ECON20001 Intermediate Macroeconomics

Tutorial 9 (Week 10)

Zheng Fan

The University of Melbourne

Introduction

Zheng Fan

- Ph.D student in Economics at Unimelb
- Consultation & Ed discussion board (your first priority)
- Email coordinators for administrative issues
- Consult Stop 1 for special consideration
- Email: fan.z@unimelb.edu.au (last resort!)

Before asking any questions, make sure you have gone through the Ed discussion board, subject guide, announcements and Q&A on Canvas!!!

Last week lectures

$① _T: \log Y_T = \log A_T + \alpha \log K_T + (1-\alpha) \log N_T \ldots$

$② _0: \log Y_0 = \log A_0 + \ldots$

$\frac{① - ②}{T}$

Growth accounting (for sources of economic growth)

- Total factor productivity (TFP): A $\leftarrow$ technology
- To apply
  - Growth rate decomposition (over time):

    ↳ time series

    country i: $0 \to T$.

    $g_Y = \frac{\log Y_T - \log Y_0}{T}$.

    $$\underbrace{g_Y - g_N = g_A + \alpha(g_K - g_N)}$$

    with TFP growth, g_A, and capital deepening, $g_K - g_N$
  - Levels decomposition (across countries)

    ↳ cross-sectional.

    $$\ln\left(\frac{Y_i}{N_i}\right) - \ln\left(\frac{Y_j}{N_j}\right) = \ln A_i - \ln A_j + \alpha\left(\ln\frac{K_i}{N_i} - \ln\frac{K_j}{N_j}\right)$$

    t:

    $i: \log Y_i = \log A_i + \alpha \log K_i + (1-\alpha) \log N_i$

    $j: \log Y_j = \log A_j + \alpha \log K_j + (1-\alpha) \log N_j$

Last week lectures

$Y = A K^{\alpha} N^{1-\alpha}.$

$60\% \quad 30\%.$

$1 - 60\% - 30\% = \underline{10\%}.$

TFP:

- As a residual: $\ln \hat{A} = \ln Y - \alpha \ln K - (1-\alpha) \ln N$
- Differences in output per worker mostly due to differences in measured TFP
- TFP is anything that effects the efficiency of production

True economy: $Y = A K^{\alpha} H^{\beta} N^{1-\alpha-\beta}$

$50\% \quad 10\% \quad 30\%$

$\to 10\%.$

Productivity and institutions

- Expropriation risk: negative
- Corruption: negative
- Contract enforceability: positive

use: $Y = A K^{\alpha} N^{1-\alpha}.$

$50\% \quad 30\%.$

$\to \underline{20\%}.$

Last week lectures

Factors of production

- Factors of income: $Y = rK + wN$
- Firm's profits: $F(K, N) - rK - wN$ $= \pi$ $\dfrac{\partial \pi}{\partial K} = 0$, $\dfrac{\partial \pi}{\partial N} = 0$.
- Profit-maximising:

$$\frac{\partial F(K,N)}{\partial K} = r; \quad \frac{\partial F(K,N)}{\partial N} = w$$

$MPK = r$, $MPL = w$.

We could then solve for the capital demand curve and labour demand curve.

Last week lectures

Wage and return in Solow model

- At steady state: $k^* = \left(\frac{s}{\delta + g_A + g_N}\right)^{\frac{1}{1-\alpha}}$ and $y^* = (k^*)^\alpha$
- Capital/Output ratio is constant
- Real wages grow at rate of productivity growth: $w = (1-\alpha) y^* A_t$
- Real return to capital is constant: $r_t = r$

In-tutorial Sheet - Q2

$$\underline{y} = \frac{Y}{AN} = \frac{K^{\alpha}(AN)^{\cancel{1}-\alpha}}{\cancel{AN}} = K^{\alpha}(AN)^{-\alpha}.$$

$$= \frac{K^{\alpha}}{(AN)^{\alpha}} = \left(\frac{K}{AN}\right)^{\alpha} = \underline{k}^{\alpha}.$$

2. Consider a Solow growth model with Cobb-Douglas production function

$$\frac{Y = K^{\alpha}(AN)^{1-\alpha}}{AN}.$$

with constant savings rate s, depreciation rate δ and no growth in productivity or labor ($g_A = g_N = 0$).

(a) Suppose $A = 1, \alpha = 1/3, s = 0.2$ and $\delta = 0.1$ (annual). Calculate the steady state capital per effective worker and steady state output per effective worker.

In-tutorial Sheet - Q2

Let $k = K/AN$ and $y = Y/AN$ denote steady state capital and output per effective worker.

Using the Cobb-Douglas production function, these are related by

$$y = k^{\alpha} \equiv f(k)$$

The steady state condition is $k_{t+1} = k_t = k^*$.

$$\Downarrow$$

$$sy^* = (\delta + g_A + g_N)k^*$$

In terms of the capital per effective worker, the steady state condition is

$$\boxed{sk^{\alpha} = (\delta + g_A + g_N)k}$$

$$\Rightarrow k^*$$

In-tutorial Sheet - Q2

Solving this gives

$$k^* = \left(\frac{s}{\delta + g_A + g_N}\right)^{\frac{1}{1-\alpha}} \tag{1}$$

And so steady state output/worker is

$$y^* = f(k^*) = (k^*)^\alpha = \left(\frac{s}{\delta + g_A + g_N}\right)^{\frac{\alpha}{1-\alpha}} \tag{2}$$

In-tutorial Sheet - Q2

Recall (a) said $A = 1, \alpha = 1/3, s = 0.2$ and $\delta = 0.1$.

With $g_A = g_N = 0, A = 1, \alpha = 1/3, s = 0.20$ and $\delta = 0.10$, from (1) steady state capital per worker is

$$k^* = \left(\frac{s}{\delta + g_A + g_N}\right)^{\frac{1}{1-\alpha}} = \left(\frac{0.2}{0.1}\right)^{3/2} = \underline{2.828} \quad = 2\sqrt{2}$$

From (2) steady state output/worker is

$$y^* = f(k^*) = (k^*)^\alpha = \left(\frac{s}{\delta + g_A + g_N}\right)^{\frac{\alpha}{1-\alpha}} = 2^{1/2} = 1.414 \quad = \sqrt{2}$$

In-tutorial Sheet - Q2

$y = k^{\alpha}$

(b) Suppose that the real wage w and real return to capital r are equal to the marginal products of labor and capital respectively. Calculate the steady state wage rate and return to capital.

$w = MPL \leftarrow \frac{\partial \Pi}{\partial N} = 0 \Rightarrow \frac{\partial F}{\partial N} = \frac{\partial K^{\alpha}(AN)^{1-\alpha}}{\partial N}$

$= (1-\alpha) K^{\alpha}(AN)^{-\alpha} \cdot A = (1-\alpha) A k^{\alpha}$

$= (1-\alpha) A y$

$\boxed{r = MPK} \leftarrow \frac{\partial \Pi}{\partial K} = 0$

$\Rightarrow \frac{\partial \Pi}{\partial K} = 0 \Rightarrow \boxed{\frac{\partial F}{\partial K} = r}$

$= \frac{\partial K^{\alpha}(AN)^{1-\alpha}}{\partial K}$

$= \alpha K^{\alpha-1}(AN)^{1-\alpha}$

$= \alpha \frac{K^{\alpha-1}}{(AN)^{\alpha-1}} = \alpha k^{\alpha-1}$

$= \alpha \frac{k^{\alpha}}{k} = \alpha \frac{y}{k}$

$\Rightarrow r^{*} = \alpha \frac{y^{*}}{k^{*}} \quad \boxed{g_r = 0}$

$w^{*} = (1-\alpha) A \cdot y^{*}$

$\Rightarrow \boxed{g_w = g_A}$

In-tutorial Sheet - Q2

(b) Suppose that the real wage w and real return to capital r are equal to the marginal products of labor and capital respectively. Calculate the steady state wage rate and return to capital.

The marginal productivity conditions can be written in per effective worker terms as

$$w = (1-\alpha)AK^{\alpha}(AN)^{-\alpha}$$

and it follows that

$$w = (1-\alpha)Ak^{*\alpha} = (1-\alpha)Ay^{*}$$

For the return to capital

$$r = \alpha K^{\alpha-1}(AN)^{1-\alpha} = \alpha k^{*\alpha-1} = \alpha \frac{y^{*}}{k^{*}}$$

Once k and y are in the steady state, the growth rate of w is the same as the growth rate of A, which is g_A and r is a constant.

In-tutorial Sheet - Q2

With $\alpha = 1/3, s = 0.20$ and $\delta = 0.10$, the return on capital is

$$r = \alpha \frac{y^*}{k^*} = (1/3)(1/2) = \underline{0.167}$$

or about 16.7% annual. Similarly, the wage rate is

$$w = (1 - \alpha)Ay^* = (2/3)(1)(1.414) = \underline{0.943}$$

In-tutorial Sheet - Q2

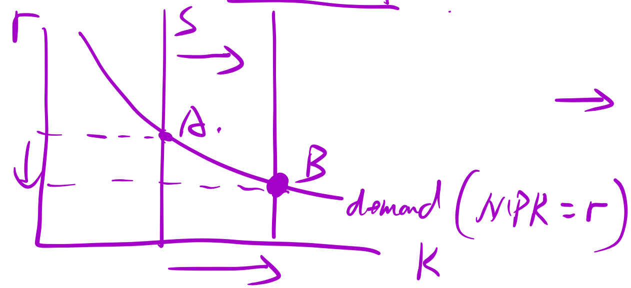
saving = investment

H.H

firms

(c) Now suppose the saving rate increases to $s = 0.25$. What happens to the steady state w and r? Do they rise or fall? Give $\boxed{\text{intuition}}$ for your results.

$s \uparrow \rightarrow$ supply of $K \uparrow \rightarrow$

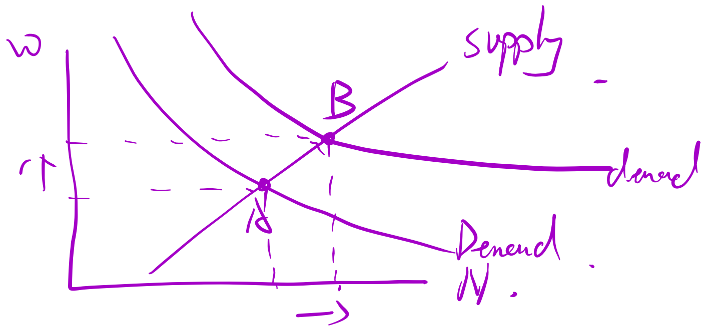


$\rightarrow r \downarrow$.

$s \uparrow \rightarrow$ supply of $K \uparrow \rightarrow K \uparrow \rightarrow$ Demand for labour $\uparrow$.

$\downarrow$

$w \uparrow$.



In-tutorial Sheet - Q2

(c) Now suppose the saving rate increases to $s = 0.25$. What happens to the steady state w and r? Do they rise or fall? Give intuition for your results.

Saving rate ↑ leads to supply of capital ↑. Think about the demand-supply analysis, so r ↓, which can be observed from

$$r = \alpha \frac{(\delta + g_A + g_N)}{s}$$

With the given numbers, the return falls from $r = 0.167$ (when $s = 0.2$) to

$$r = (1/3)\frac{(1/10)}{(1/4)} = 0.133 \approx 13.3\% \downarrow$$

The increase in capital delivered by the higher saving rate also increases the demand for labor and the wage rate increases. In particular, with the given numbers, the wage rate increases from 0.943 (when $s = 0.2$) to

$$w = (2/3)(1)(2.5)^{1/2} = 1.054 \uparrow$$

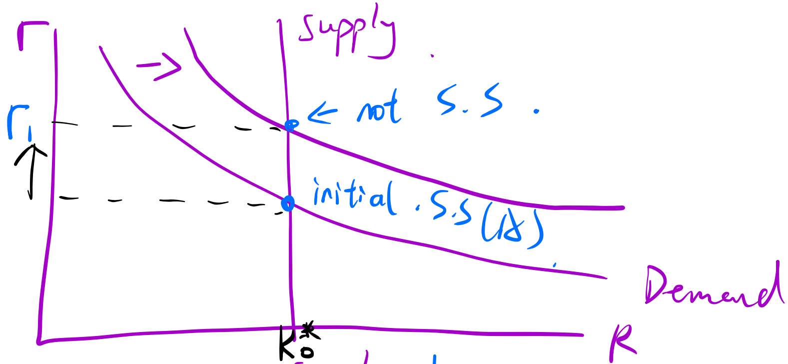
In-tutorial Sheet - Q2

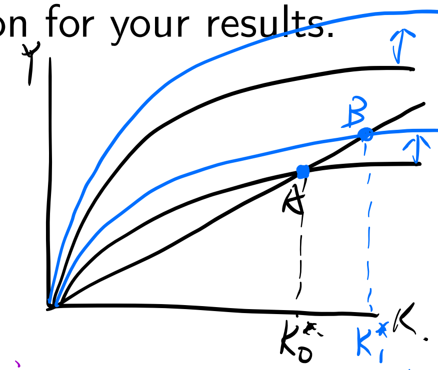
$$\boxed{r^* = \alpha \frac{y^*}{k^*}} \longrightarrow g_r = 0.$$

$$w^* = (1-\alpha) \cdot A \, y^* \longrightarrow g_w = g_A.$$

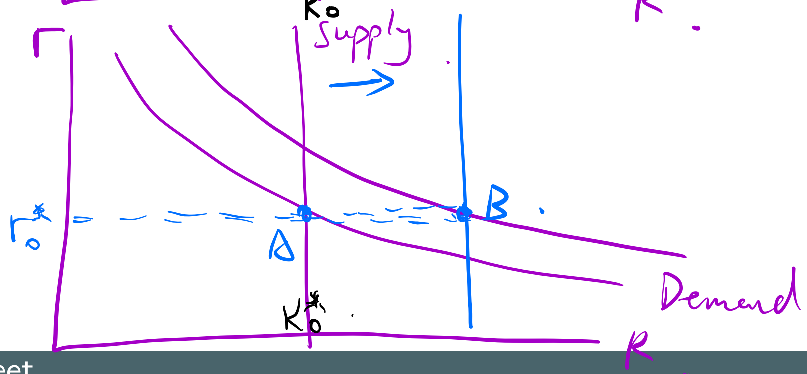
(d) What if we still have $\boxed{s = 0.2}$ but productivity increases by 10% from $A = 1$ to $A = 1.1$. How does this change the wage rate and return in the short run? What about the long run? Again, give intuition for your results.

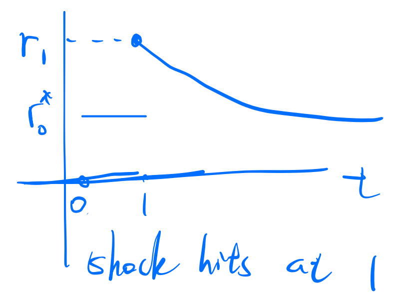
SR :





LR :





In-tutorial Sheet - Q2

(d) What if we still have $s = 0.2$ but productivity increases by 10% from $A = 1$ to $A = 1.1$. How does this change the wage rate and return in the short run? What about the long run? Again, give intuition for your results.

As above, the return to capital and wage rate are, in steady state,

$$r = \alpha \frac{(\delta + g_A + g_N)}{s}$$

and

$$w = (1-\alpha)A\left(\frac{s}{\delta + g_A + g_N}\right)^{\dfrac{\alpha}{1-\alpha}}$$

In-tutorial Sheet - Q2

The increase in productivity has no effect on the return to capital in steady state (in the long run). While the increase in productivity increases the demand for capital, the increase in productivity also increases capital accumulation so that in the long run the supply of capital rises too.

In fact, the long run supply curve for capital is perfectly horizontal (completely elastic). The wage rate rises as the increase in productivity and the resulting increase in capital both increase the demand for labor (capital and labor are complements in production). With the given numbers, the new wage rate is

$$w = (2/3)(1.1)(2)^{1/2} = 1.037$$

which is indeed higher than the $w = 0.943$ when productivity is only $A = 1$.

In-tutorial Sheet - Q2

(e) Now suppose that $\boxed{g_A = g_N = 0.05}$ and we still have $s = 0.2$. Calculate the steady state capital per effective worker and steady state output per effective worker. What is the long run growth rate of the wage rate? What is the long run growth rate of the return to capital? Explain.

In-tutorial Sheet - Q2

(e) Now suppose that $g_A = g_N = 0.05$ and we still have $s = 0.2$. Calculate the steady state capital per effective worker and steady state output per effective worker. What is the long run growth rate of the wage rate? What is the long run growth rate of the return to capital? Explain.

$g_w = g_A$.

$g_r = 0$.

Using the steady state equation for k, we can find

$$k^* = \left(\frac{0.2}{0.1 + 0.05 + 0.05}\right)^{3/2} = 1$$

and

$$y^* = 1^{1/2} = 1.$$

Notice that the long run growth rate of w is given by g_A, so the growth rate of w is 0.05.

The return to capital is constant in the long run, so the changes in g_A and g_N do not affect the long run growth rate of r.

The end

Thanks for your attention!