Supplementary Material

Theorem 2. $f(x) \geq 0, g(x) \equiv 1, x \in [a,b], t > 0$. *Define*

$$\mu_t \triangleq \frac{[\int_a^b f^t(x) g^t(x) dx]^2}{[\int_a^b f^{2t}(x)dx][\int_a^b g^{2t}(x)dx]}$$

$$Conclusion : \frac{d\mu_t}{dt} \leq 0$$

Proof.

$$when \quad g(x) \equiv 1 \Rightarrow \mu_t = \frac{1}{b-a} \frac{[\int_a^b f^t(x)dx]^2}{\int_a^b f^{2t}(x)dx}$$

$$\frac{d\mu_t}{dt} = \frac{2\int_a^b f^t dx [\int_a^b \ln f \cdot f^t dx \int_a^b f^{2t} dx - \int_a^b f^t dx \int_a^b \ln f \cdot f^{2t} dx]}{(b-a)[\int_a^b f^{2t}(x)dx]^2}$$

we just need to prove

$$\int_a^b \ln f \cdot f^t dx \int_a^b f^{2t} dx - \int_a^b f^t dx \int_a^b \ln f \cdot f^{2t} dx \leq 0$$

we can get

$$\begin{aligned}
&\int_a^b \ln f(x) \cdot f^t(x) dx \int_a^b f^{2t}(x) dx - \int_a^b f^t(x) dx \int_a^b \ln f(x) \cdot f^{2t}(x) dx \\
&= \int_a^b \ln f(x) \cdot f^t(y) dy \int_a^b f^{2t}(y) dy - \int_a^b f^t(x) dx \int_a^b \ln f(x) \cdot f^{2t}(x) dx \\
&= \int_a^b \int_a^b f^t(x) f^t(y) \ln f(x) (f^t(y) - f^t(x)) dx dy && (1*) \\
&= \int_a^b \int_a^b f^t(x) f^t(y) \ln f(y) (f^t(x) - f^t(y)) dx dy && (2*) \\
&= \int_a^b \int_a^b f^t(x) f^t(y) (\ln f(y) - \ln f(x)) (f^t(x) - f^t(y)) dx dy && \frac{(1*) + (2*)}{2}
\end{aligned}$$

obviously, we get

$$(\ln f(y) - \ln f(x)) \cdot (f^t(x) - f^t(y)) \leq 0 \Rightarrow \frac{d\mu_t}{dt} \leq 0.$$

This finishes the proof. □