FIT2014 Assignment 1

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Problem 2

a. $\emptyset$
$a_0 = 1$

$\{(0,0)\}$
$b_0 = 1$

$\emptyset, \{(0,1)\}, \{(0,0)\}$
$a_1 = 3$

$\{(1,0)\}, \{(1,0), (0,1)\}$
$b_1 = 2$

b. $\emptyset, \{(0,1)\}, \{(1,0)\}, \{(0,0)\}, \{(1,1)\}, \{(0,1), (1,0)\}, \{(0,0), (1,1)\}$
$a_2 = 7$

$\emptyset, \{(0,1)\}, \{(1,0)\}, \{(0,0)\}, \{(1,1)\}, \{(2,1)\}, \{(2,0)\}. \{(0,1), (1,0)\}, \{(0,0), (1,1)\},$
$\{(0,1), (2,0)\}, \{(0,0), (2,1)\}, \{(0,1), (2,1)\}, \{(0,0), (2,0)\}, \{(1,1), (2,0)\}, \{(1,0), (2,1)\},$
$\{(0,1), (1,0), (2,1)\}, \{(0,0), (1,1), (2,0)\}$
$a_3 = 17$

$\{(2,0)\}, \{(2,0), (0,1)\}, \{(2,0), (0,0)\}, \{(2,0), (1,1)\}, \{(2,0), (0,0), (1,1)\}$
$b_2 = 5$

$\{(3,0)\}, \{(3,0), (0,1)\}, \{(3,0), (0,0)\}, \{(3,0), (1,1)\}, \{(3,0), (1,0)\}, \{(3,0), (2,1)\},$
$\{(3,0), (0,1), (1,0)\}, \{(3,0), (0,1), (2,1)\}, \{(3,0), (0,0), (1,1)\}, \{(3,0), (0,0), (2,1)\},$
$\{(3,0), (1,0), (2,1)\}, \{(3,0), (2,1), (1,0), (0,1)\}$
$b_3 = 12$

$b_3 = 12 = 7 + 5$
$b_3 = a_2 + b_2$
$b_{n+1} = a_n + b_n \; for \; n \in \mathbb{N}$

$a_3 = 17 = 12 + 5$
$a_3 = b_3 + b_2 = a_2 + b_2 + b_2$
$a_{n+1} = a_n + 2b_n \; for \; n \in \mathbb{N}$

c. Base case: $n = 0$

$a_0 \leq \sqrt{2}((\sqrt{2}+1)^0$

$a_0 \leq \sqrt{2}$

$1 \leq \sqrt{2}$

$b_0 \leq (\sqrt{2}+1)^0$

$b_0 \leq 1$

$1 \leq 1$

Hence, the inequality is true for $n = 0$

Inductive Step:

Suppose that $a_n \leq \sqrt{2}(\sqrt{2}+1)^n$ and $b_n \leq (\sqrt{2}+1)^n$ is true for a particular n, where $n \geq 0$.

$a_{n+1} = a_n + 2b_n$

$a_{n+1} = \sqrt{2}(\sqrt{2}+1)^n + 2(\sqrt{2}+1)^n$

$a_{n+1} = (\sqrt{2}+2)(\sqrt{2}+1)^n$

$a_{n+1} = \sqrt{2}(1+\sqrt{2})(\sqrt{2}+1)^n$

$a_{n+1} = \sqrt{2}(\sqrt{2}+1)^{n+1}$

$b_{n+1} = a_n + b_n$

$b_{n+1} = \sqrt{2}(\sqrt{2}+1)^n + (\sqrt{2}+1)^n$

$b_{n+1} = (\sqrt{2}+1)(\sqrt{2}+1)^n$

$b_{n+1} = (\sqrt{2}+1)^{n+1}$

So, by Mathematical Induction, it is true for all $n \geq 0$ that $a_n \leq \sqrt{2}(\sqrt{2}+1)^n$ and $b_n \leq (\sqrt{2}+1)^n$.

d. $c_n = a_n + 2b_n$

$c_n \leq \sqrt{2}(\sqrt{2}+1)^{n+1}$