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$$(1) S = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n-1}} = \sqrt{\frac{\sum X_i^2 - n\bar{X}^2}{n-1}}$$

$$= \sqrt{\frac{1289 - 6 \times 14.33^2}{5}} = \sqrt{0.38} = 0.62$$

$\therefore \sigma$ 之點估計為 3.22

$$(2) 1 - \alpha = 0.9 \quad \frac{\alpha}{2} = 0.05 \quad n-1 = 5$$

$$\chi^2_{\frac{\alpha}{2}}(n-1) = \chi^2_{0.05}(5) = 11.07$$

$$\chi^2_{1-\frac{\alpha}{2}}(n-1) = \chi^2_{0.95}(5) = 1.15$$

$\therefore \sigma$ 之 90% 信賴區間為

$$\left(\sqrt{\frac{5 \times 10.38}{\chi^2_{0.05}(5)}} \cdot \sqrt{\frac{5 \times 10.38}{\chi^2_{0.95}(5)}} \right) = \left(\sqrt{\frac{51.9}{11.07}} \cdot \sqrt{\frac{51.9}{1.15}} \right) = (2.17, 6.72)$$

$$24 (2) 1 - \alpha = 0.9$$

$$\chi^2_{\frac{\alpha}{2}}(n-1) = \chi^2_{0.05}(8) = 15.51$$

$$\chi^2_{1-\frac{\alpha}{2}}(n-1) = \chi^2_{0.95}(8) = 2.73$$

$\therefore \sigma$ 之 90% 信賴區間為

$$\left(\sqrt{\frac{8 \times 4.27^2}{\chi^2_{0.05}(8)}} \cdot \sqrt{\frac{8 \times 4.27^2}{\chi^2_{0.95}(8)}} \right) = \left(\sqrt{\frac{687.46}{15.51}} \cdot \sqrt{\frac{687.46}{2.73}} \right)$$

$$= (6.66, 15.87)$$

$$(3) 1 - \alpha = 0.9$$

$$F_{\frac{\alpha}{2}}(n_1-1, n_2-1) = F_{0.05}(8, 8) = 3.44$$

$$F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1) = F_{0.95}(8, 8) = \frac{1}{F_{0.05}(8, 8)} = 0.29$$

$\therefore \frac{\sigma_1^2}{\sigma_2^2}$ 之 90% 信賴區間為

$$\left(\frac{\sigma_1^2}{\sigma_2^2} \times \frac{1}{F_{\frac{\alpha}{2}}(n_1-1, n_2-1)} \cdot \frac{\sigma_1^2}{\sigma_2^2} \times \frac{1}{F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)} \right) = \left(\frac{9.27^2}{21.15^2} \times \frac{1}{3.44} \cdot \frac{9.27^2}{21.15^2} \times \frac{1}{0.29} \right)$$

$$= (0.06, 0.66)$$