ECON10005 Quantitative Methods 1

Tutorial in Week 11

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Introduction

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Don't be shy if you need help

- Visit Ed Discussion Board (read others' questions first)
- Lecturer's consultation sessions: see Canvas
- In case of special considerations, consult Stop 1
- For admin issues contact Chin via qm1-economics@unimelb.edu.au

But before asking any questions, make sure you have read the Ed discussion board, subject guide, announcements and etc on Canvas!!!

Reminder

Final draft (7%)

- Due on 18th May 2pm (Thursday)
- Be mindful that this is a business report for general audience.
- Use appendix!

Chin answered on Ed discussion: “The main report should focus on providing a clear and concise presentation of the key findings, interpretations, and insights derived from the data analysis. You can provide useful tables of summary statistics, relevant visual aids.

The appendix is the appropriate place to include detailed numerical calculations, statistical tests, and any "technical" analysis that supports your findings. This allows readers to refer to the appendix if they wish to examine the calculations or delve deeper into the technical aspects of the analysis."

Linear Regression: Estimation

Linear regression model: $E\left(Y_i \mid X_i\right) = \beta_0 + \beta_1 X_i$

How to estimate β_0 and β_1 (the line that best describes the relationship)?

Linear Regression: Estimation

Linear regression model: $E\left(Y_i \mid X_i\right) = \beta_0 + \beta_1 X_i$

How to estimate β_0 and β_1 (the line that best describes the relationship)?

Minimise the sum of squared errors

$$\text{minimize} \qquad \text{SSE} = \sum_{i=1}^{n} \hat{U}_i^2 = \sum_{i=1}^{n} \left(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i\right)^2$$

yields

$$\hat{\beta}_1 = \frac{s_{XY}}{s_X^2} \qquad \text{and} \qquad \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

where

$$s_{XY} = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)$$

$$s_X^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2$$

Linear Regression: Statistical Inference

Standard errors of $\hat{\beta}_1$ and $\hat{\beta}_0$

$$\text{s.e.}\left(\hat{\beta}_1\right) = \frac{s_U}{\sqrt{(n-1)s_X^2}} \quad \text{and} \quad \text{s.e.}\left(\hat{\beta}_0\right) = \frac{s_U}{\sqrt{(n-1)s_X^2}} \cdot \sqrt{\frac{\sum_{i=1}^{n} X_i^2}{n}}$$

where

$$s_U^2 = \frac{1}{n-2}\sum_{i=1}^{n}\hat{U}_i^2 = \frac{SSE}{n-2} \quad \text{with} \quad df = n-2$$

"Goodness of Fit": $R^2 = VAR.S(\hat{Y}_i)/VAR.S(Y_i)$, which explains the proportion of the variation in Y is explained by the regression on X.

Pre-quiz questions

1. Compute the mean of Risk (X), the variance of Return (Y), and the covariance of Risk and Return
2. Compute the slope coefficient $\hat{\beta}_1$ and the intercept $\hat{\beta}_0$.
3. Compute the fitted values and their mean and variance, and R^2.
4. Compute the residuals $\hat{U}_i$ and their mean and variance (using appropriate d.f.)

Pre-quiz questions

	Return	Risk	Return.hat	U.hat
Mean	3.203	3.985	3.203	-1.7E-15
Variance	20.584	6.190	5.670	$s_u^2 = 15.010$
Covariance	5.924			
$\hat{\beta}_1$	0.957			
$\hat{\beta}_0$	-0.611			
R^2			0.275	
n	157			
SumSq/n		22.031		

Tutorial questions

1. Using the results from the Pre-Quiz questions, put together a table of the regression results of the form as in the Inference for Regression slides ("Presentation of a Regression").
2. Carry out a test to see if there is evidence that average fund returns vary with risk. (Use a p-value approach at the 5% level of significance.)
3. Carry out a test to see if there is evidence that average fund returns increase with the level risk. (Use a critical value approach at the 5% level of significance.)
4. Carry out a test to see if an additional 1% of risk corresponds to an additional 1% of average return.
5. Construct a 95% confidence interval for the slope coefficient. Interpret this interval and also relate it to your answer to question 4.
6. An investor has decided to tolerate only 2% of risk for the year. What is the estimated expected return at this level of risk? Give a 95% confidence interval.

Tutorial question 1

1. Using the results from the Pre-Quiz questions, put together a table of the regression results of the form as in the Inference for Regression slides ("Presentation of a Regression").

Recall the formula

$$\text{s.e.}\left(\hat{\beta}_1\right) = \frac{s_U}{\sqrt{(n-1)s_X^2}} \quad \text{and} \quad \text{s.e.}\left(\hat{\beta}_0\right) = \frac{s_U}{\sqrt{(n-1)s_X^2}} \cdot \sqrt{\frac{\sum_{i=1}^n X_i^2}{n}}$$

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Return	Coefficient	S.E.	t-stat	p-value
Intercept	$\hat{\beta}_0 =$ -0.611	0.585	-1.044	0.2981
Risk	$\hat{\beta}_1 =$ 0.957	0.125	7.676	1.72E-12

test-statistics $= \hat{\beta}_0/\text{s.e.}\left(\hat{\beta}_0\right)$

p-value =T.DIST.2T(ABS(t-stat),n-2)

Tutorial question 2

2. Carry out a test to see if there is evidence that average fund returns vary with risk. (Use a p-value approach at the 5% level of significance.)

Two-tail test:

$$H_0 : \beta_1 = 0 \qquad H_1 : \beta_1 \neq 0$$

Return	Coefficient	S.E.	t-stat	p-value
Intercept	$\hat{\beta}_0$ = -0.611	0.585	-1.044	0.2981
Risk	$\hat{\beta}_1$ = 0.957	0.125	7.676	1.72E-12

p-value = 1.72E-12 < 5%; reject.

Tutorial question 3

3. Carry out a test to see if there is evidence that average fund returns increase with the level risk. (Use a critical value approach at the 5% level of significance.)

Upper-tail test:

$$H_0 : \beta_1 = 0 \qquad H_1 : \beta_1 > 0$$

Return	Coefficient	S.E.	t-stat	p-value
Intercept	$\hat{\beta}_0 =$ -0.611	0.585	-1.044	0.2981
Risk	$\hat{\beta}_1 = 0.957$	0.125	7.676	1.72E-12

t-stat $= 7.676 > 1.655$(T.INV(0.95,155)); reject.

Tutorial question 4

4. Carry out a test to see if an additional 1% of risk corresponds to an additional 1% of average return.

Two-tail test:

$$H_0 : \beta_1 = 1 \qquad H_1 : \beta_1 = 1$$

Return	Coefficient	S.E.	t-stat	p-value
Intercept	$\hat{\beta}_0$ = -0.611	0.585	-1.044	0.2981
Risk	$\hat{\beta}_1 = 0.957$	0.125	7.676	1.72E-12

Tutorial question 4

4. Carry out a test to see if an additional 1% of risk corresponds to an additional 1% of average return.

Two-tail test:

$$H_0 : \beta_1 = 1 \qquad H_1 : \beta_1 = 1$$

Return	Coefficient	S.E.	t-stat	p-value
Intercept	$\hat{\beta}_0$ = -0.611	0.585	-1.044	0.2981
Risk	$\hat{\beta}_1$ = 0.957	0.125	7.676	1.72E-12

t-stat $= \frac{0.957-1}{0.125} = -0.344$;

p-value = T.DIST.2T(ABS(t-stat),n-2) = 0.731 > 5%;

H_0 is not rejected.

Tutorial question 5

5. Construct a 95% confidence interval for the slope coefficient. Interpret this interval and also relate it to your answer to question 4.

95% Confidence Interval:

$$\begin{aligned}
&\left[\hat{\beta}_1 \pm t_{0.025,155} \cdot \text{s.e.}\left(\hat{\beta}_1\right)\right] \\
=&[0.957 \pm 1.975 \times 0.125] \\
=&[0.711, 1.203]
\end{aligned}$$

This 95% confidence interval includes the value 1, which is consistent with the finding in question 4 that $H_0 : \beta_1 = 1$ cannot be rejected by a two tail test at the 5% level of significance.

Tutorial question 6

6. An investor has decided to tolerate only 2% of risk for the year. What is the estimated expected return at this level of risk? Give a 95% confidence interval.

The estimated expected return is

$$\hat{E}\left(\text{Return}_i \mid \text{Risk}_i = 2\right) = \hat{\beta}_0 + \hat{\beta}_1 \times 2 = -0.611 + 0.957 \times 2 = 1.303$$

The standard error formula

$$\text{s.e.}\left(\hat{E}\left(Y_i \mid X_i = x\right)\right) = s_U \sqrt{\frac{1}{n} + \frac{(x - \bar{X})^2}{(n-1)s_X^2}}$$

The standard error for this estimated expected value is

$$\text{s.e.}\left(\hat{E}(\text{Return}_i \mid \text{Risk}_i = 2)\right) = 3.874\sqrt{\frac{1}{157} + \frac{(2 - 3.985)^2}{(157-1)6.190}} = 0.396$$

The 95% confidence interval is therefore $[1.303 \pm 1.975 \times 0.396] = [0.521, 2.085]$

The end

Thanks for your attention! 😄

Feel free to leave and see you next week!