Cluster-based Segmentation
基于聚类的图像分割

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Contents

Clustering Analysis

Clustering is unsupervised classification: no predefined classes.

Cluster: a collection of data objects

- Similar to one another within the same cluster
- Dissimilar to the objects in other clusters

Image Segmentation with Clustering

Idea:

Cluster similar pixel features together.

How to segment images by clustering?

Image Segmentation with Clustering

Image $\Longrightarrow$ Feature space

Image Segmentation with Clustering

Image $\Longrightarrow$ Feature space

Feature space:(R,G,B),(R,G,B,X,Y),(L,U,V)$\cdots$

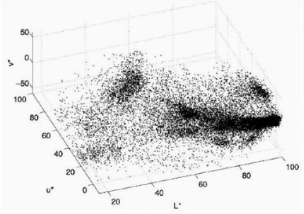
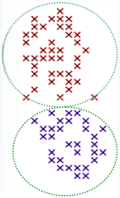


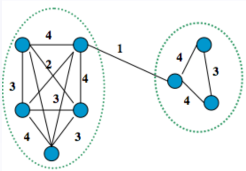
Image Segmentation with Clustering

- Vector Clustering



Each point has a vector.

- Graph Clustering



Each vertex is connected to others by edges.

Image Segmentation with Clustering

Image $\Longrightarrow$ Feature space

Feature space:(R,G,B),(R,G,B,X,Y),(L,U,V)$\cdots$

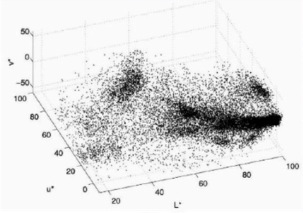


Image Segmentation with Clustering

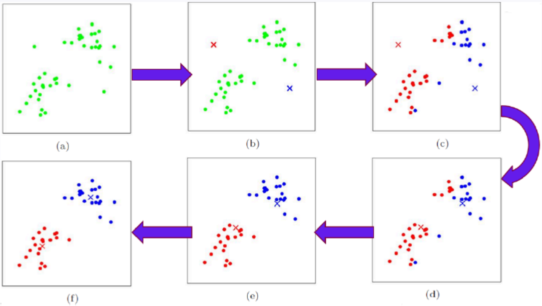
Techniques:

- K-means
- Mean Shift

K-means

Idea:

1. Randomly initialize the K cluster centers.
2. For each point, find the closest cluster centers. Put the point into the cluster.
3. Change the cluster centers.



K-means

Algorithm:

1. Choose randomly K-means $m_1, \ldots, m_k$.
2. For each vector x_i compute $D(x_i, m_k(ic)), k = 1, \ldots, K$ and assign x_i to the cluster C_j with nearest mean.
3. Update the means to get $m_1(ic), \ldots, m_K(ic)$.

$$m_i^{(t+1)} = \frac{1}{|S_i^{(t)}|} \sum_{x_j \in S_i^{(t)}} x_j$$

4. Repeat steps 2 and 3 until $C_k(ic) = C_k(ic+1)$ for all k.

K-means

Pros:

- Simple and fast
- Converges to a local minimum of the error function

Cons:

- Sensitive to initialization
- Need to pick K

K-means++

Algorithm:

1. Take one center c_1, chosen uniformly at random from X.
2. Take a new center c_i, choosing $x \in X$ with probability $\frac{D(x)^2}{\sum_{x \in X} D(x)^2}$
3. Repeat Step 1, until we have taken k centers altogether.

$D(x)$ denote the shortest distance from a data point to the closest center.

K-means

Pros:

- Simple and fast
- Converges to a local minimum of the error function

Cons:

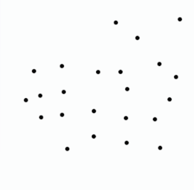
- Sensitive to initialization
- Need to pick K

Mean Shift[1]

An advanced and versatile technique for clustering-based segmentation.

Idea:

- Find the clustering center.



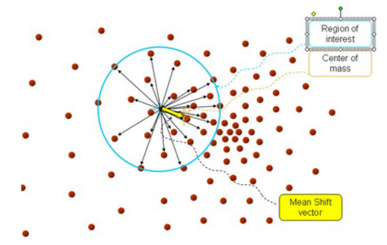
[1] D. Comaniciu and P. Meer, “Mean Shift: A Robust Approach toward Feature Space Analysis”, PAMI, 2002.

What and How?

What and How?

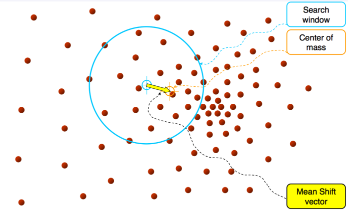
Center is maximum points of probability density function and gradient direction.

Mean Shift Vector

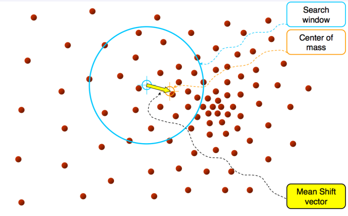


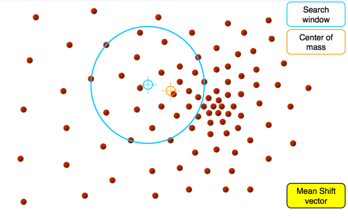
$$M_h = \frac{1}{K} \sum_{x_i S_k} (x_i - x)$$

What is Mean Shift



What is Mean Shift





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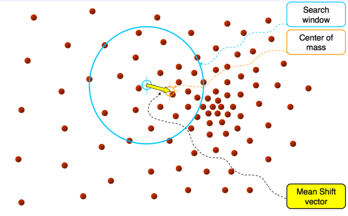

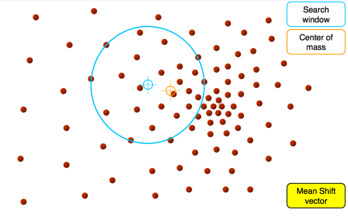

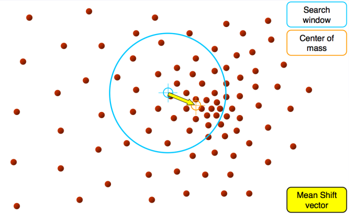

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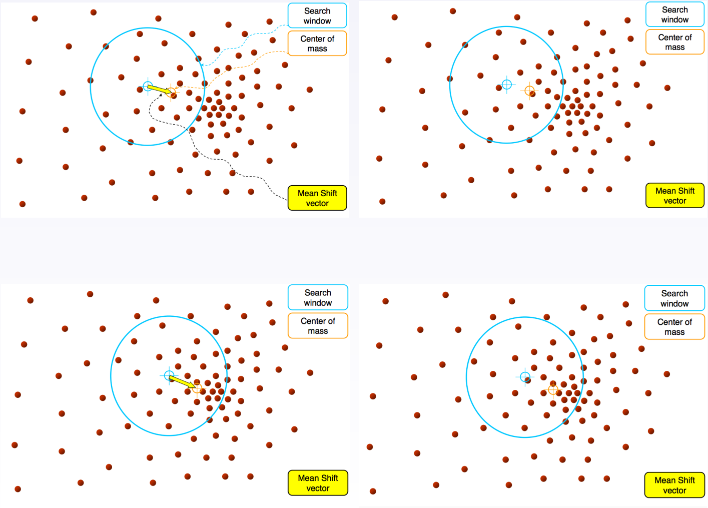

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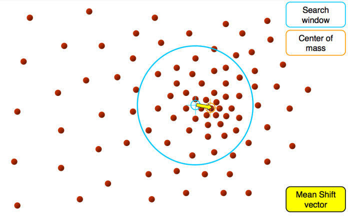

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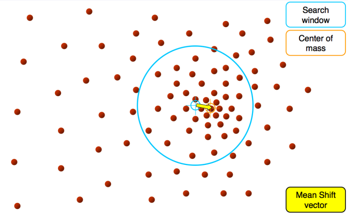

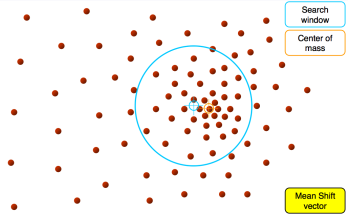

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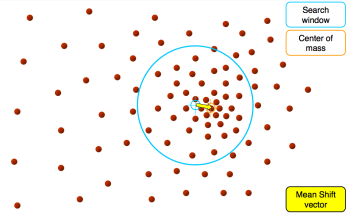

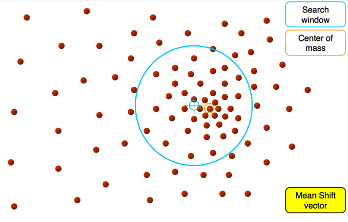

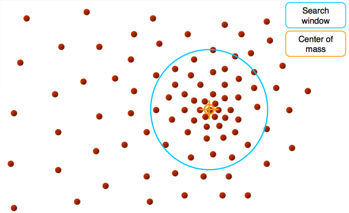

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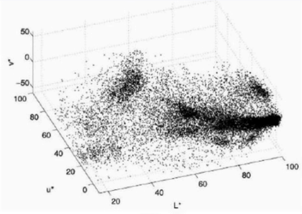
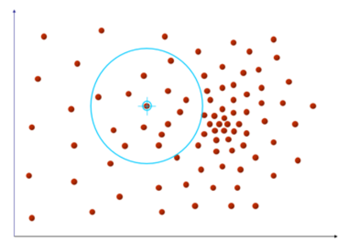


Density Estimation



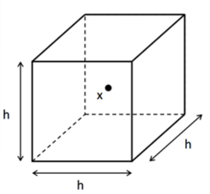
$$P = \frac{N_k}{N}$$

$$f(x) = \frac{P}{S} = \frac{N_k}{NS}$$

Kernel density estimation:

Let $(x_1, x_2, ..., x_n)$ be an independent and identically distributed sample drawn from some distribution with an unknown density $f(x)$. Its kernel density estimator is

$$\hat{f(x)}_{h,k} = \frac{1}{Nh^d} \sum_{n=1}^{N} k(\frac{x_n - x}{h})$$



Kernel function:

$$k(u) = \begin{cases} 1 & |u_i| \leqslant \frac{1}{2}, i = 1, \ldots, D \\ 0 & otherwise \end{cases}$$

Kernel density estimation

$$\hat{f}_{h,K}(x) = \frac{1}{Nh^d}\sum_{n=1}^{N} k(||\frac{x-x_n}{h}||^2)$$

Density gradient:

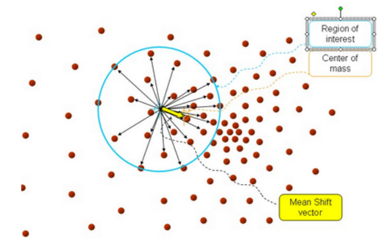
$$\hat{\nabla} f_{h,K}(x) = \frac{2}{Nh^{d+2}}\sum_{n=1}^{N}(x_n - x)[-k'(||\frac{x-x_n}{h}||^2)]$$

$$g(x) = -k'(x)$$

Kernel density estimation

$$
\begin{aligned}
\hat{\nabla} f_{h,K}(x) &= \frac{2}{Nh^{d+2}} \sum_{n=1}^{N} (x_n - x)[g(||\frac{x - x_n}{h}||^2)] \\
&= \underbrace{\frac{2}{h^2}}_{C} \underbrace{[\frac{1}{Nh^d} \sum_{n=1}^{N} g(||\frac{x_n - x}{h}||^2)]}_{f_{h,\hat{G}}(x)} \underbrace{[\frac{\sum_{n=1}^{N} x_n g(||\frac{x_n - x}{h}||^2)}{\sum_{n=1}^{N} g(||\frac{x_n - x}{h}||^2)} - x]}_{m_{h,G}(x)}
\end{aligned}
$$

Kernel density estimation

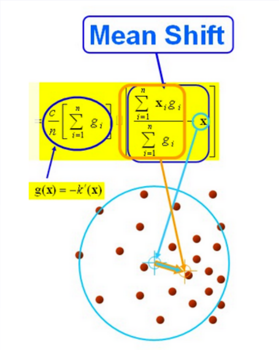


$$M_h = \frac{1}{K} \sum_{x_i S_k} (x_i - x)$$

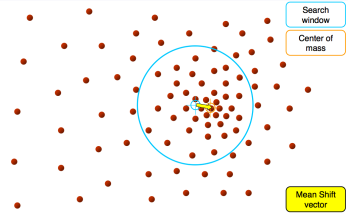
Kernel density estimation

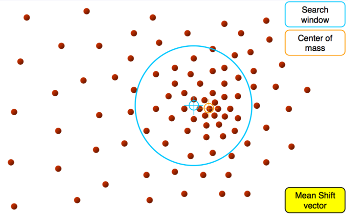
$$m_{h,G}(x) = \frac{\sum_{n=1}^{N} x_n g(||\frac{x_n - x}{h}||^2)}{\sum_{n=1}^{N} g(||\frac{x_n - x}{h}||^2)} - x$$

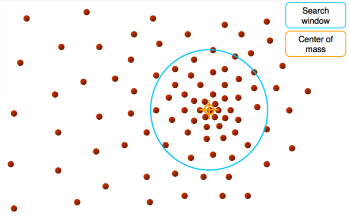
$$m_{h,G}(x) = 0$$



Kernel density estimation







Kernel density estimation

Clustering center:

$$y_{i,k+1} = \frac{\sum_{n=1}^{N} x_n g(||\frac{x_n - x}{h}||^2)}{\sum_{n=1}^{N} g(||\frac{x_n - x}{h}||^2)}$$

Mean Shift

Algorithm:

1. Find features(color, gradients, texture, etc).
2. Initialize windows at individual pixel locations.
3. Compute $y_{i,k+1}$ until convergence, $y_{i,k} = y_{i,k+1}$.
4. Assign $z_i = (x_i, y_{i,k})$.

Thanks!