Discrete Fourier transform (DFT)

$$F(u,v) = R(u,v) + jI(u,v) = |F(u,v)|e^{j\phi(u,v)}$$

Amplitude spectrum : $|F(u,v)| = [R(u,v)^2 + I(u,v)^2]^{1/2}$

Phase spectrum : $\phi(u,v) = \tan^{-1}[\frac{I(u,v)}{R(u,v)}]$

atan(I,R)

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

atan2(I,R)

$(-\pi,\pi]$

Note: The cycle of 2D-DFT is 2π , so we must use atan2.

atan (I[u][v],R[u][v]) * 180/M_PI

Wrong!

↓

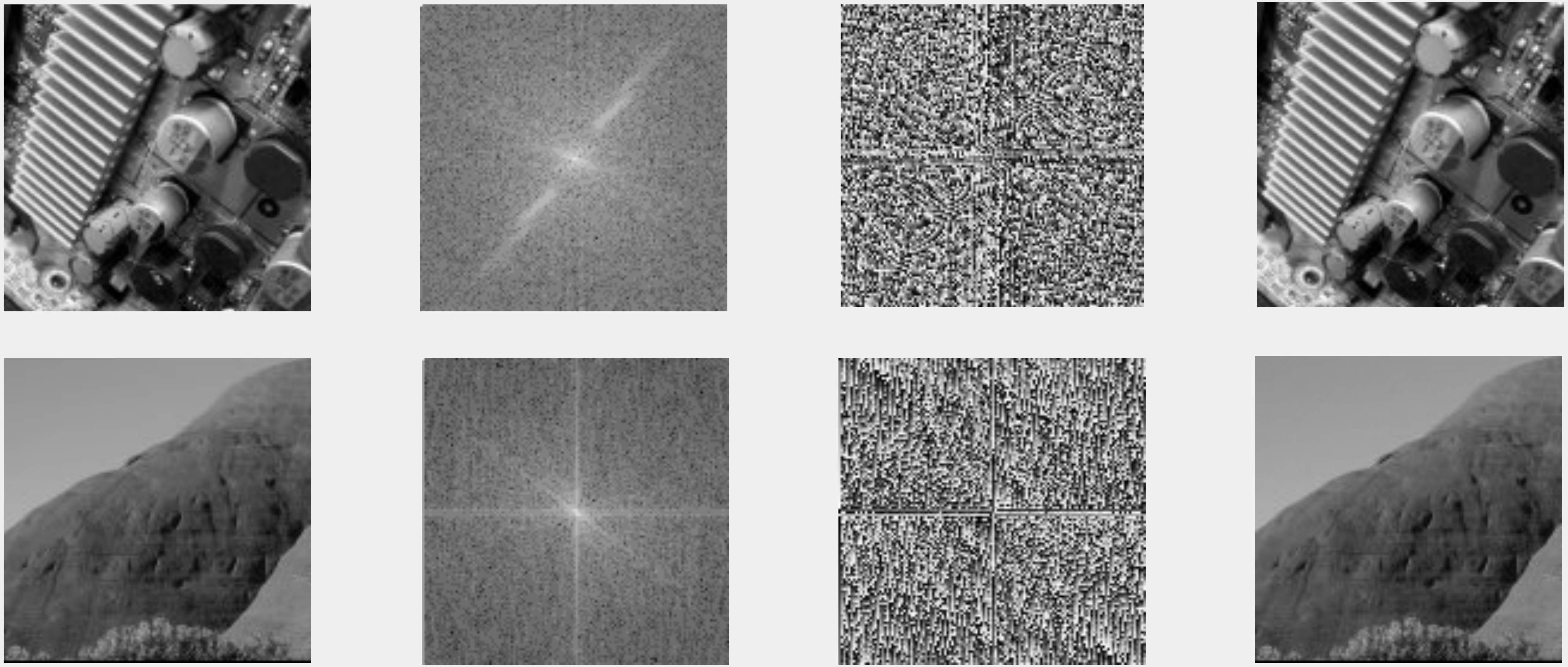
atan2(I[u][v],R[u][v])

Right~

```
void myDFT(Mat &src, int M, int N, double** R, double** I)
{

    int x, y, u, v;
    double grayValue;
    double ang;

    for (u = 0; u<M; u++){
        for (v = 0; v<N; v++){
            R[u][v] = 0.0;
            I[u][v] = 0.0;
            for (x = 0; x<M; x++){
                for (y = 0; y<N; y++){
                    grayValue = src.at<uchar>(x, y);
                    ang = 2 * M_PI*((double)x*u / (double)M + (double)y*v / (double)N);
                    R[u][v] += cos(ang)*grayValue;
                    I[u][v] += -sin(ang)*grayValue;
                }
            }
        }
    }

}
```

```
void myIDFT(int M, int N, double** r, double** i,  double** R, double** I)
{
    int x, y, u, v;
    double phase;
    double ang;

    for(x=0;x<M;x++){
        for(y=0;y<N;y++){
            r[x][y]=0.0;
            i[x][y]=0.0;
            for(u=0;u<M;u++){
                for(v=0;v<N;v++){
                    ang=2*M_PI*((double)x*u/(double)M+(double)y*v/(double)N);
                    phase=atan2(I[u][v],R[u][v]);
                    r[x][y]+=sqrt(R[u][v]  * R[u][v]  + I[u][v]  * I[u][v])*cos(phase+ang);
                    i[x][y]+=sqrt(R[u][v]  * R[u][v]  + I[u][v]  * I[u][v])*sin(phase+ang);

                }
            }
        }
    }
}
```

My Experimental Results:

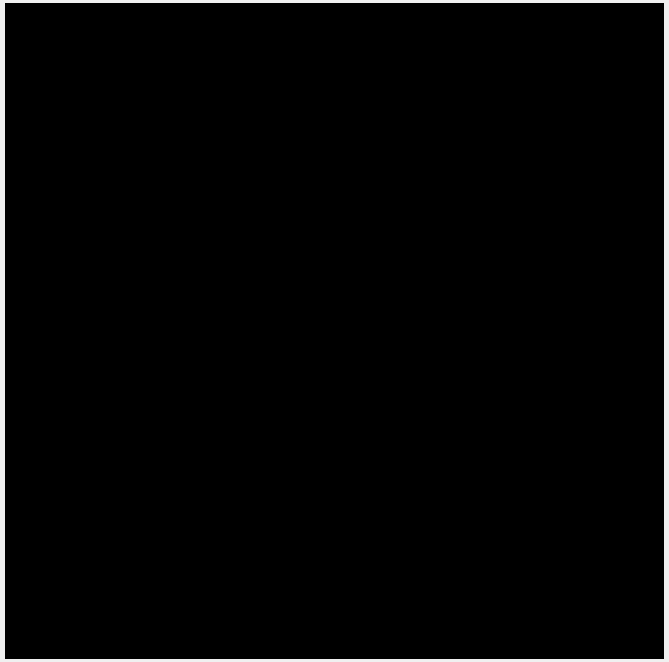


Original image　　Amplitude map　　Phase map　　IDFT result

The Physical Meaning of Amplitude Map and Phase Map:

$$F(u,v) = |F(u,v)|e^{j\phi(u,v)}$$

Original image



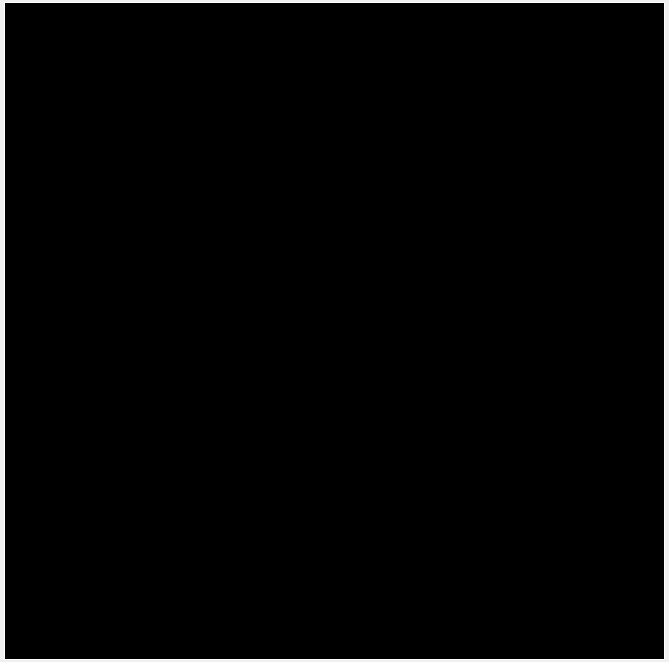
IDFT reconstructed result using only the amplitude map (set the exponential term to 1)

IDFT reconstructed result using only the phase map (with $|F(u,v)|$=1)

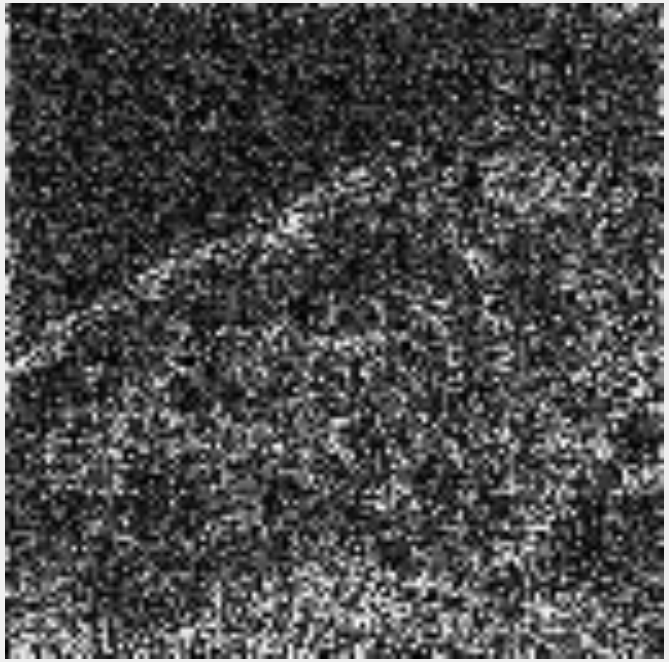
The Physical Meaning of Amplitude Map and Phase Map:

$$F(u,v) = |F(u,v)|e^{j\phi(u,v)}$$

Original image

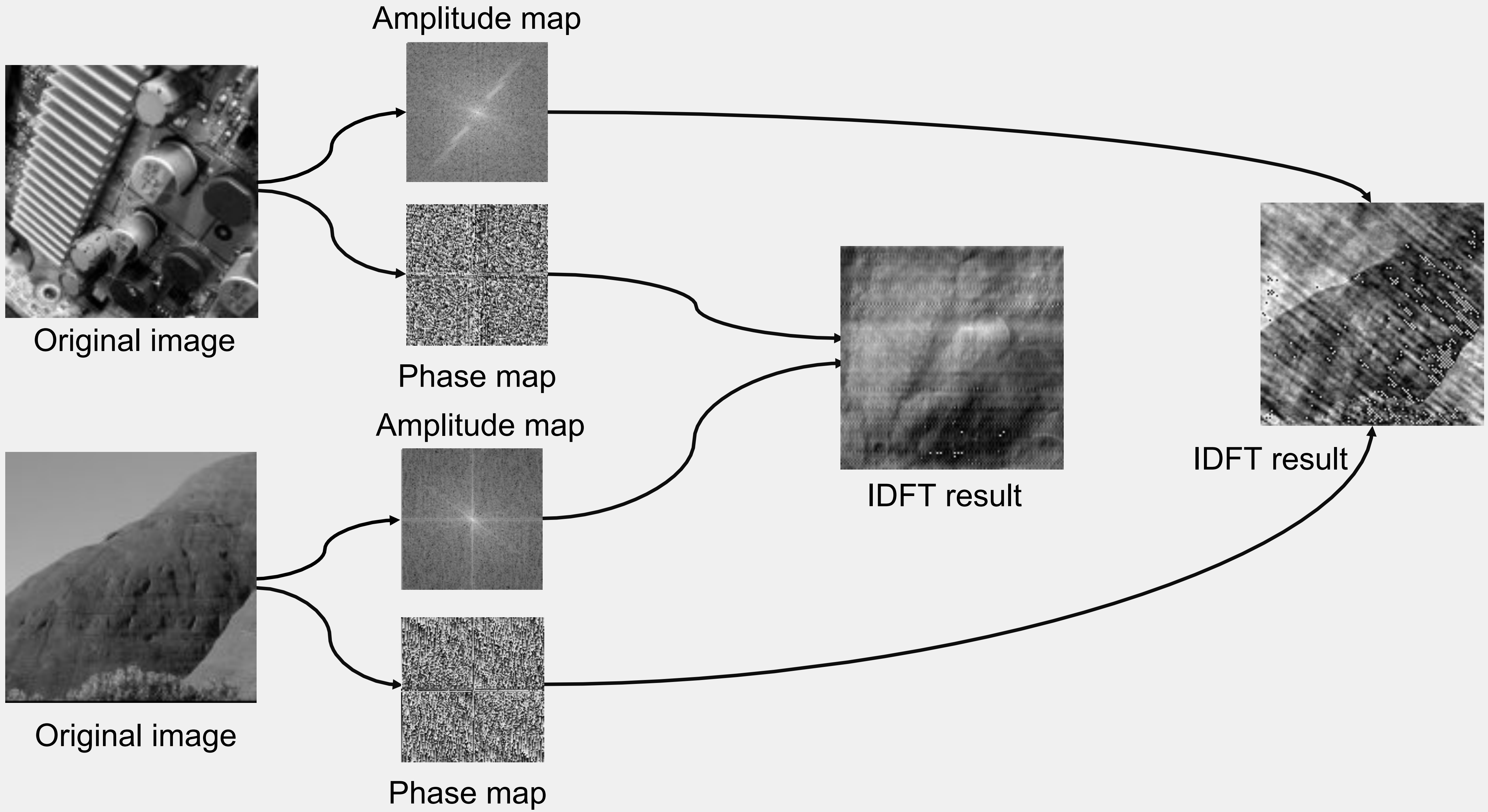


IDFT reconstructed result using only the amplitude map (set the exponential term to 1)



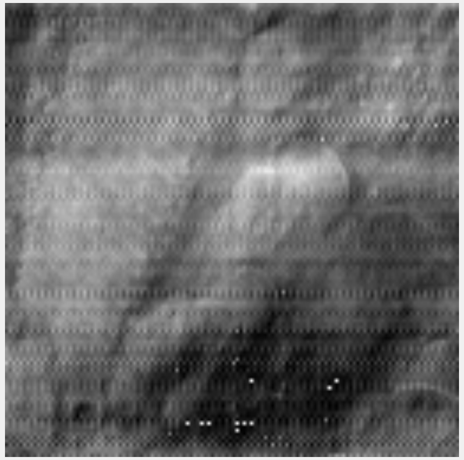
IDFT reconstructed result using only the phase map (with $|F(u,v)|$=1)

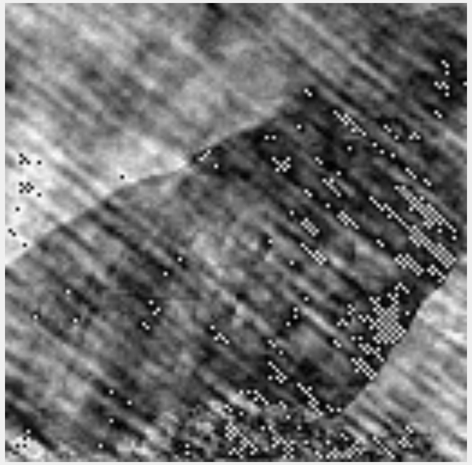
The Physical Meaning of Amplitude Map and Phase Map:



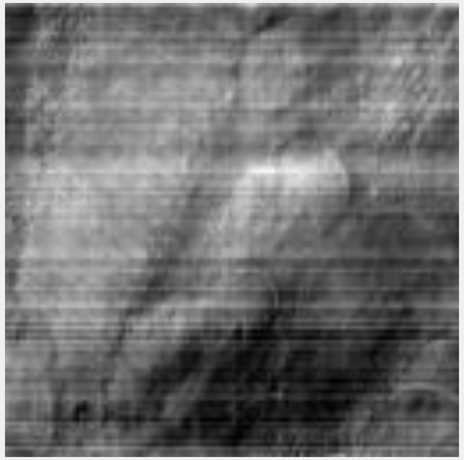
Comparison Results:

My results





MATLAB results



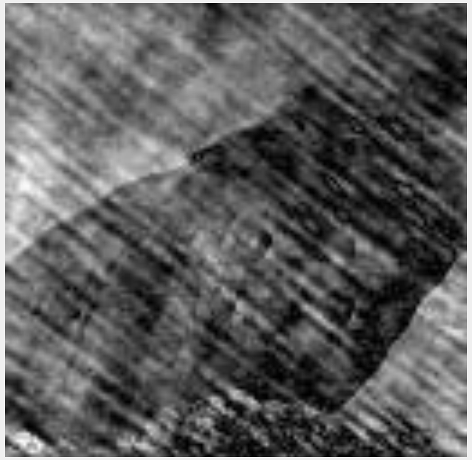


Image Smoothing Using Frequency Domain Filters

$$D(u,v) = [(u-u_0)^2+(v-v_0)^2]$$

Ideal Lowpass Filters：

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

Butterworth Lowpass Filters：

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$

Gussian Lowpass Filters：

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

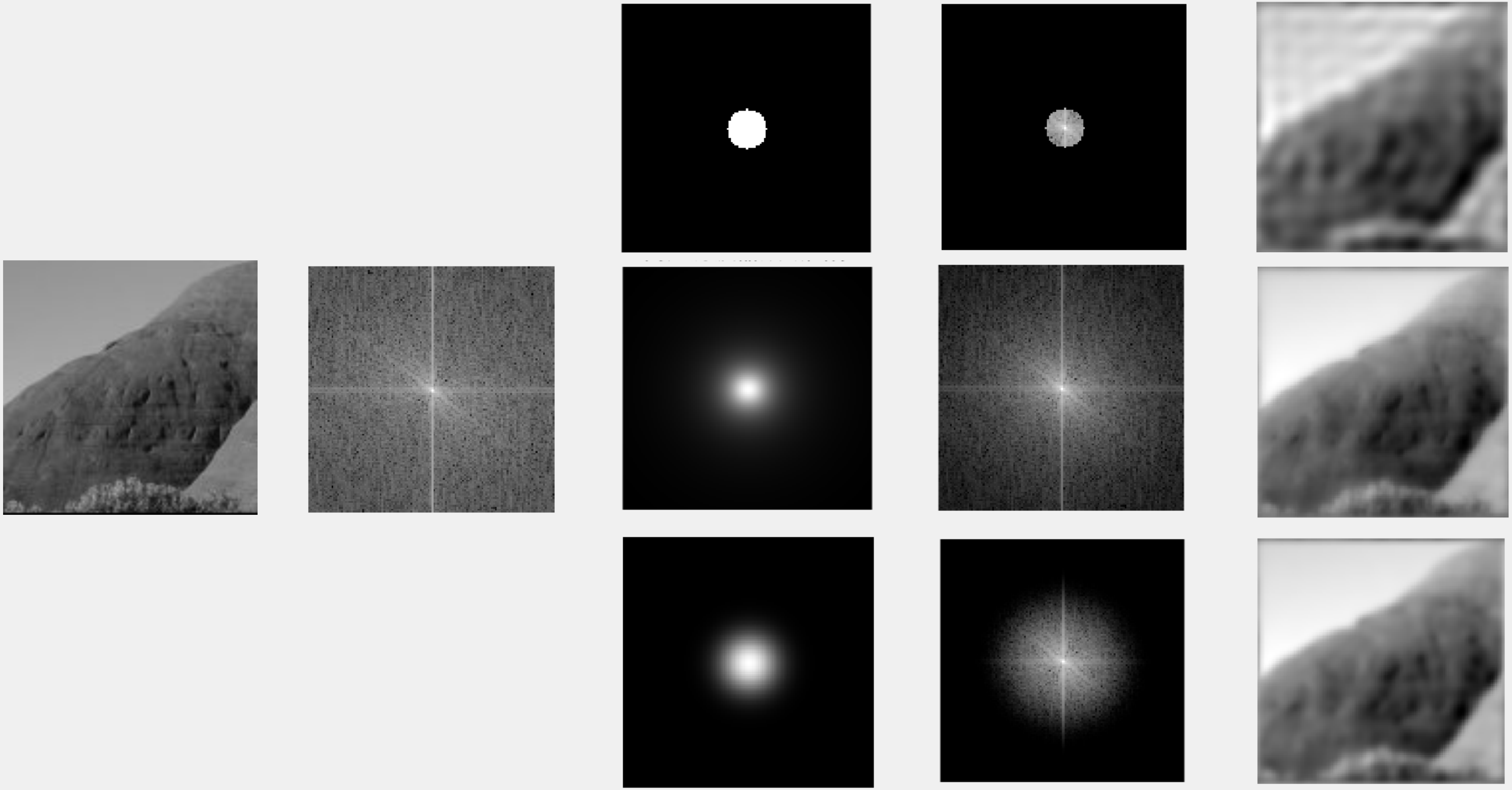


Image Sharpening Using Frequency Domain Filters

$$H_{hp}(u,v) = 1 - H_{LP}(u,v)$$

Ideal Highpass Filters：

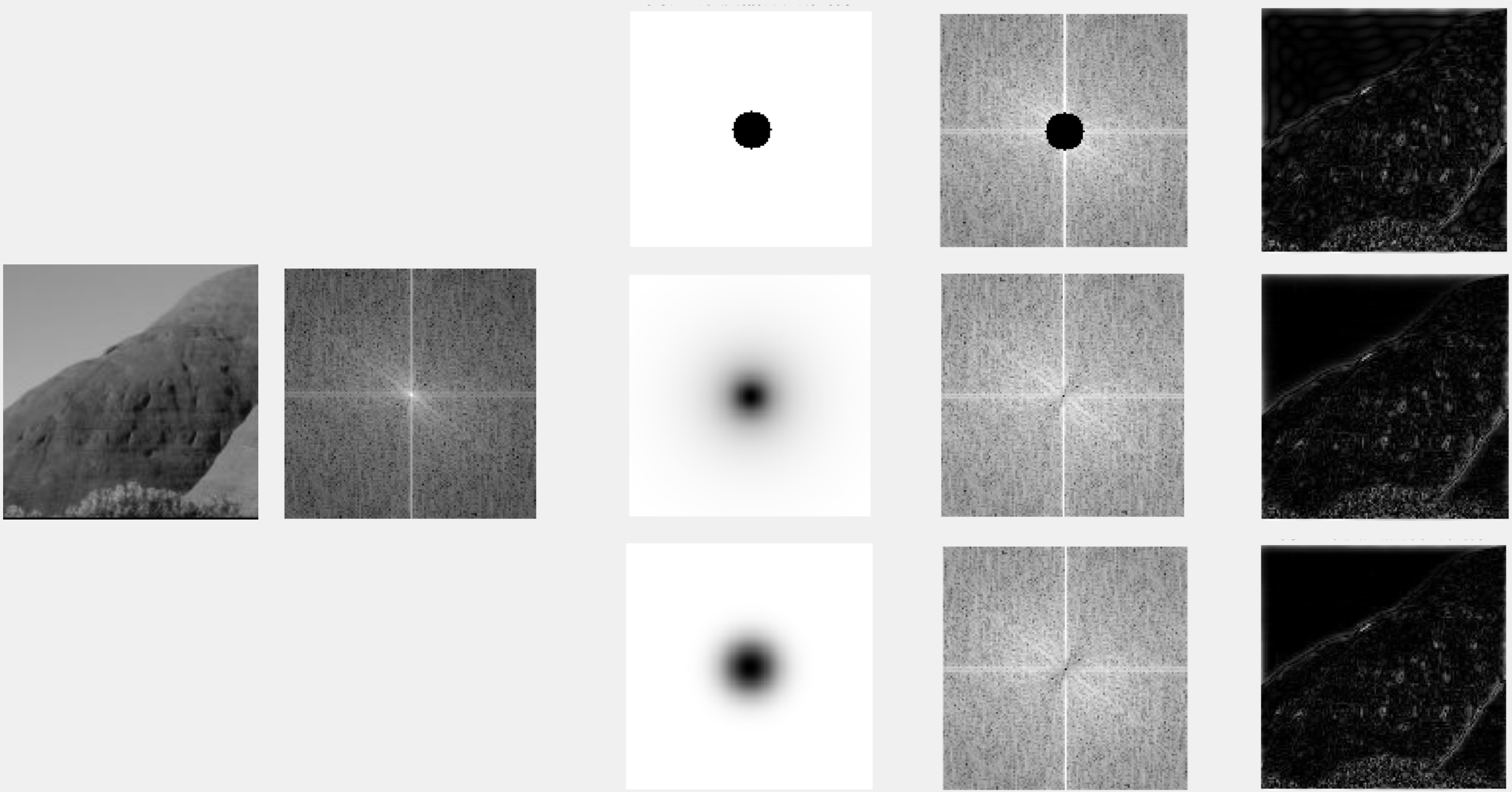
$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Butterworth Highpass Filters：

$$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$$

Gussian Highpass Filters：

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$



Thanks