例 6.4

<sol>

令 $E(X_i)=\mu$ 　　　則 $E(\bar{X})=\mu$

$V(X_i)=\sigma^2=E(X_i^2)-\mu^2$ 　　　$V(\bar{X})=\frac{\sigma^2}{n}=E(\bar{X}^2)-\mu^2$

$$E(\hat{\sigma}_1)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\frac{n-1}{n}\cdot\sigma^2$$

$$E(\hat{\sigma}_2)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\sigma^2$$

故 $\hat{\sigma}_2=\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}\right)$ 為母体變異數 σ^2 之不偏估計

而 $\hat{\sigma}_1=\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}\right)$ 為母体變異數 σ^2 之偏誤估計