ECON10005 Quantitative Methods 1

Tutorial in Week 5

Zheng Fan

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Introduction

Zheng Fan

- Ph.D student in Economics at Unimelb. Research interest in Bayesian Econometrics
- Personal website: *zhengfan.site* for some details

Don't be shy if you need help

- Discuss on Ed Discussion Board
- Attend consultation sessions: see Canvas for time and location
- Consult Stop 1, in case of special considerations,
- Contact QM-1@unimelb.edu.au for admin issues
- Send me an email: fan.z@unimelb.edu.au

Craps game

	First Die (X_1)					
Second Die (X_2)	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Table 1: Summation results S of two dice rolls (X_1 and X_2).

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6	7	8	9	10	11	12

Table 1: Summation results S of two dice rolls (X_1 and X_2).

Clearly, the probability of winning is $1/6$.

Part A 1.

1. Refer to Part A in Pre-tutorial 4. What proportion of students in the tutorial came up with:
   a. the smallest possible value for N in their Excel game?
   b. the second smallest possible value for N?
   c. the maximum of 10 plays with money still remaining?

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   a. the smallest possible value for N in their Excel game?
   b. the second smallest possible value for N?
   c. the maximum of 10 plays with money still remaining?

Tell me the number of games you played by responding in the class.

Scan the QR code or enter website: PollEv.com/zhengfan445

Make sure enter your student ID to participate

Part A 1.

1. Refer to Part A and Part B in Pre-tutorial 4

   a. What is the smallest value that N can take, and what is its probability?

   b. What is the second smallest value for N, and what is its probability?

   c. What is the probability of reaching 10 plays with money remaining?

Part A 1.

1. Refer to Part A and Part B in Pre-tutorial 4
   a. What is the smallest value that N can take, and what is its probability?
   b. What is the second smallest value for N, and what is its probability?
   c. What is the probability of reaching 10 plays with money remaining?

In a nutshell, there are only three possibilities. 1. no win. 2. one win. 3. two wins

Part A 1.

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- no win: lose 2 times and bankrupt. $P(N=2) = \frac{5}{6} \cdot \frac{5}{6} = 0.6944$
- one win: it has to be a win in the first 2 rounds, and then all lose can play 7 rounds in total.

$$P(N=7) = \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} + \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = 0.1116$$

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Three possibilities. 1. no win. 2. one win. 3. two wins

- no win: lose 2 times and bankrupt. $P(N=2)=\frac{5}{6}\cdot\frac{5}{6}=0.6944$
- one win: it has to be a win in the first 2 rounds, and then all lose can play 7 rounds in total.

$$P(N=7)=\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}+\frac{5}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}=0.1116$$

- two wins: As long as you win twice, you can play the game til the end by 10 times

$$P(N=10)=1-0.6944-0.1116=0.1940$$

Part A 2.

Refer to Part C in Pre-tutorial 4, where we have.

Let n be the fixed number of rounds. For each round, you spend \$100 playing the game. For each round, define M_i where $i \in \{1, 2, \ldots, n\}$. We define the outcomes for M_i as follows:

$$M_i = \begin{cases} 500, & \text{if "S = 7" occurs in round } i \\ 0, & \text{if "S = 7" does not occur in round } i \end{cases}$$

Thus, the total stock of money after n rounds is: $M = M_1 + M_2 + \cdots + M_n$.

a. Discuss how you got the probability distribution for M if the game is played **three times**.

b. Find the expected value of M if the game is played **ten times**.

c. Find the variance of M if the game is played **ten times**.

Probability Distribution of M

(a) Discuss how you got the probability distribution for M if the game is played three times. The outcomes and their corresponding probabilities are as follows:

- We have to consider all the possible outcomes that M could take.
- Three times, means 1. all three wins; 2. two wins; 3. one win; 4. 0 win.
- Consider all the possible cases and calculate the corresponding probability.

Probability Distribution of M

(a) Discuss how you got the probability distribution for M if the game is played three times. The outcomes and their corresponding probabilities are as follows:

- $M = 1500$ requires $M_1 = M_2 = M_3 = 500$ suggest $P(M) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = 0.0046$

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- $M = 1000$ requires two of M_1, M_2, M_3 is 500. There are three different scenarios, suggesting

$$P(M) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} + \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$$

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- $M = 500$ requires one of M_1, M_2, M_3 is 500. There are three different scenarios, suggesting

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- $M = 0$ requires $M_1 = M_2 = M_3 = 0$ suggest $P(M) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = 0.5787$

Probability Distribution of M

(a) Discuss how you got the probability distribution for M if the game is played three times. The outcomes and their corresponding probabilities are as follows:

M	1,500	1,000	500	0
P(M)	0.0046	0.0694	0.3472	0.5787

Expected Value of Playing Ten Times

b. Find the expected value of M if the game is played ten times.

Define the first dice play as M_1, second as M_2, and so on until M_{10}. The expected value from one game is calculated as $E(M_1) = 83.33$.

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The total payoff from playing ten times is:

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The total payoff from playing ten times is:

$$M = M_1 + M_2 + \cdots + M_{10}$$

The expected value for 10 games is:

$$E(M) = E(M_1) + E(M_2) + \cdots + E(M_{10}) = 10 \times 83.33 = 833.33$$

Variance of Playing Ten Times

c. Find the variance of M if the game is played ten times.

Since the outcome of each play is independent, the covariance between the outcomes of each trial is 0.The variance from one game is calculated as $\text{Var}(M_1) = 34,722.222$.

Formula: $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$

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Formula: $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$

The variance for 10 games is calculated using the addition rule for variances:

$$\text{Var}(M) = \text{Var}(M_1) + \text{Var}(M_2) + \cdots + \text{Var}(M_{10}) = 10 \times 34,722.222 = 347,222.222$$

Part B - Binomial distribution

Let X be a random variable representing the number of wins (i.e., the number of times a seven is rolled) over 10 rounds of the game. Each play of the game is an independent trial, where:

- **Success:** Getting a 7 (winning the round)
- **Failure:** Not getting a 7 (losing the round)

Given this setup, answer the following questions:

a. What is the probability of success in a single play (p)?

b. What is the probability of failure in a single play ($1 - p$ or q)?

c. What is the probability that you win exactly two out of the 10 plays?

d. What is the probability that you win more than two plays?

e. Discuss how the answers differ from the answers to pre-tutorial Part B Q2.

What is the Binomial Distribution?

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  - $Var(X) = Var(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} Var(X_i) = np(1 - p)$

Probability Mass Function (PMF)

- The probability of obtaining exactly k successes in n trials is given by:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n \tag{1}$$

- Where:
  - $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the binomial coefficient, also ${}_nC_k$.
  - p is the probability of success in each trial.
  - $(1-p)$ is the probability of failure.

Example

- Suppose a fair coin is flipped 10 times ($n = 10$, $p = 0.5$).
- The expected number of head (or tail) is $E(X) = np = 5$

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- Using the PMF:

$$P(X = 6) = \binom{10}{6}(0.5)^6(0.5)^4 = \frac{10!}{6!(4!)}(0.5)^{10} \approx 0.205$$

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- For more details please see QME-probability II_Binomial.pdf and BinomialDistribution_noBernoulli.pdf available on Canvas

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d. What is the probability that you win more than two plays?

e. Discuss how the answers differ from the answers to pre-tutorial Part B Q2.

Part B (a)

Let X be a random variable representing the number of wins (i.e., the number of times a seven is rolled) over 10 rounds of the game.

- Success = Getting a 7 (winning the game)
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- The game is played 10 times, thus $n = 10$

Part B (a)

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- Success = Getting a 7 (winning the game)
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We know that:

- The total possible outcomes of rolling two dice = 36
- The number of outcomes where the sum is 7 = 6

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We know that:

- The total possible outcomes of rolling two dice = 36
- The number of outcomes where the sum is 7 = 6

Therefore, the probability of success:

$$p = \frac{6}{36} = \frac{1}{6}$$

Part B (b)

Let X be a random variable representing the number of wins (i.e., the number of times a seven is rolled) over 10 rounds of the game.

- Success = Getting a 7 (winning the game)
- Failure = Not getting a 7 (losing the game)
- The game is played 10 times, thus $n = 10$

Part B (b)

Let X be a random variable representing the number of wins (i.e., the number of times a seven is rolled) over 10 rounds of the game.

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- Failure = Not getting a 7 (losing the game)
- The game is played 10 times, thus $n = 10$

We know that:

- The total possible outcomes of rolling two dice = 36
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Therefore, the probability of failure is

Part B (b)

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- The game is played 10 times, thus $n = 10$

We know that:

- The total possible outcomes of rolling two dice = 36
- The number of outcomes where the sum is 7 = 6

Therefore, the probability of failure is $1 - p$:

$$q = 1 - \frac{1}{6} = \frac{5}{6}$$

Part B (c)

(3) What is the probability that you win exactly two out of the 10 plays?

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Using Binomial Distribution:

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Substituting values:

$$P(X = 2) = \binom{10}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^8$$

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Substituting values:

$$P(X = 2) = \binom{10}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^8$$

$$= 45 \times 0.0278 \times 0.2326 = 0.2907$$

Part B (d)

(4) The probability of winning more than two plays can be calculated using:

$$P(X > 2) = 1 - P(X \leq 2)$$

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$$P(X > 2) = 1 - P(X \leq 2)$$

First, calculate $P(X = 0)$ and $P(X = 1)$:

$$P(X = 0) = \binom{10}{0} p^0 (1-p)^{10} = 1 \times 1 \times \left(\frac{5}{6}\right)^{10} = 0.1615$$

Part B (d)

(4) The probability of winning more than two plays can be calculated using:

$$P(X > 2) = 1 - P(X \leq 2)$$

First, calculate $P(X = 0)$ and $P(X = 1)$:

$$P(X = 0) = \binom{10}{0} p^0 (1 - p)^{10} = 1 \times 1 \times \left(\frac{5}{6}\right)^{10} = 0.1615$$

$$P(X = 1) = \binom{10}{1} p^1 (1 - p)^9 = 10 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^9 = 0.3230$$

Part B (d)

Given that the probability of winning exactly two plays is $P(X = 2) = 0.2907$, we have:

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.1615 + 0.3230 + 0.2907 = 0.7752$$

Part B (d)

Given that the probability of winning exactly two plays is $P(X = 2) = 0.2907$, we have:

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.1615 + 0.3230 + 0.2907 = 0.7752$$

$$P(X > 2) = 1 - 0.7752 = 0.2248$$

Part B (e)

(5) Discuss how the answers you have for these questions differ from the answers to pre-tutorial Part B Q2.

The probability calculations use the binomial distribution, treating each play as an independent trial and modeling the total number of wins.

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(5) Discuss how the answers you have for these questions differ from the answers to pre-tutorial Part B Q2.

The probability calculations use the binomial distribution, treating each play as an independent trial and modeling the total number of wins. **However, this approach ignores the sequence of wins and losses.**

- The sequential process in the pre-tutorial Part B models the game more realistically by tracking the exact sequence of wins and losses, and the running total of money after each round.

Any final questions?

Thanks for your attention! 😄

Let me know if you have any questions