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We prove that CW is not context-free.

Assume, by the way of contradiction, that CW is context-free, Then it has a CFG in Chomsky Normal Form with k nonterminals. Then by the Pumping Lemma for CFLs, every $w \in$ CW with $|w| > 2^{k-1}$ can be written $w = uvxyz$ where $vy \neq \varepsilon$, $|vxy| \leq 2^k$, and for all $i \geq 0$ we have $uv^ixy^iz \in$ CW.

Put $N > 2^k$, $M > 2^k$, and let w be the following string:

$$w = NN \cdots NEE \cdots ESS \cdots SWW \cdots W$$

$$w = N^M E^N S^M W^N$$

which represents the sequence of steps. This satisfies the rules of CW, so $w \in$ CW.

We also see that $|w| > 2^{k-1}$, so w satisfies the size lower bound in the Pumping Lemma for CFLs.

Now consider all possible divisions of w into five parts, $w = uvxyz$. Consider in particular, the possible locations for v and y within w. We have several cases.

Case 1:

If both v and y are directions but of different length (i.e., $v = \mathrm{NE}$ and $y = \mathrm{E}$, $v = \mathrm{EE}$ and $y = \mathrm{S}, v = \mathrm{SS}$ and $y = \mathrm{W}$), then uv^2xy^2z has an excess of a certain direction (i.e.,E, E and S respectively), so $uv^2xy^2z \notin$ CW

Case 2:

If both v and y are directions but is not the opposite of each other (i.e., $v = \mathrm{NE}$ and $y = \mathrm{EE}$, $v = \mathrm{ES}$ and $y = \mathrm{SS}$), then uv^2xy^2z has an excess of a certain direction (i.e., E and S respectively), breaking the rules of CW, so $uv^2xy^2z \notin$ CW

Case 3:

If both v and y are single direction but is not the opposite of each other (i.e., $v = \mathrm{N}$ and $y = \mathrm{E}$), then uv^2xy^2z has an excess of a certain direction (i.e., N and E), breaking the rules of CW, so $uv^2xy^2z \notin$ CW

Case 4:

If both v and y are single direction and opposite of each other (i.e., $v = \mathrm{N}$ and $y = S$), then $|vxy| > 2^k$, as $x = \mathrm{E}^N$ $|x| > 2^k$ breaking the rules of Pumping Lemma.

We have now covered all possibilities for the division of w into five parts, $w = uvxyz$ with $vy \neq \varepsilon$, $|vxy| \leq 2^k$, and for all $i \geq 0$ we have $uv^ixy^iz \in$ CW. In each case, we have found a value of i such that $uv^ixy^iz \notin$ CW. This contradicts the conclusion of the Pumping Lemma for CFLs (where $uv^ixy^iz \in$ CW always belong to the language in question). So, our initial assumption, that CW is context-free was wrong. Therefore, CW is not context-free.

References

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