Discussion 9B

Anish's 16A Discussion
Office Hours: Tuesday 3-4
@Cory 144MA Friday 4-5

Charge Sharing:

Motivation: We are working with a circuit with capacitors that get charged from a voltage source. However, the layout of the circuit dynamically changes over time through different phases, and we wish to study the behavior of this circuit, its capacitors, and how charge is shared between these capacitors.

Recipe for charge sharing:

1) Label all voltages across the capacitors in whichever direction you want, as long as you stay consistent across all phases (+ on top and − on bottom in phase 1 => + on top and − on bottom in phase 2).
2) Draw the given circuit in each phase, closing and opening phase switches appropriately and keeping the polarity chosen for each capacitor in step 1 consistent.
   → Note: ϕ_n denotes phase n (just a notation)
3) Observe the circuit in ϕ_1 (phase 1). First determine the voltage drop across each capacitor according to your arbitrarily chosen capacitor polarities. Then, use $Q = CV_C$ to determine the charge on each of the two plates on every capacitor.
4) Observe the circuit in ϕ_2 (phase 2), and determine where the charge is conserved. This will be at the "floating" nodes within the circuit. A "floating" node is a node where only capacitor plate(s) are connected to it and nothing else (i.e. voltage source or ground). At these special nodes, the total charge on the plate(s) connected to it must be conserved ($\Sigma\ \phi_1$ charges $= \Sigma\ \phi_2$ charges for "floating" nodes).
5) Finally, write the equation for charge conservation. Determine the voltages across the capacitor and calculate the charge stored in each capacitor in ϕ_2.

This will make much more sense with the examples in this worksheet and with lots of practice!

EECS 16A Designing Information Devices and Systems I
Fall 2022 Discussion 9B

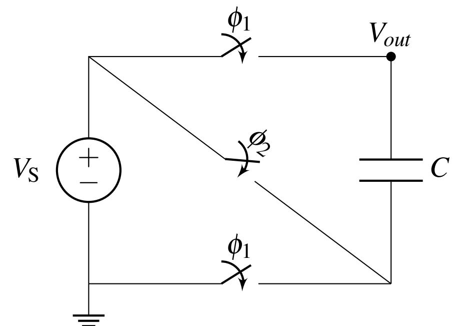
Mid Semester Survey

Please fill out the mid semester survey: https://tinyurl.com/midsemester16a

We highly appreciate your feedback!

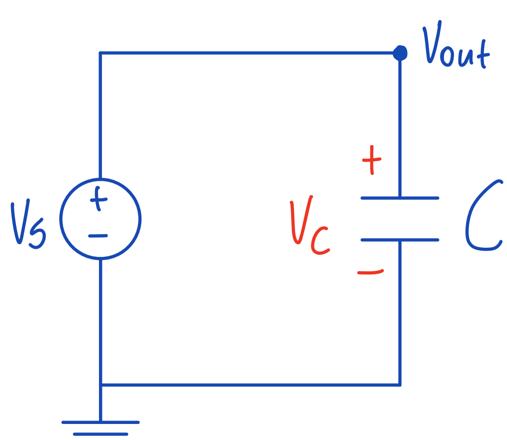
1. Voltage Booster

We have made extensive use of resistive voltage dividers to reduce voltage. What about a circuit that boosts voltage to a value greater than the supply $V_S = 5\,\text{V}$? We can do this with capacitors!



(a) In the circuit above switches ϕ_1 are initially closed and switch ϕ_2 is initially open. Calculate the value of the output voltage, V_{out} with respect to ground, and the amount of charge stored on capacitor, C, at that state (phase 1).

ϕ_1 (Phase 1):



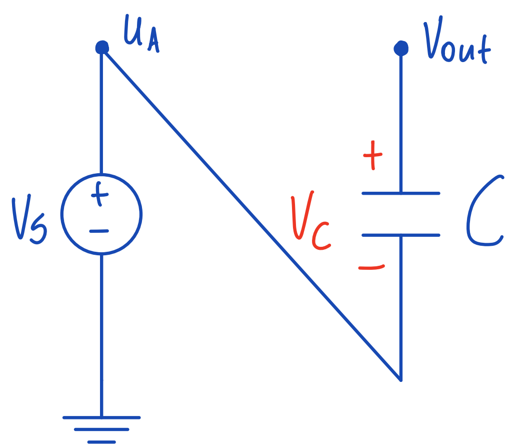
Vs and C in parallel!
=> same voltage drop

$V_S = V_C = V_{out} - 0 = V_{out}$

$Q = CV_C = CV_S$

(b) Now, after the capacitors are charged, switches ϕ_1 are opened and switch ϕ_2 is closed. Calculate the new voltage output voltage, V_{out}, at steady state (phase 2).

ϕ_2 (Phase 2):



Notice:
- In phase 2 here, the voltage source is now connected to the negative plate of the capacitor C and the positive plate is floating
  $\Rightarrow$ charge will be conserved on the top plate of C

$$V_S = u_a - 0 = u_a$$

$$V_C^{\phi_2} = V_{out} - u_a = V_{out} - V_S$$

$$Q_C^{\phi_1} = Q_C^{\phi_2}$$

$$\Rightarrow C V_S = C \cdot (V_{out} - V_S)$$

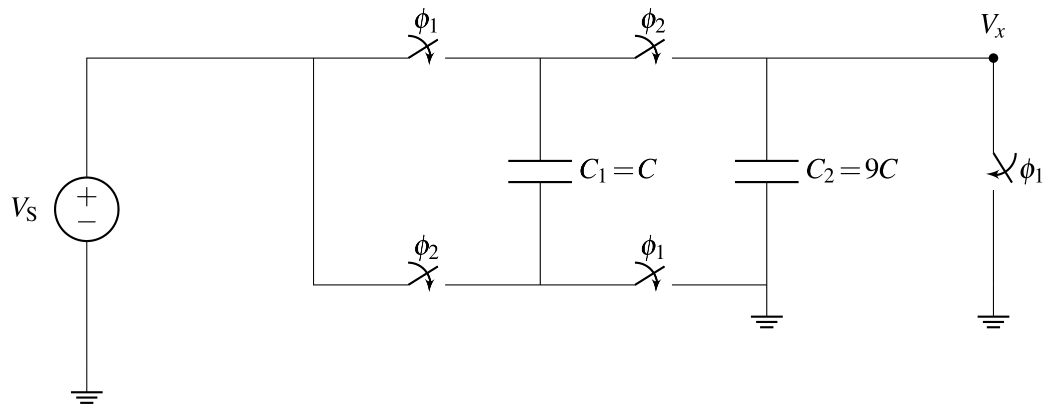
$$V_S = V_{out} - V_S$$

$$V_{out} = 2 V_S = 2 \cdot 5V = 10V$$

We have created a voltage doubler!

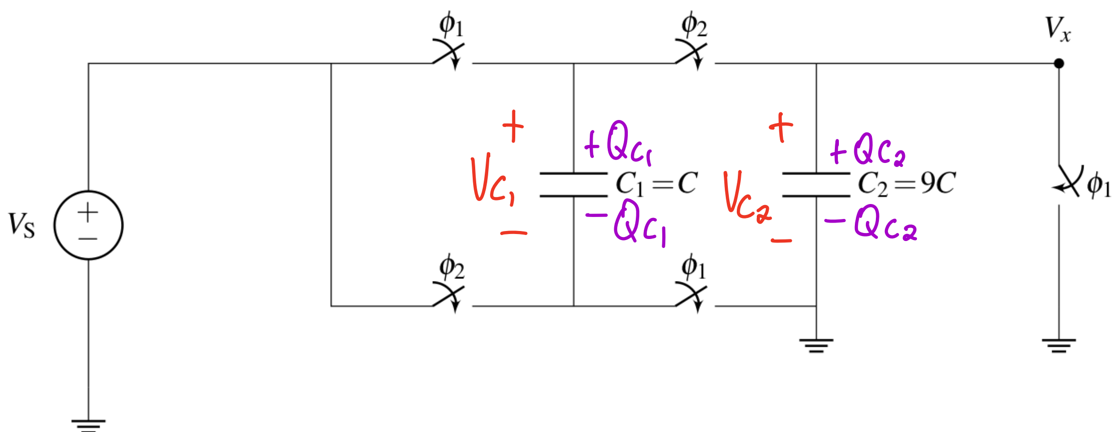
2. Charge Sharing

Consider the following circuit:



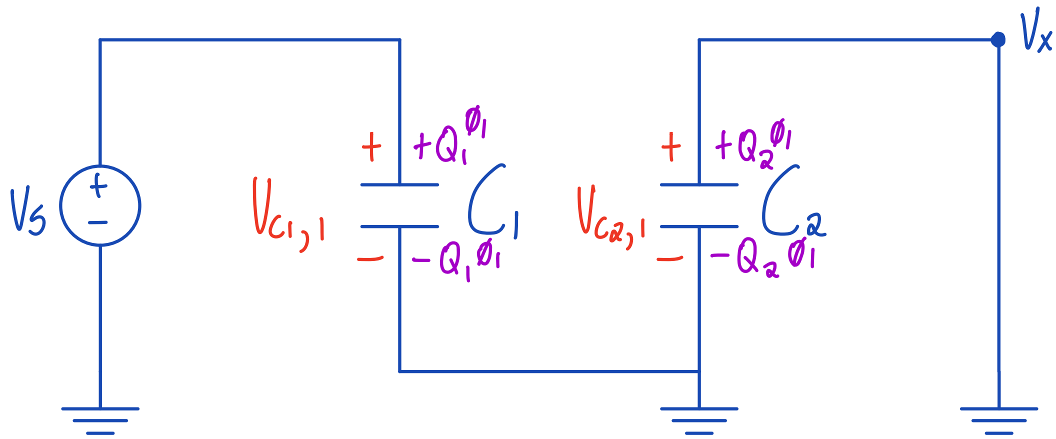
In the first phase, all of the switches labeled ϕ_1 will be closed and all switches labeled ϕ_2 will be open. In the second phase, all switches labeled ϕ_1 are opened and all switches labeled ϕ_2 are closed.

(a) Draw the polarity of the voltage (using $+$ and $-$ signs) across the two capacitors C_1 and C_2. It doesn't matter which terminal you label $+$ or $-$; just remember to keep these consistent through phase 1 and 2! Also, label the charge on at each plate: $+Q_{C_1}$, $-Q_{C_1}$, $+Q_{C_2}$, and $-Q_{C_2}$.

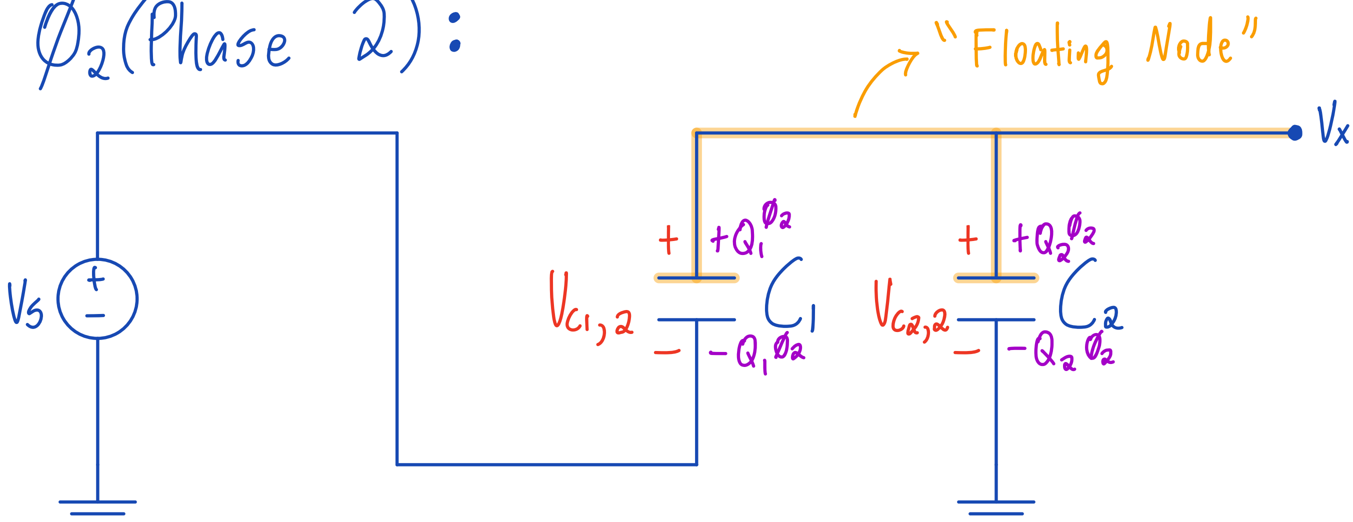


(b) Draw the circuit in the first phase and in the second phase. Keep your polarity from part (a) in mind.

ϕ_1 (Phase 1):



ϕ_2 (Phase 2):



(c) Find the voltages and charges on C_1 and C_2 in phase 1. Be sure to keep the polarities of the voltages the same!

$V_{C_1,1} = V_S$

$Q_{C_1}^{\phi_1} = C_1 V_{C_1,1} = C_1 V_S$

$V_{C_2,1} = V_X = 0$

$Q_{C_2}^{\phi_1} = C_2 V_{C_2,1} = 0$

(d) Now, in the second phase, find the voltage V_x.

Key Concept: Charge Conservation

- In phase 2, the top plates of C_1 and C_2 are "floating", as they are not connected to V_S or ground
- In phase 1, these same plates were in no way connected
  - When we transition from phase 1 to phase 2, the two plates connect to each other; therefore, in phase 2, the charges on both top plates will be shared, or distributed, as they have nowhere else to go
    - By conservation of charge, the total charge on the top of the two plates will be equivalent in phase 1 and in phase 2

$$V_{C_1}^{\phi_2} = V_x - V_S \qquad V_{C_2}^{\phi_2} = V_x$$

$$Q_{C_1}^{\phi_2} = C_1(V_x - V_S) = C(V_x - V_S) \qquad Q_{C_2}^{\phi_2} = C_2 V_x = 9C V_x$$

By charge conservation:

$$+Q_{C_1}^{\phi_1} + Q_{C_2}^{\phi_1} = +Q_{C_1}^{\phi_2} + Q_{C_2}^{\phi_2}$$

$$CV_S + 0 = C(V_x - V_S) + 9CV_x$$

$$CV_S = CV_x - CV_S + 9CV_x$$

$$2CV_S = V_x(C + 9C) = V_x(10C)$$

$$\boxed{V_x = \frac{1}{5} V_S}$$

tinyurl.com/16Anish

Password: sharing