ECON10005 Quantitative Methods 1

Tutorial in Week 6

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Introduction

Zheng Fan

- Ph.D student in Economics at Unimelb. Research interest in Bayesian Econometrics
- Personal website: *zhengfan.site* for some details

Don't be shy if you need help

- Discuss on Ed Discussion Board
- Attend consultation sessions: see Canvas for time and location
- Consult Stop 1, in case of special considerations,
- Contact QM-1@unimelb.edu.au for admin issues
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Question 1

Refer to pre-workshop question 2. Discuss as an investor if you would approve the company's decision to embark on the new venture.

Question 1: Pre-tute Q2

Evidence suggests that the annual rate of return of an ordinary share is **approximately normally distributed**. Suppose that you invested in the shares of a company for which the annual return has an **expected value of 16%** and a **standard deviation of 10%**.

1. Find the probability that your one-year return will exceed 30%.
2. Find the probability that your one-year return will be negative.
3. Suppose this company embarks on a new, high-risk, potentially highly profitable venture. As a result, the return on the share now has an **expected value of 25%** and a **standard deviation of 20%**. Recalculate the probabilities you calculated for parts (1) and (2) above if you invested in the new venture.

Question Setup

Let X be the annual percentage return on stock.
Assume $X \sim \mathcal{N}(16, 10^2)$.

We are interested in:

- $\mathbb{P}(X > 30)$
- $\mathbb{P}(X < 0)$

Probability Return Exceeds 30%

If we have excel, we can directly compute the probability based on X's distribution:
$P(X > 30) = 1 - P(X \leq 30) = 1-$NORM.DIST(30,16,10,TRUE)

Or convert to standard normal to read table:

$$\begin{aligned}
\mathbb{P}(X > 30) &= \mathbb{P}\left(Z > \frac{30 - 16}{10}\right) \\
&= \mathbb{P}(Z > 1.4) \\
&= 1 - \mathbb{P}(Z < 1.4) \\
&= 1 - 0.9192 \\
&= 0.0808
\end{aligned}$$

Note: $P(Z > 1.4) = 1 - P(Z \leq 1.4) = 1-$NORM.DIST(1.4,0,1,TRUE)
If no excel, read the table has to follow $Z \sim N(0, 1)$

Conclusion: There is an 8.08% chance that the return exceeds 30%.

Probability Return is Negative

If we have excel, we can directly compute the probability based on X's distribution:
$P(X < 0) = P(X \leq 0) =$NORM.DIST(0,16,10,TRUE)

Or convert to standard normal to read table:

$$\begin{aligned} \mathbb{P}(X < 0) &= \mathbb{P}\left(Z < \frac{0 - 16}{10}\right) \\ &= \mathbb{P}(Z < -1.6) \\ &= 0.0548 \end{aligned}$$

Note: $P(Z < -1.6) = P(Z \leq -1.6) =$NORM.DIST(-1.6,0,1,TRUE)
If no excel, read the table has to follow $Z \sim N(0, 1)$

Conclusion: There is a 5.48% chance of a negative return.

New Venture Scenario

Let Y be the annual return after embarking on a new venture.
Assume $Y \sim \mathcal{N}(25, 20^2)$.

We are interested in:

- $\mathbb{P}(Y > 30)$
- $\mathbb{P}(Y < 0)$

Probability $Y > 30$

$$
\begin{aligned}
\mathbb{P}(Y > 30) &= \mathbb{P}\left(Z > \frac{30 - 25}{20}\right) \\
&= \mathbb{P}(Z > 0.25) \\
&= 1 - \mathbb{P}(Z < 0.25) \\
&= 1 - 0.5987 \\
&= 0.4013
\end{aligned}
$$

Conclusion: There is a 40.13% chance that the return exceeds 30%.

Probability $Y < 0$

$$
\begin{aligned}
\mathbb{P}(Y < 0) &= \mathbb{P}\left(Z < \frac{0 - 25}{20}\right) \\
&= \mathbb{P}(Z < -1.25) \\
&= 0.1056
\end{aligned}
$$

Conclusion: There is a 10.56% chance of a negative return.

Question 1

Refer to pre-workshop question 2. Discuss as an investor if you would approve the company's decision to embark on the new venture.

Comparison of Return Distributions:

- **Old Return (X):** $\mathcal{N}(16, 10^2)$
  - $\mathbb{P}(X > 30) = 8.08\%$
  - $\mathbb{P}(X < 0) = 5.48\%$
- **New Return (Y):** $\mathcal{N}(25, 20^2)$
  - $\mathbb{P}(Y > 30) = 40.13\%$
  - $\mathbb{P}(Y < 0) = 10.56\%$

Discussion: Approving the New Venture

Refer to Tutorial Question 1

As an investor, the decision to approve the company's new venture depends on your level of **risk aversion**.

- The probability of achieving **very high returns** (i.e., exceeding 30%) increases from **8% to 40.13%**.
- However, the probability of experiencing a **negative return** also increases from **5.48% to 10.56%**.

Conclusion:
The new venture offers a significantly higher upside potential, but this comes at the cost of increased downside risk. A risk-neutral or risk-seeking investor may support the decision, whereas a risk-averse investor may be cautious about the added volatility.

Question 2

The Table below gives the average and standard deviation of heights, shown in cm , for a few countries worldwide. Note that these numbers may not be accurate; they provide a general idea of average heights for male and female populations. Let's assume that the heights of these populations are **normally distributed**.

Population Parameters	Australia	China	India	Indonesia	America
Average Male Height	177.8	172.1	166.3	158	175.9
Standard Deviation	9.7	8.1	8.9	6.5	8.9
Average Female Height	163.8	160.1	152.6	147	132.1
Standard Deviation	9.6	8.5	7.2	6.1	8.4

Question 2 continued

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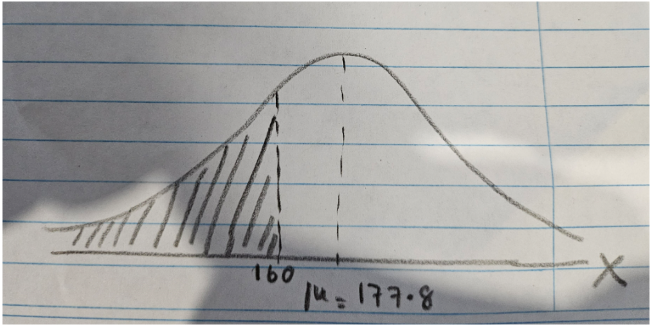
1. Provide a sketch of the distribution for the male population in Australia, including the mean, and in that diagram, also show the probability that a randomly selected person is shorter than 160 cm.
2. What percentage of the male population for each country is higher than 185 cm?
3. What percentage of the female population for each country is higher than 185 cm?
4. If a randomly selected person from each population is 175 cm tall, where does that person place within their distribution?

1. Sketching the Distribution

- Given the mean (μ) is 177.8 cm and the standard deviation (σ) is 9.7 cm.
- We'll mark the mean on the graph.
- We'll also shade the area representing the probability of a male being shorter than 160cm.



2. Probability of Being Taller than 185cm (Males)

- To find the probability of a male being taller than 185cm, we need to standardize the value using the Z-score formula:

  $Z = \frac{X-\mu}{\sigma}$

  Where:

  - X is the height (185cm)
  - μ is the mean height (e.g., 177.8cm)
  - σ is the standard deviation (e.g., 9.7cm)

2. Calculation

- Let's calculate the Z-score:

  $Z = \frac{X-\mu}{\sigma} = \frac{185-177.8}{9.7} = 0.742$

- Now, we find the probability $P(X > 185) = P(Z > 0.742)$.
- Using the standard normal table (or software), we find $P(Z < 0.742) = 0.7704$
- Therefore, $P(Z > 0.742) = 1 - P(Z < 0.742) = 1 - 0.7704 = 0.2296$

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- Therefore, $P(Z > 0.742) = 1 - P(Z < 0.742) = 1 - 0.7704 = 0.2296$
- Approximately 23% of Australian males are taller than 185cm.

3. Probability of Being Taller than 185cm (Females)

- We repeat the process for females, using the appropriate mean and standard deviation.
- Given the mean (μ) is 163.8 cm and the standard deviation (σ) is 9.6 cm.

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- We repeat the process for females, using the appropriate mean and standard deviation.
- Given the mean (μ) is 163.8 cm and the standard deviation (σ) is 9.6 cm.
- Calculate the Z-score:

$$Z = \frac{X-\mu}{\sigma} = \frac{185-163.8}{9.6} = 2.20$$

- Find the probability:

$$P(X > 185) = P(Z > 2.20) = 1 - P(Z < 2.20) = 1 - 0.9861 = 0.0139$$

- Only about 1.4% of Australian females are taller than 185cm.

4. Position of a 175cm Tall Person

- To find where a 175cm tall person places, we calculate the Z-score and find the corresponding percentile.
- Given: Chinese male population with $\mu = 172.1$ cm and $\sigma = 8.1$ cm.

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- To find where a 175cm tall person places, we calculate the Z-score and find the corresponding percentile.
- Given: Chinese male population with $\mu = 172.1$ cm and $\sigma = 8.1$ cm.
- Calculate the Z-score:

$$Z = \frac{X-\mu}{\sigma} = \frac{175-172.1}{8.1} = 0.36$$

- Find the probability:

$$P(X < 175) = P(Z < 0.36) = 0.6406$$

- A 175cm tall Chinese male is taller than 64.06% of the Chinese male population.

Question 2 continued

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5. For each population, how tall should one have to be in the top 5% of the population?
6. For each population, how tall should one have to be in the bottom 1% of the population?
7. Compare and contrast the answers you got for the questions above.

Note: we can immedicately get the results with excel '=NORM.INV(0.95,mean,std)'.

5. Top 5% Height

- To find the height for the top 5%, we need the Z-score corresponding to the 95th percentile.
- From the standard normal table, the Z-score for 0.95 is approximately 1.64.
- Use the Z-score formula to find the height (X):

$$Z = (X - \mu)/\sigma \quad \Rightarrow \quad X = \mu + Z\sigma$$

5. Top 5% Height

- To find the height for the top 5%, we need the Z-score corresponding to the 95th percentile.
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- Given: Indian male population with $\mu = 166.3$ cm and $\sigma = 8.9$ cm.
- Calculate the height:

$$X = 166.3 + 1.64 \times 8.9 = 180.9 \text{ cm}$$

- An Indian male needs to be at least 180.9cm tall to be in the top 5%.

6. Bottom 1% Height

- To find the height for the bottom 1%, we need the Z-score corresponding to the 1st percentile.
- From the standard normal table, the Z-score for 0.01 is approximately -2.33.
- Use the Z-score formula to find the height (X):

$$Z = (X - \mu)/\sigma \quad \Rightarrow \quad X = \mu + Z\sigma$$

6. Bottom 1% Height

- To find the height for the bottom 1%, we need the Z-score corresponding to the 1st percentile.
- From the standard normal table, the Z-score for 0.01 is approximately -2.33.
- Use the Z-score formula to find the height (X):

$$Z = (X - \mu)/\sigma \quad \Rightarrow \quad X = \mu + Z\sigma$$

- Given: Indian female population with $\mu = 152.6$ cm and $\sigma = 7.2$ cm.
- Calculate the height:

$$X = 152.6 + (-2.33) \times 7.2 = 135.824 \text{ cm}$$

- An Indian female needs to be 135.824 cm tall to be in the bottom 1%.

7. Comparison and Contrast

- We can now compare the probabilities and percentiles across different populations.
- For example:
  - An example would be 23% of the Australian males are taller than 185cm, but only 15% in the US and nearly no one in Indonesia.
  - Similarly for a given percentile, such as a Chinese woman should be 164.4cm tall to be in the top 5% (95th percentile) of their population, whereas it is only 157cm for a women to be in the 95th percentile of the Indonesian female population.

7. Comparison and Contrast

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- For example:
  - An example would be 23% of the Australian males are taller than 185cm, but only 15% in the US and nearly no one in Indonesia.
  - Similarly for a given percentile, such as a Chinese woman should be 164.4cm tall to be in the top 5% (95th percentile) of their population, whereas it is only 157cm for a women to be in the 95th percentile of the Indonesian female population.
- These differences highlight the variation in height distributions across different populations.

Key Takeaways

- We can use the normal distribution and Z-scores to analyze and compare heights within and across populations.
- The mean and standard deviation are crucial parameters for understanding a distribution.
- Standard normal tables (or software) are essential tools for finding probabilities and percentiles.

Any final questions?

Thanks for your attention! 😄

Let me know if you have any questions