ECON10005 Quantitative Methods 1

Tutorial in Week 5

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Introduction

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Don't be shy if you need help

- visit Ed Discussion Board
- Go for lecturer's consultation sessions: see Canvas
- In case of special considerations, consult Stop 1
- For admin issues contact Chin via qm1-economics@unimelb.edu.au
- Email me with subject code titled: fan.z@unimelb.edu.au

Section: Introduction 1

Important notes

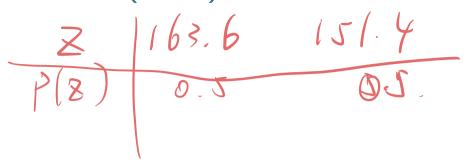
Data Analysis Report (10%): Draft Task (3%)

- Data set has be sent via Canvas announcement from me.
- Due on 6 April 2pm Thursday. No extension, penalty applies.
- You MUST check your group members in Canvas!
- Any issues has to be emailed to Chin at qm1-economics@unimelb.edu.au and cc me asap.

Mid-Semester Test 1 (10%):

- Carefully read the "MST Information Logistics" on Canvas
- This Week, 30 March Thursday at Wilson Hall (and Kwong Lee Dow)
- Check your seat number, and bring your student ID

Section: Introduction 2



From Pre-quiz, we know 7

	E(H M)	$sd(H \mid M)$	$var(H \mid M)$	P(M)
M=1	163.6	6.9	47.61	0.5
M=0	151.4	6.4	40.96	0.5

• Find E(H), hint: LIE: E(H) = E(E(H|M)) = E(Z)

• Find
$$sd(H)$$
, hint: $var(Y) = E[var(Y \mid X)] + var[E(Y \mid X)]$

$$var(H) = E\left(var(H \mid M)\right) + var\left(E(H \mid M)\right)$$

Known that

	$E(H \mid M)$	$sd(H \mid M)$	$var(H \mid M)$	P(M)
M=1	163.6	6.9	47.61	0.5
M=0	151.4	6.4	40.96	0.5

Find *E(H)*

$$E(H) = \underbrace{E(H \mid M = 1)}_{= 163.6 \times 0.5 + 151.4 \times 0.5} \times P(M = 1) + \underbrace{E(H \mid M = 0)}_{= 157.5 \text{ cm}} \times P(M = 0)$$

$$= 157.5 \text{ cm}$$

This should be the same as what you get in Pre-quiz.

Known that

	$E(H \mid M)$	$sd(H \mid M)$	$var(H \mid M)$	P(M)
M=1	163.6	6.9	47.61	?≥ 0.5
M=0	151.4	6.4	40.96	≥ 0.5

• Find sd(H), hint: $var(H) = E[var(H \mid M)] + var(E(H \mid M))$

$$E[var(H|M)] = var(H | M = 1) \times P(M = 1)$$

$$+ var(H | M = 0) \times P(M = 0)$$

$$= 6.9^{2} \times 0.5 + 6.4^{2} \times 0.5$$

$$= 44.285$$

Known that

	$E(H \mid M)$	$sd(H \mid M)$	$var(H \mid M)$	P(M)
M=1	163.6	6.9	47.61	0.5
M=0	151.4	6.4	40.96	0.5

• Find
$$sd(H)$$
, hint: $var(H) = E[var(H \mid M)] + var(E(H \mid M))$
 $var(E(H \mid M)) = E[(E(H \mid M) - E[E(H \mid M)])^2]$
 $= (163.6 - 157.5)^2 \times 0.5 + (151.4 - 157.5)^2 \times 0.5$
 $= 37.21$

Known that

	$E(H \mid M)$	$sd(H \mid M)$	$var(H \mid M)$	P(M)
M=1	163.6	6.9	47.61	0.5
M=0	151.4	6.4	40.96	0.5

• Find sd(H), hint: $var(H) = E[var(H \mid M)] + var(E(H \mid M))$

$$var(H) = E[var(H \mid M)] + var(E(H \mid M))$$

$$= 44.285 + 37.21$$

$$= 81.495$$
so $sd(H) = \sqrt{81.495} \approx 9.027$

Potions (sum of dice = 2): The cards are shuffled into random order so the probability of drawing any particular one is 1/4 each. The probability distribution of M is thus

$$E(M) = 100 \times \frac{1}{4} + (-150) \times \frac{1}{4} + 0 \times \frac{1}{4} + (-20) \times \frac{1}{4} = -17.50$$

$$var(M) = (100 - (-17.5))^{2} \times \frac{1}{4} + (-150 - (-17.5))^{2} \times \frac{1}{4}$$

$$+ (0 - (-17.5))^{2} \times \frac{1}{4} + (-20 - (-17.5))^{2} \times \frac{1}{4}$$

$$= 7918.75$$

$$sd(M) = 88.99$$

B. Harry Potter Monopoly S = Y: fliffy)s square.

Charms (sum of dice = 7): The probability distribution of C is

It follows that

$$E(C) = -25$$

$$var(C) = 15625$$

$$sd(C) = 125$$

\$1500

Define the random variable W to denote your cash balance after your first turn in Harry Potter monopoly. Calculate E(W).

The probability distribution of the sum of two dice S is

By LIE:
$$E(W) = E(W|S = 2)P(S = 2) + E(W|S = 3)P(S = 3) + ...$$

+ $E(W|S = 11)P(S = 11) + E(W|S = 12)P(S = 12)$

There are three squares with possible outcomes that may affect your cash

1. (roll 2) to reach the Ministry of Magic square, produces M, then roll 4 to reach Fluffy's square, which produces \$200, W = 1500 - 200 = 1300

$$\widetilde{W} = 1500 + M$$

$$\overline{W} = 1500 - 200 = 1300$$

- W = 1500 200 = 1300Froll 7) to reach the Charms square, which produces C, W = 1500 + C
- 4. roll anything else then W = 1500

1. roll 2 to reach the Ministry of Magic square, produces M, then

$$E(W \mid S = 2) = E(1500 + M \mid S = 2)$$

$$= 1500 + E(M \mid S = 2)$$

$$= 1500 + E(M)$$

$$= 1500 - 17.50 = 1482.50$$

The outcome of the choice of card from the Ministry of Magic pile (M) is independent of the outcome of the dice roll

2. roll 4 to reach Fluffy's square, which produces \$200,

$$W = 1500 - 200 = 1300$$

$$E(W|S=4) = E(1300|S=4) = 1300$$

there is no randomness in the outcome

3. roll 7 to reach the Charms square, which produces C, W = 1500 + C

$$E(W \mid S = 7) = E(1500 + C \mid S = 7)$$

= $1500 + E(C \mid S = 7)$ (addition rule)
= $1500 + E(C)$ (independence)
= $1500 - 25$ (calculated pre-tutorial)

4. roll anything else then W = 1500

$$E(W|S \neq 2, 4, 7) = 1500$$

$$E(W|S \neq 2, 4, 7) = 1500$$

The LIE gives the calculation

$$E(W) = E[E(W \mid S)]$$

$$= E(W \mid S = 2) \times P(S = 2) + E(W \mid S = 4) \times P(S = 4)$$

$$+ E(W \mid S = 7) \times P(S = 7) + E(W \mid S \neq 2, 4, 7) \times P(S \neq 2, 4, 7)$$

$$= E(W \mid S = 2) \times \frac{1}{36} + E(W \mid S = 4) \times \frac{3}{36}$$

$$+ E(W \mid S = 7) \times \frac{6}{36} + E(W \mid S \neq 2, 4, 7) \times \left(1 - \frac{1 + 3 + 6}{36}\right)$$

Substituting everything we computed in 1, 2, 3, 4

uting everything we computed in 1, 2, 3, 4
$$= P(S=3) + P(S=5)$$

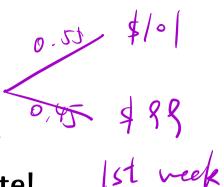
$$E(W) = 1482.50 \times \frac{1}{36} + 1300 \times \frac{3}{36} + 1475 \times \frac{6}{36} + 1500 \times \frac{26}{36} + P(S=6)$$

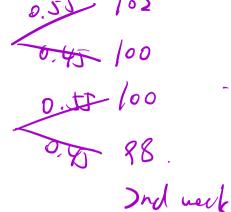
$$= 1478.68$$

$$= 1478.68$$







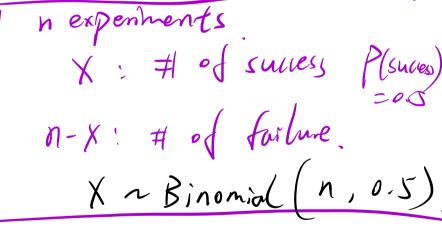


Please do read the Binomial.pdf note!

Consider the multiple period investment from the lecture video where \$100 is invested into an asset that each week might increase by \$1 with probability 0.55 or decrease by \$1 with probability 0.45. If V is the value

after 52 weeks, calculate

- 1. P(V > 110)
- 2. P(V < 90)
- 3. P(V = 100)



4. How do these answers change if the probability of increase is 0.5?

$$\chi = 26$$

$$V = 100$$

Consider the multiple period investment from the lecture video where \$100 is invested into an asset that each week might increase by \$1 with probability 0.55 or decrease by \$1 with probability 0.45. If V is the value after 52 weeks, calculate

Define X to be a binomial random variable for the number of weekly increases (success) that occur with probability p for each of n = 52 weeks.

The value of the investment by the end of the year can be expressed

$$V = 100 + 1 \cdot X - 1 \cdot (52 - X)$$

$$= 48 + 2X$$

ution
$$P(X \le 3) = P(X=0) + P(X=1) + P(X=1) + P(X=2) + P($$

1.
$$P(V > 110)$$

$$V = 48 + 2X > 110$$

 $X > 31$

In order to have more than 110 values, we need at least 31 increases, out of total 52 independent experiment.

$$P(X > 31) = 1 - P(X \leq 31)$$

Using excel, "= 1-BINOM.DIST(31,52,0.55,TRUE) = 0.210"

Csuciess

C. Binomial Distribution
$$P(\chi = x) = {n \choose s} x^{p} (n-x)^{r-p}$$

2. P(V < 90)

$$V = 48 + 2X < 90$$
$$X < 21$$

In order to have less than 90 values, we need less than 21 increases, out of total 52 independent experiment.

$$P(X < 21) = P(X \le 20) = P(X = 0) + P(X = 1)$$

$$\Delta \qquad \qquad + \cdots P(X = 0)$$
DIST(20.52.0.55 TRUE) = 0.012"

Using excel, "= BINOM.DIST(20,52,0.55,TRUE) = 0.012

3.
$$P(V = 100)$$

$$V = 48 + 2X = 100$$
$$X = 26$$

In order to have exactly 100 values, we need exactly 26 increases, out of total 52 independent experiment.

$$P(X=26) \qquad \qquad P(X=26) = \begin{pmatrix} 52 \\ 26 \end{pmatrix} = \begin{pmatrix} 52 \\$$

Using excel, "= BINOM.DIST(26,52,0.55,FALSE) = 0.085"

The end

Thanks for your attention!

Good luck with your mid semester test!

Section: End 20