ECON10005 Quantitative Methods 1

Tutorial in Week 9

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Introduction

Zheng Fan

- Ph.D student in Economics at Unimelb. Research interest in Bayesian Econometrics
- Personal website: *zhengfan.site* for some details

Don't be shy if you need help

- Discuss on Ed Discussion Board
- Attend consultation sessions: see Canvas for time and location
- Consult Stop 1, in case of special considerations,
- Contact QM-1@unimelb.edu.au for admin issues
- Send me an email: fan.z@unimelb.edu.au

Learning Objectives

- Recognise when the samples were independently drawn from two populations and when they were taken from a matched pairs experiment.
- Understand how to test hypotheses about differences in two population proportions.
- Understand how to test hypotheses about differences in two means.

Question 1: Aspirin Study Description

Check the answers for Question 1 of the pre-Tutorial questions.

A research project involving 22,000 men was conducted to investigate whether aspirin can help prevent heart attacks.

- Randomly divided into two groups:
  - 11,000 men took aspirin
  - 11,000 men took a placebo
- Over three years:
  - 104 aspirin group experienced heart attacks
  - 189 placebo group experienced heart attacks

Question 1: Sample Type

(a) What types of samples were used in this study?

Question 1: Sample Type

(a) What types of samples were used in this study?

- Independent random samples
- Random assignment to aspirin (treatment) and placebo (control) groups

Question 1: Disadvantages

(b) Potential disadvantages of these samples:

- Variation in individual characteristics (e.g., health, lifestyle)
- Large within-sample variances may mask treatment effects

Question 1: Matched-Pairs Design

How matched-pairs can help:

- Pair participants by similar characteristics
- One takes aspirin, one takes placebo (or the same person is tested twice)
- Reduces variability $\rightarrow$ easier to detect treatment effect

Question 1

1.c. Conduct a hypothesis test at the 5% significance level to determine whether aspirin is more effective than a placebo in preventing heart attacks among men. Use the critical value approach.

This is a test for population proportions using independent samples.

Let's define the population parameters. Always remember to introduce your parameters!

Question 1: Hypothesis Test (Proportions)

Step 1: Set up the hypotheses

- H_0: $p_1 = p_2$ or $p_1 - p_2 = 0$
- H_1: $p_1 < p_2$ or $p_1 - p_2 < 0$ (aspirin is more effective)

Remember: you have to define each parameter used. For example, $p_1 =$ proportion of people who used aspirin but had heart attacks, and thus $\hat{p}_1 =$ Proportion of people who used aspirin but had heart attacks in the sample. p_2 ....

Question 1: Hypothesis Test (Proportions)

Step 2: Find the decision Rule: Lower Tail Test ($\alpha = 0.05$)

- $DF_1 = n_1 - 1 = 10,999$
- $DF_2 = n_2 - 1 = 10,999$
- $DF = n_1 + n_2 - 2 = 21,998$
- $CV =$ t.inv $(0.05, 21998) = -1.645$
- Decision rule: Reject H_0 if the test statistic < -1.645

Question 1: Hypothesis Test (Proportions)

Step 3: Calculate the test statistic

The standard error is

$$\text{se}\left(\hat{p}_1 - \hat{p}_2\right) = \sqrt{\frac{0.009365}{11,000} + \frac{0.016889}{11,000}} = 0.001544838$$

and thus the test-statistics is:

$$t = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\text{se}\left(\hat{p}_1 - \hat{p}_2\right)} = \frac{(0.009455 - 0.01718) - 0}{0.00154} = -5.002$$

Question 1: Hypothesis Test (Proportions)

Step 4: Make the Decision

We are asked to use the critical value approach:

- Reject H_0 since t-statistic (-5.002) $<$ critical value (-1.645).

Step 5: Answer the question

At a 5% significance level, we can infer that aspirin is more effective than a placebo in preventing heart attacks among men.

Question 2 Part 1: Financial Literacy

In Lecture 8, we conducted a hypothesis test to examine whether the average financial literacy score for Australian-born individuals is higher than that of overseas-born individuals.

Suppose instead we were testing whether the average score for the overseas-born population is lower than that of the Australian-born population. As practice, run this alternative hypothesis test (at the 5% significance level). Use the **critical value** approach.

Although you have done this in your pre-tutorial, it is important that you can clearly show all five steps of your hypothesis test.

Question 2 Part 1: Financial Literacy

In Lecture 8, we conducted a hypothesis test to examine whether the average financial literacy score for Australian-born individuals is higher than that of overseas-born individuals.

Suppose instead we were testing whether the average score for the overseas-born population is lower than that of the Australian-born population. As practice, run this alternative hypothesis test (at the 5% significance level). Use the **critical value** approach.

	Australian-born	Overseas-born
Mean	6.763	6.596
Variance	8.971	10.417
n	219	468

Question 2 Part 1

Step 1: Set up the hypotheses

- H_0: $\mu_2 = \mu_1$ or $\mu_2 - \mu_1 = 0$
- H_1: $\mu_2 < \mu_1$ or $\mu_2 - \mu_1 < 0$ (migrants have lower financial literacy than Australian-born)

Remember: you have to define each parameter used, eg $\mu, \bar{X}$.

Question 2 Part 1

Step 2: Find the decision Rule: Lower Tail Test ($\alpha = 0.05$)

- $DF_1 = n_1 - 1 = 218$
- $DF_2 = n_2 - 1 = 467$
- $DF = n_1 + n_2 - 2 = 685$
- $CV = t.inv(0.05, 685) = -1.645$
- Decision rule: Reject H_0 if the test statistic < -1.645

Question 2 Part 1

Step 3: Calculate the test statistic

The standard error is

$$\text{se}\left(\bar{X}_2 - \bar{X}_1\right) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 0.2514$$

and thus the test-statistics is:

$$t = \frac{\left(\bar{X}_2 - \bar{X}_1\right) - \left(p_2 - p_1\right)}{\text{se}\left(\bar{X}_2 - \bar{X}_1\right)} = \frac{(6.596 - 6.763) - 0}{0.2514} = -0.66$$

Question 2 Part 1

Step 4: Make the Decision

We are asked to use the critical value approach:

- Do not reject H_0 since t-statistic (-0.66) $>$ critical value (-1.645).

Step 5: Answer the question

At a 5% significance level, the sample data does not provide strong enough evidence to support the claim that the overseas-born group has a lower average score.

Question 2 Part 2: Reversing Hypotheses

Discuss how this hypothesis test is essentially the same as the one conducted in Lecture 8, and why it leads to the same conclusion even though we are testing a different hypothesis.

Let's spend 2-3 minutes discussing this in groups.

Question 2 Part 2: Reversing Hypotheses

As you may have noted, we have come to the same conclusion as the test we conducted in the lecture.

Although the two hypothesis tests are framed differently, one tests if Group A is greater than Group B and the other if Group B is less than Group A, they are statistically equivalent when we use the same data and the same significance level.

It's fine if test is conducted either way, **but** you have to make sure everything is consistent.

Question 2 Part 3: Practice Test

Let's review the pre-tutorial exercise 2.b below and compare them with others to ensure they are correct.

The Excel file Financial Lit Scores – Tutorial 8 contains financial literacy scores for two randomly selected groups of overseas-born individuals. Conduct a hypothesis test at the 5% significance level to determine whether there is a significant difference in their average financial literacy scores.

	Group 1	Group 2
Mean	4.361	4.787
Standard Deviation	3.012	3.017
n	108	108

Question 2 Part 3

This is a test for population means using independent samples. Let's define the population parameters.

- μ_1 = average financial literacy scores of immigrant group 1.
- μ_2 = average financial literacy scores of immigrant group 2.

Then,

- $\bar{X}_1$ = sample average financial literacy scores of immigrant group 1.
- $\bar{X}_2$ = sample average financial literacy scores of immigrant group 2.

Question 2 Part 3

Step 1: Set up the hypotheses

The null and the alternative hypotheses for this test are:

$$H_0 : \mu_1 = \mu_2 \quad \text{or} \quad \mu_1 - \mu_2 = 0$$
$$H_A : \mu_1 \neq \mu_2 \quad \text{or} \quad \mu_1 - \mu_2 \neq 0$$

If we are to infer that the financial literacy levels are different from each other, $\mu_1 \neq \mu_2$, we run a two-tail hypothesis test.

Question 2 Part 3

Step 2: Find the decision Rule: Two Tail Test $(\alpha = 0.05)$

We are using a critical value approach.

$$DF_1 = n_1 - 1 = 107$$
$$DF_2 = n_2 - 1 = 107$$
$$DF = n_1 + n_2 - 2 = 214$$
$$CV = \mathrm{t.inv}(0.975, 214) = 1.9711$$

Decision rule: Reject H_0 if the $|t| > 1.9711$

Question 2 Part 3

Step 3: Calculate the test statistic

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad t \sim t_{n_1+n_2-2}$$

$$\text{s.e.} \ \left(\bar{X}_1 - \bar{X}_2\right) = 0.4103$$

$$t = \frac{(4.361 - 4.787) - 0}{0.4103} = -1.038$$

Question 2 Part 3

Step 4: Make the Decision We are asked to use the critical value approach: Do not reject H_0 since $|t|$ (1.038) $<$ critical value (1.9711).

Note that $|t| = |1.038| = 1.038$.

If we use the p-value approach,

Question 2 Part 3

Step 4: Make the Decision We are asked to use the critical value approach: Do not reject H_0 since $|t|$ (1.038) $<$ critical value (1.9711).

Note that $|t|=|1.038|=1.038$.

If we use the p-value approach, we do not reject H_0 because $0.3004 > \alpha = 0.05$.

Question 2 Part 3

Step 4: Make the Decision We are asked to use the critical value approach: Do not reject H_0 since $|t|$ (1.038) $<$ critical value (1.9711).

Note that $|t|{=}|1.038|{=}1.038$.

If we use the p-value approach, we do not reject H_0 because $0.3004 > \alpha = 0.05$.

Step 5: Answer the question

At a 5% significance level, our data does not provide strong enough evidence to support the claim that migrants in group 1 have financial literacy that is different from that in group 2.

Question 2 Part 4 and 5

4. Sketch a t-distribution showing the level of significance, the t-statistic, and the p-value for this test.

5. In Question 2b, we did not reject the null hypothesis at the 5% significance level because the test statistic was -1.038 and the critical value for a two-tail test was ± 1.97.

Would the decision change if we instead ran a lower-tail test (i.e., testing whether Group A has a lower average score than Group B)? Justify your answer.

Let's spend 5 minutes discussing this in groups.

Question 2 Part 4 and 5

4. Sketch a t-distribution showing the level of significance, the t-statistic, and the p-value for this test.

Question 2 Part 5: Changing to One-Tail

Would decision change? if we instead ran a lower-tail test (i.e., testing whether Group A has a lower average score than Group B)? Justify your answer.

Question 2 Part 5: Changing to One-Tail

Would decision change? if we instead ran a lower-tail test (i.e., testing whether Group A has a lower average score than Group B)? Justify your answer.

- Original two-tail: test stat = -1.038; critical = ± 1.97
- One-tail (lower): critical = -1.645
- Since $-1.038 > -1.645$, still **fail to reject**

Therefore, even when we test for whether Group A has a lower average score than Group B using a one-tailed (lower-tail) test, we still fail to reject the null hypothesis.

Any final questions?

Thanks for your attention! 😄

Let me know if you have any questions.