When utilizing quantum algorithms, accurately learning their outputs is crucial. Given physical quantum objects such algorithms produce, we wish to learn their mathematical description. Since measurements are destructive, doing so consumes the objects – an expended resource that we wish to minimize. This setting has been extensively studied for quantum states and channels, and we aim to *generalize* one possible approach to other objects.

We extend an RFTL-based procedure for online shadow tomography. In this setting, we learn a quantum object by proxy of its interactions with numerous co-objects (e.g. quantum state and measurements). At each time step, the adversary queries one such co-object and the learner responds with an object, subsequently being penalized through a loss function evaluated at the inner product of the object and co-object. Our extended setting allows the sets of objects and co-objects to be arbitrary convex subsets of positive semidefinite operators, thus encompassing quantum states, effects, pure state inner products, channels, interactive strategies, quantum strategies, and co-strategies. We aim to minimize *regret*

$$\mathcal{R}_T = \sum_{t=1}^{T} f_t(\omega_t) - \min_{\varphi \in \mathcal{K}} \sum_{t=1}^{T} f_t(\varphi) \ ,$$

that is, the accumulated expected loss of an algorithm compared to the best fixed action. In all these settings we achieve sublinear regret, implying that the objects are learnable. To extend the relevant proofs, we use methods from online convex optimization and complex matrix analysis.

Contributions: We establish the *learnability* of many classes of quantum objects. Given a set of objects $\mathcal{K}$ with trace bounded by A and a set of co-objects $\mathcal{E}$ with operator norm bounded by C, the algorithm guarantees

$$\mathcal{R}_T \leq 4BCD\sqrt{AT} \ .$$

Here, the loss functions are B-Lipschitz and D is the diameter of $\mathcal{K}$ relative to quantum entropy. Additionally, we have established a *generalized version of Pinsker's inequality* applicable to unnormalized distributions

$$\frac{1}{4}\|P-Q\|_{\mathrm{Tr}}^2 \leq \max\{\mathrm{Tr}(P), \mathrm{Tr}(Q)\}\,[D(P\,\|\,Q) - \mathrm{Tr}(P-Q)] \ .$$

Numerical evidence suggests the above constant may be tightened to $1/2$.

Impact: The established learnability in the online framework is directly related to a procedure known as *shadow tomography*. The latter is of particular interest in extracting data from quantum algorithms aiming for exponential speedups, and in learning states and circuits of various quantum cryptographic scenarios. For instance, shadow tomography allows for *approximate verification* of quantum software, and establishes the need for computational assumptions in *private-key quantum money* and *quantum copy-protected software* schemes. Efficiently generalizing shadow tomography to other quantum objects would extend the above results to more powerful versions of the same schemes.

Input: T, $\eta > 0$, convex and compact $\mathcal{K} \subseteq \mathrm{Pos}(\mathcal{X})$, bounded $\mathcal{E} \subseteq \mathrm{Herm}(\mathcal{X})$.

1 Set initial hypothesis $\omega_1 \leftarrow \arg\max_{\varphi \in \mathcal{K}}\{S(\varphi)\}$, where $S(X) = -\mathrm{Tr}(X \ln(X))$.

2 **for** $t \leftarrow 1$ **to** T **do**

3 Predict ω_t and incur cost $f_t(\omega_t) := \ell_t(\langle E_t, \omega_t\rangle)$ with $\ell_t : \mathcal{R} \to \mathcal{R}$ and $E_t \in \mathcal{E}$.

4 Let $\ell_t'(x)$ be a sub-derivative of ℓ_t with respect to x and define

$$\nabla_t \leftarrow \ell_t'(\langle E_t, \omega_t\rangle)E_t \ .$$

5 Update decision according to the RFTL rule

$$\omega_{t+1} \leftarrow \arg\min_{\varphi \in \mathcal{K}} \left\{ \eta \sum_{s=1}^{t} \langle \nabla_s, \varphi\rangle - S(\varphi) \right\} \ .$$

6 **end**

States: $\mathcal{R}_T \leq \left(4B\sqrt{\ln(\dim(\mathcal{X}))}\right)\sqrt{T}$

Gram matrices: $\mathcal{R}_T \leq \left(4Bn\sqrt{\ln(n)}\right)\sqrt{T}$

Effects: $\mathcal{R}_T \leq \left(4B\dim(\mathcal{X})\sqrt{e^{-1} + \ln(\dim(\mathcal{X}))}\right)\sqrt{T}$

Channels: $\mathcal{R}_T \leq \left(4B\ \dim(\mathcal{X})\sqrt{\ln(\dim(\mathcal{X})) + \ln(\dim(\mathcal{Y}))}\right)\sqrt{T}$

Interactive measurements: $\mathcal{R}_T \leq \left(4B\ \dim(\mathcal{X})\dim(\mathcal{Y})\sqrt{\ln(\dim(\mathcal{X})) + \ln(\dim(\mathcal{Y}))}\right)\sqrt{T}$

Strategies: $\mathcal{R}_T \leq \left(4B\left(\prod_{i=1}^{n}\dim(\mathcal{Y}_i)\right)\left(\prod_{i=1}^{n}\dim(\mathcal{X}_i)\right)\sqrt{\sum_{i=1}^{n}\ln(\dim(\mathcal{X}_i)) + \sum_{i=1}^{n}\ln(\dim(\mathcal{Y}_i))}\right)\sqrt{T}$

Title: Online Learning of a Panoply of Quantum Objects

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