ECON10005 Quantitative Methods 1

Tutorial in Week 5

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Introduction

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Don't be shy if you need help

- visit Ed Discussion Board
- Go for lecturer's consultation sessions: see Canvas
- In case of special considerations, consult Stop 1
- For admin issues contact Chin via qm1-economics@unimelb.edu.au
- Email me with subject code titled: fan.z@unimelb.edu.au

Important notes

Data Analysis Report (10%): Draft Task (3%)

- Data set has be sent via Canvas announcement from me.
- Due on 6 April 2pm Thursday. No extension, penalty applies.
- You MUST check your group members in Canvas!
- Any issues has to be emailed to Chin at qm1-economics@unimelb.edu.au and cc me asap.

Mid-Semester Test 1 (10%):

- Carefully read the "MST Information - Logistics" on Canvas
- This Week, 30 March Thursday at Wilson Hall (and Kwong Lee Dow)
- Check your seat number, and bring your student ID

A. Law of Iterated Expectation (LIE)

Z	163.6	151.4
$P(Z)$	0.5	0.5

From Pre-quiz, we know

	$\overset{Z}{E(H \mid M)}$	$sd(H \mid M)$	$var(H \mid M)$	$P(M)$
$M = 1$	163.6	6.9	47.61	0.5
$M = 0$	151.4	6.4	40.96	0.5

- Find $E(H)$, hint: LIE: $E(H) = E\left(\overbrace{E(H|M)}^{Z}\right)$ $= E(Z)$
- Find $sd(H)$, hint: $\underline{var(Y) = E[var(Y \mid X)] + var[E(Y \mid X)]}$ (EVVE)

$$var(H) = E\left(var(H|M)\right) + var\left(E(H|M)\right)$$

A. Law of Iterated Expectation (LIE)

Known that

	$E(H \mid M)$	$\text{sd}(H \mid M)$	$\text{var}(H \mid M)$	$P(M)$
$M = 1$	163.6	6.9	47.61	0.5
$M = 0$	151.4	6.4	40.96	0.5

- Find $E(H)$

$$\begin{aligned} E(H) &= E(H \mid M = 1) \times P(M = 1) + E(H \mid M = 0) \times P(M = 0) \\ &= 163.6 \times 0.5 + 151.4 \times 0.5 \\ &= 157.5 \text{ cm} \end{aligned}$$

This should be the same as what you get in Pre-quiz.

A. Law of Iterated Expectation (LIE)

Z	47.61	40.96
$P(Z)$	0.5	0.5

Known that

	$E(H \mid M)$	$sd(H \mid M)$	$var(H \mid M)$	$P(M)$
$M = 1$	163.6	6.9	47.61 $=6.9^2$	0.5
$M = 0$	151.4	6.4	40.96 $=6.4^2$	0.5

- Find $sd(H)$, hint: $\text{var}(H) = E[\overbrace{\text{var}(H \mid M)}^{Z}] + \text{var}(\underline{E(H \mid M)})$

$= E(Z)$

$$
\begin{aligned}
E[\text{var}(H|M)] &= \text{var}(H \mid M = 1) \times P(M = 1) \\
&\quad + \text{var}(H \mid M = 0) \times P(M = 0) \\
&= 6.9^2 \times 0.5 + 6.4^2 \times 0.5 \\
&= \underline{44.285}
\end{aligned}
$$

A. Law of Iterated Expectation (LIE)

Z	163.6	151.4
$P(Z)$	0.5	0.5

Known that

	$E(H \mid M)$	$sd(H \mid M)$	$var(H \mid M)$	$P(M)$
$M = 1$	163.6	6.9	47.61	0.5
$M = 0$	151.4	6.4	40.96	0.5

- Find $sd(H)$, hint: $var(H) = E[var(H \mid M)] + var(\underbrace{E(H \mid M)}_{Z}) = var(Z)$

$var(Z) = E\left((Z - E(Z))^2\right)$

$$var(E(H \mid M)) = E\left[(E(H \mid M) - E[E(H \mid M)])^2\right]$$
$$= (163.6 - 157.5)^2 \times 0.5 + (151.4 - 157.5)^2 \times 0.5$$
$$= 37.21$$

A. Law of Iterated Expectation (LIE)

Known that

	$E(H \mid M)$	$\text{sd}(H \mid M)$	$\text{var}(H \mid M)$	$P(M)$
$M = 1$	163.6	6.9	47.61	0.5
$M = 0$	151.4	6.4	40.96	0.5

- Find $sd(H)$, hint: $\text{var}(H) = E[\text{var}(H \mid M)] + \text{var}(E(H \mid M))$

$$\begin{aligned} \text{var}(H) &= E[\text{var}(H \mid M)] + \text{var}(E(H \mid M)) \\ &= 44.285 + 37.21 \\ &= 81.495 \end{aligned}$$

so $sd(H) = \sqrt{81.495} \approx 9.027$

B. Harry Potter Monopoly

Potions (sum of dice = 2): The cards are shuffled into random order so the probability of drawing any particular one is 1/4 each. The probability distribution of M is thus

M	100	−150	0	−20
$P(M)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$$E(M) = 100 \times \frac{1}{4} + (-150) \times \frac{1}{4} + 0 \times \frac{1}{4} + (-20) \times \frac{1}{4} = -17.50$$

$$\begin{aligned} \text{var}(M) &= (100 - (-17.5))^2 \times \frac{1}{4} + (-150 - (-17.5))^2 \times \frac{1}{4} \\ &\quad + (0 - (-17.5))^2 \times \frac{1}{4} + (-20 - (-17.5))^2 \times \frac{1}{4} \\ &= 7918.75 \end{aligned}$$

$$sd(M) = 88.99$$

B. Harry Potter Monopoly

$S = 4$: fluffy's square.

-200

Charms (sum of dice = 7): The probability distribution of C is

c	-50	150	-200	0
$P(C = c)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

It follows that

$$\boxed{E(C) = -25}$$

$$\text{var}(C) = 15625$$

$$\text{sd}(C) = 125$$

B. Harry Potter Monopoly

\$1500

Define the random variable W to denote your cash balance after your first turn in Harry Potter monopoly. Calculate E(W).

The probability distribution of the sum of two dice S is

S	2	3	4	5	6	7	8	9	10	11	12
$P(S)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

?   ?

By LIE: $E(W) = E(W|S=2)P(S=2) + E(W|S=3)P(S=3) + \dots$
$+E(W|S=11)P(S=11) + E(W|S=12)P(S=12)$

B. Harry Potter Monopoly

There are three squares with possible outcomes that may affect your cash balance:

$S=2$. $\to E(W \mid S=2)$.

1. roll 2 to reach the Ministry of Magic square, produces M, then $W = 1500 + M$

$S=2$. $\to E(W \mid S=4) = 1300$.

2. roll 4 to reach Fluffy's square, which produces \$200, $W = 1500 - 200 = 1300$

$S=7$. $\to E(W \mid S=7)$.

3. roll 7 to reach the Charms square, which produces C, $W = 1500 + C$
4. roll anything else then W = 1500

B. Harry Potter Monopoly

1. roll 2 to reach the Ministry of Magic square, produces M, then $W = 1500 + M$

$$
\begin{aligned}
E(W \mid S=2) &= E(1500 + M \mid S = 2) \\
&= 1500 + E(M \mid S = 2) \\
&= 1500 + E(M) \\
&= 1500 - 17.50 = \boxed{1482.50}
\end{aligned}
$$

The outcome of the choice of card from the Ministry of Magic pile (M) is independent of the outcome of the dice roll

B. Harry Potter Monopoly

2. roll 4 to reach Fluffy's square, which produces \$200,
   $W = 1500 - 200 = 1300$

$$E(W|S = 4) = E(1300|S = 4) = 1300$$

   there is no randomness in the outcome

3. roll 7 to reach the Charms square, which produces C, $W = 1500 + C$

$$\begin{aligned} E(W \mid S = 7) &= E(1500 + C \mid S = 7) \\ &= 1500 + E(C \mid S = 7) && \text{(addition rule)} \\ &= 1500 + E(C) && \text{(independence)} \\ &= 1500 - 25 && \text{(calculated pre-tutorial)} \\ &= 1475 \end{aligned}$$

4. roll anything else then $W = 1500$

$$E(W|S \neq 2, 4, 7) = 1500$$

$E(W|S=3) = 1500$

$E(W|S=5) = 1500$

B. Harry Potter Monopoly

The LIE gives the calculation

$$
\begin{aligned}
E(W) =& E[E(W \mid S)] \\
=& E(W \mid S = 2) \times P(S = 2) + E(W \mid S = 4) \times P(S = 4) \\
& + E(W \mid S = 7) \times P(S = 7) + E(W \mid S \neq 2, 4, 7) \times P(S \neq 2, 4, 7) \\
=& E(W \mid S = 2) \times \frac{1}{36} + E(W \mid S = 4) \times \frac{3}{36} \\
& + E(W \mid S = 7) \times \frac{6}{36} + E(W \mid S \neq 2, 4, 7) \times \left(1 - \frac{1 + 3 + 6}{36}\right)
\end{aligned}
$$

$= P(S=3) + P(S=5) + P(S=6) + P(S=8) + \dots + P(S=12)$

Substituting everything we computed in 1, 2, 3, 4

$$
\begin{aligned}
E(W) &= 1482.50 \times \frac{1}{36} + 1300 \times \frac{3}{36} + 1475 \times \frac{6}{36} + 1500 \times \frac{26}{36} \\
&= 1478.68
\end{aligned}
$$

< 1500

C. Binomial Distribution

$V = 100 + \$1 \cdot X - \$1 \cdot (n - X)$

$\$100 \xrightarrow{0.55} \$101$; $\$100 \xrightarrow{0.45} \$99$ (1st week)

$\$101 \xrightarrow{0.55} 102$; $\$101 \xrightarrow{0.45} 100$; $\$99 \xrightarrow{0.55} 100$; $\$99 \xrightarrow{0.45} 98$ (2nd week)

Please do read the $\boxed{\textbf{Binomial.pdf}}$ note!

Consider the multiple period investment from the lecture ~~video~~ where \$100 is invested into an asset that each week might increase by \$1 with probability 0.55 or decrease by \$1 with probability 0.45. If V is the value after 52 weeks, calculate

n experiments

X : # of success   P(success) = 0.5

n − X : # of failure.

$X \sim \text{Binomial}(n, 0.5)$

1. P(V > 110)
2. P(V < 90)
3. P(V = 100)
4. How do these answers change if the probability of increase is 0.5?

X : # of weeks ↑ (success) P = 0.55

52 − X : # of weeks ↓

$X \sim \text{Binom}(52, 0.55)$

C. Binomial Distribution

$X = 26$.

$V = 100$.

Consider the multiple period investment from the lecture video where \$100 is invested into an asset that each week might increase by \$1 with probability 0.55 or decrease by \$1 with probability 0.45. If V is the value after 52 weeks, calculate

Define X to be a binomial random variable for the number of weekly increases (success) that occur with probability p for each of n = 52 weeks.

The value of the investment by the end of the year can be expressed

$$\begin{aligned} V &= 100 + 1 \cdot X - 1 \cdot (52 - X) \\ &= 48 + 2X \end{aligned}$$

C. Binomial Distribution

pdf (pmf)   pmf

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$.

cdf   pmf   pmf.

1. P(V > 110)

$$V = 48 + 2X > 110$$
$$X > 31$$

(success)

In order to have more than 110 values, we need at least 31 increases, out of total 52 independent experiment.

$$P(X > 31) = 1 - P(X \leq 31)$$

$= P(X=0) + P(X=1) + \cdots P(X=31)$.

Using excel, "= 1-BINOM.DIST(31,52,0.55,TRUE) = 0.210"

X   n   P   cdf.

C. Binomial Distribution

$P(X=x) = \binom{n}{x} x^{p} (n-x)^{1-p}$

2. P(V < 90)

$$V = 48 + 2X \; < \; 90$$

$$X \; < \; 21$$

In order to have less than 90 values, we need less than 21 increases, out of total 52 independent experiment.

$$P(X < 21) = P(X \leq 20)$$

$= P(X=0) + P(X=1) + \cdots P(X=20)$

Using excel, "= BINOM.DIST(20,52,0.55,TRUE) = 0.012"

$X \quad n \quad p \quad cdf$

C. Binomial Distribution

3. P(V = 100)

$$V = 48 + 2X = 100$$
$$X = 26$$

In order to have exactly 100 values, we need exactly 26 increases, out of total 52 independent experiment.

$$P(X = 26)$$

$$P(X=26) = \binom{52}{26} p^{26} (1-p)^{26}$$

Using excel, "= ~~1-~~BINOM.DIST(26,52,0.55,FALSE) = 0.085"

pmf.

The end

Thanks for your attention! 😄

Good luck with your mid semester test!