ECON10005 Quantitative Methods 1

Tutorial in week 4

Zheng Fan

The University of Melbourne

Introduction

Zheng Fan

- Ph.D student in Economics at Unimelb

Don't be shy if you need help

- visit Ed Discussion Board
- Go for lecturer's consultation sessions: see Canvas
- In case of special considerations, consult Stop 1
- For admin issues contact Chin via qm1-economics@unimelb.edu.au
- Email me with subject code titled: fan.z@unimelb.edu.au

Important notes

Data Analysis Report (10%): Draft Task (3%)

- On Canvas: Assignment Group Registration (due by 24 March 5pm)
- Data set has be sent via Canvas announcement from me.
- Due in week 6, on 6 April 2pm Thursday

Mid-Semester Test 1 (10%):

- On Canvas: Register a exam session (due by 24 March 5pm)
- Carefully read the "MST Information - Logistics" on Canvas
- In week 5, 30 March Thursday at Wilson Hall (and Kwong Lee Dow)

Pre-quiz 3 PartA: 1-4

Outcomes

		Second roll:					
		1	2	3	4	5	6
First roll:	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

- A: outcome is even
- B: outcome is greater than 10
- C: outcome is a square number

- A = {(2; 4; 6); (2; 4; 6; 8; 10; 12); (6; 12; 18); (4; 8; 12; 16; 20; 24); (10; 20; 30); (6; 12; 18; 24; 30; 36)}
- B = {(12); (12; 15; 18); (12; 16; 20; 24); (15; 20; 25; 30); (12; 18; 24; 30; 36)}
- A ∪ B = Too long, total 30 elements
- A ∩ B = {(12); (12; 18); (12; 16; 20; 24); (20; 30); (12; 18; 24; 30; 36)}
- C = {(1; 4); (4); (9); (4; 16); (25; 36)}

Note that the list should exclude multiple listing of outcomes that occur multiple ways. The reason for listing multiple outcomes is for the purpose of calculating probability.

Pre-quiz 3 PartA: 1-4

$$P(C \mid A \cap B) = \frac{P(A \cap B \mid C) \cdot P(C)}{P(A \cap B)} = \frac{P(C \cap A \cap B)}{P(A \cap B)}$$

Outcomes **Second roll:**

		1	2	3	4	5	6
First roll:	**1**	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

- $P(A) = 27/36$
- $P(B) = 17/36$
- $P(A \cap B) = 14/36$
- $P(C) = 8/36$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 30/36$

- A = {(2; 4; 6); (2; 4; 6; 8; 10; 12); (6; 12; 18); (4; 8; 12; 16; 20; 24); (10; 20; 30); (6; 12; 18; 24; 30; 36)} 27 elements.
- B = {(12); (12; 15; 18); (12; 16; 20; 24); (15; 20; 25; 30); (12; 18; 24; 30; 36)} 17 elements.
- A ∪ B = Too long, total 30 elements
- A ∩ B = {(12); (12; 18); (12; 16; 20; 24); (20; 30); (12; 18; 24; 30; 36)}
- C = {(1; 4); (4); (9); (4; 16); (25; 36)}

Note that the list should exclude multiple listing of outcomes that occur multiple ways. The reason for listing multiple outcomes is for the purpose of calculating probability.

Pre-quiz 3 PartA: 5

$P(A) + P(\bar{A}) = 1$

$\bar{A}$

5. Consider the event consisting of outcomes that are odd numbers greater than 10. Express this in terms of A, B, C and calculate its probability.

Outcomes

		Second roll:					
		1	**2**	**3**	**4**	**5**	**6**
First roll:	**1**	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

- A: outcome is even
- B: outcome is greater than 10
- Outcomes that are odd and greater than 10 are the event $\bar{A} \cap B$

- $\bar{A} \cap B = \{(15); (15; 25)\}$
- $P(\bar{A} \cap B) = 3/36$

Pre-quiz 3 PartA: 6

$P(B|C) = \frac{P(C|B)\cdot P(B)}{P(C)}$

$= \frac{P(B \cap C)}{P(C)}$

Outcomes		Second roll:					
		1	2	3	4	5	6
First roll:	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

- Calculate $P(B \mid C)$, $P(C \mid B)$
- B: outcome is greater than 10
- C: outcome is a square number

- B = {(12); (12; 15; 18); (12; 16; 20; 24); (15; 20; 25; 30); (12; 18; 24; 30; 36)}
- C = {(1; 4); (4); (9); (4; 16); (25; 36)}
- B ∩ C = {16; 25; 36}, so $P(B \cap C) = 3/36$

Pre-quiz 3 PartA: 6

6. Calculate P(B|C) and P(C|B). Are B and C independent events?

Known that

- $P(B) = 17/36$
- $P(C) = 8/36$
- $P(B \cap C) = 3/36$

$P(B) \cdot P(C) \stackrel{?}{=} P(B \cap C)$

The conditional probability can be easily calculated

$$P(B \mid C) = \frac{P(B \cap C)}{P(C)} = \frac{3/36}{8/36} = \frac{3}{8} \neq P(B)$$

$$P(C \mid B) = \frac{P(B \cap C)}{P(B)} = \frac{3/36}{17/36} = \frac{3}{17}$$

B and C are not independent

Part A

$P(C \mid A \cap B)$

1. Calculate $P(C \mid A, B)$, and compare it with $P(C)$, $P(C|B)$.

Part A

1. Calculate $P(C \mid A, B)$, and compare it with $P(C)$, $P(C|B)$.

By the conditional probability formula

$$P(C \mid A, B) = \frac{P(C \cap A, B)}{P(A, B)} = \frac{P(C \cap A \cap B)}{P(A \cap B)}$$

Recall

- A: outcome is even
- B: outcome is greater than 10
- C: outcome is a square number; C = {(1; 4); (4); (9); (4; $\overset{>10}{16}$); (~~25~~; 36)}
- Hence, $A \cap B \cap C = \{16; 36\}$, so $P(C \cap A \cap B) = 2/36$

Part A

1. Calculate $P(C \mid A, B)$, and compare it with $P(C)$, $P(C|B)$.

Recall

- $P(C \cap A \cap B) = 2/36$
- Previously, from pre-quiz $P(A \cap B) = 14/36$

By the conditional probability formula

$$P(C \mid A, B) = \frac{P(C \cap A, B)}{P(A, B)} = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{2/36}{14/36} = \frac{2}{14} = \frac{1}{7} \approx 0.143$$

Part A

P(go to school) = 0.8.
P(go to school | sleepy) = 0.5.
P(go to school | sleepy, rain) = 0.1.

1. Calculate $P(C \mid A, B)$, and compare it with $P(C)$, $P(C|B)$.

$$P(C) = 8/36 \approx 0.222$$
$$P(C \mid B) = 3/17 \approx 0.176$$
$$P(C \mid A, B) = 1/7 \approx 0.143$$

Bayesian updating and a prominent application is in the updating of probabilities as new information arrives sequentially over time

Monty Hall problem (a interesting probability puzzle)

Part A

example

X	0	1
P	1/2	1/2

$$\mu = E(X) = 0 \times \frac{1}{2} + 1 \times \frac{1}{2} = \frac{1}{2}.$$

$$Var(X) = \frac{1}{2} \times (0 - \mu) + \frac{1}{2} \times (1 - \mu)$$

2. Define a random variable X to be the outcome from Part A of the pre-quiz, i.e. from multiplying the results from rolling two dice.

   (a) What is the probability distribution of X?

   (b) Calculate $E(X)$ and $sd(X)$.

Part A

5 times $\{1, 4, 6, 9, 36\}$

$\bar{X}$.

Outcomes **Second roll:**

		1	2	3	4	5	6
First roll:	**1**	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

2. Define a random variable X to be the outcome from Part A of the pre-quiz, i.e. from multiplying the results from rolling two dice.

   (a) What is the probability distribution of X?

X	1	2	3	4	5	6	8	...	24	25	30	36
Freq	1	2	2	3	2	4	2	...	2	1	2	1
Prob	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	...	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$= 1$.

$E(X) = \mu$.

Part A

(b) Calculate $E(X)$ and $s.d.(X)$.

X	1	2	3	4	5	6	8	...	24	25	30	36
Freq	1	2	2	3	2	4	2	...	2	1	2	1
Prob	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	...	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

By the definition of expect value

$$\mu = E(X) = 1 \cdot \frac{1}{36} + 2 \cdot \frac{2}{36} + 3 \cdot \frac{2}{36} + \cdots + 36 \cdot \frac{1}{36} \approx 12.25$$

Part A

(b) Calculate $E(X)$ and $s.d.(X)$.

X	1	2	3	4	5	6	8	...	24	25	30	36
Freq	1	2	2	3	2	4	2	...	2	1	2	1
Prob	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	...	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\begin{aligned}
\mathrm{Var}(X) = E[(X-\mu)^2] &= \sum_{i=1}^{18} \underline{P(X_i)} \cdot (X_i - \overset{\mu}{\underline{12.25}})^2 \\
&= \frac{1}{36}\cdot(1-12.25)^2 + \frac{2}{36}\cdot(2-12.25)^2 \\
&\quad + \frac{2}{36}\cdot(3-12.25)^2 + \cdots + \frac{1}{36}\cdot(36-12.25)^2 \\
&\approx \underline{79.965}
\end{aligned}$$

so $\mathrm{sd}(X) = \sqrt{79.965} \approx \underline{8.942}$

Part B

Meta Gold

1. Compare the mean and variance properties of M, G and P, where $P = 0.5M + 0.5G$. $E(P)$ $var(P)$

2. Define P more generally as $P = wM + (1 - w)G$ for any weight w between 0 and 1. Calculate $E(P)$ and $var(P)$ for $w = 0, 0.1, 0.2, \ldots,$ 0.9, 1. Which of these has the lowest variance (risk)?

Hint: addition rules for $E(Z)$ and $Var(Z)$, where $Z = aX + bY$

$$E(Z) = aE(X) + bE(Y)$$
$$Var(Z) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$$

Part B

$w = 0, 0.1 \ldots 1.$

From excel, we have

$w = 0.5.$

	M	G
mean	0.137	0.141
var	32.134	4.109
Cov	1.514	

Given $P = wM + (1 - w)G$, it is easy to calculate $E(P)$ and $Var(P)$.

$$
\begin{aligned}
E(P) =& E(wM + (1-w)G) = wE(M) + (1-w)E(G) \\
=& 0.137 \cdot w + 0.141 \cdot (1-w) \\
Var(P) =& Var(wM + (1-w)G) \\
=& w^2 Var(M) + (1-w)^2 Var(G) + 2w(1-w)Cov(M,G) \\
=& w^2 \cdot 32.134 + (1-w)^2 \cdot 4.109 + 2w(1-w) \cdot 1.514
\end{aligned}
$$

Part B

Substituting w from 0 to 1, we can have

w	$E(P)$	$Var(P)$
0	0.1408	4.11
0.1	0.1404	3.92
0.2	0.1401	4.40
0.3	0.1397	5.54
0.4	0.1394	7.35
0.5	0.1390	9.82
0.6	0.1387	12.95
0.7	0.1383	16.75
0.8	0.1379	21.21
0.9	0.1376	26.34
1	0.1372	32.13

The end

Thanks for your attention! 😄

Feel free to leave and see you next week!