CS1231S Assignment #2
AY2024/25 Semester 1
Deadline: Monday, 4 November 2024, 1:00pm
TEMPLATE FOR SUBMISSION

Q0. Full name: ZHENG JIONGJIE   Tutorial grp: **T05A**

Write your full name and tutorial group number above.

Enter your **Student Number** into the box on the right, by filling in your student number at the top row, and shading the digits and check letter.

We are going to use a software to identify your shaded Student Number, so it is very important that you shade your Student Number correctly, otherwise you will score zero mark for the assignment! The Student Number you write at the top is for us to identify you in case you have shaded your number wrongly (but do check that you have shaded it correctly!)

STUDENT NUMBER

A 0 3 0 8 4 1 1 R

Q1. Countning and Probability (Total: 7 marks)

(a) [1]

60

(b) [2]

24 ways
Both friends sit on sofa = 3 * 3! = 18
Neither friend sits on sofa = 1 * 6 = 6
18 + 6 = 24

(c) [2]

0.278
Total ways = 10 * 9 * 8 = 720
Odd even odd = 5 * 5 * 4 = 100
Even odd even = 5 * 5 * 4 = 100
(100 + 100) / 720 = 5 / 18 = 0.278

(d) [2]

(n – 1) / (2n)

Q2. Expected value (Total: 7 marks)

(a) [2]

((29/30) * 20) + ((1/30) * (-200)) = 12.67 (4s.f.)
Average profit = 12.67

(b) [2]

((13/52) * 10) + ((9/52) * 5) + ((30/52) * (-5)) = 0.4808
Expected gain = 0.484

(c) [3]

average number of rolls to get 3 = 1 / (1/6) = 6

Q3. Functions (Total: 6 marks)

(a) [1]

f(a b a a b)=b b a a b, f(b b a a b)=b b b a b
(f∘f)(a b a a b)=b b b a b

(b) [2]

No.
Counter example s1 = aa, s2 = ab
(f∘f)(aa)=bb, (f∘f)(ab)=bb. 2 unique s1 and s2 produce the same output "bb", so it is not injective

(c) [3]

Yes.
Number of a=n
Number of b=0
g(s)=n−0=n
string exists for every n∈N, so g is surjective.

Q4. Hasse Diagrams (Total: 6 marks)

(a) [2]	a = 6 D6={1,2,3,6}
(b) [2]	b = 6 D12={1,2,3,4,6,12}
(c) [2]	c = 30 D30={1,2,3,5,6,10,15,30}

Q5. Mathematical Induction: $2^{4n} - 2^n$ is divisible by 14 for all $n \in \mathbb{Z}_{\geqslant 0}$ (6 marks)

Base step
n = 0

Inductive hypothesis
2^(4k) – 2^k is divisible by 14
There exist m such that 2^(4k) – 2^k = 14m

Inductive step
2^(4(k + 1)) –2^(k+1)is divisible by 14
2^(4(k + 1)) = 16 * 2^4k
2^(k+1) = 2 * 2^k
16 * 2^4k - 2 * 2^k = (2^k)(16 * 2^3k – 2)

Q6. Function and Cardinality (Total: 6 marks)

(a) The function f has an inverse. [2 marks]

false.
for f to have an inverse, it must be bijective. Since multiple subsets can have the same smallest element, inverse is undefined, so it is not surjective.

(b) $f^{-1}(\{n\})$ is uncountable for some $n \in \mathbb{N}$. [4 marks]

True.

=== END OF PAPER ===