Degradation Modeling with TTF Data

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1

Methods Used in Degradation Modeling

Overview of Degradation Modeling

Data-Driven Degradation Modeling Frameworks
- Discrete State Degradation Models
- Continuous State Degradation Models

2

Introduction to Degradation Processes

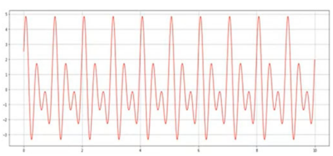
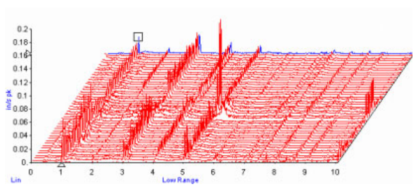
- Degradation (aka. cumulative damage) is defined as the irreversible accumulation of damage throughout the life of a component that ultimately leads to its failure or replacement.
- Unlike most conventional reliability formalisms that treat failures as a random process, degradation modeling assumes that failure is a result of a degradation process that evolves over a continuum of states.
- Degradation processes can result from chemical reactions, mechanical actions, or even a mixture of the two. Examples include corrosion, wear, fatigue, crack growth, plastic deformation, etc.

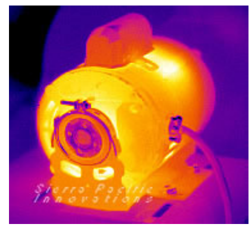
Introduction to Degradation Processes

- When it is possible to measure degradation, such measurements can provide more information than failure time data.
  - In some engineering applications, it is possible to measure physical degradation as a function of time.
  - In other applications, it may not be possible to observe the actual level of physical degradation. However, it may be possible to measure manifestations of the degradation process, e.g., power loss, increased vibration levels, higher temperature readings, etc.
  - Direct observations of the physical degradation process or some closely related surrogate may allow direct modeling of the failure causing mechanism, providing more credible and precise reliability estimates and a firmer basis for often-needed extrapolation.

Condition Monitoring

- Condition monitoring is the process of using sensor technology to monitor the degradation of engineering systems for physical and/or performance degradation.
- Raw sensor data from condition monitoring technologies can be transformed into degradation signals that are correlated with the underlying degradation process.
- Condition monitoring can be used to establish effective maintenance strategies to predict and prevent unexpected system failures.



Condition Monitoring Technologies

- There are a variety of technologies that can be used for condition monitoring; examples include
  - Thermography
  - Tribology
  - Vibration monitoring
  - Acoustic emissions
  - Current/, voltage and amperage
  - Flow rates
  - Other methods
- It is important to identify what condition monitoring technique is appropriate for each application.

Data-Driven Degradation Models

- A degradation signal is a function of condition monitoring data that captures how degradation severity evolves over time.
- If properly designed, a degradation signal can be a leading indicator of equipment failure and can be used to predict lifetime.
- Some key measure of quality of degradation signals are trend and signal-to-noise ratio (SNR).
  - Signal-to-Noise ratio often become lower under service conditions where careful control of the environment is not possible.
  - These fluctuations often lead us to rely on **probabilistic approaches** to model degradation signals.

Data-Driven Degradation Models

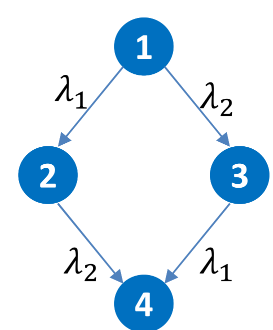
- When modeling degradation signals, there are several sources of variability;
  - **Initial state of damage:** depends on material properties, manufacturing processes, inspection techniques, installation procedures, storage procedures prior to installation.
  - **Severity and rate of degradation:** depends on usage history, material property changes during use, and environmental conditions.
  - **State of damage at failure:** degradation states at the point of failure/replacement of a component are typically spread over a wide spectrum and depend on several economic and safety considerations.

Data-Driven Degradation Models

- Developing a data-driven degradation model involves selecting a generic class of models that relate to system identification.
  - The class selected reflects our general understanding of the degradation process.
- Degradation processes involve significant variability and hence degradation models are often expected to be probabilistic.
- Statistical methods and data are used to support the selection process and aid in parameter estimation.
- **Remark:** Different investigators may focus on different generic classes. This is in sharp contrast with investigators who utilize mechanical and physical models, which tend to be more homogenous.

Overview of Markov Analysis

- Markov analysis views a system as transitioning from one state to another (according to an exponential pdf).
- The fundamental assumption in Markov analysis is that the probability that a system will undergo a transition from one state to another **depends only on the current state** and not on the previous states that the system has experienced.
- If X_n is a random variable that denotes the state of the system at time n,



$$\mathbb{P}(X_{n+1} = x_{n+1} | X_n = x_n, \dots, X_0 = x_0) = \mathbb{P}(X_{n+1} = x_{n+1} | X_n = x_n)$$

- The probability of transitioning from state n to state $n+1$ given all the historical transitions only depends on the current state n.

Overview of Markov Analysis

- Markov analysis assumes that the transition from one state to another occurs based on a constant failure rate.
  - The process is stationary, i.e., transition probabilities do not change over time.
- Consider a 2-component system with each component being in one of two states, operating or failed.
- Find the probability of the system being in each state as a function of time**.**

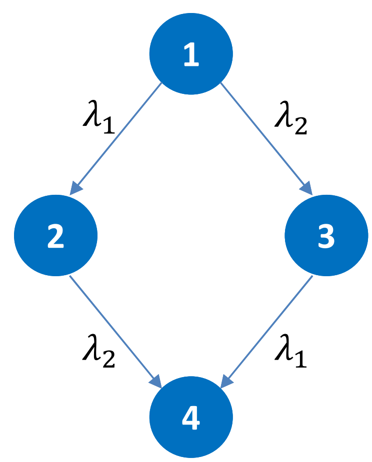
State	Component 1	Component 2
1	Operating	Operating
2	Failed	Operating
3	Operating	Failed
4	Failed	Failed

Overview of Markov Analysis

- Using the rate diagram, we can derive the following relationship.

  $$P_1(t+\Delta t) = P_1(t) - \lambda_1 \Delta t P_1(t) - \lambda_2 \Delta t P_1(t)$$

- Thus, the probability of the system being in state 1 at time $t + \Delta t$ is equal to the probability of it being in state 1 at time t **minus the probability of it being in state 1 at time t and (times) transitioning ($\lambda_i \Delta t$) to either states 2 or 3**, i.e., $(\lambda_1 \Delta t)$ and $(\lambda_2 \Delta t)$ respectively.

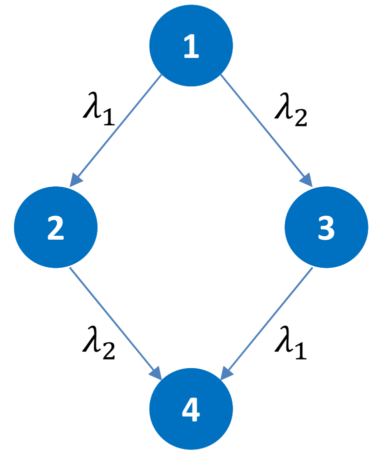


Overview of Markov Analysis

- Using the rate diagram, we can derive the following relationship.

  $P_1(t + \Delta t) = P_1(t) - \lambda_1 \Delta t P_1(t) - \lambda_2 \Delta t P_1(t)$

  - $\lambda_1 \Delta t$ is the conditional probability of a transition to state 2 occurring during time Δt given the system is currently in state 1.
  - $\lambda_1 \Delta t P_1(t)$ is the joint probability of the system being in state 1 at time t, and making a transition to state 2 during time Δt.



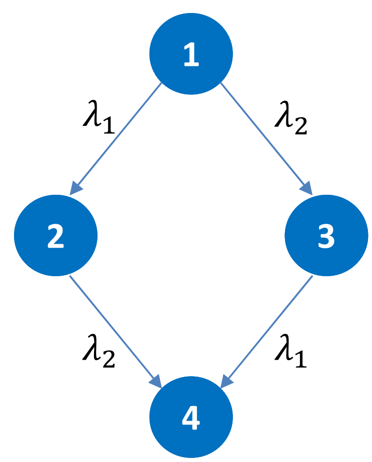
16

Overview of Markov Analysis

- Let us now consider $P_2(t + \Delta t)$, probability of the system being in state 2 at time $t + \Delta t$.

  $P_2(t + \Delta t) = P_2(t) + \lambda_1 \Delta t P_1(t) - \lambda_2 \Delta t P_2(t)$

- Translation:
  - Probability of being in state 2 at time t plus the probability of being in state 1 at time t and transitioning to state 2 at Δt, $(\lambda_1 \Delta t)$.
  - Minus the probability of being in state 2 at time t and making a transition to state 4 $(\lambda_2 \Delta t)$ at time Δt.

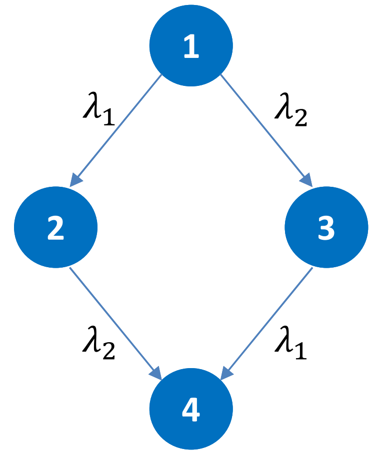


17

Overview of Markov Analysis

- Similar analysis can be derived for state 3 and 4.

$$P_3(t+\Delta t) = P_3(t) + \lambda_2 \Delta t P_1(t) - \lambda_1 \Delta t P_3(t)$$
$$P_4(t+\Delta t) = P_4(t) + \lambda_2 \Delta t P_2(t) + \lambda_1 \Delta t P_3(t)$$



18

Overview of Markov Analysis

$$P_1(t+\Delta t) = P_1(t) - \lambda_1 \Delta t P_1(t) - \lambda_2 \Delta t P_1(t)$$

- The above probability relationship can be rewritten as follows:

$$\frac{P_1(t+\Delta t) - P_1(t)}{\Delta t} = -(\lambda_1 + \lambda_2) P_1(t)$$

- Taking the limit as $\Delta t \to 0$, the above equation can be cast as a differential equation

$$\lim_{\Delta t \to 0} \frac{P_1(t+\Delta t) - P_1(t)}{\Delta t} = \frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2) P_1(t)$$

19

Overview of Markov Analysis

➢ Performing similar steps on the probability equations defining $P_2(t+\Delta t)$, and $P_3(t+\Delta t)$, we arrive at the following:

$$\frac{dP_1(t)}{dt} = -(\lambda_1+\lambda_2)P_1(t)$$
$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2 P_2(t)$$
$$\frac{dP_3(t)}{dt} = \lambda_2 P_1(t) - \lambda_1 P_3(t)$$

To obtain $P_1(t)$,

$$\frac{dP_1(t)}{dt} = -(\lambda_1+\lambda_2)P_1(t)$$

Rearranging and integrating both sides we get:

$$\ln P_1(t) = -(\lambda_1+\lambda_2)t$$
$$P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$

20

Overview of Markov Analysis

➢ To obtain $P_2(t)$, consider the following differential equation,

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2 P_2(t)$$
$$\frac{dP_2(t)}{dt} = \lambda_1 e^{-(\lambda_1+\lambda_2)t} - \lambda_2 P_2(t)$$

➢ We use an integrating factor to solve the above differential equation.
- An integrating factor is a function $\nu(x)$ by which an ordinary differential equation can be multiplied in order to make it integrable.
- $\frac{dy}{dx} + p(x)y(x) = q(x)$ can be made integrable by defining $\nu(x) = \int p(x)dx$
- where $\mathrm{e}^{\nu(x)}$ is the integrating factor
- The solution is given as $y(x) = \mathrm{e}^{-\nu(x)} \int \mathrm{e}^{\nu(x)} q(x)dx$

Overview of Markov Analysis

- For our case, we use $e^{(\lambda_2)t}$ as an integrating factor, we have

$$P_2(t)e^{(\lambda_2)t} = \lambda_1 \int e^{-(\lambda_1+\lambda_2)t}e^{(\lambda_2)t}\,dt + C$$

$$P_2(t) = e^{-(\lambda_1+\lambda_2)t} + ce^{-\lambda_2 t}$$

- Using the initial conditions $P_1(0) = 1$, $P_2(0) = 0$, and $P_3(0) = 0$ we get $c = 1$

Overview of Markov Analysis

- The solutions for the system of differential equation is found as,

$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2)P_1(t) \qquad P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2 P_2(t) \qquad P_2(t) = e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

$$\frac{dP_3(t)}{dt} = \lambda_2 P_1(t) - \lambda_1 P_3(t) \qquad P_3(t) = e^{-\lambda_1 t} - e^{-(\lambda_1+\lambda_2)t}$$

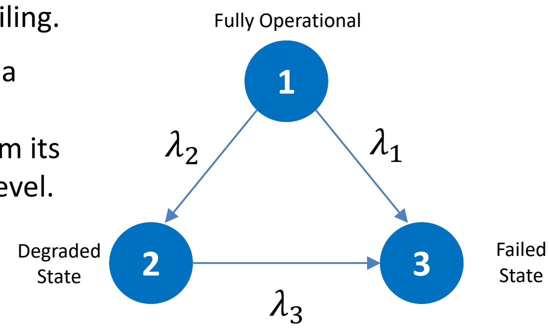
$$P_4(t) = 1 - P_1(t) - P_2(t) - P_3(t)$$

- Thus for a series system, we have $R_s(t) = P_1(t) = e^{-(\lambda_1+\lambda_2)t}$
- For a parallel system, we have $R_P(t) = P_1(t) + P_2(t) + P_3(t)$

$$R_P(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

Markov analysis for Degraded Systems

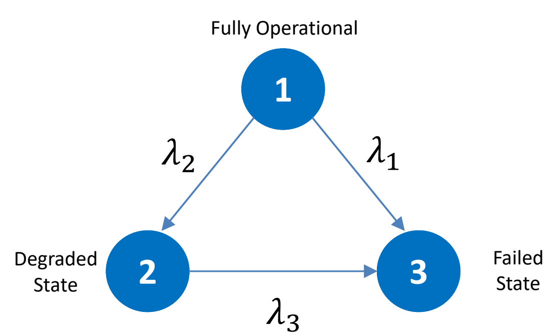
- Systems often degrade before completely failing.
- In fact, most systems continue to operate in a degraded mode following certain types of failures. The system may continue to perform its function but not at the specified operating level.



$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2)P_1(t)$$
$$\frac{dP_2(t)}{dt} = \lambda_2 P_1(t) - \lambda_3 P_2(t)$$

Degraded Systems

- The solution to the previous differential equations is as follows:



$$P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$
$$P_2(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_3}\left[e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t}\right]$$
$$P_3(t) = 1 - P_1(t) - P_2(t)$$
$$R(t) = P_1(t) + P_2(t)$$
$$MTTF = \int_0^\infty P_1(t) + P_2(t)\,\mathrm{dt} = \frac{1}{\lambda_1 + \lambda_2} + \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_3}\left[\frac{1}{\lambda_3} - \frac{1}{\lambda_1 + \lambda_2}\right]$$

Example: Degraded Systems

- A manufacturing machine experiences complete failures at a constant rate of 0.01 per day. However, the machine may degrade randomly, producing substandard parts (out of tolerances) at a constant rate of 0.05 per day. Once it has degraded, it will fail completely at a constant rate of 0.07 per day.
- What is the probability that the machine is in each of the three main states fully functional, degraded, and failed states over a one-day operation period?

$$P_1(t) = e^{-(\lambda_1+\lambda_2)t} = e^{-(0.01+0.05)t} \rightarrow P_1(1) = 0.942 \qquad P_3(1) = 0.011$$

$$P_2(t) = \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_3}\left[e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t}\right]$$

$$= \frac{0.05}{0.01+0.05-0.07}\left[e^{-0.07\times t} - e^{-(0.06)t}\right] \rightarrow P_2(1) = 0.047$$

Example: Degraded Systems

- A manufacturing machine experiences complete failures at a constant rate of 0.01 per day. However, the machine may degrade randomly, producing substandard parts (out of tolerances) at a constant rate of 0.05 per day. Once it has degraded, it will fail completely at a constant rate of 0.07 per day.
- What is the MTTF of the system?

$$MTTF = \frac{1}{\lambda_1+\lambda_2} + \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_3}\left[\frac{1}{\lambda_3} - \frac{1}{\lambda_1+\lambda_2}\right] = 28.6 \text{ days}$$

Example: Degraded Systems

- One aspect that would be of interest is the mean number of days the machine would operate in the degraded state until it fails.
- To calculate this, we need to focus on the rate that the system would transition from state 2 to state 3, i.e., 0.07 per day.
- Since $MTTF = 1/\lambda$, then $MTTF = \frac{1}{0.07} = 14.3$ days
- Such information can be valuable for a predictive maintenance program. Thus, one the machine begins to produce substandard part, we would be able to predict that the machine has approximately 14 days before it fails.
- Thus, maintenance would be needed within the 14 day period.

Example: Degraded Systems

- Given the 3 different rates, what is the average number of days that a machine would spend in a perfectly functioning state, i.e., state 1.

$$\int_0^\infty P_1(t)\,\mathrm{d}t = \int_0^\infty e^{-(\lambda_1+\lambda_2)t}\,\mathrm{d}t = \frac{1}{\lambda_1+\lambda_2}$$
$$= \frac{1}{0.01+0.05}$$
$$= 16.67 \text{ days}$$

- In other words, 16.67 days is the mean number of days the machine will spend in state 1 before either failing or becoming degraded.