ECON10005 Quantitative Methods 1

Tutorial in week 4

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Introduction

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Don't be shy if you need help

- visit Ed Discussion Board
- Go for lecturer's consultation sessions: see Canvas
- In case of special considerations, consult Stop 1
- For admin issues contact Chin via qm1-economics@unimelb.edu.au
- Email me with subject code titled: fan.z@unimelb.edu.au

Section: Introduction 1

Important notes

Data Analysis Report (10%): Draft Task (3%)

- On Canvas: Assignment Group Registration (due by 24 March 5pm)
- Data set has be sent via Canvas announcement from me.
- Due in week 6, on 6 April 2pm Thursday

Mid-Semester Test 1 (10%):

- On Canvas: Register a exam session (due by 24 March 5pm)
- Carefully read the "MST Information Logistics" on Canvas
- In week 5, 30 March Thursday at Wilson Hall (and Kwong Lee Dow)

Section: Introduction 2

Pre-quiz 3 PartA: 1-4

Outcomes		Se	cond ro	II:				
		1	2	3	4	5	6	A: outcome is even
First roll:	1	1	2	3	4	5	6	B: outcome is greater than
	2	2	4	6	8	10	12	10
	3	3	6	9	/12	15	18	/ = -
	4	4	8	12	16	20	24	C: outcome is a square
	5	5	10	15	20	25	30	number
	6	6	12	18	24	30	36	

- A = {(2; 4; 6); (2; 4; 6; 8; 10; 12); (6; 12; 18); (4; 8; 12; 16; 20; 24); (10; 20; 30); (6; 12; 18; 24; 30; 36)}
- B = $\{(12); (12; 15; 18); (12; 16; 20; 24); (15; 20; 25; 30); (12; 18; 24; 30; 36)\}$
- A \cup B = Too long, total 30 elements
- $A \cap B = \{(12); (12; 18); (12; 16; 20; 24); (20; 30); (12; 18; 24; 30; 36)\}$
- $C = \{(1; 4); (4); (9); (4; 16); (25; 36)\}$

Note that the list should exclude multiple listing of outcomes that occur multiple ways. The reason for listing multiple outcomes is for the purpose of calculating probability.

PCANBIC)-PCC)
P (ANB)

Outcomes		Se	cond ro	II:				• $P(A) = 27/36$ $P(C \cap A)$
		1	2	3	4	5	6	• $P(B) = 17/36$
First roll:	1	1	2	3	4	5	6	
	2	2	4	6	8	10	12	• $P(A \cap B) = 14/36$
	3	3	6	9	12	15	18	• $P(C) = 8/36$
	4	4	8	12	16	20	24	• $P(A \cup B) = P(A) + P(B) -$
	5	5	10	15	20	25	30	
	6	6	12	18	24	30	36	$P(A \cap B) = 30/36$

- $A = \{(2; 4; 6); (2; 4; 6; 8; 10; 12); (6; 12; 18); (4; 8; 12; 16; 20; 24); (10; 20; 30); (6; 12; 18; 24; 30; 36)\}$
- $B = \{(12); (12; 15; 18); (12; 16; 20; 24); (15; 20; 25; 30); (12; 18; 24; 30; 36)\}$
- A \cup B = Too long, total 30 elements
- $A \cap B = \{(12); (12; 18); (12; 16; 20; 24); (20; 30); (12; 18; 24; 30; 36)\}$
- $C = \{(1; 4); (4); (9); (4; 16); (25; 36)\}$

Note that the list should exclude multiple listing of outcomes that occur multiple ways. The reason for listing multiple outcomes is for the purpose of calculating probability.

Pre-quiz 3 PartA: 5

5. Consider the event consisting of outcomes that are odd numbers greater than 10. Express this in terms of A, B, C and calculate its probability.

Outcomes		Sec	ond ro	II:				A: outcome is even
		1	2	3	4	5	6	B: outcome is greater than
First roll:	1	1	2	3	4	5	6	D. Outcome is greater than
	2	2	4	6	8	10	12	10
	3	3	6	9	12	15	18	 Outcomes that are odd and
	4	4	8	12	16	20	24	groator than 10 are the
	5	5	10	15	20	(25)	30	greater than 10 are the
	6	6	12	18	24	30	36	event $\overline{A} \cap B$

- $\bar{A} \cap B = \{(15); (15; 25)\}$
- $P(\bar{A} \cap B) = 3/36$

Pre-quiz 3 PartA: 6

$$\frac{P(B|C)}{P(C)} = \frac{P(AB) \cdot P(B)}{P(C)}$$

$$= \frac{P(BAC)}{P(C)}$$

Outcomes		Se	cond ro	II:				• Calculate $P(B \mid C)$,
		1	2	3	4	5	6	$P(C \mid B)$
First roll:	1	1	2	3	4	5	6	$P(C \mid B)$
	2	2	4	6	8	10	12	B: outcome is greater than
	3	3	6	9	12	15	18	10
	4	4	8	12	16	20	24	
	5	5	10	15	20	25	30	C: outcome is a square
	6	6	12	18	24	30	36	number /

- B = {(12); (12; 15; 18); (12; 16; 20; 24); (15; 20; 25; 30); (12; 18; 24; 30; 36)}
- $C = \{(1; 4); (4); (9); (4; 16); (25; 36)\}$
- B \cap C = {16; 25; 36}, so $P(B \cap C) = 3/36$

Pre-quiz 3 PartA: 6

- 6. Calculate P(B|C) and P(C|B). Are B and C independent events? $-P(B) \cdot P(C) \stackrel{?}{=} P(B \cap C)$
- Known that

•
$$P(B) = 17/36$$

- P(C) = 8/36
- $P(B \cap C) = 3/36$

The conditional probability can be easily calculated

$$P(B \mid C) = \frac{P(B \cap C)}{P(C)} = \frac{3/36}{8/36} = \frac{3}{8}$$

$$P(C \mid B) = \frac{P(B \cap C)}{P(B)} = \frac{3/36}{17/36} = \frac{3}{17}$$

B and C are not independent

 $P(C \mid A \cap B)$ 1. Calculate $P(C \mid A \cap B)$, and compare it with P(C), $P(C \mid B)$.

1. Calculate $P(C \mid A, B)$, and compare it with P(C), $P(C \mid B)$.

By the conditional probability formula

$$P(C \mid A, B) = \frac{P(C \cap A, B)}{P(A, B)} = \frac{P(C \cap A \cap B)}{P(A \cap B)}$$

Recall

- A: outcome is even
- B: outcome is greater than 10
- C: outcome is a square number; $C = \{(1; 4); (4); (9); (4; 16); (25; 36)\}$
 - Hence, $A \cap B \cap C = \{16, 36\}$, so $P(C \cap A \cap B) = 2/36$

1. Calculate $P(C \mid A, B)$, and compare it with P(C), $P(C \mid B)$.

Recall

- $P(C \cap A \cap B) = 2/36$
- Previously, from pre-quiz $P(A \cap B) = 14/36$

By the conditional probability formula

e conditional probability formula
$$P(C \mid A, B) = \frac{P(C \cap A, B)}{P(A, B)} = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{2/36}{14/36} = \boxed{\frac{2}{14}} = \boxed{\frac{2}{14}}$$

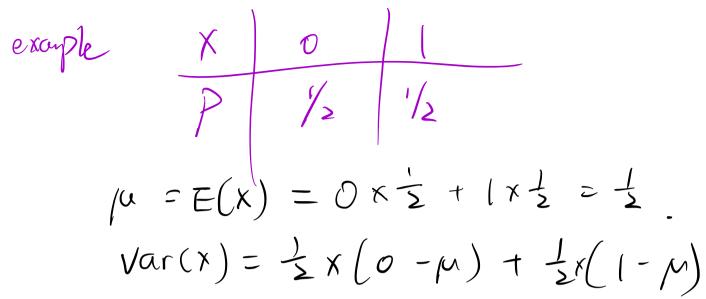
1. Calculate $P(C \mid A, B)$, and compare it with P(C), $P(C \mid B)$.

$$P(C) = 8/36 \approx 0.222$$

 $P(C \mid B) = 3/17 \approx 0.176$
 $P(C \mid A, B) = 1/7 \approx 0.143$

Bayesian updating and a prominent application is in the updating of probabilities as new information arrives sequentially over time

Monty Hall problem (a interesting probability puzzle)



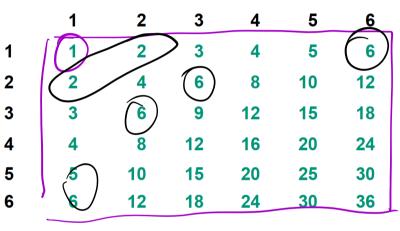
- 2. Define a random variable X to be the outcome from Part A of the pre-quiz, i.e. from multiplying the results from rolling two dice.
- (a) What is the probability distribution of X?
- (b) Calculate E(X) and sd(X).



Outcomes

Second roll:

First roll:



- 2. Define a random variable *X* to be the outcome from Part A of the pre-quiz, i.e. from multiplying the results from rolling two dice.
 - (a) What is the probability distribution of X?

X 1 2 3 4 5 6 8 ... 24 25 30 36

Freq 1 2 2 3 2 4 2 ... 2 1 2 1

Prob
$$\frac{1}{36}$$
 $\frac{2}{36}$ $\frac{2}{36}$ $\frac{3}{36}$ $\frac{2}{36}$ $\frac{4}{36}$ $\frac{2}{36}$... $\frac{2}{36}$ $\frac{1}{36}$ $\frac{2}{36}$ $\frac{1}{36}$ $\frac{2}{36}$

$$E(x) = \mu$$

(b) Calculate E(X) and s.d.(X).

By the definition of expect value

$$\mu = E(X) = 1 \cdot \frac{1}{36} + 2 \cdot \frac{2}{36} + 3 \cdot \frac{2}{36} + \dots + 36 \cdot \frac{1}{36} \approx 12.25$$

(b) Calculate E(X) and s.d.(X).

X
$$1$$
 2 3 4 5 6 8 ... 24 25 30 36
Freq 1 2 2 3 2 4 2 ... 2 1 2 1
Prob $\frac{1}{36}$ $\frac{2}{36}$ $\frac{2}{36}$ $\frac{3}{36}$ $\frac{2}{36}$ $\frac{4}{36}$ $\frac{2}{36}$... $\frac{2}{36}$ $\frac{1}{36}$ $\frac{2}{36}$ $\frac{1}{36}$

$$Var(X) = E[(X - \mu)^{2}] = \sum_{i=1}^{18} P(X_{i}) \cdot (X_{i} - 12.25)^{2}$$

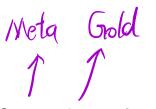
$$= \frac{1}{36} \cdot (1 - 12.25)^{2} + \frac{2}{36} \cdot (2 - 12.25)^{2}$$

$$+ \frac{2}{36} \cdot (3 - 12.25)^{2} + \dots + \frac{1}{36} \cdot (36 - 12.25)^{2}$$

$$\approx 79.965$$

so $sd(X) = \sqrt{79.965} \approx 8.942$

Part B



- 1. Compare the mean and variance properties of M, G and P, where P = 0.5M + 0.5G.
- 2. Define P more generally as P = wM + (1 w)G for any weight w between 0 and 1. Calculate E(P) and var(P) for w = 0, 0.1, 0.2, ..., 0.9, 1. Which of these has the lowest variance (risk)?

Hint: addition rules for E(Z) and Var(Z), where Z = aX + bY

$$\underbrace{E(Z) = aE(X) + bE(Y)}_{Var(Z) = a^{2}Var(X) + b^{2}Var(Y) + 2abCov(X, Y)}$$

Section: Part B Questions

Part B

From excel, we have

	M	G	W=0.5
mean	0.137	0.141	
var	32.134	4.109	
Cov	1.514		

Given
$$P = wM + (1 - w)G$$
, it is easy to calculate $E(P)$ and $Var(P)$.
$$E(P) = E(wM + (1 - w)G) = wE(M) + (1 - w)E(G)$$

$$= 0.137 \cdot w + 0.141 \cdot (1 - w)$$

$$Var(P) = Var(wM + (1 - w)G)$$

$$= w^2 Var(M) + (1 - w)^2 Var(G) + 2w(1 - w)Cov(M, G)$$

$$= w^2 \cdot 32.134 + (1 - w)^2 \cdot 4.109 + 2w(1 - w) \cdot 1.514$$

Part B

Substituting w from 0 to 1, we can have

W	E(P)	Var(P)
0	0.1408	4.11
0.1	0.1404	3.92
0.2	0.1401	4.40
0.3	0.1397	5.54
0.4	0.1394	7.35
0.5	0.1390	9.82
0.6	0.1387	12.95
0.7	0.1383	16.75
8.0	0.1379	21.21
0.9	0.1376	26.34
1	0.1372	32.13

The end

Thanks for your attention!

Feel free to leave and see you next week!

Section: End 18