Mathematical Problems from VALSE Webinars

Contents

paperreading 中提到的数学知识:

- 优化问题
- 基本数学概念
- 统计问题

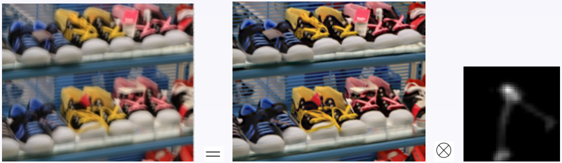
优化问题

优化问题在这些论文中有很广泛的应用，其涉及两个方面：

- 优化问题的给出
- 优化问题的求解

Separable Kernel for Image Deblurring[1]

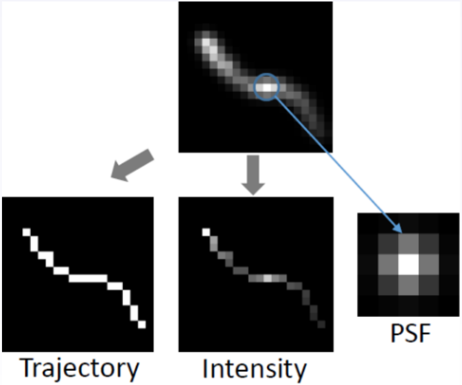
这篇文章主要考虑单张图片由于相机运动造成的模糊去除问题



$$B = K \otimes I + N \tag{1}$$

[1] L. Fang, H. Liu, F. Wu, X. Sun, H. Li. Separable Kernel for Image Deblurring. CVPR. 2014.

Separable Kernel for Image Deblurring



Separable Blur Kernel:

- **Trajectory**
  (projection of camera shake in 2D image plane)
- **Intensity**
  (staying time of shaking camera in every position)
- **Point Spread Function**
  (decided by camera focus scene depth and camera motion at the perpendicular direction of image plane)

Separable Kernel for Image Deblurring

优化函数:

$$K_p^* = arg\min_{K_p, I_p} ||\nabla B_p - \nabla I_p \otimes K_p||_2^2 + \lambda_1 \frac{\nabla ||I_p||_1}{\nabla ||I_p||_2} + \lambda_2 || W \circ K_p||_1$$
$$s.t. \quad W = 1 - G(T_p^*) \tag{2}$$

Linearized Alternating Direction Method: Two Blocks and Multiple Blocks[2]

很多计算机视觉的优化问题可归结为如下模型:

$$\begin{array}{cc} \min_{x_1,x_2} & f(x_1)+f(x_2) \\ s.t. & \mathcal{A}_1(x_1)+\mathcal{A}_2(x_2)=b \end{array} \quad (3)$$

[2] Zhouchen Lin, Risheng Liu, and Zhixun Su, Linearized Alternating Direction Method with Adaptive Penalty for Low Rank Representation, NIPS 2011, arXiv: 1109.0367.

Linearized Alternating Direction Method: Two Blocks and Multiple Blocks

增广拉格朗日函数:

$$\begin{aligned}\mathcal{L}(x_1, x_2, \lambda) &= f_1(x_1) + f_2(x_1) + < \lambda, \mathcal{A}_1(x_1) + \mathcal{A}_2(x_2) - b > \\ &+ \frac{\beta}{2}||\mathcal{A}_1(x_1) + \mathcal{A}_2(x_2) - b||_F^2\end{aligned} \tag{4}$$

Linearized Alternating Direction Method: Two Blocks and Multiple Blocks

交替迭代法求解:

$$\begin{aligned} x_1^{k+1} &= \quad arg\min_{x_1} \mathcal{L}(x_1, x_2^k, \lambda^k) \\ x_2^{k+1} &= \quad arg\min_{x_2} \mathcal{L}(x_1^k, x_2, \lambda^k) \\ \lambda^{k+1} &= \lambda^k + \beta_k[\mathcal{A}_1(x_1^{k+1}) + \mathcal{A}_2(x_2^{k+1}) - b] \end{aligned} \tag{5}$$

Linearized Alternating Direction Method: Two Blocks and Multiple Blocks

等价与下面的形式 (对两种形式求导会发现得到相同的结果):

$$\begin{aligned} x_1^{k+1} &= \arg\min_{x_1} f(x_1) + \frac{\beta_k}{2} \|\mathcal{A}_1(x_1) + \mathcal{A}_2(x_2^k) - b + \frac{\lambda_k}{\beta_k}\|^2 \\ x_2^{k+1} &= \arg\min_{x_2} f(x_2) + \frac{\beta_k}{2} \|\mathcal{A}_1(x_1^{k+1}) + \mathcal{A}_2(x_2) - b + \frac{\lambda_k}{\beta_k}\|^2 \end{aligned} \tag{6}$$

Linearized Alternating Direction Method: Two Blocks and Multiple Blocks

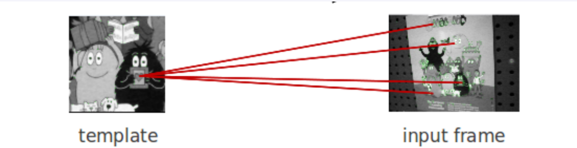
二次项做泰勒展开:

$$
\begin{aligned}
x_1^{k+1} &= arg\min_{x_1} f(x_1) + < \mathcal{A}_1^*(\lambda_k + \beta_k(\mathcal{A}_1(x_1^k) + \mathcal{A}_2(x_2^k) - b)), x_1 - x_1^k > \\
&\quad + \frac{\beta_k \eta_1}{2} ||x_1 - x_1^k||^2 \qquad (7) \\
&= arg\min_{x_1} f(x_1) \qquad (8) \\
&\quad + \frac{\beta_k \eta_1}{2} ||x_1 - x_1^k + \frac{\mathcal{A}_1^*(\lambda_k + \beta_k(\mathcal{A}_1(x_1^k) + \mathcal{A}_2(x_2^k) - b))}{\beta_k \eta_1}||^2
\end{aligned}
$$

基本数学概念

- 距离计算
- 投影
- …

Metric Learning Driven Multi-Task Structured Output Optimization for Robust Keypoint Tracking[3]



distance(d_i, d_j)

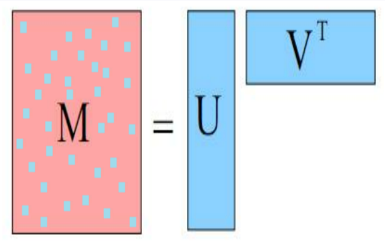
$$\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{N_1} \end{bmatrix} \longrightarrow \begin{bmatrix} d_1^* \\ d_2^* \\ \vdots \\ d_{N_2}^* \end{bmatrix} \quad (9)$$

[3] Metric Learning-Driven Multi-Task Structured Output Optimization for Robust Keypoint Tracking," Proceedings of Twenty-Ninth AAAI Conference on Artificial Intelligence (AAAI), 2015

统计问题

- 统计建模
- EM 算法
- 联合分布
- 转移概率
- ...

Matrix Factorization with Unknown Noise[4]



$$X = UV^T + \epsilon \tag{10}$$

[4] Deyu Meng, Fernando De la Torre. Robust Matrix Factorization with Unknown Noise. International Conference of Computer Vision (ICCV), 2013.

Matrix Factorization with Unknown Noise

- **L2 model is optimal to Gaussian noise**

- **L1 model is optimal to Laplacian noise**

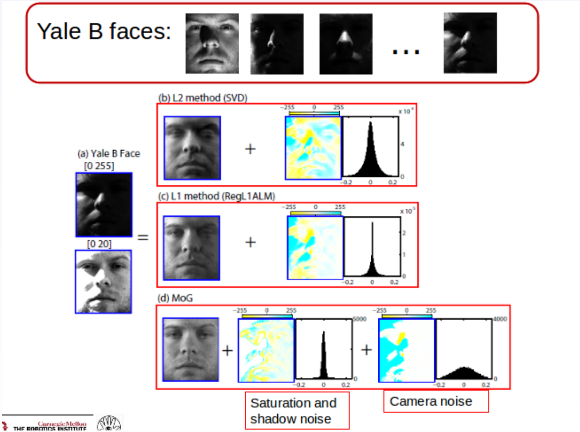
- **But real noise is generally neither Gaussian nor Laplacian**

Matrix Factorization with Unknown Noise

模型：

$$x_{ij} = u_i^T v_j + \epsilon_{ij} \quad p(\epsilon) \sim \sum_{k=1}^{K} \pi_k \aleph(0, \sigma_k^2) \tag{11}$$

Matrix Factorization with Unknown Noise



谢谢！