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Does Dockless Bike-Sharing Create a Competition for Losers?

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Abstract. We model the oligopoly competition in a dockless bike-sharing (DLB) market as a bilevel game. Each DLB operator is first committed to an action tied to a specific objective, such as maximizing profit. Then, the operators play a lower-level game to achieve their individual goals and finally reach a subgame perfect Nash equilibrium by making tactic decisions (e.g., pricing and fleet sizing). We define a Nash equilibrium under either weak or strong preference to characterize the likely outcomes of the bilevel game and formulate the demand-supply equilibrium of a DLB market that accounts for key operational features and mode choice. Using the oligopoly game model calibrated with empirical data, we show that if an operator seeks to maximize its market share with a budget constraint, all other operators must either respond in kind or be driven out of the market. When all operators compete for market dominance, even a slight efficiency edge gained by one operator can significantly shift the outcome, which signals high volatility. Moreover, even if all operators agree to focus on making money rather than ruinously seeking dominance, profitability still plunges quickly with the number of operators. Taken together, the results explain why an unregulated DLB market is often oversupplied and prone to collapse under competition. We also show that this market failure may be prevented by a fleet cap regulation, which sets an upper limit on each operator's fleet size.

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Keywords: bilevel game • dockless bike-sharing • fleet cap • oligopoly competition

1. Introduction

As a novel mode for short-distance travel, bike-sharing has gained traction in densely populated urban areas in recent years. Compared with driving, biking is cheaper, healthier, and greener (Shaheen, Guzman, and Zhang 2010). Bike-sharing offers a cost-effective solution to the first- and last-mile problem that often inconveniences transit riders (Chen, Van Lierop, and Ettema 2020). In addition to promoting public transportation, bike-sharing can also replace certain motorized trips. Because it promises to reduce vehicular traffic and mitigate environmental externalities (Fishman 2016), bike-sharing has been hailed by many as a key ingredient in a multimodal transportation system that supports the Complete and Green Streets Initiative (Jordan and Ivey 2021). Despite its appeal, however, the mass adoption of bike-sharing was hindered by the need to procure customized bikes and build stations and docks that secure them. Shaheen et al. (2014) noted that a bike station can cost as much as $40,000, and adding a dock costs another $3,000. Such a high infrastructure cost restricts the spatial coverage of these docked bike-sharing systems and serves as a natural entry barrier to would-be contenders.

Dockless bike-sharing (DLB) technology fundamentally altered this calculus (Fishman 2016). First, a DLB system enhances user experience markedly by enabling riders to locate and unlock a GPS-equipped bike using their smartphones and to lock it away wherever they find most convenient. Second, not only does this technology eliminate the need for a supporting infrastructure, but it also provides novel access to a potentially large consumer base. These desirable properties first caught the attention of venture capitalists in China, and starting from early 2015, they began to pour money into the nascent industry (Yang 2020). The result was a breathtaking expansion that culminated in 2017, when 221 million registered users completed nearly 70 million trips per day in China. At the peak, there were nearly 70 bike-sharing startups collectively holding 23 million bikes nationwide (Gu, Kim, and Currie 2019). The two unicorns produced by the industry, Ofo and Mobike, launched ambitious plans to conquer international markets, and at one point their bikes could be found on most continents around the world (Borak 2017).

However, the tide began to turn in 2018. In some way, the industry was the victim of its own success. The complete freedom the technology gives customers to retrieve and return bikes had an unintended consequence; bikes were literally everywhere, including in the way of pedestrians (Spinney and Lin 2018). The problem was exacerbated by the low cost of entry, because starting a DLB operation in a city requires virtually zero infrastructure investment. The intense competition led to massive supply expansion and then nasty price wars. In August 2017, for example, Ofo charged ¥1 per month for unlimited rides (¥1 ≈ 0.14 United States Dollar) (Yang 2020). In comparison, a single transit ride typically costs ¥2 in China. Although the public no doubt enjoyed the "free lunch" with some amusement, most startups did not have the resources to sustain the heavy losses ensued. Even the deep-pocketed players like Ofo could no longer properly look after their massive fleets. As a result, more and more bikes, broken or otherwise, were left unattended, cluttering sidewalks and other public spaces, sometimes like a giant pile of trash (Jing 2019). By 2018, the outcry became so loud that Chinese cities had to crack down on the industry with regulations that capped the total number of DLB bikes. Soon after, most startups began to fail (Kubota 2018). Even Ofo and Mobike did not survive the crash.

The rise and fall of the DLB industry in China offers a cautionary tale about the risks of an unregulated market with a low entry barrier. It is well known that although low entry barriers can promote competition and innovation, they may also lead to higher market volatility and potential challenges in achieving profitability because of intensified rivalry (Porter 1980). There are also limited economies of scale to be had, making it exceedingly difficult to establish a monopoly. As Thiel and Masters (2014) noted, "competition is for losers" in such markets, and good entrepreneurs should simply stay away from them.[1] However, writing off the DLB industry as unprofitable cannot be the only story here. After all, bike-sharing has a genuinely positive societal impact and should have its place in many of our cities that are haunted by the disease of auto-dependency. The question is, *what, if anything, can be done to foster a healthy bike-sharing market that is attractive to both users and private investors?*

There is evidence that the regulations on bike-sharing in China, hastily enacted to rein in the out-of-control growth, may be suboptimal. Take Chengdu, China, for example. The number of shared bikes in the city center, which amounts to fewer than 5 million residents, peaked at 1.1 million in September 2018. Chengdu then introduced a fleet cap of 600,000 in May 2019 and further reduced it to 450,000 a year later.[2] This might seem like a huge improvement. However, using a stylized model calibrated with real data collected in the city, Zheng et al. (2023) estimated that even if the DLB market is centrally controlled for social good, the current cap is still not effective and should be further reduced by another three quarters. If there is a monopoly operator aiming at profit, the optimal fleet size is fewer than 60,000 bikes, which is, unsurprisingly, accompanied by a lower level of service and a much higher price. Could the city achieve a better outcome by introducing regulated competition? Indeed, Chengdu did exactly that: issuing a few licenses and allocating a fleet quota to each licensed operator. However, to the best of our knowledge, no analysis exists that can tell, supported by empirical evidence, whether that is the optimal way to regulate the competition or how far the number of licenses and the quota are from the "optimal" values. The model developed by Zheng et al. (2023) is not suitable for this task because it does not allow for competition.

In this study, we propose a bilevel game structure to capture the inter-operator competition, in which each operator is first committed to an action tied to a specific objective and then makes tactic decisions to achieve it. At the upper level, their actions involve setting the targets of operation, which may be either making money (profit maximization) or dominating the market (ridership maximization). Taking operation targets as actions adds another layer of complexity to the game because preferences on payoffs now change with actions. For instance, when an operator attempts to maximize ridership, it weighs ridership more than profit in its payoff. To analyze how competition may shape the choice of actions, we define a Nash equilibrium under either weak or strong preference to characterize the likely outcomes of the bilevel game. At the lower level, the tactics include both the pricing and the fleet-sizing decisions to achieve the objective associated with the action. Note that the operator's payoffs for the action and tactic decisions are deeply intertwined with each other. Like the taxi market (Douglas 1972), this interaction is further complicated by the fact that the level of service and price level are often determined by the supply decisions of all operators *collectively* rather than that of each operator *individually*. The case in point is the access time perceived by a user, which depends on the total number of idle bikes in a city rather than any single operator's fleet size. Another complexity in the lower-level game has to do with the potential impact on the feasible set of one operator's tactic decisions by the decisions of other operators. This means that the outcome of the lower-level game has to be characterized as a generalized Nash equilibrium (GNE), which is obtained by a specialized bilevel dual gradient descent (BDGD) algorithm developed in this study.

Based on the model calibrated with empirical data, we provide a novel explanation—one of the first, to our knowledge—as to why the unregulated DLB market is frequently oversupplied and susceptible to collapse under competitive pressures. The underlying mechanism is simple and intuitive. If an operator is committed to market dominance, other operators must respond in kind. Otherwise, they would be driven out of the market.[3] Our analysis will also show the DLB market need not always fail. An effective policy instrument is fleet cap, which sets an upper limit on each operator's fleet size. If properly implemented, the policy can both prevent market failure and improve social welfare.

The rest of the paper is organized as follows. Section 2 provides a brief review of related studies and summarizes their relationship with the present work. The oligopoly competition game is introduced and analyzed in Section 3. Section 4 formulates the demand-supply equilibrium for a DLB market with multiple operators and presents several analytical results. Section 5 presents the numerical procedure to solve the GNE of the competition game. Section 6 describes a case study based on a Chinese megacity with a well-developed DLB market. Section 7 reports the results from a set of hypothetical scenarios and discusses their implications. Section 8 summarizes the main findings and comments on future research.

2. Related Studies

The rapidly expanding literature on bike-sharing can be generally categorized into descriptive and prescriptive studies. Descriptive research involves identifying patterns and deriving insights from the operational data of established bike-sharing systems, with the primary goal of understanding and interpreting the behaviors of bike users. In contrast, prescriptive research addresses mainly the strategies for designing and operating bike-sharing systems according to goals such as efficiency, market share, and service levels. Our work falls into the second category. The vast majority of the prescriptive studies cast the bike-sharing system as a network model, thereby treating bike trips as flows in a network of bike stations. Importantly, bike flow is determined not only by demand but also by spatiotemporal availability of bikes and vacant docks at stations. The latter, in turn, is affected by a host of strategic, tactical, and operational decisions whose optimization chiefly concerns these studies. Despite their popularity, network-based models are complex and computationally demanding, making them unsuitable for our purpose, namely to represent the competition between multiple operators and to analyze the impact of different regulatory policies. There has been a growing interest recently in more aggregated models of bike-sharing systems. Because these studies are more directly related to ours, they will be discussed in greater detail in the rest of this section. Those who wish to read a more comprehensive review of bike-sharing may consult Laporte, Meunier, and Wolfler Calvo (2018), Chen, Van Lierop, and Ettema (2020), and Shui and Szeto (2020).

2.1. Monopolistic Market

Chen et al. (2019) studied an idealized bike-sharing monopoly in which users can choose between bike-sharing and a generic travel mode, and the operator aims to maximize profit by setting the price and availability of bike-sharing (represented by α, or the probability of finding a functional bike within a reasonable distance). The operator's cost depends solely on availability—formulated as a simple quadratic function of α—but can be reduced by a government subsidy. The focus of the analysis was to examine how the availability function interacts with the subsidy policy. They found that subsidization is not effective for enhancing social welfare when the availability cost is high. Considering stochastic demand sequences over time, Freund, Henderson, and Shmoys (2022) developed a model to optimize the dock capacity of each bike-sharing station. The goal was to minimize the occurrence of out-of-stock events when travelers attempted to rent or return a bike. They demonstrated that the objective function in their model was multi-modular and that the design problem could be solved effectively using a discrete gradient-descent algorithm. Using a structural demand model, Kabra, Belavina, and Girotra (2020) estimated the elasticity of bike-sharing ridership to station accessibility and availability of idle bikes. They also demonstrated how these estimates could be used to evaluate different improvement strategies. In a sequel, He et al. (2021) extended the analysis by Kabra, Belavina, and Girotra (2020) to examine the impact of network effects on bike-sharing demand and to evaluate expansion strategies.

Soriguera and Jiménez-Meroño (2020) developed a continuous approximation model for the design of a bike-sharing system. As in strategic transit design, the total cost, inclusive of both the user cost and the operator cost, is minimized to determine design variables, which in the case of bike-sharing include station density, fleet size, and rebalancing intensity. Their modeling approach does not allow mode choice but can accommodate both docked and dockless systems as well as regular and electric bikes. Using the model, these authors demonstrated that bike-sharing systems enjoy economies of scale. In a similar vein, Jara-Díaz et al. (2022) studied the design of a bike-sharing system, which is, in the basic form, configured over a grid network by station space, the number of stations, the number of docks, and fleet size. The more advanced version of their model allows for "waiting time" as a function of the average availability of bikes (at origin) and docks (at destination), as well as rebalancing, which is linked to a reduction in the waiting time and an increase in the operator's cost as a result of moving bikes from oversaturated stations to empty ones. They, too, observed economies of scale in the system and argued that bike-sharing should be subsidized—the larger the city, the greater the subsidy.

Zheng et al. (2023) assumed that a bike-sharing service provided by a monopolistic operator competes for customers with walking and a generic motorized mode. The model choice is driven largely by trip length but can also be influenced by the operator's decision of fare and fleet size—the latter dictates the access time or the walking time taken to reach the nearest bike location. The cost of rebalancing is simplified as the function of an exogenous variable, which is determined by the empirically observed ratio between the number of bike trips and that of rebalancing trips. Calibrated with empirical data collected in Chengdu, China, their model indicated that the level of service of bike-sharing has rapidly diminishing returns to the investment on the fleet. This leads to the conclusion that the current fleet cap set by Chengdu is well above the optimal level. They also found that, for a regulator seeking to influence bike-sharing operations for social good, the choice of policy instruments depends on the operator's objective.

2.2. Competitive Market

Jiang, Lei, and Ouyang (2020) divided a service area into zones and an analysis horizon into multiple periods. Bike-sharing operation is then organized to serve endogenously given stochastic bike trips connecting pairs of zones in each period. Travelers may choose from one of two competing operators but have no access to other modes (mode choice may be implicitly captured using a demand function, as demonstrated in the paper). Because an operator's market share in a zone is determined by its share of bike supply in that zone, the key decision is periodic rebalancing, which dictates how many bikes need to be moved between zones at the end of each period, after the initial distribution of bikes in the network is altered by bike trips realized in the period. For either operator, this decision problem is formulated as a multiperiod two-stage stochastic program, and the authors assumed that inter-operator competition would lead to a Nash equilibrium. Given the complexity of the model, the operator's problem was solved (to a local optima) using a sample average approximation scheme, and the Nash equilibrium was taken as a stable point of a process in which each operator's problem was solved alternately. Fu et al. (2022) extended the model by Jiang, Lei, and Ouyang (2020) to consider the situation where the competition is between an incumbent and a newcomer. These authors argued that, because the newcomer cannot hope to reliably obtain information about the incumbent's operations, it should expect the worst when making the market entry decision. Based on this proposition, these authors developed a multistage max-min-max robust maximization model to maximize the worst-case revenue achievable by the newcomer. They also proposed a myopic method capable of bounding the potential optimal solution to the problem. In another follow-up study, Jiang and Ouyang (2022) incorporated the operators' dock station location decision into the model as a first-stage decision, along with the fleet size and service price, whereas rebalancing is considered as a second-stage decision. The duopoly is then modeled as a generalized Nash equilibrium (GNE) problem, which is reformulated as a linear model and solved by commercial solvers.

By inspecting the DLB trip data collected in 59 Chinese cities between 2015 and 2017 (i.e., the initial expanding phase of the industry), Cao et al. (2021) found that a monopolistic incumbent consistently benefits—in the form of greater ridership, high revenue per trip, and a higher utilization rate—when a competitor enters the same market. They explained this unexpected finding by a positive network effect, which posits that a newcomer would attract enough new users to benefit the incumbent, by widening the overall availability of bikes, thus the expected probability of finding a bike. In the theoretical model, this positive effect is captured using a Cobb-Douglas matching function that has increasing returns to scale large enough to generate a dominant market-expanding effect. By assuming the investment cost as a convex function of the fleet size, the model prevents the "winner-take-all" outcome. Although it is speculated the convexity of the function may be attributed to the "effort to balance and maintain a large and diverse network of bikes," the connection to the physical bike-sharing operation was not explicitly modeled. Nor did these authors separate the rebalancing cost from the capital investment. Wang et al. (2023) built a stylized Hotelling model of a DLB duopoly, in which travelers may subscribe to either service (single-homing) or both (multihoming), depending on their attributes (abstracted as the location on the Hotelling line). The total demand is fixed and allocated to the three options based on a utility that linearly increases with fleet size and decreases with price. On the supply side, bikes are assumed to have a negative contribution to social welfare, which incentivizes the regulator to limit the fleet size. The focus of the study was to examine three regulatory policies using a game-theoretic approach: a deployment limit (i.e., capping the fleet size for both operators), a multihoming ban, or both. The results show that the deployment limit produces more social benefits than the multihoming ban.

2.3. Summary

In summary, of the handful of studies that have considered competition in bike-sharing, few have focused on the industry's vulnerability to intense, potentially destructive competition rooted in its relatively low entry barrier. The empirical study of Cao et al. (2021) was enlightening, but it was motivated by a positive effect of competition, which may well exist in the initial development of the industry but cannot adequately explain its crash. The discussion of regulations was more scarce. The only exception to the best of our knowledge, Wang et al. (2023), justified the interventions not by the need to curtail wasteful competition but by an inherently negative externality—captured by an abstract function—that bikes supposedly impose on society. In addition, the underlying modeling approach adopted in these studies uses either a full-blown network-based specification (see, e.g., Jiang, Lei, and Ouyang 2020) or a highly stylized construct (see, e.g., Cao et al. 2021).

The present study strives to fill some of the gaps identified above. Our model, built on Zheng et al. (2023), aims at a balanced approach to the tradeoffs between tractability and realism and between empirical and theoretical investigations. The overarching goal is to explain how unhealthy competition can harm bike-sharing and to search for regulations that can effectively prevent these harms.

3. Game of Oligopoly Competition

Consider a city of an area A served by a set of $\mathbb{I} = \{1, 2, \ldots, I\}$ DLB operators. The oligopoly market is modeled as a bilevel game with a hierarchy. At the upper level, each operator $i \in \mathbb{I}$ chooses an action $s_i \in \mathbb{S} = \{S_1, \ldots, S_K\}$ characterized as optimizing a certain objective, such as maximizing ridership or profit. Let $\mathbb{T}_i = \{T_{i1}, \ldots, T_{iK}\}$ be the set of objectives for operator i, with T_{ik} being the objective associated with action S_k. The payoff of the operator i is represented as a vector-valued function $u_i : \mathbb{S}^I \to \mathbb{R}^K$. Specifically, for a given *action profile* $\boldsymbol{s}$, where $\boldsymbol{s} = (s_1, \ldots, s_i, \ldots, s_I)$, the payoff vector for operator i is $u_i(\boldsymbol{s}) = [t_{i1}, \ldots, t_{iK}]$. The payoff vector $u_i(\boldsymbol{s})$ is determined by the lower-level game, whose output is a subgame perfect Nash equilibrium (Mas-Colell, Whinston, Green 1995). Specifically, given its upper-level action s_i and those of its competitors, which can be written collectively as $\boldsymbol{s} = (s_i, \boldsymbol{s}_{-i})$, where $\boldsymbol{s}_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_I)$, the operator chooses tactics $\boldsymbol{y}_i$ to maximize the objective associated with the action chosen at the upper level. The tactics considered in this study include the fleet size (B_i) and fare rate measured by ¥ per unit distance (f_i), that is, $\boldsymbol{y}_i = (B_i, f_i)$. All operators' tactics are denoted as $\boldsymbol{y} = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_i, \ldots, \boldsymbol{y}_I)$.

We write an oligopoly game of the DLB operators as $M(\mathbb{I}, \mathbb{S}, \mathbb{T}_{i\in I}, u_i(\cdot)_{i\in I})$. In what follows, Section 3.1 defines the Nash equilibrium of the game. Section 3.2 formulates the lower-level game that determines tactics $\boldsymbol{y}_i$ and payoff function u_i, $\forall i \in \mathbb{I}$ for a given action profile $\boldsymbol{s}$. Section 3.3 gives an illustrative example.

3.1. Nash Equilibrium of the Bilevel Game

To define equilibrium, we begin by establishing the preference of an operator for actions. This is not trivial because, given the vector-valued payoff function, the operator essentially faces a multi-objective optimization problem (Deb 2013). Additionally, treating operational goals as actions introduces further complexity because the operator may prioritize different components of the payoff vector when comparing alternative actions.

Definition 1 (Weak Preference). Given the actions of other operators $\boldsymbol{s}_{-i}$, consider two action profiles, $\boldsymbol{s} = (s_i, \boldsymbol{s}_{-i})$ and $\boldsymbol{s}' = (s_i', \boldsymbol{s}_{-i})$, where $s_i = S_k \neq s_i'$. Let $u_i(\boldsymbol{s}) = [t_{i1}, \ldots, t_{iK}]$ and $u_i(\boldsymbol{s}') = [t_{i1}', \ldots, t_{iK}']$ be the corresponding payoffs for operator i. Operator i is said to weakly prefer action s_i to action s_i' if $t_{ik} \geq t_{ik}'$ holds, written as $s_i \succeq s_i'|_{\boldsymbol{s}_{-i}}$.

Definition 2 (Strong Preference). Given the actions of other operators $\boldsymbol{s}_{-i}$, consider two action profiles, $\boldsymbol{s} = (s_i, \boldsymbol{s}_{-i})$ and $\boldsymbol{s}' = (s_i', \boldsymbol{s}_{-i})$, where $s_i \neq s_i'$. Let $u_i(\boldsymbol{s}) = [t_{i1}, \ldots, t_{iK}]$ and $u_i(\boldsymbol{s}') = [t_{i1}', \ldots, t_{iK}']$ be the corresponding payoffs for operator i. Operator i is said to strongly prefer action s_i to action s_i' if $t_{ik} \geq t_{ik}'$, $\forall k = 1, \ldots, K$, and at least one inequality holds strictly, written as $s_i \succ s_i'|_{\boldsymbol{s}_{-i}}$.

The distinction between weak preference and strong preference lies in how operators evaluate their actions given the choices of others. Under weak preference, an action is preferred as long as it yields an outcome no worse that the alternative in the specific objective tied to that action, regardless of the values of other components in the payoff vector. In contrast, strong preference requires that the chosen action perform as well as the alternative across all components of the payoff vector—that is, across all potential objectives. In the terminology of multi-objective optimization, this means that a strongly preferred action must Pareto-dominate the alternative (see, e.g., Deb 2013). These preference concepts lead to the following definitions regarding an operator's choice of action:

Definition 3 (Consistent Action). If $s_i \succeq s_i'|_{\boldsymbol{s}_{-i}}$, $\forall s_i' \neq s_i$, we say that s_i is a consistent action for operator i given $\boldsymbol{s}_{-i}$.

The intuition behind consistency is that if an operator adopts a particular action, then it should not be able to improve upon the payoff value tied to that action by committing to another action. A consistent action may not exist, and when it does, it may not be unique. Unlike the related concept of weak dominance in the literature (Fudenberg and Tirole 1991), Definition 3 allows multiple actions to be consistent given $\boldsymbol{s}_{-i}$ at the same time, even if they result in entirely different payoff vectors.[4] Importantly, the existence of multiple consistent actions does not imply that the operator is indifferent to them. In such cases, additional information is needed to make a meaningful comparison among the consistent actions. We next define the concept of Pareto-dominant action.

Definition 4 (Pareto-Dominant Action). If $s_i \succ s_i'|_{\boldsymbol{s}_{-i}}$, $\forall s_i' \neq s_i$, we say that s_i is a Pareto-dominant action for operator i given $\boldsymbol{s}_{-i}$.

When other operators' actions are fixed, the existence of a Pareto-dominant action means that the operator is compelled to choose the action because it would be worse off when switching to other actions, as dictated by strong preference. Clearly, a Pareto-dominant action may not exist either. However, when it does, it must be unique. Moreover, the following property directly follows from the definition.

Property 1. *If s_i is Pareto-dominant for operator i given $\boldsymbol{s}_{-i}$, then s_i must be consistent for operator i given $\boldsymbol{s}_{-i}$.*

We are now ready to define a Nash equilibrium under weak/strong preference.

Definition 5 (Nash Equilibrium Under Weak/Strong Preference). Given an oligopoly game $M(\mathbb{I}, \mathbb{S}, \mathbb{T}_{i\in I}, u_i(\cdot)_{i\in I})$, an action profile $\boldsymbol{s}$ is a Nash equilibrium under weak preference (NEWP) (respectively, Nash equilibrium under strong preference (NESP)) for the game if, $\forall i \in \mathbb{I}$, s_i is a consistent (respectively, Pareto-dominant) action given $\boldsymbol{s}_{-i}$.

NEWP ensures that, given the actions of its competitors, an operator is content with its chosen action. However, the operator still retains the flexibility to change actions because other actions could be consistent as well. This flexibility makes NEWP less stable than a standard Nash equilibrium because an operator in a NEWP may intentionally switch its action and arrive at a different NEWP (see the later illustrative example in Figure 1(b)). In contrast, no player has an incentive to deviate under a standard Nash equilibrium. At NESP, the operator no longer has this luxury because its current action Pareto-dominates all other actions. In other words, switching to any other actions will make the operator decisively worse off. This contrast implies that NEWP is less stable than NESP, and if an NESP exists, the system is more likely to settle on it than any NEWP state. The following is the direct result of Definition 5.

Property 2. *An NESP is also an NEWP.*

The oligopoly game and its Nash equilibrium under weak/strong preference provide a framework within which the DLB market behavior is to be understood and explained. Note that the existence and uniqueness of NEWP/NESP depend on the payoff vectors emerged from the lower-level game, which itself is built on complicated market dynamics. In Section 3.3, we provide hypothetical examples in which NESP and NEWP coexist, only one exists, or neither does. The case study (Section 7) will reveal which outcomes are likely to emerge in the real world.

We set out to define the lower-level game next. The demand-supply equilibrium of the DLB market, on which the entire game structure rests, will be discussed in Section 4.

3.2. Lower-Level Game

Given an action profile $\boldsymbol{s}$, each operator determines its own tactics $\boldsymbol{y}_i$ while taking into consideration the market's demand-supply balance and the competitor's tactics. We first formulate how each operator optimizes the objective associated with its action. Before we start, it is worth emphasizing that tactics are chosen to satisfy natural constraints such as nonnegativity and finite upper bounds. The upper bound may be interpreted as either a very large number that ensures the compactness of the feasible set or a limit set by government regulations. These considerations give us the notion of *proper decisions*, defined below.

Figure 1. (Color online) Payoff Matrix Examples in a Duopoly

(a)

		Operator 2: $s_2 = \mathrm{p}$	Operator 2: $s_2 = \mathrm{r}$
Operator 1	$s_1 = \mathrm{p}$	[1, 0] [1, 0]	[0, 0] [0, 1]
	$s_1 = \mathrm{r}$	[0, 1] [0, 0]	[1, 1] [1, 1]

(b)

	Operator 2: $s_2 = \mathrm{p}$	Operator 2: $s_2 = \mathrm{r}$
$s_1 = \mathrm{p}$	[1, 0] [1, 0]	[1, 0] [0, 1]
$s_1 = \mathrm{r}$	[0, 1] [1, 0]	[0, 1] [0, 1]

(c)

	Operator 2: $s_2 = \mathrm{p}$	Operator 2: $s_2 = \mathrm{r}$
$s_1 = \mathrm{p}$	[0, 1] [0, 1]	[1, 1] [1, 1]
$s_1 = \mathrm{r}$	[1, 1] [1, 1]	[2, 0] [2, 0]

Notes. $\mathbb{I} = \{1, 2\}, \mathbb{S} = \{\mathrm{p}, \mathrm{r}\}$. In each cell, the first row reports $[t_{1\mathrm{p}}, t_{1\mathrm{r}}]$, and the second row reports $[t_{2\mathrm{p}}, t_{2\mathrm{r}}]$. (a) NEWP (upper-left cell) and NESP (lower-right cell) coexist; (b) all action profiles are NEWP; (c) neither NEWP nor NESP exists.

Definition 6 (Proper Decision). A decision vector $(\boldsymbol{y}_i)$ is said to be proper, written as $\boldsymbol{y}_i \in \mathbb{Y}_0 = \{(B_i, f_i) | B_i \in [0, \Gamma_B], f_i \in [0, \Gamma_f]\}$, where Γ_B and Γ_f are, respectively, a finite upper bound on the fleet size and fare rate.

Note that operator i's payoff is jointly determined by all operators' decisions. Thus, we write the decisions of operator i's opponents as $\boldsymbol{y}_{-i} = (\boldsymbol{y}_1, \dots, \boldsymbol{y}_{i-1}, \boldsymbol{y}_{i+1}, \dots, \boldsymbol{y}_I)$. Given the upper-level action $s_i = S_k$, operator i's decision problem in the lower-level game is formulated as Problem (P1):

$$\max_{\boldsymbol{y}_i \in \mathbb{Y}_0} \ T_{ik}(\boldsymbol{y}_i, \boldsymbol{y}_{-i})|_{s_i = S_k}, \tag{P1-a}$$

$$\text{s.t.} \ \ \boldsymbol{\Phi}(\boldsymbol{x} | \boldsymbol{y}_i, \boldsymbol{y}_{-i}) = \boldsymbol{0}, \tag{P1-b}$$

$$D_i(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) \geq 0. \tag{P1-c}$$

Here, T_{ik} is the objective function for operator i corresponding to its action at the upper level S_k.[5] (P1-b) represents the demand-supply equilibrium condition that captures the dynamics of the DLB market, given $\boldsymbol{y}$, where $\boldsymbol{x}$ is the state variable and can be obtained by solving the equilibrium (see Section 4 for details). (P1-c) captures the operational requirements that operator i needs to follow, such as maintaining a minimum level of profit. For convenience and to emphasize the feasible set of (P1) depending on $\boldsymbol{y}_{-i}$, we shall write it as $\Omega_i(\boldsymbol{y}_{-i})$ hereafter.

The lower-level game reaches a subgame perfect Nash equilibrium when the tactical decision vector $\boldsymbol{y}^*$ satisfies the following equilibrium conditions for a given action profile $\boldsymbol{s}$:

$$\begin{aligned} &T_{ik}(\boldsymbol{y}_i^*, \boldsymbol{y}_{-i}^*) \geq T_{ik}(\boldsymbol{y}_i, \boldsymbol{y}_{-i}^*), \\ &s_i = S_k, \ \forall \boldsymbol{y}_i \in \Omega_i(\boldsymbol{y}_{-i}^*), \ \forall i \in \mathbb{I}, \end{aligned} \tag{2}$$

where $\boldsymbol{y}_i^*$ and $\boldsymbol{y}_{-i}^*$ are the decisions of operators i and $-i$, respectively, at the equilibrium.

It is worth noting that Equation (2) characterizes a generalized Nash equilibrium (GNE) because both the operator's objective function and the feasible set of its decisions are affected by its competitors' decisions. Although a GNE may not exist, Equation (2) remains a necessary condition for equilibrium (Arrow and Debreu 1954). We will devise a numerical procedure to find the subgame perfect Nash equilibrium in Section 5 after we specify the demand-supply equilibrium of the DLB market (Section 4).

3.3. Illustrative Example

We now illustrate the oligopoly game using a simple example. Consider a duopoly market with two operators, denoted as $\mathbb{I} = \{1, 2\}$. The action set $\mathbb{S} = \{\mathrm{p}, \mathrm{r}\}$, where p represents profit maximization and r represents ridership maximization. Accordingly, the set of objectives for operator $i \in \mathbb{I}$ is $\mathbb{T}_i = \{T_{i\mathrm{p}}, T_{i\mathrm{r}}\}$. Given an action profile $\boldsymbol{s} = (\mathrm{r}, \mathrm{p})$, for example, the payoff of operator $i = 1, 2$ can be written as $[t_{i\mathrm{p}}|_{s_1=\mathrm{r}, s_2=\mathrm{p}}, t_{i\mathrm{r}}|_{s_1=\mathrm{r}, s_2=\mathrm{p}}]$, where $t_{i\mathrm{p}}|_{s_1=\mathrm{r}, s_2=\mathrm{p}}$ and $t_{i\mathrm{r}}|_{s_1=\mathrm{r}, s_2=\mathrm{p}}$ are obtained when Operator 1 maximizes $T_{1\mathrm{r}}$ and Operator 2 optimizes $T_{2\mathrm{p}}$, as defined by (P1).

Figure 1 showcases the payoff matrix examples in which each cell is associated with an action profile. Take Figure 1(a), as an example; the action profile corresponding to the lower-right cell (both operators choose r) is an NESP; the action r is a Pareto-dominant action for both operators when the competitor chooses r, because we have (i) given $s_2 = \mathrm{r}$, $t_{1\mathrm{p}}|_{s_1=\mathrm{r}, s_2=\mathrm{r}} = 1 > t_{1\mathrm{p}}|_{s_1=\mathrm{p}, s_2=\mathrm{r}} = 0$ and $t_{1\mathrm{r}}|_{s_1=\mathrm{r}, s_2=\mathrm{r}} = 1 > t_{1\mathrm{r}}|_{s_1=\mathrm{p}, s_2=\mathrm{r}} = 0$ and (ii) given $s_1 = \mathrm{r}$, $t_{2\mathrm{p}}|_{s_1=\mathrm{r}, s_2=\mathrm{r}} = 1 > t_{2\mathrm{p}}|_{s_1=\mathrm{r}, s_2=\mathrm{p}} = 0$ and $t_{2\mathrm{r}}|_{s_1=\mathrm{r}, s_2=\mathrm{r}} = 1 > t_{2\mathrm{r}}|_{s_1=\mathrm{r}, s_2=\mathrm{p}} = 0$. However, r is no longer a Pareto-dominant action when the competitor is committed to p. Instead, both p and r are consistent actions for the operator. This can be verified by noting that (i) given $s_2 = \mathrm{p}$, $t_{1\mathrm{p}}|_{s_1=\mathrm{p}, s_2=\mathrm{p}} = 1 > t_{1\mathrm{p}}|_{s_1=\mathrm{r}, s_2=\mathrm{p}} = 0$ and $t_{1\mathrm{r}}|_{s_1=\mathrm{r}, s_2=\mathrm{p}} = 1 > t_{1\mathrm{r}}|_{s_1=\mathrm{p}, s_2=\mathrm{p}} = 0$, and (ii) given $s_1 = \mathrm{p}$, $t_{2\mathrm{p}}|_{s_1=\mathrm{p}, s_2=\mathrm{p}} = 1 > t_{2\mathrm{p}}|_{s_1=\mathrm{p}, s_2=\mathrm{r}} = 0$ and $t_{2\mathrm{r}}|_{s_1=\mathrm{p}, s_2=\mathrm{r}} = 1 > t_{2\mathrm{r}}|_{s_1=\mathrm{p}, s_2=\mathrm{p}} = 0$. Because each action in the action profile (p, p) is consistent for the operator, the market reaches NEWP in the upper-left cell. We leave it to the reader to verify that all action profiles in Figure 1(b) are NEWP but not NESP, and no action profile in Figure 1(c), is either NEWP or NESP.

4. Demand-Supply Equilibrium of a Dockless Bike-Sharing Market

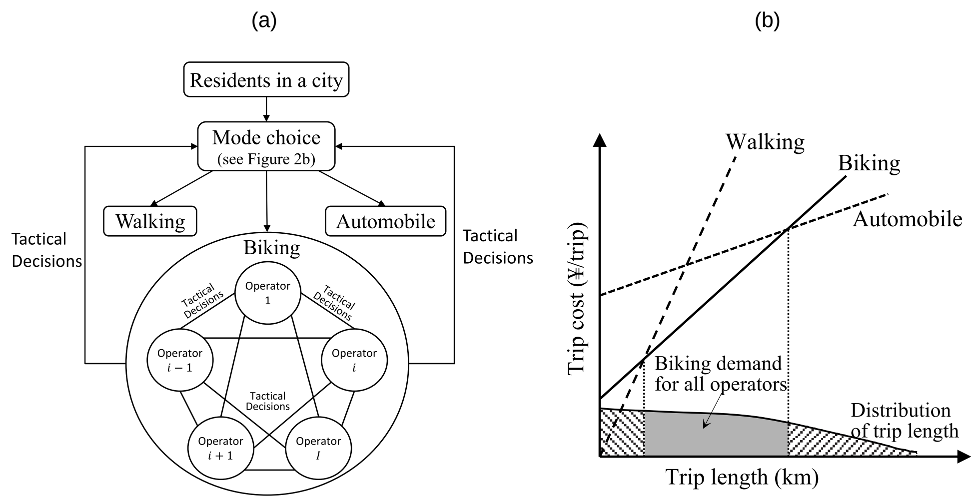
In this section, we formulate the demand-supply interaction in a dockless bike-sharing (DLB) market with multiple operators. Throughout this section, we shall assume that all tactics, namely the fleet size and fare of each operator, are fixed.

The bike-sharing market is situated in a city whose residents can either walk, bike, or choose an automobile, as illustrated in Figure 2(a). Note that "automobile" may be viewed as a composite motor mode that encompasses public transit, taxi, or ride-hail.[6] We follow Zheng et al. (2023) to assume the following.

Assumption 1. (i) *Travelers' origin and destination are uniformly distributed in space, and* (ii) *their mode choice is driven primarily by trip length.*

Assumption 1-(i) is introduced to avoid the complexities that inevitably come with spatial heterogeneity but are unlikely to change the nature of the oligopoly game. The intuition behind Assumption 1-(ii) is straightforward; biking is not preferred when the trip length is too short or too long (see Figure 2(b); a replica from Zheng et al. 2023). In the former case, the time savings are too small to justify the cost, whereas in the latter the time savings are so great that a more expensive but much faster mode (automobile) would make more sense than

Figure 2. Illustration of a Simplified Mobility Market



Note. (a) Bike-sharing market with I operators; (b) mode choice by trip length.

biking. To simplify mode choice, we further introduce the following assumption.

Assumption 2. *The total number of travelers choosing biking in the city depends on the average cost of biking.*

Note that the average cost of biking, which consists of the average fare and access time (i.e., the time it takes to reach the nearest idle bike, regardless of which DLB operator it belongs to), depends on the collective rather than individual tactics of all operators in the DLB market. A detailed justification of this assumption is provided in Online Appendix A.

Assumption 2 regulates how the total number of bikers is determined. The allocation of bikers among operators, that is, each operator's "market share," depends on their fare and share of idle bikes. In the following, we provide more details on the characterization of bike-sharing demand (Section 4.1) and supply (Section 4.2). The reader is referred to Online Appendix B for a description of main notations.

4.1. Demand

The bike-sharing demand is determined by the distribution of trip length—whose cumulative distribution function (CDF) is denoted as $G(\cdot)$—as well as the average access time, denoted as a, and the average fare rate weighted by ridership, denoted as f in ¥ per unit distance. Recall that biking is the most competitive mode for trips of medium length because, intuitively, a traveler would walk when the trip is very short and drive when it is very long. This observation suggests that there is a length range within which all trips will be made by biking. Following Assumption 2 and Zheng et al. (2023), it is easy to show that the lower and upper bounds of the range, denoted as $\underline{l}$ and $\bar{l}$, respectively, are given by

$$\underline{l} = \frac{\mu a}{\mu/v_w - f - \mu/v_b}, \qquad \bar{l} = \frac{\tau - \mu a}{f + \mu/v_b - f_d - \mu/v_d}, \tag{3}$$

where μ is the value of time, f_d is the monetary cost of automobile per unit distance, τ is the part of the automobile cost not directly proportional to the travel distance, such as acquisition and maintenance, and v_w, v_b, and v_d are operating speed for walking, biking, and automobile, respectively; see Zheng et al. (2023) for details.[7] Thus, the total demand for bike-sharing is given by

$$Q = \bar{Q}(G(\bar{l}) - G(\underline{l})), \tag{4}$$

where $\bar{Q}$ is the total travel demand. Because the focus of this study is the DLB market, we assume that the values of a and f guarantee $\bar{l} > \underline{l}$, which ensures that $Q > 0$.

We next discuss how the total biking demand Q is allocated to each operator. Let Q_i be the *market share* of operator $i \in \mathbb{I}$, and define

$$m_i = Q_i/Q \tag{5}$$

as the operator's *market proportion*. This definition allows us to estimate the fare f as a ridership-weighted average, that is,

$$f = \sum_{i \in \mathbb{I}} m_i f_i. \tag{6}$$

We shall discuss the estimation of access time a in Section 4.2 because it is a supply feature.

We assume that operator i's market share, Q_i, depends on its share in the supply of idle bikes as well as the relative difference between its fare and the fares set by the competitors. Intuitively, the larger the share of idle bikes and the lower the fare relative to those of the competitors, the greater the market share. We propose to quantify this relationship using the following function:

$$Q_i = \frac{n_i}{\sum_{j\in\mathbb{I}} n_j} Q - \sum_{j\in\mathbb{I}\setminus\{i\}} k_{ij}(f_i - f_j), \tag{7}$$

where n_i is the number of idle bikes owned by operator i, and k_{ij} measures the sensitivity of Q_i to the fare difference between operator i and j (note that, by definition, $k_{ij} = k_{ji}$). The first term in Equation (7) allocates the total biking demand to operator i in proportion to the size of its idle bikes, whereas the second term adjusts the base allocation based on fare differences. Equation (7) implies that, everything else being equal, operator i can always influence its market share through pricing. On the other hand, controlling n_i is less straightforward because, as we shall see in the next section, it is related to fleet size, bike demand, and rebalancing activity through a complicated conservation condition.

Linear demand models similar to Equation (7) have been widely used to capture price elasticity under competition in transportation systems (see e.g., Charnes et al. 1972, Kurata, Yao, and Liu 2007, Zhou and Lee 2009, Ülkü and Bookbinder 2012, Zheng, Li, and Song 2017, Choi, Chung, and Zhuo 2020, and Li, Li, and Chen 2022), largely because of the tractability they afford to the analysis. It is easy to verify that the demands assigned to the operators by Equation (7) add up to the total demand, that is, $\sum_{i\in\mathbb{I}} Q_i = Q$. In theory, the demand model (7) could produce unrealistic market shares. For example, if an operator that owns a very small fleet ($B_i \to 0$) sets a very high fare, it may end up with a "negative" demand. On the flip side, if it sets the fare close to zero, it may generate a demand larger than what its tiny fleet could plausibly serve. These corner cases are unlikely to rise in a world of rational operators, and when they do occur, they indicate that the operator in question should be excluded from the market because it would not make a meaningful contribution to the supply. We will discuss how to prevent the interference of these corner cases at the end of Section 4.2.

4.2. Supply

Each operator independently determines its fare f_i and fleet size B_i. For an operating hour, the total bike time must be conserved for each operator, namely

$$B_i = n_i + U_i + R_i, \ \forall i \in \mathbb{I}, \tag{8}$$

where the total bike time equals B_i, the idle bike time equals n_i, and U_i and R_i represent the total bike usage time and the total bike rebalancing time, respectively.

The average access time a depends on the average distance of a random traveler to the nearest idle bike, which in turn depends on the average density spatial distribution of these bikes. Note that, although idle bikes tend to scatter across the city, they naturally form clusters because of the limited suitable bike parking areas. In other words, it is the density and distribution of these bike clusters, rather than those of the bikes themselves, that play a pivotal role in determining the access time. We call the centroid of a bike cluster a *unique bike location*, and the distance to any bike included in the cluster is represented by the distance to the centroid. We further make the following assumption about unique bike locations.

Assumption 3 (Unique Bike Location). *Unique bike locations (i.e., the locations of bike clusters) distribute uniformly in space, and their density is a function of the density of idle bikes, that is,*

$$\frac{\tilde{n}}{A} = z\left(\frac{n}{A}\right), \tag{9}$$

where $\tilde{n}$ is the number of unique bike locations, and $n = \sum_{i\in\mathbb{I}} n_i$ is the total number of idle bikes.

Zheng et al. (2023) calibrated the function $z(\cdot)$ by clustering GPS locations of idle bikes from a large data set collected in Chengdu. Their results suggested that $z(\cdot)$ is an increasing and concave function.

With Assumption 3, we are ready to derive the average access time based on the average distance between a random point and the nearest unique bike location (see, e.g., Daganzo 1978):

$$a = \frac{\delta}{v_w}\sqrt{\frac{A}{\tilde{n}}}, \tag{10}$$

where v_w is the average walking speed, and δ is a parameter related to the geometry of the city.

The bike usage time U_i and rebalancing time R_i can be obtained as

$$U_i = \frac{Q_i}{Q}\frac{\bar{Q}}{v_b}\int_{\underline{l}}^{\bar{l}} x \mathrm{d}G(x), \tag{11}$$

$$R_i = \frac{L_i}{v_r}\alpha Q_i, \tag{12}$$

where L_i is the average traveling distance per rebalancing trip, v_r is a nominal rebalancing speed that converts the distance to equivalent bike time, and α is the average number of rebalancing trips needed for each "real" bike trip. Note that α can be interpreted as the rebalancing efficiency; the larger the value of α, the lower the efficiency. We treat α and v_r as exogenous parameters and L_i as an endogenous variable. Specifically, α is observed directly from empirical DLB operational data, and v_r is estimated based on demand-supply equilibrium; see Zheng et al. (2023) for details.

To estimate the average rebalancing distance L_i, we assume that each operator aims to maintain a steady spatial distribution of idle bikes by routinely moving a certain number of idle bikes to unique bike locations that require "replenishment." Because α can be observed from data, the number of idle bikes to be rebalanced per hour is simply αQ_i. By assuming that the ratio between αQ_i and the number of unique bike locations in need of replenishment equals the average number of idle bikes in each location, that is, $n_i/\tilde{n}_i$, Zheng et al. (2023) estimated that

$$L_i = \delta\sqrt{\frac{n_i}{\tilde{n}_i}\frac{A}{\alpha Q_i}}, \tag{13}$$

which is the average distance between a random idle bike and the nearest unique bike location in need of replenishment.

Finally, to rule out unrealistic equilibrium states, we impose the following constraint on the number of idle bikes:

$$0 \le n_i \le B_i, \ \forall i \in \mathbb{I}. \tag{14}$$

Although the lower bound is a natural restriction, it also ensures that an operator's bike usage never exceeds its fleet size; as discussed earlier, such a corner case could arise from the demand model (7). On the other hand, the upper bound guarantees that an operator cannot have more idle bikes than its fleet size, a situation that would occur only if its ridership were negative, as per Equation (8).

4.3. Equilibrium

Combining Equations (3)–(14) fully specifies the demand-supply interaction in a DLB oligopoly with fixed tactics. For simplicity, given all operators' tactics $\boldsymbol{y} = (y_1, \ldots, y_i, \ldots, y_I)$, we write this equation system as $\boldsymbol{\Phi}(\boldsymbol{x}|\boldsymbol{y}) = \boldsymbol{0}$, which is also the "equilibrium constraint" in (P1), that is, the operator's decision problem in the lower-level game. We proceed to establish that the system should always have a solution. Let us first define the solution as $\boldsymbol{x} = (n_1, \ldots, n_I, f)$ and $\boldsymbol{x} \in \mathbb{X}$, where $\mathbb{X}$ is the feasible set of $\boldsymbol{x}$ and includes all combinations that $n_i \in [0, B_i]$, $\forall i \in \mathbb{I}$ and $f \in [\min_{i\in\mathbb{I}} f_i, \max_{i\in\mathbb{I}} f_i]$. We choose to represent the system solution using these variables because, as we shall show, they cannot be explicitly represented, whereas all other variables can. Thus, by rewriting the system to a self-mapping function, denoted as $\boldsymbol{\Psi}: \mathbb{X} \to \mathbb{X}$, where $\boldsymbol{\Psi}(\boldsymbol{x}) = \boldsymbol{x}$, we can invoke Brouwer's fixed point theorem (Brouwer 1911) to prove solution existence.

Proposition 1. *If all decision vectors in $\boldsymbol{y}$ are proper, then there always exists a solution $\boldsymbol{x} \in \mathbb{X}$ such that $\boldsymbol{\Phi}(\boldsymbol{x}|\boldsymbol{y}) = \boldsymbol{0}$.*

Proof. See Online Appendix C for details. □

We next define each operator's profit and improved social welfare attributed to the DLB service at the demand-supply equilibrium given $\boldsymbol{y}$. Operator i's profit equals its revenue less the operating cost, that is,

$$\Pi_i = f_i\frac{Q_i}{Q}\left(\bar{Q}\int_{\underline{l}}^{\bar{l}} x\mathrm{d}G(x)\right) - \beta_0 B_i - \beta_1 \alpha L_i Q_i, \tag{15}$$

where the first term is operator i's revenue, β_0 denotes the acquisition cost per bike operation hour, and β_1 denotes the rebalancing cost per bike per unit distance. Social welfare improvement is equivalent to the social cost saved by making bike-sharing available as a new transportation mode, which can be calculated by

$$\begin{aligned} W = C_0 &- \left(\beta_0\sum_{j\in\mathbb{I}} B_j + \beta_1\alpha\sum_{j\in\mathbb{I}} L_jQ_j\right) - \left(\frac{\mu}{v_w}\int_0^{\underline{l}} xdG(x)\right)\bar{Q} \\ &- \left(\mu a(G(\bar{l}) - G(\underline{l})) + \frac{\mu}{v_b}\int_{\underline{l}}^{\bar{l}} xdG(x)\right)\bar{Q} \\ &- \left(\left(\frac{\mu}{v_d} + d\right)\int_{\bar{l}}^{+\infty} xdG(x) + \tau(1 - G(\bar{l}))\right)\bar{Q}, \end{aligned} \tag{16}$$

where C_0 denotes the social cost without the DLB system, whereas the remaining terms represent the social cost with the DLB system. This includes the costs of all DLB operators and the total travel cost, excluding monetary payments to the DLB service.[8]

4.4. Analysis

In this section, we present a few analytical results concerning the properties of the demand-supply equilibrium formulated above. To facilitate the analysis, let us first define η_n and η_f as the elasticity of total bike-sharing demand with respect to, respectively, the total number of idle bikes (n) and the average fare of DLB service (f), namely

$$\eta_n = \frac{\partial Q/Q}{\partial n/n}, \eta_f = \frac{\partial Q/Q}{\partial f/f}. \tag{17}$$

Proposition 2. *Consider a DLB duopoly, that is, $\mathbb{I} = \{1, 2\}$. Assume that $f_1 \le f_2$, and denote the difference $d_f = f_2 - f_1$. Suppose the operators maintain a steady level of service, which means that they keep their idle bike time constant.*

- *When f_1 decreases, the ridership of Operator 1 always increases, and that of Operator 2 increases if $d_f > d_1 \equiv -\frac{[Q_1n_2 + (n_1+n_2)k_{12}f/\eta_f]Q}{(n_1+n_2)k_{12}Q_2}$.*
- *When f_2 decreases, the ridership of Operator 2 increases if $d_f < d_2 \equiv \frac{[Q_2n_2 - (n_1+n_2)k_{12}f/\eta_f]Q}{(n_1+n_2)k_{12}Q_2}$, and that of Operator 1 increases if $d_f < d_3 \equiv \frac{[Q_2n_1 + (n_1+n_2)k_{12}f/\eta_f]Q}{(n_1+n_2)k_{12}Q_1}$, where $d_2 > d_3$.*

Proof. See Online Appendix D for details. □

The above result illustrates the relationship between an operator's ridership and the fares of both itself and its competitor. When an operator cuts the price, it has two opposing effects. On the one hand, the lower price attracts more travelers to the DLB service, which helps its competitor gain ridership. On the other hand, the operator can also increase its market proportion with the lowered fare, thereby taking travelers away from its competitor. Proposition 2 shows that under certain conditions, the first effect above dominates, and an operator may enjoy a "free-ride"—that is, it achieves a higher ridership when the competitor lowers the price. Moreover, when the operator charging a lower fare further reduces the price, it is guaranteed to gain ridership. However, when the operator that initially charges a higher price attempts to grab ridership by lowering the price, it has to substantially decrease the price difference for the strategy to work.

The impact of fleet size on ridership can be analyzed similarly. However, the analysis is quite complicated because the average DLB fare depends on the market share of each operator, which in turn is a function of their respective fleet sizes. For tractability, we consider only the case where all operators charge the same fare.

Proposition 3. *Consider a DLB market with $I > 1$ operators. Let n_i^* be the number of idle bikes owned by operator i at a market equilibrium (i.e., a solution to the system* (3)–(14) *defined by a proper decision vector), and suppose all other operators maintain a steady level of service (constant idle bike time). If the fare f_i is the same for each $i \in \mathbb{I}$, then n_i^* is a strictly increasing function of its own fleet size B_i,that is, $\frac{\partial n_i^*}{\partial B_i} > 0$.*

Proof. See Online Appendix E for details. □

Proposition (3) shows that, ceteris paribus, an operator may increase its holding of idle bikes, hence improving overall accessibility to bike-sharing, by acquiring a larger fleet. Zheng et al. (2023) provided a similar result for a market with a monopolizing DLB operator, and we now extend this result to hold in an oligopoly market as well.

Proposition 4. *In a DLB market with $I > 1$ operators, assume that all other operators maintain a steady level of service (constant idle bike volume). An operator's ridership always increases with its own fleet size, and decreases (respectively, increases) with its competitor's fleet size if $\eta_n < 1$ (respectively, $\eta_n > 1$).*

Proof. See Online Appendix F for details. □

Proposition 4 highlights that when η_n is too large (>1), an operator unilaterally increasing its fleet size may induce a significant demand surge, from which its competitor benefits with a larger market share (although the market portion actually decreases). This rare condition is known as the network effect in the literature (Hyland and Mahmassani 2020, Ke et al. 2020, Cao et al. 2021), which requires that the increased idle fleet disproportionally attracts more users by improving the accessibility of the DLB service. However, we expect the magnitude of η_n to be well below one in practice; that is, the increase in the total DLB demand caused by a 1% increase in n is much less than 1%. Therefore, Proposition 4 confirms the intuition that, in a DLB market with competition, an operator can always attract more travelers and diminish the market share of its competitors by putting more bikes on streets.

5. Solution Algorithm for the Lower-Level Game

In this section, we show how the subgame perfect Nash equilibrium of the lower-level game, defined in Section 3.2, may be found using a numerical procedure. We first rewrite the operator i's decision problem (P1) by replacing the equilibrium condition with Equations (3)–(14), that is,

$$\begin{aligned} \max_{\boldsymbol{y}_i \in \mathbb{Y}_0} \quad & T_{ik}(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) \\ \text{s.t.} \quad & (3)\text{–}(14), \\ & D_i(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) \geq 0. \end{aligned} \tag{P2}$$

The Lagrangian dual problem of (18) is defined as

$$\begin{aligned} \min_{\boldsymbol{\lambda}_i} \quad & \theta(\boldsymbol{\lambda}_i) = \sup_{\boldsymbol{y}_i} L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i) \\ \text{s.t.} \quad & \boldsymbol{\lambda}_i \geq \boldsymbol{0}, \\ & (3)\text{–}(14). \end{aligned} \tag{P3}$$

Here, $\boldsymbol{\lambda}_i = (\lambda_i^1, \lambda_i^2, \lambda_i^3)$ is a vector of Lagrange multiplier and $L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i) = T_{ik}(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) + \lambda_i^1 D_i(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) + \lambda_i^2(\Gamma_f - f_i) + \lambda_i^3(\Gamma_B - B_i)$. For a given λ_i and ignoring corner solutions at [0, 0],[9] the first-order condition of $\sup_{\boldsymbol{y}_i} L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i)$ implies that operator i's optimal strategy $\boldsymbol{y}_i^*$ should satisfy

$$\frac{\partial L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i^*)}{\partial \boldsymbol{y}_i^*} = 0. \tag{20}$$

By definition, we have $\theta(\boldsymbol{\lambda}_i) = L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i^*)$. Taking partial derivative with respect to $\boldsymbol{\lambda}_i$ on both sides yields $\frac{\partial \theta(\boldsymbol{\lambda}_i)}{\partial \boldsymbol{\lambda}_i} = \frac{\partial L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i^*)}{\partial \boldsymbol{\lambda}_i} + \frac{\partial L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i^*)}{\partial \boldsymbol{y}_i^*}\frac{\partial \boldsymbol{y}_i^*}{\partial \boldsymbol{\lambda}_i} = \frac{\partial L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i^*)}{\partial \boldsymbol{\lambda}_i}$. From the KKT conditions of (19), we have the following:

$$\lambda_i^z \frac{\partial L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i^*)}{\partial \lambda_i^z} = 0, \ \forall z \in \{1,2,3\}, \tag{21}$$

$$\lambda_i^z \geq 0, \frac{\partial L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i^*)}{\partial \lambda_i^z} \geq 0, \ \forall z \in \{1,2,3\}. \tag{22}$$

Equations (20)–(22) are the market equilibrium conditions for any operator $i \in \mathbb{I}$ when they maximize T_{ik} subject to a generalized constraint denoted as $D_i(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) \geq 0$.

Algorithm 1 (Bilevel Dual Gradient Descent (BDGD))

Inputs: Action s_i and objective T_{ik}, $\forall i \in \mathbb{I}$, step sizes ρ_y, ρ_λ, and convergence criterion ϵ.
Initialize $\boldsymbol{\lambda} = (\boldsymbol{\lambda}_i)_{i=1}^I$ with $\boldsymbol{\lambda}_i \geq \mathbf{0}$, $\forall i \in \mathbb{I}$
Set $g_\lambda = \infty$
while $g_\lambda \geq \epsilon$ **do**
    Set $g_y = \infty$
    Initialize $\boldsymbol{y} = (\boldsymbol{y}_i)_{i=1}^I$ with $\boldsymbol{y}_i \in \mathbb{Y}_0$, $\forall i \in \mathbb{I}$
    while $g_y \geq \epsilon$ **do**
        Find demand-supply equilibrium using Algorithm 2 in Online Appendix G.
        Calculate $\nabla_{\boldsymbol{y}} = \left(\frac{\partial L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i)}{\partial \boldsymbol{y}_i}\right)_{i=1}^I$ using the procedure in Online Appendix H.
        Set $g_y = \|\nabla_{\boldsymbol{y}}\|_\infty$
        Set $\boldsymbol{y} = (\boldsymbol{y} + \rho_y \cdot \nabla_{\boldsymbol{y}})^+$
    end while
    Calculate $\nabla_\lambda = \left(\frac{\partial L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i)}{\partial \boldsymbol{\lambda}_i}\right)_{i=1}^I$
    Set $g_\lambda = \boldsymbol{\lambda} \times (\nabla_\lambda^T)^+$
    Set $\boldsymbol{\lambda} = (\boldsymbol{\lambda} + \rho_\lambda \cdot \nabla_\lambda)^+$
end while
Output: Optimal decision vector $\boldsymbol{y}$.

We propose to solve Equations (20)–(22) using a Bilevel Dual Gradient Descent (BDGD) algorithm (see Algorithm 1). The algorithm consists of two main loops. In the inner loop, the Lagrangian multiplier vector $\boldsymbol{\lambda} = (\boldsymbol{\lambda}_i)_{i=1}^I$ is fixed as constant, and the oligopoly equilibrium, that is, the optimal decision vector $\boldsymbol{y} = (\boldsymbol{y}_i)_{i=1}^I$, is obtained by a gradient decent method based on automatic differentiation (Baydin et al. 2018).[10] After getting the optimal decision vector $\boldsymbol{y}$, the outer loop updates $\boldsymbol{\lambda}$ by adding the gradient of each operator's Lagrangian function $L_i(\boldsymbol{\lambda}_i, \boldsymbol{y}_i)$ with respect to $\boldsymbol{\lambda}_i$. By definition, the value of that gradient can be easily obtained by calculating $(D_i(\boldsymbol{y}_i, \boldsymbol{y}_{-i}), \Gamma_f - f_i, \Gamma_B - B_i)$. When $\boldsymbol{\lambda}$ converges, the algorithm is terminated, and the optimal $\boldsymbol{y}$ from the last inner loop is retained as the optimal solution. It is worth emphasizing that in the algorithm all operators simultaneously update their decisions in each loop, and when it is terminated the equilibrium solution is found for all operators in the market.

The algorithm presented herein is designed to find a stationary point that may not be unique. Strictly speaking, a physically meaningful stationary point may not even exist. In our numerical experiments, we try to detect multiple equilibria by starting from different initial points but have yet to find a single multi-equilibria instance in the case study constructed from real data. Next, we describe the case study.

6. Case Study

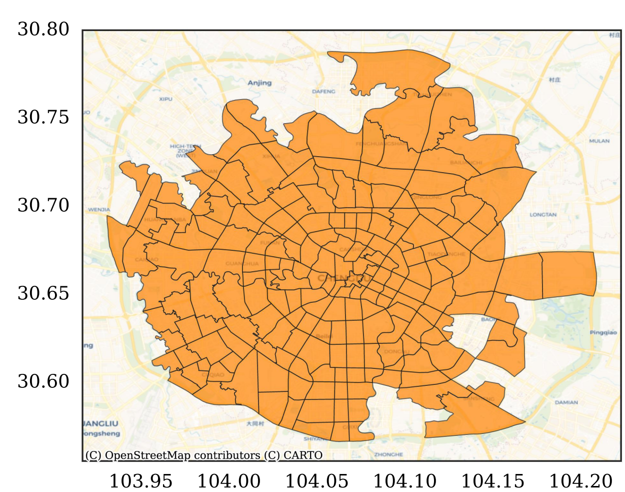
In this section, a case study is constructed to operationalize the model and to examine the likely outcome of the DLB oligopoly under various hypothetical conditions that resemble those found in a Chinese megacity with a well-developed DLB market. Below, we first describe the data set before specifying the DLB oligopoly game built from it.

6.1. Data

We employ a set of DLB trip records collected by a large DLB operator in Chengdu, China, over a period of 43 days, from March 25, 2020, to May 6, 2020.[11] The data were concentrated in the center of the city (see the colored area in Figure 3). Each trip record contains a trip ID, a bike ID, a user ID, start/end timestamps, start/end GPS coordinates, and a trip distance. In total, there are 15,349,358 recorded trips associated with 249,377 unique bike IDs. On average, a bike is utilized 1.43 times per day during this period. Considering that almost 97% of bike rides took place between 7 a.m. and 11 p.m., we utilized all trips occurring within this window to establish the analysis hour in our model—that is, all analyses are performed for an "average hour" in that period.

With a land area of 525 square kilometers (km^2), the study area is inhabited by a population of 4.23 million. In September 2018, the total number of shared bikes within this area had reached 1.1 million. However, in May 2019, a strict limit was imposed, capping the DLB bike fleet at 600,000. About a year later, shortly after the data used in this study were collected, the cap was further reduced to 450,000 bikes. Therefore, we assume that during the data collection period, there were around 600,000 bikes in the city owned by multiple operators. Accordingly, the data set we have,

Figure 3. (Color online) DLB Service Operation Area in the City Center of Chengdu



consisting of approximately a quarter million bikes, accounts for slightly more than 40% of the market share. The average number of trips made by these bikes is 20,709 per hour in the analysis period. Assuming the ridership of each operator is proportional to its fleet size, we estimate the total DLB ridership at 49,922 trips/hour for a fleet of 600,000.

Most input variables can be directly observed or estimated from publicly available sources. The notable exceptions are the unique bike location function $z(\cdot)$, the fixed automobile cost, the total travel demand, and the rebalancing speed. Zheng et al. (2023) estimated $z(\cdot)$ by clustering idle bike location data. The other three parameters were calibrated by matching a demand-supply equilibrium to real observations; see Zheng et al. (2023) for details. The default values of all input parameters are reported in Online Appendix I.

The present study does have a unique parameter—the competition factor k_{ij}—that needs to be estimated. Recall that k_{ij} captures the amount of ridership shifted between operators i and j when the difference in their fares changes. To reduce the burden of estimation, we assume that the competition factor is constant among all pairs of operators and denote it by k hereafter. To give a first-order estimation, we use the change rate in the total DLB ridership with respect to the average fare in the base model as a surrogate. The analysis suggests that when all parameters are fixed at their current level (i.e., the status quo), the DLB ridership decreases by 232 when the fare increases by ¥0.01/km. This is translated to a value of k at 23,200. We shall also test the model's sensitivity to k in Section 7.5.

6.2. Game Settings

We set the action set $\mathbb{S} = \{\mathrm{p}, \mathrm{r}\}$, where p denotes the action of profit maximization and r denotes that of ridership maximization. We choose these two actions because they are the most obvious options. An operator may prioritize ridership to grow market share, a common strategy during the early phase, when it strives to build a strong user base or drive out competition. In such cases, while keeping long-term profitability in mind, an operator may even be willing to incur short-term financial losses in pursuit of ridership growth.[12] In a mature market, where most operators have a relatively stable market share, profit maximization seems a more rational action.

For an operator i taking action p, the objective function for its decision problem is $T_{i\mathrm{p}}(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) = \Pi_i(\boldsymbol{y}_i, \boldsymbol{y}_{-i})$, as defined in Equation (15). For profit maximization, Constraint (P1-c) is not needed. If the operator chooses r, then we assume that it would be constrained by a profit target $\bar{\Pi}_i$. This is reasonable because, given an unlimited amount of money, one can always monopolize a market by flooding it with an infinite number of bikes. In this case, the operator's decision problem has an objective function $T_{i\mathrm{r}}(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) = Q_i(\boldsymbol{y}_i, \boldsymbol{y}_{-i})$ and Constraint (P1-c) that reads $\Pi_i(\boldsymbol{y}_i, \boldsymbol{y}_{-i}) - \bar{\Pi}_i \geq 0$.

In most hypotheticals, we consider a duopoly game (i.e., $\mathbb{I} = \{1, 2\}$) to avoid unnecessary complications. A sensitivity analysis will be performed to examine how the number of operators affects the outcome of the game. In our hypothetical duopoly, Operator 1 is the real operator whose data are used to build the case study (i.e., its supply of bikes amounts to about 40% of the total supply in the market), and Operator 2 is a virtual operator that supplies 60% of bikes by lumping together all other DLB operations in the city.

7. Numerical Experiments

We begin by examining various subgame perfect Nash equilibria, each corresponding to a specific action profile. The results are compared with a DLB monopoly to gain insights on the impact of competition. Section 7.2 analyzes the likely outcomes of a duopoly competition game, and Section 7.3 explores how various regulatory policies may influence them. Section 7.4 extends the analysis to the asymmetric case in which operators are heterogeneous. The results of sensitivity analyses are reported in Section 7.5.

7.1. Subgame Perfect Nash Equilibrium

In this section, we focus on the results of the lower-level game and assume that the operators in the duopoly always take the same action. Two scenarios are considered: profit maximization (DU-p in short) and ridership maximization without running a deficit (DU-r in short). The subgame perfect equilibria are compared with the status quo (i.e., as observed in the real data), defined by the following tactic decisions for the two operators: $f_1 = f_2 =$ ¥0.416/km, $B_1 = 248{,}894$, $B_2 = 351{,}106$.

Table 1 compares the system performances in the three scenarios. The performance is measured by the overall DLB market metrics (e.g., social welfare, access time, and total ridership) and each operator's metrics (e.g., tactic decisions, market share, cost, and profit). In the former category, social welfare is the social cost savings as defined in Equation (16). An operator's profit is its revenue less acquisition and rebalancing costs; the percentages in parentheses in the last four rows of the table are relative to the operator's revenue. In DU-p and DU-r, the operators have the same performance because by default they are identical (in terms of both characteristics and action). To provide another benchmark, we also include in this section the results of a monopoly, taken directly from Zheng et al. (2023); see Table 2. This gives us two more scenarios: a profit-maximizing monopoly (MO-p in short) and a ridership-maximizing monopoly (MO-r in short). A quick inspection of Tables 1 and 2 reveals that the

Table 1. DLB System Performances: Duopoly vs. the Status Quo

Scenarios:	The status quo		DU-p	DU-r[a]
Access time (min)	1.92		2.16	2.06
Total ridership (trips/hour)	49,956		40,548	46,672
Average trip distance (km)	1.26		1.17	1.23
Social welfare (¥/hr)	46,930		56,180	57,122
operator:	Operator 1	Operator 2	Operator 1/2	Operator 1/2
Price (¥/km)	0.416	0.416	0.743	0.503
No. of bikes	248,894	351,106	126,786	178,769
Utilization ratio	1.46%	1.46%	2.59%	2.23%
Ridership (trips/hour)	20,718	29,238	20,274	23,336
Market share	41.5%	58.5%	50%	50%
Profit (¥/hour)	−7,876 (−73%)	−10,826 (−71%)	6,850 (39%)	0 (0%)
Acquisition cost (¥/hour)	14,187 (131%)	20,013 (131%)	7,227 (41%)	10,190 (70%)
Rebalancing cost (¥/hour)	4,559 (42%)	6,152 (39%)	3,492 (20%)	4,264 (30%)
Revenue[b] (¥/hour)	10,869 (100%)	15,339 (100%)	17,569 (100%)	14,454 (100%)

[a]Operators 1 and 2 are identical in Scenario DU-p (DU-r).
[b]Revenue the sum of profit, acquisition cost, and rebalancing cost.

duopoly scenarios resemble the status quo much better than the monopoly scenarios in nearly all metrics (especially price, fleet size, and utilization ratio). More details to follow on this difference.

In the status quo, the operator with a smaller market share suffers a higher rebalancing cost for each bike kilometer traveled (¥0.175 for Operator 1 versus ¥0.167 for Operator 2).[13] This means that a DLB operator could reduce its rebalancing cost by serving a larger market share. Meanwhile, the model predicts that all operators are operating at a loss in the status quo, which aligns with the real-world observations.[14] In DU-p, where both operators aim to maximize profit, the price is 80% higher than the status quo. This rate of increase, however, is much smaller than what is observed in a monopoly (300% in MO-p). Similarly, there is a milder change in the total fleet size, as well as in the ridership, in a duopoly than in a monopoly. The fleet size drops by 58% in DU-p compared with the status quo, whereas a 90% reduction occurs in MO-p. Also, the total ridership drops 19% and 48% in DU-p and MO-p, respectively. Clearly, the competition means that neither operator has as much power as a monopoly to swing the market with tactic decisions. As a result, they must settle for less aggressive tactics that seem to better reflect the reality (the status quo).

Table 2. DLB System Performance in a Monopolistic Market (Results from Zheng et al. 2023)

Scenarios	MO-p	MO-r
Price (¥/km)	1.222	0.161
No. of bikes	56,962	108,427
Access time (min)	2.64	2.43
Average trip distance (km)	1.11	1.36
Utilization ratio	7.01%	8.97%
Ridership (trips/hour)	25,988	51,532
Social welfare (¥/hr)	45,461	69,672
Profit (¥/hr)	29,020 (83%)	0 (0%)
Acquisition cost (¥/hour)	3,246 (9%)	6,180 (55%)
Rebalancing cost (¥/hour)	2,876 (8%)	5,102 (45%)
Revenue (¥/hour)	35,144 (100%)	11,282 (100%)

A closer look at the differences between DU-p and MO-p reveals the inefficiencies resulting from the competition, including a lower utilization ratio and reduced profits for both operators (indeed, the total profit earned by both operators in the duopoly still falls far behind the monopolistic profit). Yet social welfare is improved in the duopoly compared with the status quo, even if both operators aim to maximize profit. In contrast, allowing a monopoly in the market hurts social welfare. This suggests that the current situation in Chengdu remains far from socially optimal despite multiple rounds of government-imposed market regulations. We shall return to the impact of competition on social welfare in Section 7.5, where more than two operators are allowed in the market.

We next turn to another action: ridership maximization without deficit. In DU-r, the fare rises but the fleet size decreases compared with the status quo. The operators in the duopoly collectively maintain a much larger fleet than does the operator in MO-r, thanks to the competitive pressure. The extra burden explains why they must charge a considerably higher price (¥0.503/km versus ¥0.161/km) to avoid running a deficit. Taken together, the duopoly generates a total ridership of around 46,700 trips/hour, about 10% lower than the monopolistic market and 7% lower than the status quo. Furthermore, social welfare in DU-r is 18% lower than that achieved by the "benign" monopoly that strives to maximize DLB ridership. Although DU-r performs better than DU-p in most system performance metrics (e.g., ridership and social welfare), it is less efficient, as measured by utilization ratio and profit. Moreover, the relative differences made by the actions are much smaller with competition than those without.

7.2. Outcomes of a DLB Duopoly Competition

We next turn to the bilevel game where the operators may choose either action at the upper level. We solve the subgame perfect Nash equilibria in four scenarios (each defined by an action profile with a different combination of actions) and obtain the payoffs of each operator in each scenario. For example, in Figure 4, the upper-right cell reports the payoff vectors (profit and ridership) of both operators and social welfare when Operator 1 maximizes profit and Operator 2 chooses ridership maximization without deficit.

If Operator 1 chooses to maximize ridership, Operator 2 will be worse off in terms of both profit and ridership if it wishes to maximize profit. Specifically, Operator 2 will break even and retain a ridership of 23,336 trips/hour (a market share of 50%) by maximizing ridership. If, instead, Operator 2 switches to profit maximization, it will run a deficit of ¥-117/hour and has a considerably smaller market share (less than half of what can be achieved had it stuck to ridership maximization). Thus, ridership maximization is a Pareto-dominant action for Operator 2 in this case.

If Operator 1's action is to maximize profit, then both profit and ridership maximization are consistent actions for Operator 2. If the action is to maximize profit, then it will find that maximizing profit indeed is better than maximizing ridership—the former gives a profit of ¥6,850/hour and the latter ¥0/hour. If the action is to maximize ridership, then maximizing ridership leads to a ridership of 39,387 trips/hour, much higher than 20,274 trips/hour obtained when it also maximizes profit. In any case, unilaterally committing to profit maximization gives one's competitor the freedom to choose based on its own preference, a strong strategic position. If the competitor happens to prioritize ridership, then Operator 1 will suffer significant losses.

Figure 4. (Color online) System Performance in a Duopoly Bilevel Game with Nondeficit Constraint

Operator 1		Operator 2: $s_2 = \mathrm{p}$	Operator 2: $s_2 = \mathrm{r}$
	$s_1 = \mathrm{p}$	6850, 20274	-117, 9539
		6850, 20274	0, 39387
		56180	62868
	$s_1 = \mathrm{r}$	0, 39387	0, 23336
		-117, 9539	0, 23336
		62868	57122

Note. In each cell, the first and second rows report the payoff vectors for Operators 1 and 2, respectively, where the first/second element is its profit (¥/hour)/ridership (trips/hour), and the third row reports social welfare (¥/hour).

The above analysis indicates that no rational operator should unilaterally commit to profit maximization because doing so puts itself in a vulnerable position where it can lose both money and ridership. Consequently, although the market reaches the Nash equilibrium under weak preference (NEWP; defined in Section 3.1) when both operators opt for profit maximization, such a scenario is improbable in the real world. Moreover, if one's competitor seeks to maximize ridership, following suit becomes the Pareto-dominant action. As a result, both operators striving to maximize ridership is the only Nash equilibrium under strong preference (NESP; defined in Section 3.1), which produces a total ridership of about 46,700 trips/hour, evenly split between the two rivals. Of course, the two operators may cooperate and agree to an accord that forbids them from attempting to maximize ridership. This will allow them to make much more money (¥6,850/hour versus ¥0/hour) while only mildly depressing the total ridership (from about 46,700 trips/hour to 40,500 trips/hour, a reduction of 13%). However, such cooperation is hard to come by in an unregulated market.

Interestingly, the outcome under NESP is desirable from the point of view of social welfare. Whereas the operators may like the outcome in the upper-left cell in Figure 4—their inability to achieve it notwithstanding—society may prefer the outcome in the lower-right cell because it yields higher social welfare. We note, however, that this need not always be the case.

Suppose one of the operators is determined to dominate the market by heavily subsidizing the DLB operation. The willingness to run a large deficit will enable the operator to expand its fleet and cut the price, thereby threatening the competitor's market position. If the competitor has the resources to respond in kind, then the market may plunge into an intensive and destructive price war, which many Chinese mega cities witnessed in 2016–2017 (Yang 2020). This could lead to a massive oversupply of bikes at an excessively low price. In the end, because the overall market potential of biking is limited, much of the resources would be wasted, which is a detriment to social good. To illustrate this point, consider a case where both operators are willing to run a deficit of ¥20,000/hour. The results are reported in Figure 5. In this case, if Operator 1 maximizes ridership and Operator 2 maximizes profit (the lower-left cell), then Operator 2 will be squeezed out of the market (highlighted as NA in Figure 5), and the system is reduced to a monopoly. The action profile corresponding to the lower-right cell is still an NESP,[15] but now social welfare drops to ¥48,480/hour, the worst of all scenarios.

Figure 5. (Color online) System Performance in a Duopoly Bilevel Game with Maximum Loss of ¥20,000/hour

		Operator 2	
		$s_2 = \mathrm{p}$	$s_2 = \mathrm{r}$
Operator 1	$s_1 = \mathrm{p}$	6850, 20274 6850, 20274 56180	NA, NA -20000, 58059 68055
	$s_1 = \mathrm{r}$	-20000, 58059 NA, NA 68055	-20000, 29012 -20000, 29012 48480

7.3. Role of Regulation

Having revealed the negative consequences of an unregulated duopoly, we proceed to examine what a regulator might do to avoid them. We assume that (i) the regulator may cap either fleet size or fare, but not both; (ii) when either restriction is imposed, the operators are still free to compete with each other by setting the objective and making tactic decisions; and (iii) when an operator's objective is to maximize ridership, the regulator assumes that it is willing to subsidize rides as much as needed.

We first consider the fleet cap policy, which sets an upper bound on the number of bikes each operator is allowed to put in the market. Figure 6 reports the market's total ridership (left y-axis) and social welfare (right y-axis) when both operators seek to maximize profit (Figure 6(a)) or ridership (Figure 6(b)). Regardless of the objective, a larger fleet cap always leads to higher ridership, although the marginal returns diminish quickly after the cap exceeds 100K. On the other hand, social welfare first increases with the fleet cap and then begins to drop precipitously once reaching a certain threshold. It peaks in both cases when the fleet cap for each operator falls between 50K and 65K.

We then assume that the regulator sets the fleet cap at the value where social welfare is maximized in Figure 6(b) (61,803). Figure 7 reports the payoffs in a duopoly bilevel game. In this case, both actions, that is, p and r, are consistent no matter what the competitor's action is. To see this, let us assume that Operator 1 adopts p. If Operator 2 also adopts p, then both earn a profit of ¥10,274/hour from serving 19,516 rides; Operator 2's profit would drop to ¥-6,552/hour by switching to r, suggesting that p is consistent for Operator 2. In the above example, when Operator 2 switches from p to r, its ridership increases from 19,516 trips/hour to 39,273 trips/hour, although its profit drops, suggesting that action r is consistent as well. The reader can verify that if Operator 1 adopts r, then p and r are still consistent actions for Operator 2. Therefore, under the "optimal" fleet cap policy, all four action profiles in Figure 7 are NEWP and no NESP exists, implying that the market may settle at any of the four competition scenarios. Importantly, the social welfare in any of the four scenarios is significantly higher than what their counterparts achieved without the regulation (see Figure 5). The fleet cap also improves profitability. Take the case where both operators adopt p. Under the fleet cap, they each make ¥10,274/hour, nearly 50% higher than what they would make without the regulation. This makes sense because the cap prevents both operators from self-destructive fleet expansion.

We next turn to the fare cap regulation, which restricts how much the operators can charge the riders. Figure 8 reports the results when the operators have the same action. We can see that when the objective is to maximize ridership, the fare cap policy affects neither ridership nor social welfare (Figure 8(b)). This is because the competition pressure would force the operators to set the fare so low that the cap is never activated. Moreover, as discussed earlier, r will be a Pareto-dominant action because it is not subject to a budget constraint per our assumption. Thus, the fare cap policy is not an effective regulatory tool. In the unlikely scenario where both operators seek to

Figure 6. (Color online) Total Ridership and Social Welfare Achieved by Different Fleet Cap Policies When Both Operators Maximize Profit (a) or Ridership (b)

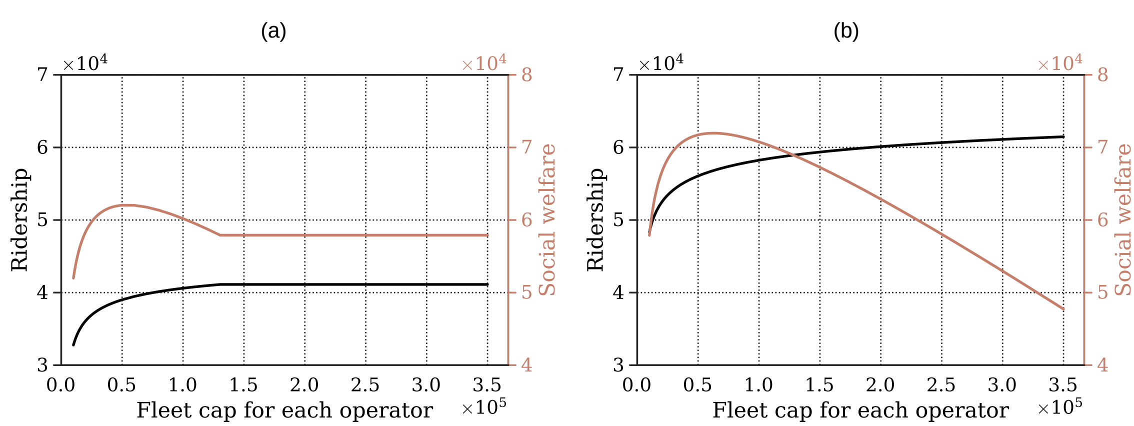


Figure 7. (Color online) System Performance in a Duopoly Bilevel Game with Fleet Cap = 61,803 for each Operator; No Budget Constraint in r

Operator 1 \ Operator 2	$s_2 = \mathrm{p}$	$s_2 = \mathrm{r}$
$s_1 = \mathrm{p}$	10274, 19516 10274, 19516 60037	5398, 12605 -6552, 39273 69480
$s_1 = \mathrm{r}$	-6552, 39273 5398, 12605 69480	-6176, 28056 -6176, 28056 69741

maximize profit, a price cap would make a difference. As shown in Figure 8(a), the price cap that attains the highest ridership and social welfare is around ¥0.2/km, which is significantly lower than the optimal fare cap in a monopolistic market with similar settings (about ¥0.4/km; see Zheng et al. 2023). The peak ridership and social welfare in a duopoly are also higher than those achieved in a monopolistic market with similar settings.

7.4. Asymmetry

Up to this point, the two operators in the duopoly are assumed to be identical. We now turn to an asymmetric duopoly that consists of two heterogeneous operators.[16] In Section 7.4.1, we consider the case where the two operators differ from each other in their ability to sustain losses. Section 7.4.2 examines how crucial operational characteristics, for example, bike acquisition cost and rebalancing efficiency, affect the system outcome.

7.4.1. Ability to Sustain Losses. Our goal is to show how the ability to sustain losses strengthens an operator's position in a duopoly. As noted before, when a lower profit target is allowed, r is a Pareto-dominant action. Thus, we assume that both operators maximize ridership. The operators are identical, except for their profit target. As a baseline, we set the profit target for Operators 1 and 2 as ¥-5,000/hour and ¥-10,000/hour, respectively. That is, Operator 2 is willing to lose ¥10,000/hour, whereas Operator 1 can afford only half of that loss. As reported in Table 3, Operator 2 acquires a fleet about 60% larger and sets the fare about 30% lower than its competitor. This is expected because Operator 2 outspends Operator 1 at a ratio of 2:1. As a result, Operator 2 captures about 66% of the DLB market and enjoys a significantly higher utilization rate (2.3% versus 1.9%). However, because of the lower price it charges, Operator 2's strong market position brings in a revenue only about 40% higher than the competitor. Moreover, the larger fleet also induces a much higher acquisition cost (61% more) and rebalancing cost (66% more). For each thousand ridership gained, Operator 2 endures a loss of ¥290/hour, whereas Operator 1 "pays" ¥281/hour. Thus, attempting to achieve dominance by overspending may not scale well because the more one spends, the less gain one can achieve at the margin. To further explore this issue, we change Operator 2's maximum loss while keeping Operator 1's fixed at ¥5,000/hour. Figure 9 reports the optimal tactic decisions and main performance metrics of Operator 2, as its maximum loss varies from ¥5,000/hour to ¥23,000/hour. As expected, the greater the deficit the operator can tolerate, the larger the fleet and the lower the fare it can afford. However, the diminishing returns to investment are evident; the curves for both the fleet size and the fare gradually level off as the profit target stretches further into the negative territory (see Figure 9(a)). Moreover, Figure 9(b) indicates that a fourfold increase in Operator 2's operating losses (from ¥5,000/hour to ¥20,000/hour) barely doubles its ridership.

7.4.2. Operational Characteristics. Two operational features are examined in this section: unit bike

Figure 8. (Color online) Total Ridership and Social Welfare Achieved by Different Fare Cap Policies When Both Operators Maximize Profit (a) or Ridership (b)

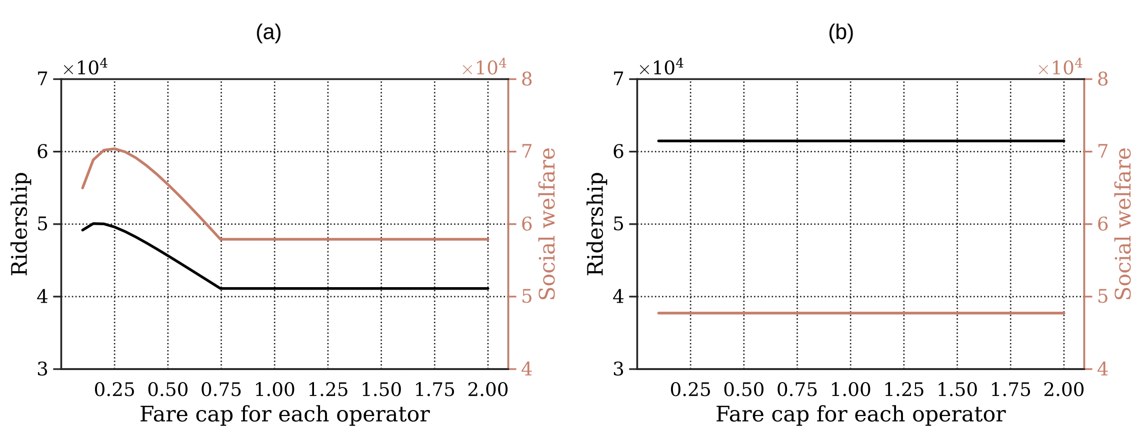


Table 3. DLB System Performances When Two Ridership-Maximizing Operators Have Different Tolerance Levels for Monetary Loss

Access time (min)	2.01	
Total ridership (trips/hour)	52,220	
Social welfare (¥/hour)	56,828	
operator:	Operator 1	Operator 2
Maximum loss (¥/hour)	−5,000	−10,000
Price (¥/km)	0.357	0.256
No. of bikes	169,245	271,755
Utilization ratio	1.90%	2.30%
Ridership (trips/hour)	17,743	34,477
Profit (¥/hour)	−5,000 (−60%)	−10,000 (−86%)
Acquisition cost (¥/hour)	9,647 (116%)	15,490 (134%)
Rebalancing cost (¥/hour)	3,649 (44%)	6,051 (52%)
Revenue (¥/hour)	8,296 (100%)	11,541 (100%)

acquisition cost and rebalancing efficiency. The former includes the cost of purchasing and maintaining a shared bike amortized over its life cycle (i.e., the parameter β_0), whereas the latter is measured by the average number of rebalancing trips needed to maintain a stable level of service per each bike trip (i.e., the parameter α). A smaller α corresponds to a greater rebalancing efficiency, hence a lower rebalancing cost. For simplicity, the two operators are assumed to have the same upper-level action in this section.

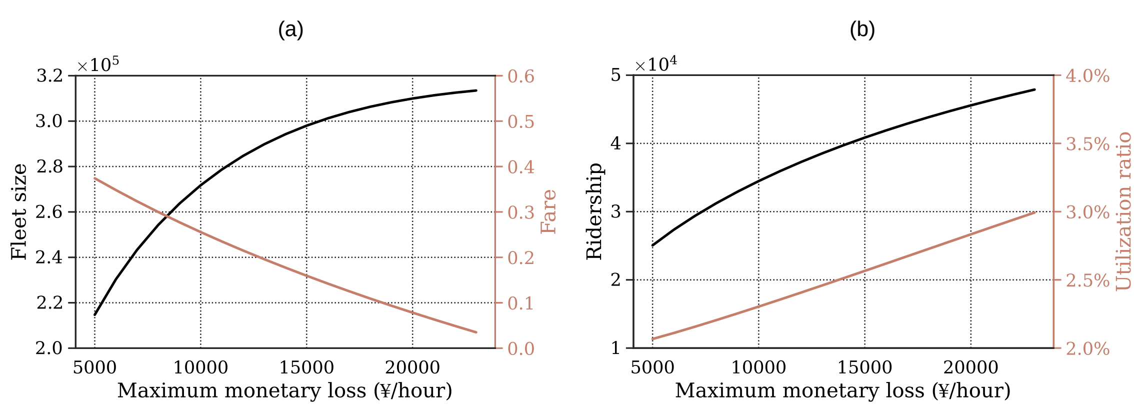
The first experiment considers profit-seeking operators and consists of three scenarios: (i) the base scenario where the two operators are identical; (ii) the acquisition cost of Operator 1 is half of Operator 2's; and (iii) the rebalancing efficiency of Operator 1 doubles that of Operator 2. Figure 10 reports four performance metrics in the three scenarios: profit, ridership, fare, and fleet size.

As expected, in Scenario (i), all performance metrics of the operators are the same. When Operator 1's acquisition cost is reduced by half (Scenario (ii)), it raises its fare from ¥0.743/km to ¥0.826/km (see Figure 10(c)) and its fleet size from 126,786 to 213,765 (see Figure 10(d)). Accordingly, Operator 1's ridership increases by 22% (Figure 10(b)) and its profit nearly doubles, rising from ¥6,850/hour to ¥12,966/hour (see Figure 10(a)). In contrast, under competitive pressure, Operator 2 is forced to reduce its fleet size to control the cost and to lower its fare to compensate for the degraded level of service. As a result, although the overall DLB market produces more profit and serves more trips, Operator 2 is much worse off compared with the benchmark. A similar pattern emerges in Scenario (iii). Equipped with better rebalancing efficiency, Operator 1 rakes in greater profit and ridership by raising the price and expanding the fleet. However, the relative advantage enjoyed by Operator 1 is noticeably smaller than that in Scenario (ii).

Figure 11 reports the results of the second experiment where both operators maximize ridership without running a deficit. We similarly construct three scenarios: (iv) the base scenario where the two operators are identical; (v) the acquisition cost of Operator 1 is 1% lower than that of Operator 2; and (vi) the rebalancing efficiency of Operator 1 is 1% better than that of Operator 2. In all scenarios, the profit earned by either operator is zero because that is the profit target. We can see that Operator 1 enjoys a compelling competitive advantage across the board over Operator 2, even with a seemingly minute improvement in operational efficiency. With a slightly lower acquisition cost, for example, Operator 1 achieves a market share 65% higher than its competitor (see Figure 11(b)). The effect of rebalancing cost is less dramatic but still substantially stronger than that revealed in the first experiment: an 8% gain in the market share for a 1% cost reduction.

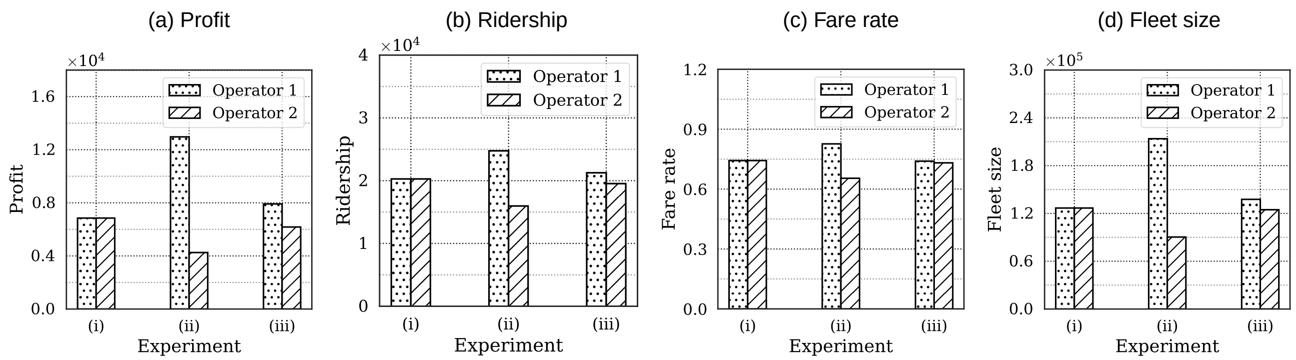
Although many of the above results were expected, the last finding is surprising. It suggests that the operational characteristics matter much more when the

Figure 9. (Color online) Tactic Decisions and Performance Metrics of Operator 2 Against Maximum Loss (Both Operators Maximize Ridership and Operator 1's Maximum Loss is ¥5,000/hour)



Note. (a) Fleet size and fare; (b) ridership and bike utilization rate.

Figure 10. System Performance Metrics in an Asymmetric Duopoly with Profit-Seeking Operators



Note. Three tested scenarios are as follows: (i) operators are identical; (ii) the acquisition cost of Operator 1 is half of that of Operator 2; and (iii) the rebalancing efficiency of Operator 1 is twice that of Operator 2.

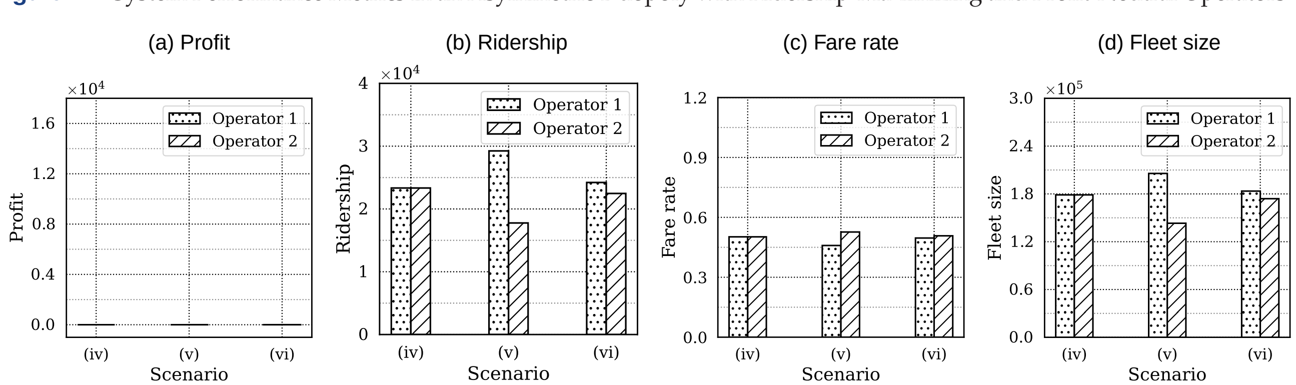
operators seek market dominance and that a small operational advantage could swing the market position wildly. On the bright side, such a hyperintense environment may motivate the operators to focus on the betterment of their operations (e.g., bike design and manufacturing and rebalancing strategies). However, for regulators, the situation poses a challenge because it may lead to instability and unpredictability.

7.5. Sensitivity Analysis

We first test the sensitivity of Scenarios DU-p and DU-r (defined in Section 7.1) to the competition factor k introduced in Equation (7). Recall that the value of k measures how sensitive the demand distribution between the two operators is to fare discrepancy. Figure 12 reports the results corresponding to k being set to 20%, 100%, and 500% of the default value (i.e., 23,200 trips/¥). When travelers are highly sensitive to price (i.e., large k), profit-maximizing operators are forced to lower fares to protect their market share, which depresses the total profit achievable (see Figure 12(a)). The same mechanism is at work when the operators seek to maximize ridership. That is, high price sensitivity drives down price through competition, which then attracts more riders (see Figure 12(b)).

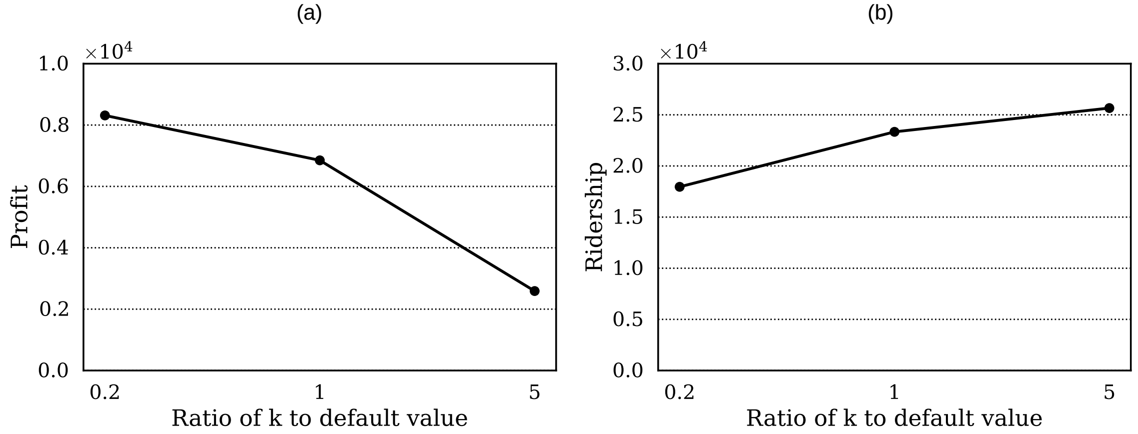
We next consider how the number of operators in a DLB market might affect its performance, especially the total ridership and profit for the operators. We consider a relatively mature market in which all operators seek to maximize profit. Figure 13(a) reports ridership and social welfare corresponding to different numbers of operators. As the number of operators increases, both ridership and social welfare first rise sharply, indicating that the system benefits from competition. However, the incremental advantages of introducing an additional operator diminish rapidly, and the benefit derived from market expansion becomes negligible once the market reaches a total of four operators. In the case of social welfare, the additional operator actually becomes counterproductive once the market is saturated by competition. Thus, one may argue that, in this case, there is an optimal number of operators that a city may wish to allow to enter its market. Of course, a regulator may also cap the fleet size (or fare) for each operator, which may affect the trend observed here. In

Figure 11. System Performance Metrics in an Asymmetric Duopoly with Ridership-Maximizing and Profit-Neutral Operators



Note. Three tested scenarios are as follows: (iv) the base scenario where the two operators are identical; (v) the acquisition cost of Operator 1 is 1% lower than that of Operator 2; and (vi) the rebalancing efficiency of Operator 1 is 1% better than that of Operator 2.

Figure 12. Sensitivity of the System Performance to the Competition Factor (k) in a Duopoly Where Both Operators Maximize Profit (a) or Ridership Without a Deficit (b)



the interest of space, we leave a more in-depth study to future research.

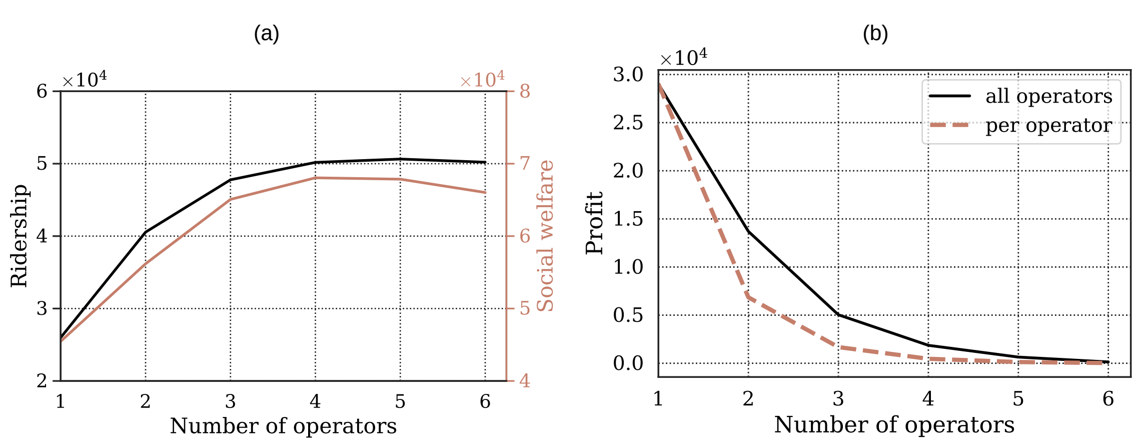
From the perspective of the operators, the competition is a curse. Figure 13(b) shows that the total profit shared by all operators declines precipitously with the number of operators in the market. With four operators, which are "socially optimal," each operator's profit is less than 2% of that in a monopolistic market. Thus, if an operator's ultimate target is to make money, the only way to achieve it is by driving most if not all rivals out of the market. This logic might explain why the major DLB startups in China were committed to the pipedream of "winner-take-all."

8. Conclusions

In this paper, we modeled the inter-operator competition in a dockless bike-sharing (DLB) market as a bilevel game. Each DLB operator is first committed to an action tied to a specific objective, such as maximizing profit. Then, the operators play a lower-level game to reach a subgame perfect Nash equilibrium by making tactic decisions (e.g., pricing and fleet sizing). We defined the Nash equilibrium under weak preference (NEWP) and under strong preference (NESP) to characterize the likely outcomes of the bilevel game and formulated the demand-supply equilibrium of a DLB market that accounts for key DLB operational features and travelers' mode choice. Our analysis of the demand-supply equilibrium confirmed the intuition that an operator can always expect to gain market share by putting more bikes on streets. We also found a free-rider effect. That is, lowering the price could be a double-edged sword because the induced demand could benefit not only the operator that does cut the price but also those that do not.

Using the oligopoly competition game model calibrated with empirical data from Chengdu, China, we sought to explain the instability of an unregulated

Figure 13. (Color online) System Performance Metrics in an Oligopoly Market with Different Numbers of Profit-Maximizing Operators



Note. (a) Total ridership and social welfare; (b) profit.

DLB market that underlay the spectacular rise and quick fall of the industry in China. We showed that when an operator is determined to dominate the market at any cost, other operators have no choice but to follow suit if they wish to stay in the game. This dilemma creates a race to the bottom in the sense that everyone is basically competing to lose more money. That is not the end of the story. Remarkably, if everyone attempts to maximize ridership, even a slight improvement in the acquisition cost of bikes can swing the outcome wildly, an ominous sign of high volatility. Moreover, even if every operator agrees to focus on making money rather than seeking dominance, the profitability still plunges quickly with the number of players. Thus, left to its own devices, the DLB market may be doomed to implode under competitive pressure, as we have witnessed in China.

Having thus demonstrated that the DLB industry creates a competition for losers, we then explore the possibility of regulating it. We conducted experiments with two regulatory policies: a fleet cap and a price control. We found that a fleet cap, if implemented properly, can be effective for not only preventing market failure but also improving social welfare. In contrast, price control is not very useful simply because the real problem was never the high price but too many bikes. Moreover, for the benefit of society, a city should not have more than a handful of operators. Our results suggest that social welfare peaks when the number of operators is four in Chengdu, although at that point, profitability has already sunk so low that it is doubtful that any private investors would be impressed. Encouragingly, Chengdu's existing regulatory practices already reflect the two recommendations—implementing fleet caps and restricting the number of operators—albeit with different thresholds.

Our model was calibrated with a cross-sectional trip data set collected by a single operator, which does not capture any market dynamics associated with competition. Therefore, to a certain extent, the predictions given by the model are but theoretical speculations. It does correctly predict the collapse of an unregulated market, but one cannot know for sure whether the mechanisms embedded in our model agree with the real ones. A future study that strives to fix this shortcoming will need to recalibrate the model with time-series data of multiple operators. Another promising direction is to extend the current model to account for operator's dynamic entry/exit decisions. This would involve analyzing how potential entrants make their decisions in the presence of an incumbent's advantages or disadvantages. Incorporating such dynamics would require a more complex framework—such as a repeated game with imperfect information—to capture how the market evolves with new entrants and how travelers shift among different modes and DLB service operators. Although this extension is beyond the scope of this study, it represents a natural and valuable next step toward a more comprehensive understanding of competition in DLB markets.

The present study assumes that DLB operators are committed to maximizing either profit or ridership. In reality, their objectives may encompass both profitability and market share; a "public" operator may prioritize social welfare over these measures. More likely than not, the business decisions of the operators are driven by a combination of all the conflicting objectives. As a result, the action at the upper level is not to choose one from several targets (a discrete choice) but how to best weigh them (a continuous one). A future study may extend the current game-theoretic framework to allow such a general objective. Lastly, our demand-supply model is tailored to dockless systems, where competition depends primarily on idle bike availability and pricing, allowing us to abstract away spatial heterogeneity. This is a reasonable simplification for dockless systems, where bikes can be picked up or dropped off anywhere. In contrast, docked systems require decisions on the spatial placement of docks in addition to fleet size and price. In such settings, Assumption 3 (joint uniqueness of bike locations) would no longer hold, and the current modeling framework would have to be extended to account for spatial competition.

Endnotes

[1] The point is that a market with hypercompetition is no place to make money. Thiel's favorite example is the restaurant industry, which features intense competition, low entry barriers, and a high failure rate.

[2] https://sichuan.scol.com.cn/cddt/202005/57817949.html, in Chinese, accessed on August 8, 2024.

[3] Although multiple bike-sharing companies are still operating in China, all of them are suffering from significant financial losses. For more details, please see https://www.bjnews.com.cn/detail/1722437196129044.html, in Chinese, accessed on August 8, 2024.

[4] In contrast, under standard weak dominance, if multiple actions weakly dominate all others besides themselves, then they must yield an identical payoff, and the player is indifferent to them.

[5] Here, operator i must select its tactics that maximize the objective associated with its action at the upper level. Because tactics such as pricing and fleet sizing can be easily observed in the market, they are assumed to be public information for all operators.

[6] Public transit, taxi, and ride-hail are grouped into a composite mode because they share similar operating speeds compared to walking and biking. This setting aligns with our assumption that trip length is the primary factor driving mode choice.

[7] In this study, the automobile mode is treated as exogenous to the DLB market, and its associated parameters are assumed to remain constant throughout the analysis. Modeling the dynamic interaction between DLB services and other transportation modes is left for future work.

[8] DLB fare payments are offset by operators' revenue and therefore do not contribute to net social cost.

[9] Setting either the price or the fleet size to zero is evidently a suboptimal decision.

[10] The detailed procedure is included in Online Appendix H.

[11] Although the data for our case study were collected during the COVID-19 pandemic, its impact on Chengdu was minimal at the time. Fewer than 25 cases in total were confirmed during the entire data collection period, which is negligible relative to the city's population (see https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Sichuan\#March_2020, in Chinese, accessed on May 20, 2025), and the city was not under any lockdown order or travel restrictions (see https://www.sc.gov.cn/10462/c103046/2020/3/25/e345d4686b8b443399373ba6823bceb6.shtml, in Chinese, accessed on May 20, 2025). Therefore, we believe that the data reasonably reflect normal travel behavior. Even if some pandemic-related behavioral deviations were present, they would have had limited impact on our main findings, which focus not on estimating the exact market share of DLB services but on analyzing how competition among DLB operators influences overall market profitability.

[12] This strategy has been observed during the early stages of DLB competition in major Chinese cities, where operators focused on competing for local markets before seeking sustainable business models. (see https://www.adlittle.com/sites/default/files/viewpoints/adl_bike_sharing.pdf, accessed on May 20, 2025).

[13] At ¥0.17/km, the rebalancing cost is about ¥0.3/bike per day, very close to the estimation reported in https://m.36kr.com/p/2150789257283847, in Chinese, accessed on August 8, 2024.

[14] See, for example, https://www.scmp.com/tech/start-ups/article/3114932/rise-and-fall-mobike-and-ofo-chinas-bike-sharing-twin-stars, accessed on May 20, 2025.

[15] When Operator 2 is forced out by choosing action p, both its ridership and profit are reduced to zero. If it chooses r, it will lose ¥20,000. However, because this deficit must be subsidized externally (e.g., by investors), Operator 2 is assumed to be indifferent between (i) losing ¥20,000 when choosing r and (ii) earning a zero profit when choosing p. Thus, the lower-right cell corresponds to an NESP.

[16] The asymmetry experiments are designed to analyze the market outcomes of a hypothetical duopoly market rather than to explain why, in practice, some DLB companies succeed whereas others fail. Although the latter is an important and interesting question, addressing it would require modeling, among other things, dynamic competition over time and entry/exit decisions, which is beyond the scope of the current study and hence, left for future research.

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