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A discrete-flow form of the point-queue model

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***Abstract*—This paper proposes a discrete-flow form of the well-known point-queue model. Different from the traditional continuous-flow form with non-integral flow, the discrete-flow form treats a vehicle as a basic unit. The treatment simplifies the measurements of link travel times, and expands the applications of the point-queue model, such as into agent-based applications.**

I. INTRODUCTION

The well-known point-queue model [1] is a macroscopic bottleneck-type link model, which ignores the physical lengths of vehicles and allows vehicles to move at free-flow speed until reach the point of congestion. The most attractive characteristics of the model are simplicity and realistic representation of traffic delays. Due to these characteristics, this model has been widely used in transportation researches, in particular in the dynamic network loading in dynamic traffic assignment (see [2]–[12], for example).

In the traditional point-queue model, the traffic flow is continuous, i.e., non-integral flow is allowed. The continuous-flow form of the point-queue model allows accurate analysis. However, the non-integral flow ignoring individual vehicles limits its applications. For example, in some applications we focus on individual driver's route choices. Travel time is just simply needed as an input, and traffic dynamics are of no importance. To meet the demand, this paper proposes a discrete-flow form of the point-queue, in which an individual vehicle is the minimum unit. The discrete-flow form retains the simple representation of travel time, and expands the applications of the point-queue model.

The remainder of the paper is organized as follows: The following section introduces the proposed discrete-flow form of the point-queue model; Section III proposes the methods of measuring link travel time in the discrete-flow form model in real time; In Section IV, a simulation scenario is constructed, and the methods of measurements are tested and compared with different time step lengths; A conclusion is made at last.

II. A DISCRETE-FLOW FORM OF THE POINT-QUEUE MODEL

In the discrete-flow form of the point-queue model, a vehicle is treated as the minimum unit. A simulated link implementing the discrete-flow form contains two sections: free-flow and congested sections. A vehicle first enters the free-flow section, and then joins in the congested section after experiencing given free-flow travel time. The first-in vehicles in the congested section are first discharged based on the exit capacity. To reflect the fractional part of a non-integral variable without losing values, we propose the following function with a probability,

$$f(x) = \lfloor x \rfloor + \xi \tag{1}$$

where x is the non-integral variable; $\xi = 0$ or 1, which is determined by the following probability,

$$\Pr\{\xi = 1 | \xi = 0, 1\} = x - \lfloor x \rfloor \tag{2}$$

Usually, the exit capacity is non-integral, and the arrival rate generated by continuous models is non-integral.

Thus, the point-queue model in a discrete-flow form can be written as

$$d(t) = \begin{cases} f(a(t-\tau)) + l(t) & , \quad \text{if } f(a(t-\tau)) + l(t) < f(\mu) \\ f(\mu) & , \quad \text{otherwise} \end{cases} \tag{3}$$

where $a(t)$, $d(t)$, and $l(t)$ are arrival rates (veh/step), departure rates (veh/step), and the total number of queueing vehicles (veh/step) at time step t, respectively; μ and τ are an exit capacity (veh/step) and free-flow travel time (step) of a link, respectively; $\tau, d(t), l(t) \in \mathbb{N}$, and $a(t), \mu \in \mathbb{R}^+$.

III. THREE TYPES OF LINK TRAVEL TIMES AND THEIR MEASUREMENTS IN REAL TIME

A. *Cumulative vehicle count curves*

To better illustrate link travel times, we first introduce the cumulative vehicle count curves, which are an important tool widely used in transportation studies (see [12]–[17], for example). In the curves, the cumulative arrival and departure count curves (called A-curve and D-curve, respectively) make records of cumulative counts of arrival and departure vehicles, respectively. The virtual cumulative arrival count curve (V-curve) is built by shifting A-curve along the direction of the time-axis by free-flow travel time of a link. Denote by $A(t)$, $D(t)$, and $V(t)$ A-curve, D-curve, and V-curve, respectively, the equations can be written as

$$A(t) = \sum_{i=1}^{t} a(i), \tag{4}$$

$$D(t) = \sum_{i=1}^{t} d(i), \tag{5}$$

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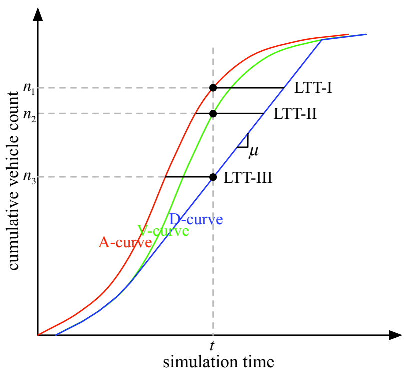

Fig. 1. Three types of LTTs presented using the cumulative vehicle count curves. A-curve, D-curve, and V-curve represent the cumulative arrival, departure, and virtual arrival count curves, respectively

$$V(t) = A(t - \tau). \tag{6}$$

In a typical queue system such as a link with an exit bottleneck, the vertical and horizontal distance between A-curve and D-curve are the vehicle number on the link and travel time of corresponding vehicles, respectively. The vertical and horizontal distance between V-curve and D-curve are the vehicle number in a queue and the delay of corresponding vehicles, respectively.

B. Three types of link travel times

Measuring link travel time (LTT) is one of the most important basis for applying the point-queue model. At step t, we usually have the following three types of LTTs represented by different vehicles (see Figure 1):

• LTT-I is the travel time that the arrival vehicle (vehicle n_1) at step t will experience. It is called experienced or predictive travel time in relevant literature (see [18] and [19], for example).

• LTT-II is the travel time that experienced by vehicle n_2, which is the vehicle at the tail of the existing queue. It is also the travel time that a vehicle would experience under current traffic conditions at step t, which is called instantaneous or reactive travel time in relevant literature (see [18] and [19], for example).

• LTT-III is the travel time experienced by the vehicle (vehicle n_3) just leaving the link at step t (see [20]–[23] for its applications).

The reminder of the section introduces the methods of real-time measurement of the LTTs in the discrete-flow form, i.e., in the middle of a simulation run we measure the LTTs in the condition that only the current and past traffic conditions are known.

C. Real-time measurements of LTTs in the discrete-flow form

To predict LTT-I in the discrete-flow form, we clone the link with current traffic conditions, and send a virtual vehicle at the entry of the cloned link. The travel time taken by the virtual vehicle is LTT-I at current time step.

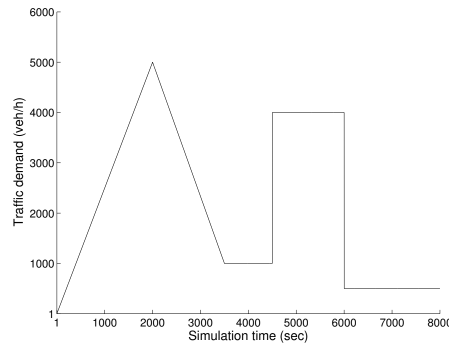

Fig. 2. A traffic demand for testing

LTT-II is measured based on its definition. The existing queue at the current step represents current traffic conditions. The equation is thus written as $\tau + l(t)/\mu$.

LTT-III in the discrete-flow form is taken as the travel time experienced by the latest departure vehicle at step t. If more than one vehicle leave the link at a step, the experienced travel time of the last vehicle is considered as LTT-III. If no vehicle leaves at a step, we take the link travel time at the last step as LTT-III at this step.

IV. A SIMULATION EXPERIMENT

A simulation scenario is constructed to test the methods of measuring the three types of LTTs in real time. Under this scenario, a point-queue link with free-flow travel time of 200 sec and a constant exit capacity of 3000 veh/h is employed. A traffic demand presented in Figure 2 is taken as inputs of the link. The demand contains two peaks: a regular peak in which the congestion forms and vanishes gradually, and an extreme peak in which the demand changes steeply. During these peaks the demands are greater than the exit capacity, and the link is congested. We test these methods of measurements with four different time step lengths, i.e., 1 sec, 10 sec, 20 sec, and 50 sec.

To obtain a reference for comparisons, we run the continuous-flow form of the point-queue model with a time step length of 1 sec, which is considered as the accurate LTT-I. Figure 3-5 present the results of measuring the three types of LTTs in the discrete-flow form. The results with different time steps basically overlap in measuring each type of LTTs. It turns out the consistency of the results with different time steps.

Fluctuations, however, can be observed in these results in particular in the one with the time step length of 1 sec (the blue curves in Figure 3-5). The fluctuations are resulted from the introduction of probability in the discrete-flow form. With the increase of time step lengths, the uncertainty decreases due to the cumulation of arriving and departing vehicles within a time step.

The measured LTT-I with the time step length of 1 sec (the blue curve in Figure 3) shows more fluctuations than the

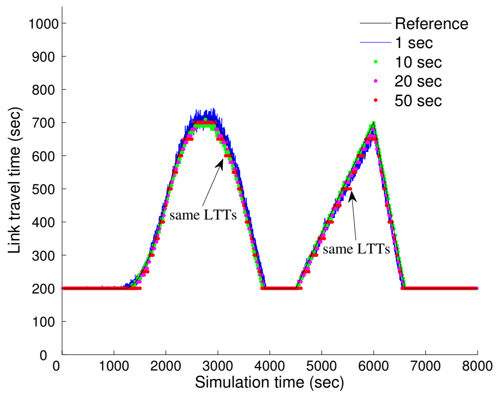

Fig. 3. The results of measuring LTT-I in the discrete-flow form with different time step lengths

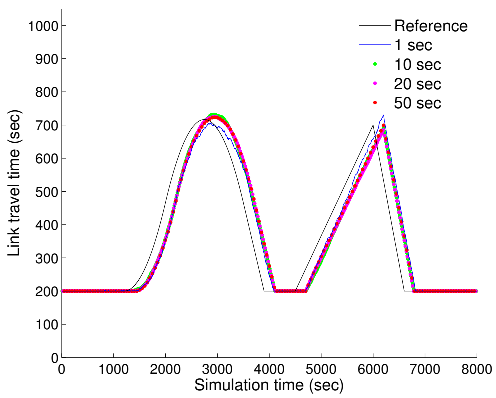

Fig. 4. The results of measuring LTT-II in the discrete-flow form with different time step lengths

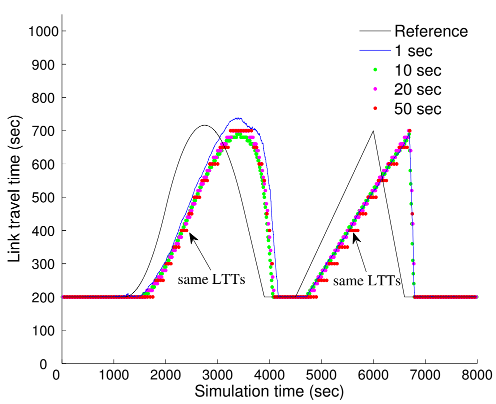

Fig. 5. The results of measuring LTT-III in the discrete-flow form with different time step lengths

measured LTT-II and LTT-III with the time step length of 1 sec (the blue curves in Figure 4 and 5). LTT-I is predicted by running the cloned link. Different random numbers are used in producing the probabilities. It results in the difference in measured LTT-I even when the traffic conditions are the same. In contrast, LTT-II and LTT-III are measured directly based on current traffic conditions. Therefore, LTT-II and LTT-III show more continuity in values, while LTT-I shows more fluctuations. If we employ the same random numbers in the prediction and the real run, the fluctuations will be reduced. However, due to the existence of probabilities, the measured LTT-III in the discrete-flow form is not the same in different simulation runs.

It is observed in Figure 3 and 5 that the measured LTT-I and LTT-III show the same values in some periods, in particular when the time step lengths are 20 sec and 50 sec. Arriving vehicles cumulate in a time step. The relative difference of total numbers of arriving vehicles between two adjacent time steps becomes smaller with the increase of the time step lengths. Since we take the last vehicle in a group of departure vehicles as LTT-I and LTT-III, the same LTTs appear when the number of the arrivals in adjacent time steps are relatively close. More LTT-I and LTT-III with the same values appear in the second peak, because the traffic demands during this peak are the same. The phenomena do not appear in LTT-II in Figure 4, because LTT-II is measured based on the existing queue instead of taking the travel time of a vehicle.

In general, all methods in the continuous-flow and discrete-flow forms give satisfied estimations of LTTs in real time. The continuous-flow form provides continuous and deterministic traffic conditions, while the methods of measurements need to consider more factors even an extra tool, such as the cumulative count vehicle curves. In contrast, the discrete-flow form takes individual vehicles as the basic unit, which suits the application based on individual vehicles, such as agent-based applications. The methods of measuring LTTs in the discrete-flow form are straightforward, although the measured LTTs locally fluctuate due to the introduction of probabilities.

In addition, Figure 6 presents all queues in the continuous-flow and discrete-flow forms under the testing scenario. No negativity reported in [9], [10], [24] is observed.

Computational costs are of importance to apply the point-queue model and the methods of measuring the LTTs. Here we test the computational times of the measurements under the scenario. To reduce random effects, the scenario are run 100 times, and each time the discrete-flow form is run with different random sequences. The average running time is presented in Table I. It is noted that the times are influenced by the hardware configuration and the coding quality. However, it is still clear that these computational times are short except the time in measuring LTT-I with a time step length of 1 sec. They meet the requirement of large-scale simulation applications (such as DTA), in particular when the time step lengths are set to be large. The computational time in measuring LTT-I with a 1-sec time step is acceptable in small-scale agent-based simulation models. It can be seen that the computational time in a large time step length is quite shorter than those in a small time step length. It implies that increasing the time step length is capable of reducing the computational time remarkably. In programming the codes, the discrete-flow

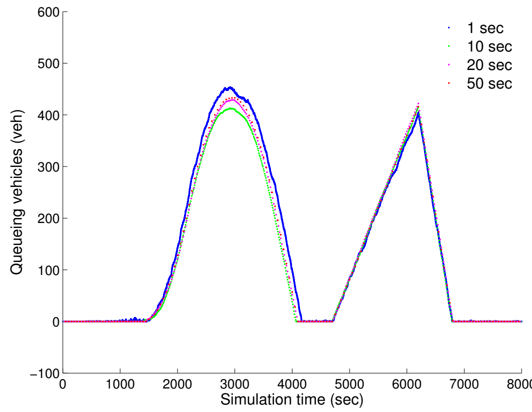

Fig. 6. All queues under the simulation scenario with different time step lengths: (a) the continuous-flow form; (b) the discrete-flow form

TABLE I. COMPUTATIONAL TIMES OF MEASURING LTTS IN THE DISCRETE-FLOW FORM

time step (sec)	Discrete-flow (msec)		
	LTT-I	LTT-II	LTT-III
1	2645.90	13.50	13.69
10	39.57	1.72	1.73
20	14.17	1.19	1.18
50	4.53	0.84	0.80

form treats an vehicle as an object in the object-oriented programming. Although measuring LTTs is straightforward in the discrete-flow form, most of its computational time is spent on adding/removing vehicle objects in/from the vehicle array in the simulated link. These processes increase the computational time in measuring LTTs in the discrete-flow form.

V. CONCLUSION

To expand the applications of the well-known point-queue model, a discrete-flow form treating a vehicle as a basic object is proposed. A probability is incorporated into the discrete-flow form to cope with the fractional part of non-integral variables. The methods of measuring three types of LTTs in the discrete-flow form are introduced. Measuring LTT-I and LTT-III in the discrete-flow form is to directly take the travel time of the corresponding vehicles. LTT-II is calculated simply according to it definition. These methods are tested under a single-link scenario with different time step lengths. The results show the accuracy of these methods even when the time step length is set to be large. Computational times show that these methods, except that of measuring LTT-1 with a time step length of 1 sec, meet the requirements of large-scale applications. Measuring LTT-1 with a time step length of 1 sec in the discrete-flow form suits for small-scale agent-based applications. Computational time is remarkably reduced when the time step length is increased. The accuracy and short computational time in the large time step demonstrate that the methods allow us to expedite DTA by enlarging the time step lengths.

It is interesting to treat an vehicle in the point-queue model as an object. It allows us to track the movements of a vehicle. Combining with other considerations, the point-queue model might be capable of reproducing congestion propagation and trajectories.

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