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Delays caused by motorized vehicles unable to clear intersections in China: Graphical analysis

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Abstract: In many Chinese cities, motorized vehicles (M-vehicles) move slowly at intersections due to the interference of a large number of non-motorized vehicles (NM-vehicles). The slow movement makes a part of M-vehicles fail to leave intersections timely after the traffic signal turns red, and thereby conflicts between vehicles from two directions occur. The phenomenon was analyzed graphically by using the cumulative vehicle curve. Delays in three cases were modeled and compared: NM-vehicle priorities and M-vehicle priorities with all-red intervals unable to release all vehicles, and longer all-red intervals ensuring release all vehicles. Marginal delays caused by two illegal behaviors that occasionally happened in mixed traffic intersections were also investigated. It is concluded that increasing the speed of M-vehicles leaving intersections and postponing the entering of NM-vehicles are the keys in mathematics, although they are uneasy in disordered mixed traffic intersections due to a dilemma between efficiency and orders in reality. The results could provide implications for the traffic management in the cities maintaining a large number of M- and NM-vehicles.

Key words: conflict delay; marginal delay; mixed traffic intersection; priority; cumulative count curve of vehicles

1 Introduction

Vehicle behaviors in mixed traffic intersections are complex, while the complexity is amplified greatly in the intersections with high demand and (or) loose supervision. Non-motorized vehicles (NM-vehicles) violate the red light, cut off a platoon of motorized vehicles (M-vehicles), and could even appear at any place of intersections at any time. M-vehicle drivers sometimes have to make quicker and extra manipulation in order to avoid NM-vehicles and pass through intersections. These are the scenes of intersections in many Chinese cities.

The chaos at mixed traffic intersections has gained attention of researchers, in particular of those in the developing countries of Asia. In China, SU et al [1] reported that urban traffic flow was often hindered by pedestrians at intersections so that intelligent traffic control systems did not work effectively. A simplified delay estimation model considering vehicle types and positions was proposed correspondingly. WEI et al [2] depicted serious congestion and capacity drop led by mixed traffic at at-grade signalized intersections in Beijing, and a method was presented to quantitatively measure non-motorized effects. WANG et al [3] investigated on bicycle conversion factors in the mixed traffic. Through and left-turn bicycle conversion factors for mixed traffic intersections were calibrated based on the field data collected from three Chinese cities.

In India, CHANDRA and AGRAWAL [4] proposed a service delay model based on microscopic analysis of delay data collected from five uncontrolled mixed traffic intersections. The results showed that the proportion of heavy vehicles in the conflicting traffic greatly affected the service delay. By considering passenger cars, three wheelers, two wheelers and heavy vehicles, ASHALATHA et al [5] investigated service delay under different priority of the four types at unsignalized mixed intersections.

Moreover, WANG and NIHAN [6] developed a method to estimate bicycle−motor vehicle accident risks based on probability theory. The method was then demonstrated using a 4-year data set collected from a number of signalized intersections in Tokyo, Japan. In order to depict the movements on virtual lanes, MINH et al [7] proposed a model framework for motorcycles at

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signalized intersections, and the data collected from Vietnam were used to validate the model. PRASETIJO et al [8] emphasized the various differences of intersections in Indonesia from those in developed countries, and introduced a new method of capacity analysis for developing countries based on possible conflict streams.

In the models to depict vehicle movements at mixed traffic intersections, LAN [9] developed inhomogeneous cellular automata models with both deterministic and stochastic rules. In the models, different types of vehicles were represented by non-identical numbers of cell units. Compared with cellular automata models, a two-dimensional car-following model was proposed by XIE et al [10]. Velocity difference terms were introduced as important factors for the traffic behavior. The results of numerical simulations showed that the straight-going M-vehicle flow just next to the NM-vehicle lane was disturbed more seriously than others.

This work carries on reporting and investigating a practical problem occurring at disordered mixed traffic intersections in China, that is, M-vehicles that cannot be timely released from mixed traffic intersections in signal transitions conflict with NM- and M-vehicles at the other direction. These disordered mixed traffic intersections exist in one of the biggest Chinese cities, Tianjin, and we also believe that they are common in many other Chinese cities and even other developing countries with a large number of NM-vehicles (see the traffic conditions in Beijing, China [11], and in Nairobi, Kenya [12] for example). One might suspect that the problem could be practically solved by arranging bicycle lanes or changing the intersection geometry. To the most of Chinese cities, however, it might be less likely to occur in the coming years because of the high costs and the social impacts. Besides, the effect might not be as expected due to the disorder at intersections.

The cumulative count curve (see, for example, Refs. [13−16]) is employed as a graphical analysis tool. Briefly introducing applications of the curve, in a typical queue system, the vertical and horizontal distances between an arrival count curve and a departure count curve are the queue length and the delay of corresponding vehicles, respectively, and the areas enclosed by the two curves are the total delay and the total queue length. Results of this work provide implications for the traffic management in the cities containing a large number of M- and NM-vehicles.

2 Problem formulation

When traffic lights turn red, M-vehicles that are not able to timely clear mixed traffic intersections might conflict with NM-vehicles entering the intersections from the other direction. The conflict frequently occurs when the traffic stream is heavy in Tianjin, a city of 10.43×10^6 inhabitants (in 2005), over 6×10^6 bicycles, and 1.7×10^6 M-vehicles (in 2010) (see Fig. 1 for the situations). We also believe that it happens in many other Chinese cities and probably other developing countries based on the following facts:

1) NM-vehicles enter intersections fast. NM-

Fig. 1 Conflicts caused by M-vehicles that cannot clear intersections when signals turn red in Tianjin, China: (a) Intersection between Jieyuan Road and Hongqi Road at 7:45am, Friday, May 27, 2011; (b) Intersection between Anshan Xidao and Santan Road at 6:11pm, Wednesday, May 18, 2011; (c) Intersection between Anshan Xidao and Baidi Road at 5:46pm, Thursday, July 14, 2011

vehicles have higher starting speed than M-vehicles most time; sometimes NM-vehicles even enter intersections before the signals turn green due to the difficulty of supervision for NM-vehicles.

2) M-vehicles move slowly at disordered mixed traffic intersections. Since NM-vehicles or pedestrians could appear at any place of intersections, M-vehicles have to slow down or even stop inside intersections to avoid collision. Moreover, the authorities suggest or require that M-vehicle drivers steer slowly at mixed traffic intersections. Therefore, vehicles cannot pass through intersections quickly particularly when the number of NM-vehicles is large, and the experience length of all-red intervals (for example, 2−3 s recommended by ROESS and PRASSAS [17] and Nation Research Council [18]) are sometimes too short for clearance.

To graphically analyze the problem, we use x and y to indicate two directions of intersections (see Fig. 2(a)); use M_x, M_y and NM to denote M-vehicles in direction x, M-vehicles in direction y, and NM-vehicles in direction y; use A, B and C to denote the potential conflicts between M_x and M_y, M_x and NM, and the stop line in direction y; call sections each from A to B, from C to A, and from C to B as "servers" for M_x, M_y and NM. Moreover, the starting time of the investigation is the time when the traffic signal in direction y turns green. The following basic notations will be used:

L_x, L_y: The distances from A to B, A (or B) to C;

n_{M_x}, n_{M_y}, n_{NM}: The numbers of M_x, M_y and NM in front of each server. Note that vehicles usually enter intersections with multi-stream from multi-lane, and for simplifications here we only consider one stream of vehicles;

d_{M_x}, d_{M_y}, d_{NM}: Spacing headways in M_x, M_y and NM, respectively.

v_{M_x}, v_{M_y}, v_{NM}: The speeds of M_x, M_y and NM, respectively, moving at intersections.

We suppose that 1) NM- and M-vehicles move at constant speeds, and also keep constant spacing headways at intersections either when moving or stopping. It is because that NM- and M-vehicles can only move quite slowly at the disordered mixed traffic intersections, and the propagation of congestions caused by acceleration and deceleration of vehicles are not significant as traffic flow in a link, and the differences of spacing headways between moving and stopping are also relatively small. In addition, the particular interest of this

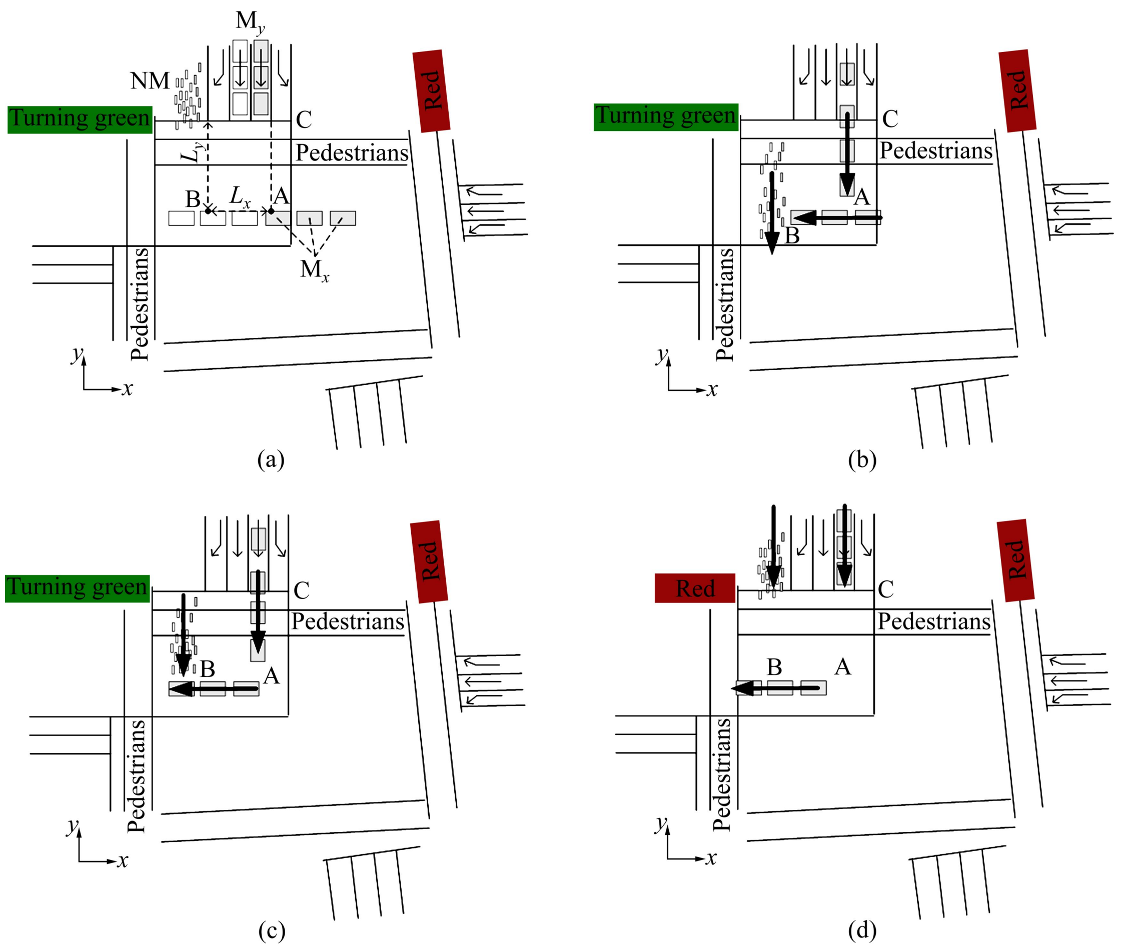

Fig. 2 Notations and illustrations for cases demonstrated using intersection between Anshan Xidao and Baidi Road shown in Fig. 1(c): (a) Notations; (b) An illustration for Case 1; (c) An illustration for Case 2; (d) An illustration for Case 3

work is the delay of NM- and M-vehicles instead of traffic dynamics at intersections; 2) the speed of NM is larger than that of M_y, i.e., $v_{NM}>v_{M_y}$; 3) M-vehicles in a platoon have priorities when two platoons of M-vehicles in two directions conflict.

3 Delays in three cases

This section compares delays in three different cases: Case 1 and 2: NM- and M-vehicle priorities with all-red intervals unable to release all vehicles, respectively; Case 3: longer all-red intervals ensuring release all vehicles. Case 1 and 2 do happen in Tianjin (see Figs. 1(a) and (c) for the situation in Case 1, and Fig. 1(b) for that in Case 2). Setting all-red intervals to prevent conflicts such as in Case 3 is a strategy usually recommended by signal control guides.

To better demonstrate the delays, we create an idealized base case in which vehicles encountering conflicts can pass through the conflicts at the same time, i.e., no conflict delay happens at intersections; it is unrealistic but could be a reference to measure delays. As illustrated in Fig. 3(a), we have the flow rates of M_x, M_y and NM:

$$\begin{cases} \lambda_{M_x}=\dfrac{v_{M_x}}{d_{M_x}} \\ \lambda_{M_y}=\dfrac{v_{M_y}}{d_{M_y}} \\ \lambda_{NM}=\dfrac{v_{NM}}{d_{NM}} \end{cases} \tag{1}$$

The storage capacities of the servers for M_x, M_y and NM are

$$\begin{cases} n^*_{M_x}=\dfrac{L_x}{d_{M_x}} \\ n^*_{M_y}=\dfrac{L_y}{d_{M_y}} \\ n^*_{NM}=\dfrac{L_y}{d_{NM}} \end{cases} \tag{2}$$

The times when the first vehicle of M_x, M_y and NM departs from each server are

$$t_1=\frac{L_x}{v_{M_x}} \tag{3}$$

$$t_2=\frac{L_y}{v_{M_y}} \tag{4}$$

$$t_3=\frac{L_y}{v_{NM}} \tag{5}$$

The times when the last vehicle of M_x, M_y and NM arrives at each server are

$$t_4=\frac{n_{M_x}}{\lambda_{M_x}}=\frac{n_{M_x}d_{M_x}}{v_{M_x}} \tag{6}$$

$$t_5=\frac{n_{M_y}}{\lambda_{M_y}}=\frac{n_{M_y}d_{M_y}}{v_{M_y}} \tag{7}$$

$$t_6=\frac{n_{NM}}{\lambda_{NM}}=\frac{n_{NM}d_{NM}}{v_{NM}} \tag{8}$$

Hence, the times when the last vehicle of M_x, M_y and NM leaves each server can be expressed as follows:

$$t_7=t_1+t_4 \tag{9}$$

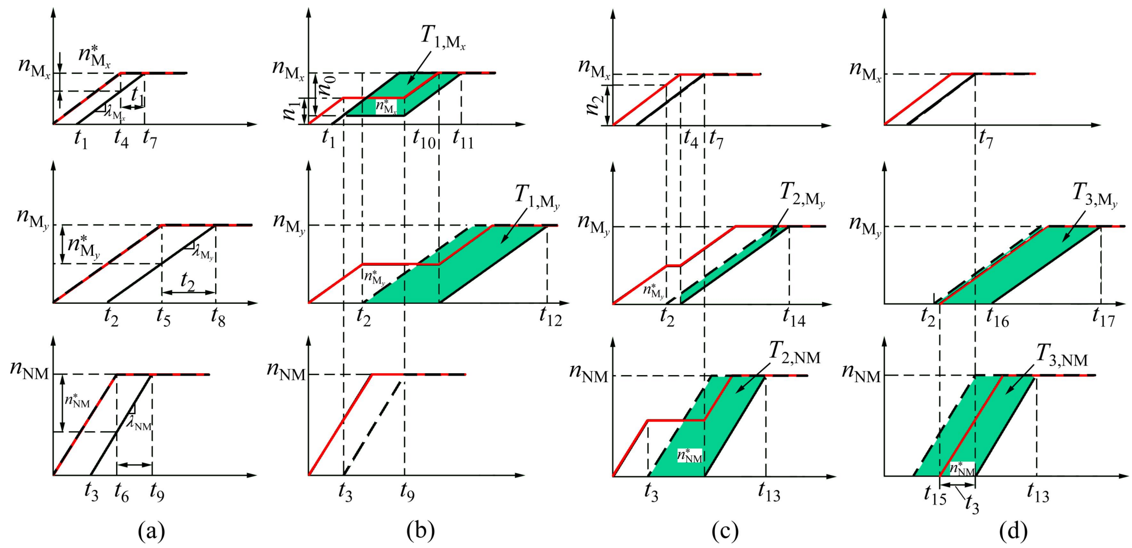

Fig. 3 Cumulative vehicle curves and delay analysis of cases: (a) Base case−Vehicles can pass through conflicts at same time; (b) Case 1−NM-vehicle priorities with all-red intervals unable to release all vehicles; (c) Case 2−M-vehicle priorities with all-red intervals unable to release all vehicles; (d) Case 3−Long-enough all-red intervals ensuring release all vehicles (Dotted line: cumulative departure curve in base case)

$$t_8=t_2+t_5 \tag{10}$$

$$t_9=t_3+t_6 \tag{11}$$

3.1 Case 1: NM-vehicle priorities with all-red intervals unable to release all vehicles

In the case, NM cut off M_x at B if not all M_x can pass B timely (see Fig. 2(b) for an illustration and Fig. 3(b) for graphical analysis). The number of M_x stuck by NM in front of B is $n_0=n_{M_y}-\lambda_{M_x}(t_3-t_1)$. Obviously, the conflict occurs if $n_0>0$. The condition in term of time can also be written as $t_1+t_4>t_3$, i.e., the time that the last of M_x passes B is later than that the first of NM arrives at B. The two conditions are the same and we can derive one from the other by simply expanding these equations.

Once the conflict occurs, only after all NM leave, M_x can go on moving and clear the intersection. In Fig. 3(b), M_x are stuck at t_3 and restart to move at t_9. The last vehicle of M_x arrives at and departs from its server at

$$t_{10}=t_4+(t_9-t_3)=t_4+t_6 \tag{12}$$

$$t_{11}=t_{10}+t_1=t_1+t_4+t_6 \tag{13}$$

Furthermore, if n_0 is larger than the maximum storage capacity of M_x, M_y will be delayed, i.e., $n_0>n_{M_x}$. M_y then stop in front of M_x at t_2 and leave its sever at

$$t_{12}=t_{10}+t_5=t_4+t_5+t_6 \tag{14}$$

Notice that the condition that M_y are stuck by M_x can also be written as follows:

$$\begin{aligned} & n_0 > n^{*}_{M_x} \\ \Rightarrow\ & n_{M_x} - \frac{v_{M_x}}{d_{M_x}}\left(\frac{L_y}{v_{NM}} - \frac{L_x}{v_{M_x}}\right) - \frac{L_x}{d_{M_x}} > 0 \\ \Rightarrow\ & n_{M_x} - \frac{v_{M_x} L_y}{d_{M_x} v_{NM}} > 0 \\ \Rightarrow\ & n_{M_x} - \lambda_{M_x} t_3 > 0 \\ \Rightarrow\ & n_{M_x} > n_1 \end{aligned} \tag{15}$$

where t_3 is the time NM take to arrive at B, and denote $n_1=\lambda_{M_x}t_3$. Therefore, the physical meaning can also be interpreted as that M_y will be stuck if M_x cannot leave A before NM reach B, and the part of that $n_{M_x}-n_1>0$ is the number of M_x that block M_y in front of A.

We take the departure curves in the base case as references, and the total delay of the case can thus be obtained as follows:

$$T_1 = T_{1,M_x} + T_{1,M_y} + T_{1,NM} \tag{16}$$

where the total delays of M_x, M_y and NM due to the cut of NM are

$$\begin{cases} T_{1,M_x} = \begin{cases} n_0(t_9-t_3)=n_0 t_6,\ t_1+t_4>t_3 \\ 0,\ \text{otherwise} \end{cases} \\ T_{1,M_y} = \begin{cases} n_{M_y}(t_{10}-t_2)=n_{M_y}(t_4+t_6-t_2),\ n_{M_x}>n_1 \\ 0,\ \text{otherwise} \end{cases} \\ T_{1,NM}=0 \end{cases} \tag{17}$$

Note that the delays could also be weighted with coefficients to represent differences in values of time, priorities of vehicles, etc.

3.2 Case 2: M-vehicle priorities with all-red intervals unable to release all vehicles

Similarly, if $t_1+t_4>t_3$, a conflict between NM and M_x occurs (see Fig. 2(c) for an illustration and Fig. 3(c) for graphics). In the case, comparably, NM have to wait in front of B at t_3 until all M_x leave their server at t_7. The last NM departs from the server at time

$$t_{13}=t_7+t_6=t_1+t_4+t_6 \tag{18}$$

If $n_{M_x}>n_2$ where $n_2=\lambda_{M_x}t_2$, M_y will be stuck from t_2 until t_4 because a part of M_x are not able to pass A before the time when M_y arrive at A, i.e., t_2. All M_y will be released at

$$t_{14}=t_4+t_5 \tag{19}$$

Therefore, the total delay is

$$T_2 = T_{2,M_x} + T_{2,M_y} + T_{2,NM} \tag{20}$$

where the delays of M_x, M_y and NM are

$$\begin{cases} T_{2,M_x}=0 \\ T_{2,M_y} = \begin{cases} n_{M_y}(t_4-t_2), n_{M_x}>n_2 \\ 0, \text{otherwise} \end{cases} \\ T_{2,NM} = \begin{cases} n_{NM}(t_1+t_4-t_3), t_1+t_4>t_3 \\ 0, \text{otherwise} \end{cases} \end{cases} \tag{21}$$

3.3 Case 3: Longer all-red intervals ensuring release all vehicles

If $t_1+t_4>t_3$, a conflict between NM and M_x would occur. In Case 3, all-red intervals are set long enough to prevent the conflict (see Fig. 2(d)). To the end, M_y and NM are held by the red light until the time that enables the last vehicle of M_x to leave B and the first vehicle of NM to just arrive at B (after experiencing travel time t_3). All-red intervals should be thus enlarged (see Fig. 3(d)) by

$$t_{15}=t_7-t_3=t_1+t_4-t_3 \tag{22}$$

where t_7 is the time when the last M_x leaves B and the first NM arrives at B. The time when all NM depart from the server is also t_{13}. The first and last vehicle of M_y will depart from its server at times

$$t_{16}=t_{15}+t_2=t_1+t_2+t_4-t_3 \tag{23}$$

$$t_{17}=t_{16}+t_5=t_1+t_2+t_4+t_5-t_3 \tag{24}$$

Correspondingly, the total delay is

$$T_3=T_{3,\mathrm{M}_x}+T_{3,\mathrm{M}_y}+T_{3,\mathrm{NM}} \tag{25}$$

where the delays of M_x, M_y and NM are

$$\begin{cases} T_{3,\mathrm{M}_x}=0 \\ T_{3,\mathrm{M}_y}=\begin{cases} n_{\mathrm{M}_y}(t_{16}-t_2)=n_{\mathrm{M}_y}(t_1+t_4-t_3),\ t_1+t_4>t_3 \\ 0,\ \text{otherwise} \end{cases} \\ T_{3,\mathrm{NM}}=T_{2,\mathrm{NM}}=\begin{cases} n_{\mathrm{NM}}(t_1+t_4-t_3),\ t_1+t_4>t_3 \\ 0,\ \text{otherwise} \end{cases} \end{cases} \tag{26}$$

The delays of M_y in Case 2 and Case 3 are the same, because they all rely on M_x. The difference is that M_y are stopped by M_x in Case 2 and M_y are held by all-red intervals which are set according to M_x in Case 3.

3.4 Delay comparisons

Since L_x and L_y are physical attributes of an intersection and n_{M_x}, n_{M_y}, n_{NM}, d_{M_x}, d_{M_y} and d_{NM} generally depend on traffic flow (or demands) on a network, they could be seen as invariants if the intersection and the flow are given. We thus only focus on the influence of v_{M_x}, v_{M_y} and v_{NM} on the conflict occurrence and delays in the cases. In particular, v_{M_x} could be great differences at situations such that if it is interferred by pedestrians or NM-vehicles.

3.4.1 Conflict occurrence and length of all-red intervals

We first check the condition that $t_1+t_4>t_3$, that is, NM block M_x in Case 1; M_x stop NM in Case 2; all-red intervals need to be enlarged in Case 3. Expanding t_1, t_4 and t_3 in the inequality using the relevant equations, it can be easily seen that greater v_{M_x} and smaller v_{NM} may prevent the occurrence in the three cases.

Then, we focus on the conflict occurrence of M_y stuck by M_x in Case 1 and Case 2, i.e., the conditions in Eqs. (17) and (21), because it is always unexpected that M-vehicles in one direction block those in the other. For Case 1, it can be seen mathematically from Inequality (15) that improving v_{M_x} or decreasing v_{NM} benefits to avoid blocking M_y; this is also logical. For Case 2, the condition is similar with that in Case 1 except for using t_2 instead of t_3, and improving v_{M_x} or decreasing M_y helps to avoid blocking M_y. Notice that decreasing v_{M_y} can help to avoid M_y stuck, but it doesn't really reduce the delay of v_{M_y} because slower moving or stopping in front of M_x makes no distinction to the delay.

For Case 3, we look into prolonged time that all-red intervals need, i.e., t_{15}. With Eqs. (3), (5), (6) and (9) substituted into Eq. (22), $t_{15}=(L_x+n_{\mathrm{M}_x}d_{\mathrm{M}_x})/v_{\mathrm{M}_x}-L_y/v_{\mathrm{NM}}$. It is clear to see that bigger v_{M_x} and smaller v_{NM} need shorter prolonged time, which makes sense because increasing v_{M_x} makes M_x released faster and decreasing v_{NM} is of service to avoid blocking. Table 1 presents the influences together.

Table 1 Influence factors of conflict occurrence and all-red intervals

Situation	v_{M_x}	v_{M_y}	v_{NM}
M_x stuck by NM in Case 1	↑	---	↓
M_y stuck by M_x in Case 1	↑	---	↑
M_y stuck by M_x in Case 2	↑	---	---
Shorter all-red intervals in Case 3	↑	---	↓

↑: Increasing variable can relieve situation described left; ↓: Decreasing variable can relieve situation; ---: Variable is unrelated to situation.

3.4.2 Comparisons of total delays in cases

To compare the cases, we focus on the total delays in the three cases. Expand the expressions of delays, i.e., Eqs. (17), (21) and (26) as follows (note that the parts equivalent to zero are not listed here for clarity of exposition):

$$T_{1,\mathrm{M}_x}=\frac{n_{\mathrm{NM}}}{v_{\mathrm{NM}}d_{\mathrm{M}_x}}(d_{\mathrm{M}_x}d_{\mathrm{NM}}n_{\mathrm{M}_x}-L_y v_{\mathrm{M}_x}+L_x d_{\mathrm{NM}}),\quad t_1+t_4>t_3 \tag{27}$$

$$T_{1,\mathrm{M}_y}=n_{\mathrm{M}_y}\left(\frac{n_{\mathrm{M}_x}d_{\mathrm{M}_x}}{v_{\mathrm{M}_x}}+\frac{n_{\mathrm{NM}}d_{\mathrm{NM}}}{v_{\mathrm{NM}}}-\frac{L_y}{v_{\mathrm{M}_y}}\right),\ n_{\mathrm{M}_x}>n_1 \tag{28}$$

$$T_{2,\mathrm{M}_y}=n_{\mathrm{M}_y}\left(\frac{n_{\mathrm{M}_x}d_{\mathrm{M}_x}}{v_{\mathrm{M}_x}}-\frac{L_y}{v_{\mathrm{M}_y}}\right),\ n_{\mathrm{M}_x}>n_2 \tag{29}$$

$$T_{2,\mathrm{NM}}=n_{\mathrm{NM}}\left(\frac{L_x+n_{\mathrm{M}_x}d_{\mathrm{M}_x}}{v_{\mathrm{M}_x}}-\frac{L_y}{v_{\mathrm{NM}}}\right),\ t_1+t_4>t_3 \tag{30}$$

$$T_{3,\mathrm{M}_y}=n_{\mathrm{M}_y}\left(\frac{L_x+n_{\mathrm{M}_x}d_{\mathrm{M}_x}}{v_{\mathrm{M}_x}}-\frac{L_y}{v_{\mathrm{NM}}}\right),\ t_1+t_4>t_3 \tag{31}$$

$$T_{3,\mathrm{NM}}=n_{\mathrm{NM}}\left(\frac{L_x+n_{\mathrm{M}_x}d_{\mathrm{M}_x}}{v_{\mathrm{M}_x}}-\frac{L_y}{v_{\mathrm{NM}}}\right),\ t_1+t_4>t_3 \tag{32}$$

Observing v_{M_x}, v_{M_y} and v_{NM} in the equations, we conclude the relations in Table 2. For T_{1,M_x}, higher v_{M_x} lessens the total delays of M_x by reducing the number of M_x stuck by NM, and higher v_{NM} shortens the duration of M_y stuck (referring to the top subfigure in Fig. 3(b) and Eq. (17)). For T_{1,M_y}, higher v_{NM} and v_{M_x} make intersections clear sooner. Slower v_{M_y} reduces the duration of M_y stuck in front of M_x by delaying the time when M_y arrive at A; it cannot really save time for M_y,

Table 2 Influence factors of total delays in three cases

Total delay	v_{M_x}	v_{M_y}	v_{NM}
T_{1,M_x}	↑	---	↑
T_{1,M_y}	↑	↓	↑
$T_{1,\mathrm{NM}}$	---	---	---
T_{2,M_x}	---	---	---
T_{2,M_y}	↑	↓	---
$T_{2,\mathrm{NM}}$	↑	---	↓
T_{3,M_x}	---	---	---
T_{3,M_y}	↑	---	↓
$T_{3,\mathrm{NM}}$	↑	---	↓

but it reduces the waiting time in front of M_x, i.e., T_{1,M_y}. Due to space limitations, we do not explain one by one. The basic rule is that increasing speed in Table 2 can really shorten the delay, while decreasing speed just reduces the time of stopping and waiting, and the duration of crossing intersections is not really changed.

In addition, we can see that $T_{1,\mathrm{M}_y} \geq T_{2,\mathrm{M}_y}$ and $T_{3,\mathrm{M}_y} \geq T_{2,\mathrm{M}_y}$. For the first statement, $n_1=\lambda_{\mathrm{M}_x}t_3>\lambda_{\mathrm{M}_x}t_2=n_2$ with the assumption that $v_{\mathrm{NM}}>v_{\mathrm{M}_y}$ and $T_{1,\mathrm{M}_y}=n_{\mathrm{M}_y}(t_4+t_6-t_2)>n_{\mathrm{M}_y}(t_4-t_2)=T_{2,\mathrm{M}_y}$, if $n_{\mathrm{M}_x}>n_2$. Similarly, it can be verified that $T_{3,\mathrm{M}_y} \geq T_{2,\mathrm{M}_y}$ in the assumption that $v_{\mathrm{NM}}>v_{\mathrm{M}_y}$. It turns out that M-vehicles in the other direction are affected least under M-vehicle priorities, compared with those in the other two cases.

Since $T_{3,\mathrm{M}_y} \geq T_{2,\mathrm{M}_y}$, $T_{2,\mathrm{M}_x}=T_{3,\mathrm{M}_x}=0$ and $T_{2,\mathrm{NM}}=T_{3,\mathrm{NM}}$, the total delay in Case 2 is not greater than that in Case 3, i.e., $T_2 \leq T_3$. This conclusion is sound in mathematics. The safety, however, should be concerned in practice. In the cases, only long-enough all-red intervals prevent all conflicts. Other comparisons of the total delays rely on specific parameter values, and we will present them in a practical illustration in Section 5.

In conclusion of the section, increasing the speed of M_x can effectively shorten or prevent the delay of M_y. Also, it can decrease the length of all-red intervals and thereby shorten the delay of all vehicles. Hence, the key in mathematics to deal with the delay is to increase the speed or flow traversing intersections. However, it is uneasy because of safety concern at disordered mixed traffic intersections. In Case 3, long-enough all-red intervals are beneficial to the speed increase because it makes intersections clear, but better supervision of NM-vehicles is essential to the effect. We will discuss it in Section 6.

4 Marginal delay of illegal behaviors

Two illegal behaviors happen occasionally in Tianjin: M-vehicles in a platoon violate the red light, and NM-vehicles enter intersections earlier before the light turning green. To measure the impact, this section presents the marginal delay caused by the illegal behaviors.

4.1 M-vehicles in direction *x* violating red light

In Case 1 where NM have priorities, the red-light violation of M_x will only result in M_y delay (see Fig. 4(a)). It turns out the marginal delay of NM and M_y:

$$\Delta T_1=\begin{cases} n_{\mathrm{M}_y}\dfrac{\Delta n_{\mathrm{M}_x}}{\lambda_{\mathrm{M}_x}}, & n_{\mathrm{M}_x}+\Delta n_{\mathrm{M}_x}>n_1 \\ 0, & \text{otherwise} \end{cases} \tag{33}$$

where Δn_{M_x} is the number of vehicles violating the red light of M_x.

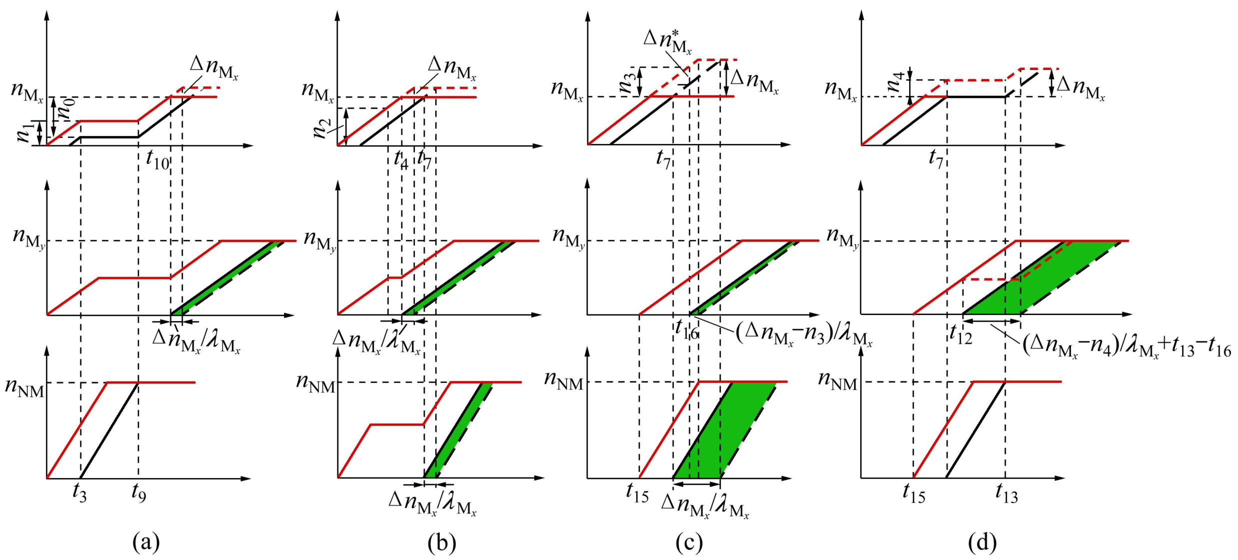

Fig. 4 Marginal delay of NM and M_y when Δn_{M_x} vehicles in M_x violate red light: (a) Case 1; (b) Case 2; (c) Case 3 with M-vehicle priorities; (d) Case 3 with NM-vehicle priorities (Dotted line: new curves caused by illegal behaviors)

In Case 2 where M_x have priorities, both NM and M_y can be affected by the behavior of violation (see Fig. 4(b)), and the marginal delay of NM and M_y is

$$\Delta T_2=\begin{cases} n_{M_y}\dfrac{\Delta n_{M_x}}{\lambda_{M_x}}+n_{NM}\dfrac{\Delta n_{M_x}}{\lambda_{M_x}},\ n_{M_x}+\Delta n_{M_x}>n_2 \\ n_{M_y}\dfrac{\Delta n_{M_x}}{\lambda_{M_x}},\ \text{otherwise}\end{cases} \tag{34}$$

Conflicts in Case 3 occur due to the red-light violation. If it is in M-vehicle priorities (see Fig. 4(c)), NM will be delayed first; when the number of vehicles violating the red light of M_x exceeds $n_3=n^*_{M_x}+(t_{16}-t_7)\cdot\lambda_{M_x}=n^*_{M_x}+(t_2-t_3)\lambda_{M_x}$, M_y will be stuck. The marginal delay of NM and M_y is

$$\Delta T_3=\begin{cases} n_{NM}\dfrac{\Delta n_{M_x}}{\lambda_{M_x}}+n_{M_y}\dfrac{\Delta n_{M_x}-n_3}{\lambda_{M_x}},\ \Delta n_{M_x}>n_3 \\ n_{NM}\dfrac{\Delta n_{M_x}}{\lambda_{M_x}},\ \text{otherwise}\end{cases} \tag{35}$$

For NM-vehicle priorities (see Fig. 4(d)), the vehicles violating the red light of M_x will be cut off by NM, and the marginal delay of NM and M_y is

$$\Delta T_4=\begin{cases} n_{M_y}\left[\dfrac{\Delta n_{M_x}-n_4}{\lambda_{M_x}}+(t_{13}-t_{16})\right]= \\ \qquad n_{M_y}\left[\dfrac{\Delta n_{M_x}-n_4}{\lambda_{M_x}}+(t_6+t_3-t_2)\right],\ \Delta n_{M_x}>n_4 \\ 0,\ \text{otherwise}\end{cases} \tag{36}$$

where $n_4=n^*_{M_x}$.

4.2 NM-vehicles violating red light

If NM violate the red light and enter intersections by Δt_{NM} earlier in Case 1 (see Fig. 5(a)), more M_x will be stuck while M_y will not be affected because the time when the last M_x leaves B is not changed; it is still t_{10}. In Case 2 and Case 3 with M-vehicle priorities, NM will not be stuck by the red-light violation of NM; thereby M_y will not be affected (see Figs. 5(b) and (c)). In Case 3 with NM-vehicle priorities (see Fig. 5(d)), if NM violate the red light earlier by the time smaller than $\tau_1=t_7-t_4=t_1$, all M_x will pass A before being stuck by NM and no M_y will be blocked by the stuck part of M_x; otherwise, if M_y arrive at A before all NM and M_x are released, i.e., $\tau_2=t_4+t_6-t_{16}=t_3+t_6-t_1-t_2>0$, the delay of M_y will be generated. Accordingly, the marginal delay of M_x and M_y is

$$\Delta T_5=\begin{cases} t_6\lambda_{M_x}\Delta t_{NM}+(t_3+t_6-t_1-t_2)n_{M_y}, \\ \qquad \Delta t_{NM}>\tau_1 \text{ and } \tau_2>0 \\ t_6\lambda_{M_x}\Delta t_{NM},\ \text{otherwise}\end{cases} \tag{37}$$

5 Practical illustrations

To better illustrate the problem discussed in this work, we take a set of data close to the observation from the intersection shown in Fig. 1(c) to demonstrate the cases (see Fig. 2 for the sketch of the intersection): L_x=9 m, L_y=18 m; v_{NM}=8 km/h, v_{M_y}=6.5 km/h, $v_{M_x}\in$[5 km/h, 15 km/h]; d_{M_x}=7 m, d_{M_y}=7 m, d_{NM}=2 m; n_{M_x}=3, n_{M_y}=10, n_{NM}=8. Although the values related to vehicles are absolutely not the same to every signal cycle, we believe that they could still shed light on the problem. Specifically, we focus on the influence of v_{M_x} on other

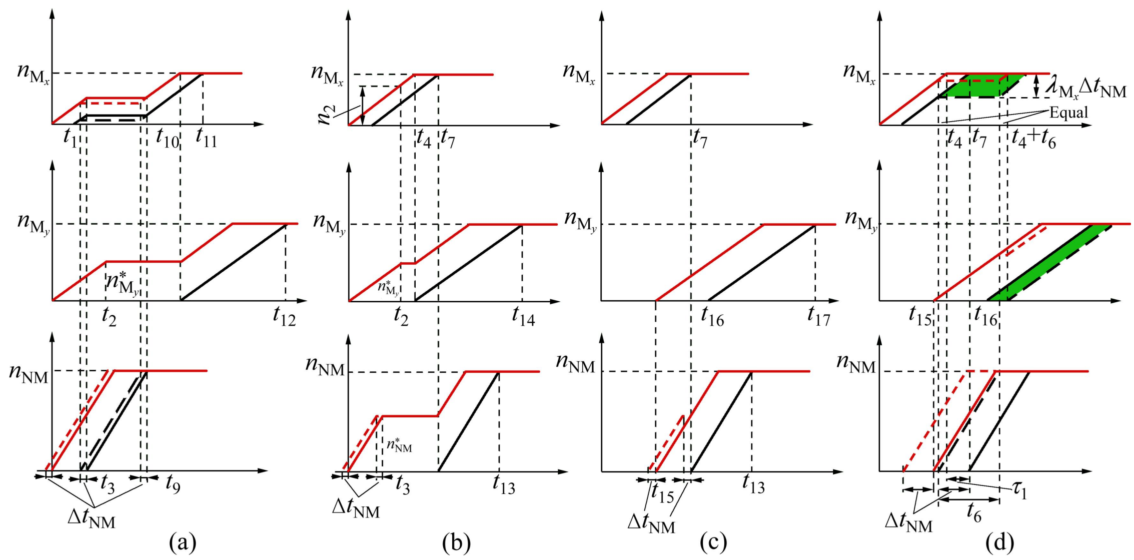

Fig. 5 Marginal delay of M_x and M_y when NM enter intersections earlier by Δt_{NM}: (a) Case 1; (b) Case 2; (c) Case 3 with M-vehicle priorities; (d) Case 3 with NM-vehicle priorities (Dotted line: new curves caused by illegal behaviors)

vehicles, total delays, and marginal delays.

Figure 6 presents the total delays in the cases with different v_{M_x}. We can see that under the given data:

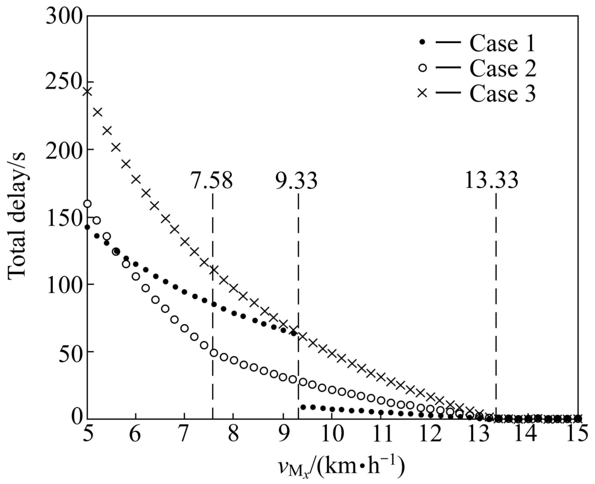

Fig. 6 Total delays with different v_{M_x} in three cases considering practical parameter values

1) The total delay in Case 3 is always greater than or equivalent to that in other cases. As will be discussed in Section 6, however, we cannot accordingly judge that Case 3 is the worst.

2) When v_{M_x} is greater than 7.58 km/h and 9.33 km/h, the delays of M_y in Case 2 and Case 1 will be prevented, respectively, i.e., T_{1,M_y}=0 and T_{1,M_y}=0. This implies that the intersections with M-vehicle priorities can accept slower M_x and will not make the M_y delay; it is also easy to understand logically, because NM are able to stop M_x in Case 1, but they are not in Case 2.

3) If v_{M_x}>13.33 km/h, no conflict between M_x and NM at B happens in Case 1 and Case 2, and the all-red intervals in Case 3 do not need to be prolonged. There is thus no delay for all cases.

4) A breakpoint in Case 1 implies that the delay of M_y dramatically increases the total delay in comparison with that in Case 2. Hence, it is more essential to prevent blocking M_y if NM-vehicle priorities are implemented.

To show actions of M_x, M_y and NM in each case, Table 3 lists the values of all times and delays calculated using the corresponding equations with v_{M_x}=6, 9, 12 km/h, respectively. It can be seen that the prolonged time needed by all-red intervals (i.e., t_{15}) are 9.9, 3.9 and 0.9 s, respectively; it turns out that all-red intervals need to be prolonged by a quite long time when the speed of M_x is low.

Figure 7 displays the marginal delays of M_x violating the red light in the cases with Δn_{M_x}=1. It can be usually seen that one M-vehicle violates the red (or amber) light within a platoon in reality. Breakpoints on ΔT_1 and ΔT_2 are due to the blocking of M_y, at M_x speeds of 12.44 and 10.11 km/h, respectively. Compared with Fig. 6, no M_y would have been delayed if v_{M_x}>9.33 km/h in Case 1 (v_{M_x}>7.58 km/h in Case 2), but if one M-vehicle in direction x violates the red light, the speed has to be enlarged to 12.44 km/h (10.11 km/h in Case 2)

Table 3 All parameters and values in practical illustration

Parameter	v_{M_x}/(km·h^{-1})			Parameter	v_{M_x}/(km·h^{-1})		
	6	9	12		6	9	12
t_1	5.4	3.6	2.7	$n^*_{M_x}$	1.29	1.29	1.29
t_2	9.97	9.97	9.97	$n^*_{M_y}$	2.57	2.57	2.57
t_3	8.1	8.1	8.1	n^*_{NM}	9	9	9
t_4	12.6	8.4	6.3	n_0	2.36	1.39	0.43
t_5	38.77	38.77	38.77	n_1	1.93	2.89	3.86
t_6	7.2	7.2	7.2	n_2	2.37	3.56	4.75
t_7	18	12	9	n_3	1.73	1.95	2.18
t_8	48.74	48.74	48.74	n_4	1.29	1.29	1.29
t_9	15.3	15.3	15.3	T_{1,M_x}	16.97	10.03	3.09
t_{10}	19.8	15.6	13.5	T_{1,M_y}	98.31	56.31	0
t_{11}	25.2	19.2	16.2	$T_{1,NM}$	0	0	0
t_{12}	58.57	54.37	—	T_{2,M_x}	0	0	0
t_{13}	25.2	19.2	16.2	T_{2,M_y}	26.31	0	0
t_{14}	51.37	—	—	$T_{2,NM}$	79.2	31.2	7.2
t_{15}	9.9	3.9	0.9	T_{3,M_x}	0	0	0
t_{16}	19.87	13.87	10.87	T_{3,M_y}	99	39	9
t_{17}	58.64	52.64	49.64	$T_{3,NM}$	79.2	31.2	7.2

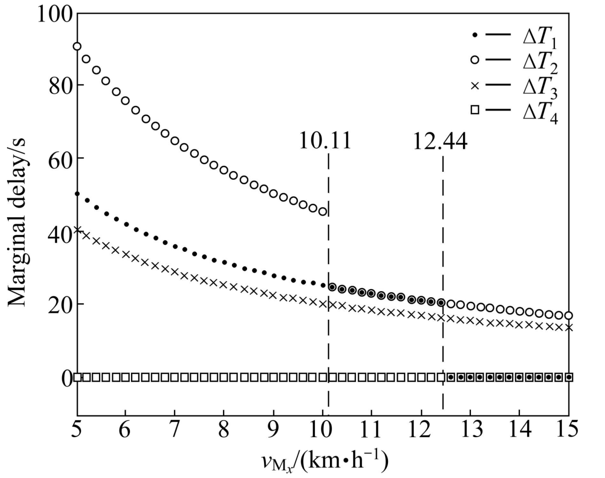

Fig. 7 Marginal delays of M_x violating red light in three cases (Δn_{M_x}=1)

and about 30 s (about 50 s in Case 2) marginal delay is increased in Case 1. In general, if v_{M_x}<10.11 km/h, the violation of one M_x produces the highest delay in Case 2, and no marginal delay is generated in Case 3 with NM-vehicle priorities in the situation, i.e., ΔT_4=0.

Figure 8 presents the marginal delays caused by NM violating the red light with v_{M_x}=6, 9 and 12 km/h, respectively. With the three speeds, τ_1 and τ_2 are respectively obtained as 5.4, 3.6, 2.7 s and −0.07, 1.73, 2.63 s. Hence, when Δt_{NM}>2.7 or 3.6 s with v_{M_x}=12 and 9 km/h, M_y will be stuck and large marginal delays are made; while for v_{M_x}=6 km/h, all M_x and NM have been released before M_y arrive at A. Additionally, it is noticed that the greater the v_{M_x} is, the larger the marginal delay caused by the same Δt_{NM}; it looks unusual. This is because the all-red interval discussed in this work is set based on v_{M_x}, i.e., the greater the v_{M_x} is, the shorter the all-red intervals will be, and then the relative negative effects of the same violation will be larger to shorten all-red intervals.

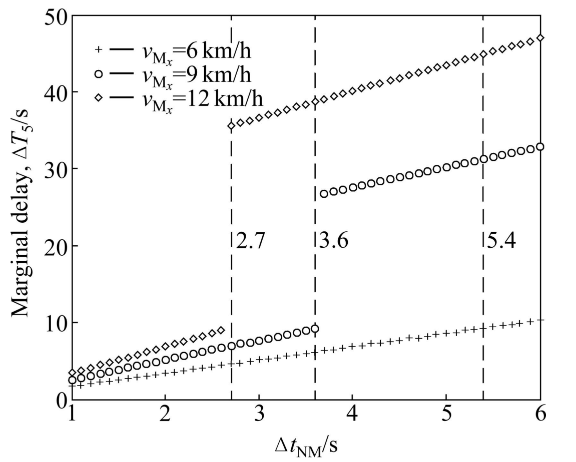

Fig. 8 Marginal delays of NM violating red light in three cases with v_{M_x}=6, 9, 12 km/h

6 Conclusions

Streams of both NM-vehicles and M-vehicles are heavy in rush hours in Tianjin, China. M-vehicles sometimes fail to clear intersections timely when traffic signals turn red. It would result in extra conflict delay of NM- and M-vehicles in the other direction. To straighten out the mess, we graphically investigate and model the delays in three cases by using the cumulative vehicle curve: 1) NM-vehicle priorities with all-red intervals unable to release all vehicles; 2) M-vehicle priorities with all-red intervals unable to release all vehicles; 3) Long-enough all-red intervals ensuring all vehicles to be released. These models could be used in measuring delay and conflicts at mixed traffic intersections. In addition, we analyze the marginal delays of M_x and NM violating the red light that occasionally happen in Tianjin. Results both directly from models and from a practical illustration show that improving the speed of M-vehicles traversing intersections and postponing the entering of NM-vehicles are the keys; it is just a conclusion in mathematics, however.

In practice, such as in Tianjin, improving the speed is uneasy, because NM-vehicles and pedestrians could appear at any place at intersections. The situation is that M-vehicle drivers steer slowly and carefully at intersections; once NM-vehicles or pedestrians appear, they have to drive slower or stop, and honk. Additionally, when conflicts occur between NM-vehicles/pedestrians and M-vehicles, the priorities do not go to one side by following some clear rules; instead, the priorities seem to more depend on “courage”. Hence, the situations in Case 1 and Case 2 are usually seen randomly at intersections with heavy traffic.

A dilemma gains the attention when we talk with the transportation authorities; it is that the works of transportation planners and engineers are based on M-vehicle priorities, while correct behaviors of NM-vehicle drivers are difficult to be enforced. The difficulties are further increased since a law has been passed, in which M-vehicle drivers are required to take full responsibility in the M- and NM-vehicle collisions. For example, to improve efficiency and reduce the stop-and-go of M-vehicles, some cycle times are set longer (such as 2−3 min determined based on the configuration of intersections), but it is unacceptable to NM-vehicles and pedestrians. In contrast, shorter cycle time satisfies NM-vehicles and pedestrians, and is more beneficial to make intersections ordered; the efficiency of M-vehicles and further, that of the traffic network is sacrificed, however. This is the reason that we emphasize that better supervision is essential to longer all-red intervals, because they would make NM-vehicles and pedestrians more impatient. Therefore, the authorities call it as a dilemma between efficiency and orders, which might be able to explain the problem from the policy perspective. Further influence of the conflict delay and the dilemma on adjacent intersections and network gridlock could be the future works.

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