ECON20001 Intermediate Macroeconomics

Tutorial 4 (Week 5)

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Semester 2, 2022

The University of Melbourne

Introduction

Zheng Fan

- Ph.D student in Economics at Unimelb
- Consultation & Ed discussion board (your first priority)
- Email Dr David Moreton for all administrative issues
- Consult Stop 1 for special consideration
- Email: fan.z@unimelb.edu.au (last resort!)

Before asking any questions, make sure you have gone through the Ed discussion board, subject guide and Q&A on Canvas!

Slides: github.com/zhengf1/InterMa2022

Last week lectures

Model of flows

$$U_{t+1} - U_t = sN_t - fU_t$$

Dividing both side by the constant labor force $\bar{L}$

$$u_{t+1} - u_t = s(1 - u_t) - fu_t$$

job finding rate

In steady state: $u_t = u_{t+1} = \bar{u}$

$\theta = \frac{v}{u}$

s. s.:

$$\bar{u} = \frac{s}{s + f} \longrightarrow f(\theta)$$

$s \uparrow$ or $f \downarrow \rightarrow \bar{u} \uparrow$

$\boxed{\text{Beveridge Curve}}$

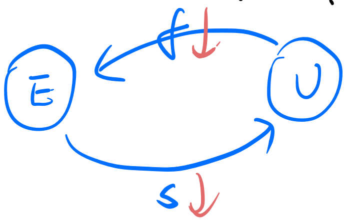
Last week lectures

Unemployment dynamics

- From an initial value u_0: $\boxed{u_t - \bar{u} = \lambda^t (u_0 - \bar{u})}$
- $\lambda = 1 - (s + f)$
- $\lambda \uparrow$ or $s \downarrow$ or $f \downarrow \rightarrow$ speed of adjustment $\downarrow$

$t=1$ $u_1 = ?$
$t=2$ $u_2 = ?$
$\vdots$
$t=?$ converge.

$t=T$ $u_T = ?$



Last week lectures

Unemployment dynamics

- From an initial value u_0: $u_t - \bar{u} = \lambda^t(u_0 - \bar{u})$
- $\lambda = 1 - (s + f)$
- $\lambda \uparrow$ or $s \downarrow$ or $f \downarrow \rightarrow$ speed of adjustment $\downarrow$

CRS.

Job matches

$\theta = \frac{V_t}{U_t}$

- Matching function: $M_t = F(U_t, V_t) = AU_t^{1-\alpha} V_t^{\alpha}$
- Jobs finding rate: $f = \frac{M_t}{U_t} = \frac{F(U_t, V_t)}{U_t} \overset{\text{why?}}{=} F\left(\frac{U_t}{U_t}, \frac{V_t}{U_t}\right) = F(1, \theta_t)$

  $= f(\theta_t)$.
- Labor market tightness: $\theta = \frac{v}{u} \rightarrow f = f(\theta) = A\theta^{\alpha}$

$\theta \uparrow$, $f(\theta) \uparrow$.

$\theta = \frac{v}{u} \uparrow$. $v \uparrow > u$

Last week lectures

Unemployment dynamics

- From an initial value u_0: $u_t - \bar{u} = \lambda^t (u_0 - \bar{u})$
- $\lambda = 1 - (s + f)$
- $\lambda \uparrow$ or $s \downarrow$ or $f \downarrow \rightarrow$ speed of adjustment $\downarrow$

Job matches

- Matching function: $M_t = F(U_t, V_t) = AU_t^{1-\alpha} V_t^{\alpha}$
- Jobs finding rate: $f = \dfrac{M_t}{U_t} = \dfrac{F(U_t, V_t)}{U_t}$
- Labor market tightness: $\theta = \dfrac{v}{u} \rightarrow f = f(\theta) = A\theta^{\alpha}$

Beveridge curve

- Implicit function: $\boxed{u = \dfrac{s}{s + f(v/u)}}$

Last week lectures JC: $v = \theta^* \cdot u$.

$\theta\uparrow \rightarrow f(\theta)\uparrow$
$\theta\downarrow \rightarrow f(\theta)\downarrow$.

$f = \frac{m}{u}$

Job creation curve

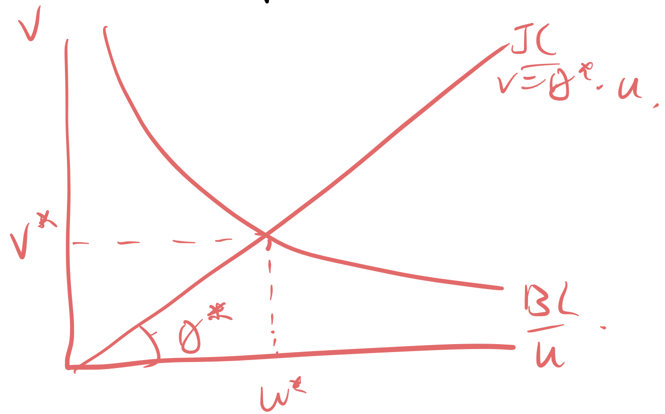
$\xi = \frac{m}{v}$

- Vacancy filling rate: $\boxed{q_t} = q(\theta_t) = A\theta_t^{\overset{\alpha-1}{\cancel{1-\alpha}}}$ $\theta\uparrow \rightarrow \xi(\theta)\downarrow$.
- Expected value of a vacancy: qJ
- Expected payoff from posting vacancy: $qJ - c$
- Job creation function: $v = \theta^* u$ with

$$q(\theta^*)J = c \rightarrow \theta^* = \left(\frac{AJ}{c}\right)^{\frac{1}{1-\alpha}}$$

free-entry

$$\boxed{\xi J - c = 0}$$



Last week lectures

Job creation curve

- Vacancy filling rate: $q_t = q(\theta_t) = A\theta_t^{1-\alpha}$
- Expected value of a vacancy: qJ
- Expected payoff from posting vacancy: $qJ - c$
- Job creation function: $v = \theta^* u$ with

$$q(\theta^*)J = c \rightarrow \theta^* = \left(\frac{AJ}{c}\right)^{\frac{1}{1-\alpha}}$$

Duration:

$v < u$

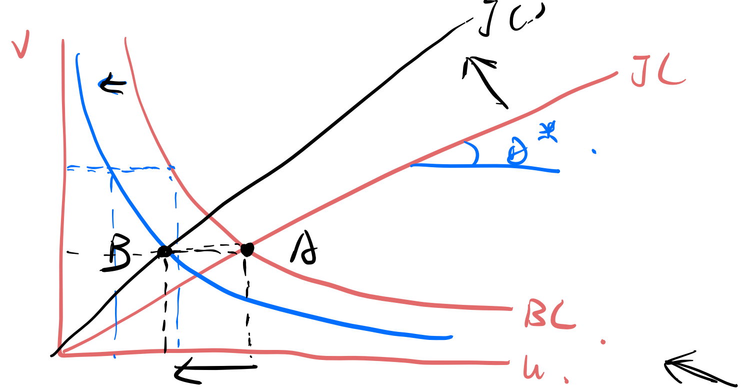
$\theta \downarrow$

- Recession: labor market is more slack, $f(\theta)\downarrow \rightarrow$ unemployment duration $1/f(\theta)\uparrow$

$\theta \uparrow$ $v > u$

- Boom: labor market is tighter, $f(\theta)\uparrow \rightarrow$ unemployment duration $1/f(\theta)\downarrow$

In-tutorial Sheet - Q2



Consider our model of labor market fluctuations. Label each of the following statements true, false or uncertain. Explain briefly:

(i) An increase in matching efficiency reduces both unemployment and vacancies proportionately and so has no effect on labor market tightness.

$A \uparrow \longrightarrow$ BC shift in

$\longrightarrow \theta^* = \left(\frac{AJ}{c}\right)^{\frac{1}{1-\alpha}} \uparrow \rightarrow$ JC up

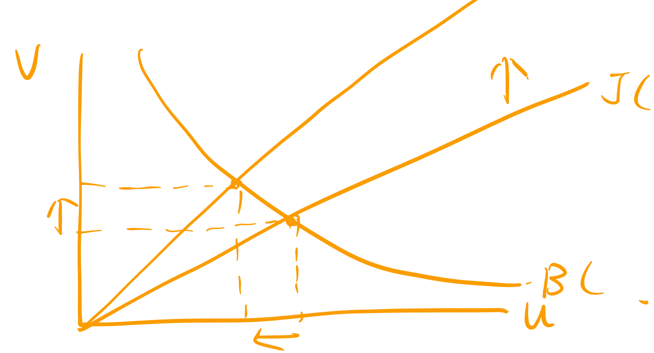
In-tutorial Sheet - Q2

Consider our model of labor market fluctuations. Label each of the following statements true, false or uncertain. Explain briefly:

(i) An increase in matching efficiency reduces both unemployment and vacancies proportionately and so has no effect on labor market tightness.

False. An increase in matching efficiency shifts in the Beveridge curve, but it also rotates the job creation curve to the left so that labor market tightness actually increases.

In-tutorial Sheet - Q2



Consider our model of labor market fluctuations. Label each of the following statements true, false or uncertain. Explain briefly:

(ii) An increase in the value of a filled position J increases a firm's incentive to create jobs, but, because it also increases labor market tightness θ, there is ~~no effect on~~ the rate at which a firm actually fills jobs in equilibrium.

$$J \uparrow \Rightarrow \theta^* = \left(\frac{\delta J}{c}\right)^{\frac{1}{1-\alpha}} \uparrow \Rightarrow q(\theta) \downarrow .$$

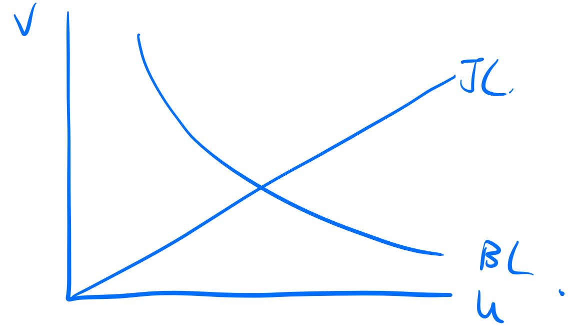
In-tutorial Sheet - Q2

Consider our model of labor market fluctuations. Label each of the following statements true, false or uncertain. Explain briefly:

(ii) An increase in the value of a filled position J increases a firm's incentive to create jobs, but, because it also increases labor market tightness θ, there is no effect on the rate at which a firm actually fills jobs in equilibrium.

False. An increase in J increases labor market tightness θ but because the vacancy filling rate $q(\theta)$ is decreasing in θ, the vacancy filling rate (i.e., the rate at which a firm fills jobs) falls in equilibrium.

In-tutorial Sheet - Q2



Consider our model of labor market fluctuations. Label each of the following statements true, false or uncertain. Explain briefly:

(iii) A fall in the cost of posting vacancies c reduces the average duration of unemployment but increases the average duration of an unfilled job opening.

$c \downarrow \Rightarrow \theta^* = \left(\frac{\delta J}{c\downarrow}\right)^{\frac{1}{1-\alpha}} \uparrow \Rightarrow f(\theta)\uparrow \Rightarrow \frac{1}{f(\theta)} \downarrow$

$q(\theta) \downarrow \Rightarrow \frac{1}{q(\theta)} \uparrow$

In-tutorial Sheet - Q2

Consider our model of labor market fluctuations. Label each of the following statements true, false or uncertain. Explain briefly:

(iii) A fall in the cost of posting vacancies c reduces the average duration of unemployment but increases the average duration of an unfilled job opening.

True. A fall in c increases labor market tightness θ. The job finding rate $f(\theta)$ also increases because $f(\theta)$ is increasing in θ. So unemployment duration $1/f(\theta)$ decreases. Similarly, the increase in θ decreases $q(\theta)$ and so increases vacancy duration $1/q(\theta)$.

In-tutorial Sheet - Q3

Consider the Cobb-Douglas matching function

$$m = F(u, v) = A\sqrt{uv}, \qquad A > 0$$

(a) Show that this matching function has constant returns to scale.

$$\lambda u, \lambda v \longrightarrow \lambda m$$

In-tutorial Sheet - Q3

$f(\theta) = \frac{m}{u} = \frac{A\sqrt{uv}}{u} = A\sqrt{\frac{v}{u}} = A\sqrt{\theta}$

$g(\theta) = \frac{m}{v} = \frac{A\sqrt{uv}}{v} = A\sqrt{\frac{u}{v}} = A\sqrt{\frac{1}{\theta}}$

$\theta = \frac{v}{u}$

Consider the Cobb-Douglas matching function

$f(\theta) = \frac{m}{u} = \frac{A\sqrt{uv}}{u}$

$= A\frac{\sqrt{uv}}{\sqrt{u^2}}$

$$m = F(u, v) = A\sqrt{uv}, \qquad A > 0$$

$= A\sqrt{\frac{uv}{u^2}} = A\sqrt{\frac{v}{u}}$

(a) Show that this matching function has constant returns to scale.

CRS

If we scale u and v by some common $X > 0$ we have

$$F(uX, vX) = A\sqrt{uXvX} = A\sqrt{uv}\sqrt{XX} = A\sqrt{uv}X = mX$$

so this matching function does have constant returns.

In-tutorial Sheet - Q3

$$f(\theta) = \frac{m}{u} = \frac{A\sqrt{uv}}{u} = \boxed{A\sqrt{\frac{v}{u}}} = A\sqrt{\theta}$$

$$q(\theta) = \frac{m}{v} = \frac{A\sqrt{uv}}{v} = A\sqrt{\frac{u}{v}} = A\sqrt{\frac{1}{\theta}}$$

(b) Use this matching function to show that the Beveridge curve is downward sloping, i.e., solve for v in terms of u and show that an increase in u reduces v.

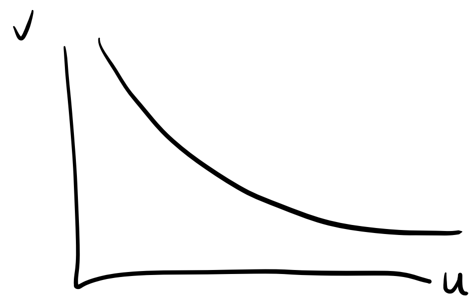
$$\text{BC}: \quad u = \frac{s}{s+f} = \frac{s}{s + A\sqrt{\frac{v}{u}}} \Rightarrow s + A\sqrt{\frac{v}{u}} = \frac{s}{u}.$$

$$\Rightarrow A\sqrt{\frac{v}{u}} = \frac{s - us}{u} = \frac{s(1-u)}{u}.$$

$$\Rightarrow \sqrt{\frac{v}{u}} = \frac{s(1-u)}{Au}$$

$$\Rightarrow \frac{v}{u} = \frac{s^2(1-u)^2}{A^2u^2}$$

$$\Rightarrow v = \frac{s^2(1-u)^2}{A^2 u}$$



$u\uparrow \rightarrow v\downarrow.$

In-tutorial Sheet - Q3

(b) Use this matching function to show that the Beveridge curve is downward sloping, i.e., solve for v in terms of u and show that an increase in u reduces v.

The job finding rate with this matching function is

$$f = \frac{F(u,v)}{u} = \frac{A\sqrt{uv}}{u} = A\sqrt{\frac{v}{u}}$$

where v/u is the labor market tightness ratio. The steady state unemployment rate satisfies

$$u = \frac{s}{s+f}$$

where the job finding rate is increasing in labor market tightness, $f = A\sqrt{v/u}$. Plugging this in

$$u = \frac{s}{s + A\sqrt{v/u}}$$

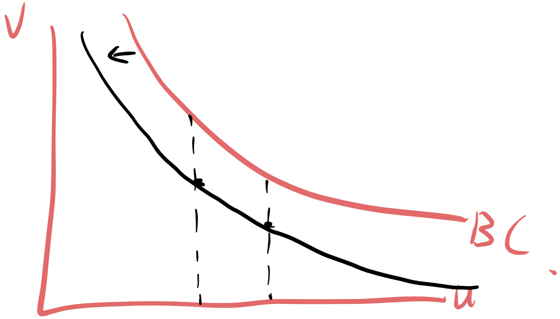
In-tutorial Sheet - Q3

Multiplying both sides by $s + A\sqrt{v/u}$ and rearranging

$$su + A\sqrt{vu} = s \Leftrightarrow A\sqrt{vu} = s(1-u)$$

In steady state the flow out of unemployment $fu = s(1-u)$ the flow into unemployment. Hence solving for v we get the following expression for the Beveridge curve

$$v = \left(\frac{s}{A}\right)^2 \frac{(1-u)^2}{u}$$



downward

- An increase in u reduces v

shift

- An increase in A (a more efficient labor market) shifts in the Beveridge curve, reducing vacancies at any level of unemployment
- An increase in s (a greater flow out of employment) shifts out the Beveridge curve, increasing unemployment at any level of vacancies.

In-tutorial Sheet - Q3

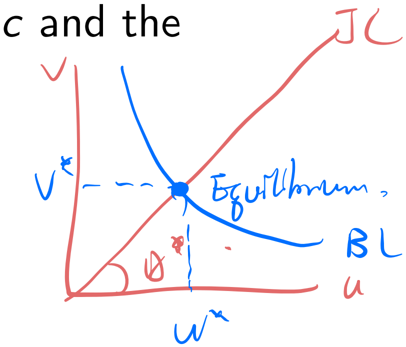
$$f(\theta) = \frac{m}{u} = \frac{A\sqrt{uv}}{u} = A\sqrt{\frac{v}{u}} = A\sqrt{\theta}$$

$$q(\theta) = \frac{m}{v} = \frac{A\sqrt{uv}}{v} = A\sqrt{\frac{u}{v}} = A\sqrt{\frac{1}{\theta}}$$

(c) Solve for the equilibrium labor market tightness ratio θ^* in terms of the value of a filled vacancy J, the cost of posting a vacancy c and the matching efficiency parameter A.

$$V = \theta^* \cdot u.$$

?



Free-entry $\Rightarrow \quad qJ = c.$

$$\Rightarrow A\sqrt{\frac{1}{\theta}} \cdot J = c.$$

$$\Rightarrow \sqrt{\frac{1}{\theta}} = \frac{c}{AJ}.$$

$$\Rightarrow \frac{1}{\theta} = \left(\frac{c}{AJ}\right)^2.$$

$$\Rightarrow \theta^* = \left(\frac{AJ}{c}\right)^2. \longrightarrow \text{slope of JC.}$$

In-tutorial Sheet - Q3

(c) Solve for the equilibrium labor market tightness ratio θ^* in terms of the value of a filled vacancy J, the cost of posting a vacancy c and the matching efficiency parameter A.

With this matching function, the vacancy filling rate is

$$q = \frac{F(u,v)}{v} = A\frac{\sqrt{uv}}{v} = A\sqrt{\frac{u}{v}}$$

Vacancies are created until the expected value of a filled vacancy qJ equals the cost of creating them c, that is

$$qJ = c \Leftrightarrow A\sqrt{u/v}J = c$$

Solving this for the labor market tightness ratio $v/u = \theta^*$ we get

$$v/u = \theta^* = (AJ/c)^2$$

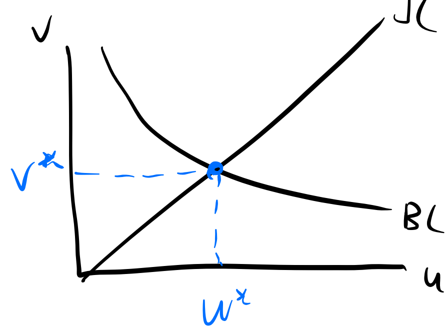
An increase in J or A makes for a tighter labor market (rotating the job creation curve to the left), an increase in c does the reverse.

In-tutorial Sheet - Q3

(d) Using your solution for labor market tightness and the Beveridge curve, solve for equilibrium v and u. Explain how an increase in the value J of a filled vacancy affects $\boxed{\text{equilibrium } v \text{ and } u.}$ Suppose J rises in a boom, what does this do to labor market tightness, vacancies and unemployment? Suppose A also rises. Does this affect vacancies and unemployment in the same way? Explain.

$$BC: \quad u^* = \frac{s}{s+f} = \frac{s}{s+A\sqrt{\theta^*}} = \frac{s}{s+A\cdot\frac{AJ}{c}} = \frac{s}{s+A^2J/c} = \frac{cs}{cs+A^2J}.$$

$$JC: \quad v^* = \theta^* \cdot u^* = \left(\frac{AJ}{c}\right)^2 \cdot \frac{cs}{cs+A^2J}.$$



In-tutorial Sheet - Q3

(d) Using your solution for labor market tightness and the Beveridge curve, solve for equilibrium v and u. Explain how an increase in the value J of a filled vacancy affects equilibrium v and u. Suppose J rises in a boom, what does this do to labor market tightness, vacancies and unemployment? Suppose A also rises. Does this affect vacancies and unemployment in the same way? Explain.

At this equilibrium labor market tightness ratio, the job finding rate is $f^* = A\sqrt{\theta^*}$ or $= A^2 J/c$

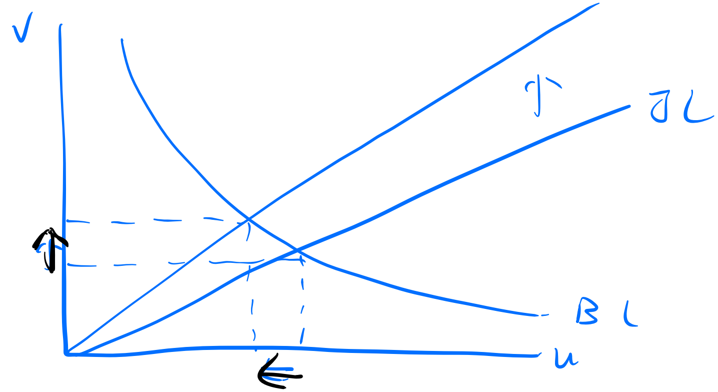
The associated equilibrium unemployment rate is therefore

$$u^* = \frac{s}{s + f^*} = \frac{sc}{sc + A^2 J}$$

and the corresponding equilibrium vacancy rate is

$$v^* = \theta^* u^* = \left(\frac{JA}{c}\right)^2 \frac{sc}{sc + A^2 J}$$

In-tutorial Sheet - Q3



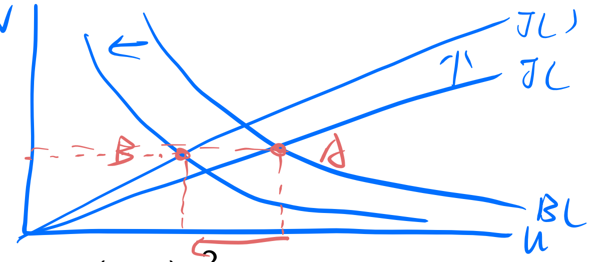
$$u^* = \frac{s}{s+f^*} = \frac{sc}{sc + A^2 J} \qquad v^* = \theta^* u^* = \left(\frac{JA}{c}\right)^2 \frac{sc}{sc + A^2 J}$$

$$= \frac{J^2 A^2}{c^2} \cdot \frac{sc}{sc + A^2 J} \cdot \frac{1/J^2}{1/J^2}$$

$$= \frac{A^2}{c^2} \cdot \frac{sc}{\frac{sc}{J^2} + \frac{A^2}{J}}$$

An increase in J: T/F (ii) $\rightarrow$ θ^* ↑.

In-tutorial Sheet - Q3



$$u^* = \frac{s}{s+f^*} = \frac{sc}{sc + A^2 J}$$

$$v^* = \theta^* u^* = \left(\frac{JA}{c}\right)^2 \frac{sc}{sc + A^2 J}$$

An increase in A: T/F (i). $\longrightarrow$ $A \uparrow \to \theta^* \uparrow$ $\to$ JC up, $\to$ BL in

θ

θ^*

$\to$ equilibrium.

$$v^* = \frac{J^2 A^2}{c^2} \cdot \frac{sc}{sc + A^2 \cdot J} \cdot \frac{1/A^2}{1/A^2}$$

$$= \frac{J^2}{c^2} \cdot \frac{sc}{\frac{sc}{A^2} + J}.$$

Y $\quad$ Y^*

not a general case.

$m = A\sqrt{uv}$

The end

Thanks for your attention! 😃