FIT3080: Artificial intelligence

Assignment 2:
Knowledge Representation and Reasoning

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Question 1: Logic

Part a:

i. $(A \to B) \leftrightarrow (\neg B \to \neg A)$

Truth table:

A	B	$\neg A$	$\neg B$	$A \to B$	$\neg B \to \neg A$	$(A \to B) \leftrightarrow (\neg B \to \neg A)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Since $(A \to B) \leftrightarrow (\neg B \to \neg A)$ is always true, this propositional logic sentence is valid.

Valid.

ii. $(A \land B) \leftrightarrow ((B \to C) \to (C \land D))$

Truth table:

A	B	C	D	$A \land B$	$B \to C$	$C \land D$	$(B \to C) \to (C \land D)$	$(A \land B) \leftrightarrow ((B \to C) \to (C \land D))$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	F	F
T	T	F	T	T	F	F	T	F
T	T	F	F	T	F	F	T	F
T	F	T	T	F	T	T	T	F
T	F	T	F	F	T	F	F	T
T	F	F	T	F	T	F	F	T
T	F	F	F	F	T	F	F	T
F	T	T	T	F	T	T	T	F
F	T	T	F	F	T	F	F	T
F	T	F	T	F	F	F	T	F
F	T	F	F	F	F	F	T	F

As shown in the truth table, $(A \land B) \leftrightarrow ((B \to C) \to (C \land D))$ is not valid as it is not always true. However, it is satisfiable as it is not always false.

Satisfiable but not valid.

iii. $(\neg A \to \neg B) \to (B \to A)$

$\equiv \neg(\neg A \to \neg B) \vee (B \to A)$ (implicative law)

$\equiv \neg(A \vee \neg B) \vee (\neg B \vee A)$ (implicative law)

$\equiv (\neg A \wedge B) \vee (\neg B \vee A)$ (De Morgan's law)

$\equiv ((\neg B \vee A) \vee \neg A) \wedge ((\neg B \vee A) \vee B)$ (distributive law)

$\equiv (\neg B \vee A \vee \neg A) \wedge (\neg B \vee A \vee B)$

$\equiv (\neg B \vee T) \wedge (T \vee A)$ (complement law)

$\equiv T \wedge T$ (identity law)

$\equiv T$

iv. $(A \wedge B) \leftrightarrow ((B \to C) \to ((C \wedge A) \vee A))$

$\equiv (A \wedge B) \leftrightarrow (\neg(B \to C) \vee ((C \wedge A) \vee A))$ (implicative law)

$\equiv (A \wedge B) \leftrightarrow (\neg(B \to C) \vee A)$ (absorption law)

$\equiv (A \wedge B) \leftrightarrow (\neg(\neg B \vee C) \vee A)$ (implicative law)

$\equiv (A \wedge B) \leftrightarrow ((B \wedge \neg C) \vee A)$ (De Morgan's law)

$\equiv ((A \wedge B) \to ((B \wedge \neg C) \vee A)) \wedge (((B \wedge \neg C) \vee A) \to (A \wedge B))$ (equivalence law)

$\equiv (\neg(A \wedge B) \vee ((B \wedge \neg C) \vee A)) \wedge (\neg((B \wedge \neg C) \vee A) \vee (A \wedge B))$
(implicative law)

$\equiv (\neg A \vee \neg B \vee ((B \wedge \neg C) \vee A)) \wedge ((\neg(B \wedge \neg C) \wedge \neg A) \vee (A \wedge B))$
(De Morgan's law)

$\equiv (\neg A \vee \neg B \vee (B \wedge \neg C) \vee A) \wedge (((\neg B \vee C) \wedge \neg A) \vee (A \wedge B))$ (De Morgan's law)

$\equiv (T \vee \neg B \vee (B \wedge \neg C)) \wedge (((\neg B \vee C) \wedge \neg A) \vee (A \wedge B))$ (complement law)

$\equiv ((\neg B \vee C) \wedge \neg A) \vee (A \wedge B)$ (identity law)

$\equiv (\neg A \vee B) \wedge (A \vee (\neg B \vee C))$ (factoring)

$\equiv (\neg A \vee B) \wedge (A \vee \neg B \vee C)$

Part b:

Let:

- P: $Daniel\ has\ a\ good\ resume$
- Q: $There\ is\ an\ economic\ downturn$
- R: $There\ are\ fewer\ jobs$
- a: $Daniel\ will\ get\ a\ good\ job$

KB:

- $Q \rightarrow R$
- $(R \wedge P) \rightarrow a$
- $\neg R \rightarrow a$
- P
- Q

i. Checking whether a is true in every model in which KB is true.

P	Q	R	a	$Q \rightarrow R$	$(R \wedge P) \rightarrow a$	$\neg R \rightarrow a$	KB
F	F	F	F	T	T	F	F
F	F	F	T	T	T	T	F
F	F	T	F	T	T	T	F
F	T	F	F	F	T	F	F
T	F	F	F	T	T	F	F
F	F	T	T	T	T	T	F
F	T	F	T	F	T	T	F
F	T	T	F	T	T	F	F
T	F	F	T	T	T	T	F
T	F	T	F	T	F	T	F
T	T	F	F	F	T	F	F
F	T	T	T	T	T	T	F
T	F	T	T	T	T	T	F
T	T	F	T	F	T	T	F
T	T	T	F	T	F	T	F
T	T	T	T	T	T	T	T

By our definition of entailment, if a is true in all models where KB is true, then $KB \vDash a$. Therefore, Daniel will get a good job.

ii.

$$\frac{\begin{array}{c} Q \to R \\ Q \end{array}}{R}$$

Given that Q is true, by modus ponens, R is true.

$$\frac{\begin{array}{c} (R \wedge P) \to a \\ R, P \end{array}}{a}$$

Given that R and P is true, by modus ponens, a is true.

Therefore, Daniel will get a good job.

iii. $KB \wedge \neg a$
$\equiv (Q \to R) \wedge ((R \wedge P) \to a) \wedge (\neg R \to a) \wedge P \wedge Q \wedge \neg a$
$\equiv(\neg Q \vee R) \wedge (\neg(R \wedge P) \vee a) \wedge (R \vee a) \wedge P \wedge Q \wedge \neg a$ (implication law)
$\equiv(\neg Q \vee R) \wedge (\neg R \vee \neg P \vee a) \wedge (R \vee a) \wedge P \wedge Q \wedge \neg a$ (De Morgan's law)
$\equiv R \wedge a \wedge R \wedge P \wedge Q \wedge \neg a$ (absorption law)
$\equiv R \wedge R \wedge P \wedge Q \wedge F$ (absorption law)
$\equiv F$

Therefore, $KB \wedge \neg a$ is unsatisfiable.

Question 2: Bayesian Network

Part a:

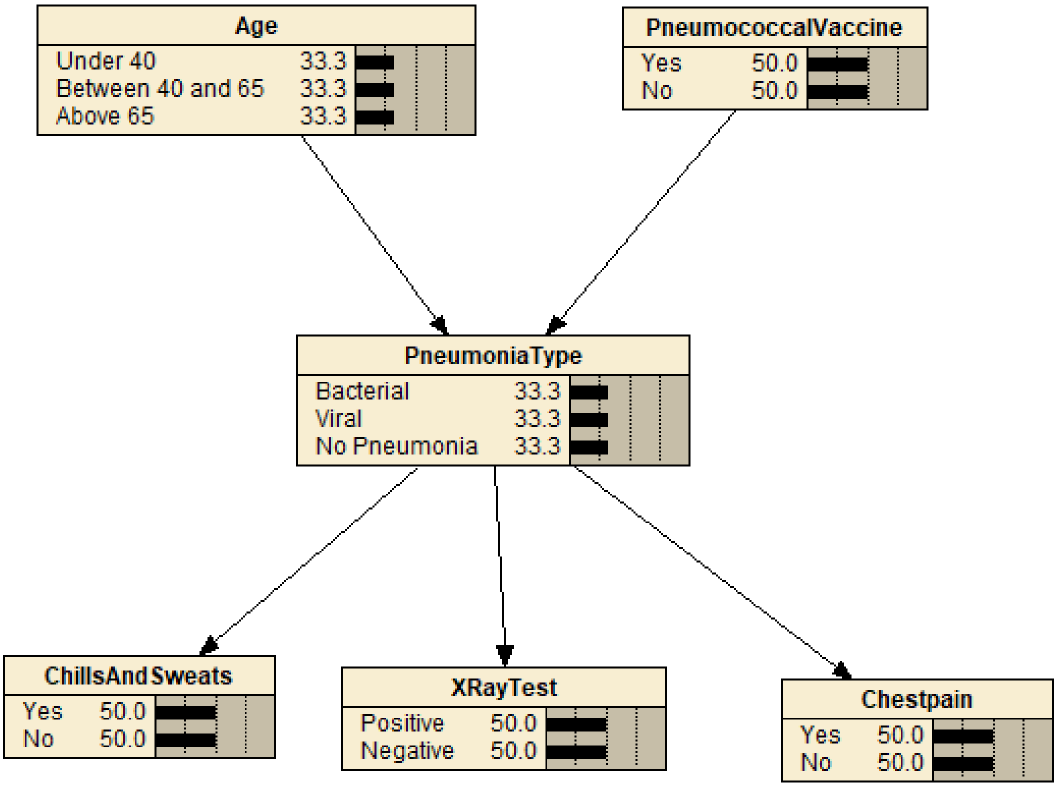

Figure 1: BN for determining Pneumonia

Node	Type	Values
PneumoniaType	Category	{Bacterial, Viral, No Pneumonia}
Age	Category	{Under 40, Between 40 and 65, Above 65}
ChillsAndSweats	Binary	{Yes, No}
ChestPain	Binary	{Yes, No}
XRayTest	Binary	{Positive, Negative}
PneumococcalVaccine	Binary	{Yes, No}

Table 1: Variable table for BN

Justification:

- Arcs added:
  - $PneumoniaType \rightarrow ChestPain$: PneumoniaType influences the possibility of ChestPain. For example, if a patient has bacterial pneumonia, they might have chest pains and on the other hand, if another patient has no pneumonia, they might not have any chest pains.

- $Age \rightarrow PneumoniaType$: PneumoniaType depends on Age as a younger patient might not have pneumonia while an older patient is more likely to have pneumonia.

- Arcs omitted:
    - $PneumococcalVaccine \rightarrow ChestPain$: PneumococcalVaccine does not influence ChestPain. This is because a person that has pneumococcal vaccine might or might not have chest pain and knowing their vaccination does not provide more information regarding chest pain.
    - $Age \rightarrow XRayTest$: XRayTest is not dependent on Age as both young or old people might test positive or negative from the XRayTest for pneumonia.

Part b:

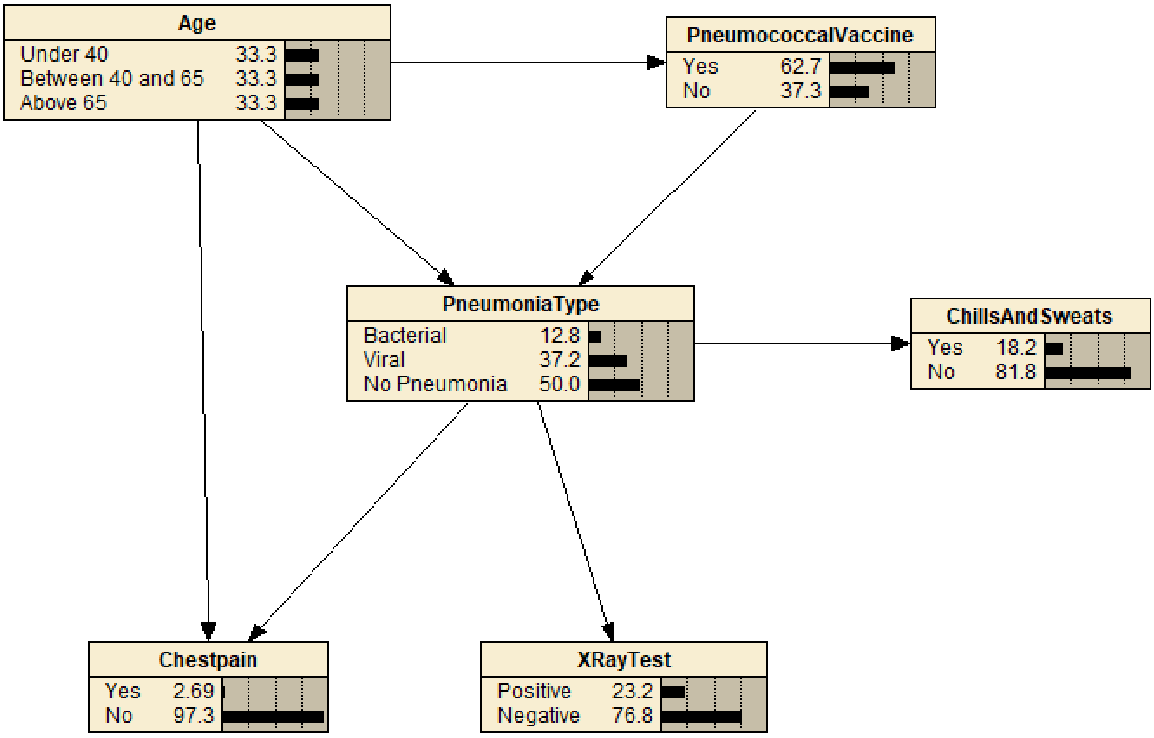

Figure 2: BN for determining Pneumonia with CPTs

Node	Type	Values	Relationship
PneumoniaType	Category	{Bacterial, Viral, No Pneumonia}	The nature of the relationship between having a bacterial pneumonia is increasing with age but vaccinated individuals are more protected against bacterial pneumonia. However, the probability for not having pneumonia is 50% across age and vaccination status. This

			information is obtained from the ME and LIT. I have made a rough assumption for the values as certain values are not specified by ME.
Age	Category	{Under 40, Between 40 and 65, Above 65}	The ME states the model would only be used for someone whose age is known so no distribution is required.
ChillsAndSweats	Binary	{Yes, No}	Chills and sweats have a decreasing relationship with pneumonia type where it is highest when the pneumonia is bacterial and decreases for viral and more for no pneumonia. This information was obtained from the ME.
ChestPain	Binary	{Yes, No}	Chest pain has an increasing relationship with age for bacterial pneumonia type. However, it is constant for viral pneumonia and is extremely rare for people without pneumonia. This information was obtained from LIT and an assumption was made for the value of extremely rare.
XRayTest	Binary	{Positive, Negative}	The X-Ray test has a highest positive rate for bacterial pneumonia and decreases for viral to almost 0 for no pneumonia as there is a very low false positive rate. This information was according to the LIT.
PneumococcalVaccine	Binary	{Yes, No}	There is a decreasing relationship for vaccination with age where the older the age, the lower the vaccination probability. This information is obtained from the ME and from a foreign data source UNICEF (2023).

Table 2: Variable table for BN with relationship column

Part c:

i.

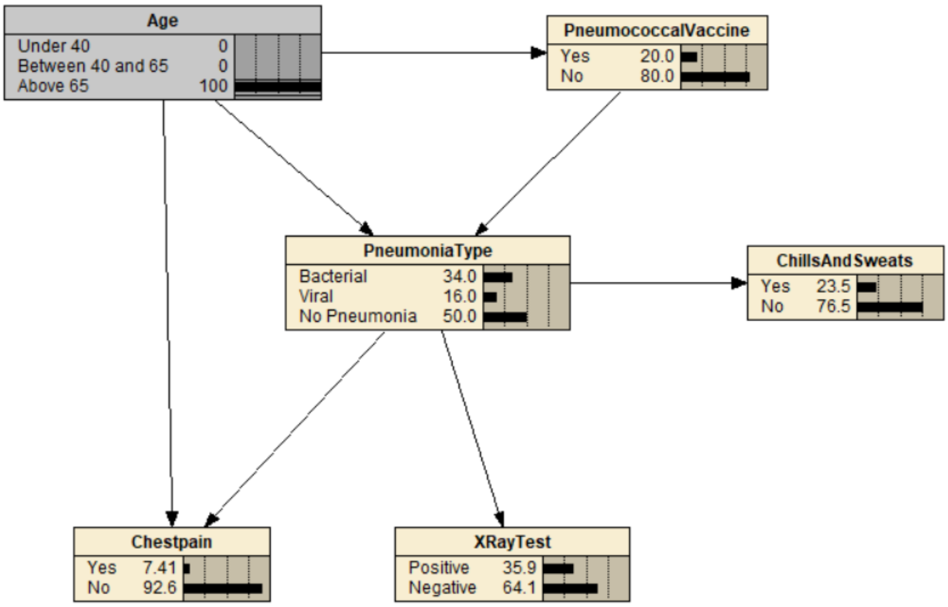

Figure 3: BN with 75-year-old female

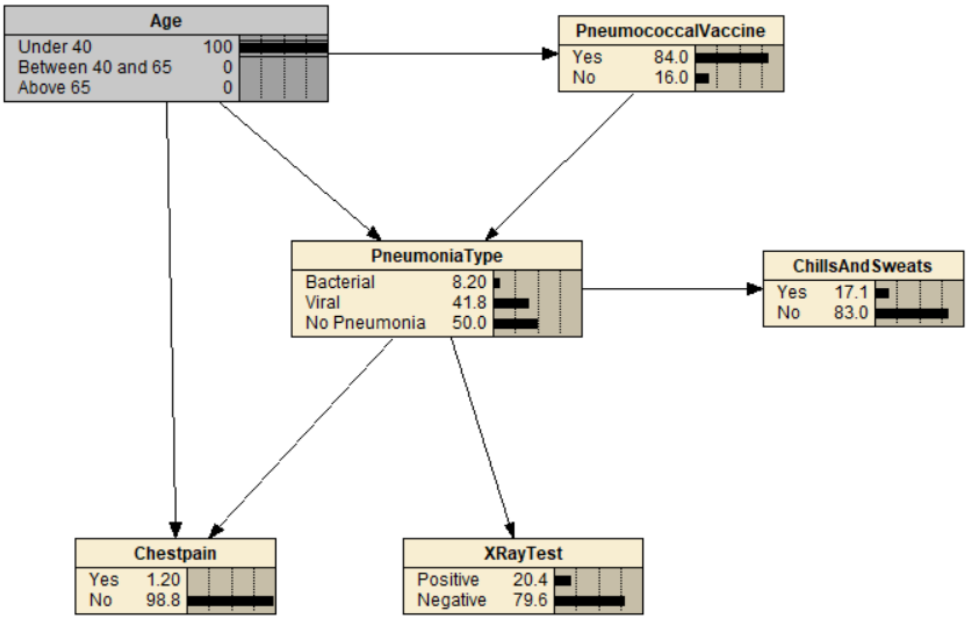

Figure 4: BN with 30-year-old male

The female and the male are equally likely to have any type of pneumonia. The chance of the female having any type of pneumonia is 50%.

ii.

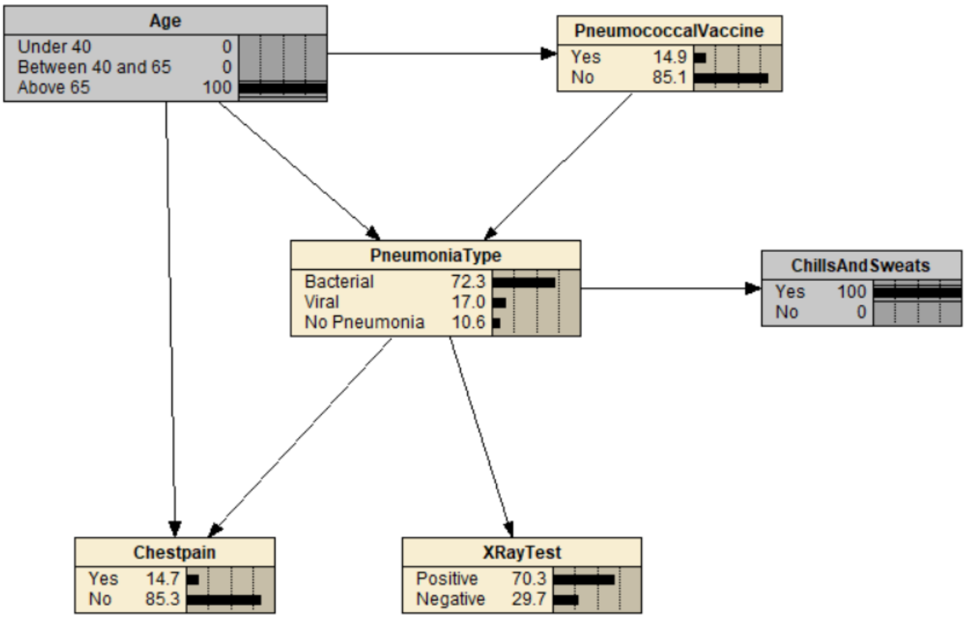

Figure 5: BN with 75-year-old female with chills and sweats

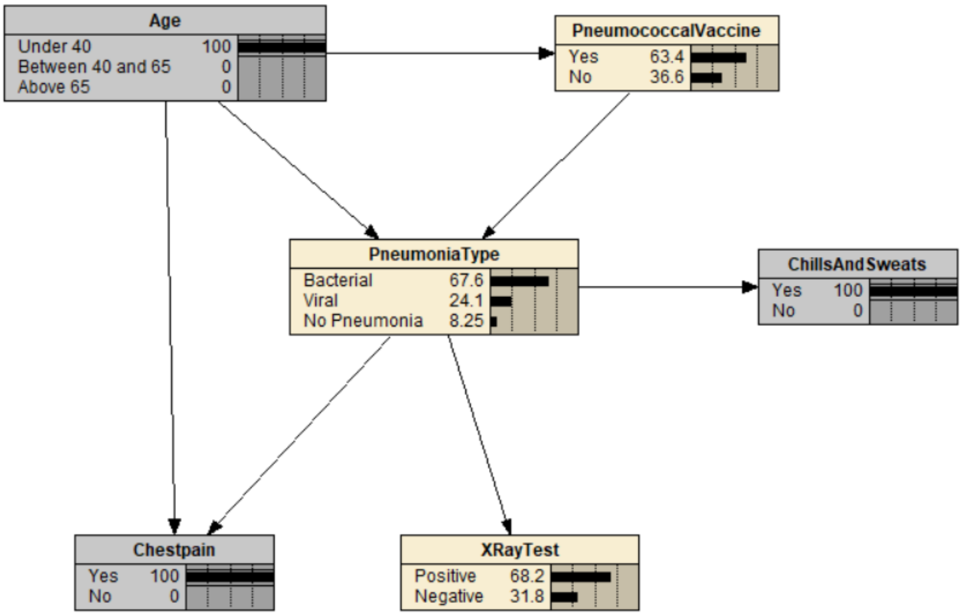

Figure 6: BN with 30-year-old male with chills and sweats and chest pain

The probability of the female having bacterial pneumonia is 72.3% while the probability of the male having bacterial pneumonia is 67.6%. On the other hand, the probability of the female having no pneumonia is 10.6% while the probability for the male not having pneumonia is 8.25%.

iii.

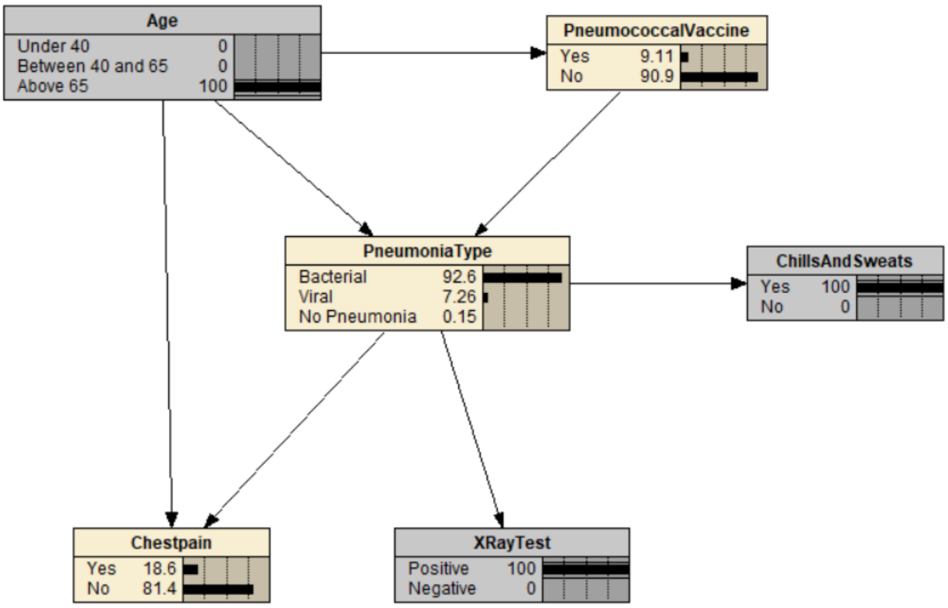

Figure 7: BN with 75-year-old female with chills and sweats and positive X-ray test

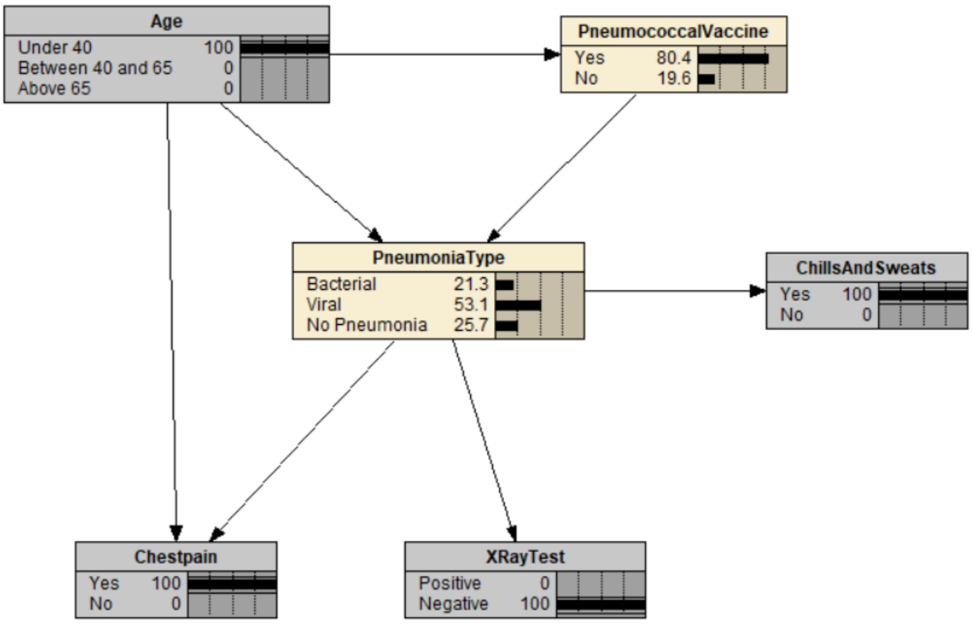

Figure 8: BN with 30-year-old male with chills and sweats, chest pain and negative X-ray test

The female is much more likely to have bacterial pneumonia at 92.6% compared to the male probability of 21.3%. The probability of the female having no pneumonia is 0.15% while the probability of the male having no pneumonia is 25.7%.

iv.

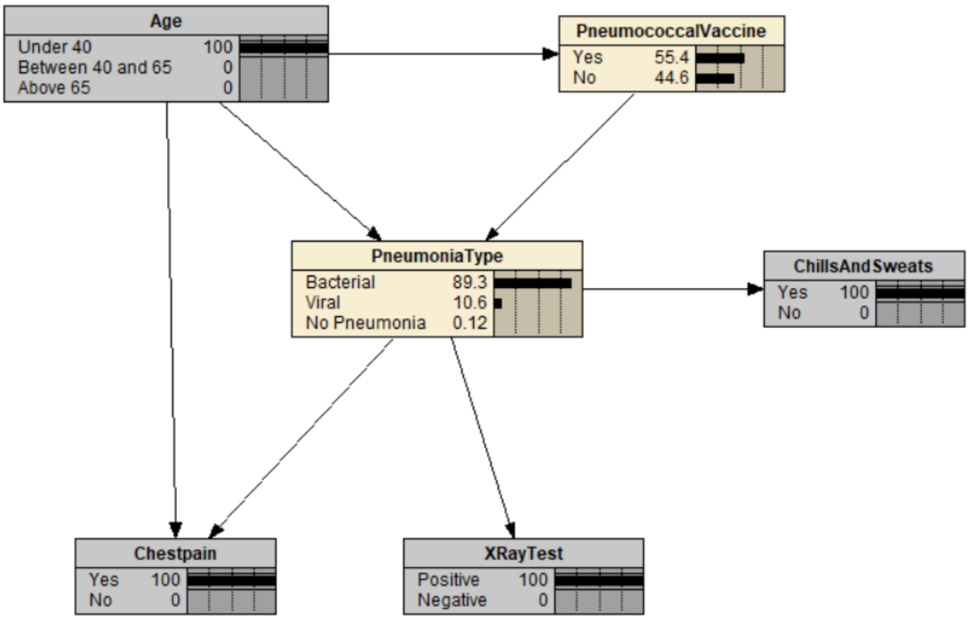

Figure 9: BN Network with 30-year-old male with chills and sweats, chest pain and positive X-ray test

If the male tested positive instead, his chances for having bacterial pneumonia would instead be 89.3%.

v. The posterior probabilities for bacterial pneumonia for (iii) and (iv) rely on the age, chills & sweats, chest pain, and X-ray test results. The sources of these information are from the ME, LIT, foreign reputable sources, and some assumptions. The posteriors to the parameters in the model are very sensitive to the X-ray test results as the CPTs it describes a very strong relationship. This information was described in the information from LIT where the true positives rates were obtained, and rare cases of false positives occur.

Part d:

i.

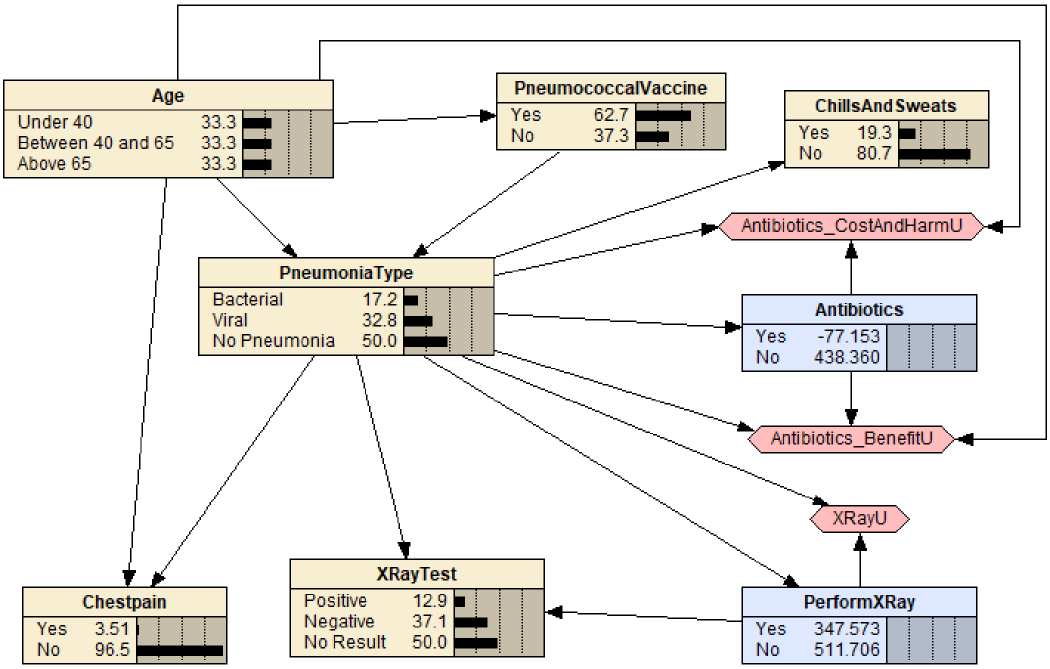

Figure 10: BDN with two new decision nodes and three new utility nodes

Node	Type	Values	Relationship
PneumoniaType	Category	{Bacterial, Viral, No Pneumonia}	The nature of the relationship between having a bacterial pneumonia is increasing with age but vaccinated individuals are more protected against bacterial pneumonia. However, the probability for not having pneumonia is 50% across age and vaccination status. This information is obtained from the ME and LIT. I have made a rough assumption for the values as certain values are not specified by ME.
Age	Category	{Under 40, Between 40 and 65, Above 65}	The ME states the model would only be used for someone whose age is known so no distribution is required.
ChillsAndSweats	Binary	{Yes, No}	Chills and sweats have a decreasing relationship with pneumonia type where it is highest when the pneumonia is bacterial and decreases for viral and more for no pneumonia. This information was obtained from the ME.
ChestPain	Binary	{Yes, No}	Chest pain has an increasing relationship with age for bacterial pneumonia type. However, it is constant for viral pneumonia and is extremely rare for people without pneumonia. This information was obtained from LIT and an assumption

			was made for the value of extremely rare.
XRayTest	Binary	{Positive, Negative}	The X-Ray test has a highest positive rate for bacterial pneumonia and decreases for viral to almost 0 for no pneumonia as there is a very low false positive rate. This information was according to the LIT. The X-Ray test also has a relationship with the PerformXRay decision node as the decision to not perform an X-ray might produce a 'no result'.
PneumococcalVaccine	Binary	{Yes, No}	There is a decreasing relationship for vaccination with age where the older the age, the lower the vaccination probability. This information is obtained from the ME and from a foreign data source UNICEF (2023).
Antibiotics	Binary	{Yes, No}	Antibiotics is a decision node that has a relationship with two utility nodes Antibiotics_CostAndHarmU and Antibiotics_BenefitU. Both these utility nodes represent the utility values of the cost and harm for using the antibiotics treatment and the benefits of using it.
Antibiotics_CostAndHarmU	Utility	-	The Antibiotics_CostAndHarmU utility node has the utility function for the cost and harm of applying the antibiotics treatment method to various different types of pneumonia and to different age groups.
Antibiotics_BenefitU	Utility	-	The Antibiotics_BenefitU utility node has the utility function for the benefits of applying the antibiotics treatment method to various different types of pneumonia and to different age groups.
PerformXRay	Binary	{Yes, No}	PerformXRay is a decision node that has a relationship with the utility node XRayU that computes the utility value for the cost and harm of performing an x-ray test for various different types of pneumonia.
XRayU	Utility	-	XRayU is a utility node that has the utility function for the cost and harm of performing an x-ray test. The utility values are the highest for performing the x-ray test for bacterial pneumonia, and lowest for when not performing the x-ray test for bacterial pneumonia.

Table 3: Variable table for BN with new nodes

ii.

PneumoniaType	Antibiotics	Age	Antibiotics_CostAndHarmU
Bacterial	Yes	Under 40	300
Bacterial	Yes	Between 40 and 65	400
Bacterial	Yes	Above 65	300
Bacterial	No	Under 40	-300
Bacterial	No	Between 40 and 65	-400
Bacterial	No	Above 65	-500
Viral	Yes	Under 40	-300
Viral	Yes	Between 40 and 65	-300
Viral	Yes	Above 65	-300
Viral	No	Under 40	300
Viral	No	Between 40 and 65	300
Viral	No	Above 65	300
No Pneumonia	Yes	Under 40	-300
No Pneumonia	Yes	Between 40 and 65	-300
No Pneumonia	Yes	Above 65	-300
No Pneumonia	No	Under 40	300
No Pneumonia	No	Between 40 and 65	300
No Pneumonia	No	Above 65	300

Table 4: Utility table for Antibiotics_CostAndHarmU utility node

Antibiotics	PneumoniaType	Age	Antibiotics_BenefitU
Yes	Bacterial	Under 40	300
Yes	Bacterial	Between 40 and 65	400
Yes	Bacterial	Above 65	500
Yes	Viral	Under 40	-200
Yes	Viral	Between 40 and 65	-250
Yes	Viral	Above 65	-300
Yes	No Pneumonia	Under 40	-300
Yes	No Pneumonia	Between 40 and 65	-300
Yes	No Pneumonia	Above 65	-300
No	Bacterial	Under 40	0
No	Bacterial	Between 40 and 65	0
No	Bacterial	Above 65	0
No	Viral	Under 40	0
No	Viral	Between 40 and 65	0
No	Viral	Above 65	0
No	No Pneumonia	Under 40	0
No	No Pneumonia	Between 40 and 65	0
No	No Pneumonia	Above 65	0

Table 5: Utility table for Antibiotics_BenefitU utility node

PneumoniaType	PerformXRay	XRayU
Bacterial	Yes	300
Bacterial	No	-300
Viral	Yes	200
Viral	No	100
No Pneumonia	Yes	-300
No Pneumonia	No	300

Table 6: Utility table for XRayU utility node.

The utility values in Table 4 are decided according to the cost and harm of antibiotics treatment to different age groups and type of pneumonia. Using antibiotics without the correct reason to do so is harmful to the human body as the body will be more resistance towards the antibiotics in the future. Thus, wrongfully taking antibiotics will cause harm to the body. However, not administrating antibiotics to a bacterial pneumonia will cause significant harm to the body especially to higher age groups as their natural immune system is weaker.

For Table 5, the utility values represent the benefits of administering antibiotics to patients of different types of pneumonia and age group. The utility value is highest when antibiotics treatment is provided to patients with bacterial pneumonia as antibiotics are only effective against bacterial pneumonia. The utility value is higher as the age group increases as people of greater age has a weaker immune system. The utility value of providing antibiotics to people without bacterial pneumonia is negative as instead of any benefits, it instead causes harm. The utility value is 0 when no antibiotics is prescribed as there will be no effect.

Lastly, Table 6 shows the utility values for performing an X-ray test for patients with different pneumonia type. The X-ray test is able to confirm the presence of bacterial pneumonia, therefore, performing an x-ray test will have the highest utility value to patients with bacterial pneumonia. However, it can also be used for patients with viral pneumonia to confirm that it is not a bacterial pneumonia, thus having a high utility value as well. Not conducting an X-ray test for patients without pneumonia is also important and on the other hand, conducting the X-ray test for patients without pneumonia is extremely bad, having the lowest utility value. In addition, not performing the X-ray test for patients with pneumonia is also extremely dangerous, which has the equally low utility value.

Part e

i.

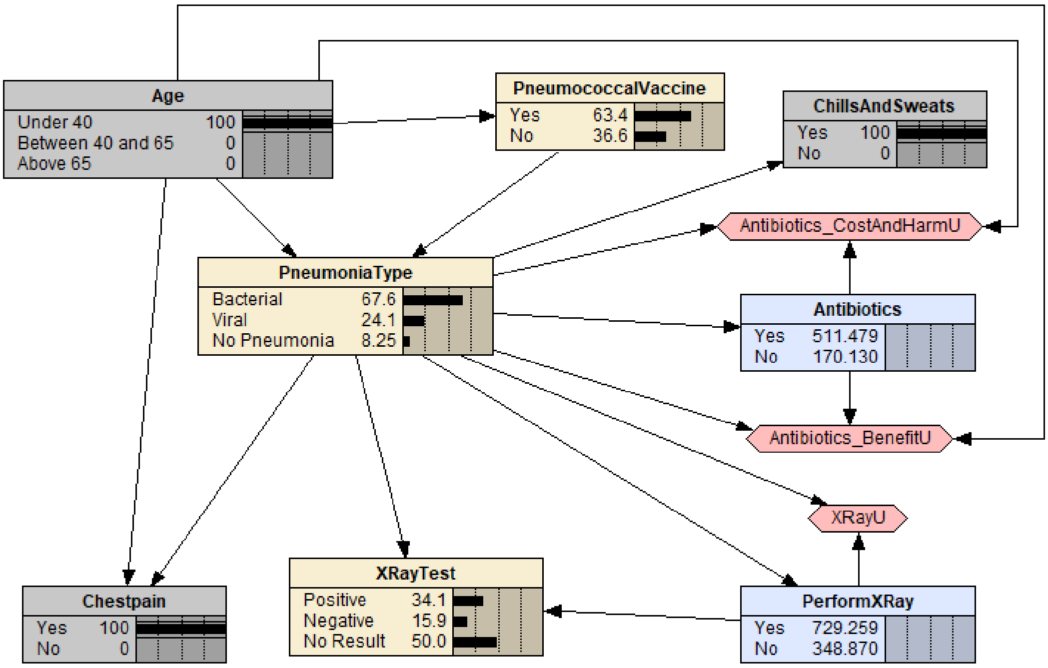

Figure 11: BDN showing the decision to perform X-ray for a 30-year-old man with chills & sweats and chest pain

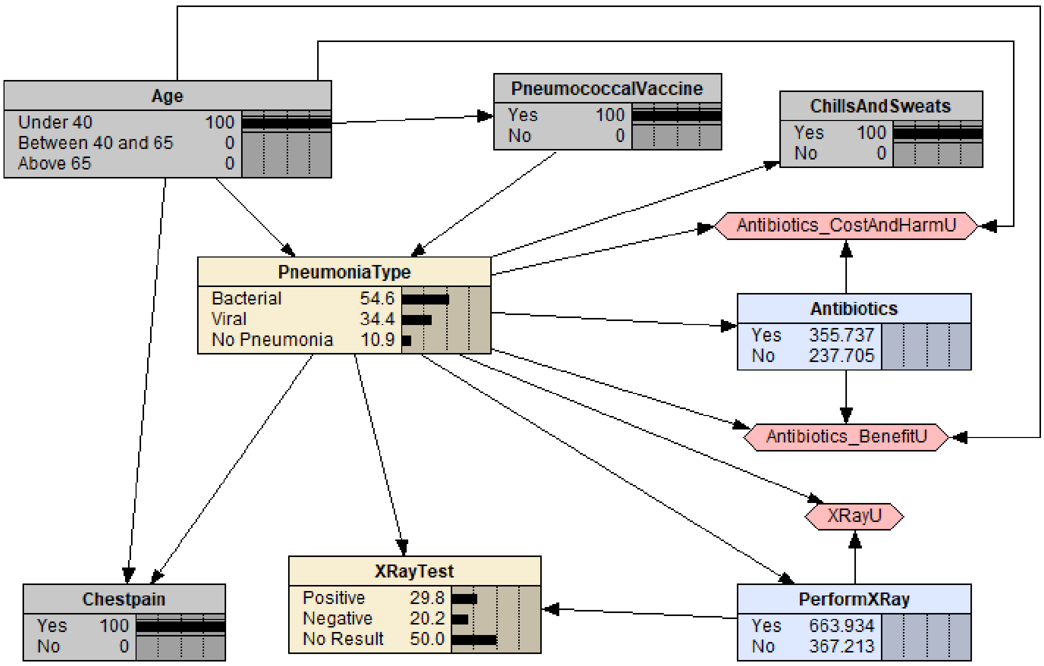

Figure 12: BDN showing the decision to perform X-ray with a recent pneumococcal vaccine

The expected utility for performing the X-ray is 729.259. When the fact that he had a recent pneumococcal vaccine is learnt, the expected utility dropped to 663.934 for performing the X-ray. Knowing that the male had a recent pneumococcal vaccine decreased the expected utility, but the BDN still computes that the 30-year-old male should perform the X-ray test.

ii. $EU(DoXray \mid ChillsSweats = Yes, ChestPain = Yes, PneumococcalVaccine = Yes)$
$= \Pr(Age < 40 \mid ChillsSweats = Yes, ChestPain = Yes, PneumococcalVaccine = Yes)U(A < 40, DoXray)$
$+\Pr(40 < Age < 65 \mid ChillsSweats = Yes, ChestPain = Yes, PneumococcalVaccine = Yes)U(40 < Age < 65, DoXray)$
$+\Pr(Age > 65 \mid ChillsSweats = Yes, ChestPain = Yes, PneumococcalVaccine = Yes)U(A > 65, DoXray)$

$= 0.256 * 282.8 + 0.452 * 305.92 + 0.292 * 454$
$= 343.24064$

iii. The sensitivity of doing an X-ray test is directly related to the type of suspected pneumonia. The decision is sensitive in a way that if the patient is suspected to have bacterial or viral pneumonia, the expected utility for performing an X-ray test would be higher and vice versa, if the patient is suspected to have no pneumonia, then the expected utility for performing an X-ray test would be lower.

The sensitivity of doing an X-ray test to antibiotic treatment exists but is of a lower sensitivity. The BDN would have a higher utility for not performing an X-ray test if a patient has already been prescribed or not prescribed any antibiotics.

References

UNICEF. (2023, July). *Immunization*. UNICEF Data. Retrieved from https://data.unicef.org/topic/child-health/immunization/