Discussion 9A ⌣̈

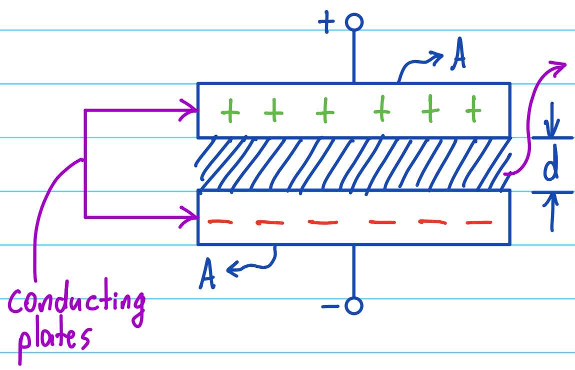
Anish's 16A Discussion
Office Hours: Tuesday 3-4
@Cory 144MA Friday 4-5

Capacitance:

Capacitance is the ability to store charge {Unit = Farads}

$$C = \frac{Q \rightarrow \text{charge stored}}{V_C \rightarrow \text{voltage drop across capacitor}} \Bigg\} \; Q = CV_C$$

★ Note: In this class, we mainly look at parallel plate capacitors



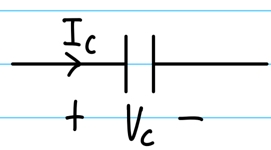
$$C = \varepsilon \frac{A}{d}$$

A → surface area of each plate

d → separation between plates

ε → "permitivity" of the material between the plates
− air (vacuum) has $\varepsilon = 8.85 \times 10^{-12}$ F/m

I-V Relation of a Capacitor:

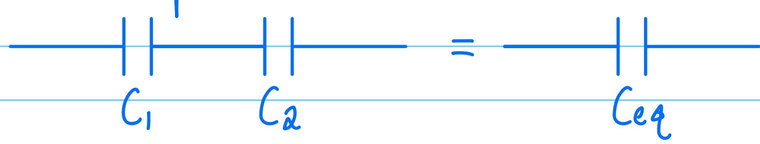


$$I_C = \frac{dQ}{dt} = \frac{d(CV_C)}{dt} = C\frac{dV_C}{dt}$$

$$I_C = C \cdot \frac{dV_C}{dt}$$

Capacitor Equivalence:

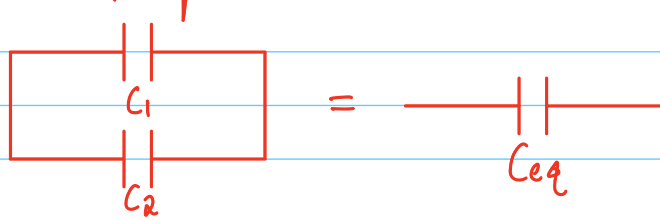
Series Capacitors:



Recall: "parallel operator"

$$C_{eq} = C_1 \| C_2 = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Parallel Capacitors:



$$C_{eq} = C_1 + C_2$$

EECS 16A Designing Information Devices and Systems I
Fall 2022 Discussion 9A

Mid Semester Survey

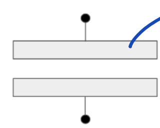
Please fill out the mid semester survey: https://tinyurl.com/midsemester16a

We highly appreciate your feedback!

1. Capacitance Equivalence

For the structures shown below, assume that the plates have a depth L into the page and a width W and are always a distance d apart. The dielectric between the plates has absolute permitivity ε. For the following calculations, assume the capacitance is purely parallel plate, i.e. ignore fringing field effects.

(a) What is the capacitance of the structure shown below?



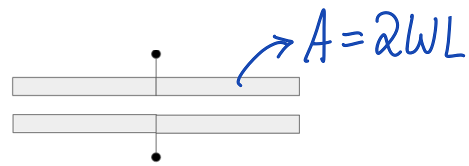
$A = WL$

$$C = \varepsilon \frac{A}{d} \quad \text{(general formula)}$$

cross-sectional area $A = W \cdot L$

$$\Rightarrow C = \varepsilon \frac{WL}{d}$$

(b) Suppose that we take two such structures and put them next to each other as shown below. What is the capacitance of this new structure?

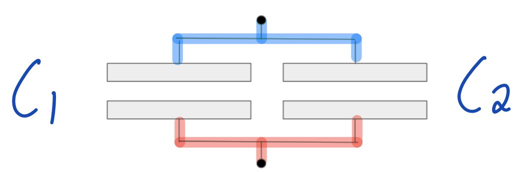


We have just doubled the width of the plates

$$C = \varepsilon \frac{2W \cdot L}{d} = 2 \cdot \varepsilon \frac{WL}{d}$$

Capacitance doubles!

(c) Now suppose that rather than connecting them together as shown above, we connect them with an ideal wire as shown below. What is the capacitance of this structure?



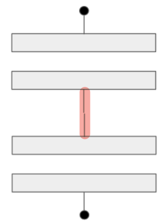
Parallel Capacitors!

$$C_{eq} = C_1 + C_2$$

$$C_{eq} = \varepsilon \frac{WL}{d} + \varepsilon \frac{WL}{d}$$

$$C_{eq} = 2 \cdot \varepsilon \frac{WL}{d}$$

(d) Suppose that we now take two capacitors and connect them as shown below. What is the capacitance of the structure?



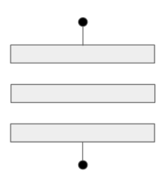
Uninterrupted Connection!
=> Series capacitors

$$C_{eq} = C_1 \| C_2 = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = \frac{\varepsilon \cancel{\frac{WL}{d}} \cdot \varepsilon \frac{WL}{d}}{2 \cdot \varepsilon \cancel{\frac{WL}{d}}}$$

$$C_{eq} = \frac{1}{2} \varepsilon \frac{WL}{d}$$

(e) What is the capacitance of the structure shown below?



same as 2 capacitors in series

Total distance now 2d

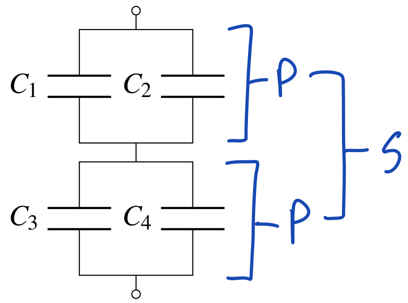
$$C_{eq} = \varepsilon \frac{WL}{2d} = \frac{1}{2} \varepsilon \frac{WL}{d}$$

(same as above)

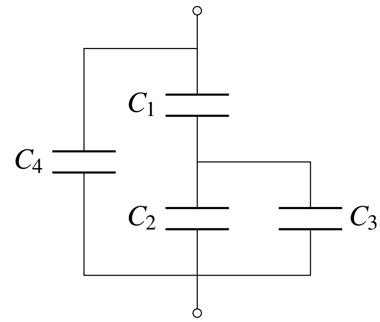
2. Series And Parallel Capacitors

Derive C_{eq} for the following circuits.

(a)



(b)



$$a)\ C_{eq} = ((C_1 + C_2) \parallel (C_3 + C_4))$$

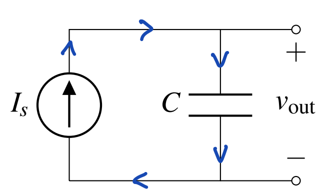
$$= \frac{(C_1 + C_2) \cdot (C_3 + C_4)}{(C_1 + C_2) + (C_3 + C_4)} = \frac{C_1 C_3 + C_2 C_3 + C_1 C_4 + C_2 C_4}{C_1 + C_2 + C_3 + C_4}$$

$$b)\ C_{eq} = (C_4 + (C_1 \parallel (C_2 + C_3)))$$

$$= \frac{C_4(C_1 + C_2 + C_3) + C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

3. Current Sources And Capacitors

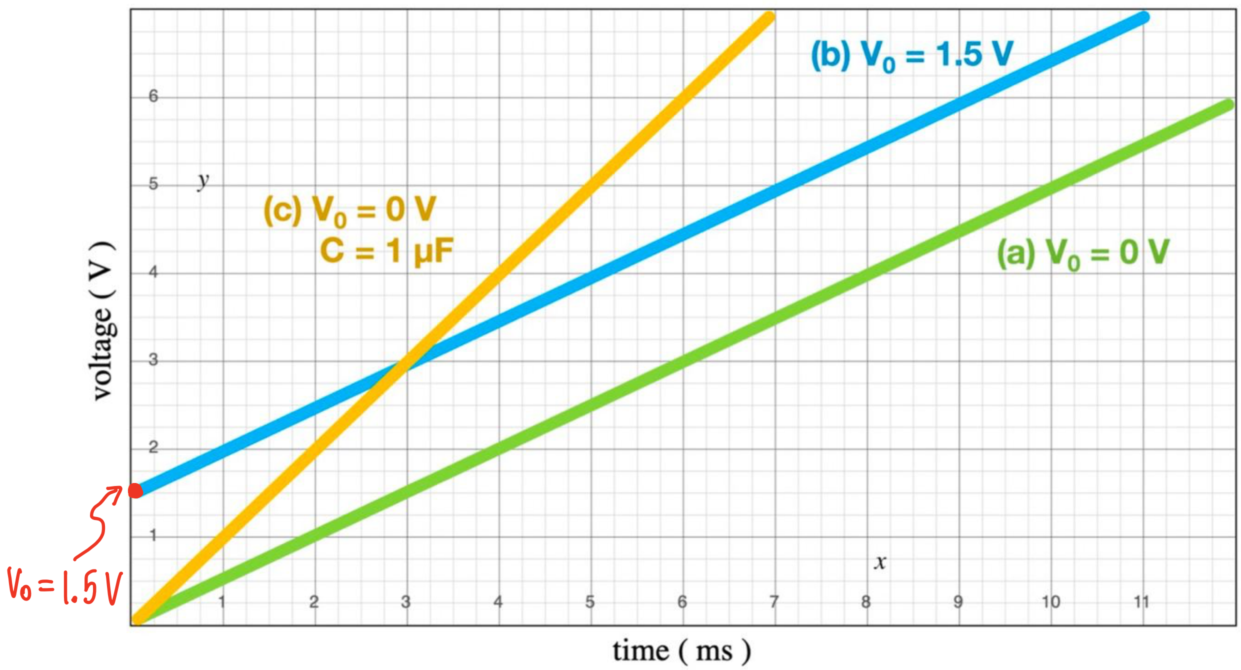
Given the circuit below, find an expression for $v_{\text{out}}(t)$ in terms of I_s, C, V_0, and t, where V_0 is the initial voltage across the capacitor at $t = 0$.



Then plot the function $v_{\text{out}}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\,\mu\text{F}$.

(a) Capacitor is initially uncharged, with $V_0 = 0$ at $t = 0$.
(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.
(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\,\mu\text{F}$, which is initially uncharged, with $V_0 = 0$ at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.



Recall: $\int_a^b f'(x)dx = f(b) - f(a)$ $\left\{ \begin{matrix} \text{Second fundamental} \\ \text{theorem of calculus} \end{matrix} \right\}$

$$I_s = C\frac{dV_c}{dt} \longrightarrow \frac{I_s}{C} = \frac{dV_c}{dt} \qquad \{V_c = V_{out}\}$$

$$\int_0^t \frac{I_s}{C}dt = \int_{V_{out}(0)}^{V_{out}(t)} \frac{dV_{out}}{\cancel{dt}}\cancel{dt} \Rightarrow \frac{I_s}{C}(t-0) = V_{out}(t) - V_{out}(0)$$

$$\Rightarrow \frac{I_s}{C}t = V_{out}(t) - V_0$$

$$\Rightarrow \left\{ V_{out}(t) = V_0 + \frac{I_s}{C}t \right\}$$

Linear function of form $y = mx + b$

y: $V_{out}(t)$, m: I_s/C, x: t, b: V_0

tinyurl.com/16Anish

Password: charge