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DDA 3005 Assignment 2.

Problem

$$\text{Cond.} = \frac{(f(\hat{x}) - f(x))/f(x)}{(\hat{x} - x)/x} \approx \frac{\Delta x f'(x)/f(x)}{\Delta x / x}$$

$$= \frac{x f'(x)}{f(x)}$$

$$= \frac{x \frac{1}{1+x}}{\log(1+x)}$$

$$= \frac{x+1-1}{(1+x)\log(1+x)}$$

when $x \geq -\frac{1}{2}$ $x+1 \geq \frac{1}{2}$, set $u = x+1$

$$\text{Cond} = \frac{u-1}{u \log u} \geq \frac{u-1}{u(u-1)} = \frac{1}{u} \leq 2$$

So it is well-conditioned.

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(6) For $\frac{\sin(x)}{x}$, $f'(x) = \frac{x\cos x - \sin x}{x^2}$

$$\text{cond}_{f(x)} = \left|\frac{x f'(x)}{f(x)}\right| = \left|\frac{\frac{x\cos x - \sin x}{x}}{\frac{\sin x}{x}}\right|$$

$$= \frac{x\cos x - \sin x}{\sin x}$$

At $x=0$, $\text{cond}_f(0) = \left|\frac{\cos x + (-\sin x)x - \cos x}{\cos x}\right|_{x=0}$

$$= \frac{1-1}{1} = 0$$

well-conditioned!

for all x: $\text{cond}_{f(x)} = \left|\frac{x}{\tan x} - 1\right|$

when x near zero, $\tan x = x + \frac{x^3}{3} + \cancel{O(x^5)}\ O(x^5)$

$$\text{cond}_{f(x)} = \left|\frac{1}{1+\frac{x^2}{3}+o(x^4)} - 1\right| \approx 0$$

well conditioned!

but when x near $\frac{\pi}{2}+k\pi$, not well-conditioned

If x are close to $k\pi$, well-conditioned

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Problem 2

(a) Step 1: the computing of $\|x^i - z^j\|$ needs $n + n + n-1 + 1 = 3n$ flops

For each x^i, we need to calculate k distances, and find the minimum, until now, we needs $3nk + O(k)$

There are m vectors, So we need $3mnk + O(mk)$

Step 2: For each group j, we need to update z^j by computing the mean, which costs $m_j \times n + n$ flops, where m_j is the number of vectors the group have.

Since there're k groups, costs $=$

$$\sum_{j=1}^{k} (m_j + 1)n = mn + kn \text{ flops}$$

$\therefore$ It costs $3mnk + O(mk) + mn + kn$ flops $\approx O(mnk)$

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Problem 3.

Given $f(x) = (1+O(x))(1+O(x))$

$= 1+2O(x)+O(x^2)$

$= 1+O(x)+O(x^2)$

$\le 1+O(x)+C(x^2)$

(According to the definition of "O",

$(C \ge 0)$)

$\le 1+O(x)+Cx$ (when $x \to 0$)

$= 1+O(x)+O(x)$

$= 1+O(x)$

Problem 4

(a)(i) $fl(x) = x(1+\varepsilon_1) \quad fl(y) = y(1+\varepsilon_2)$

$$\hat{F}(z) = \begin{bmatrix} \left\{ [x(1+\varepsilon_1)]^2 (1+\varepsilon_3) - [(y(1+\varepsilon_2))]^2 (1+\varepsilon_4) \right\} (1-\varepsilon_5) \\ 2(1+\varepsilon_1)x \cdot (1+\varepsilon_2)y \cdot (1+\varepsilon_6)(1+\varepsilon_7) \end{bmatrix}$$

$$= \begin{bmatrix} x^2(1+O(\varepsilon_{mach})) - y^2(1+O(\varepsilon_{mach})) \\ 2xy(1+O(\varepsilon_{mach})) \end{bmatrix}$$

(ii) $$\frac{\|\hat{F}(z) - F(z)\|}{\|F(z)\|} = \frac{\left[(x^2-y^2)O(\varepsilon_{mach}),\ 2xyO(\varepsilon_{mach})\right]^T}{\|F(z)\|}$$

$$= \frac{\left\| \begin{bmatrix} (x^2-y^2)O(\varepsilon_{mach}) \\ 2xyO(\varepsilon_{mach}) \end{bmatrix} \right\|}{\left\| \begin{matrix} x^2-y^2 \\ 2xy \end{matrix} \right\|} = \frac{O(\varepsilon_{mach})(x^2+y^2)}{(x^2+y^2)} = O(\varepsilon_{mach})$$

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(b)

$$\frac{\|\hat{f}(z)-f(\hat{z})\|}{\|f(z)\|}, \quad \|f(z)\|=(x^2+y^2)$$

$$\|\hat{f}(z)-f(\hat{z})\| = \|\hat{f}(z)-f(z)+(f(z)-f(\hat{z}))\|$$

$$\le \|\hat{f}(z)-f(z)\| + \|f(z)-f(\hat{z})\|$$

$$\le O(\varepsilon_{mach})(x^2+y^2) + ?$$

$$\|f(z)-f(\hat{z})\| = \left\| \begin{matrix} -(\varepsilon_1^2+2\varepsilon_1)x^2+(\varepsilon_2^2+2\varepsilon_2)y^2 \\ -2xy\,O(\varepsilon_{mach}) \end{matrix} \right\|$$

$$= \sqrt{O(\varepsilon_{mach}^2)(x^2+y^2)^2}$$

$$= (x^2+y^2)\,O(\varepsilon_{mach})$$

$$\frac{\|\hat{f}(z)-f(\hat{z})\|}{\|f(z)\|} \le \frac{2(x^2+y^2)\,O(\varepsilon_{mach})}{x^2+y^2}$$

$$= O(\varepsilon_{mach})$$

stable!

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Problem 5:

$$\det(A) = 0 \cdot \det\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 1 \det\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + 0 \det\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= -1$$

$\therefore$ A is ~~none~~ singular

We try LU factorization on A:

$A' = M_1 A$, but we find $A_{11} = 0$,

so we can't continue LU