ECON10005 Quantitative Methods 1

Tutorial in Week 4

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Introduction

Zheng Fan

- Ph.D student in Economics at Unimelb. Research interest in Bayesian Econometrics
- Personal website: *zhengfan.site* for some details

Don't be shy if you need help

- Discuss on Ed Discussion Board
- Attend consultation sessions: see Canvas for time and location
- Consult Stop 1, in case of special considerations,
- Contact QM-1@unimelb.edu.au for admin issues
- Send me an email: fan.z@unimelb.edu.au

Part A

1. Discuss the solutions for pre-tutorial Part A questions.
2. What is the relationship between E(M) and E(X1), E(X2)?

Part A - 1

	First Roll (X_1)					
Second Roll (X_2)	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Table 1: Multiplication results of two dice rolls (X_1 and X_2).

Part A - 1

M	1	2	3	4	5	6	8	9	10	12	15	16	18	20	24	25	30	36
$P(M)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Table 2: Probability distribution of M.

By the definition of expect value

$$E(M) = 1 \cdot \frac{1}{36} + 2 \cdot \frac{2}{36} + 3 \cdot \frac{2}{36} + \cdots + 36 \cdot \frac{1}{36} \approx 12.25$$

Part A - 1

M	1	2	3	4	5	6	8	9	10	12	15	16	18	20	24	25	30	36
$P(M)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Table 3: Probability distribution of M.

Following the formula

$$\begin{aligned}\mathrm{Var}(M) = E[(M - E(M))^2] &= \sum_{i=1}^{18} P(M_i) \cdot (M_i - 12.25)^2 \\ &= \frac{1}{36} \cdot (1 - 12.25)^2 + \frac{2}{36} \cdot (2 - 12.25)^2 \\ &\quad + \frac{2}{36} \cdot (3 - 12.25)^2 + \cdots + \frac{1}{36} \cdot (36 - 12.25)^2 \\ &\approx 79.965\end{aligned}$$

Part A - 1

X_1	1	2	3	4	5	6	X_2	1	2	3	4	5	6
$P(X_1)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$P(X_2)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Table 4: Probability distribution of X_1 and X_2 respectively.

By the definition of expect value

$$E(X_1) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5$$

and variance

$$\begin{aligned} \mathrm{Var}(X_1) = E[(M - E(X_1))^2] &= \frac{1}{6} \cdot (1 - E(X_1))^2 + \frac{1}{6} \cdot (2 - E(X_1))^2 + \frac{1}{6} \cdot (3 - E(X_1))^2 \\ &\quad + \frac{1}{6} \cdot (4 - E(X_1))^2 + \frac{1}{6} \cdot (5 - E(X_1))^2 + \frac{1}{6} \cdot (6 - E(X_1))^2 \\ &\approx 2.917 \end{aligned}$$

Part A - 2

By the independence of X_1 and X_2, we know:

$$E(M) = E(X_1) \times E(X_2) = 3.5 \times 3.5 = 12.25$$

Part B

1. Compare the mean and variance properties of M, G and P.
2. Define P more generally as $P = wM + (1 - w)G$, where w represents any weight between 0 and 1. Calculate $E(P)$ and $var(P)$ for $w = 0, 0.1, 0.2, \ldots, 0.9, 1$. Which of these portfolios has the lowest risk?

Part B

From excel, we have

	M	G
mean	0.1328	0.1241
sd	6.4136	2.3577
var	41.1345	5.5588
Cov	2.5508	

Given $P = wM + (1-w)G$, it is easy to calculate $E(P)$ and $Var(P)$.

$$\begin{aligned} E(P) =& E(wM + (1-w)G) = wE(M) + (1-w)E(G) \\ =& 0.1328 \cdot w + 0.1241 \cdot (1-w) \\ Var(P) =& Var(wM + (1-w)G) \\ =& w^2 Var(M) + (1-w)^2 Var(G) + 2w(1-w)Cov(M,G) \\ =& w^2 \cdot 41.1345 + (1-w)^2 \cdot 5.5588 + 2w(1-w) \cdot 2.5508 \end{aligned}$$

Part B

Substituting w from 0 to 1, we can have

w	$E(P)$	$Var(P)$
0	0.1241	5.5588
0.1	0.1249	5.3731
0.2	0.1258	6.0193
0.3	0.1267	7.4972
0.4	0.1276	9.8071
0.5	0.1284	12.9487
0.6	0.1293	16.9222
0.7	0.1302	21.7275
0.8	0.1311	27.3647
0.9	0.1319	33.8337
1	0.1328	41.1345

Group report

- Discuss the progress of the assignment
- Finalize the contract if not yet done last week
- Q&A

Any final questions?

Thanks for your attention! 😄

Let me know if you have any questions