CS1231S Assignment #2

AY2024/25 Semester 1

Deadline: Monday, 4 November 2024, 1:00pm

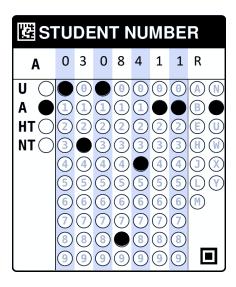
TEMPLATE FOR SUBMISSION

| Q0. Full name: ZHENG JIONGJIE | Tutorial grp: T05A |
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Write your full name and tutorial group number above.

Enter your **Student Number** into the box on the right, by filling in your student number at the top row, <u>and</u> shading the digits and check letter.

We are going to use a software to identify your shaded Student Number, so it is very important that you shade your Student Number correctly, otherwise you will score zero mark for the assignment! The Student Number you write at the top is for us to identify you in case you have shaded your number wrongly (but do check that you have shaded it correctly!)



Q1. Countning and Probability (Total: 7 marks)

- (a) ₆₀
- [1]
- (b) 24 ways

Both friends sit on sofa = 3 * 3! = 18 Neither friend sits on sofa = 1 * 6 = 6 18 + 6 = 24

- (c)
 [2]
 0.278
 Total ways = 10 * 9 * 8 = 720
 Odd even odd = 5 * 5 * 4 = 100
 Even odd even = 5 * 5 * 4 = 100
 (100 + 100) / 720 = 5 / 18 = 0.278
- (d) (n-1)/(2n) [2]

| Q2. Expected value (Total: 7 marks) |
|-------------------------------------|
|-------------------------------------|

- (a) ((29/30) * 20) + ((1/30) * (-200)) = 12.67 (4s.f.)
- [2] Average profit = 12.67
- (b) ((13/52) * 10) + ((9/52) * 5) + ((30/52) * (-5)) = 0.4808
- [2] Expected gain = 0.484
- (c) average number of rolls to get 3 = 1 / (1/6) = 6
- [3]

Q3. Functions (Total: 6 marks)

- (a) | f(a b a a b)=b b a a b, f(b b a a b)=b b b a b
- [1] (fof)(a b a a b)=b b b a b
- (b) No.
- [2] Counter example s1 = aa, s2 = ab (fof)(aa)=bb, (fof)(ab)=bb. 2 unique s1 and s2 produce the same output "bb", so it is not injective
- (c) Yes.
- Number of a=n
 Number of b=0
 g(s)=n-0=n
 string exists for every n∈N, so g is surjective.

Q4. Hasse Diagrams (Total: 6 marks)

(a) a = 6

[2] $D6=\{1,2,3,6\}$

(b) b = 6

[2] D12={1,2,3,4,6,12}

(c) c = 30

[2] D30={1,2,3,5,6,10,15,30}

Q5. Mathematical Induction: $2^{4n} - 2^n$ is divisible by 14 for all $n \in \mathbb{Z}_{\geqslant 0}$ (6 marks)

Base step

n = 0

Inductive hypothesis

 $2^{4k} - 2^k$ is divisible by 14

There exist m such that $2^{(4k)} - 2^k = 14m$

Inductive step

 $2^{(4(k+1))} - 2^{(k+1)}$ is divisible by 14

 $2^{(4(k+1))} = 16 * 2^{4k}$

 $2^{(k+1)} = 2 * 2^{k}$

 $16 * 2^4k - 2 * 2^k = (2^k)(16 * 2^3k - 2)$

| Q6 | . Function and Cardinality (Total: 6 marks) | |
|-----|---|-----------|
| (a) | The function f has an inverse. | [2 marks] |
| | false. for f to have an inverse, it must be bijective. Since multiple subsets can have the same smallest inverse is undefined, so it is not surjective. | element, |
| (b) | $f^{-1}(\{n\})$ is uncountable for some $n \in \mathbb{N}$. | [4 marks] |
| | True. | |

=== END OF PAPER ===