VI.

II. Literature Review

Traditionally, reliability testing has been used to estimate general failure distributions for a population of components [7], [8]. The time available for testing is often considerably less than the expected lifetime of the component. This is apparent in highly reliable components. Testing such components under typical use conditions may not generate enough failures. In accelerated life testing, components are subjected to increased levels of stress to accelerate the failure process, and reduce testing durations [1]. Elsayed *et al.* [9] used failure time data obtained from accelerated conditions to estimate reliability under typical operating conditions. The authors developed a generalized Extended Linear Hazard Regression (ELHR) model with linear time-varying coefficients for this purpose. Huang & Askin [10] presented a generalized reliability model for stress-strength interference (SSI) that considers stochastic loading, and strength aging degradation.

In some systems, a measurable performance indicator changes as the system degrades. In such applications, it may be possible to predict the time to failure by extrapolating the performance measure over time. Most of the degradation modeling literature focuses on the development of random coefficients growth models and time series techniques to model the path of a component's degradation signal. Whitmore & Schnekelberg [3] modeled an accelerated degradation process using Brownian motion with a time scale transformation to account for varying operating conditions. Tseng *et al.* [11] studied the degradation of fluorescent lamps by monitoring their luminosity. The authors developed a linear random coefficients model of luminosity with experimental design to improve the reliability of fluorescent lamps. The degradation model was used to identify the combination of manufacturing settings that provided the slowest rate of luminous degradation. Yang & Jeang [12] also used a random coefficients model to study the effect of cutting tool flank wear on surface roughness in metal cutting. Tool degradation was quantified by the surface roughness value of the machined part. The degradation model was used to develop an inspection strategy for optimal tool replacement. Yang & Yang [13] developed a random-coefficient-based approach that uses the lifetimes of failed devices, plus degradation information from operating devices, to estimate parameters of lifetime distributions. Goode *et al.* [14] developed a predictive model for monitoring the condition of a hot strip mill. The authors developed a vibration-based degradation signal that grows exponentially as the component degrades. They concluded that the degradation model outperforms the reliability model. Swanson [15] used time series techniques, namely Kalman filters, to track changes in vibration characteristics of degraded devices. Wiener processes have also been used to model degradation signals from accelerated tests [2], [3]. In [2], drift and diffusion parameters were altered according to the stress level. The authors illustrated how to estimate mean life under typical stress levels. Whitmore [3] illustrated how the Wiener process can be used to account for measurement errors in degradation signals.

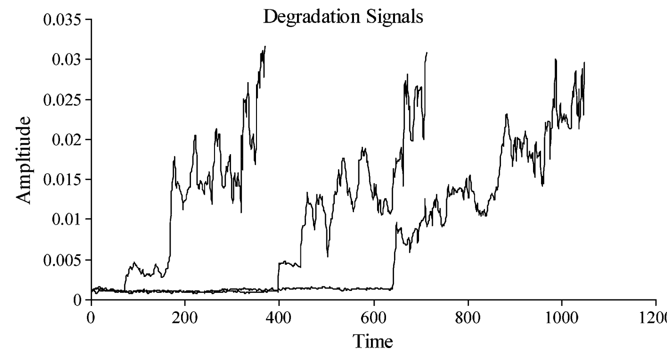

Fig. 1. Bearing degradation signals.

Lu & Meeker [4] developed degradation models to estimate time-to-failure distributions, and demonstrated some special cases for which it is possible to obtain closed-form expressions for the distributions. The authors assumed that the degradation path can be defined accurately if the values of specific degradation parameters are known. However, these parameters were evaluated across the components' population, and are not unique to individual components. Gebraeel *et al.* [5] used a Bayesian updating methodology to continuously predict & update the remaining life distributions of individual components using a random coefficients exponential model.

The following section discusses the general degradation modeling framework. We present two examples of degradation models: linear, and exponential. We also elaborate on the sensor-based updating technique presented in Gebraeel *et al.* [5].

III. Degradation Modeling Framework

The degradation signals of similar components operating under similar environmental conditions generally exhibit a unique functional form. However, it is common to find that the individual components have different degradation rates. The differences in degradation signals may be attributed to various factors, such as material inhomogeneity, processing variations, and tolerance differences. For example, Fig. 1 presents three vibration-based degradation signals of three bearings that were run to failure under the same testing conditions [5].

The approach used in this paper is to mathematically model the functional form of the degradation signal. The degradation signal is assumed to evolve according to a continuous-time stochastic process. We define the level of the degradation signal $S(t_i)$ of a component at time $t_i,\ i = 1, 2, \ldots,$ as $S(t_i) = \eta(t_i; \beta, \phi) + \varepsilon(t_i)$, where $\eta(\cdot)$ describes the path followed by the degradation signal. The deterministic parameter ϕ is constant across all the units of a given population of components. The parameter β is a stochastic coefficient that characterizes the differences in the degradation rates among the individual units of the population. This parameter is assumed to follow some prior distribution, $\pi(\beta)$, across the component's population. The error term $\varepsilon(t_i)$ is used to model signal noise and transients, and is assumed to be i.i.d. $N(0, \sigma^2)$, across the population of devices.

The same assumption was used in Lu & Meeker [4], Gebraeel *et al.* [5], and Wang [6].

We define T_R as a random variable that represents the residual life. The distribution of T_R is obtained by evaluating the distribution of the time until the degradation signal reaches a predetermined failure threshold, D, as illustrated in (1).

$$F_{T_R}(t) = P\{T_R \leq t\} = P\left\{\eta(t;\, \beta, \phi) + \varepsilon(t) \geq D\right\} \quad (1)$$

In this paper, we consider two types of degradation models. The first model is used to characterize degradation signals that follow a linear path, whereas the second considers signals that grow exponentially.

A. Linear Degradation Model

The linear degradation model is typically used for modeling degradation processes where cumulative damage does not have a significant effect on the rate of degradation. For example, Christer & Wang [18] modeled the wear of brake pads as a linear function of time where the thickness of the break pad decreases linearly with time. For a given component, the linear degradation model is expressed as

$$S(t_i) = \phi + \beta t_i + \varepsilon(t_i) \quad (2)$$

where, $S(t_i)$ is the value of the signal at time t_i, ϕ is the deterministic parameter, and β is the random coefficient.

In this work, we focus on modeling degradation signals that begin at zero amplitude. Our underlying assumption is that, because a new component has no degradation, the path of the degradation signal is assumed to begin at zero amplitude level. Furthermore, components that do not exhibit any level of degradation are considered new. To model this property, the fixed parameter ϕ is assumed to be equal to zero. The stochastic parameter β is assumed to follow a prior distribution $\pi(\beta)$ that is generally unknown. We assume that $\pi(\beta)$ is s-normally distributed with mean μ_β, and variance σ_β^2. As mentioned earlier, the error term captures signal transients, and is assumed to be i.i.d. $N(0, \sigma^2)$.

Gebraeel *et al.* [5] developed a Bayesian updating methodology where the prior distribution $\pi(\beta)$ can be updated using real-time degradation signals. Thus, the initial residual life distribution computed by (1) can now be updated given the posterior distribution of β.

The following subsection discusses the updating procedure for the linear model.

1) Computing & Updating the Residual Life Distribution: We define $S_i = S(t_i)$ (the value of the degradation signal at time t_i). Next, assume that a partial degradation signal $S_1, \ldots, S_k$ has been observed up to time $t_k \ldots$ We are interested in updating the prior distribution of β given the observed degradation signals

$$p(\beta | S_1, \ldots, S_k) \propto f(S_1, \ldots, S_k | \beta)\, \pi(\beta) \quad (3)$$

Because the error terms $\varepsilon(t_i)$, $i = 1, \ldots, k$ are i.i.d. normal random variables with mean 0, and variance σ^2, the joint distribution of the partially observed signal can be expressed as a product of k normal distributions:

$$f(S_1, \ldots, S_k | \beta) = \frac{1}{\prod_{i=1}^{k} \sqrt{2\pi\sigma^2}} \times \exp\left(-\sum_{i=1}^{k} \frac{(S_i - \beta t_i - \phi)^2}{2\sigma^2}\right) \quad (4)$$

Given that $\pi(\beta)$ is normally distributed with mean μ_β, and variance σ_β^2, we use (3) and (4) to evaluate the parameters of the posterior distribution of β:

$$\tilde{\mu}_\beta = \frac{\sigma_\beta^2 \sum_{i=1}^{k} \{(S_i - \phi) t_i\} + \mu_\beta \sigma^2}{\sigma_\beta^2 \sum_{i=1}^{k} (t_i^2) + \sigma^2} \quad (5)$$

$$\tilde{\sigma}_\beta^2 = \frac{\sigma^2 \sigma_\beta^2}{\sigma_\beta^2 \sum_{i=1}^{k} (t_i^2) + \sigma^2} \quad (6)$$

We define the random variable $S(t + t_k)$ as the level of the degradation signal at some time t in the future, given we are currently at time t_k,. The mean, and variance of the predictive distribution of $S(t + t_k)$ are given by

$$\tilde{\mu}(t + t_k) = \phi + \tilde{\mu}_\beta t \quad (7)$$

$$\tilde{\sigma}^2(t + t_k) = \tilde{\sigma}_\beta^2 t^2 + \sigma^2 \quad (8)$$

Thus, each time a signal is observed, the trajectory of the degradation signals is modified accordingly. This predictive distribution is used to compute the distribution of the residual life T_R. In other words, T_R satisfies $S(T_R + t_k) = D$, where D is a predetermined failure threshold associated with the degradation signal

$$\begin{aligned} &P(T_R \leq t | S_1, \ldots, S_k) \\ &\quad = P\left(S(t + t_k) \geq D | S_1, \ldots, S_k\right) \\ &\quad = 1 - \left(P\left(S(t + t_k)\right) \leq D | S_1, \ldots, S_k\right) \\ &\quad = 1 - P\left(Z \leq \frac{D - \tilde{\mu}(t + t_k)}{\tilde{\sigma}(t + t_k)}\right) \\ &\quad = \Phi\left(\frac{\tilde{\mu}(t + t_k) - D}{\tilde{\sigma}(t + t_k)}\right) \end{aligned} \quad (9)$$

where $\Phi(\cdot)$ is the cdf of a standardized normal random variable Z.

Note that $\lim_{t \to -\infty} g(t) = -(\mu_\theta / \sigma_\theta)$, thus, the domain of the remaining life is $(-\infty, \infty)$. To preclude negative values of the remaining life, we compute the truncated cdf,

$$P(T_R \leq t | S_1, \ldots, S_k) = \frac{\Phi\left(g(t)\right) - \Phi\left(g(0)\right)}{1 - \Phi\left(g(0)\right)}. \quad (10)$$

The truncated pdf is evaluated by differentiating $F_{T_R | S_1, \ldots, S_k, T_R \geq 0}(t)$ with respect to t:

$$f_{T_R | S_1, \ldots, S_k, T \geq 0}(t) = \frac{\phi\left(g(t)\right) g'(t)}{1 - \Phi\left(g(0)\right)} \quad (11)$$

where $\phi(.)$ is the pdf of a standardized normal random variable.

B. Exponential Degradation Model

The exponential degradation model is well suited for applications where cumulative damage has a significant effect on the rate of degradation, i.e. cumulative damage accelerates or decelerates the degradation process. Examples of such processes include corrosion, deterioration of civil structures, and bearing degradation [12], [14], [19]. The general form of the exponential model is expressed as

$$S(t_i) = \phi e^{\left(\beta t_i + \varepsilon(t_i) - \frac{\sigma^2}{2}\right)} \tag{12}$$

where ϕ is assumed to be constant. The parameter β is random; and similar to the linear model, it is assumed to follow a s-normal distribution $\pi(\beta)$ with mean μ_β, and variance σ_β^2. The error terms $\varepsilon(t_1), \varepsilon(t_2), \ldots$ are also assumed to be i.i.d. $N(0, \sigma^2)$.

Next, we define $L_i = \ln S(t_i)$. By taking the logarithm of the degradation signal, we can express the exponential model in a more convenient linear form as

$$L_i = \phi' + \beta t_i + \varepsilon(t_i) \tag{13}$$

where $\phi' = \ln \phi - \sigma^2/2$. Because ϕ' is assumed to be constant, (13) is similar to the linear degradation model in (2). The two models follow a linear functional form with a fixed intercept. Whereas the linear degradation model represents the actual amplitude of the degradation signal, the exponential model characterizes the evolution of the logarithm of the degradation signal.

The distribution of the stochastic parameter β, given that we observed a partial degradation signal, $L_1, \ldots, L_k$ can be evaluated in a similar manner as the linear model

$$p(\beta | L_1, \ldots, L_k) \propto f(L_1, \ldots, L_k | \beta)\, \pi(\beta) \tag{14}$$

The posterior mean of β is given by (15), whereas the variance is the same as that of the linear model.

$$\tilde{\mu}_\beta = \frac{\sigma_\beta^2 \sum_{i=1}^{k} \{(L_i - \phi')t_i\} + \mu_\beta \sigma^2}{\sigma_\beta^2 \sum_{i=1}^{k} (t_i^2) + \sigma^2} \tag{15}$$

The first step in implementing the degradation models proposed in this paper requires that we estimate the prior distribution of the stochastic parameters. To do this, we use a database of degradation signals that constitute a complete realization of the degradation process, i.e. degradation signals acquired from the "as good as new" state until the system reaches the "failed" state. This approach has been widely used in [2]–[5], [11]–[15]. Each degradation signal is fitted with the desired functional form of the degradation model. The coefficients of the fitted model are then computed using the least squares method. These coefficients are then used to estimate the distribution of the stochastic parameters.

IV. Two-Stage Prognostic Methodology

The prognostic methodology proposed in this paper is divided into two stages. In the first stage, we focus on estimating the prior distributions of the degradation model stochastic parameters in the absence of prior degradation signals. Specifically, we use failure time information, coupled with the characteristics of the Bernstein distribution, to derive these prior distributions. This is an important deviation from the conventional degradation modeling frameworks presented in [5], [16], [17]. The rationale is that the process of building a database of degradation signals requires significant degradation testing, which can be very time-consuming, and/or economically unjustifiable. However, it is safe to say that most of the maintenance records of these systems may contain repair, and/or failure times related to the systems themselves, or their components. From a different perspective, a wide range of application domains consists of legacy systems that are not equipped with sensor technology. We may consider a subset of these systems for which it is possible to perform health monitoring as a result of the recent advances in sensor technology. For these applications, failure and/or repair times might be the only available information.

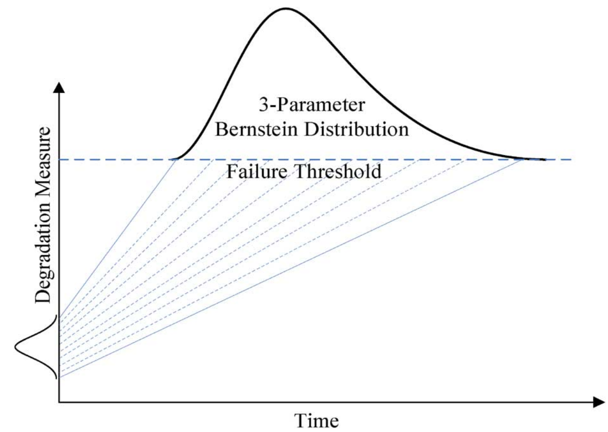

Fig. 2. The Bernstein distribution.

The second stage of the prognostic methodology uses real-time degradation signals, coupled with the prior distributions evaluated in the first stage, to compute residual life distributions. These distributions are continuously updated as subsequent signals are acquired.

A. First-Stage of Prognostic Methodology

This section discusses the estimation of the prior distribution of the stochastic parameters by statistically transforming failure time. This transformation is established through the Bernstein distribution.

1) Bernstein Distribution: The Bernstein distribution has been widely used to model the lifetime of components whose wear follows a non-stationary linear path [20]. It is derived as the probability distribution of the time required for the degradation path to reach a predetermined failure threshold, as shown in Fig. 2. The 3-paramter Bernstein distribution is derived from a linear model whose slope and intercept are assumed to follow independent normal distributions. Some examples include machine tools, and cutting tools, where the wear at time t, $W(t)$, is modeled as

$$W(t) = \lambda + \omega t \tag{16}$$

The random variables, λ, and ω, are normally distributed, thus $W(t) \sim N(\mu_\lambda + \mu_\omega t, \sigma_\lambda^2 + \sigma_\omega^2 t)$.

The component is assumed to fail once its wear reaches a maximum allowable threshold, D. We define the random variable, T_L, as the component's life. Hence, the probability of the event $(T_L > t)$ is equivalent to the probability of the event $(W(t) \leq D)$, and can be expressed as

$$P(T_L > t) = P\left(W(t) \leq D\right) = P(\lambda + \omega t \leq D) = F_{W(t)}(D). \tag{17}$$

The failure time distribution is expressed as

$$\begin{aligned} F(t) &= P(T_L \leq t) \\ &= 1 - F_{W(t)}(D) \\ &= 1 - \Phi\left\{\frac{D - (\mu_\lambda + \mu_\omega t)}{\sqrt{\sigma_\lambda^2 + \sigma_\omega^2 t}}\right\}. \end{aligned} \tag{18}$$

The failure time distribution can also be expressed in terms of the three parameters that define the Bernstein distribution

$$F(t; c, \alpha, \gamma) = \Phi\left\{\frac{t - \frac{D-\mu_\lambda}{\mu_\omega}}{\sqrt{\frac{\sigma_\lambda^2}{\mu_\lambda^2} + \frac{\sigma_\omega^2}{\mu_\omega^2} t^2}}\right\} = \Phi\left\{\frac{t - c}{\sqrt{\gamma + \alpha t^2}}\right\} \tag{19}$$

where, $|t| > 0, c = (D - \mu_\lambda)/\mu_\omega, \alpha = \sigma_\omega^2/\mu_\omega^2$, and $\gamma = \sigma_\lambda^2/\mu_\lambda^2$.

The corresponding density function of the 3-parameter Bernstein distribution [20] is given as

$$f(t) = \frac{1}{\sqrt{2\pi}}\left[\frac{\gamma + c\alpha t}{(\gamma + \alpha t)^{3/2}}\right] \exp\left\{-\frac{1}{2}\left(\frac{t - c}{\sqrt{\alpha t^2 + \gamma}}\right)^2\right\} \tag{20}$$

where, $-\infty < t < \infty, c > 0, \alpha > 0, \gamma \geq 0$.

The 3-paramater Bernstein distribution is reduced to the 2-parameter Bernstein distribution (21) in the special case where the intercept of the linear model is constant (i.e., $\mu_\lambda = \lambda$, $\sigma_\lambda^2 = 0$, and $\gamma = 0$):

$$f(t) = \frac{c}{\sqrt{2\pi\alpha}} \frac{1}{t^2} \exp\left\{-\frac{1}{2\alpha}\left(1 - \frac{c}{t}\right)^2\right\} \tag{21}$$

The degradation models presented earlier in (2), and (13) are similar to the linear model used to derive the 2-parameter Bernstein distribution. Hence, the 2-parameter Bernstein distribution can be used to estimate the mean, and variance of the stochastic parameters (slopes) for these models.

2) Estimation of the Stochastic Parameters: The first step in estimating the prior distribution of the stochastic parameters is to hypothesize a functional form for the (unobserved) degradation signal. This hypothesis will be re-evaluated once we have observed one or more degradation signals. Whether the trend should be increasing or decreasing can be established by the engineering knowledge of the application. For example, the degradation of an automotive braking system is governed by the wear of the brake pads. Thus, if we consider the thickness of the pad as a measure of the level of degradation, then a linear model with a decreasing trend (negative slope) may be used to characterize the degradation process.

Next, we assume that component failure times can be characterized by a 2-parameter Bernstein distribution. We focus on the 2-parameter Bernstein because the degradation models presented in this paper assume a constant intercept. Failure time data are then used to estimate the parameters, c and α, of the Bernstein distribution using Maximum Likelihood Estimation. The parameters of the Bernstein distribution can be obtained by using

$$L = n \ln c - \frac{n}{2} \ln \alpha - \frac{n}{2} \ln(2\pi) - 2\sum_{i=1}^{n} t_i - \frac{1}{2\alpha} \sum_{i=1}^{n} \left(1 - \frac{c}{t_i}\right)^2 \tag{22}$$

where $t_i, i = 1, \ldots n$ represents the data set of failure times.

Next, we compute the partial derivatives of the likelihood function with respect to the terms c, and α. The derivatives are then set equal to zero, and solved for c, and α.

$$\frac{\partial L}{\partial c} = \frac{n}{c} + \frac{1}{\alpha} \sum_{i=1}^{n} \left[\frac{1}{t_i}\left(1 - \frac{c}{t_i}\right)\right] \tag{23}$$

$$\frac{\partial L}{\partial \alpha} = -\frac{n}{2\alpha} + \frac{1}{2\alpha^2} \sum_{i=1}^{n} \left(1 - \frac{c}{t_i}\right)^2 \tag{24}$$

The parameters c, and α are then used to evaluate the mean μ_ω, and variance σ_ω^2 of the stochastic parameters as

$$\hat{\mu}_\omega = \frac{D - \lambda}{c}, \quad \text{and} \tag{25}$$

$$\hat{\sigma}_\omega^2 = \hat{\mu}_\omega^2 \alpha \tag{26}$$

Note that $\hat{\mu}_\omega$, and $\hat{\sigma}_\omega^2$ can then be used to define the distribution of the stochastic parameter of the hypothesized degradation models (linear or exponential), with β replacing the random variable ω.

B. Second-Stage of Prognostic Methodology

In the second stage of the prognostic methodology, real-time degradation signals are used to update residual life distributions. This is performed by computing the posterior distribution of the stochastic parameters given the observed degradation signals. The updating process is achieved using the Bayesian updating methodology presented earlier in Section III, (3), and (14).

After one or more degradation signals have been observed, the characteristic patterns of these signals will be used to reject or fail to reject the initial hypothesis of the functional form. Due to the characteristics of the Bernstein distribution, the choices will be limited to the linear, and exponential degradation models. In this work, we choose the model that results in the lower mean square error between the actual degradation signal, and the functional form of the degradation model.

In the event that the hypothesized degradation model is not suitable for characterizing the degradation signal, the prior distributions are reevaluated for the new degradation model using the parameters of the Bernstein distribution. In the next section, we validate this methodology using real-world degradation data from rolling element bearings.

C. Summary of Prognostic Methodology

There are several steps involved in implementing the proposed methodology. These steps are outlined below.

Step 1) Collect a set of failure times for the component.

Step 2) Hypothesize a functional form for the degradation signal.
Step 3) Use the failure time data to calculate the parameters of the Bernstein distribution using (23), and (24).
Step 4) Given the functional form of the degradation model, linear or exponential, use the parameters of the Bernstein distribution to estimate the prior distributions of the degradation model stochastic parameters using (25), and (26), with β replacing ω. For the linear model, those statistics refer to the slope of the linear model that characterizes the amplitude of the degradation signal. But for the exponential model, those statistics refer to the slope of the linear model that characterizes the *logarithm* of the amplitude of the degradation signal.
Step 5) Use a partially observed degradation signal to estimate the variance of the error terns.
Step 6) Predict/update the residual life distribution of the first component being monitored.
Step 7) Once a complete degradation signal has been observed, conduct a hypothesis test to decide whether to reject or fail to reject the initial hypothesis regarding the functional form of the degradation model.
Step 8) If the outcome of step 7 is to fail to reject the initial hypothesis, this implies that a suitable degradation model was chosen. Otherwise, return to step 2, choose a different model, and proceed through the remaining steps as before.

Next, we demonstrate the implementation of this methodology using a rolling element bearing application.

V. Case Study: Degradation of Rolling Element Thrust Bearing

To evaluate the two-stage prognostic methodology, we evaluate the accuracy of its predictions using real-world bearing failure time data. We then compare the performance of the methodology with a conventional degradation modeling approach, which uses a degradation database to estimate the prior distributions of the stochastic model parameters.

The following section explains the experimental setup used to conduct the tests. We discuss the data acquisition system used to acquire in-situ condition-based sensory information, and the development of bearing-specific degradation signals. Finally, we present our implementation results.

A. Experimental Setup, and Accelerated Degradation Testing

Fig. 3 presents a schematic diagram of the experimental setup used to perform accelerated degradation tests on thrust ball bearings. The setup consists of four main subsystems. First, each test bearing is installed in a testing chamber that provides continuous lubrication for the test bearing. The lower race of the bearing is fixed to a stationary housing, and the upper race is fastened to a rotating shaft. Second, a lubrication system consisting of an oil reservoir, pump, and filter provides the necessary cooling and lubrication for the test bearing. Third, a closed-loop speed control system ensures that the

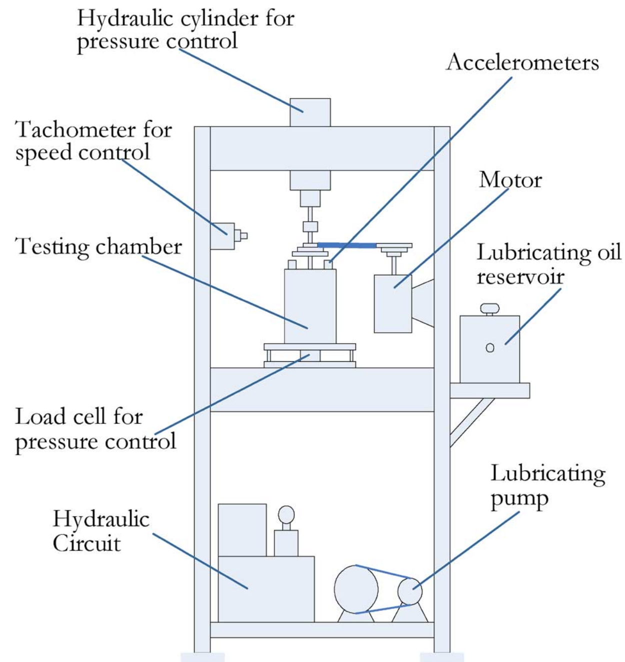

Fig. 3. Setup used for accelerated degradation testing.

shaft rotates at the designated speed throughout the duration of the test. Rotational motion is provided by the DC motor, and a tachometer measures the actual rotational speed. The motor controller, and tachometer are connected to a DAQ board installed on a PC to ensure constant rotational speed. Finally, a closed-loop load control system ensures that a constant load is maintained throughout the duration of the testing experiment. This subsystem consists of a hydraulic system that controls a hydraulic cylinder used to provide constant load. A load cell is used to measure the actual load applied on the bearing, which is then used to control the hydraulic pressure in the system, and ensure constant loading conditions.

Each test bearing consists of 10 hardened steel balls held by a copper cage between two races (upper and lower). The bearings are rated for 320 lbs according to the manufacturer's catalogue. Twenty-five bearings were tested under accelerated conditions. To facilitate acceleration, every other rolling element was removed from the cage. Consequently, the applied load is distributed among five rolling elements (steel balls) instead of ten.

During testing, each test bearing is subjected to a load of 200 lbs, and speed of 2200 rpm. The following section compares the accelerated conditions to the manufacturer's rated conditions.

1) Accelerated Loading: To evaluate the acceleration factor, we calculate the Hertzian stress at the point of contact between the rolling element, and the surface of the raceway. Note that the contact will be a point contact because the bearing raceway is flat, and the rolling element has the shape of a sphere.

The maximum contact stress (Hertz stress) is given by

$$\text{Max}\sigma_c = 0.918\sqrt[3]{\frac{P}{K_D^2 \times C_E^2}}. \tag{27}$$

$\mathrm{Max}\sigma_c$ is the maximum contact stress between the ball, and the bearing raceway; P is the load per rolling element; K_D is the ball diameter (0.156 inches for the thrust ball bearing used in the experiment); and $\mathrm{C_E}$ is the contact modulus given by

$$C_E = \frac{1-\delta_1^2}{E_1} + \frac{1-\delta_2^2}{E_2} = 6.07 \times 10^{-5}. \tag{28}$$

δ_1, and δ_2 are Poisson's ratios of the ball, and raceway materials, respectively (both equal to 0.3); and E_1, and E_2 represent the modulus of elasticity of the ball, and raceway materials, respectively (both equal to 30,000 psi).

The following analysis presents the maximum rated contact stress per ball versus the contact stress per ball induced by our experimental setup.

Case I: Maximum Contact Stress → Bearing Rating

The maximum rated load per ball for this bearing in its original configuration with 10 balls is

$$P = \frac{365\ \mathrm{lbs}}{10\ \mathrm{balls}} = 36.5\ \mathrm{lbs/ball}.$$

Substituting the values of P, C_E, and K_D in (27), we get the Maximum contact stress as $\mathrm{Max}\sigma_c = 6806$ psi.

Case II: Maximum Contact Stress → Accelerated Conditions

Under the accelerated testing conditions, the rolling elements are reduced to half (5 balls), and each bearing is subjected to a load of 200 lbs. Thus, the load per ball under accelerated testing conditions is

$$P = \frac{200\ \mathrm{lbs}}{5\ \mathrm{balls}} = 40\ \mathrm{lbs/ball}.$$

The corresponding maximum contact stress is $\mathrm{Max}\sigma_c = 7017$ psi. The acceleration factor with respect to the load is 3.1%.

2) Accelerated Rotational Speed: The maximum rotational speed associated with the thrust ball bearing is 1500 rpm (based on the manufacturer's catalogue). Under the accelerated testing conditions, each bearing is tested at 2200 rpm. The acceleration factor with respect to the rotational speed is 47%.

B. Bearing Degradation

The degradation of rolling element bearings typically begins with the formation of subsurface micro-cracks inside the raceway material. The crack propagates towards the surface of the raceway. Once the cracks reach the surface, they dislodge pieces of the raceway material causing small pits, also known as spalls, on the surface. Spall formation increases the friction between the rolling elements and bearing raceways, which is typically accompanied by increased temperature. More importantly, spall formation and propagation along the surface of the raceway results in increased levels of vibration. The passage of the rolling elements over these spalls creates a repetitive impact, which results in a bearing-specific vibration frequency. This defective frequency is a function of bearing rotational speed, number of rolling elements, bearing dimensions, and geometry. There are several types of defective frequencies related to rolling element bearings [22]:

Cage Rotating Frequency

$$FTF = \frac{1}{2}\frac{RPM}{60}\left(1 - \frac{d_r}{d_c}\cos\nu\right) \tag{29}$$

Rotational Frequency of the Rolling Elements

$$BSF = \frac{1}{2}\frac{RPM}{60}\frac{d_c}{d_r}\left(1 - \frac{d_r^2}{d_c^2}\cos^2\nu\right) \tag{30}$$

Ball-pass Frequency on the Outer Race

$$BPFO = \frac{1}{2}\frac{RPM}{60}\left(1 - \frac{d_r}{d_c}\cos^2\nu\right) \times z, \quad \text{and} \tag{31}$$

Ball-pass Frequency on the Inner Race

$$BPFI = \frac{1}{2}\frac{RPM}{60}\left(1 + \frac{d_r}{d_c}\cos^2\nu\right) \times z \tag{32}$$

where $d_c = (d_{out} + d_{in})/2$, d_{out}.

For thrust bearings, the inner, and outer races (as would be found in the radial bearing) are replaced by upper, and lower races. The raceways of the bearing are flat, thus the angle of contact, ν, between the rolling element and the rolling surface is 90°. Based on the rotational speed of the test bearing, the defective frequency is calculated as 92.5 Hz.

C. Vibration-Based Degradation Signals

Our underlying assumption is that the evolution of the degradation signal is correlated with the level of physical degradation associated with the test bearing. Two accelerometers (to ensure fidelity) are used to acquire vibration data resulting from the degradation of the test bearings. The signals are acquired by a Dynamic Signal Analysis DAQ board, NI 4452, installed on the PC. The signals are first passed through analog filters to remove any signal frequency components beyond the range of the ADC (Analog-to-Digital Converter). The signal is then passed through a digital anti-aliasing filter that automatically adjusts its cutoff frequency to remove any frequency component above half the programmed sampling rate. The data acquisition software is programmed in Labview. The software displays the vibration signal in time, and frequency domains. Acquisition is performed every two minutes.

To develop a degradation signal for the test bearing, bearing-specific frequency information is extracted from the vibration spectrum. Although there are several ways to develop a bearing-specific degradation signal, for the purpose of this study, the degradation signal is taken to be the average amplitude of the defective frequency, and its first six harmonics. The same degradation signal was used in [5], [16]. Fig. 4 presents the evolution of the average amplitude over the life cycle of one of the test bearings.

There are several prominent characteristics of the vibration-based degradation signal. First, the non-defective operation of the test bearing is represented by the steady (flat) component of the degradation signal. The instance of spall formation is characterized by a spike in amplitude. From that point onwards, the bearing operates in a partially degraded phase. This part shows an increasing trend in the amplitude of the degradation signal until the point of failure.

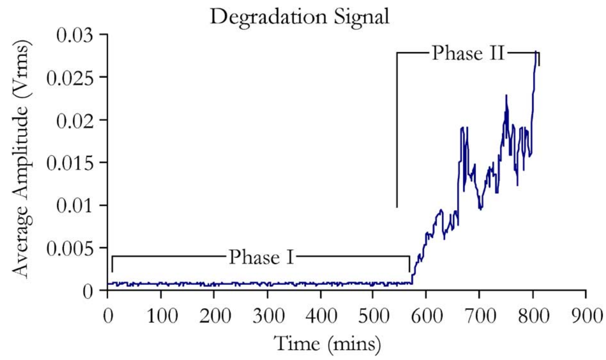

Fig. 4. Bearing degradation signal.

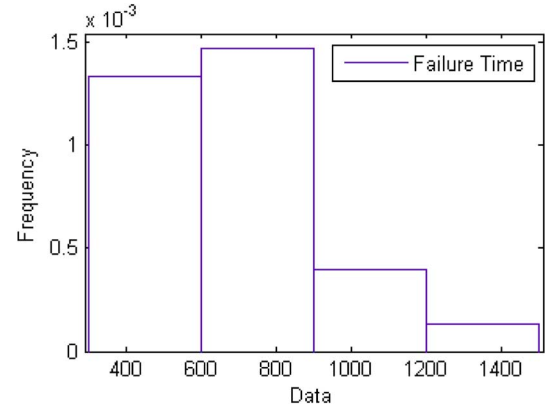

Fig. 5. Bearing failure times.

The fluctuations result from the random phenomena associated with degradation. The physical interpretation of these fluctuations is attributed to the underlying geometry of the spalls, and their propagation. Newly formed spalls have sharp edges. As rolling elements impact these edges, the vibration amplitude tends to be large.

We define bearing failure once its degradation signal crosses a predetermined failure threshold. In defining the appropriate failure threshold, we resort to ISO 2373 industrial standards for acceptable machine vibration [21].

These standards are based on the degree of precision of the application, and represent the overall Root Means Square (RMS) vibration level. For the purpose of this study, we classified our application as general-purpose machinery. Based on this classification, the danger-level (catastrophic failure) is identified as having an RMS vibration level between 2.0 to 2.2 Gs. At an overall machine vibration level of 2.2 Gs, we observed that the amplitude of all the degradation signals crossed 0.025 $\mathrm{V_{rms}}$. Consequently, 0.025 $\mathrm{V_{rms}}$ was defined as the bearing-specific failure threshold.

D. Implementation

The performance of the proposed methodology is evaluated using real world vibration-based degradation data obtained from the experimental setup discussed in the previous section. We are interested in comparing the prediction accuracy of the linear, and exponential models under two scenarios. The first scenario considers the case where there is no degradation database, and the prior distribution of the stochastic parameters are evaluated using the Bernstein failure time distribution. We refer to the models in this category as the Bernstein-Linear, and the Bernstein-Exponential degradation models. The second scenario utilizes a prior database of degradation signals, which will be used to estimate the distribution of the stochastic parameters of the degradation models. We refer to the models in this scenario as the Empirical-Linear, and Empirical-Exponential degradation models.

Twenty-five thrust ball bearings (bearings 1 to 25) are run to failure. The failure times, and degradation signals of the 25 bearings are used to estimate the distribution of the stochastic parameter in each scenario. An additional set of twenty-five bearings (bearings 26 to 50) are then used to validate the models, and compare the prediction accuracy of the Bernstein, and the Empirical approaches.

1) First Approach: Bernstein Failure-Time Distribution: As mentioned earlier, this approach assumes that there are no prior degradation signals that can model degradation, and that the only information available is historical failure times. To implement the proposed degradation methodology, we follow the steps presented in the Summary subsection of Section IV.

Step 1) Assume that the failure times follow a Bernstein distribution. Fig. 5 shows a histogram of the failure times of the first 25 bearings (bearing 1 to 25).

Step 2) Choose between modeling the degradation signal using a linear, or exponential degradation model. For the purpose of this application, assume that we choose a linear degradation model with a fixed intercept.

Step 3) The observed failure times are used to compute the coefficients of a 2-parameter Bernstein distribution using the Maximum Likelihood principle (25), and (26). Based on the data, $c = 322.38$, and $\alpha = 0.12525$.

Step 4) Given a failure threshold $D = 0.025\ \mathrm{V_{rms}}$, the mean, and variance of the stochastic parameters of the degradation model are $\hat{\mu}_\beta = 4.1634 \times 10^{-5}$, and $\hat{\sigma}^2_\beta = 2.9419 \times 10^{-10}$.

Step 5) The variance of the error terms, σ^2, is estimated using a sequence of initial observations of the degradation signal (during the degradation, phase II of Fig. 4, which has the highest level of fluctuations). This partially observed degradation signal is assumed to be from one of the validation bearings (bearing 26), and the error variance is estimated as $\hat{\sigma}^2 = 7.78006 \times 10^{-5}$.

Step 6) We use this prior information with subsequent sensory information to compute & update residual life distributions in real-time for the first component being monitored. Fig. 6 illustrates the evolution of the updated residual life distributions. Except for the error variance, the parameters of the linear degradation model have been estimated using failure time data.

Step 7) We continue monitoring the first component until the degradation signal crosses the failure threshold. A formal hypothesis test is now conducted to check

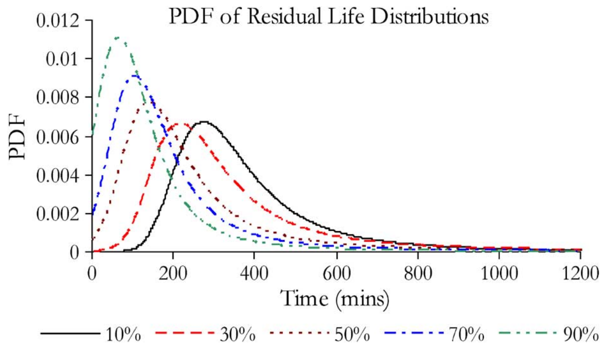

Fig. 6. Sensory-updated residual life distributions.

whether the linear functional form is suitable for modeling the degradation signal. This is done by testing the hypothesis [23]

$$H_o : b = 0$$
$$H_1 : b \neq 0 \tag{33}$$

where b is the slope of the fitted regression line. If we reject the null hypothesis, then there is statistical evidence that the slope is not equal to zero, which suggests that the linear model is suitable for modeling the degradation signal. The computed p-value $\approx$0; thus we reject the null hypothesis at the 5% significance level, with $R^2 = 0.22$.

Step 8) Based on step 7, we use the linear degradation model to make residual life predictions for 24 more validation bearings.

To assess the prediction accuracy, each time a signal is observed, the percentage difference between the actual, and predicted bearing failure times is computed using (33). The predicted failure time is equal to the time the bearing has been operating, plus the predicted residual life. We define the predicted residual life as the median of the residual life distribution. There are two reasons for choosing the median. First, the form of the residual life distributions is similar to the Bernstein distribution, and it is not possible to compute moments for this distribution [20]. Second, the distribution of the residual life is highly skewed. Under these conditions, it is common to use the median as a measure of central tendency.

$$D_j^i = \frac{(p_j^i + \widetilde{t}_j^i) - t_j^F}{t_j^F} \tag{34}$$

where D_j^i is the prediction error associated with bearing B_j evaluated at updating epoch i, t_j^F is its actual (observed) failure time, p_j^i is the total service time of B_j up to the ith updating epoch, and $\widetilde{t}_j^i$ is the updated median of the residual life distribution of B_j computed at the ith updating epoch.

For further validation, we check whether the exponential functional form with a fixed intercept is suitable for modeling the degradation signal. The hypothesis test in [33] is performed for the logarithm of the degradation signal, resulting in a p-value $\approx$0, and $R^2 = 0.25$. Although the results of the hypothesis tests fail to reject both functional forms, the higher R^2 justifies using the exponential model for more validation.

We perform the same implementation procedure using the Bernstein-exponential degradation model. Because this model operates on the logarithm of the degradation signal, we choose a very small value (less than 0.025 $\mathrm{V_{rms}}$) to represent our fixed intercept. In this implementation, we choose the value 0.001 $\mathrm{volts_{rms}}$ as the initial amplitude of the degradation signal for a new bearing. Given that the Y-intercept is ln (0.001), and the failure threshold is now ln (0.025), we can reevaluate the parameters of the prior distribution of the stochastic parameters using (24), and (25). The mean, and variance of the prior distribution is $\hat{\mu}_\beta = 0.009985$, and $\hat{\sigma}_\beta^2 = 1.2486 \times 10^{-5}$. The variance of the error terms is $\hat{\sigma}^2 = 0.45488$. Finally, the same error analysis is performed, and the results are outlined in the discussion section.

In the following section, we evaluate the prediction accuracy of the Empirical-linear, and exponential degradation models; and then compare them with the Bernstein models.

2) Second Approach: Database of Degradation Signals: In this section, we assume that historical degradation signals are available. We use a database of vibration-based degradation signals to estimate the distribution of the stochastic parameters of two different degradation models. The first has a linear functional form, whereas the second follows an exponential functional form. Because the parameters will be estimated using an existing degradation database, we will refer to these models as Empirical models.

First, we consider the Empirical-linear degradation model. We use 25 vibration-based degradation signals associated with the degradation of bearings 1-25. We fit each degradation signal with a linear regression model. We use the slopes of the twenty-five linear models to estimate the distribution of the stochastic parameters of the linear degradation model; $\hat{\mu}_\beta = 3.14752 \times 10^{-5}$, and $\hat{\sigma}_\beta^2 = 2.8343 \times 10^{-10}$ compared to $\hat{\mu}_\beta = 4.1634 \times 10^{-5}$, and $\hat{\sigma}_\beta^2 = 2.9419 \times 10^{-10}$ in the Bernstein-Linear degradation model. The residuals of the fitted regression, and the actual degradation signals are used to estimate the variance of the error terms, $\hat{\sigma}^2 = 0.00638$ compared to $\hat{\sigma}^2 = 7.78006 \times 10^{-5}$. The performance of the model is evaluated using 25 degradation signals from the validation bearings 26-50. The prediction accuracy of the Empirical-linear, and the Bernstein degradation models are shown in Fig. 7

Next, we consider the Empirical-exponential model. The database of degradation signals is also used to estimate the prior distribution of the stochastic parameter β. The prior distribution is normal with mean $\hat{\mu}_\beta = 0.006529$, and variance $\hat{\sigma}_\beta^2 = 7.7283 \times 10^{-6}$. Similar to the empirical-linear model, the residuals are used to estimate the variance of the error terms, $\hat{\sigma}^2 = 0.6251$. The twenty-five validation signals (associated with bearings 26-50) are also used to evaluate the performance of the Empirical-exponential model.

3) Discussion of Validation Results: Fig. 7 compares the performances of the Bernstein-linear, and the Empirical-linear models. The 95% confidence interval plots of the prediction errors for all 25 validation bearings at various degradation percentiles are shown for both models. To avoid overlapping intervals, the plots have been slightly indented to the left (Bernstein),

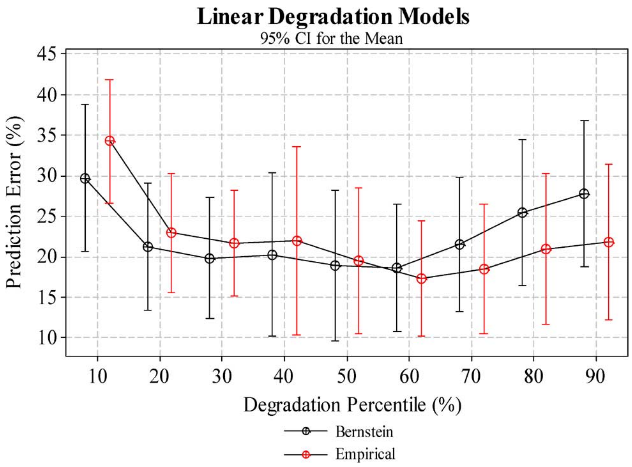

Fig. 7. Prediction error interval plots: linear degradation models.

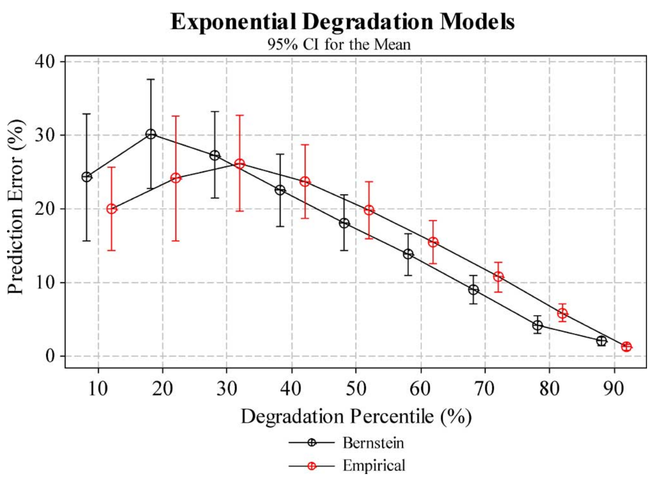

Fig. 8. Prediction error interval plots: exponential degradation models.

and to the right (empirical) of the reference grid lines for each degradation percentile. It is clear that the prediction errors are generally large for both models, indicating that the linear model is not the best choice for modeling the degradation of the rolling element thrust bearings. However, what is more important is that both types of models provided relatively similar results. The average prediction errors over all degradation percentiles are equal to 22.5% (for the Bernstein model), and 22.1% (for the Empirical model). This result can be attributed to the fact that the failure times used to estimate the stochastic model parameter for the Bernstein model represent a good fit for the Bernstein distribution (Fig. 5).

Fig. 8 compares the performances of the Bernstein-exponential and the Empirical-exponential models. The first observation is that the exponential models provide better prediction accuracy when compared to the linear models. This implies that the exponential functional form better characterizes the evolution of the bearing degradation signals. Indeed, by inspecting the shape of the degradation signal shown in Fig. 4, see that an exponential functional form is a better fit of the signal. The average prediction errors over all degradation percentiles for the Bernstein-exponential model was 16.7%, and that of the Empirical-exponential model was 16.2%. This observation demonstrated that, whether the model parameters were estimated using failure time data (Bernstein approach), or an exhaustive set of degradation signals, both approaches yielded almost similar predictions.

The results demonstrate that using the Bernstein approach to estimate the parameters of linear and exponential stochastic degradation models is a reasonable starting point for applications that do not have a database of degradation signals. Thus, if we consider applications that have been recently retrofitted with sensor technology, or in which degradation testing is very expensive, we can rely on failure time data, and use the two-stage prognostic methodology to estimate and continuously update residual life distributions using real-time sensor-based degradation signals. The main constraint of this approach is that the failure time data should reasonably fit a Bernstein distribution. The fact still remains that better accuracy will be obtained if a degradation database is used to estimate the distribution of the model parameters.

VI. Conclusion

We propose a degradation modeling framework for computing and updating residual life distributions of partially degraded components in the absence of prior knowledge about the stochastic parameters of the random coefficients degradation model. Traditionally, databases of degradation signals are used to estimate the random coefficients/stochastic parameters of these degradation models. We propose a mathematical framework that transforms failure time distributions into a form that can be used to estimate the characteristics of the stochastic parameters of the degradation models. The main benefit of this approach is that sensory-based degradation models, such as the ones presented herein, can be rapidly deployed without necessarily building a significant degradation knowledge-base.

We consider two base case models: the linear, and exponential. The models are used to compute residual life distributions of partially degraded components. The prediction accuracy of the proposed methodology is validated using real-world bearing degradation data, and then compared with the empirical approach for evaluating the stochastic parameters using a degradation database. Results indicate the feasibility of our framework. For the particular real-world application presented, our approach performed as good as the empirical approach. This suggests that the proposed methodology represents a good starting point for implementing the degradation models until a degradation database is built.

For applications where linear or exponential degradation models apply, there is need for developing additional models for cases where the failure time data do not fit the Bernstein distribution. Further developments are also required for degradation signals that do not follow either a linear or an exponential functional form.

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Nagi Gebraeel is an Assistant Professor in the Milton H. Stewart School of Industrial and Systems Engineering at Georgia Tech. Prior to joining Georgia Tech in 2007, he was an assistant professor in the Mechanical and Industrial Engineering Department at the University of Iowa. He received his MS, and PhD from Purdue University in 1998, and 2003, respectively. His research focuses on improving the accuracy of predicting unexpected failures of engineering systems by leveraging sensor-based data streams. His major research interests are in the areas of degradation modeling, and sensor-based prognostics. He is a member of IIE, and INFORMS Quality, Statistics, and Reliability Society.

Alaa Elwany is a graduate Ph.D. student in the H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology. He received his B.Sc. in Production Engineering, and M.Sc. in Industrial Engineering from Alexandria University, Egypt in 2002, and 2004, respectively. His major research interests are in the areas of Applied Statistics, Applied Operations Research (Emphasis on applications in Manufacturing and Production Systems), Reliability Engineering, Degradation Modeling, Maintenance Management, Replacement and Spare Parts Inventory Models, Quality Control, Operations Management, and Supply Chain Management.

Jing Pan is a PhD candidate in the Department of Statistics and Actuarial Science at the University of Iowa. She received her Masters from the University of Iowa in 2007. Some of her research interests include development of statistics-based degradation models.