Round5: a KEX based on learning with rounding over the rings

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Areas I have been working on

- Theoretical results
  - Signature schemes: Falcon (NTRUSign+GPV), pqNTRUSign
  - Security proofs: Computational R-LWR problem
  - Fully homomorphic encryptions
  - Raptor: lattice based linkable ring signature (Blockchains!)
- Practical instantiations
  - NTRU, Round5
  - Cryptanalysis and parameter derivation for lattices
  - Efficient implementations: AVX-2
  - Constant time implementations
- Standardization: NIST, IETF, ETSI, ISO, CACR PQC process
- Deployment: enabling PQC for TLS, Tor, libgcrypt
- Under the radar: lattice based DAA, NIZK

This talk

- Theoretical results
  - Signature schemes: Falcon (NTRUSign+GPV), pqNTRUSign
  - Security proofs: Computational R-LWR problem
  - Fully homomorphic encryptions
  - Raptor: lattice based linkable ring signature (Blockchains!)
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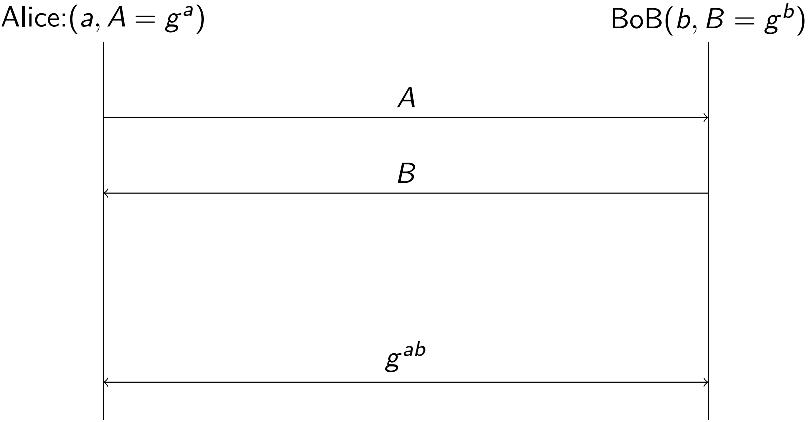
Diffie-Hellman



- A, B and g^{ab} are group elements over $\mathbb{Z}_q^*$.

Alice:$(a, A = Ga + e_a)$ BoB$(b, B = Gb + e_b)$

$$\xrightarrow{A}$$

$$\xleftarrow{B}$$

$$\xleftrightarrow{aGb + ae_b \approx bGa + be_a}$$

- Every element is a ring element over $\mathcal{R} := \mathbb{Z}_q[x]/f(x)$.

RLWE-KEX

Alice:$(a, X = Ga + e_a)$ BoB$(b, B = Gb + e_b)$

$$\xrightarrow{\quad A \quad}$$

$$\xleftarrow{\quad B \quad}$$

$$\xleftarrow{\quad \text{Reconciliation } r \quad}$$

$$\textsc{Extract}(aGb + ae_b, r) = \textsc{Extract}(bGa + be_a, r)$$

- Every element is a ring element over $\mathcal{R} := \mathbb{Z}_q[x]/f(x)$.

NTRU-KEM

Alice:$(a = 2t + 1, A = 2g/a)$ BoB:(b, m)

$$\xrightarrow{\quad A \quad}$$

$B = 2bA + m$

$$\xleftarrow{\quad B \quad}$$

$$\begin{aligned} m = Ba &= 2bg + ma \\ &= m \bmod 2 \end{aligned}$$

$$\xleftrightarrow{\quad \text{m} \quad}$$

- Every element is a ring element over $\mathcal{R} := \mathbb{Z}_q[x]/(x^N - 1)$.

NTRU-KEM vs RLWE-KEX

	NTRU	R-LWE
Ring	$\mathbb{Z}_q[x]/(x^N - 1)$	$\mathbb{Z}_q[x]/(x^N + 1)$
Provable security	No	Yes
Secrets	Trinary: $\{-1, 0, 1\}^{\text{dim}}$	Gaussian: $\chi_{\sqrt{q}}^{\text{dim}}$
Errors	Rounded, binary	Gaussian: $\chi_{\sqrt{q}}^{\text{dim}}$
Trapdoor	Yes	No
KeyGen	Slow	Fast
CT size	Small	Large

Major concerns on NTRU

- No provable security
- Slow key generation c.f. New Hope

NTRU-KEM vs RLWE-KEX

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- RLWE is hard for any ring of integers [PRS17]

NTRU-KEM vs RLWE-KEX

	NTRU	R-LWE
Ring	$\mathbb{Z}_q[x]/(\phi_N(x))$	$\mathbb{Z}_q[x]/(x^N+1)$
Provable security	Yes(?)	Yes
Secrets	Trinary: $\{-1,0,1\}^{\text{dim}}$	Gaussian: $\chi_{\sqrt{q}}^{\text{dim}}$
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- RLWE is hard for any ring of integers [PRS17]
- Hardness of dec R-LWR is an open problem (more on this later)

The new *NTRU*, a.k.a. R-LWR-KEX

Alice:$(a, A = Round(Ga))$ BoB$(b, B = Round(Gb))$

$\xrightarrow{\quad A \quad}$

$\xleftarrow{\quad B \quad}$

$\xleftarrow{\text{Reconciliation } r}$

$\textsc{Extract}(Round(aB), r) = \textsc{Extract}(Round(bA), r)$

- a, b are ring elements over $\mathcal{R} := \mathbb{Z}_q[x]/f(x)$;
- A, B are rounded over $\mathbb{Z}_p[x]$.

R-LWR-KEX

Improvements

- Prime cyclotomic ring, i.e., $\phi_{743}(x) = (x^{743} - 1)/(x - 1)$
- Rounding instead of errors

"*Disadvantages*"

- Parameters not compatible with number theoretic transform (NTT)
- Noise dependency

Improvements I - PC ring

Prime Cyclotomic ring

- PC ring as secure as power-of-2 cyclotomics;
  - i.e., $\phi_{2048}(x) = x^{1024} + 1$;
- Degree $\approx$ 700 offers enough security against BKZ attacks with quantum sieving;
- NewHope - has to be 512, 1024, etc.;
- Kyber - a multiple of 256;
- PC - any prime > 700;
  - Also used in LIMA, NTRU-KEM, etc.


ONE
RING
TO
RULE
THEM
ALL

Improvements II - Rounding

Rounding

- Less randomness sampling - e_a and e_b;
- Ciphertext reduced to $n \log p$, c.f. $n \log q$;
- Small enough to be in an MTU for TLS;
- Introduces new assumptions.

"*Disadvantages*" I - Ring multiplications

Rule of Thumb

School book $\gg$ Karatsuba/Toom-Cook $\gtrapprox$ NTT $>$ Index based

"*Disadvantages*" I - Ring multiplications

Rule of Thumb

School book $\gg$ Karatsuba/Toom-Cook $\gtrapprox$ NTT $>$ Index based

Karatsuba and Toom-Cook

- Divide and Conquer;
- Parameter dependent optimizations;
  - Improving NTRU-743 reference implementation by 2.3x;
- Constant time; strong side channel resistance;
- Slightly slower than NTT for similar N.

"*Disadvantages*" I - Ring multiplications

Rule of Thumb

School book $\gg$ Karatsuba/Toom-Cook $\gtrapprox$ NTT $>$ Index based

Index based

- Super friendly with a trinary polynomial;
- Even faster than NTT;
- Constant time iff $\textsc{Ham}(a)$ and $\textsc{Ham}(b)$ are constant;
- Memory leakage.

"*Disadvantages*" II - Noise management

Rational: Use ECC to control errors

- Consider $c(x) = a(x)b(x) \bmod x^{N-1} + x^{N-2} + \cdots + 1$
- Let $c'(x) = a(x)b(x) \bmod (x^N - 1)$, then $c'(x) = c(x) \bmod \phi_N(x)$
- $\Rightarrow c_i = c'_i - c'_N$
  - Noise, i.e., $\|xe_y\|_\infty$ is "doubled";
  - Every coefficient is "lifted" by c'_N - creates dependency;
- ECC doesn't work on dependent errors.

"*Disadvantages*" II - Noise management

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- ECC doesn't work on dependent errors.

Solution - ring switching

- multiply over $\phi_N(x)$, lift the final results to $x^N - 1$ ring;
- $c'(x) = c(x)(x-1)$
- Only use the coefficients of $1, x^2, x^4, x^6, \ldots$
- Security? $\mathbb{Z}[x]/\phi_N(x) \cong \mathbb{Z}[x]/(x^N - 1) \cap \{\text{Poly with root } 1\}$

Round2

The team

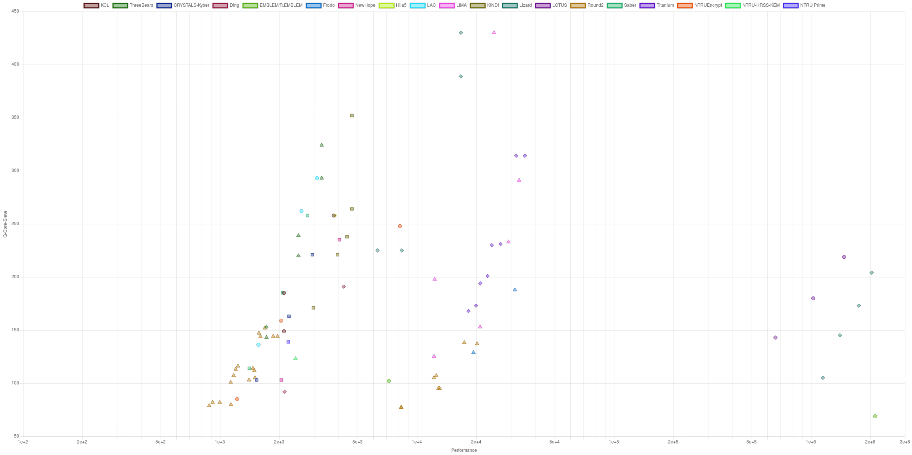
- Philips: Hayo Baan, Sauvik Bhattacharya, Oscar Garcia-Morchon, Ronald Riemann, Ludo Tolhuizen, Jose Luis Torre Arce
- OnBoard Security: Zhenfei Zhang

Round5 = Round2 + HILA5's ECC

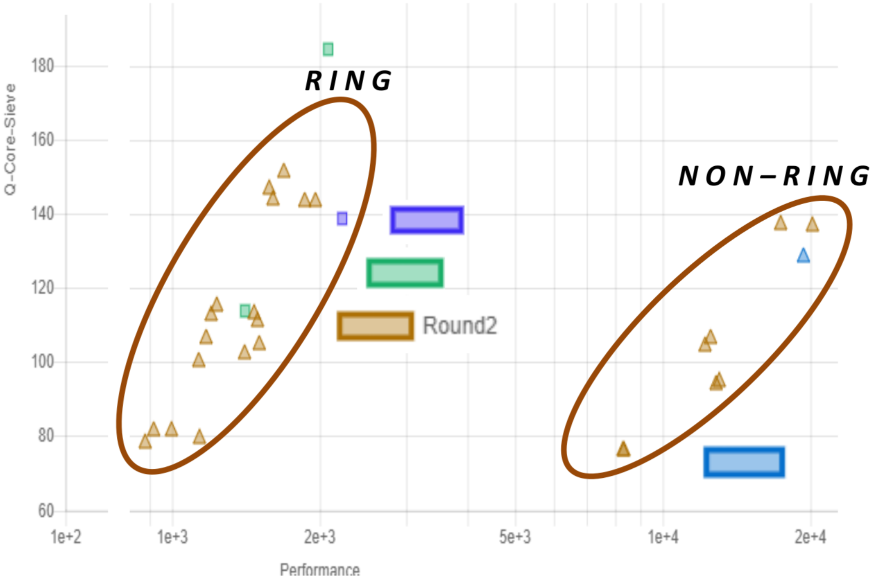
The team

- Philips: Hayo Baan, Sauvik Bhattacharya, Oscar Garcia-Morchon, Ronald Riemann, Ludo Tolhuizen, Jose Luis Torre Arce
- Cisco: Scott Fluhrer
- Rambus: Mike Hamburg
- TU/e: Thijs Laarhoven
- PQShield: Markku-Juhani Olavi Saarinen
- OnBoard Security: Zhenfei Zhang

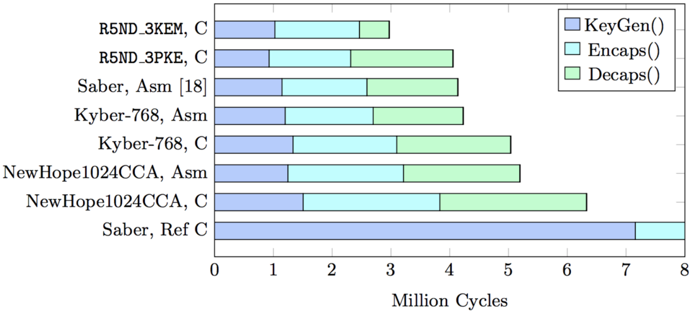
Performance



Performance



Performance



Deployment: TLS

```
Client                                          Server

ClientHello
ClientKeyShare            -------->
                          <--------      HelloRetryRequest

ClientHello
ClientKeyShare            -------->
                                              ServerHello
                                           ServerKeyShare
                                   {EncryptedExtensions*}
                                           {Certificate*}
                                   {CertificateRequest*+}
                                     {CertificateVerify*}
                          <--------            {Finished}
{Certificate*+}
{CertificateVerify*+}
{Finished}                -------->
[Application Data]        <------->      [Application Data]

      * message is not sent under some conditions
      + message is not sent unless client authentication
        is desired
```

Deployment: Hybrid solutions

```
INTERNET-DRAFT                                              W. Whyte
Intended Status: Experimental                    Security Innovation
Expires: 2017-XX-YY                                         Z. Zhang
                                                 Security Innovation
                                                          S. Fluhrer
                                                       Cisco Systems
                                                  O. Garcia-Morchon
                                                             Philips
                                                          2017-03-31


               Quantum-Safe Hybrid (QSH) Key Exchange
           for Transport Layer Security (TLS) version 1.3
                    draft-whyte-qsh-tls13-04.txt

  Client                                                  Server

  ClientHelloExtensions
  + qshDataExtension
    (QSHPKList)
  + qshNegotiateExtension
    (QSHSchemeIDList)        -------->
                                   HelloRetryRequestExtensions
                                       + qshNegotiateExtension
                             <-------- (AcceptQSHSchemeIDList)

  ClientHelloExtensions
  + qshDataExtension

    (QSHPKList)              -------->
                                          EncryptedExtensions*
                                            + qshDataExtension
                                               (QSHCipherList)
                             <--------              {Finished}
  {Finished}                 -------->

  ClassicSecret|QSHSecret    <-------> ClassicSecret|QSHSecret

        * previously known as SeverKeyShareExtensions
```

ARE WE DONE YET?

Another look at the security

$$
\begin{aligned}
\text{Round5} &\Rightarrow \text{Dec R-LWR over PC} \\
&\Rightarrow \text{Search R-LWE over PC} \\
&\Rightarrow \text{Search R-LWE over any ring} \\
&\Rightarrow \text{BDD over Ideal Lattices}
\end{aligned}
$$

Another look at the security

Round5 $\Rightarrow$ Dec R-LWR over PC

$\Rightarrow$ Search R-LWE over PC

$\Rightarrow$ Search R-LWE over any ring

$\Rightarrow$ BDD over Ideal Lattices

What about the hardness of Dec R-LWR?

Dec R-LWE as hard as search R-LWE

- Given $(a, b = as + e) \in \mathcal{R}^2$;
- Increase the first coefficient of b gradually and call Dec R-LWE oracle;
- Oracle will keep returning true till overflow - this tells us e_0;
- Repeat for all coefficients to learn e;
- Extract s from a noisy free sample $b' = as$.

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Dec R-LWR?

- We can't modify e - it is deterministic.

Our approach

Intuition

Search Problem $\geq$ Computational Problem $\geq$ Decisional Problem

- Computational Diffie-Hellman: given $\{g, g^x, g^y\}$, find g^{xy};
- Similarly, given $\{a, Round(as_1), Round(as_2)\}$, find $Round(as_1 s_2)$;
- This is the underlying problem for R-LWR-KEX
    - Dec R-LWR problem isn't essential.

The whole picture

Computational R-LWR $(a, b = Round(as))$

$\Rightarrow$ Computational Rounded R-LWE $(a, b = Round(as + e))$

$\Rightarrow$ Search R-LWE $(a, b = as + e)$

	R-LWE	R-LWR
Samples - KeyGen	2	1
Samples - Encrypt	3	1
Sampler	Gaussian	Uniform & Invertible
Modulus	$\Omega(n^{5.5} \log^{0.5} n)$	$\Omega(n^{3.75} \log^{0.25} n)$

Table: Performance comparison

If I have an unlimited fund

- Lattice based zero knowledge proofs;
- Lattice based group signatures;
- Sieving algorithms.


OBRIGADO
gracias
どうも
ARIGATO
hvala
DANK U
takk
MERSI
merci
謝謝
obrigado
danke schön
köszi
PALDIES
muchas gracias
ありがとう
TEŞEKKÜR EDERIM
MOLTE GRAZIE
GO RAIBH MAITH AGAT
danke
grazas
GRAZZI
THANK YOU
благодаря
TAK
どうも
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qujan
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muchas gracias
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DZIEKI
TEŞEKKÜR EDERIM
muchas gracias
obrigado
спасибо
MULTUMESC
多謝
NA GODE

Backup materials: Lattice basics

Figure source: Wendy Cordero's High School Math Site

Lattice

Definition of a Lattice

- All the integral combinations of $d \leq n$ linearly independent vectors over $\mathbb{R}$

$$\mathcal{L} = \mathbb{Z}\,\boldsymbol{b}_1 + \cdots + \mathbb{Z}\,\boldsymbol{b}_d = \{\lambda_1 \boldsymbol{b}_1 + \cdots + \lambda_d \boldsymbol{b}_d \;:\; \lambda_i \in \mathbb{Z}\}$$

- d dimension.
- $\boldsymbol{B} = (\boldsymbol{b}_1, \ldots, \boldsymbol{b}_d)$ is a *basis*.

An example

$$\boldsymbol{B} = \begin{pmatrix} 5 & \frac{1}{2} & \sqrt{3} \\ \frac{3}{5} & \sqrt{2} & 1 \end{pmatrix}$$

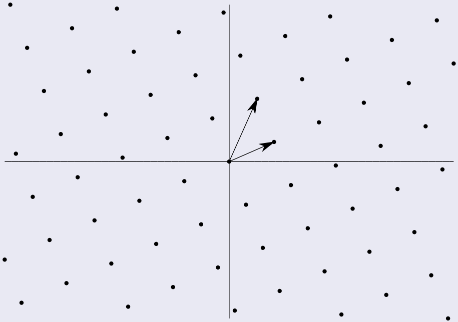
- $d = 2 \leq n = 3$
- In this talk, full rank integer Basis: $\boldsymbol{B} \in \mathbb{Z}^{n,n}$.

Example

A lattice $\mathcal{L}$

$$\boldsymbol{B} = \begin{pmatrix} 8 & 5 \\ 5 & 16 \end{pmatrix}$$

All lattice crypto talks start with an image of a dim-2 lattice

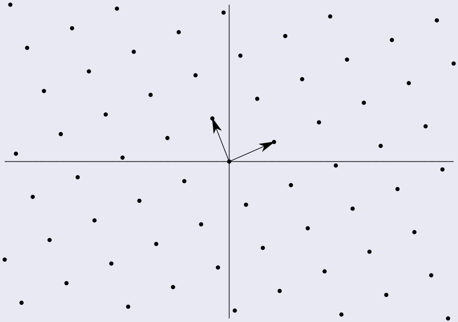


Example

A lattice $\mathcal{L}$

$$\boldsymbol{UB} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 8 & 5 \\ 5 & 16 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -3 & 11 \end{pmatrix}$$

An infinity of basis

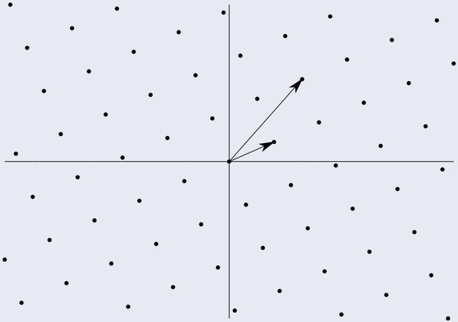


Example

A lattice $\mathcal{L}$

$$\boldsymbol{UB} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 8 & 5 \\ 5 & 16 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ 13 & 21 \end{pmatrix}$$

An infinity of basis

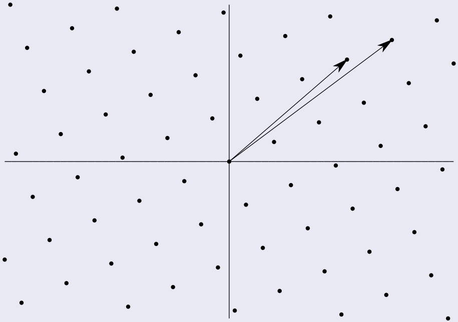


Example

A lattice $\mathcal{L}$

$$\boldsymbol{UB} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 8 & 5 \\ 5 & 16 \end{pmatrix} = \begin{pmatrix} 29 & 31 \\ 21 & 26 \end{pmatrix}$$

An infinity of basis

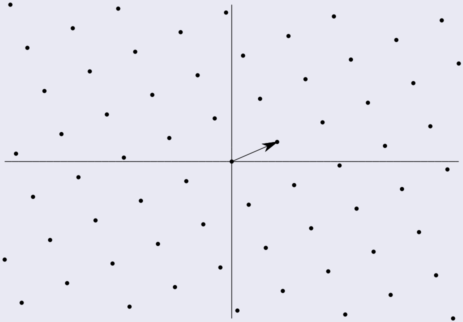


Example

The Shortest Vector and The First Minima

$$\boldsymbol{v} = \begin{pmatrix} 8 & 5 \end{pmatrix}, \text{ with } \lambda_1 = \sqrt{8^2 + 5^2} = 9.434$$

The Shortest Vector

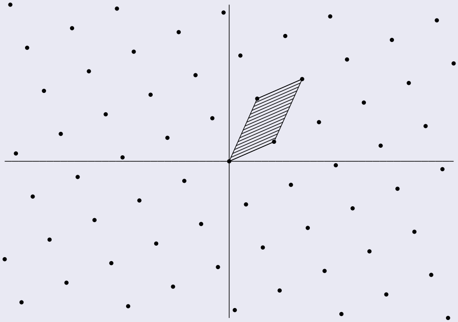


Example

The Determinant

$$\det \mathcal{L} = \sqrt{\det \left(\boldsymbol{B}\boldsymbol{B}^T \right)} = 103$$

The Fundamental Parallelepiped



NTRU lattice

NTRU ring

- Originally: $\mathbb{Z}_q[x]/(x^N - 1)$, q a power of 2, N a prime;
- Alternative 1: $\mathbb{Z}_q[x]/(x^N - x - 1)$, q a prime;
- Alternative 2: $\mathbb{Z}_q[x]/(x^N + 1)$, q a prime, N a power of 2

NTRU lattice

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Ring multiplications: $h(x) = f(x) \cdot g(x)$

- Compute $h'(x) = f(x) \times g(x)$ over $\mathbb{Z}[x]$
- Reduce $h'(x)$ mod $(x^N - 1)$ mod q

NTRU lattice

NTRU ring

- Originally: $\mathbb{Z}_q[x]/(x^N - 1)$, q a power of 2, N a prime;
- Alternative 1: $\mathbb{Z}_q[x]/(x^N - x - 1)$, q a prime;
- Alternative 2: $\mathbb{Z}_q[x]/(x^N + 1)$, q a prime, N a power of 2

Ring multiplications: $h(x) = f(x) \cdot g(x)$, alternatively

$$\langle h_0, \dots, h_{N-1} \rangle = \langle f_0, \dots, f_{N-1} \rangle \times \begin{bmatrix} g_0 & g_1 & g_2 & \cdots & g_{N-1} \\ g_{N-1} & g_0 & g_1 & \cdots & g_{N-2} \\ g_{N-2} & g_{N-1} & g_0 & \cdots & g_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_1 & g_2 & g_3 & \cdots & g_0 \end{bmatrix} \bmod q$$

NTRU lattice

NTRU assumption

- Decisional: given two small ring elements f and g; it is hard to distinguish $h = f/g$ from a uniformly random ring element;
- Computational: given h, find f and g.

NTRU lattice

NTRU assumption

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NTRU lattice

$$\begin{bmatrix} qI_N & 0 \\ H & I_N \end{bmatrix} := \begin{bmatrix} q & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & q & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q & 0 & 0 & \dots & 0 \\ h_0 & h_1 & \dots & h_{N-1} & 1 & 0 & \dots & 0 \\ h_{N-1} & h_0 & \dots & h_{N-2} & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_1 & h_2 & \dots & h_0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

NTRU lattice

NTRU assumption

- Decisional: given two small ring elements f and g; it is hard to distinguish $h = f/g$ from a uniformly random ring element;
- Computational: given h, find f and g.

NTRU lattice $\mathcal{L} = \begin{bmatrix} qI_N & 0 \\ H & I_N \end{bmatrix}$

- $\langle g, f \rangle$ (and its cyclic rotations) are unique shortest vectors in $\mathcal{L}$;
- Decisional problem: decide if $\mathcal{L}$ has unique shortest vectors;
- Computational problem: find those vectors.
- Both are hard for random lattices.

NTRU lattice

The real NTRU assumption

- NTRU lattice behaves the same as random lattices.

NTRU lattice $\mathcal{L} = \begin{bmatrix} qI_N & 0 \\ H & I_N \end{bmatrix}$

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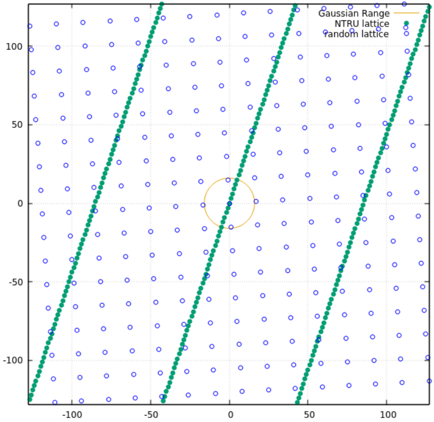
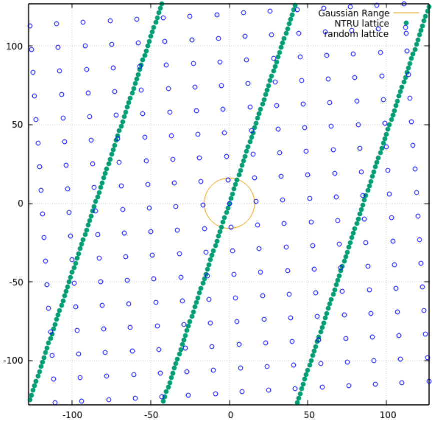
NTRU lattice vs random lattice



$$\begin{pmatrix} 256 & 0 \\ 172 & 1 \end{pmatrix} \qquad \begin{pmatrix} 256 & 0 \\ 17 & 1 \end{pmatrix}$$

$$(g, f) = (1, 3) \qquad v = (17, 1)$$

NTRU lattice vs random lattice



- Random lattice, SV $\approx$ Gaussian Heuristic length $= \sqrt{\frac{\dim}{2\pi e}} \det^{\frac{1}{\dim}}$
- NTRU lattice, unique shortest vectors $= \|g, f\|_2$

Lattice signatures

GGHSign	hash-then-sign	generic lattice
NTRUSign	hash-then-sign	NTRU lattice
Fiat Shamir with abort	FS, Rejection sampling	generic lattice
GPV	hash-then-sign	generic lattice
BLISS	FS, Rejection sampling	NTRU lattice
Dilithium	FS, Rejection sampling	generic lattice
Falcon	hash-then-sign	NTRU lattice
pqNTRUSign	HTS, Rejection sampling	NTRU lattice

GGHSign

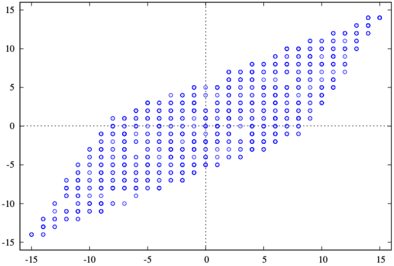
- Signing key: a good basis B
- Verification key a bad basis H
- Sign
  - Hash message to a vector $\mathbf{v}$
  - Use B to find the closest vector $\mathbf{c}$ (Babai's algorithm)
- Verification
  - Check $Dist(\mathbf{v} - \mathbf{c})$ is small

NTRUSign

- Good basis: (g,f)
- Bad basis: h

Transcript security

- Breaks GGHSign, NTRUSign;
- Each signature is a vector close to the lattice (info leakage);
- Recover enough of distance vectors (blue dots) gives away a good basis of the lattice;
- Seal the leakage with rejection sampling.



GPV sampler: a randomized Babai function

How it works

- A trapdoored lattice $\mathcal{L}$, i.e.

$$\mathcal{L}_A^{\perp} := \{v : Av = 0 \bmod q\}, \qquad \mathcal{L}_h := \{(u, v) : uh = v \bmod q\}$$

- A trapdoor S, or (g, f), and a smooth parameter $\eta_{\varepsilon}(\mathcal{L})$
- A target lattice point $\mathbf{v}$
- Outputs another vector $\mathbf{s}$, s.t.
  - $\mathbf{s}$ is uniform over $\mathcal{L}$
  - $dist(\mathbf{s}, \mathbf{v})$ Gaussian over $\mathbb{Z}^n$

Bottle neck: trapdoor generation

- Bonsai Tree, Gadget matrix, NTRU lattices ...

Falcon

- Falcon = GPV + NTRUSign + more ticks

Falcon

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Falcon in a Nutshell

We work over the cyclotomic ring $\mathcal{R} = \mathbb{Z}_q[x]/(x^n + 1)$.

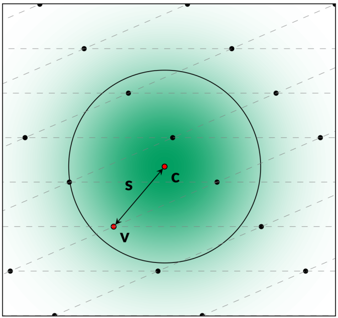
- **Keygen()**
  1. Generate matrices **A**, **B** with coefficients in $\mathcal{R}$ such that
     - $\mathbf{BA} = 0$
     - **B** has small coefficients
  2. pk $\leftarrow$ **A**
  3. sk $\leftarrow$ **B**

- **Sign(m,sk)**
  1. Compute **c** such that $\mathbf{cA} = H(\mathrm{m})$
  2. $\mathbf{v} \leftarrow$ "a vector in the lattice $\Lambda(\mathbf{B})$, close to **c**"
  3. $\mathbf{s} \leftarrow \mathbf{c} - \mathbf{v}$

  The signature sig is $\mathbf{s} = (s_1, s_2)$

- **Verify(m,pk sig)**
  Accept iff:
  1. **s** is short
  2. $\mathbf{sA} = H(\mathrm{m})$

$\Rightarrow$



Performance comparison

	A	B	I	J	K	L
1		Category	sk	pk	bytes	
2	Dilithium_medium	Lattices	2,800	1,184	2,044	
3	Dilithium_recommended	Lattices	3,504	1,472	2,701	
4	Dilithium_very_high	Lattices	3,856	1,760	3,366	
5						
6	falcon1024	Lattices	8,193	1,793	1,330	
7	falcon512	Lattices	4,097	897	690	
8	falcon768	Lattices	6,145	1,441	1,077	
9						
10	qTesla_128	Lattices	2,112	4,128	3,104	
11	qTesla_192	Lattices	8,256	8,224	6,176	
12	qTesla_256	Lattices	8,256	8,224	6,176	
13						
14	Gaussian-1024	Lattices	2,604	2,065	2,065	
15						
16						

Raptor

- Raptor = Falcon + anonymity (stealth mode)

Raptor

The scheme

- **Setup** Output a hash function $\mathcal{H} : \{*\} \rightarrow D_b$ and a random $\mathbf{h}$.
- **KeyGen** Return $pk = \mathbf{a} := \mathbf{g}/\mathbf{f}$ and $sk = (\mathbf{f}, \mathbf{g})$.
- **Signing**
  - Input $\{pk_1, \ldots, pk_\ell\}$, sk_π and μ;
  - For $i \in [1, \cdots, \ell]$ and $i \neq \pi$, pick $\mathbf{m}_i$ and $\mathbf{r}_i$. Compute $\mathbf{c}_i = \mathbf{h}\mathbf{m}_i + \mathbf{a}_i\mathbf{r}_i$.
  - For $i = \pi$, pick $\mathbf{c}_\pi$.
  - Compute $\mathbf{m}_\pi$ such that

$$\mathbf{m}_1 \oplus \cdots \oplus \mathbf{m}_\pi \oplus \cdots \oplus \mathbf{m}_\ell = \mathcal{H}(\mu, \mathbf{c}_1, \cdots, \mathbf{c}_\ell, pk_1, \ldots, pk_l). \qquad (1)$$

  - Set $\mathbf{u} = \mathbf{c}_\pi - \mathbf{h}\mathbf{m}_\pi$
  - Set $\mathbf{r}_\pi = \text{Falcon.sign}(\mathbf{u}, sk_\pi)$
  - Signature $(\mathbf{r}_1, \ldots, \mathbf{r}_\pi, \mathbf{m}_i, \ldots, \mathbf{m}_\pi)$
- **Verify**
  - For $i \in [1, \cdots, \ell]$, generate $\mathbf{c}_i = \mathbf{h}\mathbf{m}_i + \mathbf{a}_i\mathbf{r}_i$
  - Check Equation (1).

Raptor

Security

- Public key security: as hard as breaking NTRU
- Strong Unforgeability: as hard as forging a Falcon signature
- Strong Anonymity: $(\mathbf{m}_\pi, \mathbf{r}_\pi)$ statistically IND from $(\mathbf{m}_i, \mathbf{r}_i)$

Linkable Raptor

- Use one time signature to generate a tag for each pk
- Tag is enforced in verification
- Same tag = linked.

Wrap up

Features

- First lattice based linkable ring signature
- Do not require NIZK
- Competitive performance to classical solutions
- 50 to 100x smaller than known (none-implementable) PQC solutions