ECON20001 Intermediate Macroeconomics

Tutorial 7 (Week 8)

Zheng Fan

The University of Melbourne

Introduction

Zheng Fan

- Ph.D student in Economics at Unimelb
- Consultation & Ed discussion board (your first priority)
- Email coordinators for administrative issues
- Consult Stop 1 for special consideration
- Email: fan.z@unimelb.edu.au (last resort!)

Before asking any questions, make sure you have gone through the Ed discussion board, subject guide, announcements and Q&A on Canvas!!!

Slides: github.com/zhengf1/InterMa2022

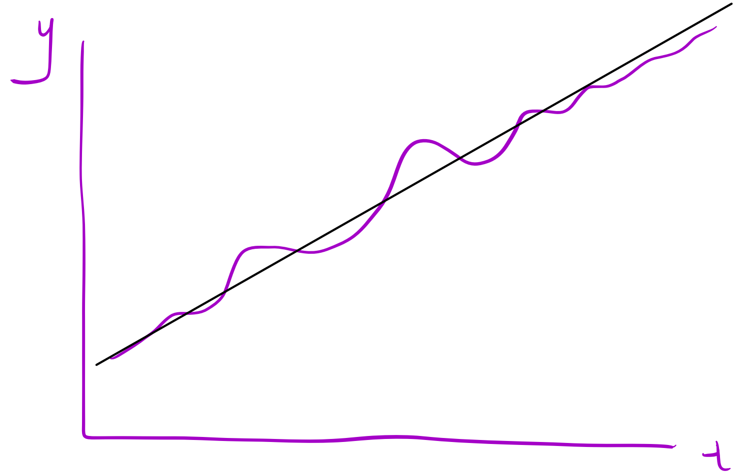
Public holidays on next Thursday and Friday

Please carefully read the announcement released at 11am today.

- Include all the details about replacement tutorials for next week.
- Campus students, you may attend any on campus tutorials between Monday and Wednesday. There are also some replacement tutorials scheduled.
- Online students, you may attend any online tutorials between Monday and Wednesday. There are also some replacement online tutorials scheduled.

Also notice that, after next week, we will be in a non-teaching week, so no tutorials on the week after next week. In short, no teaching activities from 22-Sep (Thursday) to 2 Oct (Sunday).

Long run growth



Long-run growth:

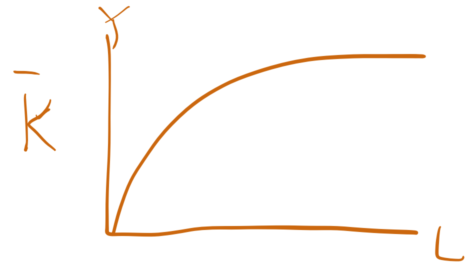
- Trend dominates (fluctuations dominate the SR)
- Driven by supply-side considerations, such as technological progress A

Convergence: levels of output per person across (rich) countries have become closer.

$A \xrightarrow{s,\delta,n} S.S_A.$

$B \xrightarrow{s,\delta,n} S.S_B.$

Aggregate production function



Aggregate production function

- $Y = F(K, AN) = F(K, N)$ when $A = 1$
- Diminishing marginal returns to capital (labor)
- Constant return to scale $\rightarrow \times K, \times N \Rightarrow \times Y$.

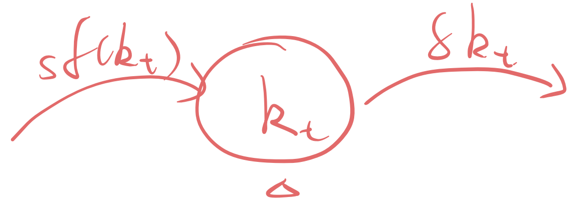
Intensive variables (output per worker)

$$\frac{Y}{N} = \frac{F(K,N)}{N} = F\left(\frac{K}{N}, \frac{N}{N}\right) = f(k, 1)$$

- $y = f(k, 1) = f(k)$
- $f(k)$ increasing in k at a diminishing rate
- Technology improvements shifts it up

A

The Solow model



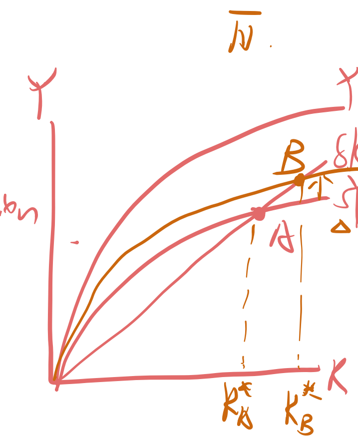
Solow model:

- Closed economy with balanced budget: $I_t = S_t = sY_t = sF(K_t, N_t)$
- Capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t$
- In per worker terms: $k_{t+1} - k_t = sf(k_t) - \delta k_t$ $= 0$.
- Steady-state where capital per worker not changing

$$k_{t+1} = k_t = k^* \Leftrightarrow sf(k^*) = \delta k^*$$

saving = depreciation

$\Rightarrow y^*$.



Savings rate (s) and output

- No effect on long-run growth rate of output per worker
- Determines long-run level of output per worker
- Increase in s leads to higher growth of output per worker in the SR
- Golden rule: choose s to maximise consumption per worker

$s \leq 100\%$.

In-tutorial Sheet - Q2

2. Consider the production function

$$Y = \sqrt{K}\sqrt{N} = K^{1/2} N^{1/2} = K^{\frac{1}{2}}$$

Assume that N is constant and equal to 1.

(i) Derive the relation between the growth rate of output and the growth rate of capital.

In-tutorial Sheet - Q2

$MV = PY$.

$\log Y_t = \frac{1}{2} \log K_t$ ... ①

$\log Y_{t-1} = \frac{1}{2} \log K_{t-1}$ ... ②

① - ② : $g_Y = \frac{1}{2} \cdot g_K$.

2. Consider the production function

$$Y_t = \sqrt{K}\sqrt{N} = K^{1/2} N^{1/2} = K_t^{\frac{1}{2}}$$

Assume that N is constant and equal to 1.

(i) Derive the relation between the growth rate of output and the growth rate of capital.

$g_Y = \log Y_t - \log Y_{t-1}$

$g_K = \log K_t - \log K_{t-1}$.

The growth rate of a variable X_t between dates $t-1$ and t is

$$g_X = \frac{X_t - X_{t-1}}{X_{t-1}} \approx \boxed{\ln X_t - \ln X_{t-1}}$$

The approximation holds when X is small.

In-tutorial Sheet - Q2

Now lets apply this to the production function. When $N = 1$ output per worker is $Y = \sqrt{K}$. Take logs of this equation to write

$$\ln Y = 0.5 \ln K$$

(remember $\ln(X^a) = a \ln X$ etc). Therefore

$$(\ln Y_t - \ln Y_{t-1}) = 0.5(\ln K_t - \ln K_{t-1})$$

And so in growth rates

$$\boxed{g_Y \approx 0.5 g_K}$$

In-tutorial Sheet - Q2

$g_Y = \frac{1}{2} g_K$. $g_Y = 2\%$.

(ii) Suppose that we want to achieve output growth equal to 2 per cent per year. What is the required rate of capital growth? $g_K = ?$

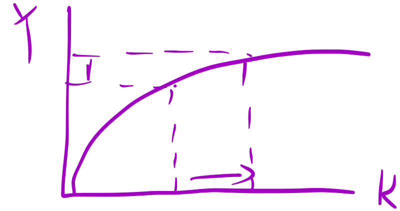
In-tutorial Sheet - Q2

$g_K > g_Y$.

(ii) Suppose that we want to achieve output growth equal to 2 per cent per year. What is the required rate of capital growth?

We want $g_Y = 2\%$, but $g_Y = 0.5 g_K$ so we need $\boxed{g_K = 4\%.}$

In-tutorial Sheet - Q2



(ii) Suppose that we want to achieve output growth equal to 2 per cent per year. What is the required rate of capital growth?

We want $g_Y = 2\%$, but $g_Y = 0.5 g_K$ so we need $g_K = 4\%$.

(iii) In (ii), what happens to the ratio of capital to output K/Y over time?

$$\frac{K_{t+1}}{Y_{t+1}} = \boxed{\frac{\cancel{K_t}(1+g_K)}{\cancel{Y_t}\cdot(1+g_Y)} > \frac{\cancel{K_t}}{\cancel{Y_t}}}$$

$$\Rightarrow \quad (1+g_K) > (1+g_Y)$$

$$\Rightarrow \quad g_K > g_Y.$$

In-tutorial Sheet - Q2

(ii) Suppose that we want to achieve output growth equal to 2 per cent per year. What is the required rate of capital growth?

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K/Y will rise over time as capital is growing faster than output. Capital must grow faster than output because there are diminishing returns to capital in the production function.

In-tutorial Sheet - Q2

$Y = \sqrt{K}$

(ii) Suppose that we want to achieve output growth equal to 2 per cent per year. What is the required rate of capital growth?

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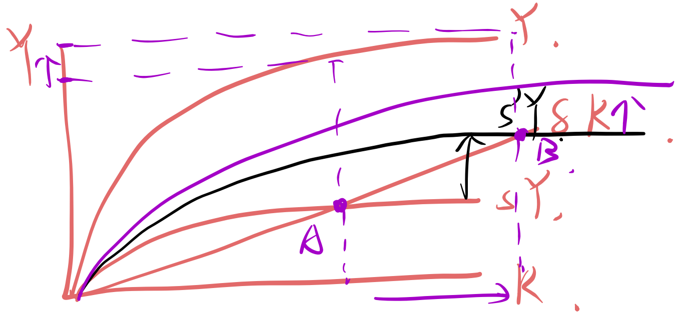
$g_Y = 2\%$?

(iv) Is it possible to sustain output growth of 2% forever in this economy? Justify your answer.

$g_K = 4\%$



$A \uparrow$

$s \leq 1$

In-tutorial Sheet - Q2

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(iv) Is it possible to sustain output growth of 2% forever in this economy? Justify your answer.

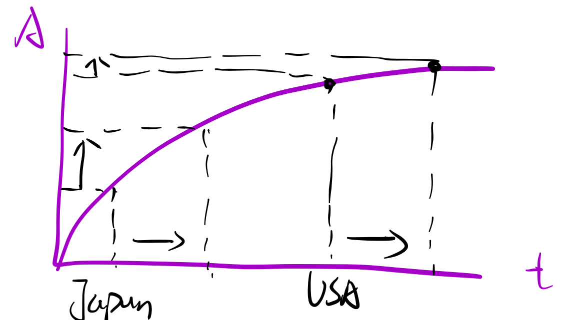
$g_K = 4\%$.

No. Since capital is growing faster than output, the saving rate will have to increase to maintain the same pace. The required saving rate will keep increasing and eventually exceed 1. So it is not possible to maintain 2% output growth forever in this economy.

In-tutorial Sheet - Q3

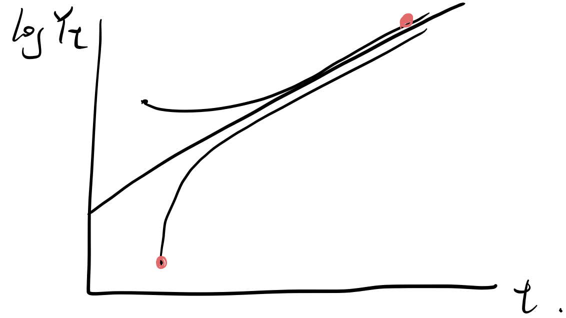
▷ 50 $\xrightarrow{3m}$ 80 .

▷ 95 $\xrightarrow{3m}$ 98 .



3. Between 1950 and 1973, Germany and Japan experienced growth rates of that were at least two percentage points higher than those in the USA. In the same periods the major technological advances occurred in the USA. Can you reconcile these facts?

In-tutorial Sheet - Q3



3. Between 1950 and 1973, Germany and Japan experienced growth rates of that were at least two percentage points higher than those in the USA. In the same periods the major technological advances occurred in the USA. Can you reconcile these facts?

Even though the United States was making the most important technical advances, the other countries were growing faster because they were importing technologies previously developed in the United States.

In other words, they were reducing their technological gap with the United States.

In-tutorial Sheet - Q~~3~~4

4. Suppose that an economy's aggregate production function is given by

$$Y = K^{1/3}N^{2/3}$$

Please attempt the following questions in a group!

$\times K, \times N \longrightarrow \times Y.$

1. Show this production function is characterized by CRS?
2. Are there decreasing returns to capital?
3. Are there decreasing returns to labor?
4. Transform the aggregate production function into a relationship between output per worker and capital per worker.
5. For a given saving rate s and depreciation rate δ, solve for capital per worker in the steady state (and output per worker).

In-tutorial Sheet - Q3

4. Suppose that an economy's aggregate production function is given by

$$Y = K^{1/3}N^{2/3}$$

(i) Is this production function characterized by constant-returns-to-scale? Justify your answer.

In-tutorial Sheet - Q3

4. Suppose that an economy's aggregate production function is given by

$$Y = K^{1/3}N^{2/3}$$

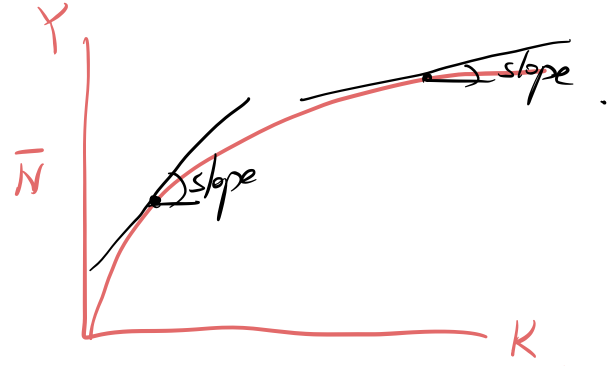
(i) Is this production function characterized by constant-returns-to-scale? Justify your answer.

Yes. See the answer to blue sheet question 2 part (iii) above. Notice that for any $x > 0$ this production function satisfies

$$(xK)^{1/3}(xN)^{2/3} = x^{1/3}K^{1/3}x^{2/3}N^{2/3} = x^{1/3+2/3}K^{1/3}N^{2/3} = xK^{1/3}N^{2/3} = xY$$

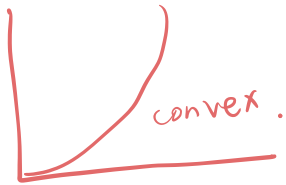
so that scaling both inputs up by $x > 0$ (i.e., in equal proportion) leads to a scaling up output by exactly the same amount.

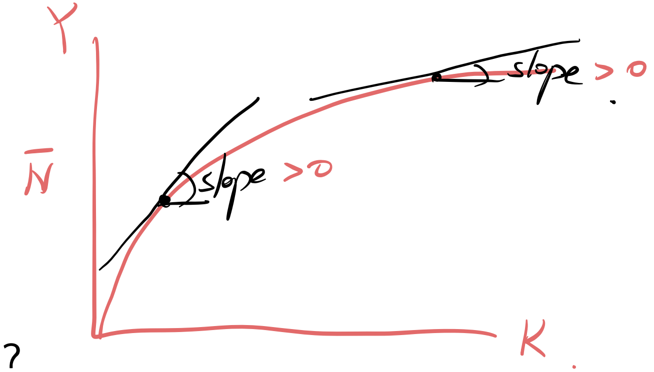
In-tutorial Sheet - Q3



(ii) Are there decreasing returns to capital?

In-tutorial Sheet - Q3





(ii) Are there decreasing returns to capital?

Yes. The marginal product of capital (the return to capital) is positive

$$\frac{\partial Y}{\partial K} = \frac{1}{3} K^{-2/3} N^{2/3} > 0$$

$\boxed{\text{Y increasing with K}}$

slope

but this return, while positive, is lower the larger the amount of capital used, that is

$$\frac{\partial^2 Y}{\partial K^2} = -\frac{2}{9} K^{-5/3} N^{2/3} < 0$$

$\boxed{\text{at a } \downarrow \text{ rate}}$ $\Rightarrow$ concave.

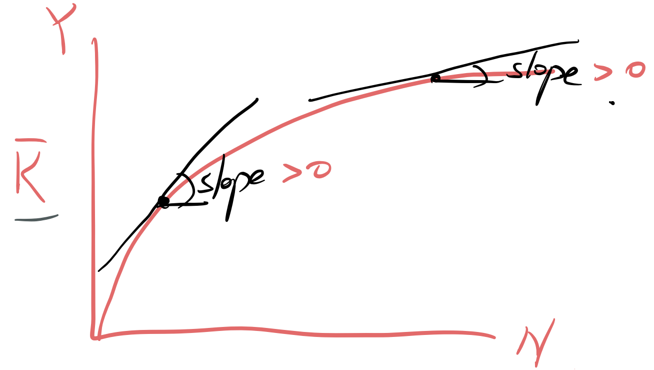
$$\frac{\partial \text{ slope}}{\partial K} < 0.$$

$\rightarrow$ slope decreases.

In-tutorial Sheet - Q3

(iii) Are there decreasing returns to labor?

In-tutorial Sheet - Q3



(iii) Are there decreasing returns to labor?

Yes. The marginal product of labor is positive

$$\frac{\partial Y}{\partial N} = \frac{2}{3} K^{1/3} N^{-1/3} > 0$$

slope > 0.

$\boxed{Y \uparrow \text{ in } N}$

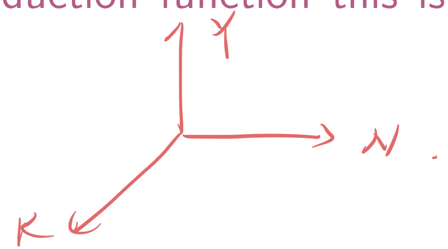
and similarly this is lower the larger the amount of labor used, that is

$$\frac{\partial^2 Y}{\partial N^2} = -\frac{2}{9} K^{1/3} N^{-4/3} < 0$$

slope ↓.

$\boxed{\text{at a} \downarrow \text{rate.}}$

Graphically, this shows up as a production function this is increasing but concave in capital and labor.



In-tutorial Sheet - Q3

(iv) Transform the aggregate production function into a relationship between output per worker and capital per worker.

In-tutorial Sheet - Q3

(iv) Transform the aggregate production function into a relationship between output per worker and capital per worker.

This comes from dividing both sides by N to get

$$y = \frac{Y}{N} = \frac{K^{2/3}N^{2/3}}{N} = \frac{K^{1/3}\cancel{N^{2/3}}}{N^{1/3}\cancel{N^{2/3}}} = \frac{K^{1/3}}{N^{1/3}} = \left(\frac{K}{N}\right)^{1/3} = k^{1/3}$$

In-tutorial Sheet - Q3

(v) For a given saving rate s and depreciation rate δ, solve for capital per worker in the steady state.

$$\underline{k_{t+1} - k_t} = \underline{sy} - \underline{\delta k} = 0.$$

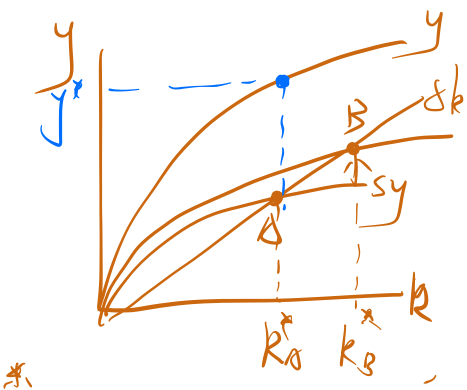
$$\Rightarrow sy = \delta k.$$

In-tutorial Sheet - Q3

$\underline{k_{t+1} - k_t} = \underline{sy} - \underline{\delta k} = 0$.

$\Rightarrow sy = \delta k$

(v) For a given saving rate s and depreciation rate δ, solve for capital per worker in the steady state.



Steady state capital per worker satisfies the condition

saving depreciation

$$\underbrace{sy}_{} = \underbrace{\delta k}_{}$$

where $y = k^{1/3}$. So we want to find k that solves

$$\boxed{sk^{1/3} = \delta k} \Rightarrow k^*$$

Graphically, this solution is found by the intersection of the concave function of k given by aggregate savings and the straight line through the origin with slope δ. One solution is at the origin with $\underline{k = 0}$, but that's not economically interesting. The unique positive solution is given by

$$\boxed{k = \left(\frac{s}{\delta}\right)^{3/2}}$$

k is increasing in the saving rate s, but decreasing in depreciation rate δ.

In-tutorial Sheet - Q3

(vi) For a given saving rate s and depreciation rate δ, solve for output per worker in the steady state.

In-tutorial Sheet - Q3

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Steady state output per worker is given by

$$y = k^{1/3} = \left(\underline{\left(\frac{s}{\delta}\right)^{3/2}}\right)^{1/3} = \underline{\left(\frac{s}{\delta}\right)}^{1/2}$$

In-tutorial Sheet - Q3

(vi) For a given saving rate s and depreciation rate δ, solve for output per worker in the steady state.

Steady state output per worker is given by

$$y = k^{1/3} = \left(\left(\frac{s}{\delta}\right)^{3/2}\right)^{1/3} = \left(\frac{s}{\delta}\right)^{1/2}$$

(vii) Calculate the steady state level of output per worker when $s = 0.36$ and $\delta = 0.04$.

In-tutorial Sheet - Q3

(vi) For a given saving rate s and depreciation rate δ, solve for output per worker in the steady state.

Steady state output per worker is given by

$$y = k^{1/3} = \left(\left(\frac{s}{\delta}\right)^{3/2}\right)^{1/3} = \left(\frac{s}{\delta}\right)^{1/2}$$

(vii) Calculate the steady state level of output per worker when $s = 0.36$ and $\delta = 0.04$.

Plug in the values $s = 0.3\cancel{2}6$ and $\delta = 0.04$ to get

$$y = \left(\frac{0.36}{0.04}\right)^{1/2} = 3$$

In-tutorial Sheet - Q3

(viii) Suppose that the depreciation rate remains constant at $\delta = 0.04$ while the savings rate decreases to $s = 0.16$. What is the impact of this reduction in the savings rate on the steady state level of output per worker?

In-tutorial Sheet - Q3

(viii) Suppose that the depreciation rate remains constant at $\delta = 0.04$ while the savings rate decreases to $s = 0.16$. What is the impact of this reduction in the savings rate on the steady state level of output per worker?

Plug in the values $s = 0.16$ and $\delta = 0.04$ to get

$$y = \left(\frac{0.16}{0.04}\right)^{1/2} = 2 < 3$$

So the fall in the savings rate s from 0.36 to 0.16 reduces steady state (long run) output per worker.

The end

Thanks for your attention! 😄