ECON10005 Quantitative Methods 1

Tutorial in Week 6

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Introduction

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Don't be shy if you need help

- visit Ed Discussion Board
- Go for lecturer's consultation sessions: see Canvas
- In case of special considerations, consult Stop 1
- For admin issues contact Chin via qm1-economics@unimelb.edu.au
- Email me with subject code titled: fan.z@unimelb.edu.au

Important notes

Data Analysis Report (10%): Draft Task (3%)

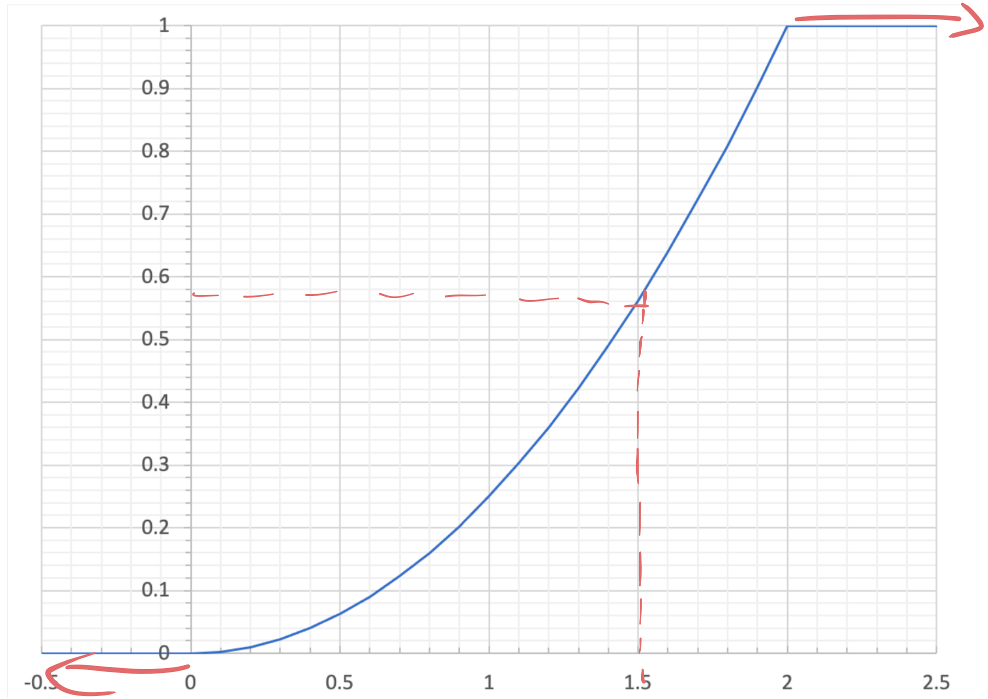
- Data set has be sent via Canvas announcement from me.
- Due on 6 April 2pm THIS Thursday. No extension, penalty applies.
- You MUST check your group members in Canvas!
- You MUST check your file before submit!
- You may not be in the same group for the final 7%.

A. Cumulative Distribution Function (Continuous)

(Pre-quiz) Suppose a continuous random variable X has the CDF

$P(X \le x) = F(x)$,

pdf: $\frac{dF(x)}{dx} = f(x)$.



1. What is the range of possible outcomes for X? $[0, 2]$.
2. As far as you can tell from the graph, what is $P(X \le 1.5)$? $= 0.56$
3. What feature of the CDF tells you that X is a continuous random variable? no steps.

A. Cumulative Distribution Function (Continuous)

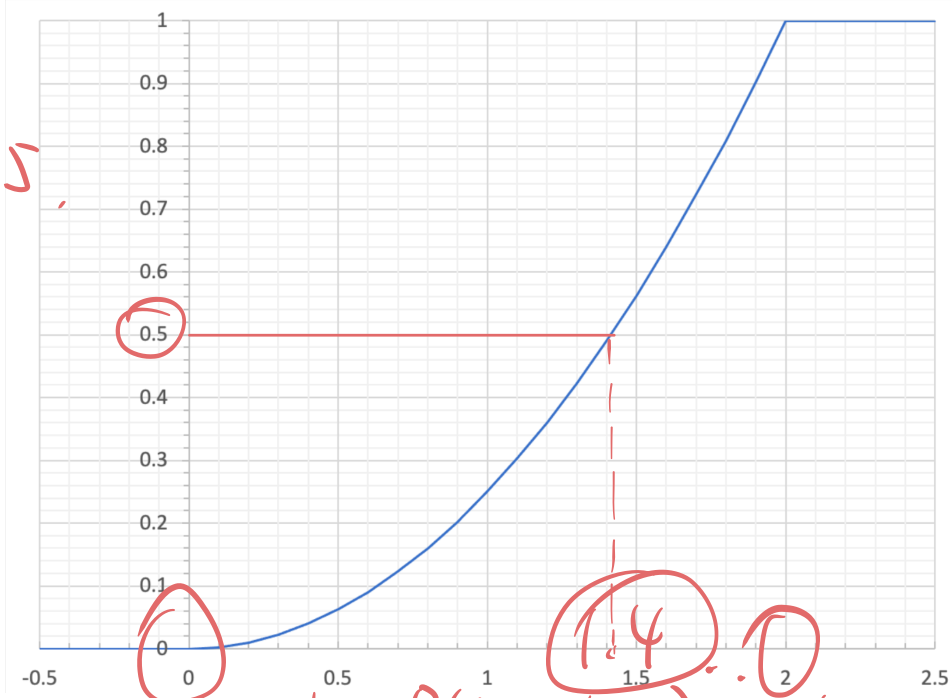
$P(X = x) = 0$.

(In-tute Part A) Suppose a continuous random variable X has the CDF

$F(x)$.

$P(X \leq x) = 0.5$.

median.



$= 1 - P(X \leq 1.5) = 1 - 0.56 = 0.44$.

4. What are $P(X > 1.5)$ and $P(X \geq 1.5)$?

$= 0.44$.

5. Do you think the probability density function of X would be symmetric?

pdf $= f(x)$.

A. Cumulative Distribution Function (Discrete)

Suppose X is the number of sixes from two independent dice rolls.

What are the possible outcomes?

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What are the corresponding probabilities (probability mass function)?

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Discrete random variable: Binomial distribution with 2 trials and probability of "success" (six) of 1/6

$n = 2$

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C_n^x

So the probability for each possible outcome (distribution table)?

$$P(X = x) = \binom{n}{x} \cdot p^x (1-p)^{n-x}$$

$$\frac{n!}{x!(n-x)!}$$

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So the probability for each possible outcome (distribution table)?

X	0	1	2
P	0.694	0.278	0.028

$$P(X=0) = \binom{2}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^2 = 0.694$$

$$P(X=1) = \binom{2}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^1 = 0.278$$

$$P(X=2) = \binom{2}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^0 = 0.028$$

A. Cumulative Distribution Function (Discrete)

pdf → continuous

→ discrete

PMF is therefore

$$P(X = x) = \binom{2}{x}\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{2-x} = \begin{cases} 0.694 & \text{if } x = 0 \\ 0.278 & \text{if } x = 1 \\ 0.028 & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

$x = -0.001$ $\quad P(X \leq -0.001) = 0$

$x = 0$ $\quad P(X \leq 0) = P(X=0) = 0.694$

The CDF is therefore

$$P(X \leq x) = F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 0.694, & \text{if } 0 \leq x < 1 \\ 0.972, & \text{if } 1 \leq x < 2 \\ 1, & \text{if } 2 \leq x \end{cases}$$

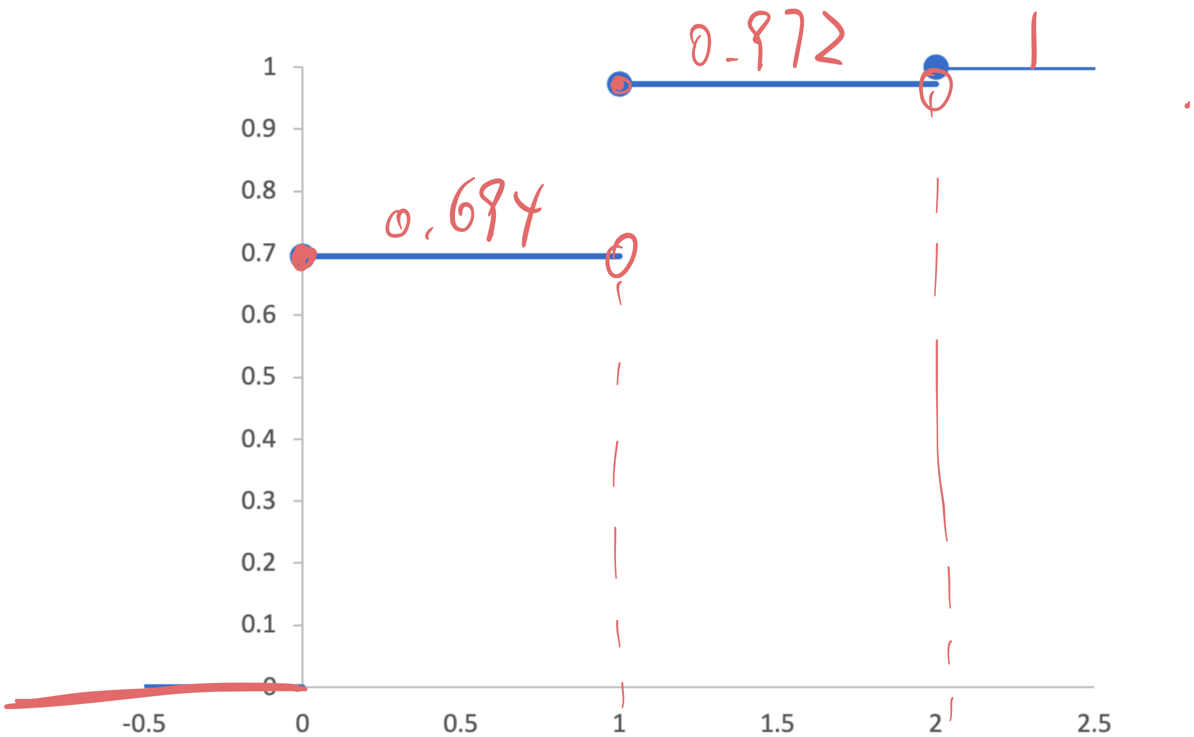
$x = 0.99$ $\quad P(X \leq 0.99) = P(X=0) = 0.694$

Which CDF figure appears to represent the distribution of X?

A. Cumulative Distribution Function (Discrete)

The CDF plot is therefore

$$P(X \leq x) = F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 0.694, & \text{if } 0 \leq x < 1 \\ 0.972, & \text{if } 1 \leq x < 2 \\ 1, & \text{if } 2 \leq x \end{cases}$$



B. Continuous Random Variable

$P(X \leq x) = 50\%$

median

Cumulative distribution function (CDF)

$$F(x) = P(X \leq x) = p$$

Inverse CDF (to obtain percentile)

$P(X \leq x) = 25\%$

median $x = F^{-1}(p)$

$x = F^{-1}(25\%)$

Probability density function (if F(x) is continuous and differentiable)

$$f(x) = dF(x)/dx$$

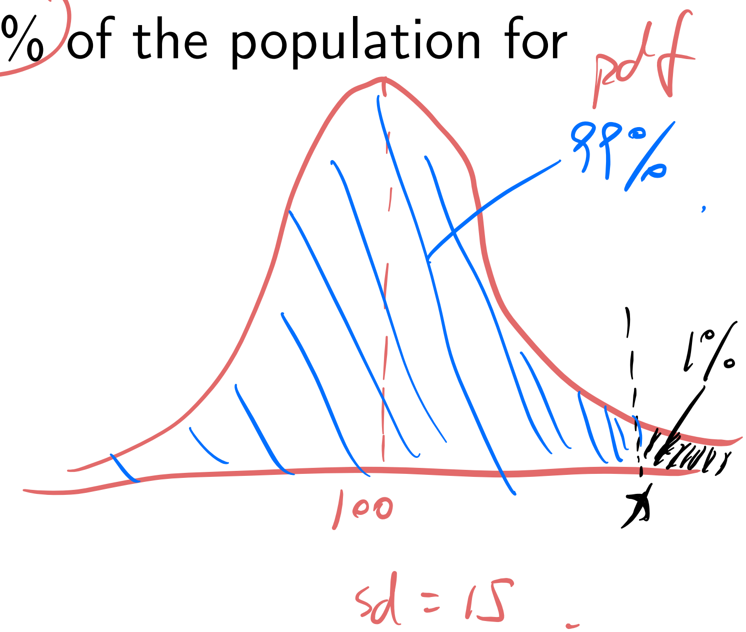
B. Continuous Random Variable

→ sd = 15.

variance

1. What IQ score is necessary to be in the top 1% of the population for intelligence? $X \sim N(100, 15^2)$

mean



Hint: in other words, find the 99% percentile

$$F(x) = P(X \leq x) = 0.99$$

Inverse CDF to obtain percentile

$$x = F^{-1}(p) = F^{-1}(0.99)$$

In Excel, "NORM.INV(0.99,100,15)" gives the answer $x_{0.99} = 134.90$

p mean sd.

in which case we have

$$P(X \leq 134.9) = 0.99 \quad \Rightarrow \quad P(X > 134.9) = 0.01$$

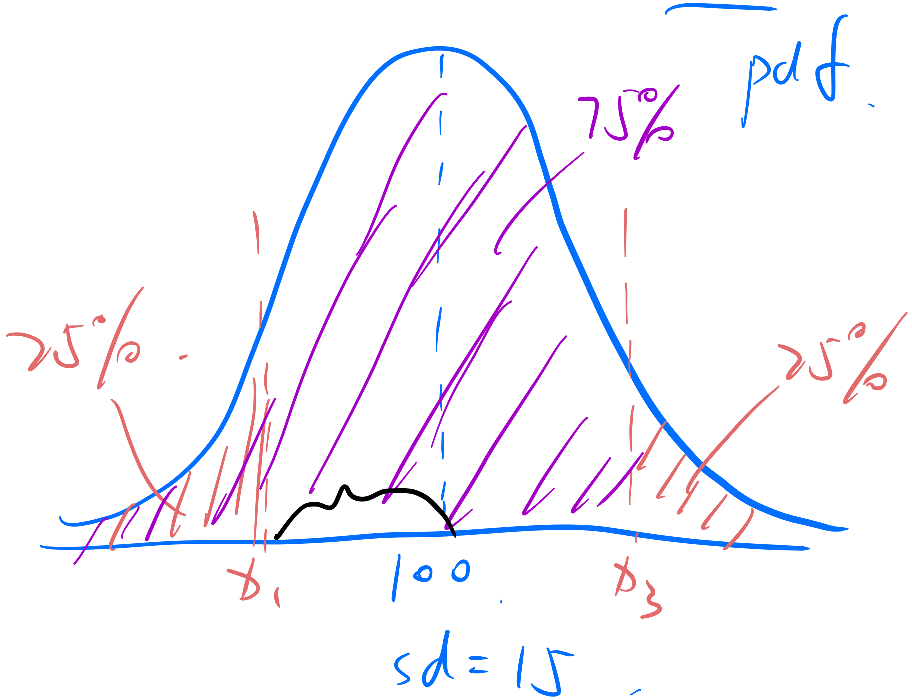
B. Continuous Random Variable

2. What is the range within which the "middle 50%" of the IQ scores fall? (i.e. $\underbrace{\text{leaving 25\% above}}_{\text{3rd}}$ the range and $\underbrace{\text{25\% below}}_{\text{1st}}$.)

B. Continuous Random Variable

2. What is the range within which the "middle 50%" of the IQ scores fall? (i.e. leaving 25% above the range and 25% below.)

Hint: in other words, find the 25% and 75% percentile, and IQR



$X \sim N(100, 15^2)$

$P(X \leq x_1) = 25\%$

$\rightarrow x_1 = F^{-1}(25\%)$

$P(X \leq x_3) = 75\%$

$\rightarrow x_3 = F^{-1}(75\%)$

B. Continuous Random Variable

2. What is the range within which the “middle 50%" of the IQ scores fall? (i.e. leaving 25% above the range and 25% below.)

Hint: in other words, find the 25% and 75% percentile, and IQR

$$F(x) = P(X \leq x) = 0.25$$

Inverse CDF to obtain percentile

$$x = F^{-1}(p) = F^{-1}(0.25)$$

$(100 - 89.88) \times 2$
$\parallel$
$20.24.$

In Excel, “NORM.INV(0.25,100,15)" gives the answer $x_{0.25} = 89.88$
In Excel, “NORM.INV(0.75,100,15)" gives the answer $x_{0.75} = 110.12$

in which case we have IQR

$$110.12 - 89.88 = 20.24$$

B. Continuous Random Variable

3. John Wayne tested his true grit on Angela Duckworth's test, but it turned out he was only acting gritty all that time and in real life scored 3.10. Discuss how you could estimate his position in the grittiness distribution.

$$10 + \frac{x - 2.51}{2.83 - 2.51} \times 10$$

$$20 + \frac{x - 2.83}{3.06 - 2.83} \times 10$$

Percentile	Male	Female
10	2.51	2.51
20	2.83	2.88
30	3.06	3.13
40	3.25	3.25
50	3.38	3.50
60	3.54	3.63
70	3.75	3.79
80	3.92	4.00
90	4.21	4.25
100	5.00	5.00

$$30 + \frac{3.1 - 3.06}{3.25 - 3.06} \times 10$$

$$\approx 32\%$$

B. Continuous Random Variable

The simplest way to do this is to linearly interpolate:

$$30 + \frac{3.10 - 3.06}{3.25 - 3.06} \times 10 = 32.11\%$$

That is, John Wayne is approximately at the 32nd percentile for grittiness.

If we do this for any $x \in [0, 5]$, we can obtain a continuous CDF. Try it!

B. Continuous Random Variable

Figure 1. CDF for grit scores by linear interpolation

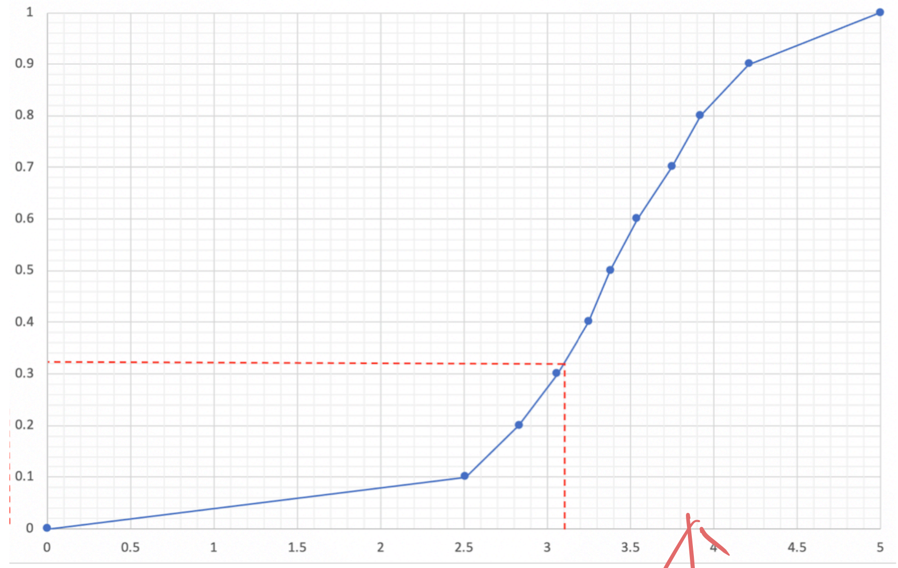
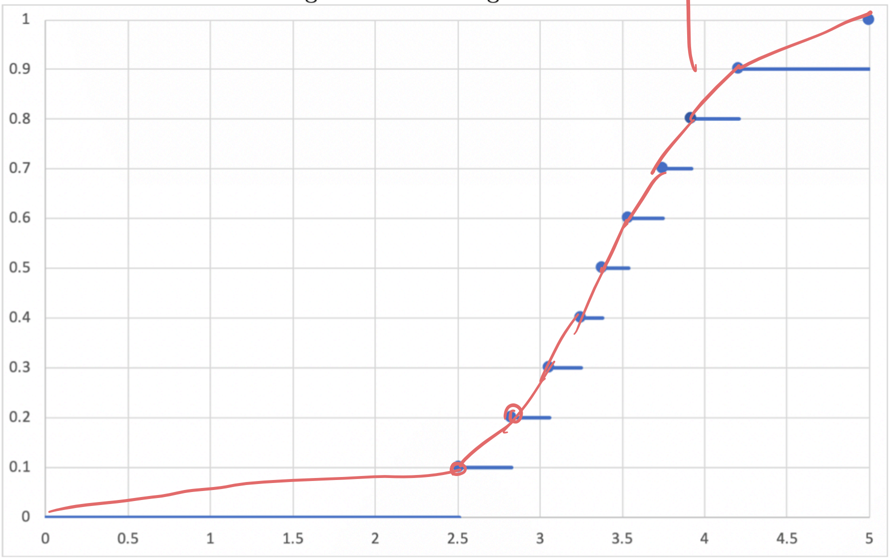


Figure 2. CDF for grit scores?



$y = ax + b$

The end

Thanks for your attention! 😄

Feel free to leave and enjoy the non-teaching week!