Accuracy of Square Root of Positive Definite Matrix

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In [1, sec. 6.8.4] I give an algorithm for computing the Hermitian positive definite square root of a Hermitian positive definite matrix. Does this algorithm have any special accuracy properties compared with, say, computing $A^{1/2}$ using the spectral decomposition of A? That it might is suggested by the work on the high accuracy properties of Jacobi's method for positive definite matrices and for the SVD. Of course, the answer will depend on how the Hermitian polar factor is computed, which could be via the one-sided Jacobi SVD method (I am not worried about the cost at this point).

A related question is whether perturbation results for $A^{1/2}$ for positive definite A can be expressed in terms of $\min\{\kappa_2(D^{-1}AD) : D \text{ diagonal}\}$ instead of $\kappa_2(A)$.

References

[1] Nicholas J. Higham. *Functions of Matrices: Theory and Computation*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2008. xx+425 pp. ISBN 978-0-898716-46-7.

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