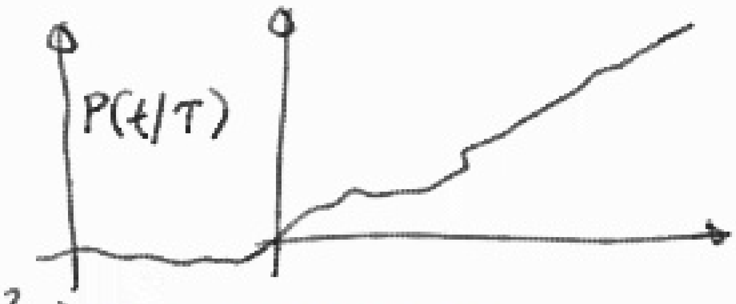
Bayesian Degradation Modelling — M8

Linear prognostic degradation model

Single parameter linear model



$S(t) = \phi + \theta t + \varepsilon(t)$

- $\phi = S(0)$
- θ is stochastic, $\pi(\theta) \sim N(\mu_0, \sigma_0^2)$ ← Historical Data

- $\varepsilon(t), t > 0 \sim N(0, \sigma^2)$, iid
- let $S_j = S(t_j)$
- We observe $S_1, \dots, S_k$ to update model & prediction
- Use Bayesian to estimate θ

- Note that given θ, $S_1, \dots, S_k$ are independent due to iid error terms

$$f(S_1, \dots, S_k | \theta) = \prod_{i=1}^{k} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\left(\frac{(S_i - \theta t_i - \phi)^2}{2\sigma^2} \right) \right\}$$

- find the posterior dist of θ as $p(\theta | S_1, \dots, S_k) \propto f(S_1, \dots, S_k)\, \pi(\theta)$

$$p(\theta | S_1, \dots, S_k) \propto \prod_{i=1}^{k} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\left(\frac{(S_i - \theta t_i - \phi)^2}{2\sigma^2} \right) \right\} * \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left\{ -\left(\frac{(\theta - \mu_0)^2}{2\sigma_0^2} \right) \right\}$$

$$\propto \exp\left\{ \frac{-1}{2\sigma^2\sigma_0^2} \left(\sigma_0^2 \sum_{i=1}^{k} t_i^2 + \sigma^2 \right) * \left[\theta^2 - 2\theta \left(\frac{\sigma_0^2 \sum_{i=1}^{k} (S_i - \phi) t_i + \mu_0 \sigma^2}{\sigma_0^2 \sum_{i=1}^{k} t_i^2 + \sigma^2} \right) \right] \right\}$$

$$\mu_p = \frac{\sigma_0^2 \sum_{i=1}^{k} (S_i - \phi) t_i + \mu_0 \sigma^2}{\sigma_0^2 \sum_{i=1}^{k} t_i^2 + \sigma^2}$$

$$\propto \exp\left\{ \frac{-1}{2\sigma_p^2} (\theta - \mu_p)^2 \right\}, \quad \sigma_p^2 = \frac{\sigma^2 \sigma_0^2}{\sigma_0^2 \sum_{i=1}^{k} t_i^2 + \sigma^2}$$

Distribution of remaining life $\quad$ T denotes distribution of remaining life

$$S(T+t_k) = \phi + \theta(T+t_k) + \varepsilon(T+t_k) \approx D_2$$

$F_{T|S_1,\dots,S_k}(t)$ $\quad$ conditional CDF of T

$$= P\{T \le t \mid S_1,\dots,S_k\} \approx 1 - P\{\theta(\overset{N}{t}+t_k) + \varepsilon(\overset{N}{t}+t_k) < D_2 - \phi \mid S_1,\dots,S_k\}$$

$$= P\left\{Z \geq \frac{D_2 - \phi - \mu_p(t+t_k)}{\sqrt{(t+t_k)^2\sigma_p^2 + \sigma^2}}\right\} = P(Z \le g(t)) = \Phi(g(t))$$

$$P(T \le t \mid S_1,\dots,S_k) = \frac{\Phi(g(t)) - \Phi(g(0))}{1 - \Phi(g(0))}$$

Given the posterior distribution of θ, we can find $S(t)$ for $t > k$

Since $S(t) = \phi + \theta t + \varepsilon(t)$

$$S(t) \mid S_1,\dots,S_k \sim N(\phi + \mu_p t,\ t^2\sigma_p^2 + \sigma^2)$$

$$f(S(t) \mid S_1,\dots,S_k) = \int f(S(t) \mid \theta, S_1,\dots,S_k)\, f(\theta \mid S_1,\dots,S_k)\, d\theta$$

Exponential Degradation Model w/ Brownian Error Term

$$L(t) - \theta' - \beta' t = \varepsilon(t)$$

$$\Delta(L(t) - \theta' - \beta' t) \sim N(0, \sigma^2) \quad \text{iid increments}$$

$$L_i = L(t_i) - L(t_{i-1})$$

$$\varepsilon(t_i) - \varepsilon(t_{i-1}) \sim N$$